Investigation of ideal and viscous Mach Cones

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in collaboration with A. El, O. Fochler, H. Niemi, Z. Xu and C. Greiner

I. Bouras et al., Phys. Rev. Lett. 103:032301 (2009)

I. Bouras et al., PRC 82, 024910 (2010)

I. Bouras et al., in preparation

HGS-HIRe for FAIR
Helmholtz Graduate School for Hadron and Ion Research

HIC for FAIR
Helmholtz International Center

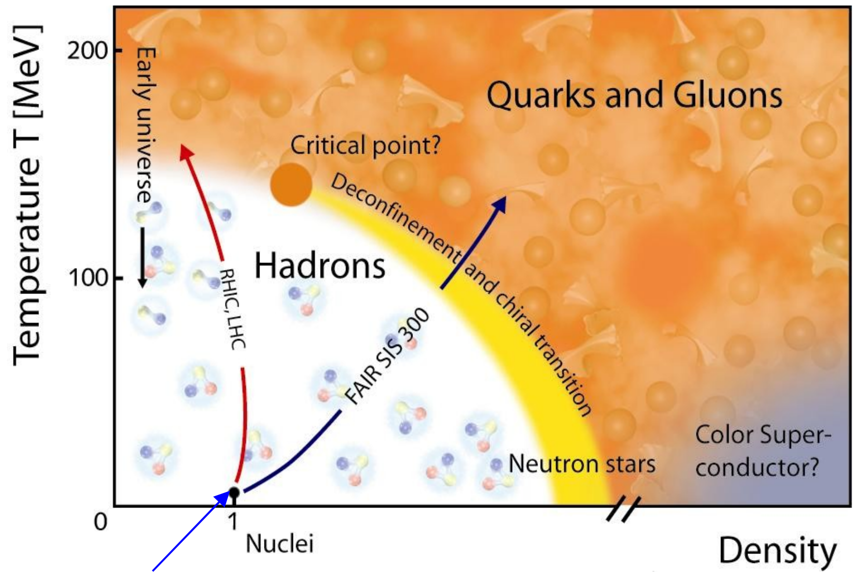
GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Toric Workshop
Heraklion, Crete, Greece

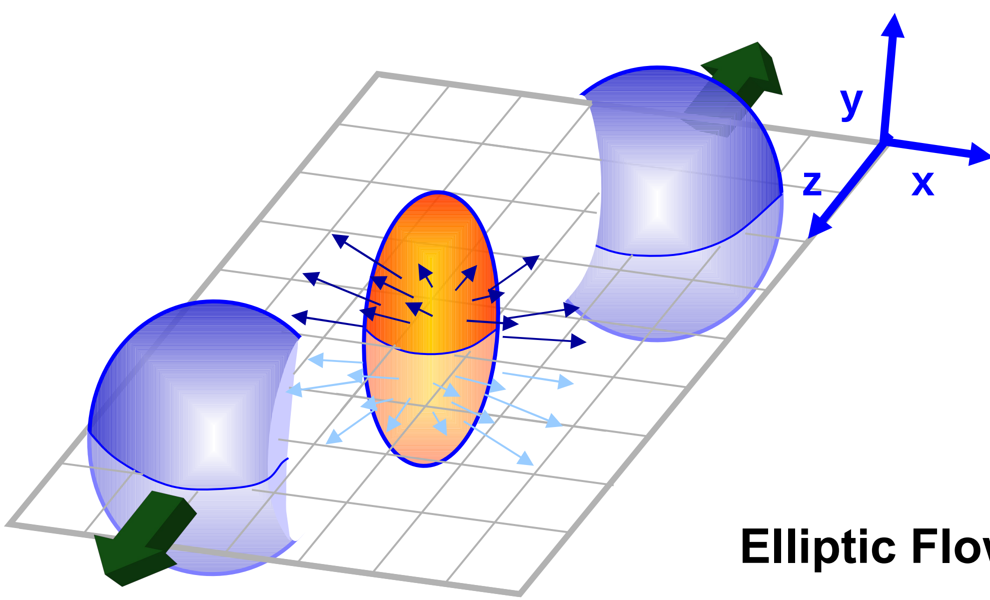
September, 2011

Motivation

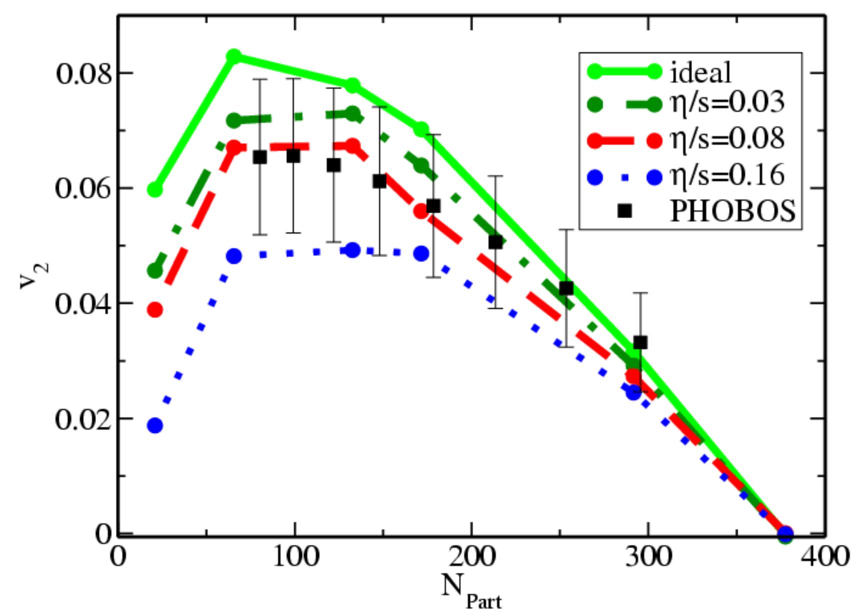
Phase diagram



ordinary world

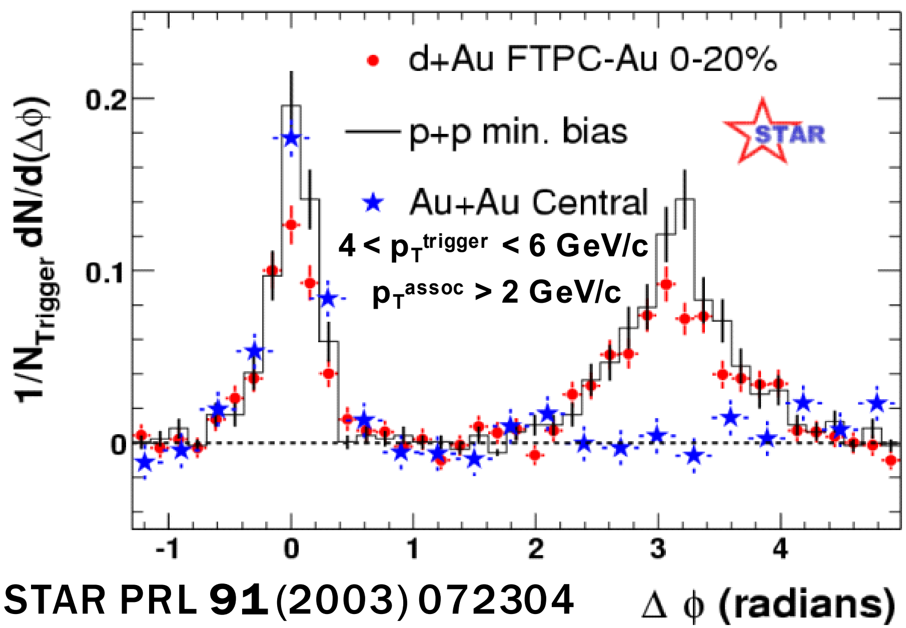


Elliptic Flow

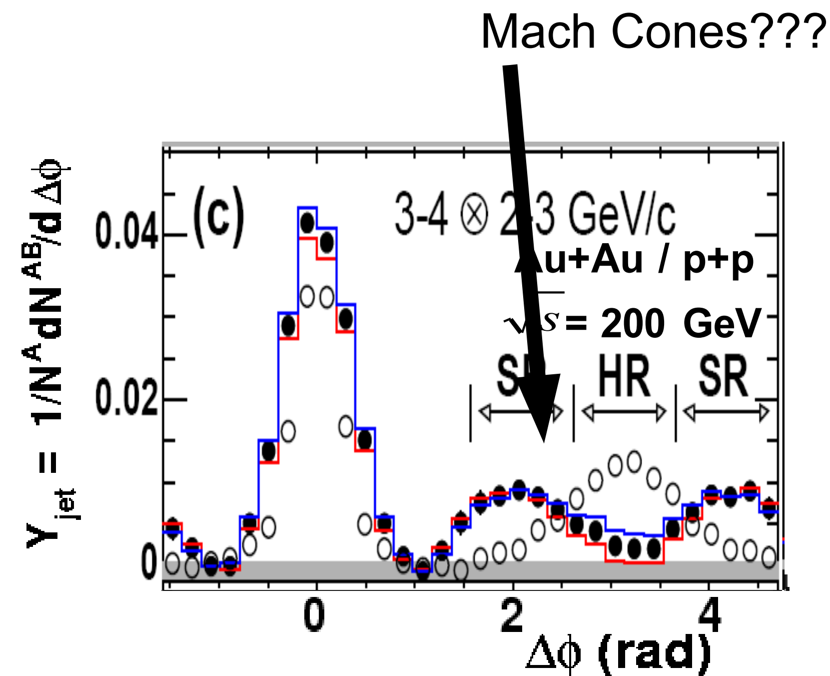


Motivation

Two-particle correlations



STAR PRL **91** (2003) 072304



PHENIX arXiv:0705.3238 [nucl-ex]

The Parton Cascade BAMPS

- Transport algorithm solving the **Boltzmann equation** using Monte Carlo techniques

$$p^{\mu} \partial_{\mu} f(x,p) = C_{22} + C_{23} + \ldots$$

- Stochastic interpretation of collision rates

$$P_{2i} = v_{rel} \frac{\sigma_{2i}}{N_{test}} \frac{\Delta t}{\Delta^3 x}$$

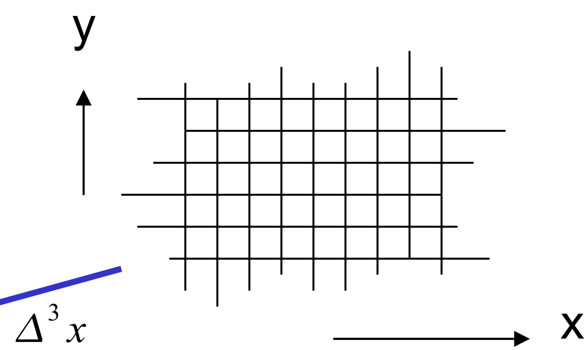
- In general:
  pQCD interactions, 2 $\leftrightarrow$ 3 processes,
  quarks and gluons

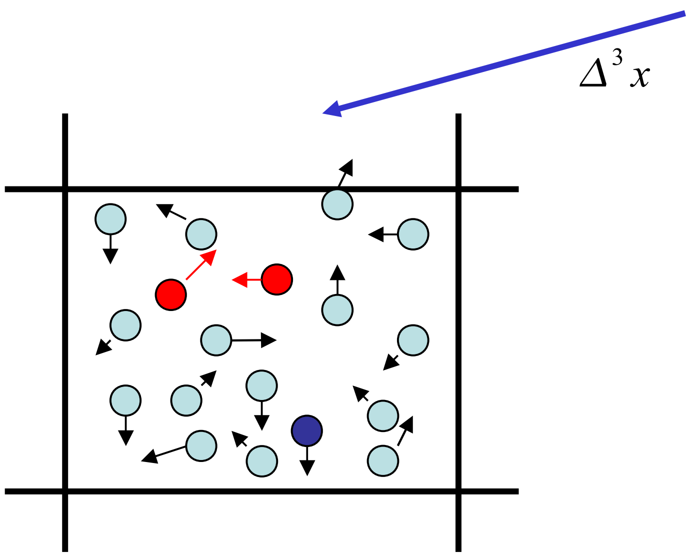
Boltzmann
Approach for
Multi-
Parton
Scatterings

Z. Xu & C. Greiner,
Phys. Rev. C 71 (2005) 064901

The Parton Cascade BAMPS

Space is divided into small cells !





Boltzmann
Approach for
Multi-
Parton
Scatterings

for $2 \rightarrow 2$ $\quad P_{22} = v_{rel} \dfrac{\sigma_{22}}{N_{test}} \dfrac{\Delta t}{\Delta^3 x}$

for $2 \rightarrow 3$ $\quad P_{23} = v_{rel} \dfrac{\sigma_{23}}{N_{test}} \dfrac{\Delta t}{\Delta^3 x}$

for $3 \rightarrow 2$ $\quad P_{32} = \dfrac{1}{8 E_1 E_2 E_3} \dfrac{I_{32}}{N_{test}^2} \dfrac{\Delta t}{(\Delta^3 x)^2}$

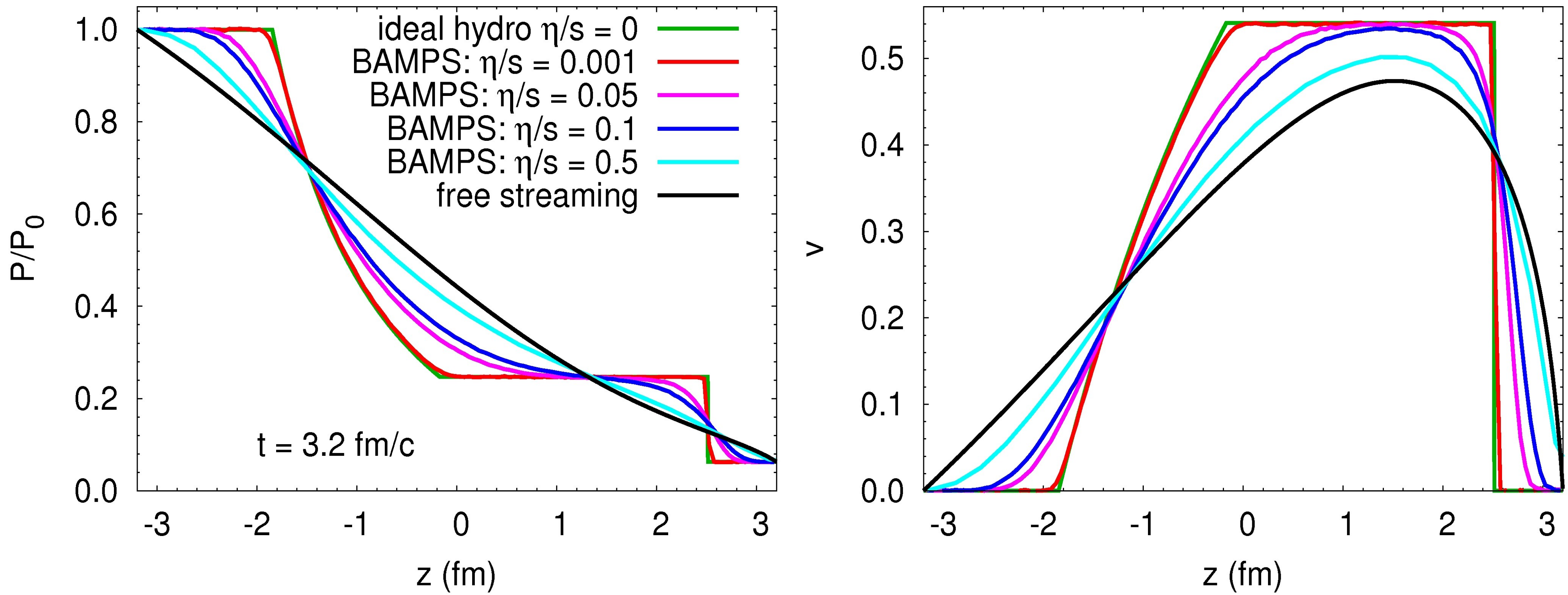
Z. Xu & C. Greiner,
Phys. Rev. C 71 (2005) 064901

$$I_{32} = \frac{1}{2} \int \frac{d^3 p'_1}{(2\pi)^3 2E'_1} \frac{d^3 p'_2}{(2\pi)^3 2E'_2} |M_{123 \rightarrow 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1 + p_2 + p_3 - p'_1 - p'_2)$$

