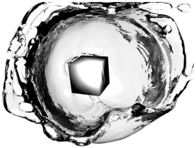

Calculation of shear viscosity within transport models

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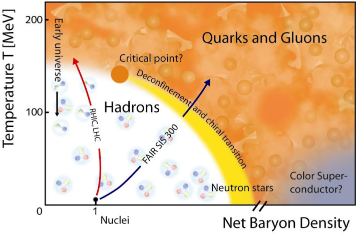
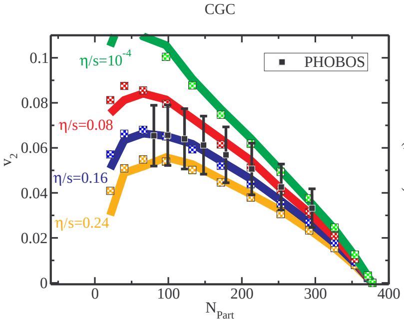
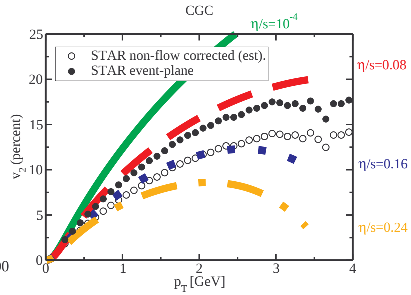
ITP Uni Frankfurt

TORIC, September 2011, Crete

- RHIC data show QGP signatures
- hot and collective medium
- nearly perfect liquid
- elliptic flow v_2 in central collisions
- employment of hydro- and transport-models



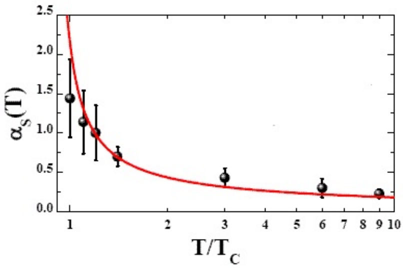


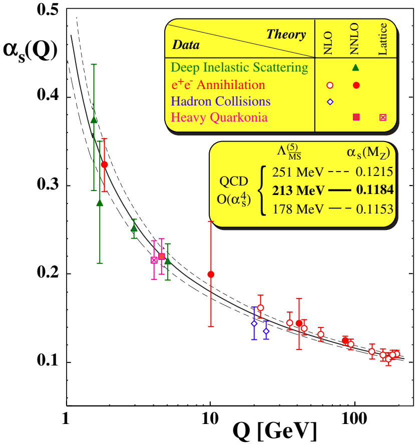


Motivation

for the early phase of heavy ion collisions:

- transport model based evolution / quasi-particle description
- pQCD cross sections for: $T > T_c$
- resonances, fragmentation, ...





Models

used models:

- BAMPS
  - partonic transport model
  - isotropic / constant cross sections
  - pQCD-based cross sections
- UrQMD
  - hybrid transport cascade / hydro model
  - isotropic / constant cross sections
  - Breit–Wigner resonance

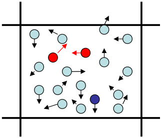
- transport algorithm: solve Boltzmann-equation with MC techniques

$$p^\mu \partial_\mu f(x,p) = C_{22} + C_{23} + \ldots$$

- on-shell partons
- stochastic interpretation of collision rates

$$P_{2i} = v_{\mathrm{rel}} \frac{\sigma_{(2i)}}{N_{\mathrm{test}}} \frac{\Delta t}{\Delta^3 x}$$

- cross sections are modeled via pQCD or isotropic cross sections
- elastic $2 \to 2$ and inelastic $2 \to 3$ / $3 \to 2$ processes included



- elastic pQCD-gluon cross sections:

$$\frac{d\sigma^{(gg\to gg)}}{dt} \approx \frac{d\sigma^{(gg\to gg)}}{dq_\perp^2} \approx \frac{9\pi\alpha_s^2}{2(q_\perp^2 + m_D^2)^2}$$

