



**UNIVERSITÀ DEGLI STUDI DI CATANIA**  
**INFN-LNS**



**ANISOTROPIC FLOWS AND SHEAR VISCOSITY WITHIN A  
TRANSPORT APPROACH**

**S. PLUMARI, V. GRECO**

- **Transport approach at fixed  $\eta/s$ .**
- **Effect of  $\eta/s(T)$  on the generation of  $v_2$  at RHIC and LHC.**
- **Impact of high  $p_T$  partons on the build-up of  $v_2$ .**
  - **Scaling properties of  $v_2(p_T)/\epsilon$ .**
  - **$\eta/s(T)$ : large  $v_4/(v_2)^2$  .**
- **Conclusions**

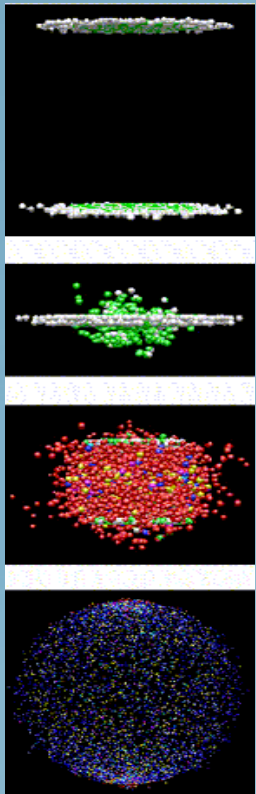
## Motivation for a kinetic approach:

$$\left\{ p^\mu \partial_\mu + \left[ p_\nu F^{\mu\nu} + M \partial^\mu M \right] \partial_\mu^p \right\} f(x, p) = C_{22} + C_{23} + \dots$$

Free  
streaming

Field Interaction  $\rightarrow \epsilon \neq 3P$

Collisions  $\rightarrow \eta \neq 0$



- It is not a gradient expansion in  $\eta/s$ .
- Valid at intermediate  $p_T$  out of equilibrium.
- Valid at high  $\eta/s$  (cross over region).
- Include hadronization by coalescence + fragmentation.
- Allows to study the jet-bulk talk.

# Parton Cascade model

$$p^\mu \partial_\mu f(X, p) = C = C_{22} + C_{23} + \dots$$

Collisions

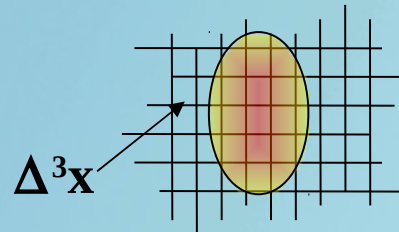


$$\left\{ \begin{array}{l} \varepsilon - 3p = 0, \\ \eta \neq 0 \end{array} \right.$$

$$C_{22} = \frac{1}{2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \frac{1}{v} \int \frac{d^3 p'_1}{(2\pi)^3 2E'_1} \frac{d^3 p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 |M_{1'2' \rightarrow 12}|^2 (2\pi)^4 \delta^{(4)}(p'_1 + p'_2 - p_1 - p_2)$$

**For the numerical implementation of the collision integral we use the stochastic algorithm. ( Z. Xu and C. Greiner, PRC 71 064901 (2005) )**

$$P_{22} = \frac{\Delta N_{coll}^{2 \rightarrow 2}}{\Delta N_1 \Delta N_2} = v_{rel} \sigma_{22} \frac{\Delta t}{\Delta^3 x}$$



$\Delta t \rightarrow 0$

$\Delta^3 \mathbf{x} \rightarrow 0$



**right solution**

**Passed several numerical test on the box.**

## Simulating a constant $\eta/s$

$$\eta(\vec{x}, t) = \frac{4}{15} \lambda(\vec{x}, t) n_{tot}(\vec{x}, t) \langle p \rangle \longrightarrow \sigma_{tr}^{\eta/s} = \frac{4}{15} \frac{\langle p \rangle}{n} \frac{1}{\eta/s} = K \sigma_{tr}^{pQCD}$$

$\sigma$  is evaluated in such way to keep the  $\eta/s = \text{const.}$  during the dynamics. (similar to D. Molnar, arXiv:0806.0026[nucl-th] but our approach is local.)

With this approach we have large cross-section

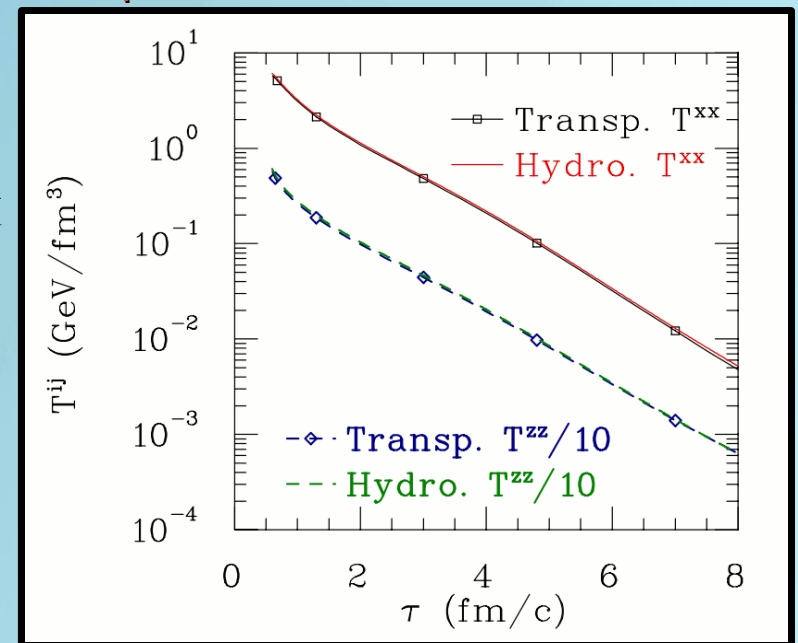
$$\sigma_{Tot} = K \sigma_{pQCD} \gg \sigma_{pQCD}$$

V.Greco, et al. PLB 670 (2009), PPNP 62 (2009) 562.

**Take into account the non-perturbative effects**

**BUT NEVER REACH THE pQCD LIMIT**  
**WHATEVER IS THE  $p_T$  OF THE COLLIDING**  
**PARTICLE – EQUIVALENT TO HYDRO**

**At low  $p_T$  equivalent to Hydro dynamics**



D. Molnar et al., JPG 35, (2008). QM08

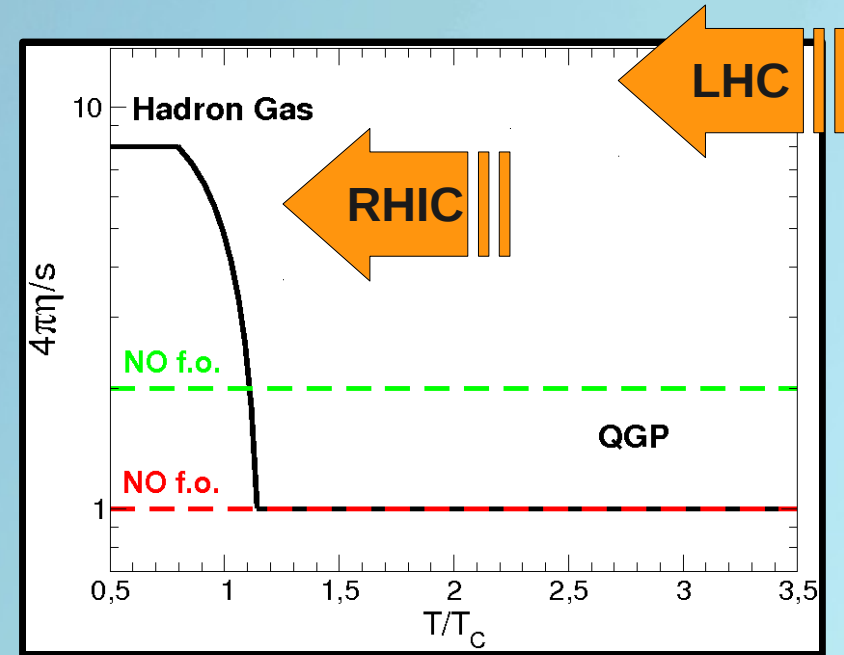
## Initial condition of our simulation

- In coordinate space we use a Glauber model
- In momentum space:
  - For  $p_T < p_0$  GeV thermal distribution.
    - 2  $T_c$  at RHIC for Au+Au @ 200 GeV
    - 3.5  $T_c$  at LHC for Pb+Pb @ 2.76 TeV
  - For  $p_T > p_0$  GeV spectra of minijets.

$$\frac{dN_i}{d^2 p_T} \approx \frac{A_i}{(1 + p_T/B_i)^n}$$

# kinetic freeze-out scheme

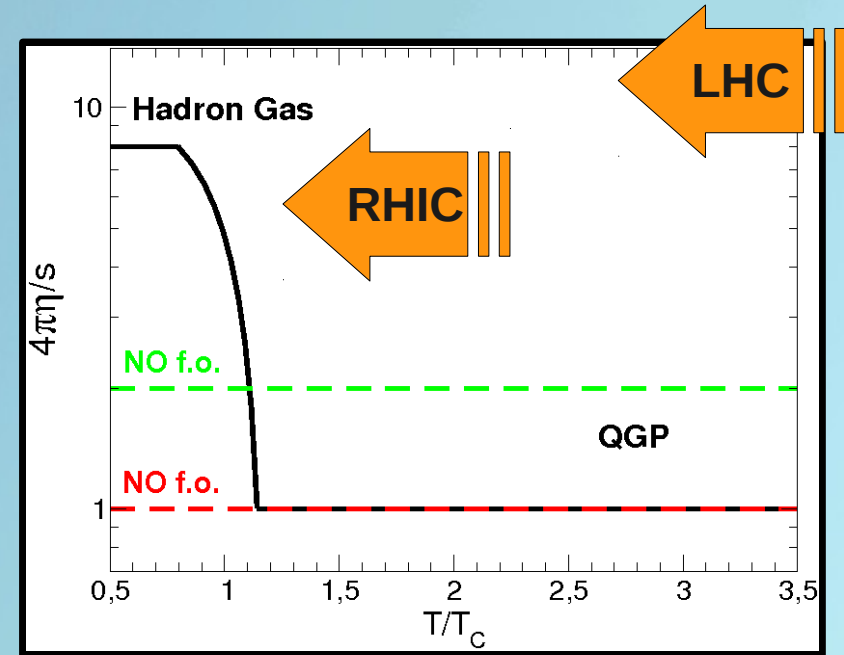
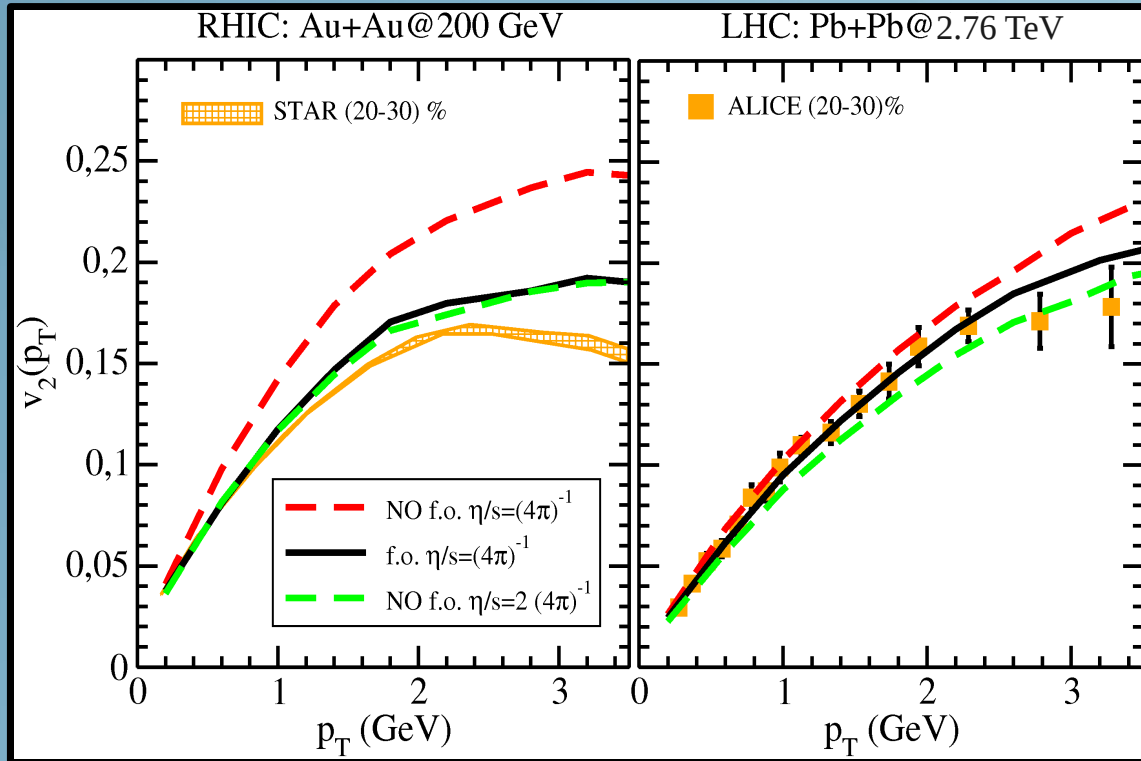
- $\eta/s$  increases in the cross-over region, with a smooth transition between the QGP and the hadronic phase, the collisions are switched off.