The Relativistic Riemann Problem

Investigation of Shock Waves in one dimension

Boltzmann solution of the relativistic Riemann problem



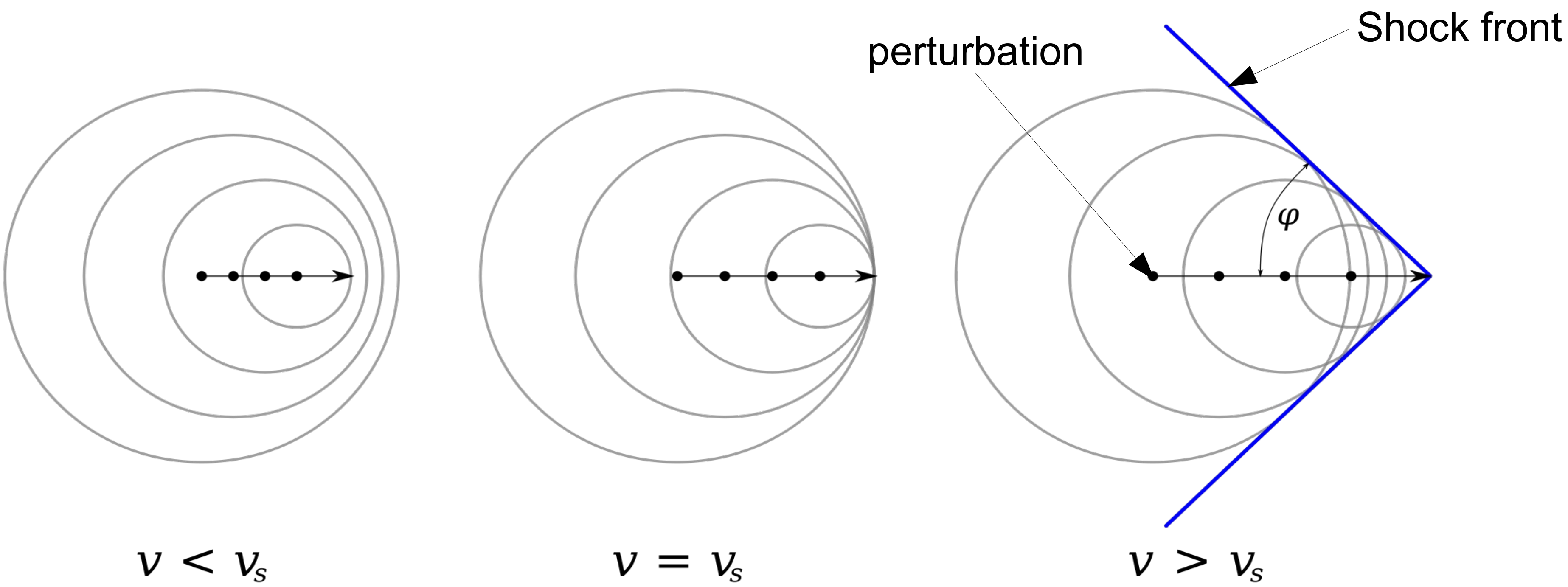
Transition from ideal hydro to free streaming

I. Bouras et al., **Phys. Rev. Lett. 103:032301 (2009)**

I. Bouras et al., **PRC 82, 024910 (2010)**

Mach Cones

- If source (perturbation) is propagating faster than the speed of sound, then a Mach Cone structure is observed



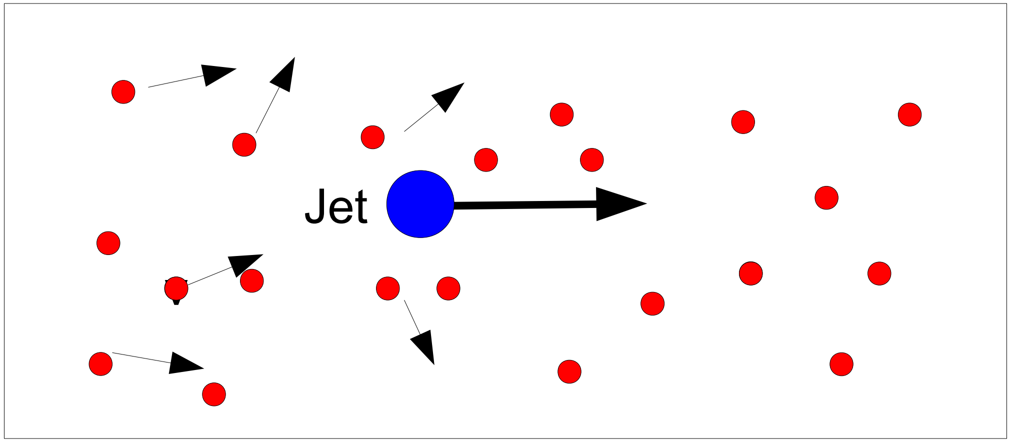
”Source” Terms

1) Punch Through Scenario

2) Pure energy deposition scenario

Punch Through Scenario

A scenario usefull to investigate the shape and development of ideal Mach Cones



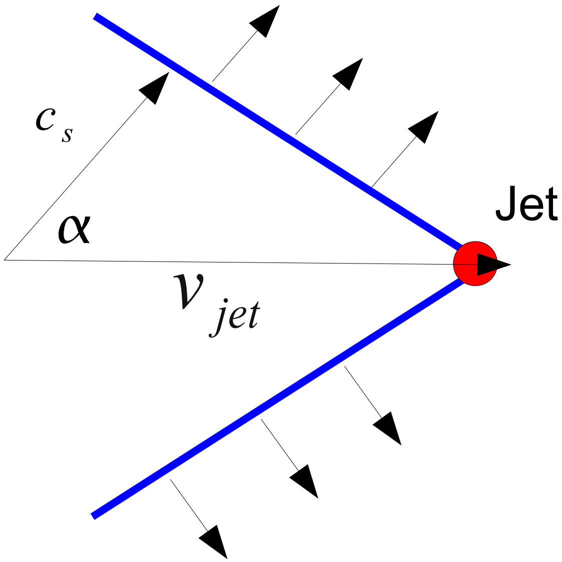
- Jet has finite initial energy and momentum E = pz and is massless;
  no transverse momentum → px = py = 0
- The Jet deposits energy to the medium due to binary collisions with particles
- After every collision with a thermal particle of the medium the energy of the jet gets recharged to its inital value

Movie:
Evolution of Mach Cones in BAMPS

Mach Cones in BAMPS

Mach angle dependence

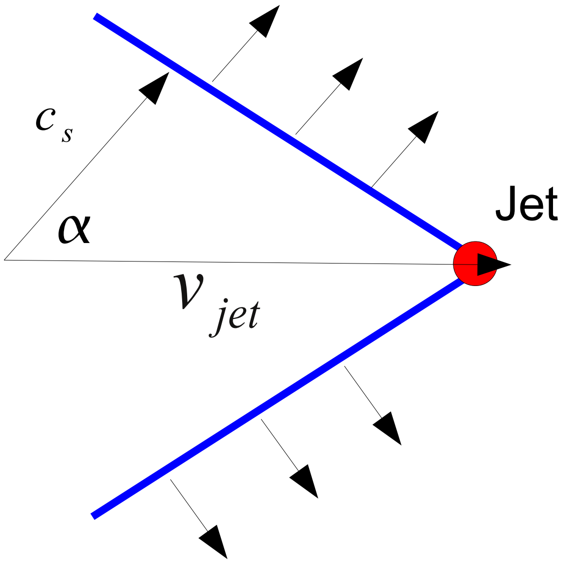
Scenario for a very weak perturbation



Mach Cones in BAMPS

Mach angle dependence

Scenario for a very weak perturbation



- In the case of a perfect fluid, i.e. $\eta = 0$, the Mach angle is

$$\alpha = \arccos \frac{c_s}{v_{jet}} \approx 54.7^o$$

for a massless Boltzmann gas, i.e. $e = 3P$, with $c_s = 1/\sqrt{3}$ and $v_{jet} = 1$

- This is only valid for small perturbation, i.e. energy of the jet is infinite small

Mach Cones in BAMPS

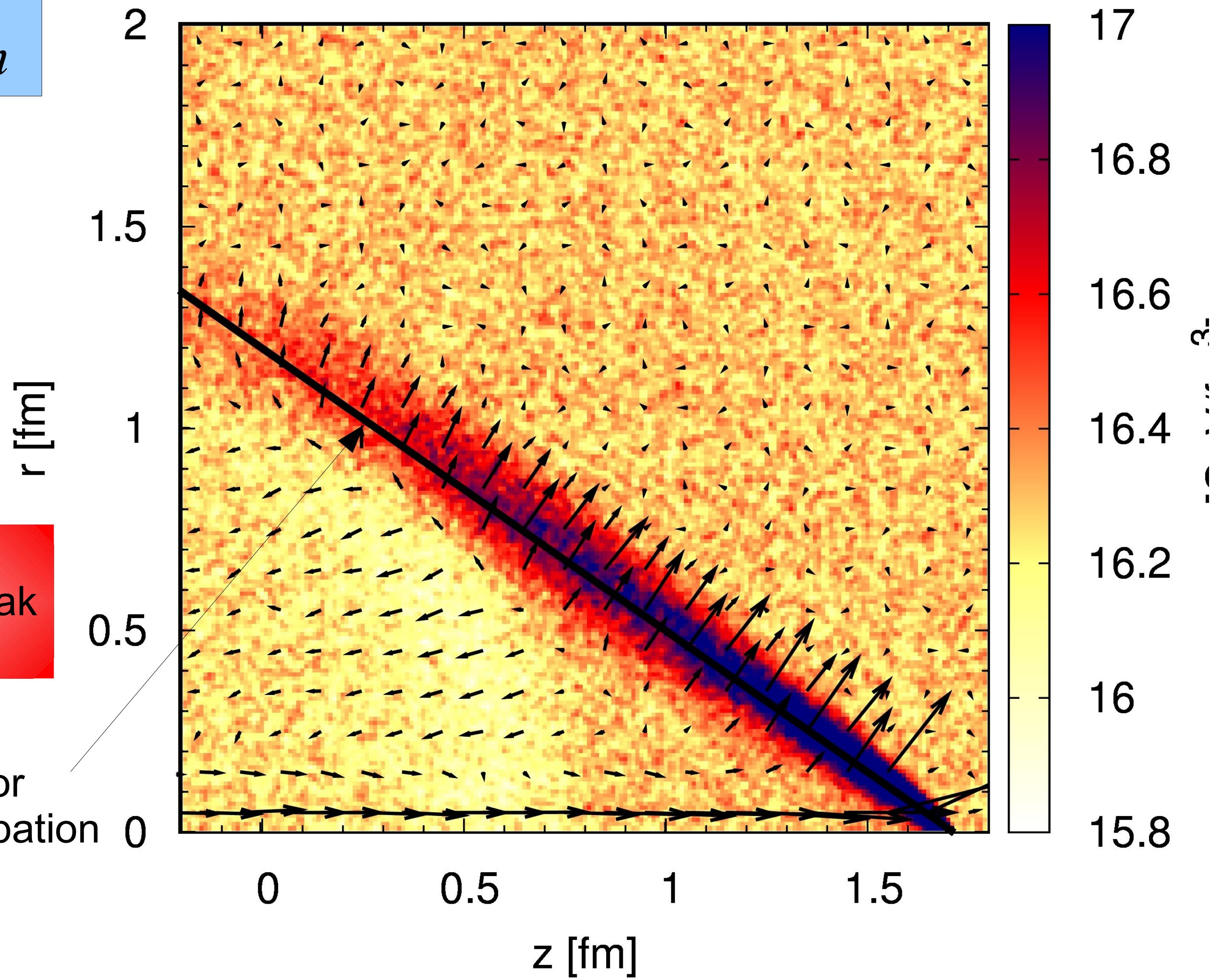
Mach angle dependence

Punch Through Scenario

$$E_{Jet} = 5\,GeV$$
$$dE/dx = 0.4\,GeV/fm$$

$$\eta/s = 0.005$$

BAMPS reproduces the ideal angle for a very weak perturbation

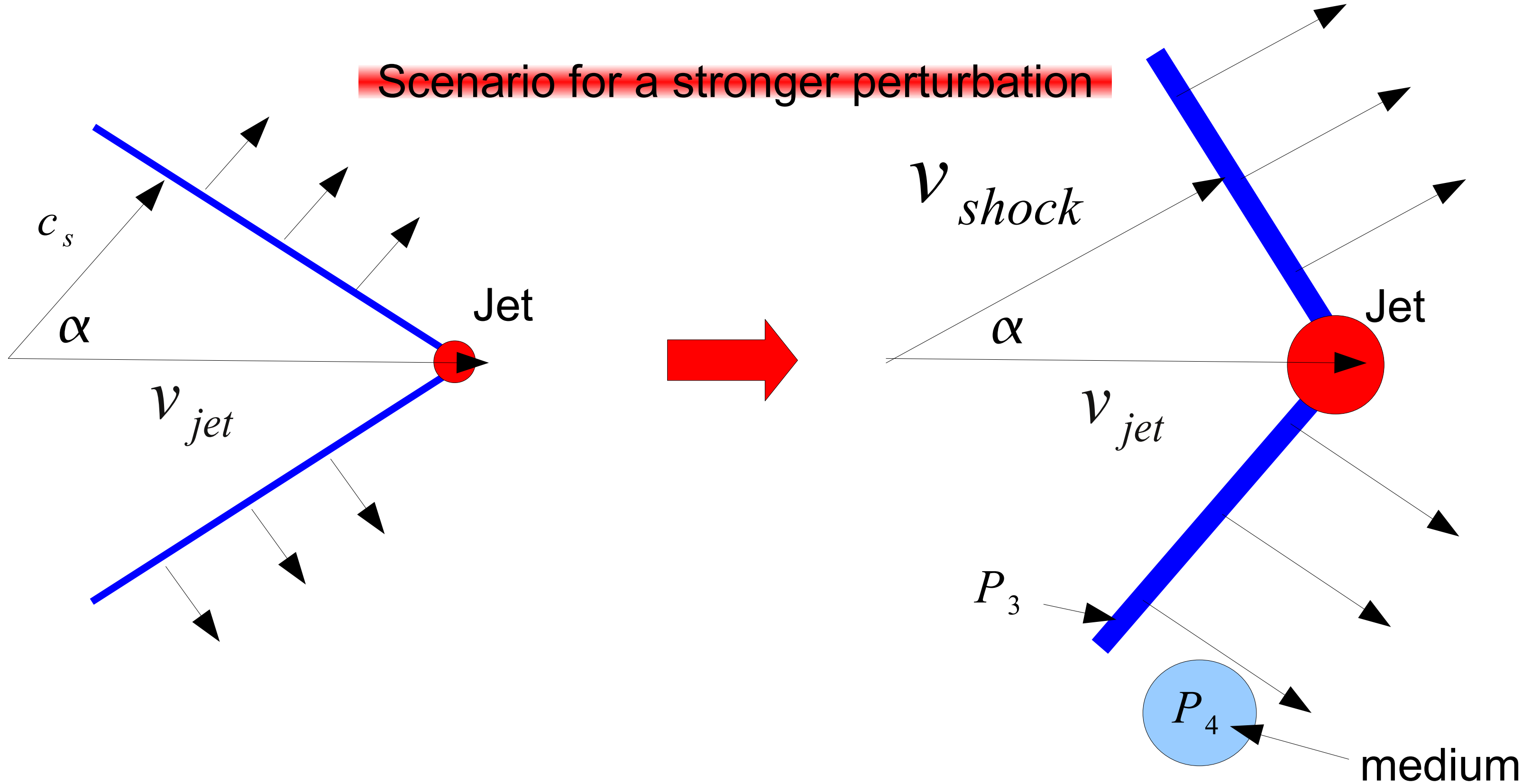


Ideal Mach angle for
a very weak perturbation

Mach Cones in BAMPS

Mach angle dependence

Scenario for a stronger perturbation



- In the case of a stronger perturbation the energy deposition is larger and therefore shock waves develop which exceed the speed of sound. Therefore the angle is approximately given by

$$\alpha = \arccos \frac{v_{shock}}{v_{jet}} \qquad v_{shock} = \left[\frac{(P_4 - P_3)(e_3 + P_4)}{(e_4 - e_3)(e_4 + P_3)} \right]^{\frac{1}{2}}$$

- The emission angle α changes to smaller values than in the weak perturbation case

