Recent results from chiral effective models

Dirk H. Rischke

**Institut für Theoretische Physik and
Frankfurt Institute for Advanced Studies**

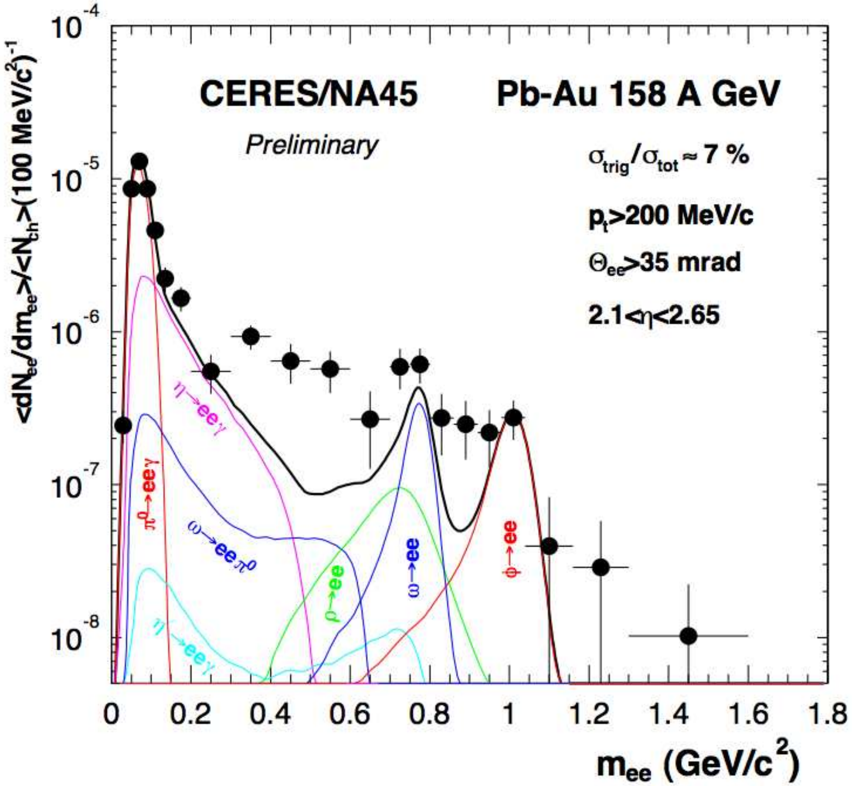
Johann Wolfgang Goethe-Universität
Frankfurt am Main

FIAS Frankfurt Institute
for Advanced Studies

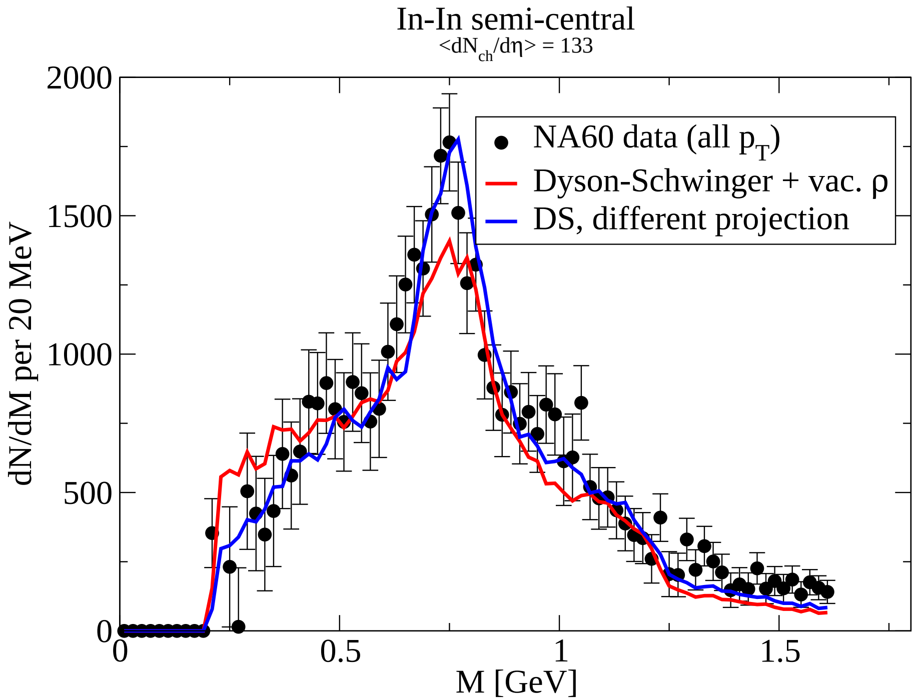
with:
**Walaa Eshraim, Anja Habersetzer,
Martin Grahl, Achim Heinz, Stanislaus Janowski, Denis Parganlija, Elina Seel,
Werner Deinet, Susanna Gallas, Francesco Giacosa,**
and
Peter Kovacs, Gyuri Wolf

Motivation

Dileptons carry information from hot and dense stages of heavy-ion collisions:



CERES/NA45 collaboration



NA60 collaboration

(fig. courtesy of Thorsten Renk)

$\Longrightarrow$ **learn about chiral symmetry restoration in hot and dense hadronic matter!**

The chiral effective model

Chiral symmetry of QCD: global $U(N_f)_r \times U(N_f)_\ell$ symmetry (classically)

$\Longrightarrow$ spontaneously broken in the vacuum by nonvanishing quark condensate $\langle \bar{q} q \rangle \neq 0$

$\Longrightarrow$ restored at nonzero temperature T and chemical potential μ

$\Longrightarrow$ degeneracy of hadronic chiral partners in the chirally restored phase

$\Longrightarrow$ for this application: chiral symmetry must be linearly realized

$\Longrightarrow$ Linear sigma model

$$\mathcal{L} = \mathcal{L}_S + \mathcal{L}_V + \mathcal{L}_B + \mathcal{L}_{SV} + \mathcal{L}_{VB} + \mathcal{L}_{SB}$$

$\mathcal{L}_S$: scalar and pseudoscalar mesons

$\mathcal{L}_V$: vector and axial-vector mesons

$\mathcal{L}_B$: baryons, incl. chiral partners

$\mathcal{L}_{SV}$: scalar/pseudoscalar – vector/axial-vector interactions

$\mathcal{L}_{VB}$: vector/axial-vector – baryon interactions

$\mathcal{L}_{SB}$: scalar/pseudoscalar – baryon interactions

Disclaimer

No attempt to describe precision data for hadron vacuum phenomenology!