For the  $v_2$  similar to cut-off at  $\varepsilon_0 = 0.7 \text{ GeV}/\text{fm}^3$

# Effect of kinetic freeze-out

K. Aamodt et al. [ALICE Collaboration], Phys. Rev.Lett. 105, 252302 (2010).



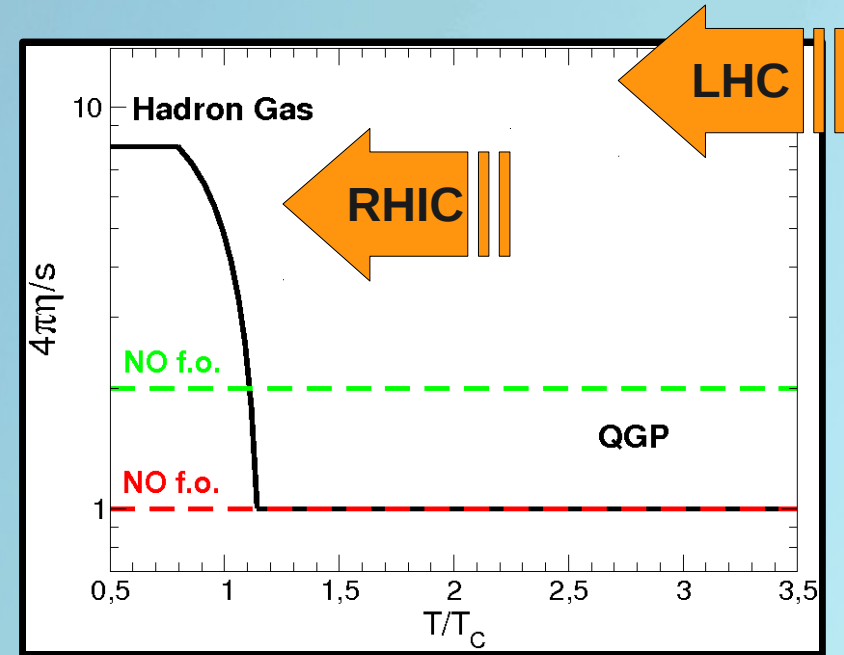
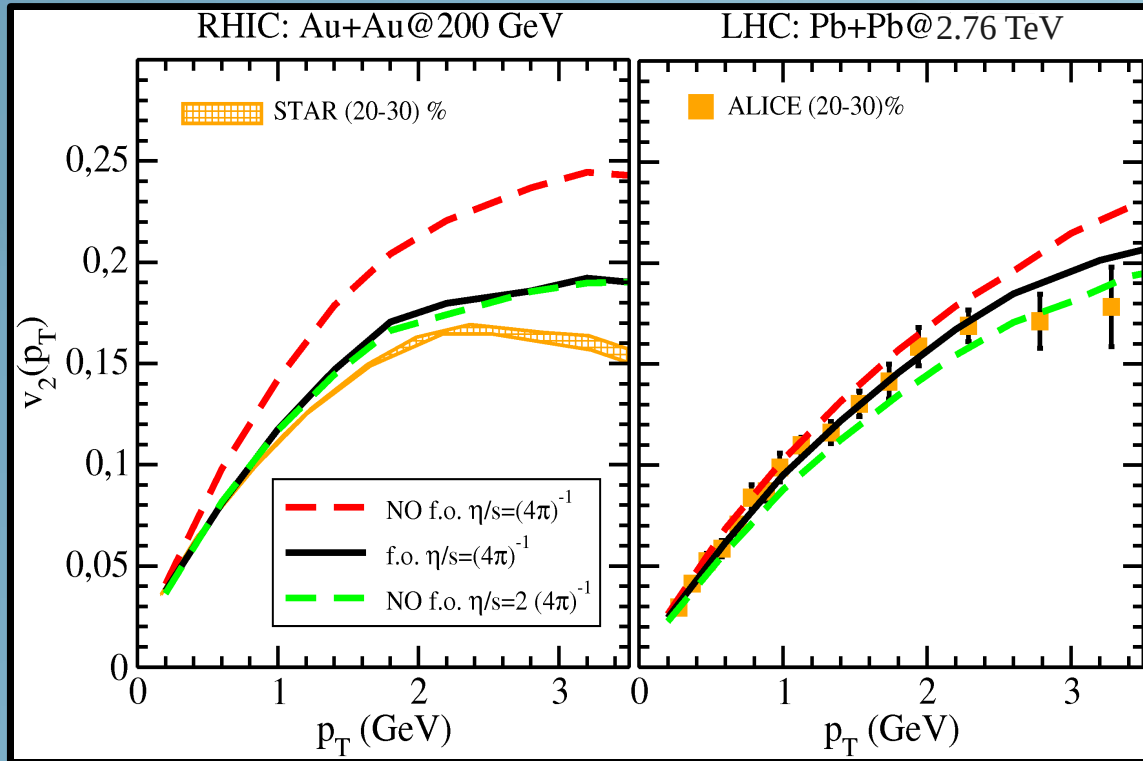
## RHIC:

- Like viscous hydro the data are close to  $\eta/s=1/(4\pi) + \text{f.o.}$
- Sensitive reduction of the  $v_2$  when the f.o. is included the effect is about of 20%.
- $p_T > 2 \text{ GeV}$   $v_2$  decrease not explained (similar to hydro).



# Effect of kinetic freeze-out

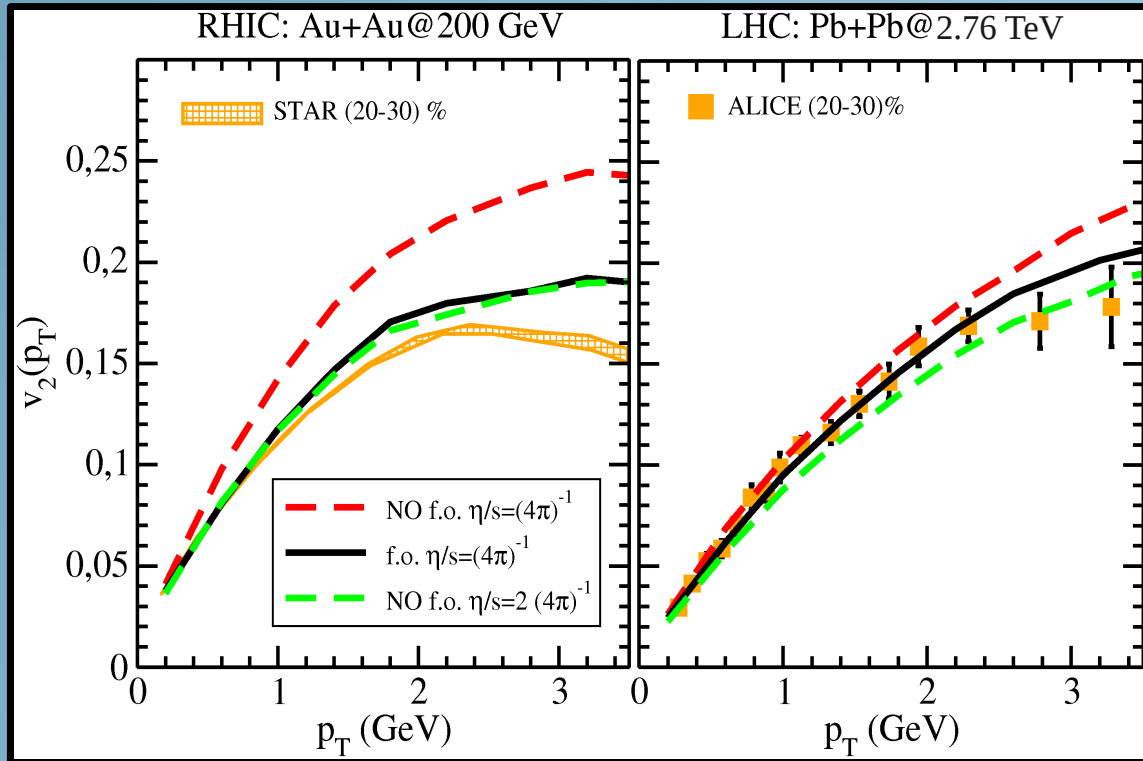
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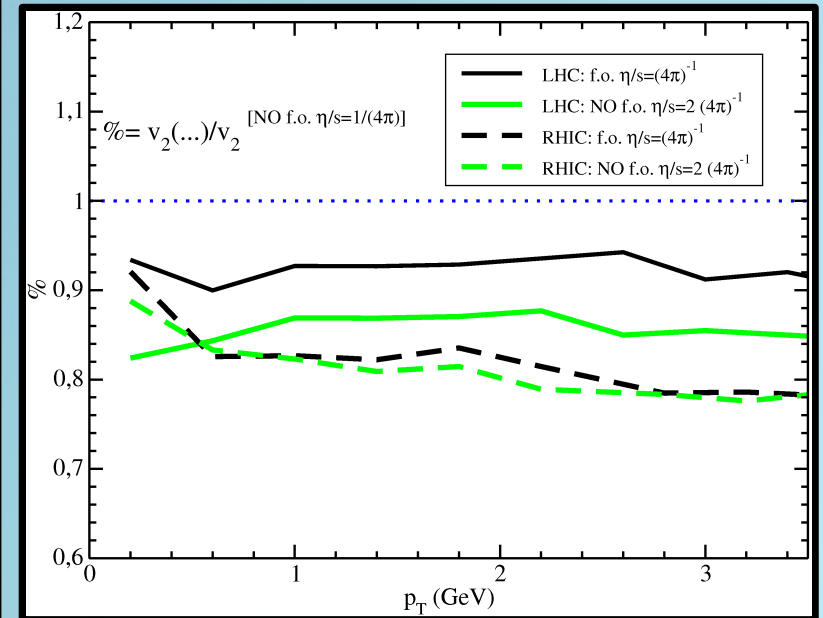
## LHC:

- $p_T < 2$  GeV like hydro data described with  $\eta/s = 1/(4\pi) + \text{f.o.}$
- Smaller effect on the reduction of the  $v_2$  when the f.o. is included an effect of about 10%.
- Without the kinetic freezout the effect of a constant  $\eta/s = 2(4\pi)^{-1}$  is to reduce the  $v_2$  of 20%.

# Effect of kinetic freeze-out



Suppression of  $v_2$  respect to the ideal case  $\eta/s=(4\pi)^{-1}$



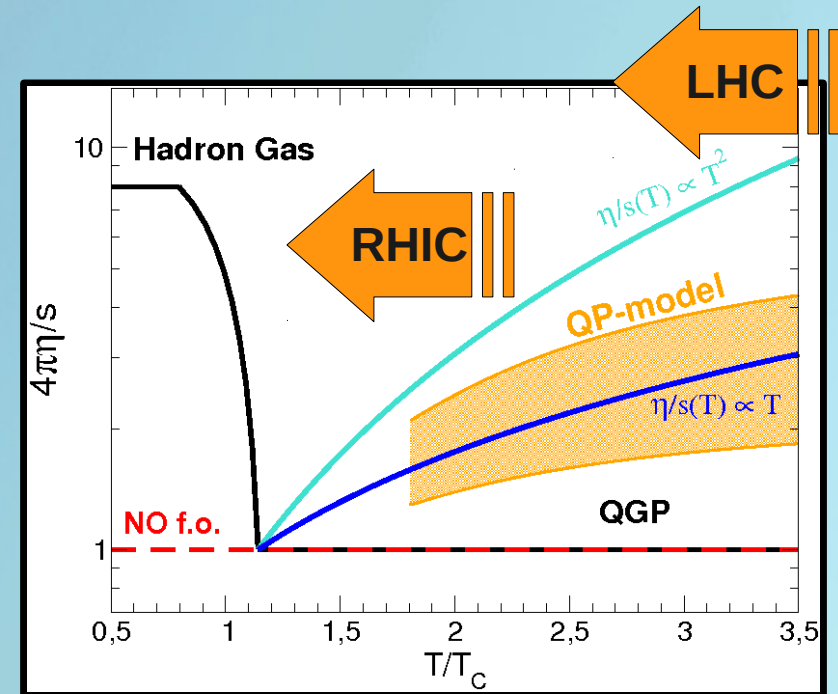
At LHC the contamination of mixed and hadronic phase becomes negligible

Longer life time of QGP  $\rightarrow v_2$  completely developed in the QGP phase  
(at least up to 3 GeV)