Mach Cones in BAMPS

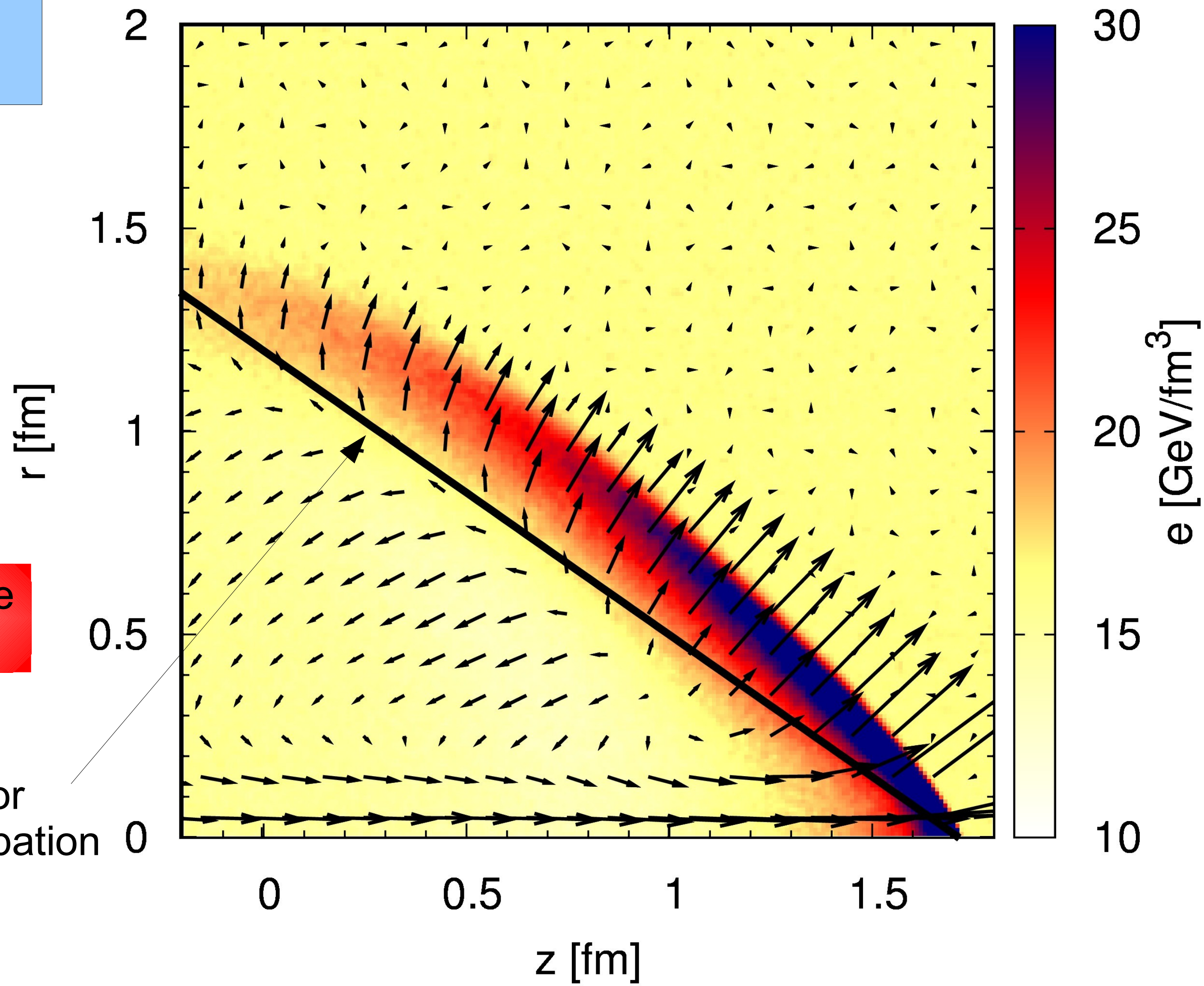
Mach angle dependence

Punch Through Scenario

$$E_{Jet} = 200\,GeV$$
$$dE/dx = 11\,GeV/fm$$

$$\eta/s = 0.005$$

Shock front exceeds the speed of sound



Mach Cones in BAMPS

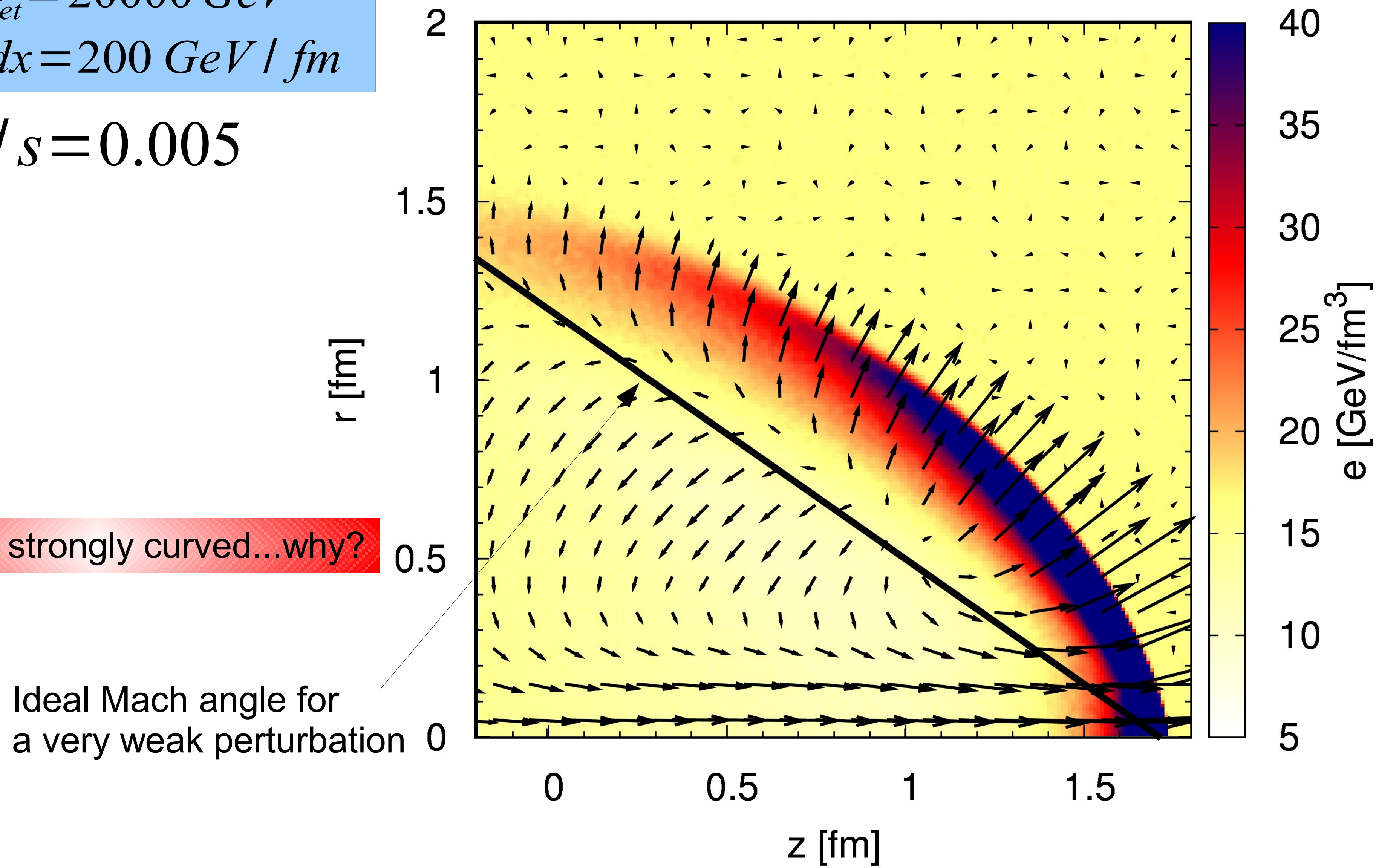
Mach angle dependence

Punch Through Scenario

$$E_{Jet} = 20000\, GeV$$
$$dE/dx = 200\, GeV/fm$$

$$\eta/s = 0.005$$

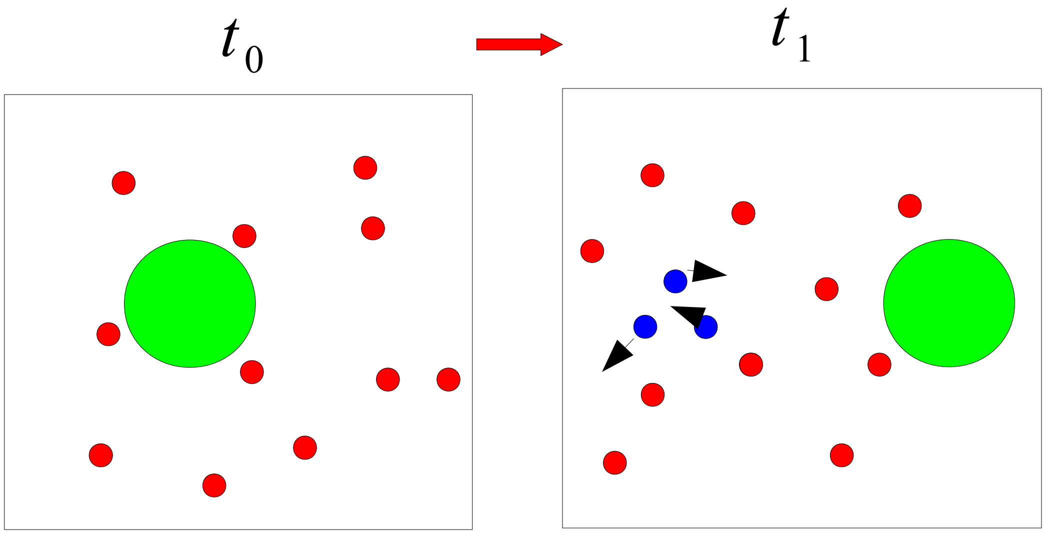
Profile strongly curved...why?



Mach Cones in BAMPS

Pure energy deposition scenario

Energy deposition via the creation of thermal distributed particles



- The source (green) propagates with the speed of light and generates new particles (blue) at different timesteps
- The advantage of that method: a constant energy deposition but no momentum deposition, because new particles are thermal distributed

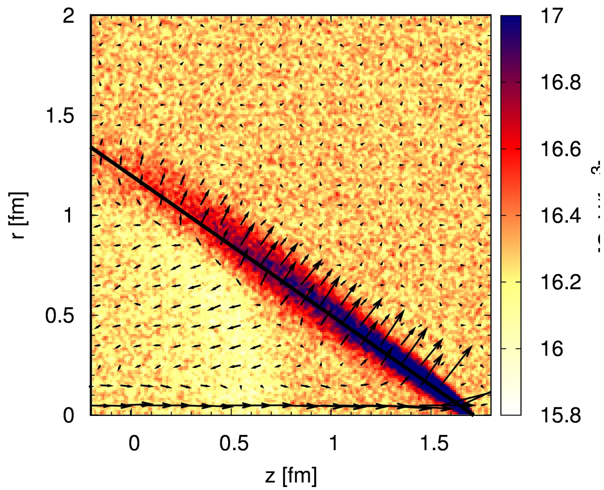
$$\longrightarrow f_{ped}(x,p)=e^{-E/T}$$

