Jet quenching and elliptic flow in partonic transport simulations including gluons and light quarks

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Heavy Ion Collisions are Complicated!

Models are needed for:
- Initial state
- Evolution of the medium
- High-$p_T$ physics ("jet physics")
- Phase transition

Some tools:
- Parameterizations (e.g. Bjorken)
- Hydrodynamics
- Transport models
- Lattice QCD
- AdS / CFT
- pQCD (BDMPS, ASW, AMY, ...)

The problem
No model can describe all (most) aspects of the medium evolution.
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Elliptic Flow and Suppression of Jets

Jet suppression

\[ R_{AA} = \frac{d^2 N_{AA}/dy \, dp_T}{T_{AA} \, d^2 \sigma_{NN}/dy \, dp_T} \]

- Strong suppression of jets compared to \( p + p \) reference

Collective behavior of the medium

\[ E \frac{d^3 N}{d^3 p} \sim \frac{d^2 N}{dy \, dp_T} \left( 1 + \sum_{n=1}^{\infty} 2v_n \cos[n(\phi - \Psi_R)] \right) \]

- Elliptic flow: Fourier coefficient \( v_2 \)
- Hydrodynamic behavior

Common description of \( R_{AA} \) and \( v_2 \) is difficult
Elliptic Flow and Suppression of Jets

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Outline

1. Motivation
2. The Model - BAMPS
3. Validity of the Gunion-Betsch approximation
4. Static Medium - Brick Scenario
5. Simulations of Heavy Ion Collisions

OF, Z. Xu, C. Greiner, PRL 102 (2009)
OF, Z. Xu, C. Greiner, PRC 82 (2010)
Partonic Transport Model - BAMPS

BAMPS = Boltzmann Approach to Multiple Particle Scattering

Microscopic transport simulations with full dynamics

Attack various problems within one model.
(elliptic flow, $R_{AA}$, thermalization, ...)

Solve Boltzmann equation for $2 \rightarrow 2$ and $2 \leftrightarrow 3$ processes based on LO pQCD matrix elements.

$$p^\mu \partial_\mu f (x, p) = c_{2\rightarrow 2} (x, p) + c_{2\leftrightarrow 3} (x, p)$$

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Visualization by Jan Uphoff
Visualization framework courtesy MADAi collaboration funded by the NSF under grant NSF-PHY-09-41373

\(^1\)Z. Xu, C. Greiner, Phys. Rev. C71 (2005)
Monte Carlo sampling of interactions

- Massless Boltzmann particles (gluons, quarks)
- Discretize:
  - Spatial cells $\Delta V$
  - Time steps $\Delta t$
- Sampling of interaction probabilities from LO pQCD
  - $2 \rightarrow 2$ Small angle cross sections
  - $2 \leftrightarrow 3$ Gunion Bertsch matrix element
- Fixed coupling ($\alpha_s = 0.3$)

$gg \rightarrow gg$ cross section

$$\frac{d\sigma_{gg\rightarrow gg}}{dq_{\perp}^2} \simeq \frac{9\pi\alpha_s^2}{2(q_{\perp}^2 + m_D^2)^2}$$

Gunion Bertsch matrix element

$$|\mathcal{M}_{gg\rightarrow ggg}|^2 = \frac{72\pi^2 \alpha_s^2 s^2}{(q_{\perp}^2 + m_D^2)^2} \frac{48\pi \alpha_s q_{\perp}^2}{k_{\perp}^2 [(k_{\perp} - q_{\perp})^2 + m_D^2]}$$

Debye screening (dynamic):

$$m_D^2 = d_G \pi \alpha_s \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p} (N_{c f_g} + N_{f f_q})$$
## Implemented Processes (gluons and light quarks)

### 2 → 2 processes

**Original BAMPS version** ($N_f = 0$):

$gg \rightarrow gg$

Including light quarks ($N_f = 3$):

- $gg \rightarrow q\bar{q}$
- $q\bar{q} \rightarrow gg$ and $q\bar{q} \rightarrow q'\bar{q}'$
- $qg \rightarrow qg$ and $\bar{q}g \rightarrow \bar{q}g$
- $q\bar{q} \rightarrow q\bar{q}$
- $qq \rightarrow qq$ and $\bar{q}\bar{q} \rightarrow \bar{q}\bar{q}$
- $qq' \rightarrow qq'$ and $q\bar{q}' \rightarrow q\bar{q}'$

### 2 ↔ 3 processes

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- $qq' \leftrightarrow qq'g$ and $q\bar{q}' \leftrightarrow q\bar{q}'g$

- Emission of gluon factorizes: $|M_{GB}|^2 = |M_{coll}|^2 P^g$
- Re-use $gg \rightarrow ggg$ matrix element
  $$|M_{x \rightarrow xg}|^2 = |M_{x \rightarrow x}|^2 \left[ |M_{gg \rightarrow ggg}|^2 / |M_{gg \rightarrow gg}|^2 \right]$$
- Use small angle cross sections for scaling $\rightarrow$ simple prefactors