# Temperature dependent $\eta/s(T)$

Phase transition physics suggest a  $T$  dependence of  $\eta/s$  also in the QGP phase

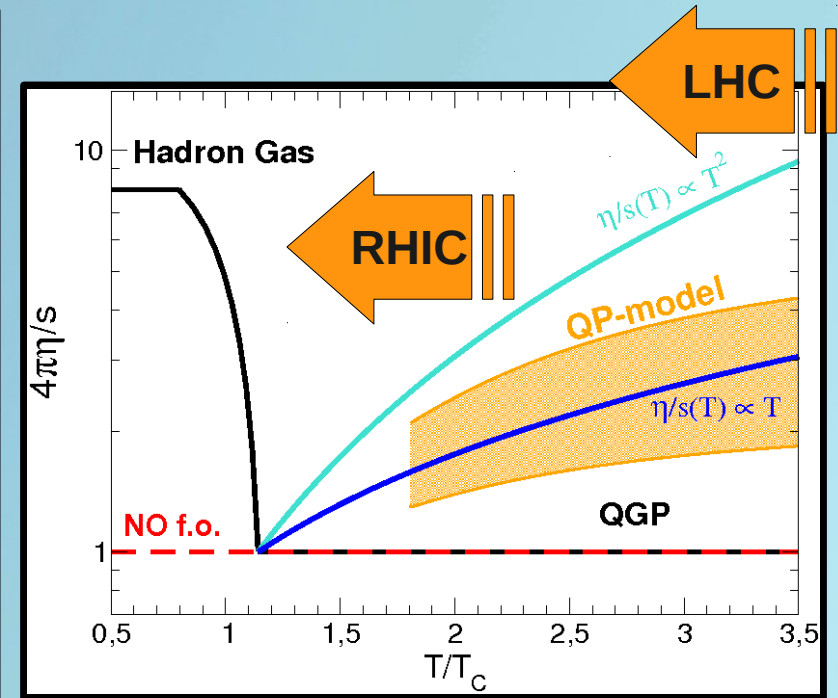
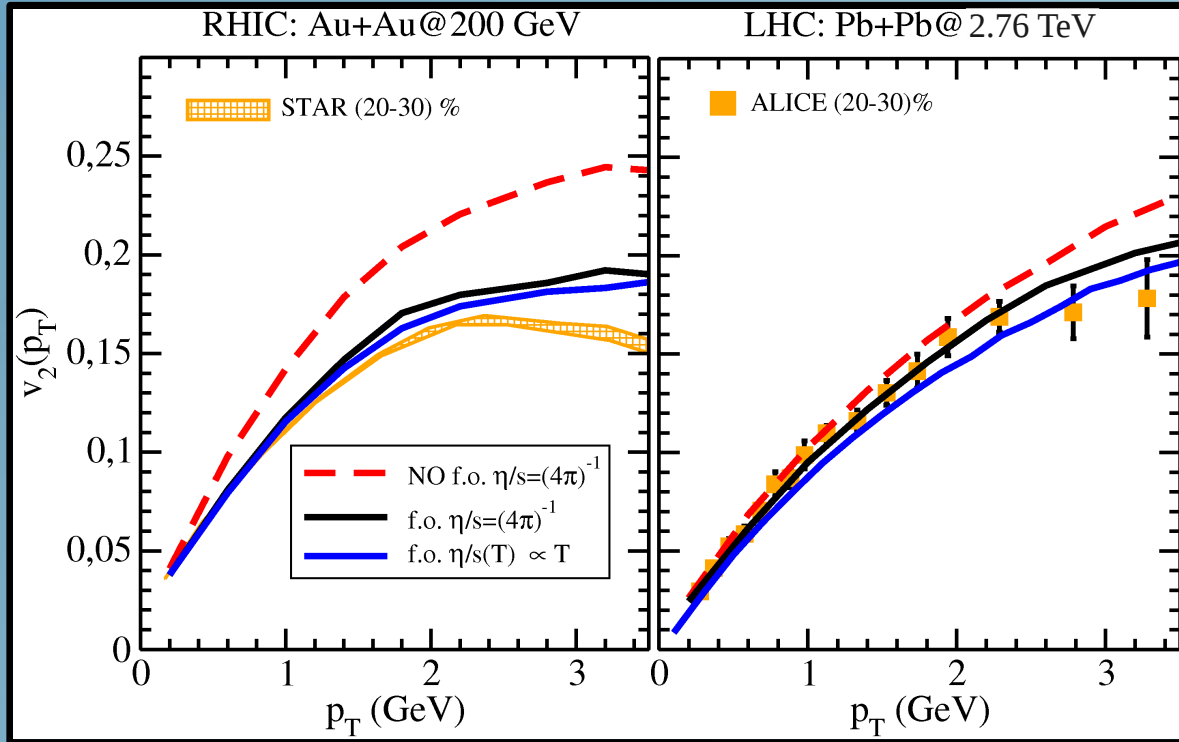
- In the QGP phase  $\eta/s$  is expected to have a minimum value  $T_c$ .
- Quasi-particle models seem to suggest a  $\eta/s \sim T^\alpha$   $\alpha \sim 1 - 1.5$ .



S.Plumari, W.M.Alberico, V.Greco, C.Ratti, arxiv:1103.5611[hep-ph].

# Temperature dependent $\eta/s(T)$

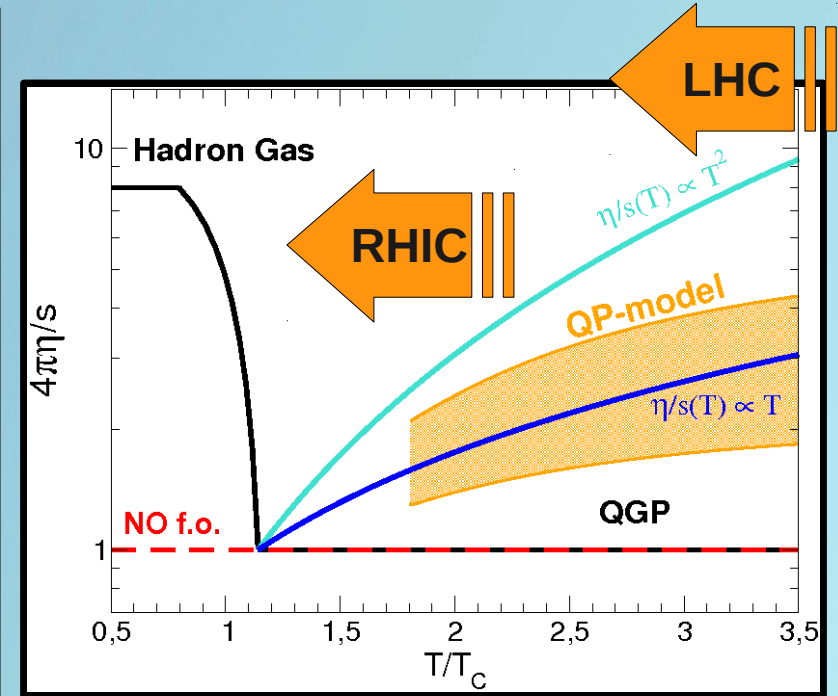
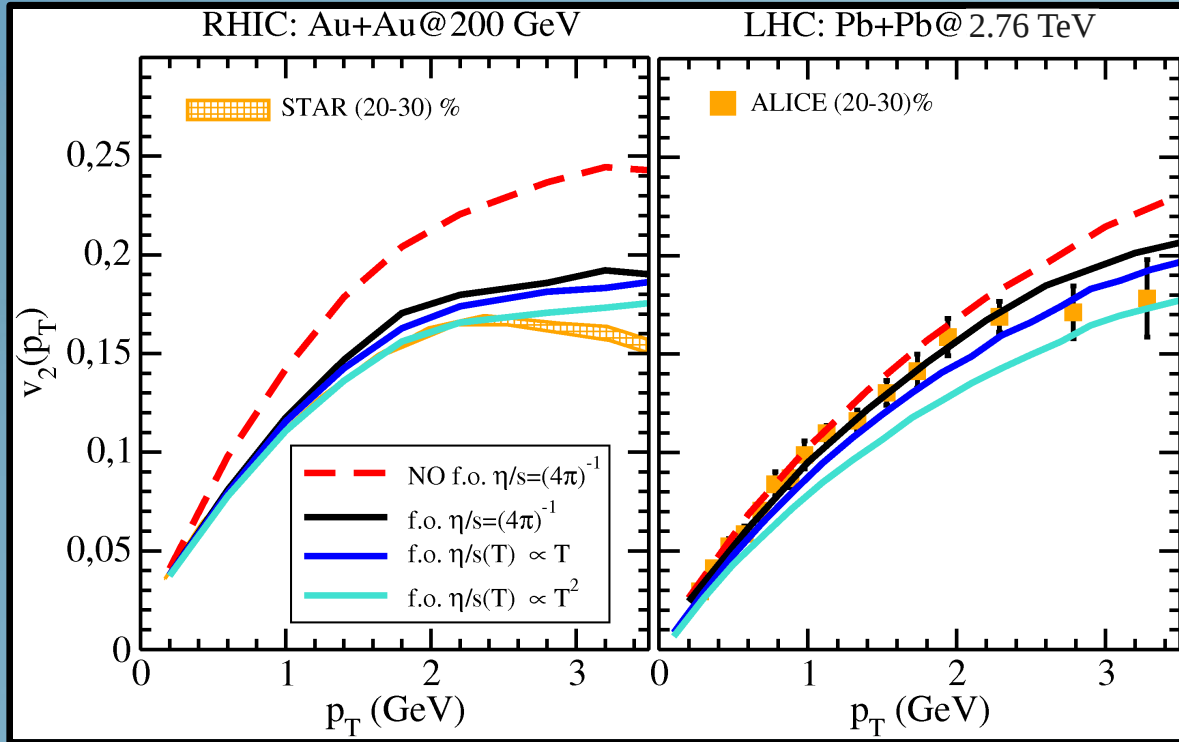
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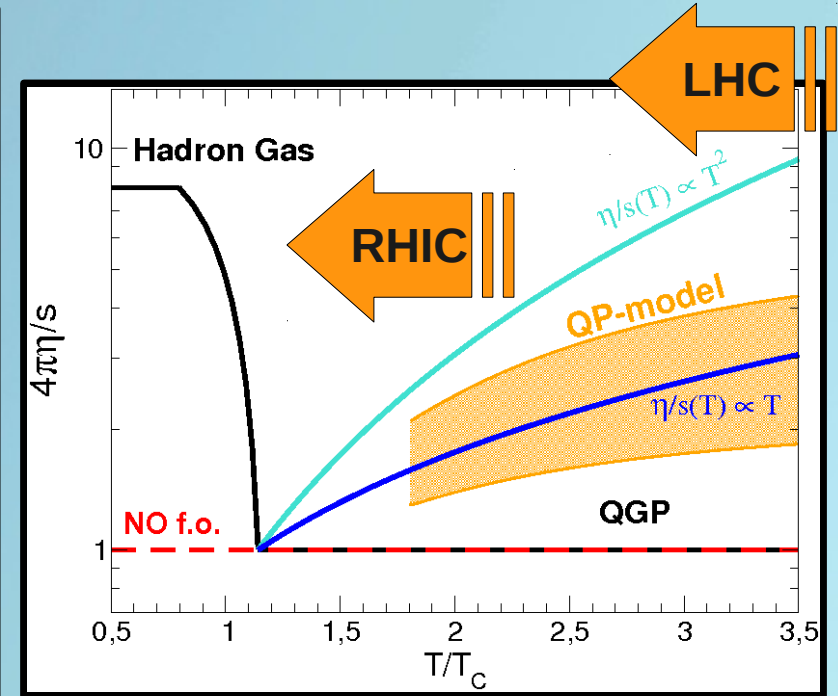
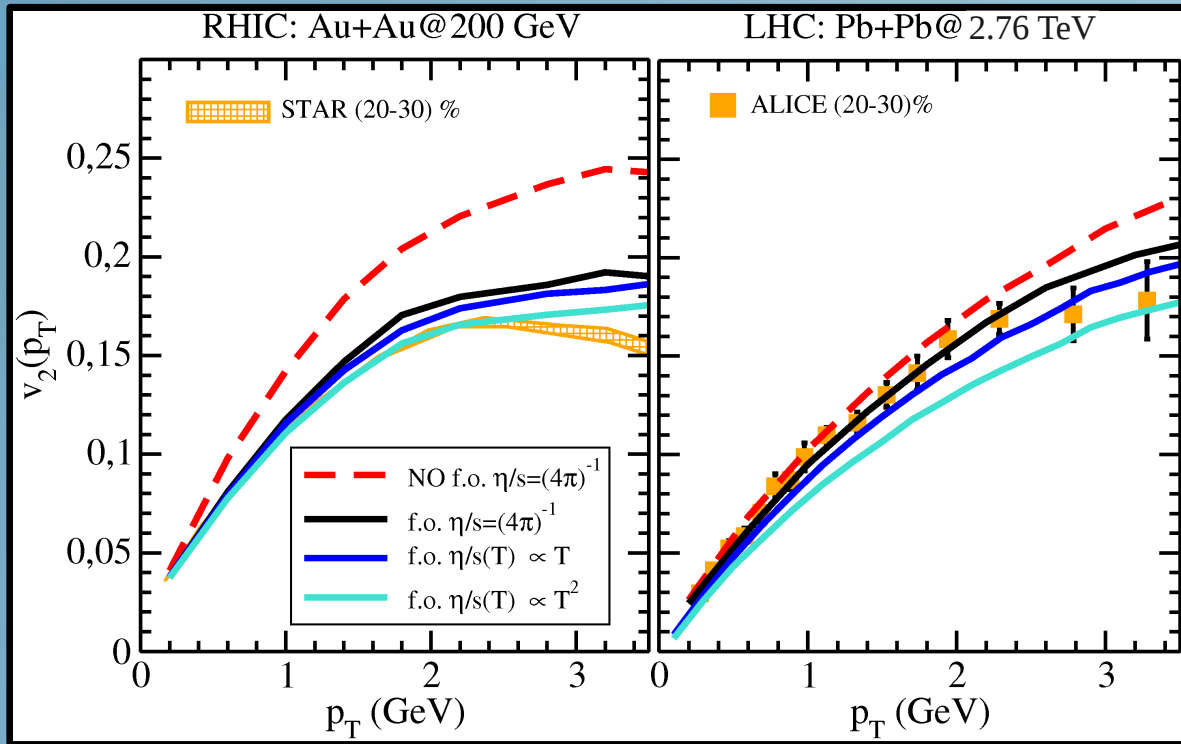
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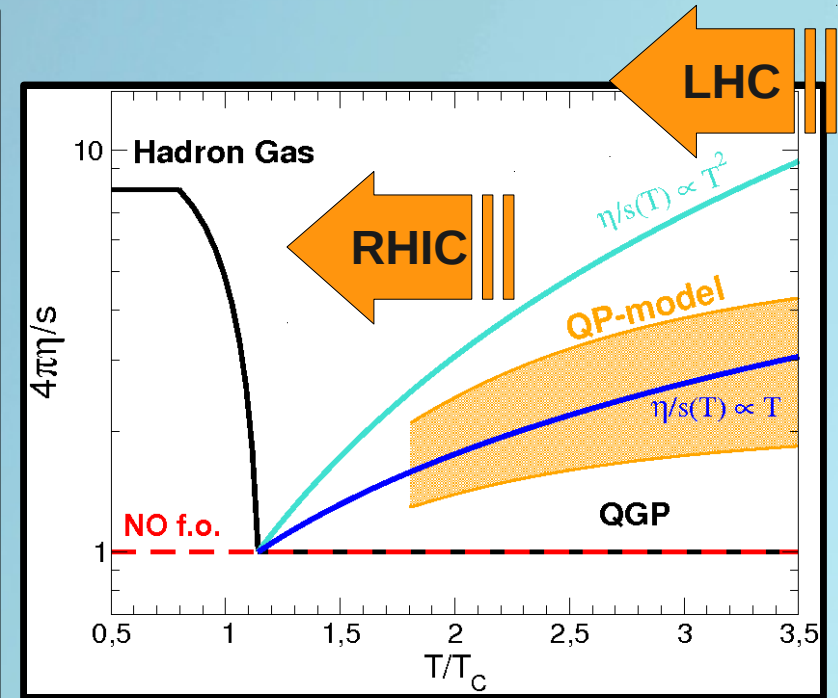
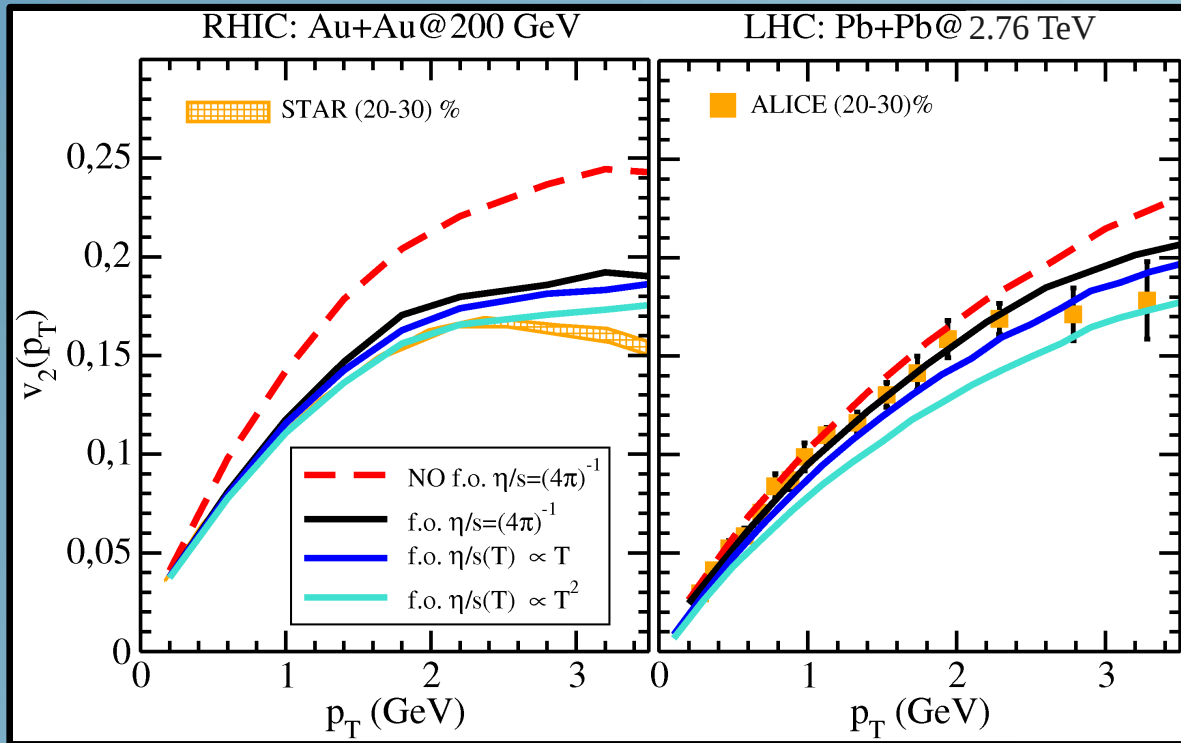
S.Plumari, W.M.Alberico, V.Greco, C.Ratti, arxiv:1103.5611[hep-ph].