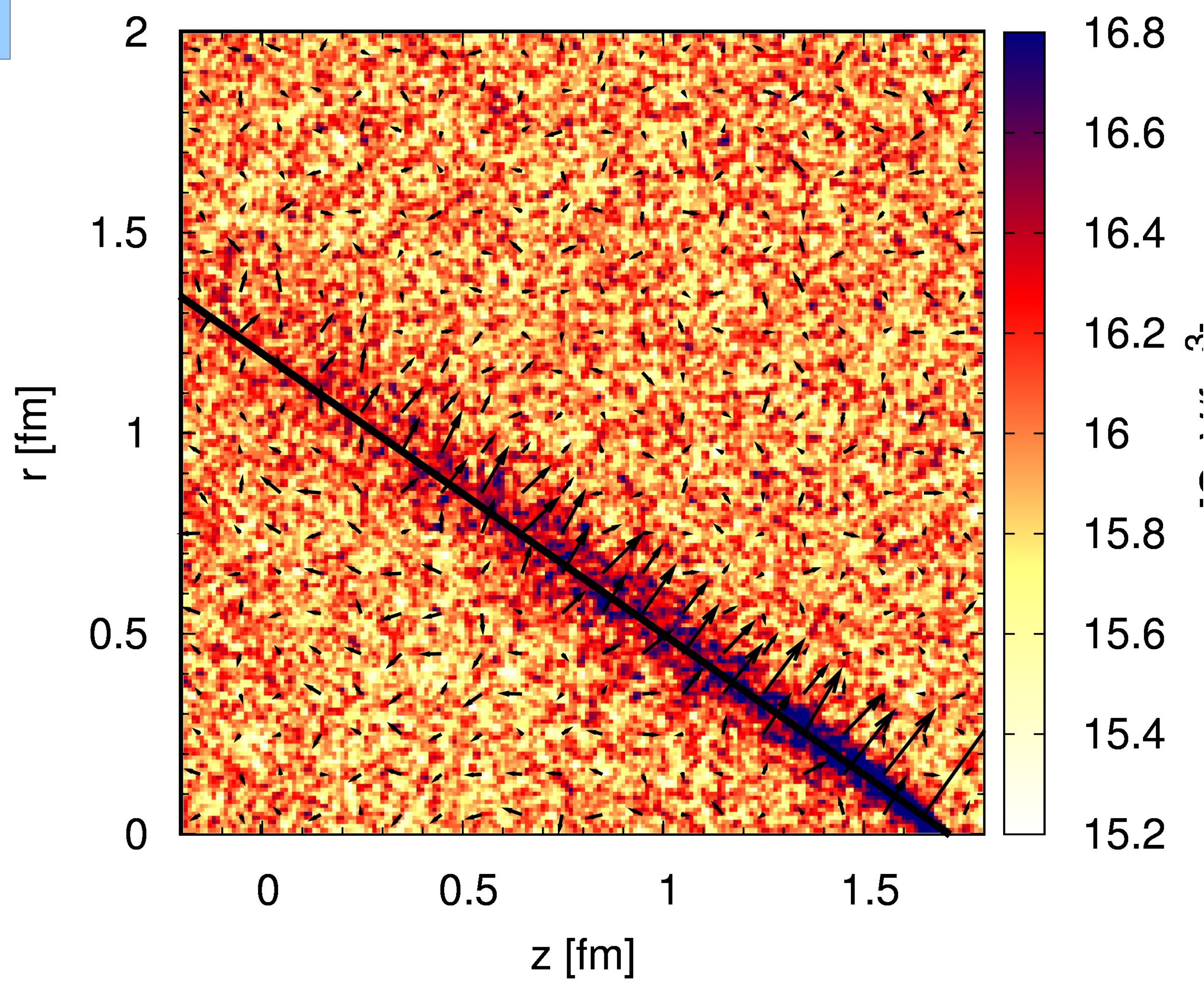
Mach Cones in BAMPS

Mach Angle Dependence
Pure energy deposition scenario

$dE/dx = 1\,GeV/fm$

$\eta/s = 0.005$



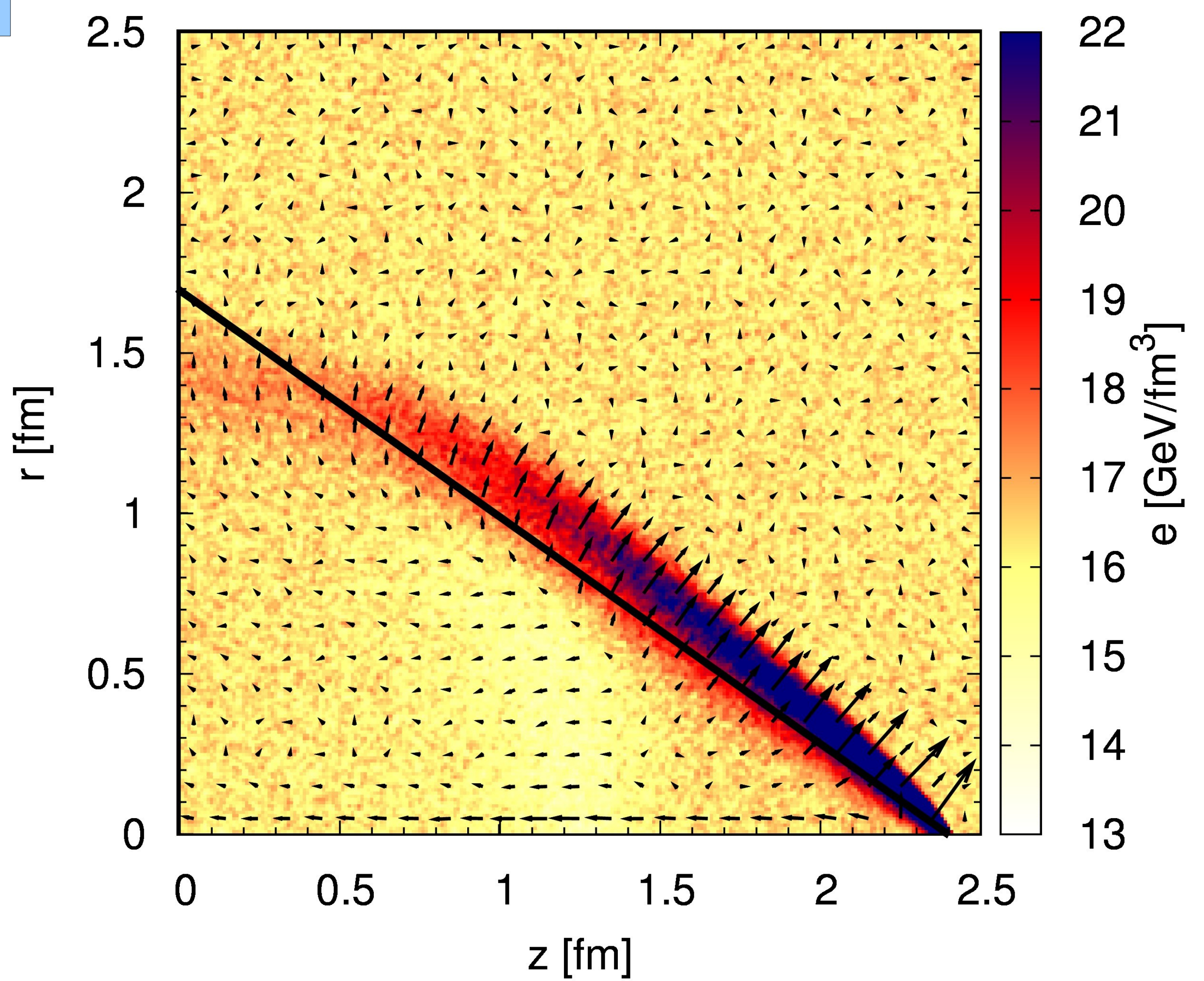


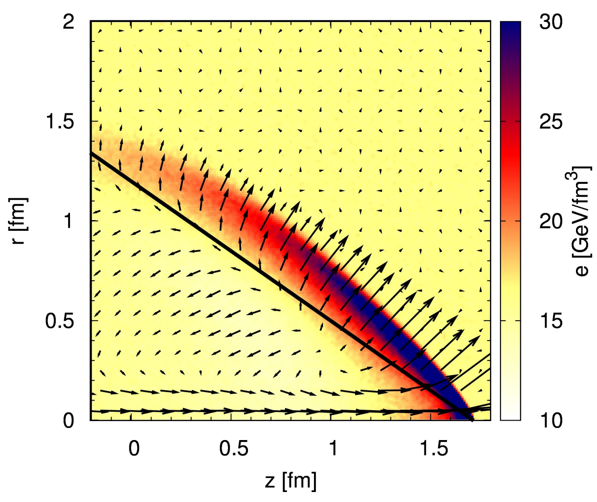
Mach Cones in BAMPS

Mach Angle Dependence
Pure energy deposition scenario

$$dE/dx = 10\,GeV/fm$$

$$\eta/s = 0.005$$





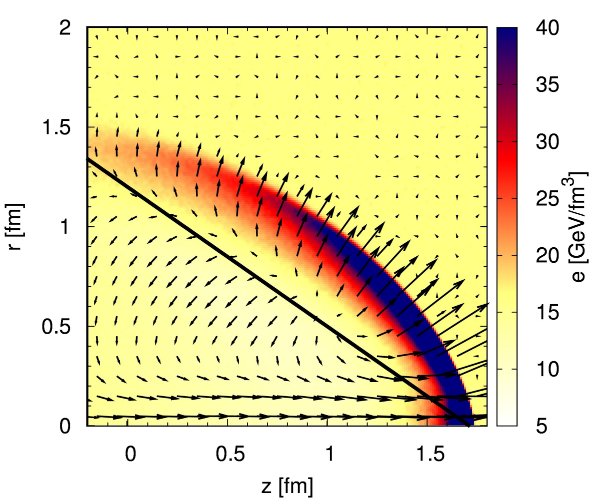
Mach Cones in BAMPS

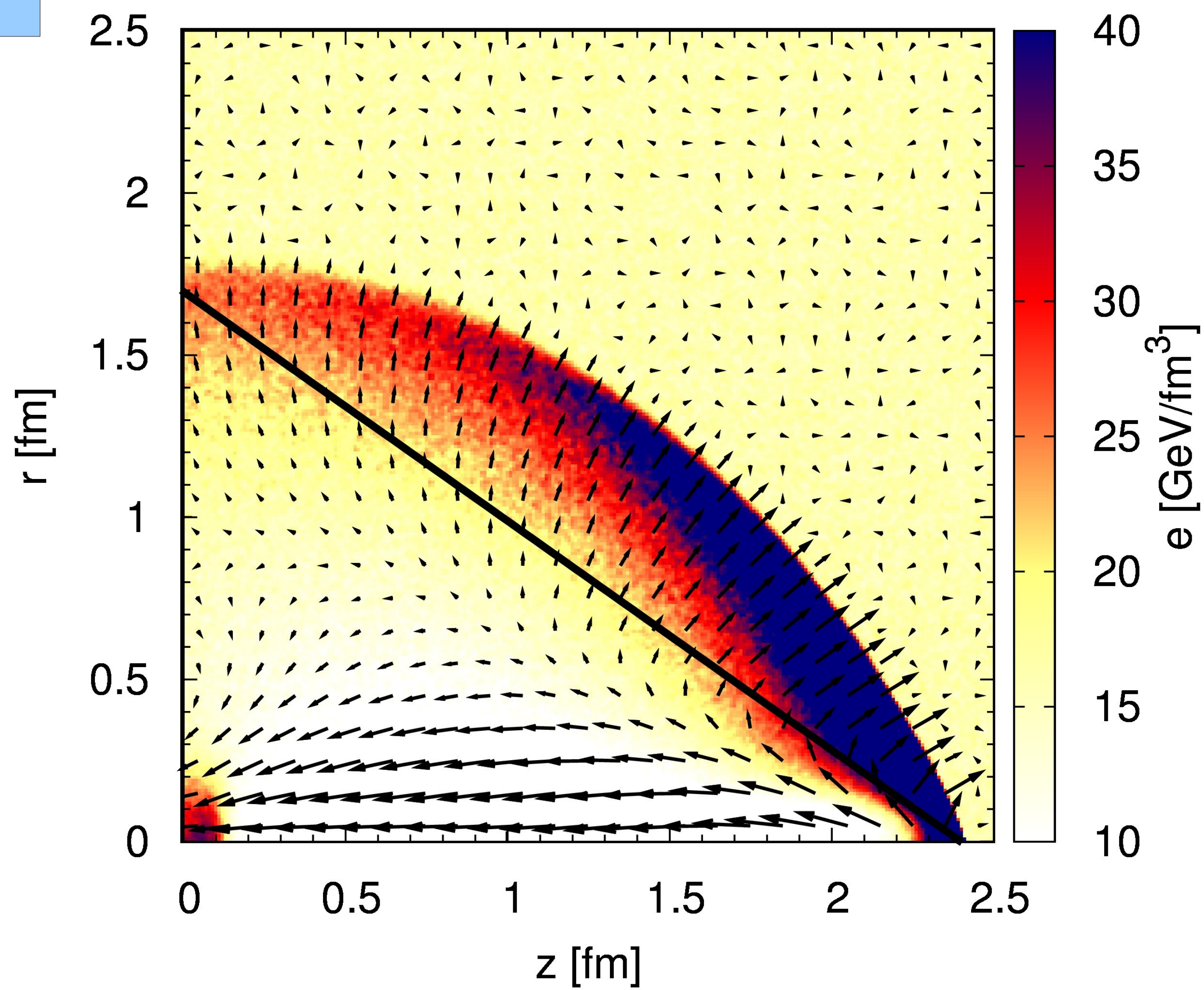
Mach Angle Dependence
Pure energy deposition scenario

$$dE/dx = 200\ GeV/fm$$

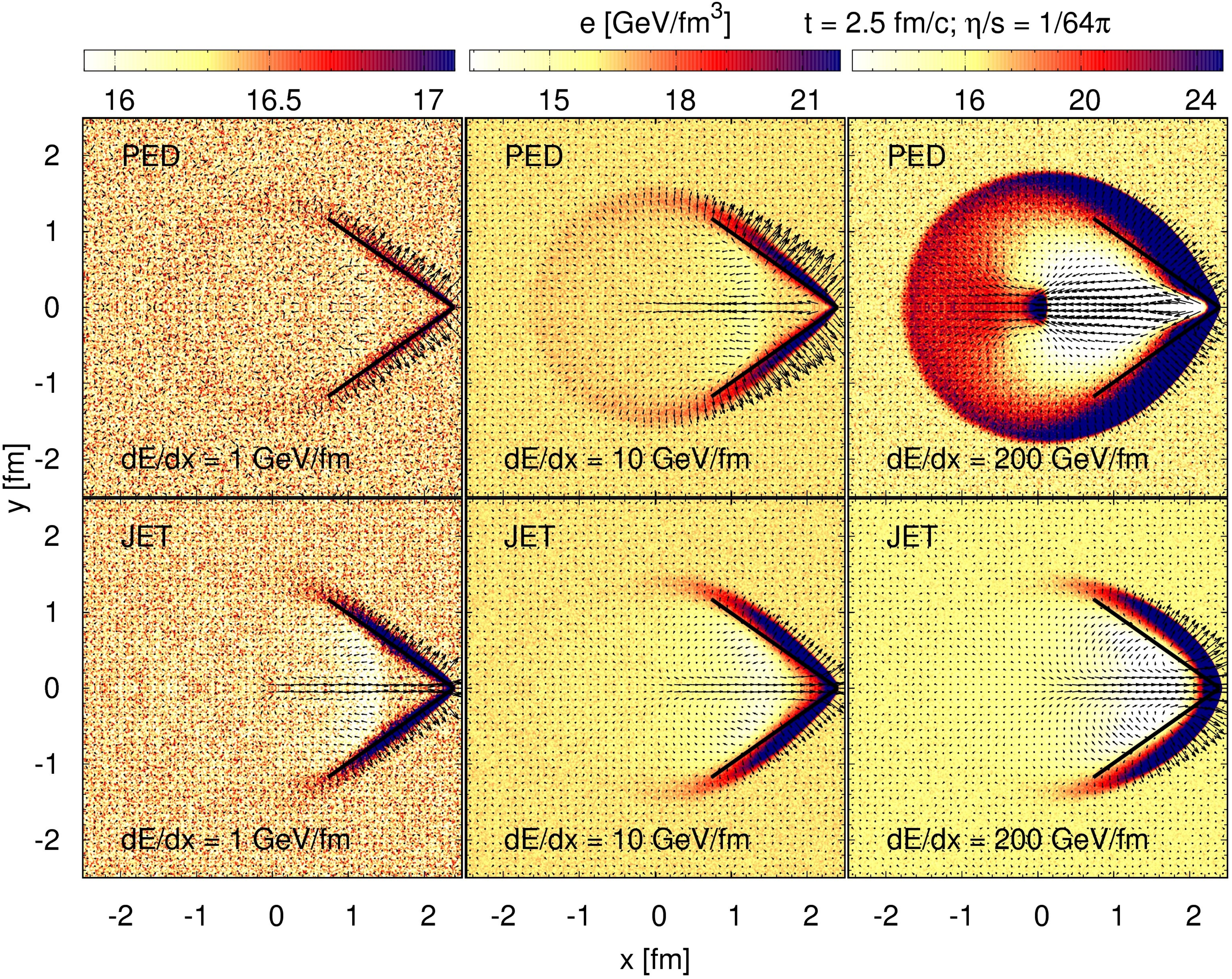
$$\eta/s = 0.005$$

Angle depends for sure
On the energy deposition!!!





Movie:
Evolution of Mach Cones
in BAMPS
Pure energy deposition scenario



e [GeV/fm^3]
t = 2.5 fm/c; η/s = 1/64π
16
16.5
17
15
18
21
16
20
24
PED
dE/dx = 1 GeV/fm
PED
dE/dx = 10 GeV/fm
PED
dE/dx = 200 GeV/fm
JET
dE/dx = 1 GeV/fm
JET
dE/dx = 10 GeV/fm
JET
dE/dx = 200 GeV/fm
y [fm]
x [fm]
2
1
0
-1
-2

Mach Cones in BAMPS

Transition from ideal to viscous

Punch Through Scenario

$$\eta / s = 0.005$$

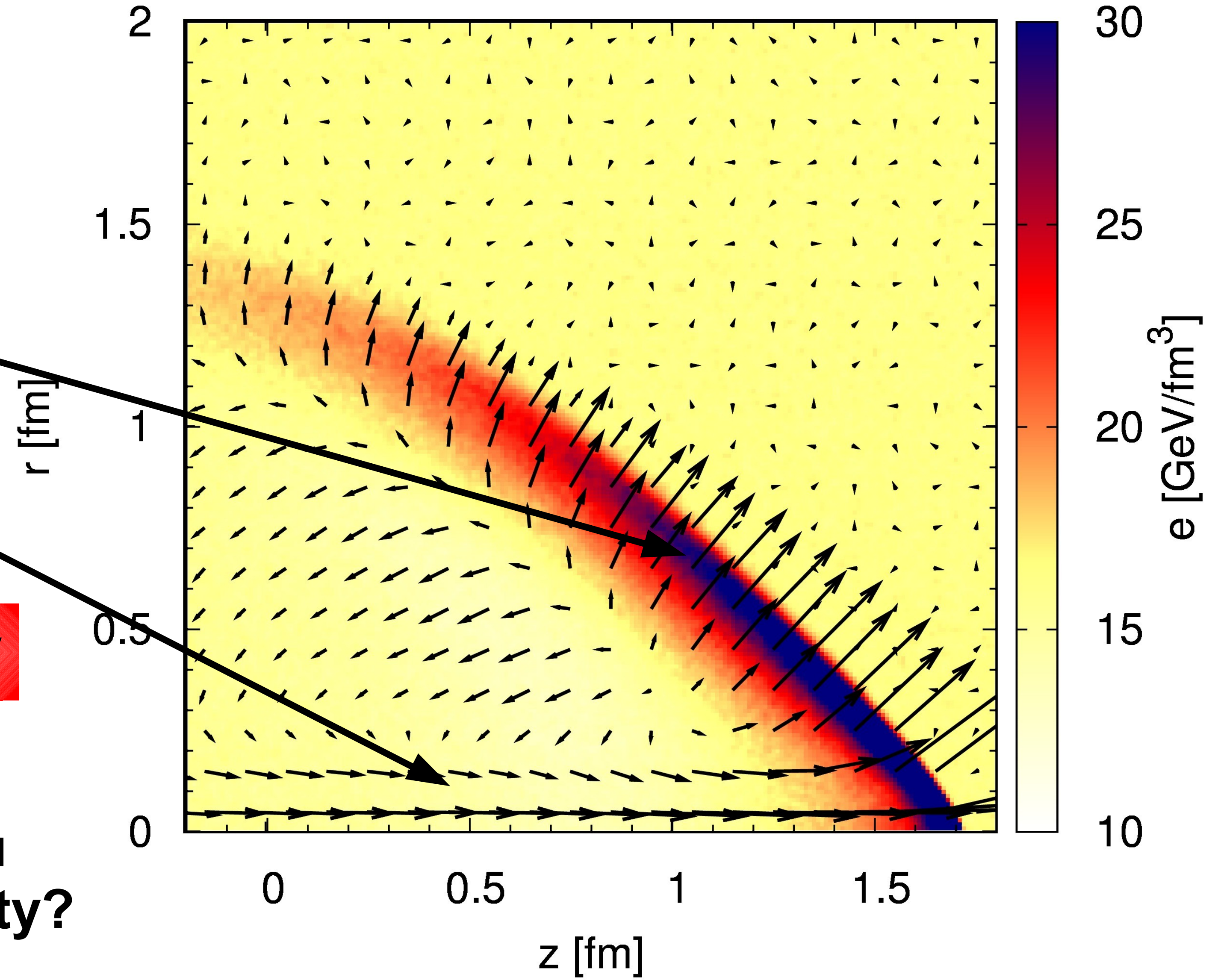
$$E_{Jet} = 200\,GeV$$

$$dE/dx = 11\,GeV/fm$$

Shock front

Diffusion Wake

Small Viscosity



What happens if you increase the viscosity?