$\Longrightarrow$ **No attempt to compete with chiral perturbation theory (which is based on nonlinear realization of chiral symmetry)**

Nevertheless: after fitting coupling constants of the model to hadron vacuum properties

$\Longrightarrow$ **reasonable description of hadron vacuum phenomenology!**

Moreover: strong statement on the nature of the scalar mesons!

scalar-meson puzzle: too many scalar states to fit into a $q\bar{q}$ meson octet

$$f_0(600),\ f_0(980),\ f_0(1370),\ f_0(1500),\ f_0(1710)$$

$\Longrightarrow$ **Jaffe's conjecture:** R.L. Jaffe, PRD 15 (1977) 267, 281

two scalar $[qq][\bar{q}\bar{q}]$ tetraquark states mix with two scalar $q\bar{q}$ meson states

$\Longrightarrow$ **fifth scalar meson could be due to mixing with glueball**

Scalar and pseudoscalar mesons

$$\mathcal{L}_S = \mathrm{Tr}\left(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2\, \Phi^\dagger \Phi\right) - \lambda_1 \left[\mathrm{Tr}\left(\Phi^\dagger \Phi\right)\right]^2 - \lambda_2 \mathrm{Tr}\left(\Phi^\dagger \Phi\right)^2 + c\left(\det\Phi + \det\Phi^\dagger\right) + \mathrm{Tr}\left[H\left(\Phi + \Phi^\dagger\right)\right]$$

$\Phi \in (N_f^*, N_f) \implies \Phi \equiv \phi_a T_a$, T_a generators of $U(N_f)$, $\phi_a \equiv \sigma_a + i\pi_a$, $H \equiv h_a T_a$

$h_a = c = 0$, $m^2 > 0$: $U(N_f)_r \times U(N_f)_\ell$ symmetry

$h_a = c = 0$, $m^2 < 0$: v.e.v. $\langle \Phi \rangle = \phi\, N_f\, T_0$, $\phi \equiv \langle \sigma_0 \rangle > 0$

Spontaneous symmetry breaking (SSB):

$U(N_f)_r \times U(N_f)_\ell \longrightarrow U(N_f)_V \quad (V \equiv \ell + r)$

$h_a = 0$, $c \neq 0$: $U(1)_A$ anomaly $(A \equiv \ell - r)$

Explicit symmetry breaking (ESB):

$U(N_f)_r \times U(N_f)_\ell \longrightarrow SU(N_f)_r \times SU(N_f)_\ell \times U(1)_V$

$m^2 < 0$: SSB: $SU(N_f)_r \times SU(N_f)_\ell \longrightarrow SU(N_f)_V$

$\dim[SU(N_f)_r \times SU(N_f)_\ell / SU(N_f)_V] = 2\,(N_f^2 - 1) - (N_f^2 - 1) = N_f^2 - 1$

$\implies$ $N_f^2 - 1$ Goldstone bosons $\implies$ pseudoscalar mesons!

h_a, $c \neq 0$, $m^2 < 0$: ESB $\implies$ $N_f^2 - 1$ pseudo - Goldstone bosons

Vector and axial-vector mesons

$$\begin{aligned}\mathcal{L}_V = & -\frac{1}{4}\,\mathrm{Tr}(\mathcal{L}^0_{\mu\nu}\mathcal{L}_0^{\mu\nu} + \mathcal{R}^0_{\mu\nu}\mathcal{R}_0^{\mu\nu}) + \frac{m_1^2}{2}\,\mathrm{Tr}(\mathcal{L}_\mu\mathcal{L}^\mu + \mathcal{R}_\mu\mathcal{R}^\mu) \\ & + i\,\frac{g_2}{2}\,\mathrm{Tr}\left\{\mathcal{L}^0_{\mu\nu}[\mathcal{L}^\mu,\mathcal{L}^\nu] + \mathcal{R}^0_{\mu\nu}[\mathcal{R}^\mu,\mathcal{R}^\nu]\right\} \\ & + g_3\,\mathrm{Tr}\,(\mathcal{L}^\mu\mathcal{L}^\nu\mathcal{L}_\mu\mathcal{L}_\nu + \mathcal{R}^\mu\mathcal{R}^\nu\mathcal{R}_\mu\mathcal{R}_\nu) - g_4\,\mathrm{Tr}\,(\mathcal{L}^\mu\mathcal{L}_\mu\mathcal{L}^\nu\mathcal{L}_\nu + \mathcal{R}^\mu\mathcal{R}_\mu\mathcal{R}^\nu\mathcal{R}_\nu) \\ & + g_5\,\mathrm{Tr}\,(\mathcal{L}^\mu\mathcal{L}_\mu)\;\mathrm{Tr}\,(\mathcal{R}^\nu\mathcal{R}_\nu) \\ & + g_6\,[\mathrm{Tr}\,(\mathcal{L}^\mu\mathcal{L}_\mu)\;\mathrm{Tr}\,(\mathcal{L}^\nu\mathcal{L}_\nu) + \mathrm{Tr}\,(\mathcal{R}^\mu\mathcal{R}_\mu)\;\mathrm{Tr}\,(\mathcal{R}^\nu\mathcal{R}_\nu)]\end{aligned}$$

$$\mathcal{L}^0_{\mu\nu} \equiv \partial_\mu\mathcal{L}_\nu - \partial_\nu\mathcal{L}_\mu,\;\; \mathcal{R}^0_{\mu\nu} \equiv \partial_\mu\mathcal{R}_\nu - \partial_\nu\mathcal{R}_\mu,\;\; \mathcal{L}_\mu \equiv L^a_\mu T_a,\;\; \mathcal{R}_\mu \equiv R^a_\mu T_a$$

vector mesons: $V^a_\mu \equiv \frac{1}{2}\left(L^\mu_a + R^a_\mu\right)$, axial-vector mesons: $A^a_\mu \equiv \frac{1}{2}\left(L^\mu_a - R^a_\mu\right)$