## RHIC:

- The  $v_2$  is insensitive to the value of  $\eta/s$  in the QGP phase
- $\eta/s \sim T^2$  cannot account for the  $v_2$  decrease for  $p_T < 2$  GeV.

# Temperature dependent $\eta/s(T)$

K. Aamodt et al. [ALICE Collaboration], Phys. Rev.Lett. 105, 252302 (2010).



S.Plumari, W.M.Alberico, V.Greco, C.Ratti, arxiv:1103.5611[hep-ph].

## LHC:

- The  $v_2$  more sensitive to the QGP phase but still a strong temperature dependence in  $\eta/s$  has a small effect in the  $v_2(p_T)$ .

What happens in a wider  $p_T$  range?

Transport approach allows for such an extension naturally



**Of course there is a minimal consideration to do ...**

$$\sigma_{Tot}(s) = K \sigma_{pQCD}(s)$$



**Some non perturbative effect also for high  $p_T$  particles**

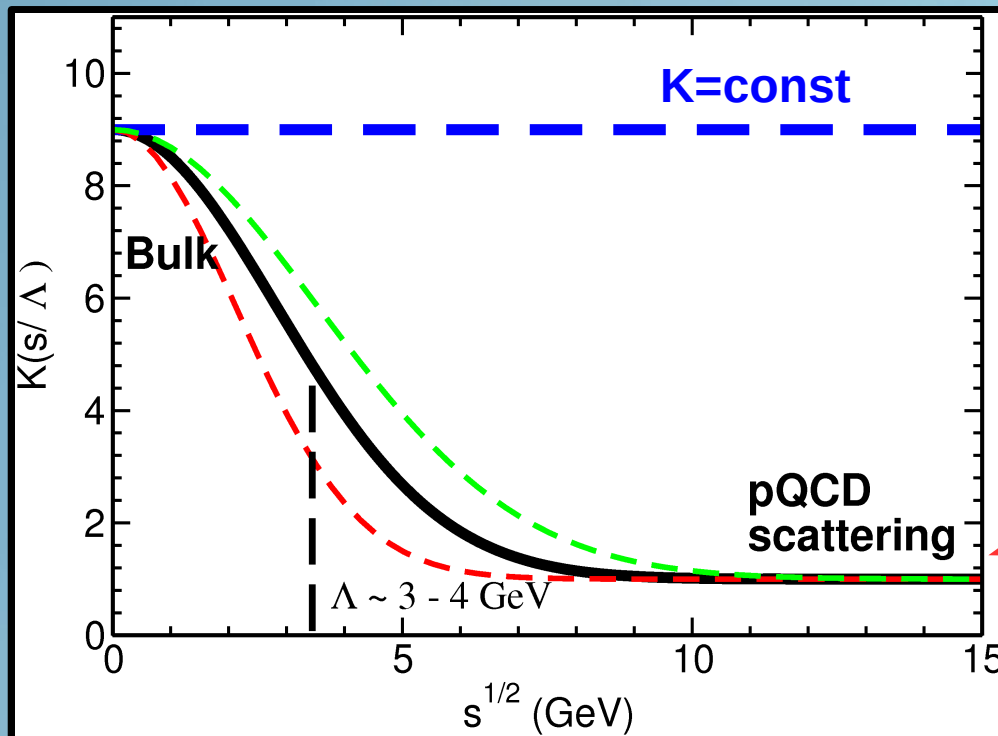
**and high  $p_T$  particle should collide with  $\sigma_{pQCD}$**