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Jets and $\nu_2$ in Partonic Transport

TORIC 2011 7 / 19
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- Emission of gluon factorizes: $|M_{GB}|^2 = |M_{coll}|^2 P^g$
- Re-use $gg \rightarrow ggg$ matrix element
  
  $|M_{X \rightarrow xg}|^2 = |M_{X \rightarrow x}|^2 \left|\frac{|M_{gg \rightarrow ggg}|^2}{|M_{gg \rightarrow gg}|^2}\right|$  
- Use small angle cross sections for scaling $\rightarrow$ simple prefactors
Modelling of the LPM Effect

**LPM effect**

Multiple gluon emission $\Rightarrow$ interference

- Difficult to realize in a semi-classical transport model
- Ansatz: Discard possible interference processes (Bethe-Heitler)

Parent must not scatter during formation time of emitted gluon

\[ |M_{gg\rightarrow ggg}|^2 \rightarrow |M_{gg\rightarrow ggg}|^2 \Theta (\lambda - \tau) \]

Comparison of $\lambda$ und $\tau$ requires consideration of different Lorentz frames

\[ \Theta (\lambda - \tau) = \Theta \left( k_\perp \lambda - \frac{\cosh y}{\sqrt{1 - \beta'^2}} (1 + \beta' \tanh y \cos \theta) \right) \]
Modelling of the LPM Effect

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Some Results from BAMPS

- **Fast thermalization**, $\lesssim 1 \text{ fm/c}$ Z. Xu, C. Greiner, PRC 71 (2005)
- **Small viscosity**, $\eta/s \simeq 0.1 - 0.2$
  

- **Investigate heavy quark production and dynamics**
  
  J. Uphoff, OF et al., PRC 82 (2010)

- **Can serve as reference for viscous hydro** I. Bouras et al. PRL 103 (2009)

- **Investigate hydrodynamic shocks / Mach cones**
  
  I. Bouras et al. PRL 103 (2009) / 1008.4072
Things that won’t be covered in this talk

... but can learned from going to these:

**Heavy flavor in BAMPS**

⇒ Jan Uphoff, this afternoon

**Shear viscosity in transport models**

⇒ Christian Wesp, this afternoon

**Hydrodynamic behavior within BAMPS and collective excitations**

⇒ Ioannis Bouras, Thursday
Comparison of GB to the Exact Matrix Element

How good is the approximation by the Gunion-Bertsch matrix element?

- Gunion Bertsch matrix element for \( gg \rightarrow ggg \)

\[
|M_{gb\rightarrow ggg}^\text{GB}|^2 = \frac{72\pi^2\alpha_s^2s^2}{(q^2_\perp)^2} \left( \frac{48\pi\alpha_s q^2_\perp}{k^2_\perp[(k_\perp - q_\perp)^2]} \right)
\]

- Exact solution by Berends et al. (PLB 103, 1981) and by Ellis / Sexton (Nucl.Phys.B269, 1986)

\[
|M_{gb\rightarrow ggg}^\text{full}|^2 = \frac{g^6}{2} \left[ \frac{N^3}{(N^2 - 1)} \right] \left[ (12345) + (12354) + (12435) + (12453) + (12534) + (13254) + (13452) + (13524) + (14235) + (14325) \right]
\]

\[
\times \frac{[(p_1p_2)^4 + (p_1p_3)^4 + (p_1p_4)^4 + (p_1p_5)^4 + (p_2p_3)^4]}{(p_1p_2)(p_1p_3)(p_1p_4)(p_1p_5)(p_2p_3)(p_2p_4)(p_2p_5)(p_3p_4)(p_3p_5)(p_4p_5)}
\]

\[
+ \frac{[(p_2p_4)^4 + (p_2p_5)^4 + (p_3p_4)^4 + (p_3p_5)^4 + (p_4p_5)^4]}{(p_1p_2)(p_1p_3)(p_1p_4)(p_1p_5)(p_2p_3)(p_2p_4)(p_2p_5)(p_3p_4)(p_3p_5)(p_4p_5)}
\]

with \((ijklm) = (p_ip_j)(p_jp_k)(p_kp_l)(p_lp_m)(p_mp_i)\)
Comparison of GB to the Exact Matrix Element

Comparison done for *bare* matrix elements, $k_\perp = q_\perp = m_D$ fixed.