Mach Cones in BAMPS

Transition from ideal to viscous

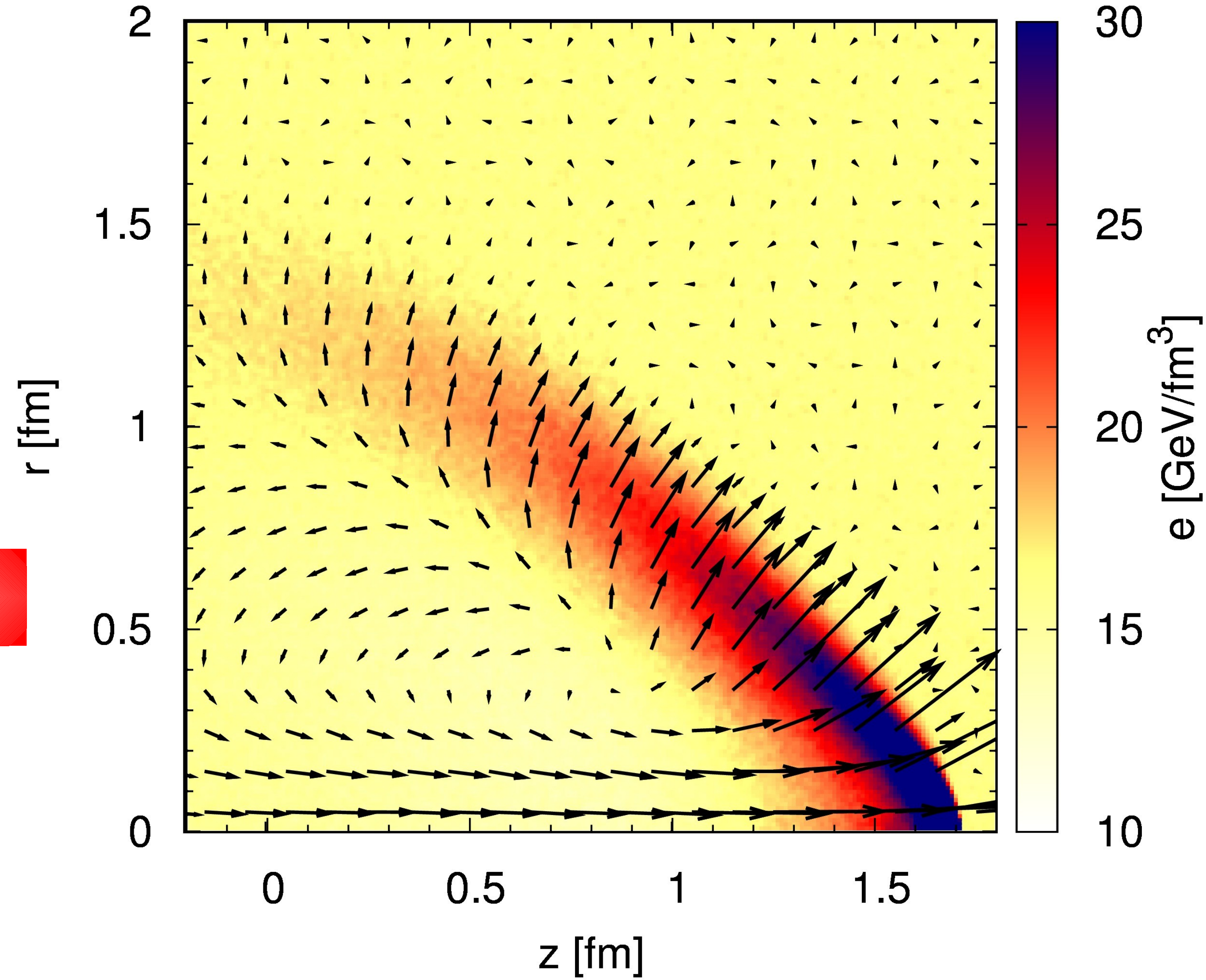
Punch Through Scenario

$\eta / s = 0.025$

$E_{Jet} = 200\,GeV$

$dE / dx = 11\,GeV / fm$

Larger Viscosity



Mach Cones in BAMPS

Transition from ideal to viscous

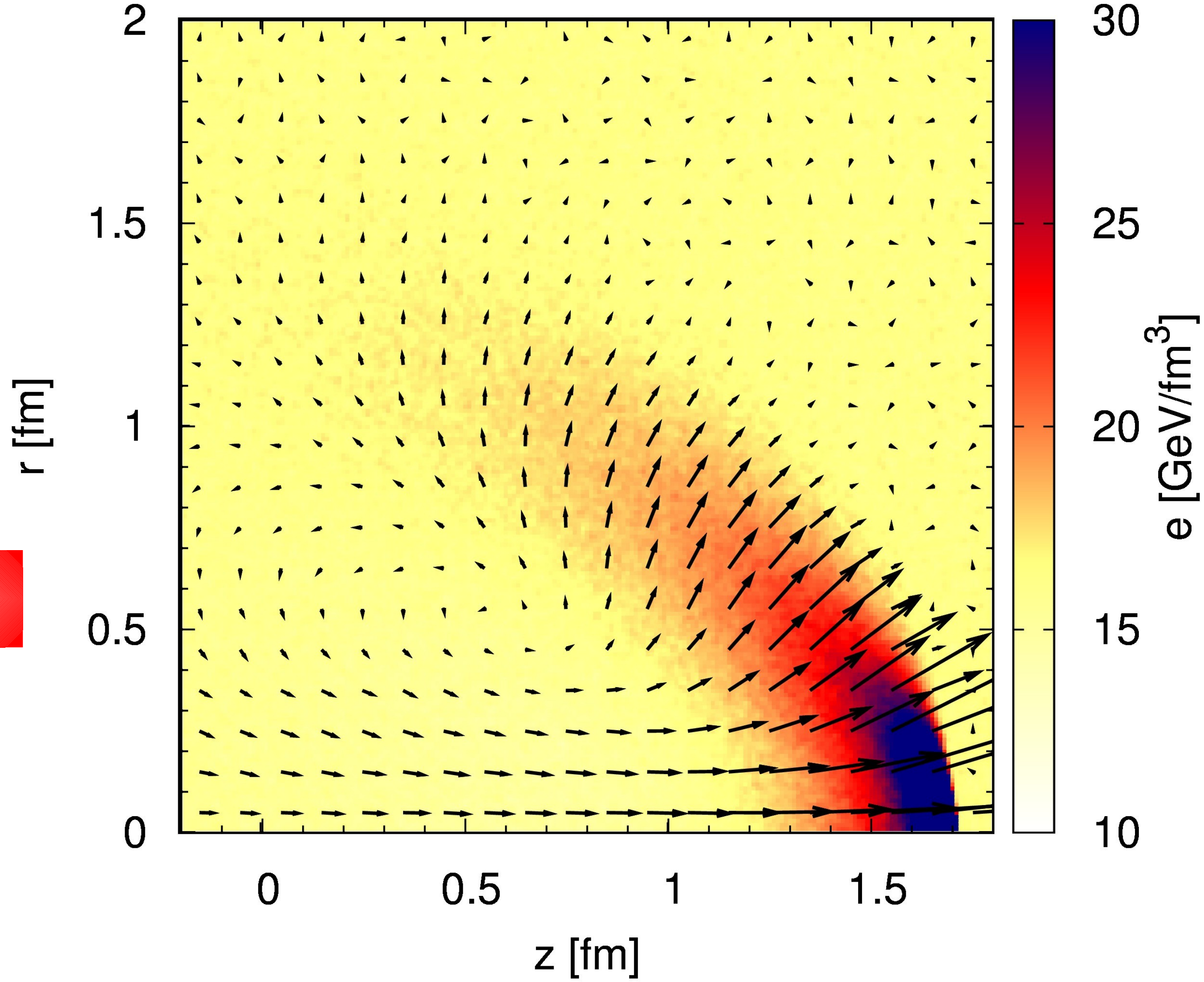
Punch Through Scenario

$\eta / s = 0.08$

$E_{Jet} = 200\, GeV$

$dE / dx = 11\, GeV / fm$

Larger Viscosity



Mach Cones in BAMPS

Transition from ideal to viscous

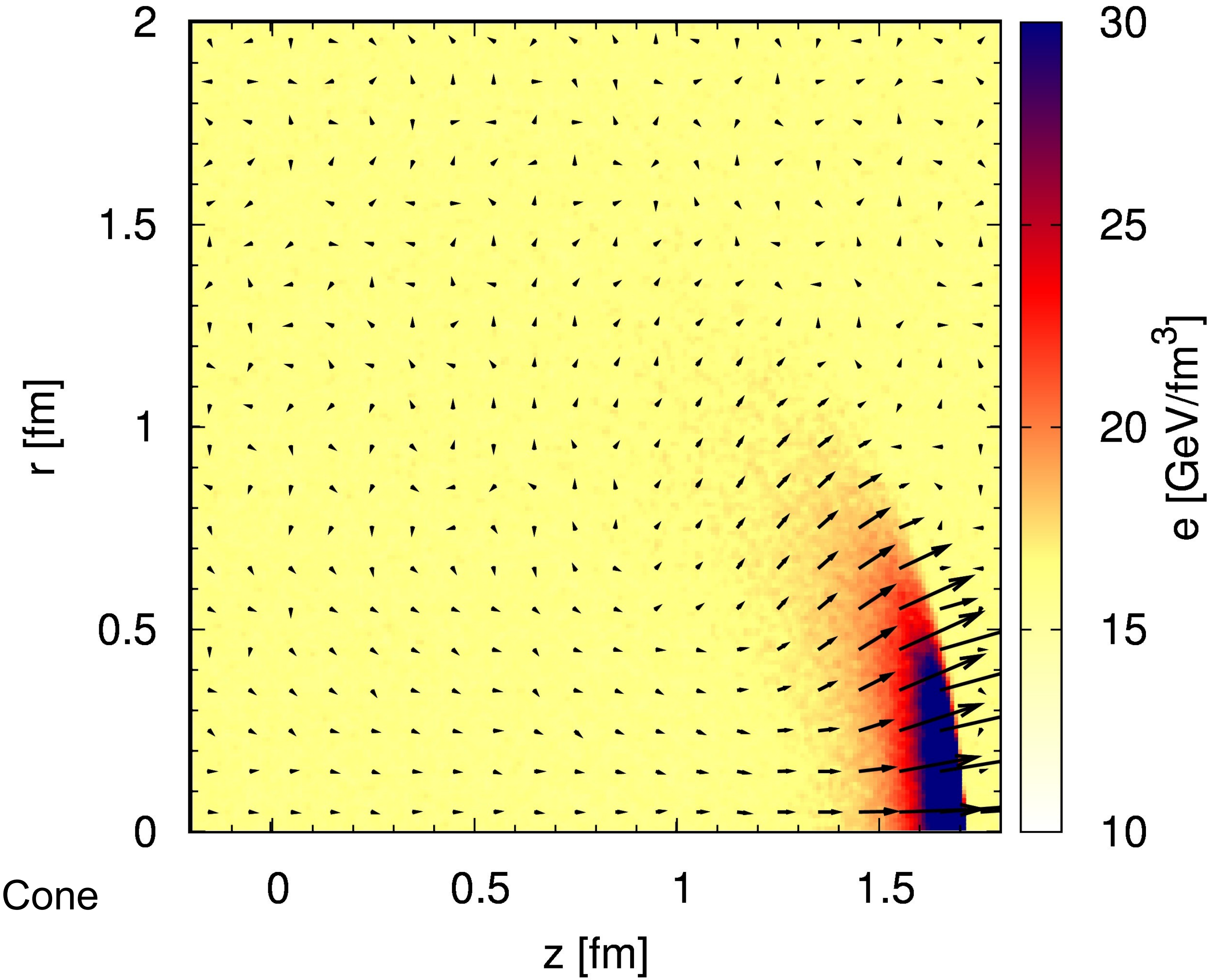
Punch Through Scenario

$$\eta / s = 0.32$$

$$E_{Jet} = 200\, GeV$$

$$dE / dx = 11\, GeV / fm$$

High Viscosity



Should the angle of the Mach Cone change with viscosity?