- inelastic three-gluon pQCD-based cross section:

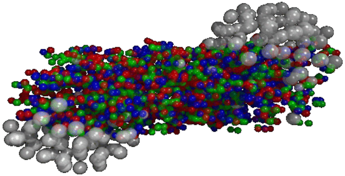
$$\left|\mathcal{M}_{(gg\to ggg)}\right|^2 = \frac{72\pi^2\alpha_s^2 s^2}{\left(\mathbf{q}_\perp^2 + m_D^2\right)^2} \frac{48\pi\alpha_s \mathbf{q}_\perp^2 \Theta\left(\Lambda_g - \tau\right)}{\mathbf{k}_\perp^2 \left[\left(\mathbf{k}_\perp - \mathbf{q}_\perp\right)^2 + m_D^2\right]}$$

- constant and isotropic cross sections

Ultrarelativistic Quantum Molecular Dynamics

- Non-equilibrium transport model with parameterized cross sections
- hadrons, resonances, string excitation and fragmentation
- via AQM or calculated by detailed balance
- pQCD hard scattering at high energies

In this work UrQMD is confined to a box with periodic boundary conditions.



used methods to extract η from transport models:

- Green Kubo Relation
  - simulation of equilibrated medium
  - viscosity proportional to thermal fluctuations
- static gradient method
  - simulation of static velocity gradient
  - viscosity is proportional to non-linear slope
- both methods are model and medium independent.

- Green-Kubo relation for shear viscosity:

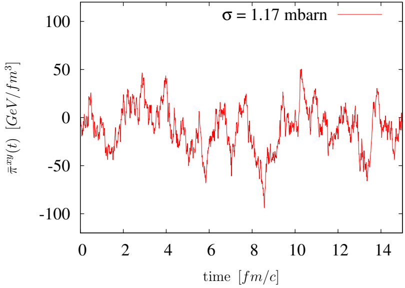
$$\eta = \frac{1}{10\ T} \int_V \mathrm{d}^3 r \int_\infty \mathrm{d}t \ \left\langle \pi^{ij}(0,0)\ ;\ \pi^{ij}(\vec{r},t) \right\rangle$$

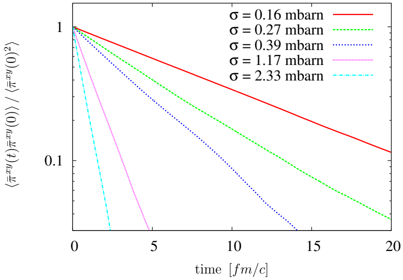
- $\pi^{ij}(\vec{r},t)$ fluctuates in thermal equilibrium

- for a Boltzmann-gas:

$$\langle \pi^{ij}(t)\ ;\ \pi^{ij}(0) \rangle = \langle \pi^{ij}(0)^2 \rangle \cdot \exp^{-t/\tau}$$

$$\langle \pi^{ij}(0)^2 \rangle = \frac{4}{15} \frac{eT}{V}$$





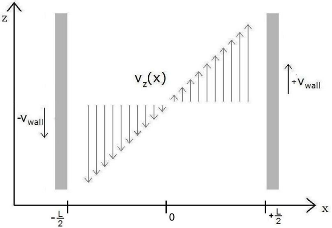
set up thermal medium → simulation → $\pi(t) \rightarrow \tau \rightarrow \eta$

only relevant timescale: t/τ !

$$S_{xx}(f) = \left[\tau(\tau^2 + f^2)\right]^{-1}$$

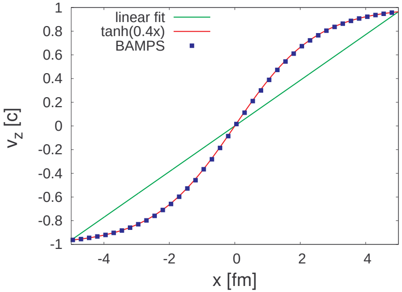
- method by Felix Reining (arXiv:1106.4210v1)
- simulation of static gradient
- extraction of shear stress from the slope of the gradient

$$F = -\eta \frac{\partial v_z}{\partial x}$$



- shape of gradient: $v(x) = tanh(\theta x) \qquad | \; \theta = 2v_{\mathrm{wall}}/L$
- using Navier-Stokes: $\pi^{xy} = -\eta \, \partial_x v_z(x)$