$E = 40$ GeV jet + thermal ($T = 0.4$ GeV)

Approximations by GB are reasonable

GB overestimates the exact matrix element by a factor 1.2 to 2.
Energy Loss in a Static Medium

Static Medium (brick): $T = \text{const, no expansion}$

Energy loss from elastic processes

Energy loss including $2 \leftrightarrow 3$ processes

- Strong energy loss from $2 \rightarrow 3$ processes
  - Complex interplay of GB matrix element and LPM cutoff
  - Prefered gluon radiation into $y < 0$ (backward) direction
- Only small difference between quarks and gluons
  - Iterative computation of rates due to LPM restriction

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Jets and $\nu_2$ in Partonic Transport
Energy Loss in a Static Medium

Distribution of $\Delta E$

$\Delta E$ versus radiated energy $\omega$

- Broad distribution of energy loss $\Delta E$ per collision
- $\Delta E$ is larger than the energy of the radiated gluon
  - $\Delta E = E_{in} - \max(E^i_{out}) \geq \omega$
Elliptic Flow at RHIC

Parameters:
- Coupling $\alpha_s = 0.3$ to $\alpha_s = 0.6$
- Freeze-out energy density $\varepsilon_c = 0.6 \text{ GeV fm}^{-3}$ to $\varepsilon_c = 1.0 \text{ GeV fm}^{-3}$
- $N_f = 0$ (purely gluons)
- Mini jet initial conditions ($p_0 = 1.4 \text{ GeV}$)

Medium develops strong collectivity using pQCD-based interactions Xu, Greiner, PRC 79 (2009)

$\langle v_2 \rangle$ can be described over a large range of centrality
Elliptic flow at RHIC

Differential $v_2(p_T)$ at RHIC from BAMPS

- Differential elliptic flow of gluons and quarks is (almost) the same
- NCQ scaling the experimental data, the magnitude of quark $v_2(p_T)$ is ok, but peak shifts $\Rightarrow$ hadronization mechanisms?
- Qualitative features of high-$p_T$ $v_2$ agree with PHENIX $\pi^0$ data
  - fitted using $v_2(p_T) = \left( a + \frac{1}{p_T^2} \right) \frac{(p_T/\lambda)^m}{1+(p_T/\lambda)^m}$
Jet Suppression in BAMPS Simulations at RHIC

$R_{AA}$, Au + Au at 200 A GeV, 0 %–10 %

- Hadronization via AKK fragmentation functions
- Suppression in BAMPS is too strong
  - Strong mean energy loss in 2 $\rightarrow$ 3 processes
  - Sizeable conversion of quark jets into gluon jets
  - Small difference in the energy loss of quarks and gluons

$R_{AA}$, Au + Au at 200 A GeV, 40 %–50 %
Jet Suppression and Elliptic Flow at LHC

$R_{AA}$, Pb + Pb at 2.76 A TeV, 0 %–5 %

PYTHIA initial conditions \(^{(\text{Uphoff, OF et al. PRC 82 (2010)})}\), $\alpha_s = 0.3$

$R_{AA}$ almost identical to RHIC, does not reproduce rise towards large $p_T$

Integrated $v_2$ shows increase, drops below data at about 50 % centrality
Summary

- Partonic transport provides means of:
  - exploring the dynamics of the medium evolution based on pQCD processes
  - exploring different observables within a common framework
- Strong collective flow of the medium is reproduced
- Suppression of jets is too strong using the same parameters
Summary

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Possible improvements:

- Implementation of running coupling $\alpha_s(Q^2)$
- Revisit LPM effect, explore prospects of Monte Carlo implementation?
- Hadronization scheme for low and medium $p_T$ range
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Ongoing work:
- Restructuring and improving code
- Preparing code for publication
Additional material
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Rates and Mean Free Paths

Interaction rates ($T = 0.4$ GeV)

Mean free path ($T = 0.4$ GeV)
Radiation Spectra

Radiation spectrum ($E = 50$ GeV)
Radiation Distributions

Angular distribution ($E = 50$ GeV)

Distribution of momentum transfers

$m_D \approx 0.6$ GeV
Sensitivity on the LPM Cut-Off

Parameters in BAMPS

- Coupling strength \( \alpha_s \)
- Critical freeze-out energy density \( \varepsilon_c \)
- LPM cut-off

The effective implementation of the LPM cut-off requires \( \Lambda_g > \tau \).

Only qualitative argument, introduce factor \( X \) to test sensitivity.

\[
\Theta \left( k_\perp - \frac{\gamma}{\Lambda_g} \right) \rightarrow \Theta \left( k_\perp - X \frac{\gamma}{\Lambda_g} \right)
\]
Sensitivity on the LPM Cut-Off

\[ \sigma_{gg\rightarrow ggg} \sim \int dq_{\perp}^2 \int dk_{\perp}^2 \int y \cdots \Theta \left( k_{\perp} - X \frac{\gamma}{\Lambda_g} \right) \]

- Large \( X \) reduces total cross section
- Sampling of outgoing particles affected in non-trivial way
- Energy loss per collision only slightly affected, main contribution to the change in energy loss from change in \( \sigma \).
Sensitivity on the LPM Cut-Off

$R_{AA}$ for different $X$ ($b = 7$ fm)

$\nu_2$ for different $X$ ($b = 7$ fm)