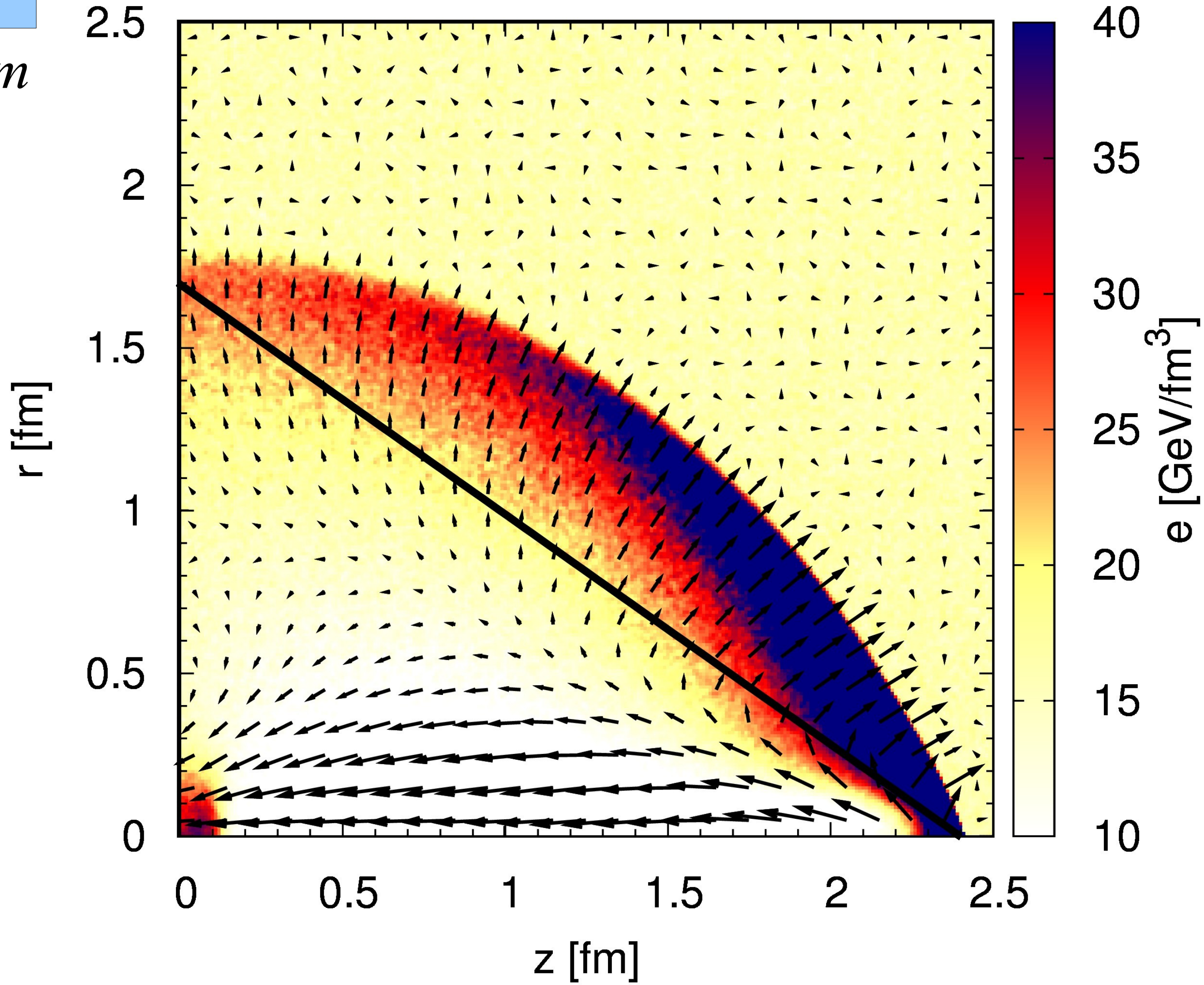
Mach Cones in BAMPS

Mach Angle Dependence

Pure energy deposition scenario

$$\eta / s = 0.005$$

$$dE / dx = 200\ GeV / fm$$

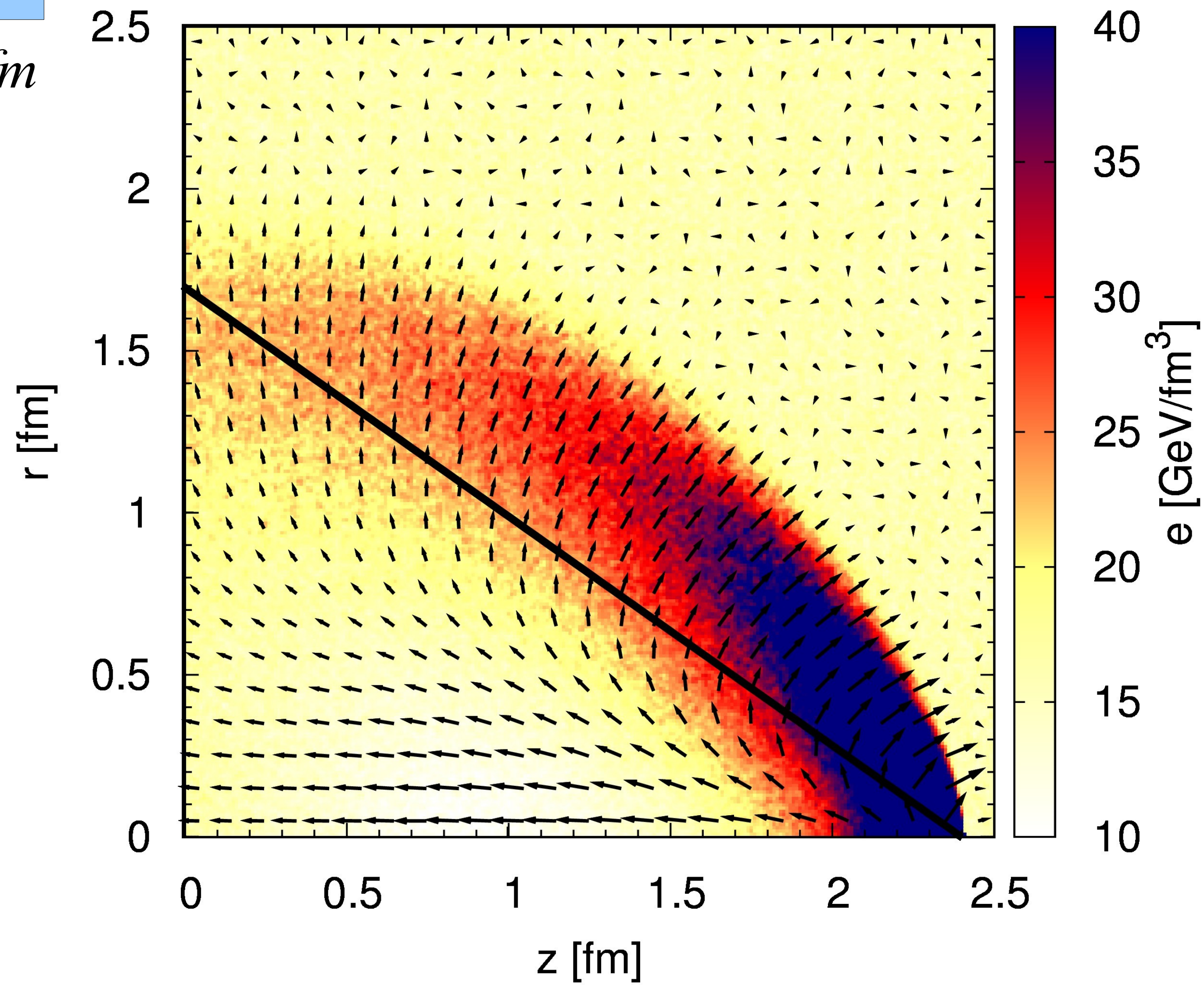


Mach Cones in BAMPS

Mach Angle Dependence
Pure energy deposition scenario

$\eta / s = 0.05$

$dE / dx = 200\ GeV / fm$

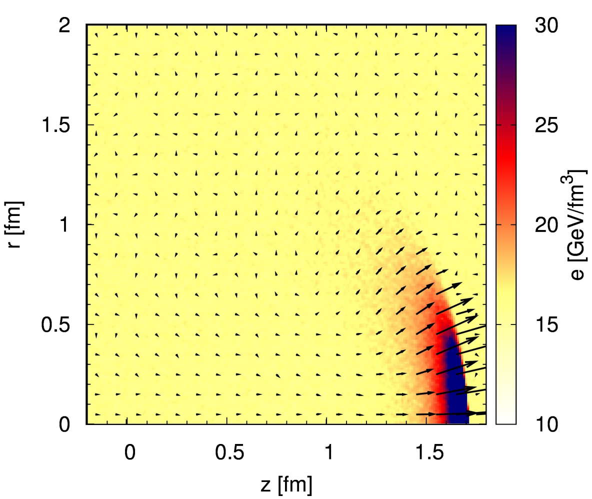


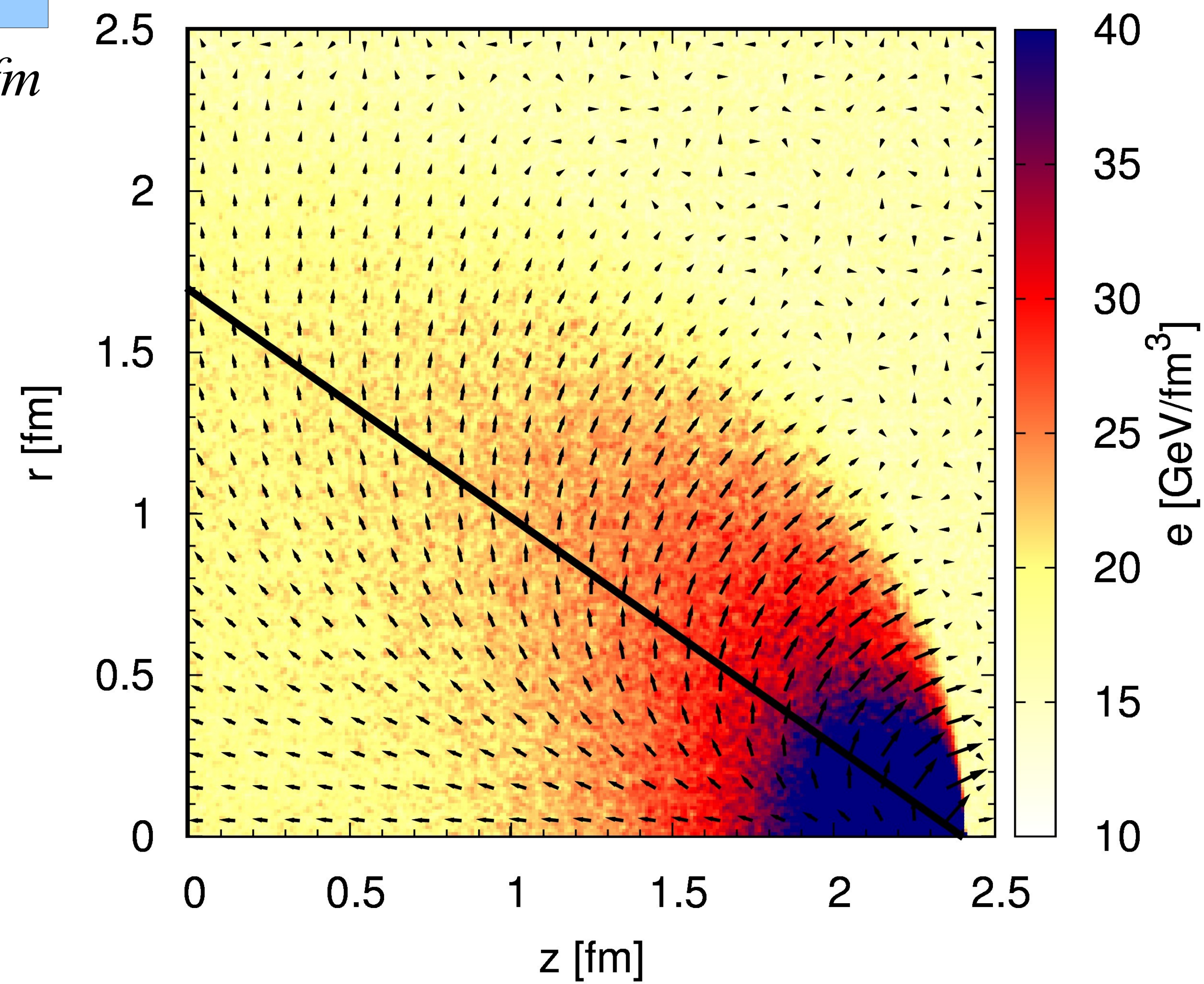
Mach Cones in BAMPS

Mach Angle Dependence
Pure energy deposition scenario

$$\eta / s = 0.5$$

$$dE / dx = 200\ GeV / fm$$

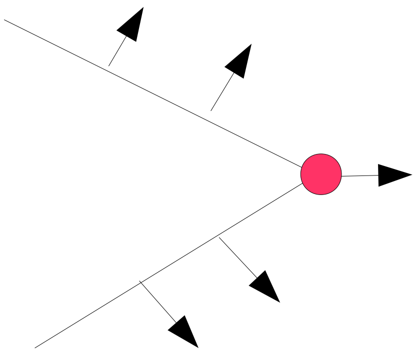


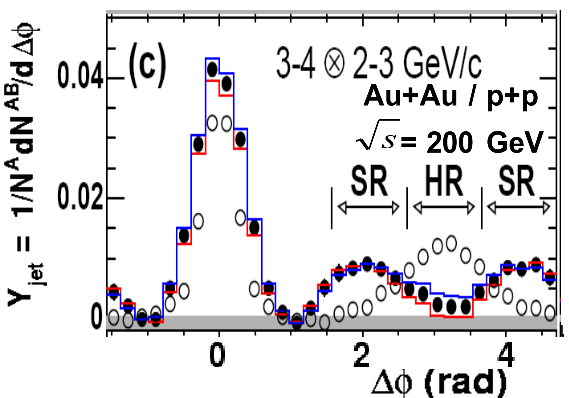


Mach Cones in BAMPS

Two Particle Correlations

- First, we (have) expect(ed) that the double peak observed in experimental data is a hint for a conical structure...because of the naive picture

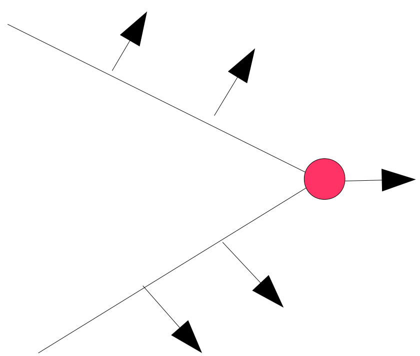


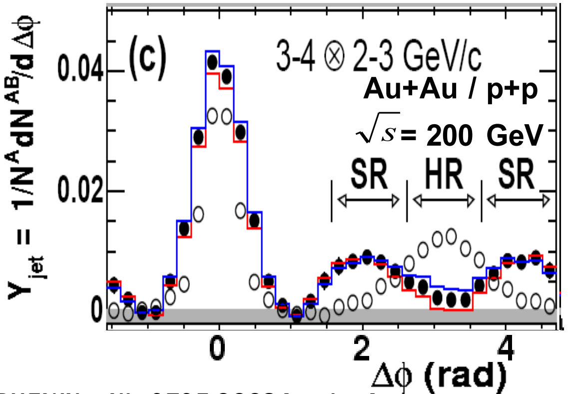


PHENIX arXiv:0705.3238 [nucl-ex]

Mach Cones in BAMPS
Two Particle Correlations

- First, we (have) expect(ed) that the double peak observed in experimental data is a hint for a conical structure...because of the naive picture





PHENIX arXiv:0705.3238 [nucl-ex]

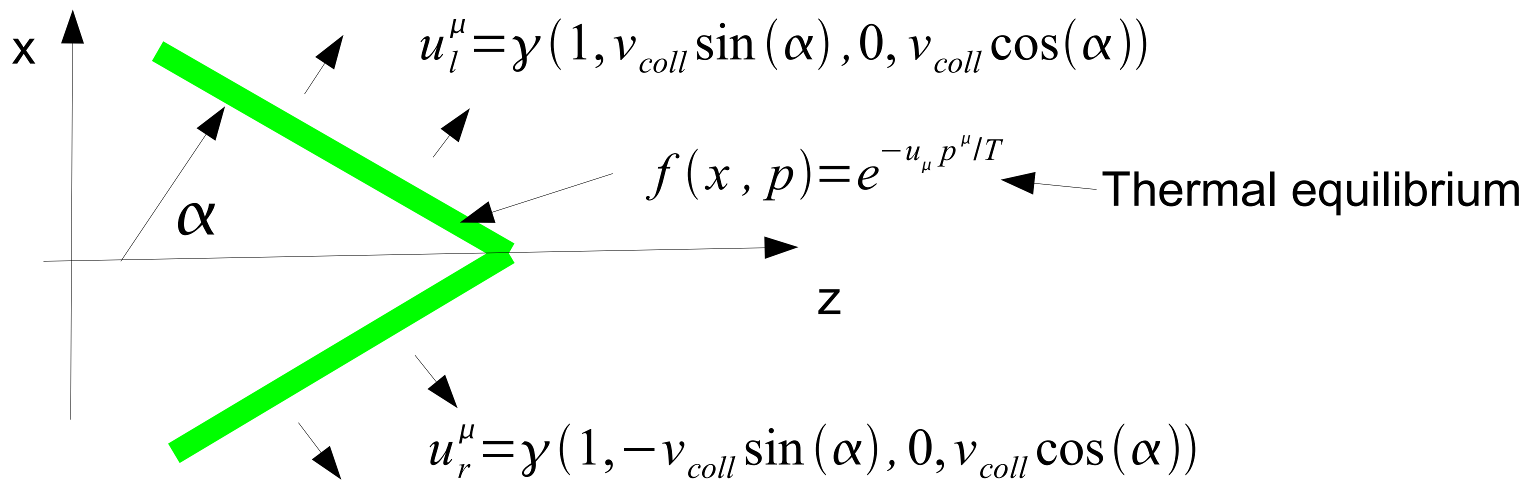
- But....
  1) viscosity is not zero in heavy-ion collisions (HIC)...and as we have already seen, viscosity in order expected in HIC destroys the conical structure to a very weak signal
  2) The jet in reality has not infinite energy....and the formation-time is finite
  3) The angle changes of the Mach Cone changes depending on the energy deposition
  4) The diffusion wake and head shock will have a big contribution...as we will see..

- However, one can can find an analytical expression for the two-particle correlations of Mach Cones....

Mach Cones in BAMPS

Two Particle Correlations
Analytical solution

Assume two wings in thermal equilibrium

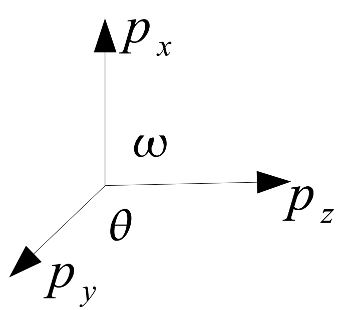


alpha is a const and corresponds to the Mach angle, where v_coll is the collective velocity of matter velocity in the wings

Mach Cones in BAMPS

Two Particle Correlations
Analytical solution

- We are looking for the angle ω, which is the angle in the p_x and p_z plane

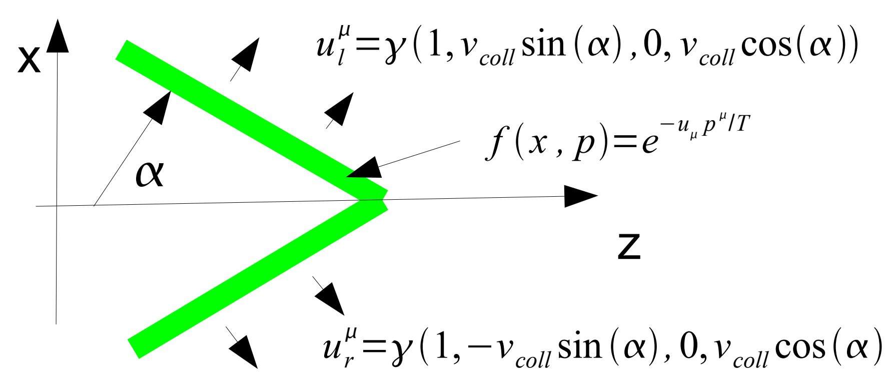


$$p_z = p\cos(\omega)\sin(\theta)$$
$$p_x = p\sin(\omega)\sin(\theta)$$
$$p_y = p\cos(\theta)$$

One calculate for each wing the particle distribution

$$\Rightarrow \quad \frac{dN}{d\omega} = \frac{V}{(2\pi)^3}\iint p^2\sin(\theta)e^{-u_\mu p^\mu/T}\,dp\,d\theta$$

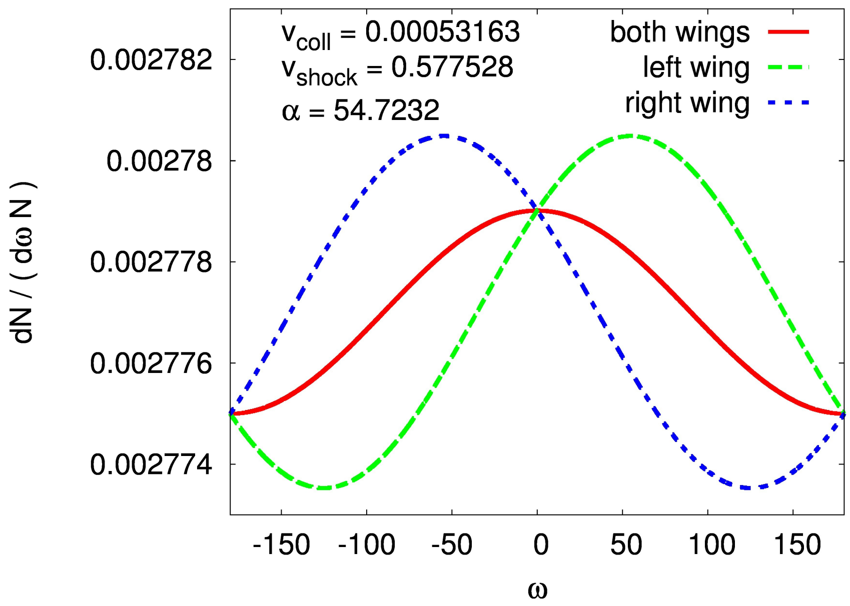
In the end one has to add both contributions!