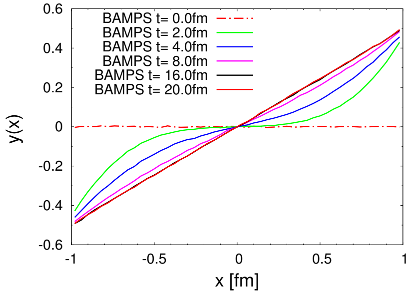
$$\eta = -\pi^{xy} \, \frac{\gamma \; L}{2y_{\mathrm{wall}}}$$

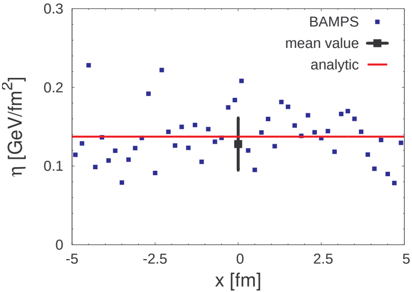


$$\text{set up geometry} \rightarrow \text{simulation} \rightarrow \text{evolve gradient} \rightarrow \frac{\gamma \; L}{2y_{\mathrm{wall}}} \rightarrow \eta$$

gradient relaxation time can be estimated:

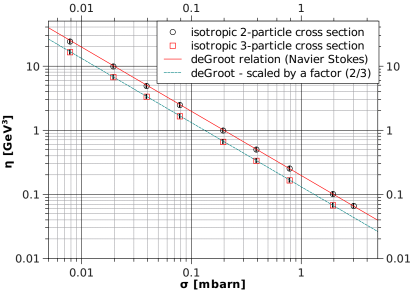
$$t_\tau = \frac{L^2 e}{2\eta} \approx \frac{L^2}{0.82\lambda_{\mathrm{mfp}}}$$

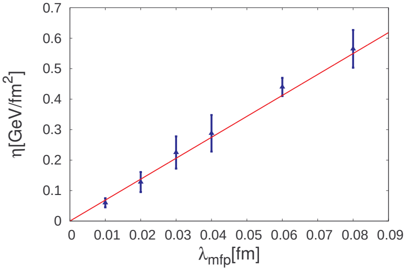




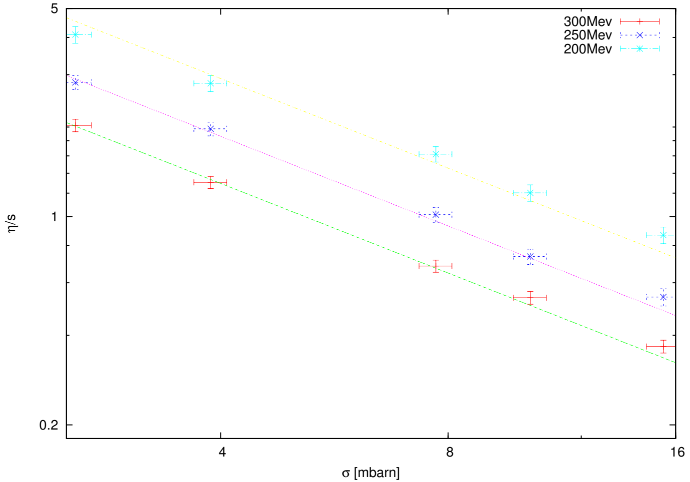
cross-check with an analytical relation (de Groot):