$m_1^2 = 0$, $g_2 = g$, $g_3 = g_4 = \frac{g^2}{2}$, $g_5 = g_6 = 0$:

$\Longrightarrow$ local $U(N_f)_r \times U(N_f)_\ell$ symmetry, g: gauge coupling

$\Longrightarrow$ $\boxed{\mathcal{L}_V \to -\frac{1}{4}\,\mathrm{Tr}(\mathcal{L}_{\mu\nu}\mathcal{L}^{\mu\nu} + \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu})}$ $\mathcal{L}_{\mu\nu} = \mathcal{L}^0_{\mu\nu} - ig[\mathcal{L}_\mu,\mathcal{L}_\nu]$, $\mathcal{R}_{\mu\nu} = \mathcal{R}^0_{\mu\nu} - ig[\mathcal{R}_\mu,\mathcal{R}_\nu]$

$m_1^2 \neq 0$: global $U(N_f)_r \times U(N_f)_\ell$ symmetry

$m_1^2 \neq 0$, $g_2 = g$, $g_3 = g_4 = \frac{g^2}{2}$, $g_5 = g_6 = 0$:

$\Longrightarrow$ Sakurai model S. Gasiorowicz, D.A. Geffen, Rev. Mod. Phys. 41 (1969) 531

In general, all $g_i \neq 0$ and different from each other

Baryons and their chiral partners

Inclusion of baryons and their chiral partners:

$\Longrightarrow$ **Mirror assignment:** C. DeTar and T. Kunihiro, PRD 39 (1989) 2805

$$\Psi_{1,r} \longrightarrow U_r\,\Psi_{1,r}\;, \quad \Psi_{1,\ell} \longrightarrow U_\ell\,\Psi_{1,\ell}\;,$$

but: $$\Psi_{2,r} \longrightarrow U_\ell\,\Psi_{2,r}\;, \quad \Psi_{2,\ell} \longrightarrow U_r\,\Psi_{2,\ell}$$

$\Longrightarrow$ **new, chirally invariant mass term:**

$$\mathcal{L}_B = \bar{\Psi}_{1,\ell}\, i\partial\!\!\!/\, \Psi_{1,\ell} + \bar{\Psi}_{1,r}\, i\partial\!\!\!/\, \Psi_{1,r} + \bar{\Psi}_{2,\ell}\, i\partial\!\!\!/\, \Psi_{2,\ell} + \bar{\Psi}_{2,r}\, i\partial\!\!\!/\, \Psi_{2,r} + m_0 \left(\bar{\Psi}_{2,\ell}\,\Psi_{1,r} - \bar{\Psi}_{2,r}\,\Psi_{1,\ell} - \bar{\Psi}_{1,\ell}\,\Psi_{2,r} + \bar{\Psi}_{1,r}\,\Psi_{2,\ell}\right)$$

Note: Chiral symmetry restoration:

chiral partners become degenerate, but not necessarily massless!

$\Longrightarrow$ m_0 **models contribution from gluon condensate to baryon mass**

$\Longrightarrow$ **allows for stable nuclear matter ground state!**

Scalar – vector interactions

$$\mathcal{L}_{SV} = i\, g_1\, \mathrm{Tr}\left[\partial_\mu \Phi \left(\Phi^\dagger \mathcal{L}^\mu - \mathcal{R}^\mu \Phi^\dagger\right) - \partial_\mu \Phi^\dagger \left(\mathcal{L}^\mu \Phi - \Phi \mathcal{R}^\mu\right)\right]$$
$$+ \tfrac{h_1}{2}\, \mathrm{Tr}\left(\Phi^\dagger \Phi\right) \mathrm{Tr}\left(\mathcal{L}_\mu \mathcal{L}^\mu + \mathcal{R}_\mu \mathcal{R}^\mu\right) + (g_1^2 + h_2)\, \mathrm{Tr}\left(\Phi^\dagger \Phi \mathcal{R}_\mu \mathcal{R}^\mu + \Phi \Phi^\dagger \mathcal{L}_\mu \mathcal{L}^\mu\right)$$
$$- 2(g_1^2 - h_3)\, \mathrm{Tr}\left(\Phi^\dagger \mathcal{L}_\mu \Phi \mathcal{R}^\mu\right)$$

$g_1 = g$, $h_1 = h_2 = h_3 = 0$ $\Longrightarrow$ $\mathcal{R}_\mu$, $\mathcal{L}_\mu$ gauge bosons of $U(N_f)_r \times U(N_f)_\ell$

$\Longrightarrow$ covariant derivative: $\partial_\mu \Phi \to D_\mu \Phi = \partial_\mu \Phi - i\, g\, \mathcal{L}_\mu\, \Phi + i\, g\, \Phi\, \mathcal{R}_\mu$

$\Longrightarrow$ $\mathrm{Tr}\left[(\partial_\mu \Phi)^\dagger\, \partial^\mu \Phi\right] + \mathcal{L}_{SV} \to \mathrm{Tr}\left[(D_\mu \Phi)^\dagger\, D^\mu \Phi\right]$

SSB:

- induces mass splitting $m_A^2 - m_V^2 = (g_1^2 - h_3)\phi^2$
- induces bilinear term $\sim g_1 \phi\, A_a^\mu\, \partial_\mu \pi_a$:

$\Longrightarrow$ eliminate by shift $A_a^\mu \to A_a^\mu + w(\phi)\, \partial^\mu \pi_a$, $w(\phi) \equiv \dfrac{g_1 \phi}{m_A^2}$

$\Longrightarrow$ wave function renormalization of pseudoscalar fields

$\pi_a \to Z\, \pi_a$, $Z^2 \equiv \left(1 - \frac{g_1^2 \phi^2}{m_A^2}\right)^{-1}$ (KSFR : $Z \equiv \sqrt{2}$)

$\Longrightarrow$ v.e.v. $\phi \equiv Z\, f_\pi$

Vector – baryon interactions

remember: in the mirror assignment, $\Psi_{2,r} \to U_\ell \, \Psi_{2,r}$, $\Psi_{2,\ell} \to U_r \, \Psi_{2,\ell}$

$$\Longrightarrow \quad \mathcal{L}_{VB} = c_1 \left(\bar{\Psi}_{1,\ell} \, \not{\mathcal{L}} \, \Psi_{1,\ell} + \bar{\Psi}_{1,r} \, \not{\mathcal{R}} \, \Psi_{1,r} \right) + c_2 \left(\bar{\Psi}_{2,\ell} \, \not{\mathcal{R}} \, \Psi_{2,\ell} + \bar{\Psi}_{2,r} \, \not{\mathcal{L}} \, \Psi_{2,r} \right)$$

$c_1 = c_2 = g \quad \Longrightarrow \quad \mathcal{R}_\mu \, , \; \mathcal{L}_\mu$ gauge bosons of $U(N_f)_r \times U(N_f)_\ell$

$\Longrightarrow$ covariant derivative: $D_\ell^\mu \equiv \partial^\mu - i g \mathcal{L}^\mu \, , \quad D_r^\mu \equiv \partial^\mu - i g \mathcal{R}^\mu$

$$\Longrightarrow \quad \begin{aligned} \mathcal{L}_B + \mathcal{L}_{VB} = & \; \bar{\Psi}_{1,\ell} \, i \not{D}_\ell \, \Psi_{1,\ell} + \bar{\Psi}_{1,r} \, i \not{D}_r \, \Psi_{1,r} + \bar{\Psi}_{2,\ell} \, i \not{D}_r \, \Psi_{2,\ell} + \bar{\Psi}_{2,r} \, i \not{D}_\ell \, \Psi_{2,r} \\ & + m_0 \left(\bar{\Psi}_{2,\ell} \, \Psi_{1,r} - \bar{\Psi}_{2,r} \, \Psi_{1,\ell} - \bar{\Psi}_{1,\ell} \, \Psi_{2,r} + \bar{\Psi}_{1,r} \, \Psi_{2,\ell} \right) \end{aligned}$$

Scalar – baryon interactions

Yukawa interaction:

$$\mathcal{L}_{SB} = -\hat{g}_1 \left(\bar{\Psi}_{1,\ell}\, \Phi\, \Psi_{1,r} + \bar{\Psi}_{1,r}\, \Phi^\dagger\, \Psi_{1,\ell} \right) - \hat{g}_2 \left(\bar{\Psi}_{2,r}\, \Phi\, \Psi_{2,\ell} + \bar{\Psi}_{2,\ell}\, \Phi^\dagger\, \Psi_{2,r} \right)$$

$N_f = 2$ **mass eigenstates:**

$$\begin{pmatrix} N \\ N^* \end{pmatrix} \equiv \begin{pmatrix} N^+ \\ N^- \end{pmatrix} = \frac{1}{\sqrt{2\cosh\delta}} \begin{pmatrix} e^{\delta/2} & \gamma_5\, e^{-\delta/2} \\ \gamma_5\, e^{-\delta/2} & -e^{\delta/2} \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}, \quad \sinh\delta = \frac{\phi}{4\, m_0}(\hat{g}_1 + \hat{g}_2)$$

$$m_\pm = \sqrt{m_0^2 + \frac{\phi^2}{16}(\hat{g}_1 + \hat{g}_2)^2} \pm \frac{\phi}{4}(\hat{g}_1 - \hat{g}_2) \quad \longrightarrow \quad m_0 \qquad (\phi \to 0)$$

axial coupling constant:

$$g_A = +\tanh\delta \left[1 - \frac{c_1 + c_2}{2\, g_1}\left(1 - \frac{1}{Z^2}\right)\right] - \frac{c_1 - c_2}{2\, g_1}\left(1 - \frac{1}{Z^2}\right)$$

$$g_A^* = -\tanh\delta \left[1 - \frac{c_1 + c_2}{2\, g_1}\left(1 - \frac{1}{Z^2}\right)\right] - \frac{c_1 - c_2}{2\, g_1}\left(1 - \frac{1}{Z^2}\right) \neq -g_A\, !$$

$\Longrightarrow$ for $c_1 \neq c_2$ compatible with $g_A \simeq 1.26\,,\ g_A^* \simeq 0\,!$

T.T. Takahashi, T. Kunihiro, PRD 78 (2008) 011503

Vacuum phenomenology: The scalar meson puzzle (I)

Meson sector ($N_f = 2$): D. Parganlija, F. Giacosa, DHR, PRD 82 (2010) 054024

Determine parameters of the model through masses and decay widths:

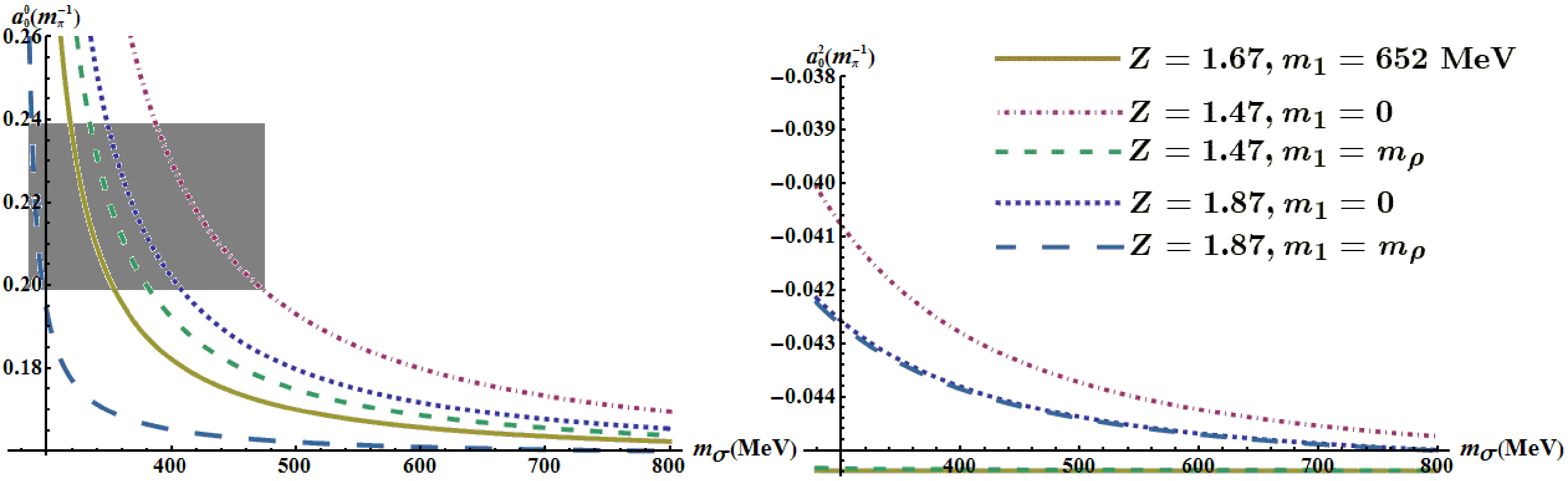
- $g_i\,,\ i = 3, \ldots, 6,$ do not contribute to masses or decay widths
- using tree-level relations for the meson masses, replace set
  $$m^2,\ \lambda_1,\ \lambda_2,\ c,\ h_0,\ m_1^2,\ g_1,\ g_2,\ h_1,\ h_2,\ h_3$$
  by $\quad m_\sigma,\ m_\pi,\ m_{a_0},\ m_{\eta_N},\ m_\rho,\ m_{a_1},\ Z,\ \phi,\ g_2,\ h_1,\ h_2$
- use $m_\pi = 139.57$ MeV, $m_\rho = 775.49$ MeV, $m_{a_1} = 1230$ MeV
- $\eta\eta'$ mixing: $\quad \eta = \eta_N \cos\varphi + \eta_S \sin\varphi\,,\ \eta' = -\eta_N \sin\varphi + \eta_S \cos\varphi$
  $\Longrightarrow$ with $\varphi \simeq -36^o \quad \Longrightarrow \quad m_{\eta_N} \simeq 716$ MeV
- pion decay constant $f_\pi = 92.4$ MeV fixes $\phi = Z\, f_\pi$ in terms of Z
  $\Longrightarrow$ remaining set: $\quad m_\sigma,\ m_{a_0},\ Z,\ g_2,\ h_1,\ h_2$
- determine g_2 via decay width $\Gamma(\rho \to \pi\pi)$
- determine h_2 via decay width $\Gamma(f_1 \to a_0\pi)$
- determine Z via decay width $\Gamma(a_1 \to \pi\gamma) \quad \Longrightarrow \quad Z = 1.67 \pm 0.2$

Vacuum phenomenology: The scalar meson puzzle (II)

(i) Scenario I: Light scalar mesons $f_0(600)$, $a_0(980)$ are $q\bar{q}$ states

$\Longrightarrow$ $\sigma \equiv f_0(600)$, $a_0 \equiv a_0(980)$

$\Longrightarrow$ fix $m_{a_0} = 980$ MeV,
perform χ^2 fit of $\pi\pi$ scattering lengths a_0^0, a_0^2 to determine m_σ, h_1