Mach Cones in BAMPS

Two Particle Correlations
Analytical solution - Results

Taking the very weak perturbation case in account, we do not observe a double peak structure as we expected.

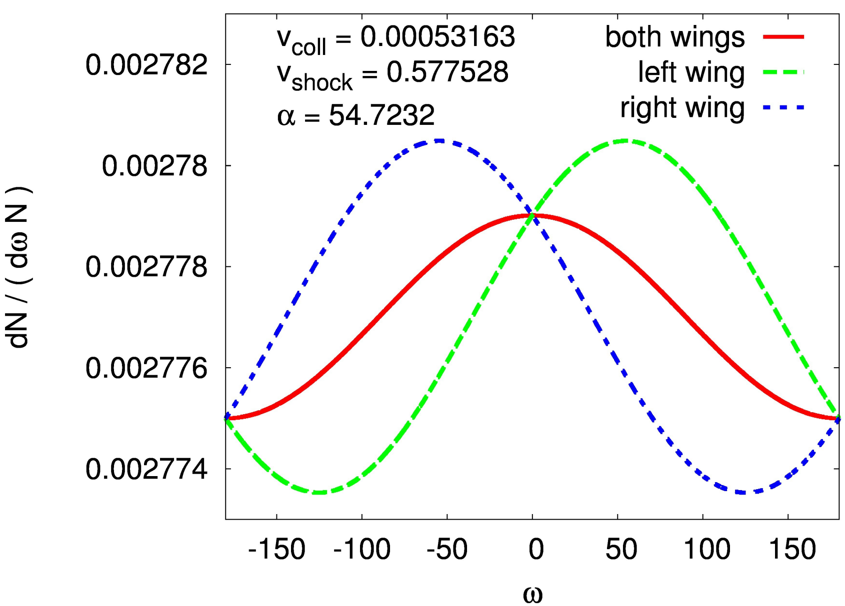


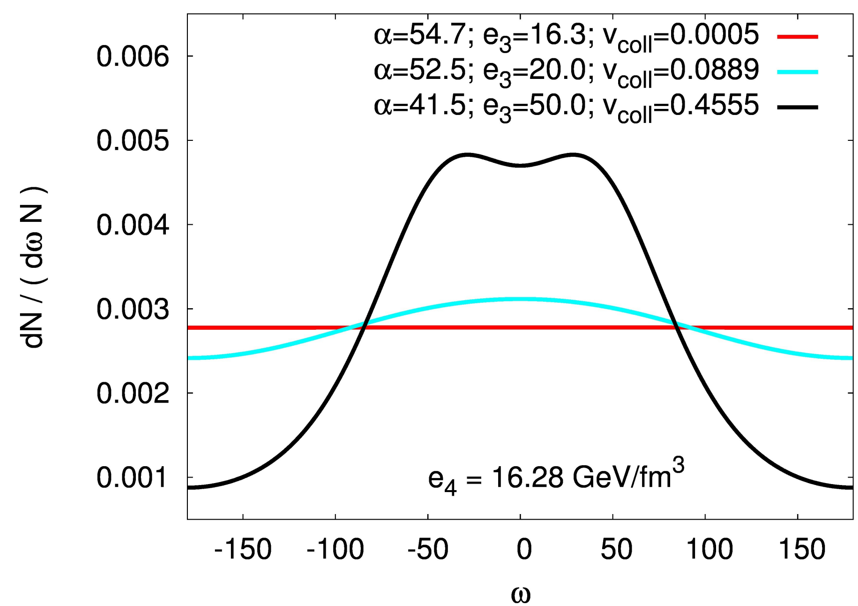
alpha and v_coll depends on the ratio of density in the wing and medium in rest

Mach Cones in BAMPS

Two Particle Correlations
Analytical solution - Results

Taking the very weak perturbation case in account, we do not observe a double peak structure as we expected.
→ Only if the shock gets stronger a double peak is observed
→ If the shock gets stronger, also v_coll gets larger and therefore the double peak is clearer

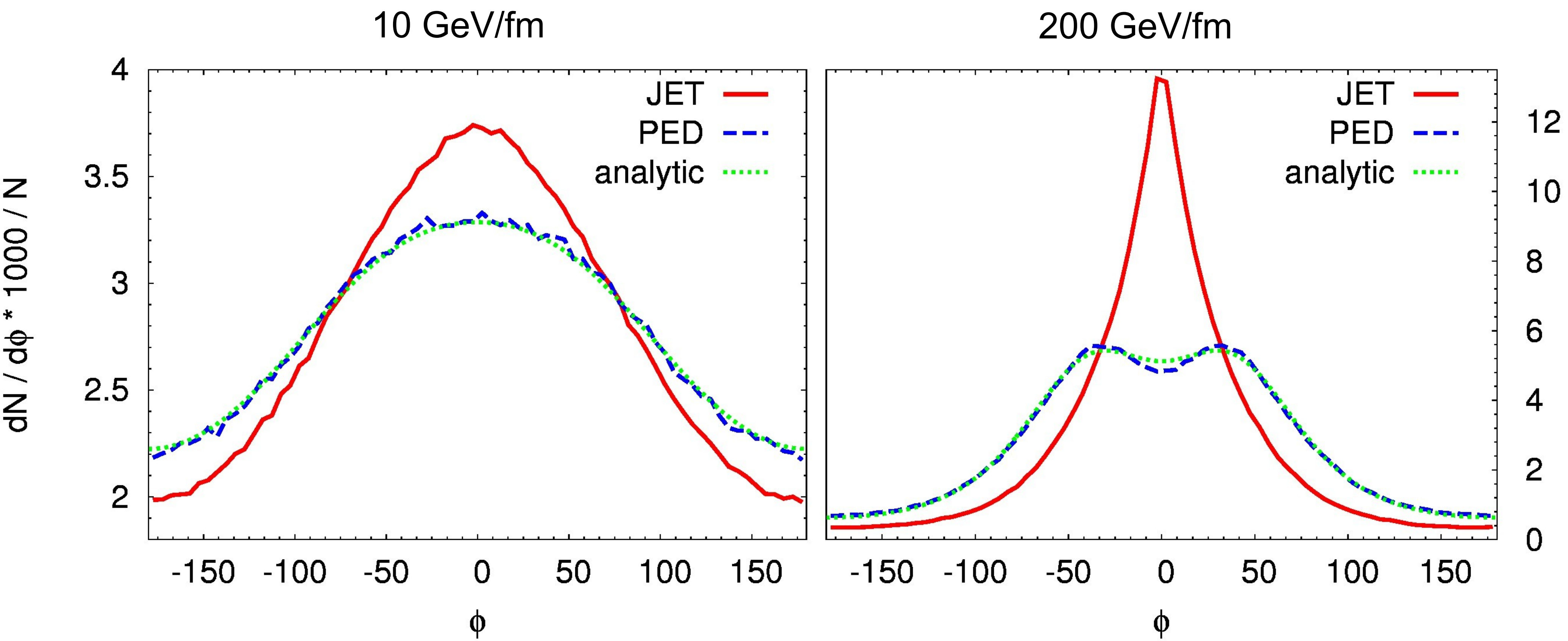




alpha and v_coll depends on the ratio of density in the wing and medium in rest

Mach Cones in BAMPS

Two Particle Correlations
Numerical Results



The source term plays a big role for observation a double peak structure

Conclusion

- BAMPS is an excellent benchmark to investigate phenomena like shock waves and Mach Cones in the ideal and viscous region

- Mach Cones might exist in heavy-ion collisions...

  ...but have **NOT** to be the origin of the famous ”double peak structure”....

Thank you

The Parton Cascade BAMPS

For this setup :

- **Boltzmann gas, isotropic cross sections, elastic processes only**
- **Implementing a constant η/s, we locally get the cross section σ_{22}:**

$$\eta = \frac{4}{15}\frac{\epsilon}{R^{tr}}$$

Transport collision rate R^{tr}

For isotropic elastic collisions:

$$R_{22}^{tr} = n\frac{2}{3}\sigma_{22}$$

$$\epsilon = 3nT$$

$$s = 4n - n\ \ln(\lambda_{fug})$$

$$\lambda_{fug} = \frac{n}{n_{eq}} \qquad n_{eq} = \frac{g}{\pi^2}T^3$$

g = 16 for gluons

Z. Xu & C. Greiner,
Phys.Rev.Lett.100:172301,2008

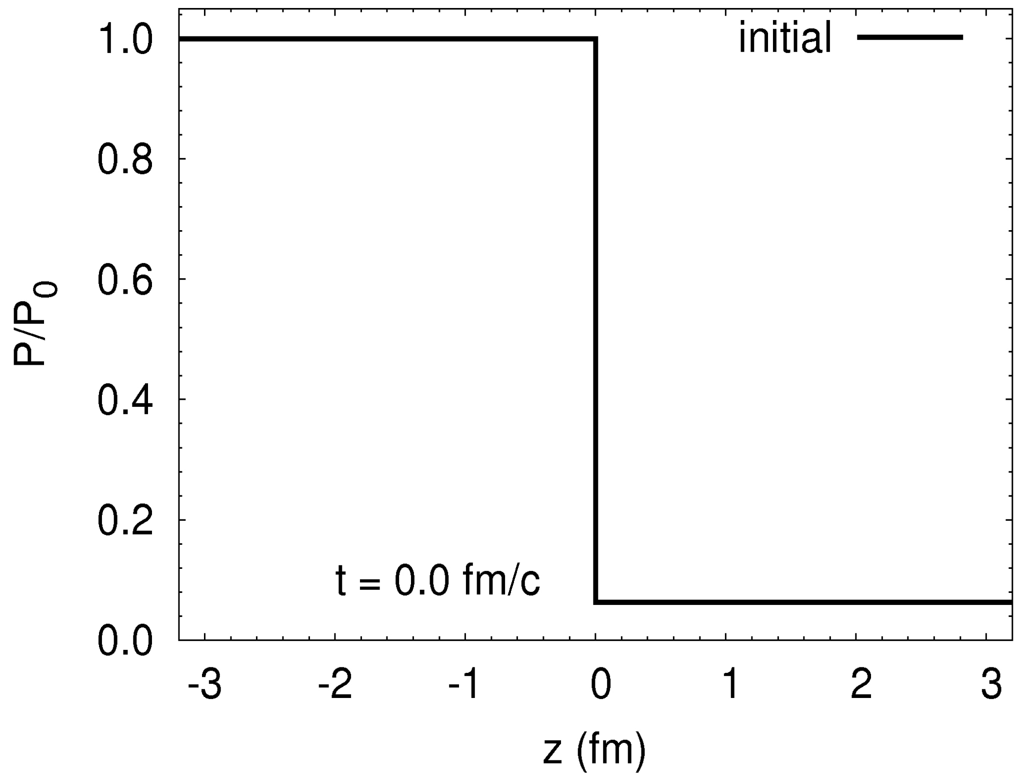
$$\sigma_{22} = \frac{6}{5}\frac{T}{s}\left(\frac{\eta}{s}\right)^{-1}$$

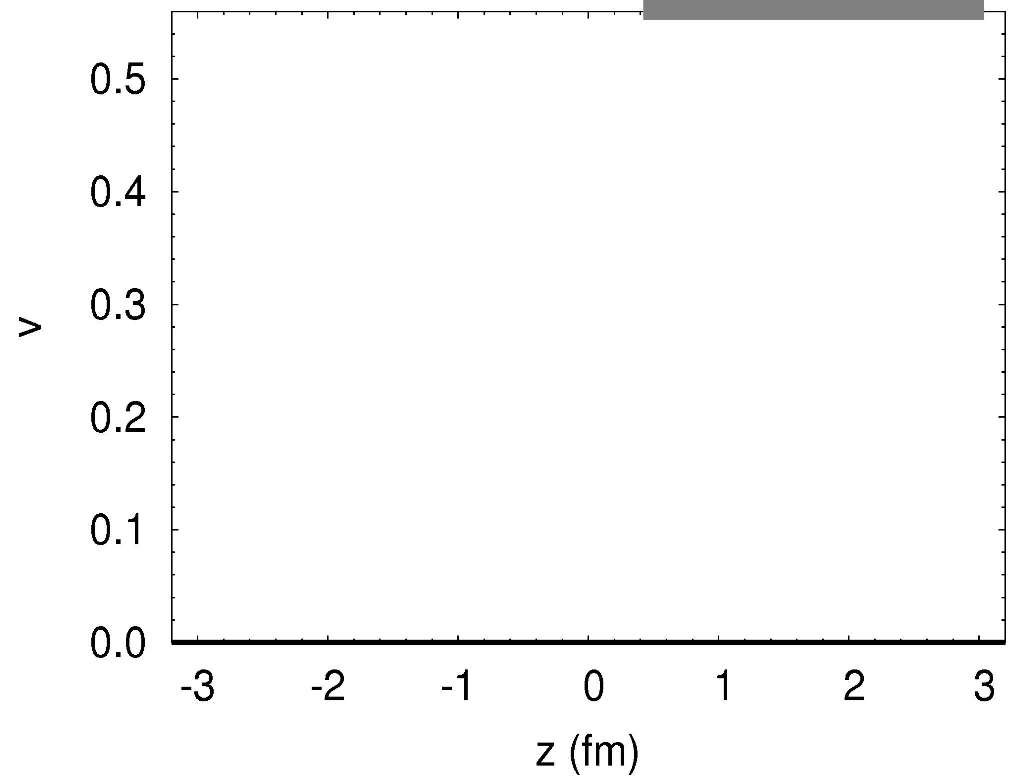
The Relativistic Riemann Problem

Investigation of Shock Waves in one dimension

$$T_L = 400\ MeV$$
$$T_R = 200\ MeV$$
$$t = 0\ fm/c$$

Initial conditions





- Two pressure regions seperated by a membran
- The velocities on both sides are zero

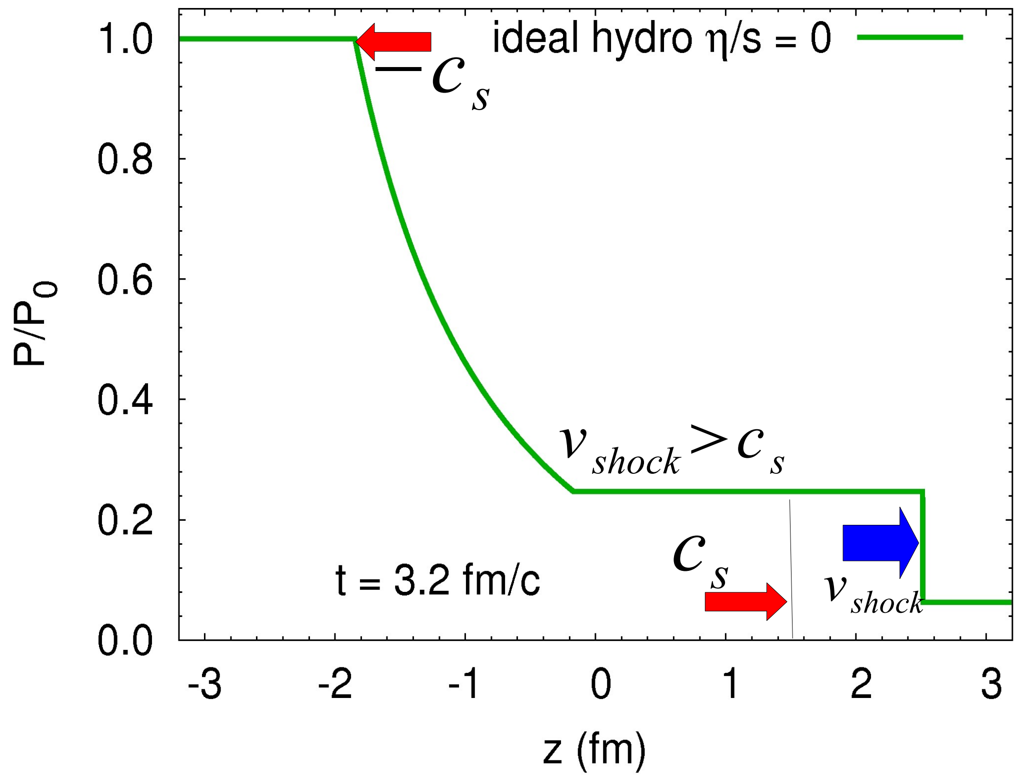
→ What happens if you remove the membran?