$$K = K(s/\Lambda^2)$$

$$\sigma_{Tot}(s) = K(s/\Lambda^2) \sigma_{pQCD}(s)$$

**$K(s)$  is such that  $\sqrt{s} \gg \Lambda$  ( $\sim 3 - 4$  GeV)  $K(s) \rightarrow 1$**

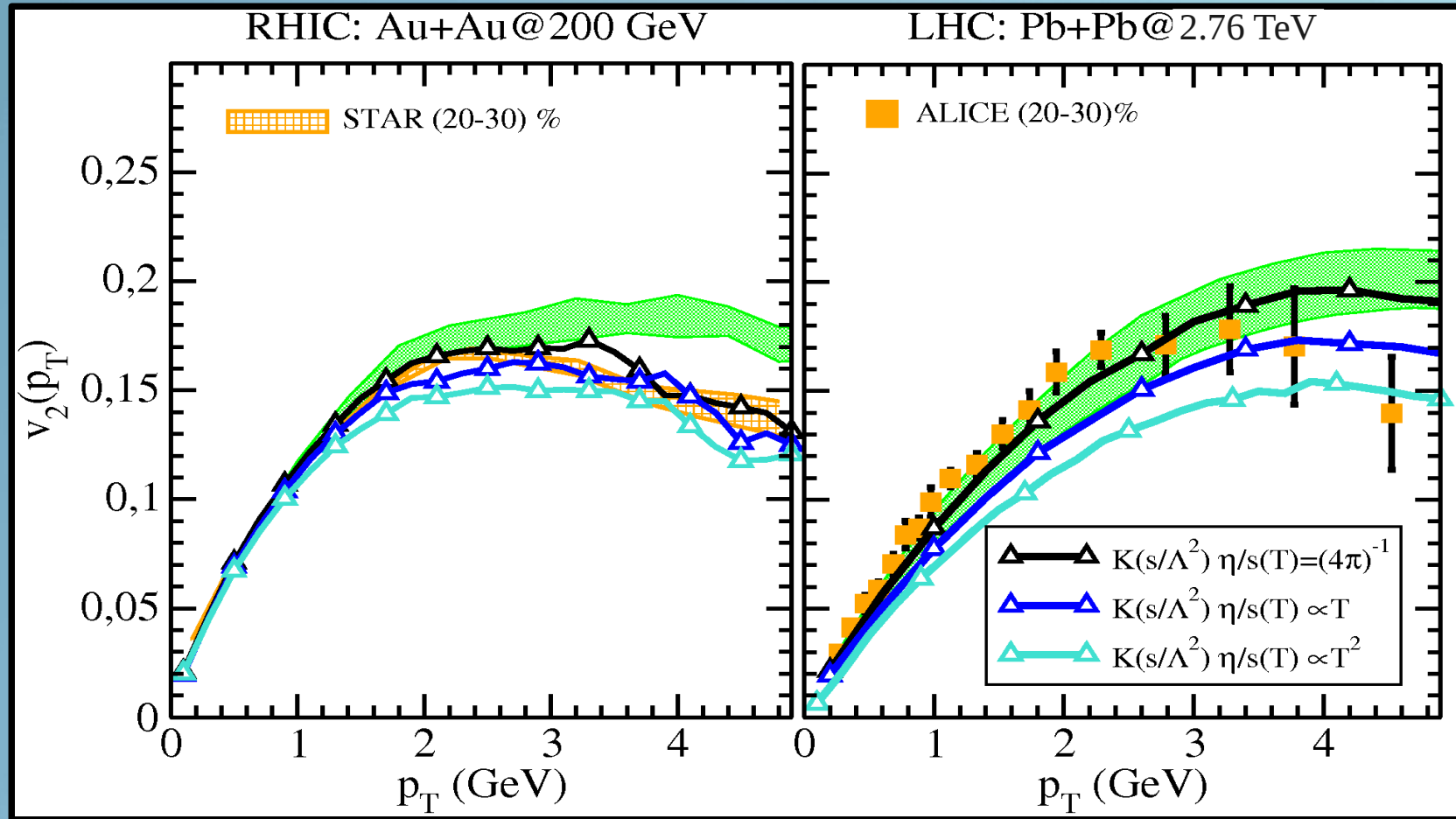


**Equivalent to viscous hydro**



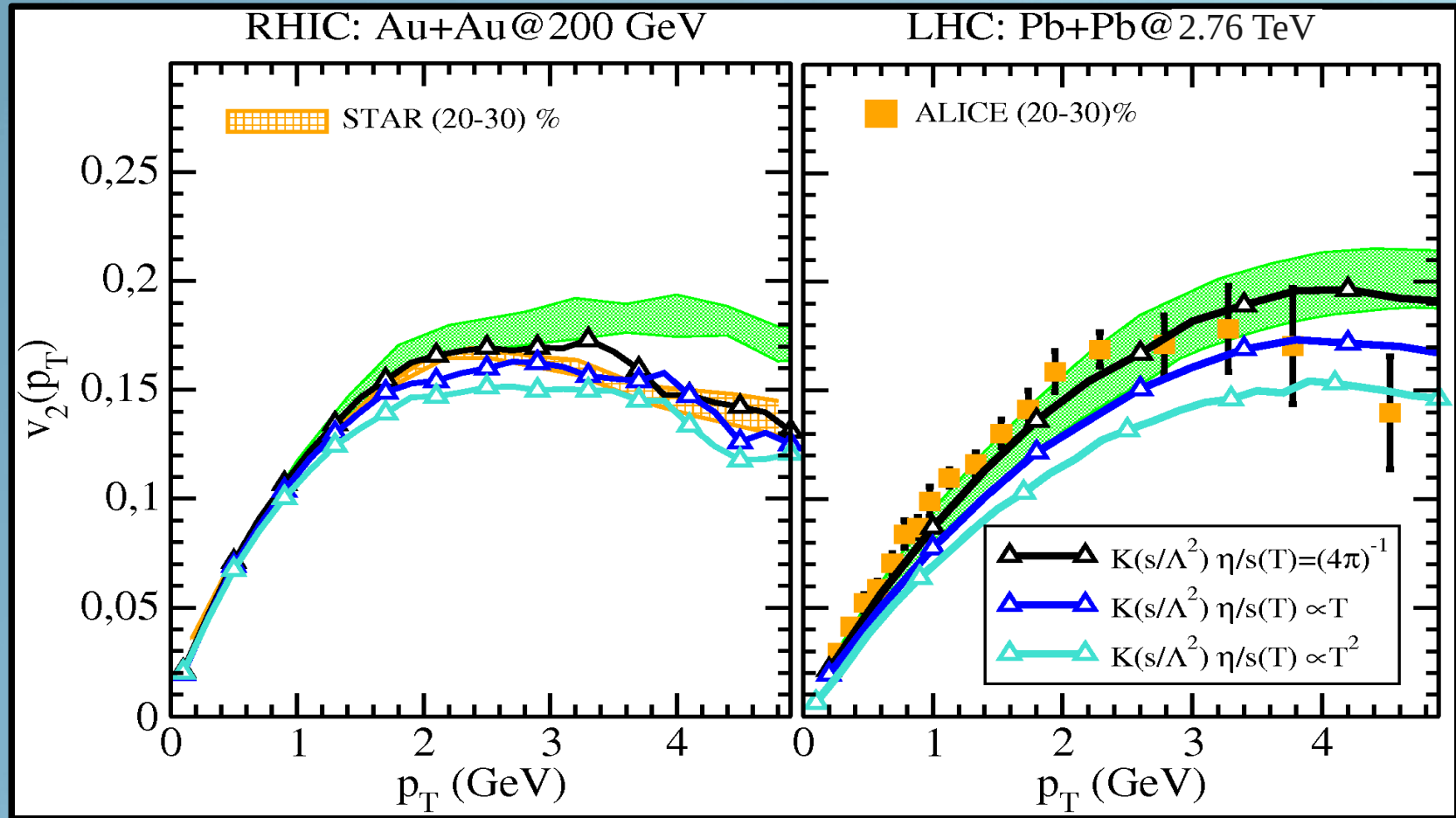
**At high  $p_T$  “correct” pQCD limit to be defined but certainly far from  $\eta/s \sim 0.1$**





### RHIC:

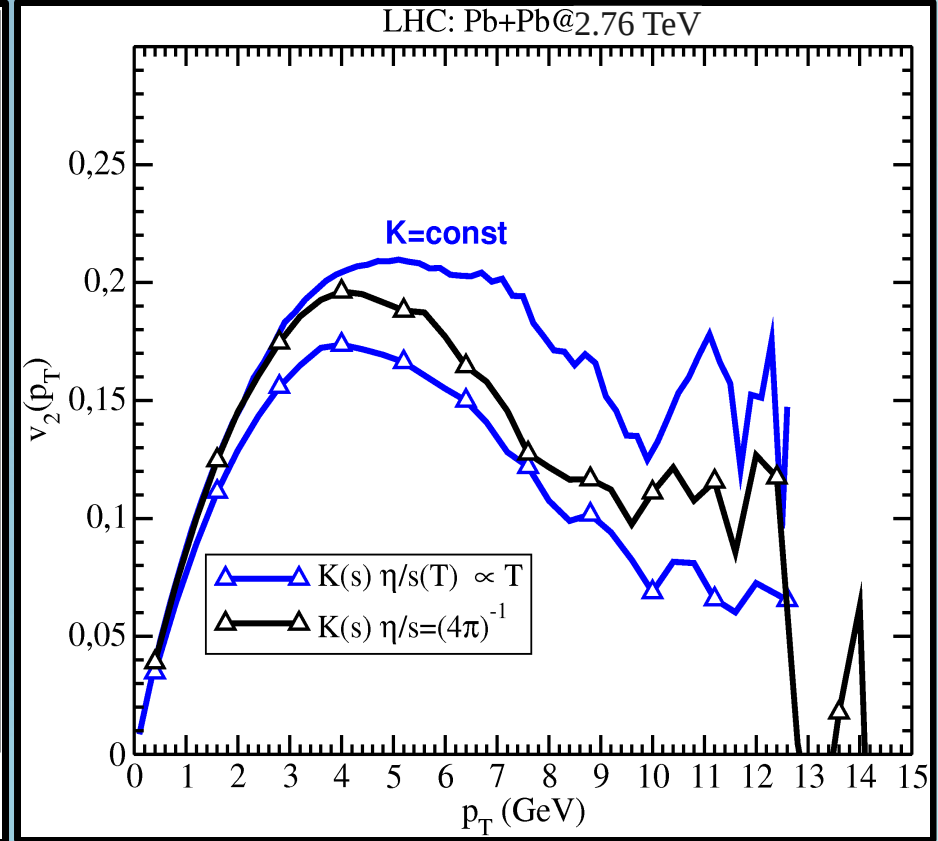
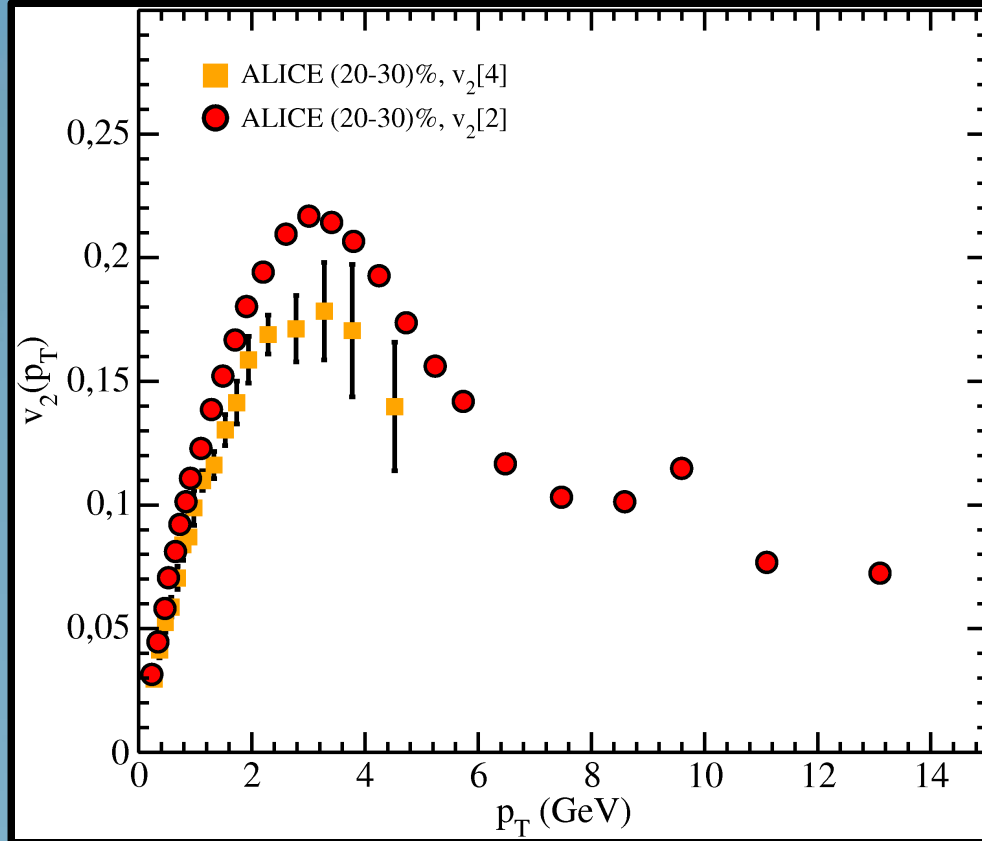
- For  $\Lambda > 3$  GeV the  $v_2$  is not very sensitive to the scale parameter  $\Lambda$ .
- $K(s/\Lambda)$  does not affect  $v_2$  for  $p_T < 2.5$  GeV and high  $p_T$  does not affect the bulk.
- A reduction of  $v_2$  only for  $p_T < 3$  GeV .