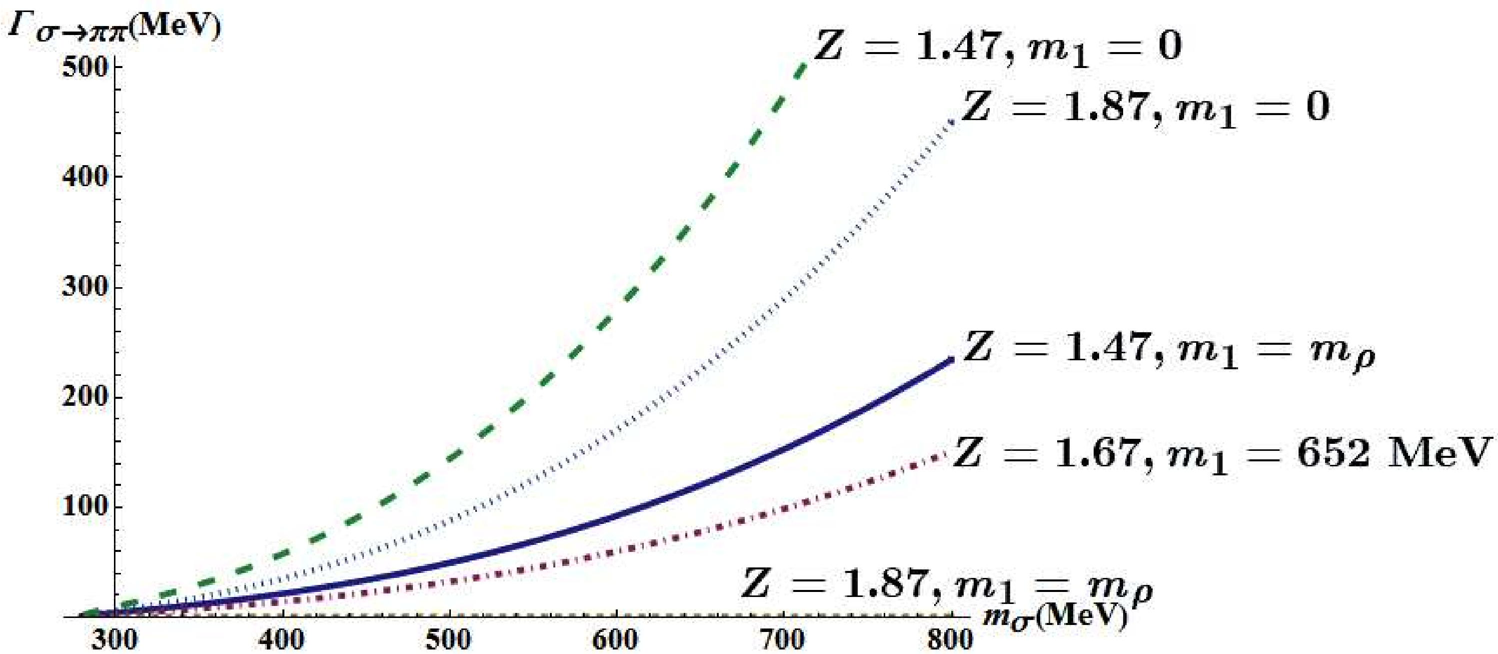


$\Longrightarrow$ $m_\sigma = 332^{+145}_{-44}$ MeV

c.f. e.g. $m_\sigma = 441^{+16}_{-8}$ MeV from analysis of Roy eqs. in chiral perturbation theory   H. Leutwyler, AIP Conf. Proc. 1030 (2008) 46

Vacuum phenomenology: The scalar meson puzzle (III)

However: decay width $\Gamma(\sigma \longrightarrow \pi\pi)$ is much too small!



PDG: $\Gamma(\sigma \longrightarrow \pi\pi) = (600 - 1000)$ MeV C. Amsler et.al., PLB 667 (2008) 1

Roy eqs.: $\Gamma(\sigma \longrightarrow \pi\pi) = 544^{+18}_{-25}$ MeV H. Leutwyler, AIP Conf. Proc. 1030 (2008) 46

$\Gamma(\sigma \longrightarrow \pi\pi) = (510 \pm 32)$ MeV R. Kaminski et al., NPB PS 186 (2009) 318

too small $\Gamma(\sigma \longrightarrow \pi\pi)$ is due to interference of diagrams arising from vector mesons $\Longrightarrow$ no problem in linear sigma models without vector mesons

Vacuum phenomenology: The scalar meson puzzle (IV)

(ii) Scenario II: Heavy scalar mesons $f_0(1370)$, $a_0(1450)$ are $q\bar{q}$ states

$\Longrightarrow$ $\sigma \equiv f_0(1370)$, $a_0 \equiv a_0(1450)$

$\Longrightarrow$ fix $m_{a_0} = 1450$ MeV

Note: $\pi\pi$ scattering length a_0^0 cannot be used for the fit, due to absence of light $f_0(600)$ state in the model

and: decay width $\Gamma(f_1 \rightarrow a_0\pi) \equiv 0$ due to absence of phase space

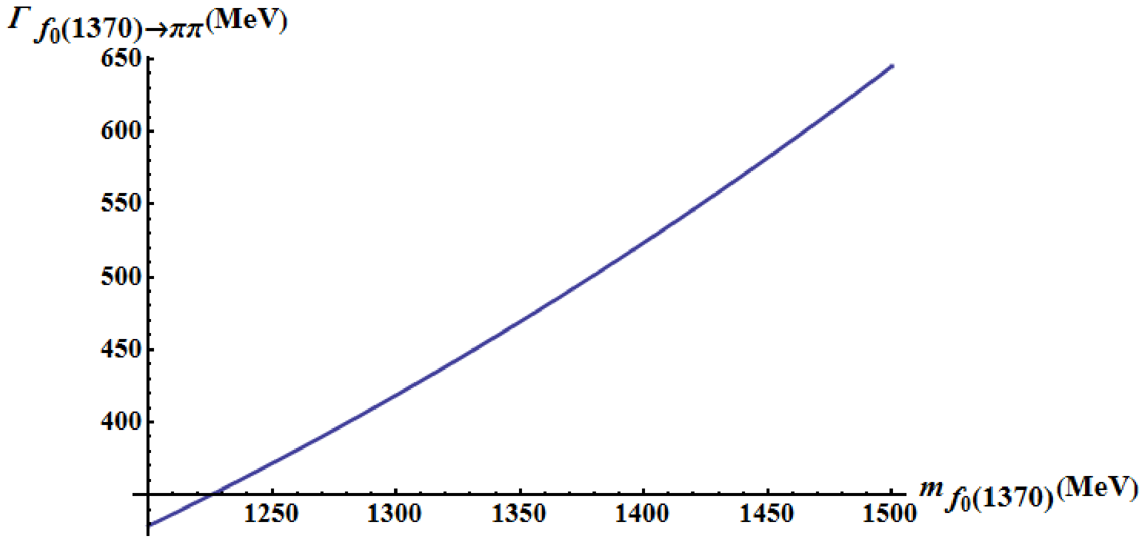
$\Longrightarrow$ h_2 cannot be determined from this decay

$\Longrightarrow$ change fit strategy:

- set $h_1 \equiv 0$ (anyhow large$-N_c$ suppressed)
- determine h_2 from matching total decay width $\Gamma_{a_0(1450)}$ to exp. value

Vacuum phenomenology: The scalar meson puzzle (V)

⟹ compute decay width as a function of m_σ



cf. exp. values: $m_\sigma = (1200 - 1500)$ MeV, $\Gamma(\sigma \to \pi\pi) = (200 - 500)$ MeV

⟹ good agreement with data!