$$\eta = 1.264\, Tn\lambda = 1.264 \frac{T}{\sigma_{22}}$$



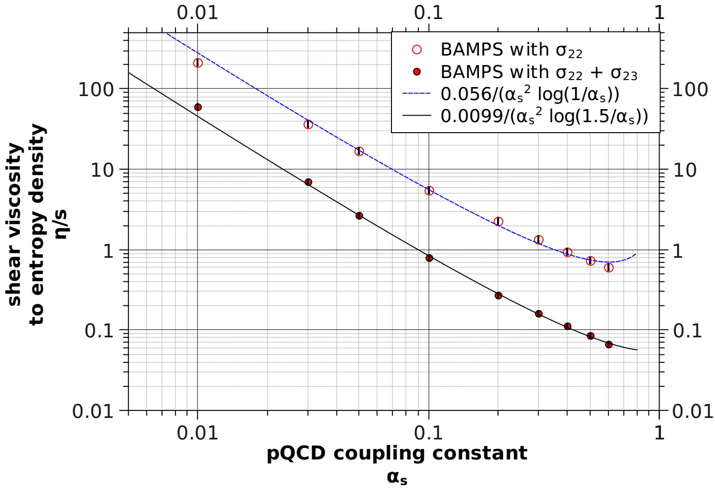


Model: BAMPS, massless gluon gas, $T = 400 MeV$

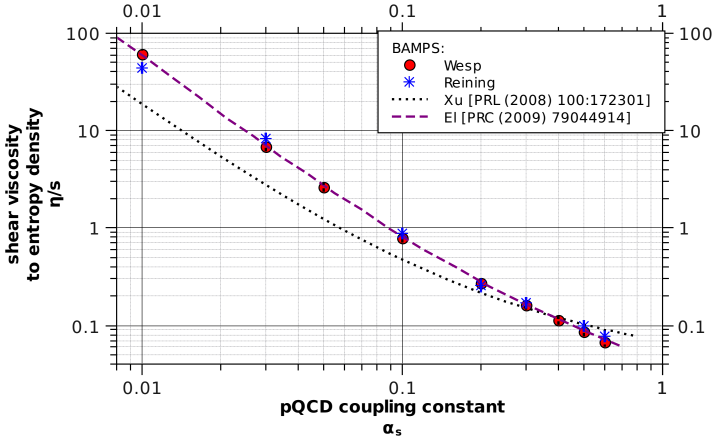


Model: UrQMD, massless pion gas $\eta = 1.264\frac{T}{\sigma_{22}}$

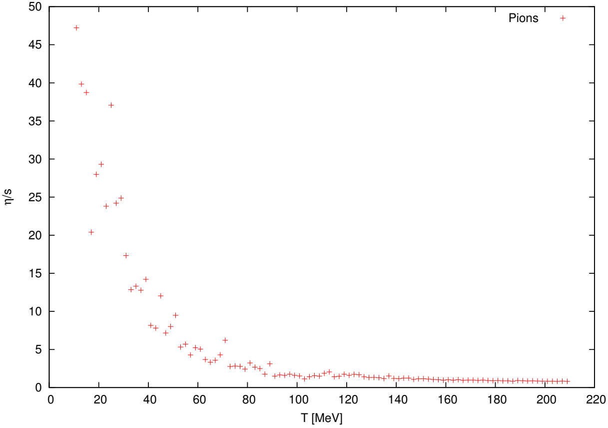
η/s for BAMPS with pQCD-based cross sections (elastic and inelastic):



η/s for BAMPS with pQCD-based cross sections (elastic and inelastic):



η/s for UrQMD with constant cross sections
and massive pions $m = 138 MeV$, $\sigma_{22} = 5\mathrm{mbarn}$ [preliminary]



Green-Kubo relation

- allows a precise extraction of η
- numerical error can be estimated
- equilibrium can be used for η/s

Static -Gradient method

- numerical convergency easily controllable
- fast runtime

both methods in agreement,
both methods are model independdent,
numerical and physical independent methods for robust cross-check

Thanks your attention!