### LHC:

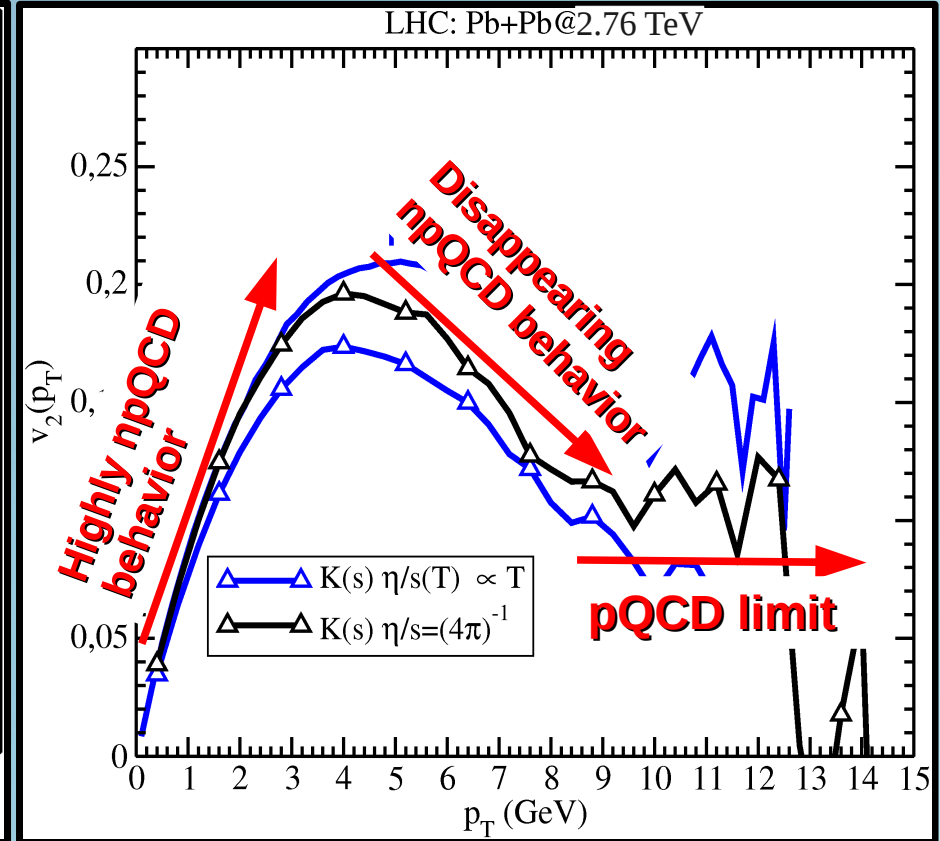
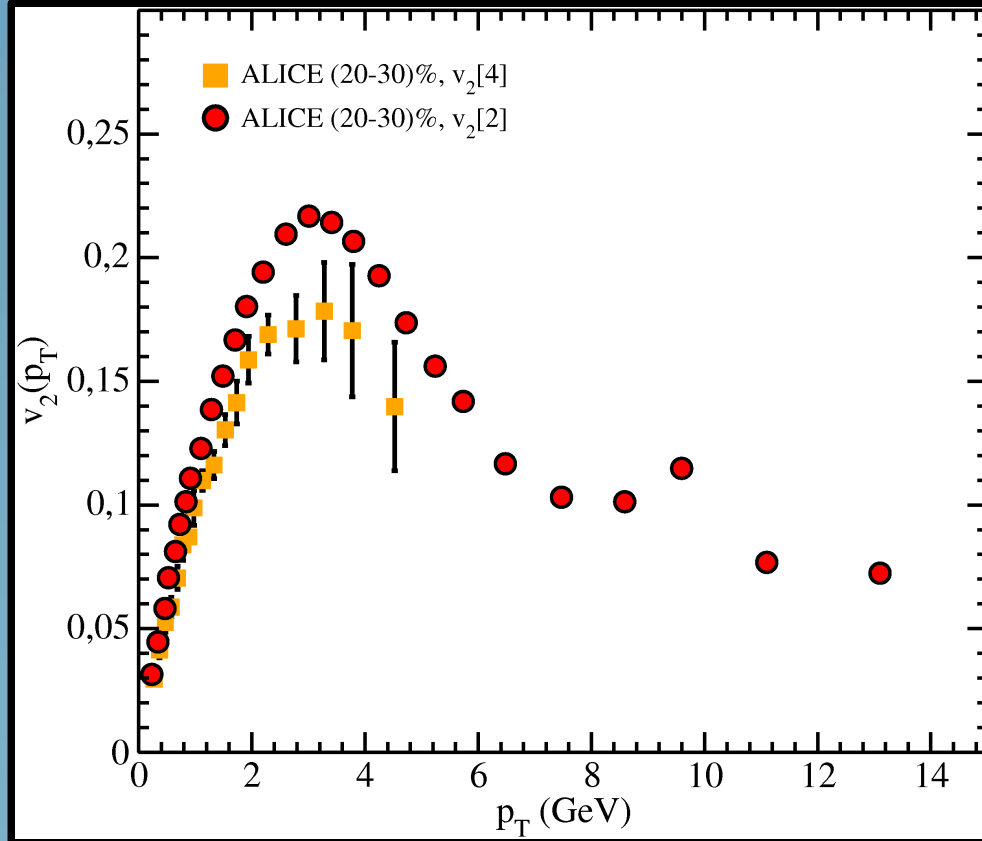
- The effect of  $K(s/\Lambda)$  even at  $p_T \sim 1 - 3$  GeV (much more high partons that affect also the bulk).
- $v_2$  remains sensitive to the  $\eta/s(T)$  in the QGP phase.

# Going to high $p_T$



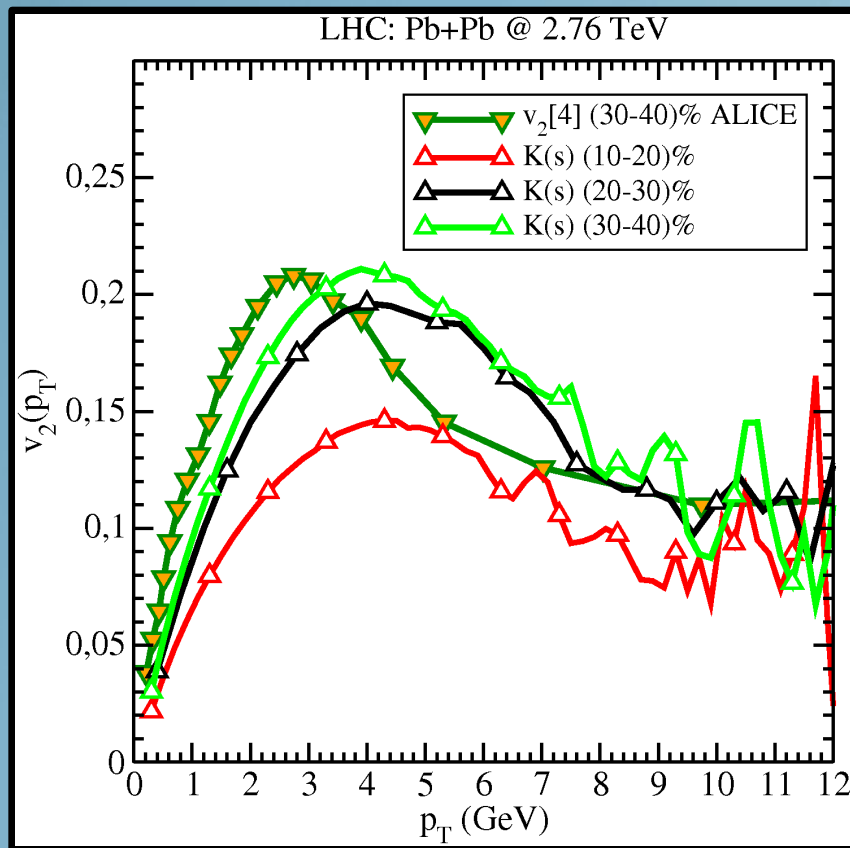
- We get a sizeable  $v_2$  from collisional energy loss ( $v_2 \sim 7\%$ ) (and a behaviour similar to the one observed experimental).
- Above 10 GeV sets in flat behavior typical of path length.
- This is not hydro+jet model, because we have only collisional energy loss and we treat on the same footing the high and low energy particles.

# Going to high $p_T$

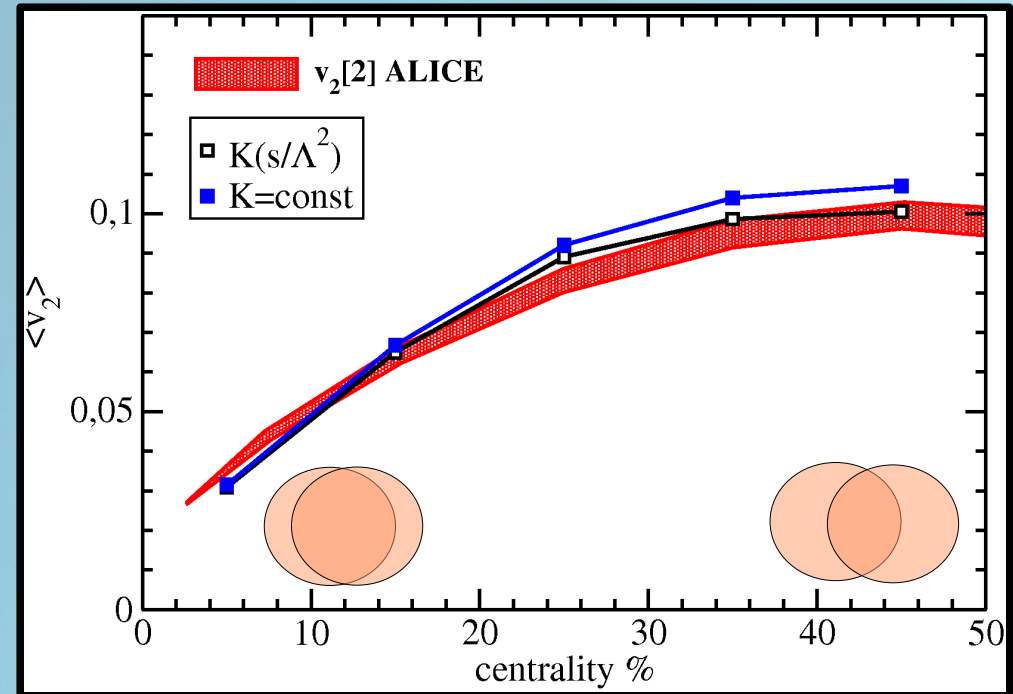


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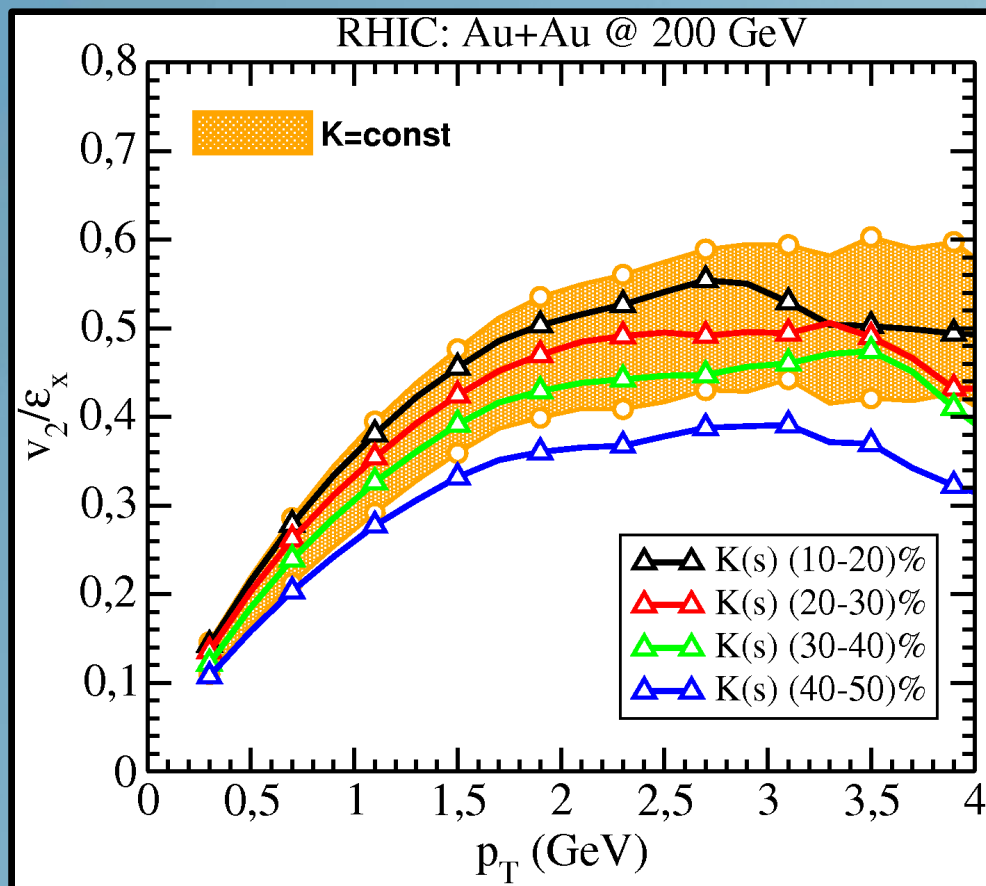


K. Aamodt et al. [ALICE Collaboration],  
Phys. Rev.Lett. 105, 252302 (2010).



- Shift of  $v_2$  to low  $p_T$  expected for fragmentation.
- The saturation persist also for more central collisions.
- Good agreement with the experimental data for the  $\langle v_2 \rangle$  as function of the centrality. The effect of  $K(s)$  is to reduce the  $\langle v_2 \rangle$ .