⟹ heavy scalar states are (predominantly) $q\bar{q}$ states ...

⟹ ...and thus the chiral partners of the π!

⟹ light scalar states are (predominantly) tetraquark states!

⟹ extension to $N_f = 3$ seems to confirm this!

D. Parganlija, P. Kovacs, F. Giacosa, Gy. Wolf, DHR (in preparation)

Vacuum phenomenology: The scalar meson puzzle (VI)

Another confirmation of the (predominantly) $q\bar{q}$ assignment for the heavy scalar mesons: $\Longrightarrow$ coupling to the glueball/dilaton field!

S. Janowski, D. Parganlija, F. Giacosa, DHR, arXiv:1103.3238 (accepted for publication in PRD)

- dilatation symmetry $\Longrightarrow$ dynamical generation of tree-level meson mass parameters through glueball field G: $m^2 \longrightarrow m^2 \left(\frac{G^2}{G_0^2}\right)$, $m_1^2 \longrightarrow m_1^2 \left(\frac{G^2}{G_0^2}\right)$
- add glueball Lagrangian: $$\mathcal{L}_G = \frac{1}{2}\,(\partial_\mu G)^2 - \frac{1}{4}\,\frac{m_G^2}{\Lambda^2}\,G^4 \left(\ln\left|\frac{G}{\Lambda}\right| - \frac{1}{4}\right)$$

  $\Lambda \sim$ gluon condensate $\langle G^a_{\mu\nu} G_a^{\mu\nu}\rangle$

  $\Longrightarrow$ $\mathcal{L} \longrightarrow \mathcal{L} + \mathcal{L}_G$
- shift σ and G by their v.e.v.'s, $\sigma \longrightarrow \sigma + \phi$, $G \longrightarrow G + G_0$

  $\Longrightarrow$ v.e.v. G_0 given by $$-\frac{m^2}{m_G^2}\,\phi^2\,\Lambda^2 = G_0^4\,\ln\left|\frac{G_0}{\Lambda}\right|$$

  $\Longrightarrow$ glueball mass given by $$M_G^2 = m^2\,\frac{\phi^2}{G_0^2} + m_G^2\,\frac{G_0^2}{\Lambda^2}\left(1 + 3\ln\left|\frac{G_0}{\Lambda}\right|\right)$$

  $\Longrightarrow$ bilinear term $\sim \sigma G$ $\Longrightarrow$ eliminate by $O(2)$ transformation
  $$\begin{pmatrix} \sigma' \\ G' \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \sigma \\ G \end{pmatrix}$$

Vacuum phenomenology: The scalar meson puzzle (VII)

(i) Scenario I: $\sigma' = f_0(600)$, $G' = f_0(1500)$

$\Longrightarrow$ as before, $\Gamma(\sigma' \to \pi\pi)$ comes out too small $\Longrightarrow$ unfavored scenario!

(ii) Scenario II: $\sigma' = f_0(1370)$, $G' = f_0(1500)$

$\Longrightarrow$ χ^2 fit of Λ, M_σ, m_G^2, m_1^2 to the following experimental quantities:

Quantity	Our Value [MeV]	Experiment [MeV]
$M_{\sigma'}$	1191 ± 26	1200-1500
$M_{G'}$	1505 ± 6	1505 ± 6
$G' \to \pi\pi$	38 ± 5	38.04 ± 4.95
$G' \to \eta\eta$	5.3 ± 1.3	5.56 ± 1.34
$G' \to K\bar{K}$	9.3 ± 1.7	9.37 ± 1.69

$\chi^2/\text{d.o.f.} = 0.29$

$\Longrightarrow$ $\theta = (29.7 \pm 3.6)^o$ $\Longrightarrow$ $f_0(1500)$ is 76% glueball!

$\Longrightarrow$

Quantity	Our Value [MeV]	Experiment [MeV]
$G' \to \rho\rho \to 4\pi$	30	54.0 ± 7.1
$G' \to \eta\eta'$	0.6	2.1 ± 1.0
$\sigma' \to \pi\pi$	284 ± 43	325
$\sigma' \to \eta\eta$	72 ± 6	61.8 ± 22.8

$\Longrightarrow$ favored scenario!

(iii) Scenario III: $\sigma' = f_0(1370)$, $G' = f_0(1710)$

$\Longrightarrow$ large $\Gamma(G' \to 4\pi)$, not seen in experiment $\Longrightarrow$ unfavored scenario!

Vacuum phenomenology: The chiral partner of the nucleon (I)

Baryon sector ($N_f = 2$): S. Gallas, F. Giacosa, DHR, PRD 82 (2010) 014004

Determine m_0 , c_1 , c_2 , $\hat{g}_1$, $\hat{g}_2$ **through** χ^2 **fit to**

$$M_N\,,\ M_{N^*}\,,\ g_A = 1.267 \pm 0.004\,,\ g_A^*\,,\ \Gamma(N^* \to N\pi)$$

(i) Scenario A: $N = N(940),\ N^* = N(1535)$

$\Longrightarrow$ $g_A^* = 0.2 \pm 0.3$ T.T. Takahashi, T. Kunihiro, PRD 78 (2008) 011503

$\Gamma(N^* \to N\pi) = (67.5 \pm 23.6)$ MeV

(ii) Scenario B: $N = N(940),\ N^* = N(1650)$

$\Longrightarrow$ $g_A^* = 0.55 \pm 0.2$ T.T. Takahashi, T. Kunihiro, PRD 78 (2008) 011503