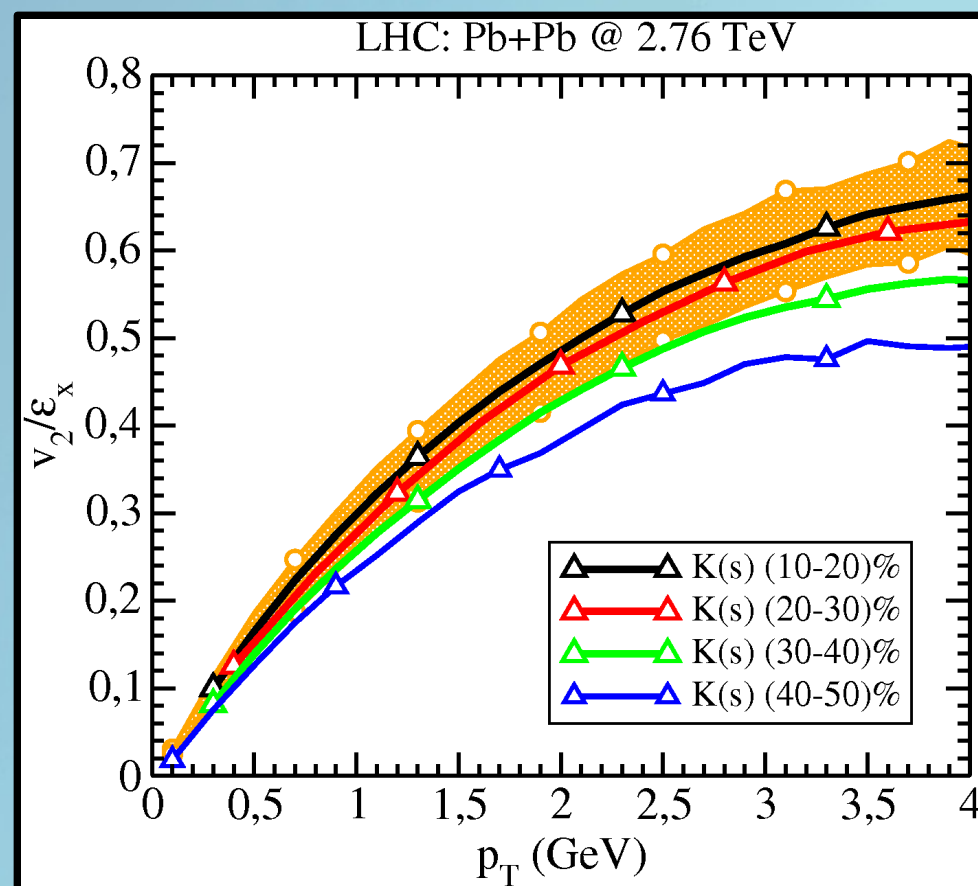
# Effect on the breaking of the $v_2/\epsilon$ scaling



## RHIC

Similar breaking w/o K(s): the amplitude is of about **30 %** (from f.o.).

The effect of K(s) is in general to reduce the value of  $v_2/\epsilon$ .

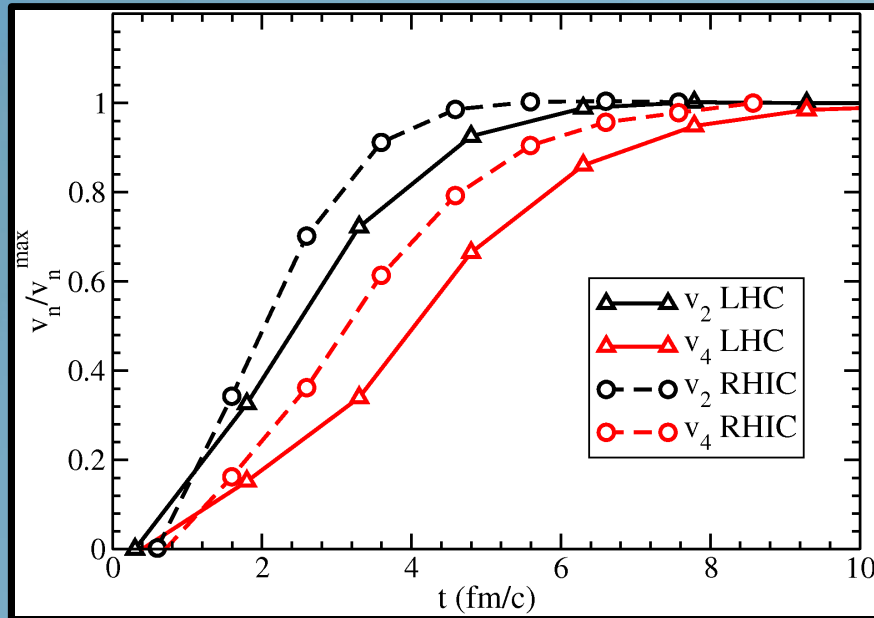


## LHC

For K=const the system is more efficient and the breaking is of about **17 %**.

The effect of K(s) is in general to increase the value of the breaking of about **25 %**.

# Effect of $\eta/s(T)$ and $K(s)$ on the ratio $v_4/(v_2)^2$

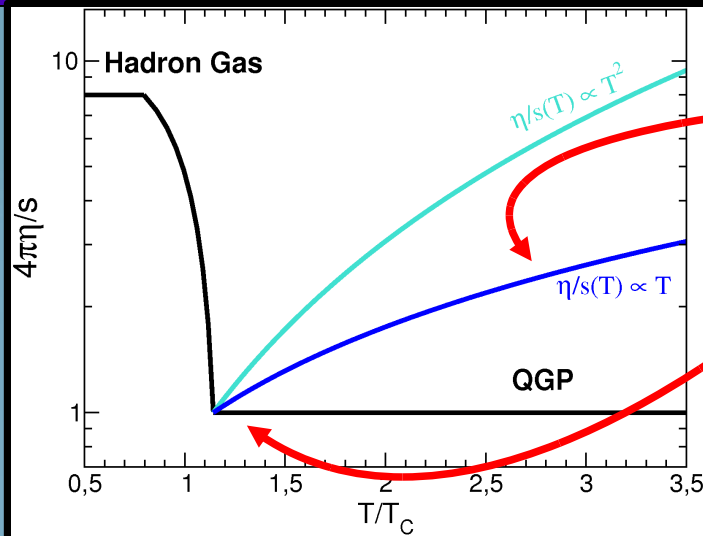


- $V_2$  develops earlier at higher  $\eta/s$ .
- $V_4$  develops later at lower  $\eta/s$ .

S. Plumari et al., J.Phys.: Conf. Ser. 270 (2011) 012061

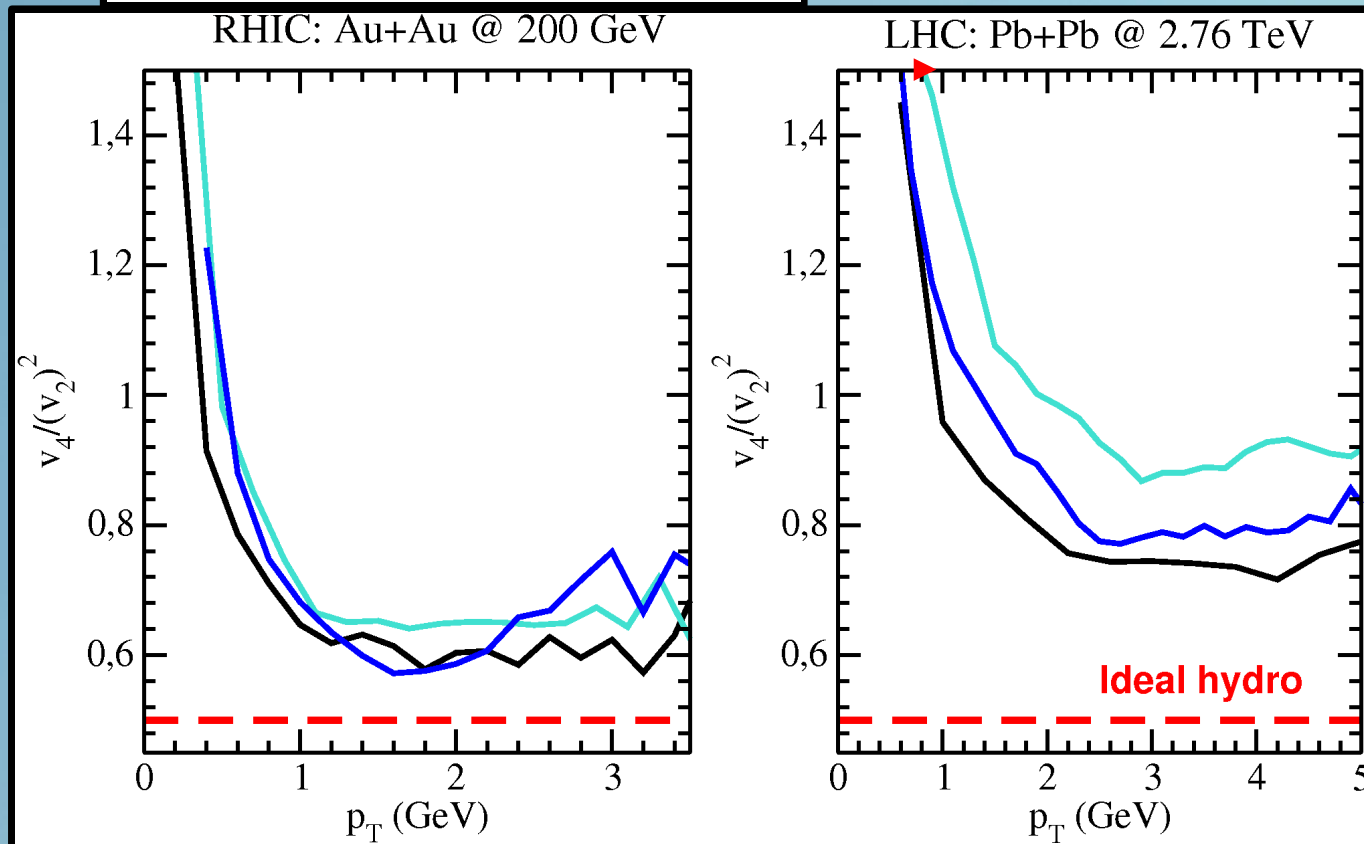


# Effect of $\eta/s(T)$ and $K(s)$ on the ratio $v_4/(v_2)^2$



- $V_2$  develops earlier at higher  $\eta/s$ .
- $V_4$  develops later at lower  $\eta/s$ .
- This gives an higher value of the ratio  $V_4/(V_2)^2$ .

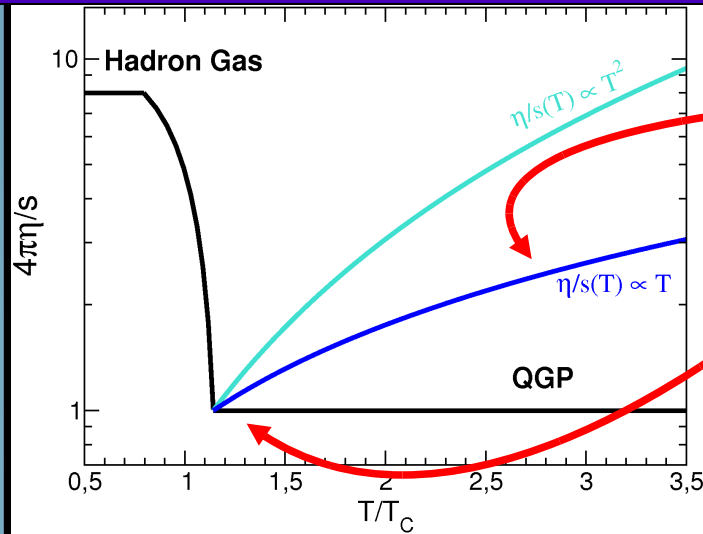
S. Plumari et al., J.Phys.: Conf. Ser. 270 (2011) 012061



At LHC the  $v_4/(v_2)^2$  is more sensitive to the QGP phase than at RHIC energies



# Effect of $\eta/s(T)$ and $K(s)$ on the ratio $v_4/(v_2)^2$

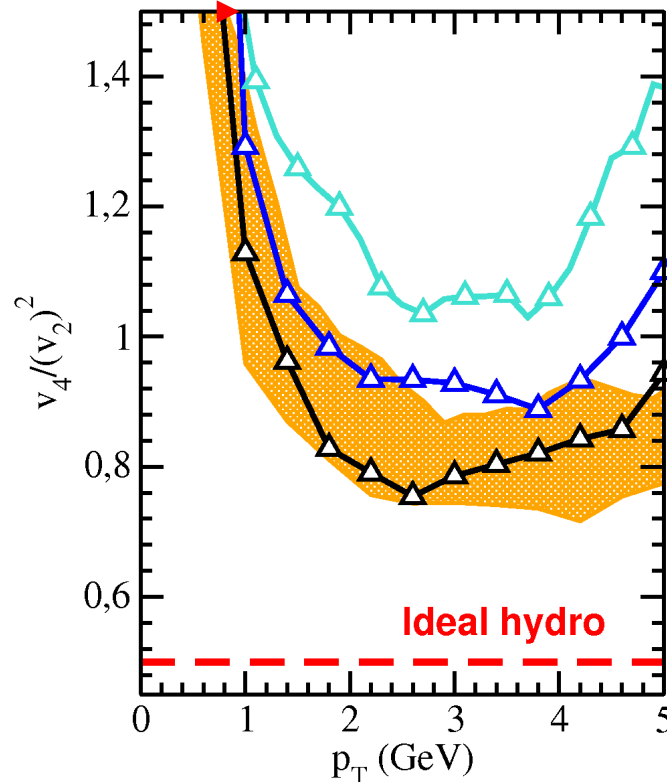
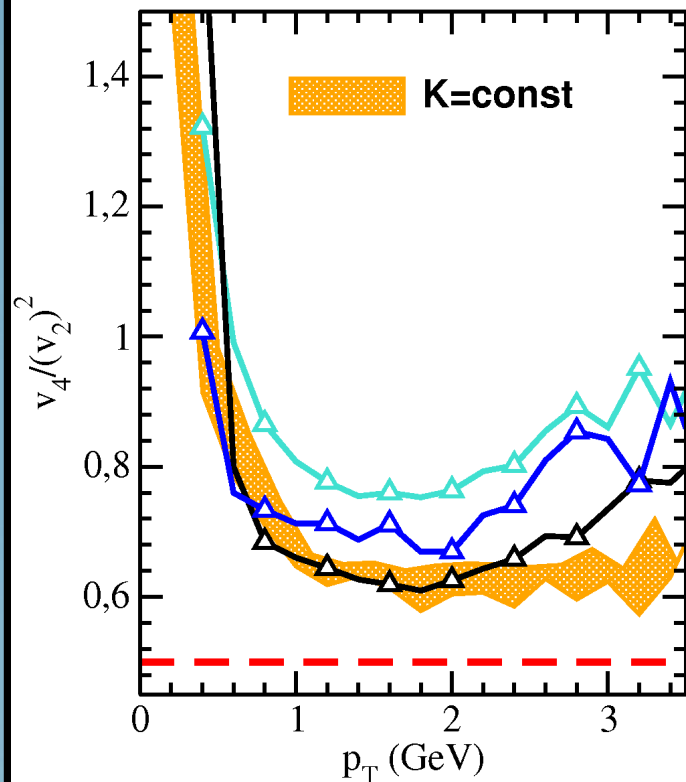


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- This gives an higher value of the ratio  $V_4/(V_2)^2$ .