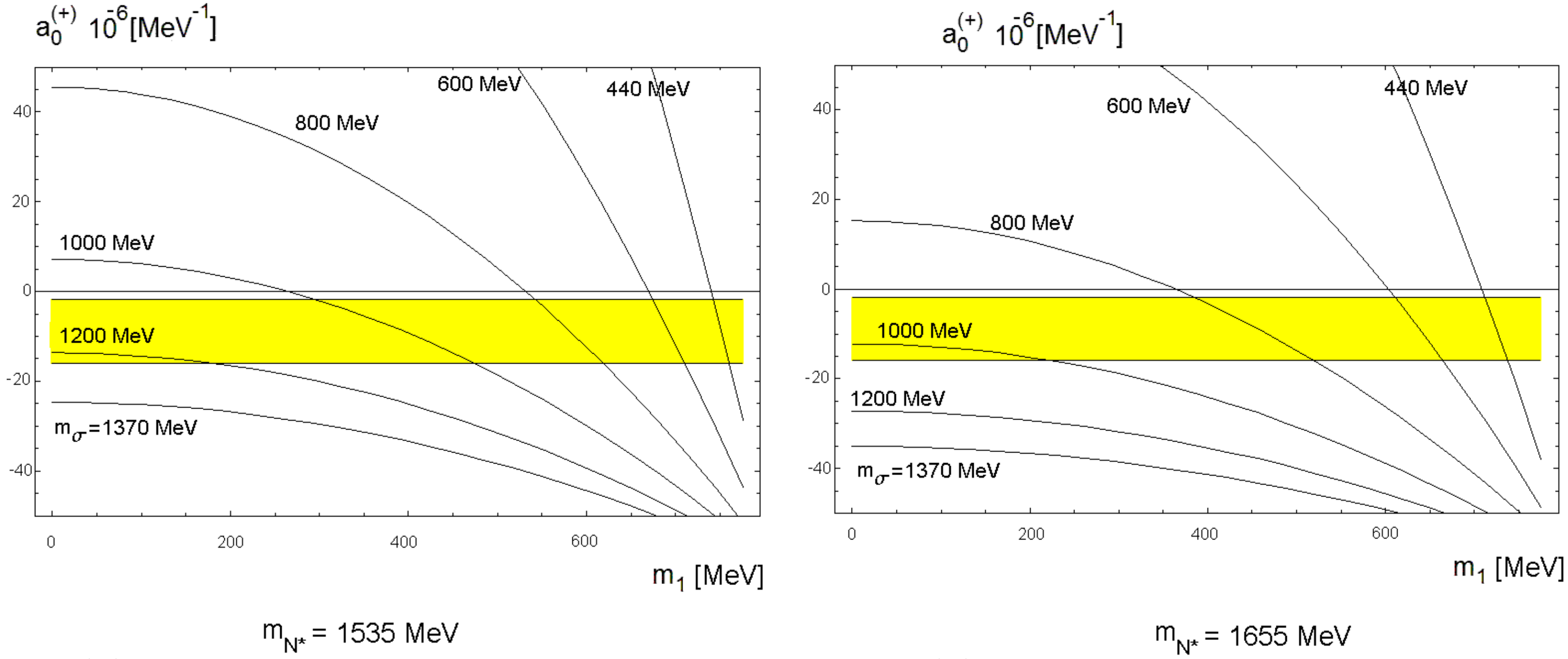
$\Gamma(N^* \to N\pi) = (128 \pm 44)$ MeV

Test validity of the two scenarios through comparison to:

- πN scattering lengths
- decay width $\Gamma(N^* \to N\eta)$

Vacuum phenomenology: The chiral partner of the nucleon (II)

πN scattering lengths $a_0^{(\pm)}$:



$a_0^{(-)} = (6.04 \pm 0.63) \cdot 10^{-4}$ MeV^{-1} $\qquad$ $a_0^{(-)} = (5.90 \pm 0.46) \cdot 10^{-4}$ MeV^{-1}

for comparison: $a_{0,\mathrm{exp}}^{(-)} = (6.4 \pm 0.1) \cdot 10^{-4}$ MeV^{-1}

However: $\Gamma(N^* \to N\eta) = (10.9 \pm 3.8)$ MeV $\qquad$ $\Gamma(N^* \to N\eta) = (18.3 \pm 8.5)$ MeV

$\Gamma_{\mathrm{exp}}(N^* \to N\eta) = (78.7 \pm 24.3)$ MeV! $\qquad$ $\Gamma_{\mathrm{exp}}(N^* \to N\eta) = (10.7 \pm 6.7)$ MeV

$\Longrightarrow$ Scenario B seems to be favored!

$\Longrightarrow$ But then: what is the chiral partner of $N(1535)$?

Conclusions

I. Linear σ model with $U(N_f)_r \times U(N_f)_\ell$ symmetry with scalar, vector mesons, baryons and their chiral partners

II. Vacuum phenomenology:

1. **The scalar meson puzzle: evidence for tetraquark assignment for the light scalar mesons**
2. **The chiral partner of the nucleon: is it $N(1650)$ instead of $N(1535)$?**
   $\Longrightarrow$ **$N_f = 3$ study mandatory!**

Outlook: Further studies

1. Vacuum:

(i) Extension to $N_f = 3$ D. Parganlija, P. Kovacs, Gy. Wolf

(ii) Extension to $N_f = 4$ W. Eshraim

(iii) Full scalar mixing scenario including $q\bar{q}$, tetraquark, and glueball states
S. Janowski

(iv) electroweak interactions, τ decay A. Habersetzer, 9:50 talk on Thursday!

(v) Δ resonance S. Gallas

(vi) NN scattering W. Deinet

(vii) HADES dilepton data for elementary NN collisions

2. Nonzero T, μ:

(i) $q\bar{q}-$tetraquark mixing
A. Heinz, S. Strüber, F. Giacosa, DHR, PRD 79 (2009) 037502

(ii) . . . dilepton rates for heavy-ion collisions