S. Plumari et al., J.Phys.: Conf. Ser. 270 (2011) 012061

RHIC: Au+Au @ 200 GeV

LHC: Pb+Pb @ 2.76 TeV



- $v_4/(v_2)^2$  becomes more sensitive to the QGP phase also at RHIC
- $v_4/(v_2)^2$  becomes an increasing function at high  $p_T$

## BEFORE A QUANTITATIVE ESTIMATE

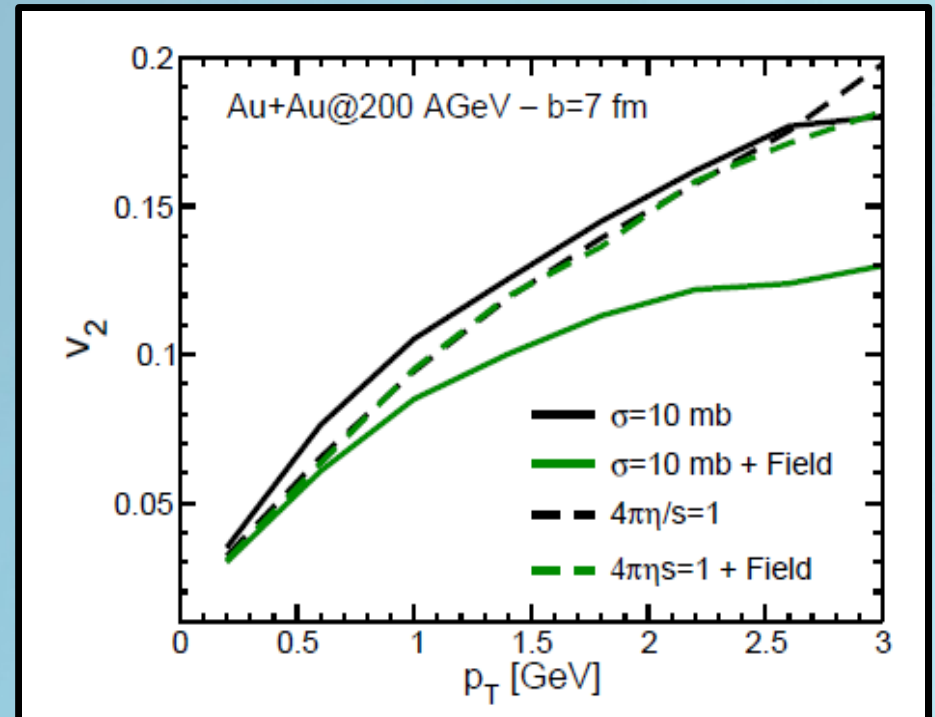
- Include lattice QCD-EoS  $P(\epsilon)$   
→ quasi particle model (Near FUTURE)
- Include fragmentation (of high  $p_T$ ) + coalescence
- At high  $p_T$  gluon radiation

## Conclusions and Outlook

- We have investigate within a parton cascade model the effect of a  $\eta/s(T)$  at RHIC and LHC energies.
  - At RHIC  $v_2$  still 20% of  $v_2$  depend on the cross-over region.
  - At LHC nearly all the  $v_2$  from the QGP phase.
  - Agreement with Viscous Hydrodynamics  $\eta/s \sim 0.1$  but not large sensitivity to T dependence ( $\eta/s \sim T^2$  could be excluded)
- At LHC the large ammount of particle with  $p_T > 4$  GeV interacting nearly perturbatively cannot be neglected.
  - Affect the  $\eta/s$  estimates ( $v_2$  at  $p_T < 3$  GeV).
  - Could explain the rise and fall of  $v_2(p_T)$  in  $0 < p_T < 8$  GeV.
  - Breaking of  $v_2/\epsilon$  similar to RHIC .
  - Signature of  $\eta/s(T)$  and  $K(s)$ : large  $v_4/(v_2)^2$  and increasing function of  $p_T$  at higher  $p_T$ .

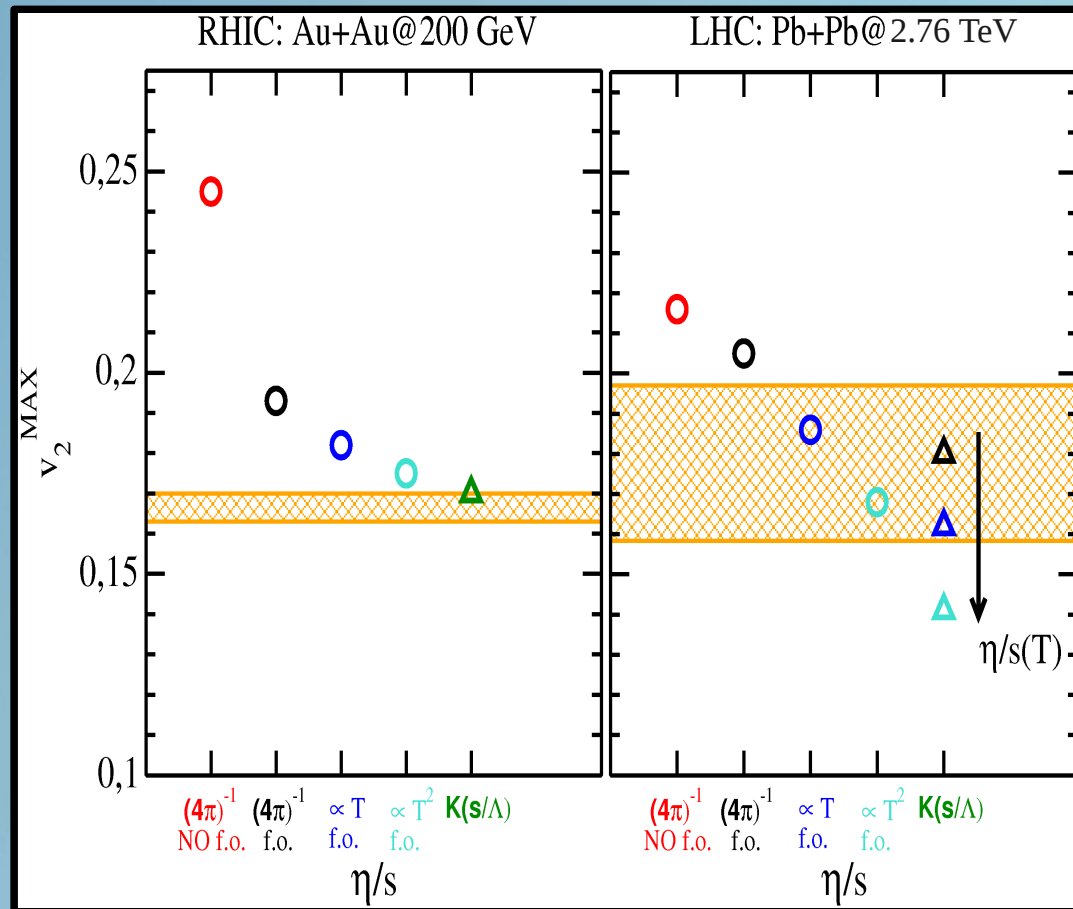


# Does the NJL chiral phase transition affect the elliptic flow of a fluid at fixed $\eta/s$ ?



S. Plumari et al., PLB689(2010)

- The NJL mean field reduce the  $v_2$ : attractive field (the effect is about 15 %).
- if  $\eta/s$  is fixed the effect of NJL mean field is compensated by the increase of  $\sigma$ .
- $v_2 \leftrightarrow \eta/s$  not modified by the NJL mean field dynamics.

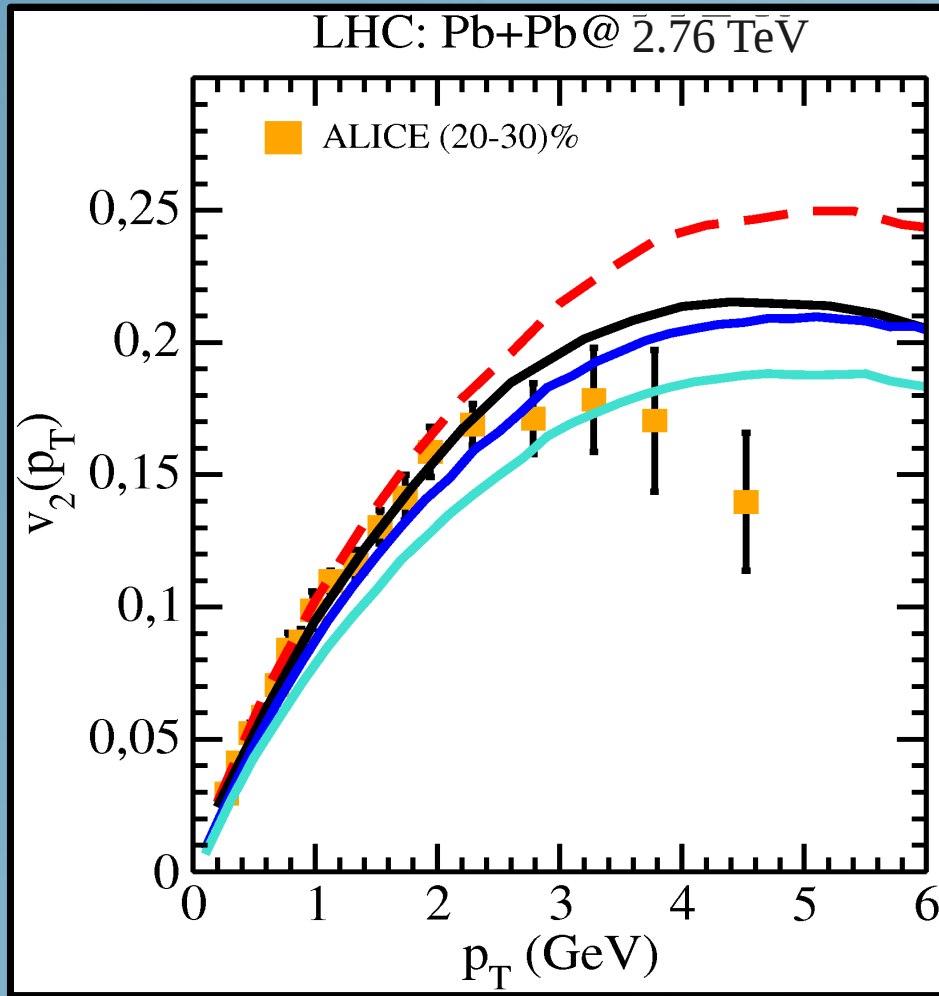


## Simulating a constant $\eta/s$

$$\eta(\vec{x}, t) = \frac{4}{15} \lambda(\vec{x}, t) n_{tot}(\vec{x}, t) \langle p \rangle \longrightarrow \sigma_{tr}^{\eta/s} = \frac{4}{15} \frac{\langle p \rangle}{n} \frac{1}{\eta/s} = K \sigma_{tr}^{pQCD}$$

$$\sigma_{Tot} = K \sigma_{pQCD} \gg \sigma_{pQCD}$$

$$\frac{d\sigma_{gg}}{dt} = \frac{9\pi\alpha_s^2}{2} \frac{1}{(t+m_D^2)^2} \left( 1 + \frac{m_D^2}{s} \right)$$



**At LHC the maximum of the  $v_2$  is at  $p_T \sim 5.5$  GeV strong efficiency in converting the initial asymmetry also at large  $p_T$ .**