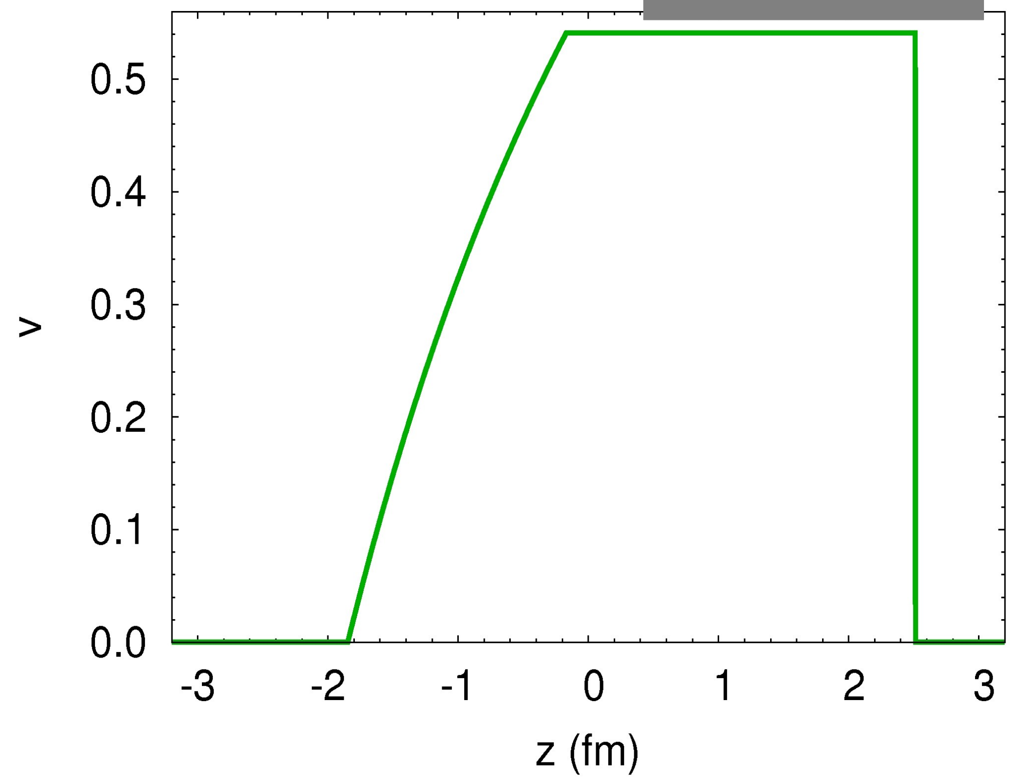
The Relativistic Riemann Problem

Investigation of Shock Waves in one dimension

Analytical Solution for a massless Boltzmann gas → e = 3P

$T_L = 400\ MeV$
$T_R = 200\ MeV$
$t = 3.2\ fm/c$

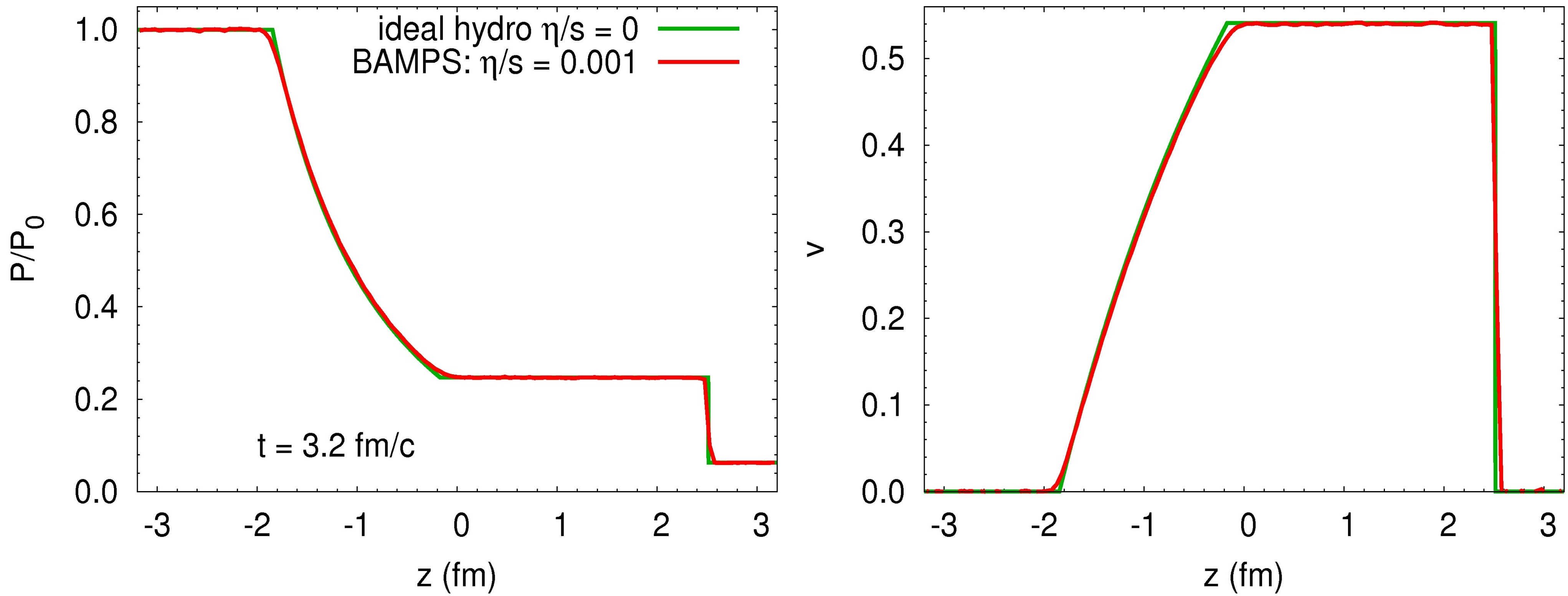




- **Analytical Solution for a perfect fluid**
  → A shock wave travels to the right with a speed <u>higher</u> than the speed of sound
  → A rarefaction wave travels to the left with the speed of sound

The Relativistic Riemann Problem
Investigation of Shock Waves in one dimension

Boltzmann solution of the relativistic Riemann problem



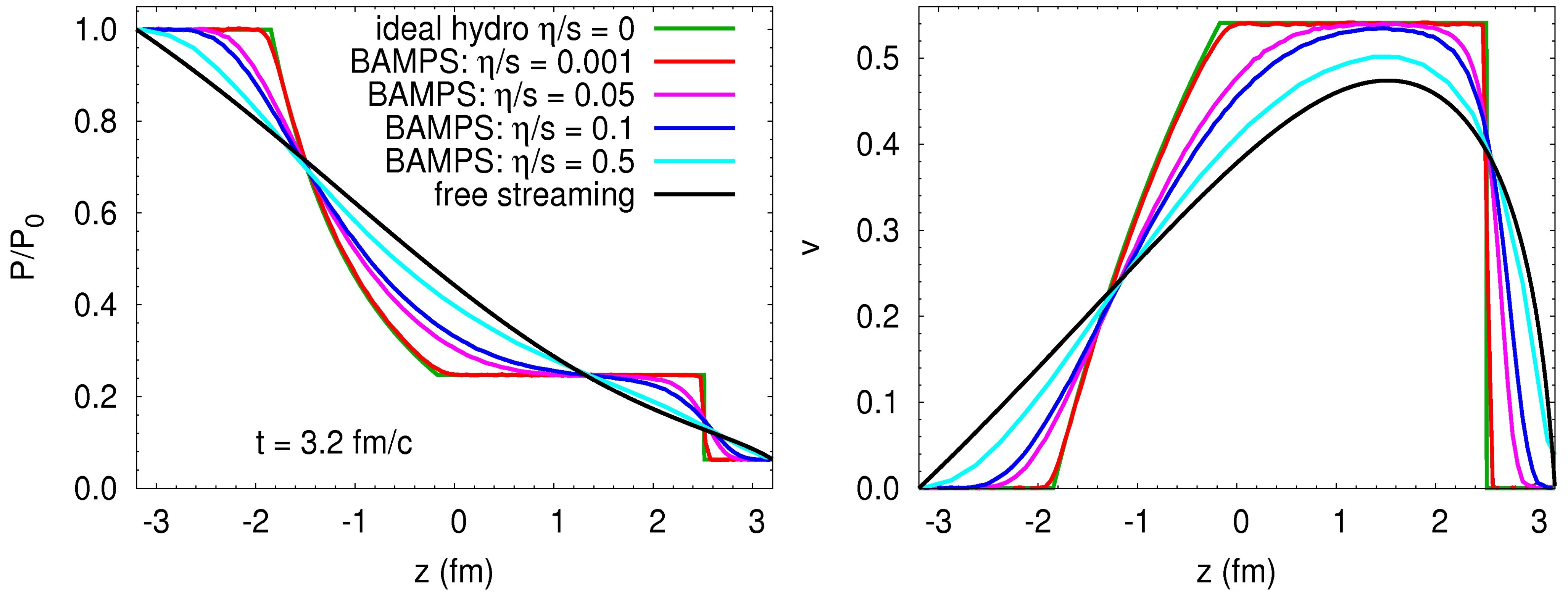
BAMPS reproduces the correct hydro limit

I. Bouras et al., Phys. Rev. Lett. 103:032301 (2009)

The Relativistic Riemann Problem

Investigation of Shock Waves in one dimension

Boltzmann solution of the relativistic Riemann problem



Transition from ideal hydro to free streaming

I. Bouras et al., **Phys. Rev. Lett. 103:032301 (2009)**

Dissipative Hydro for One-Component Systems

Comparison of kinetic theory to viscous hydrodynamics

$$u^{\mu}\partial_{\mu}\pi^{\alpha\beta}=-\frac{\pi^{\alpha\beta}}{2\beta_2\eta}-\pi^{\alpha\beta}\frac{T}{\beta_2}\partial_{\mu}\left(\frac{\beta_2}{2T}u^{\mu}\right)+\frac{\nabla^{<\alpha}u^{\beta>}}{\beta_2}$$

Israel-Stewart Eq.

Static one-dimensional setup, no spatial gradients

$$\dot{\pi}=-\frac{\pi}{2\beta_2\eta}$$

$\eta\rightarrow$ Shear viscosity

$$\beta_2=\frac{9}{4e}$$

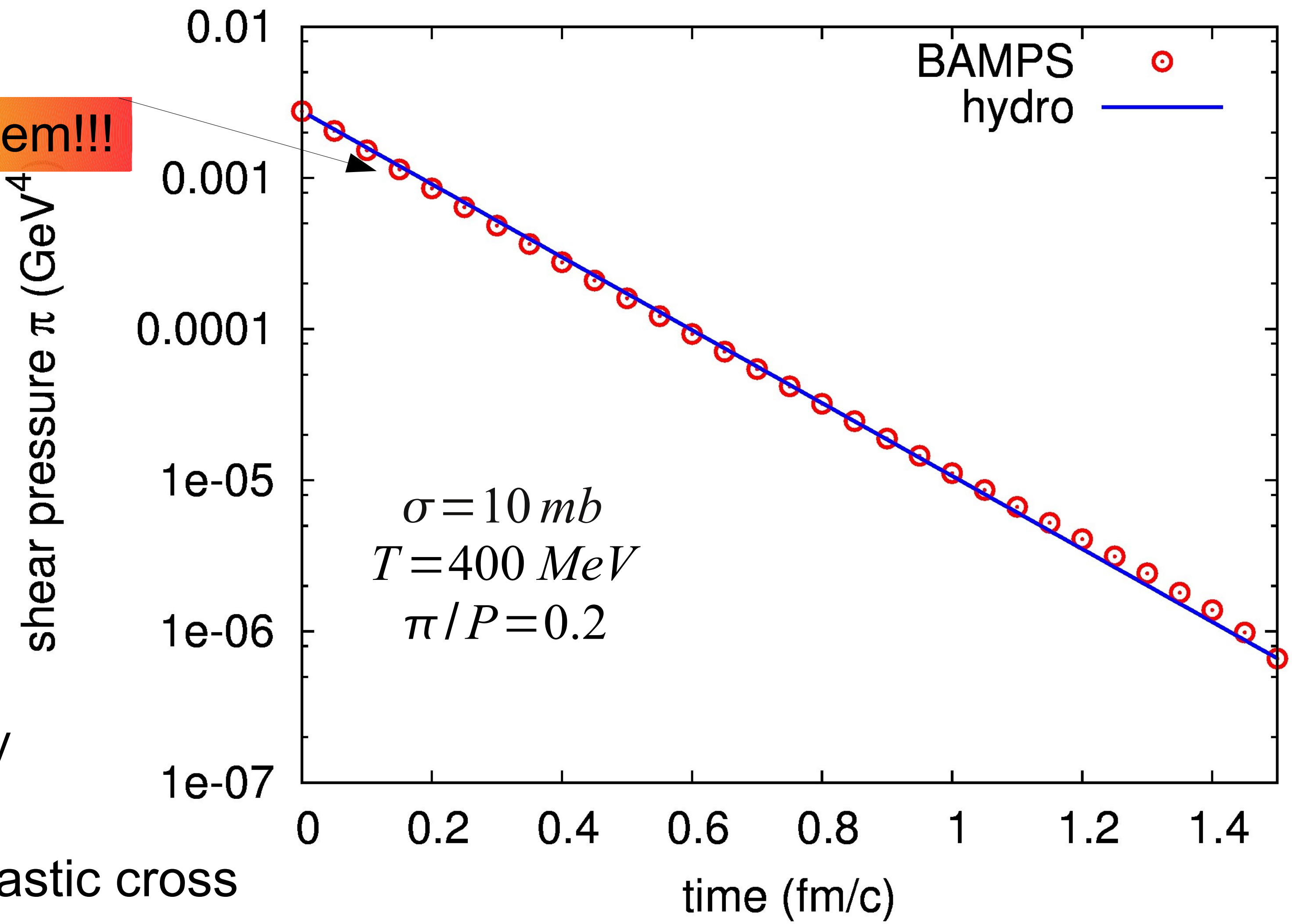
See also the next talk of G. Denicol!!!

Dissipative Hydro for One-Component Systems

Comparison of kinetic theory to viscous hydrodynamics

Initialization with Grad

Relaxation of the system!!!



$$\dot{\pi} = -\frac{\pi}{2\beta_2 \eta}$$

$\eta \rightarrow$ Shear viscosity

Constant cross elastic cross sections

Isotropic angle distribution: $\eta = \frac{5}{6}\frac{T}{\sigma}$

See also the next talk of G. Denicol!!!

Dissipative Hydro for Multi-Component Systems

Comparison of kinetic theory to viscous hydrodynamics

Consider a mixture of *N* components → *arXiv:1103.4038v1 [hep-ph]*

$$u^{\mu}\partial_{\mu}\pi_i^{\alpha\beta} = -\frac{\pi_i^{\alpha\beta}}{2\beta_{2,i}\eta_i} - \pi_i^{\alpha\beta}\frac{T}{\beta_{2,i}}\partial_{\mu}\left(\frac{\beta_{2,i}}{2T}u^{\mu}\right) + \frac{\nabla^{<\alpha}u^{\beta>}}{\beta_{2,i}}$$

Static one-dimensional setup, no spatial gradients
Isotropic cross sections

$$\dot{\pi}_i = -\pi_i \cdot \left(\frac{5}{9}\sigma_{ii}n_i + \frac{7}{9}\sigma_{ij}n_j\right) + \pi_j \cdot \left(\frac{2}{9}\sigma_{ij}n_i\right)$$

> all dissipative fields are coupled
> Viscosities $\boldsymbol{\eta_i}$ depend on ratios of the shear pressures $\boldsymbol{\pi_i, \pi_j}$
> Effective viscosity of a mixture can be defined only in a quasi-static limit $\boldsymbol{\pi_i/\pi_j = const}$

Dissipative Hydro for Multi-Component Systems

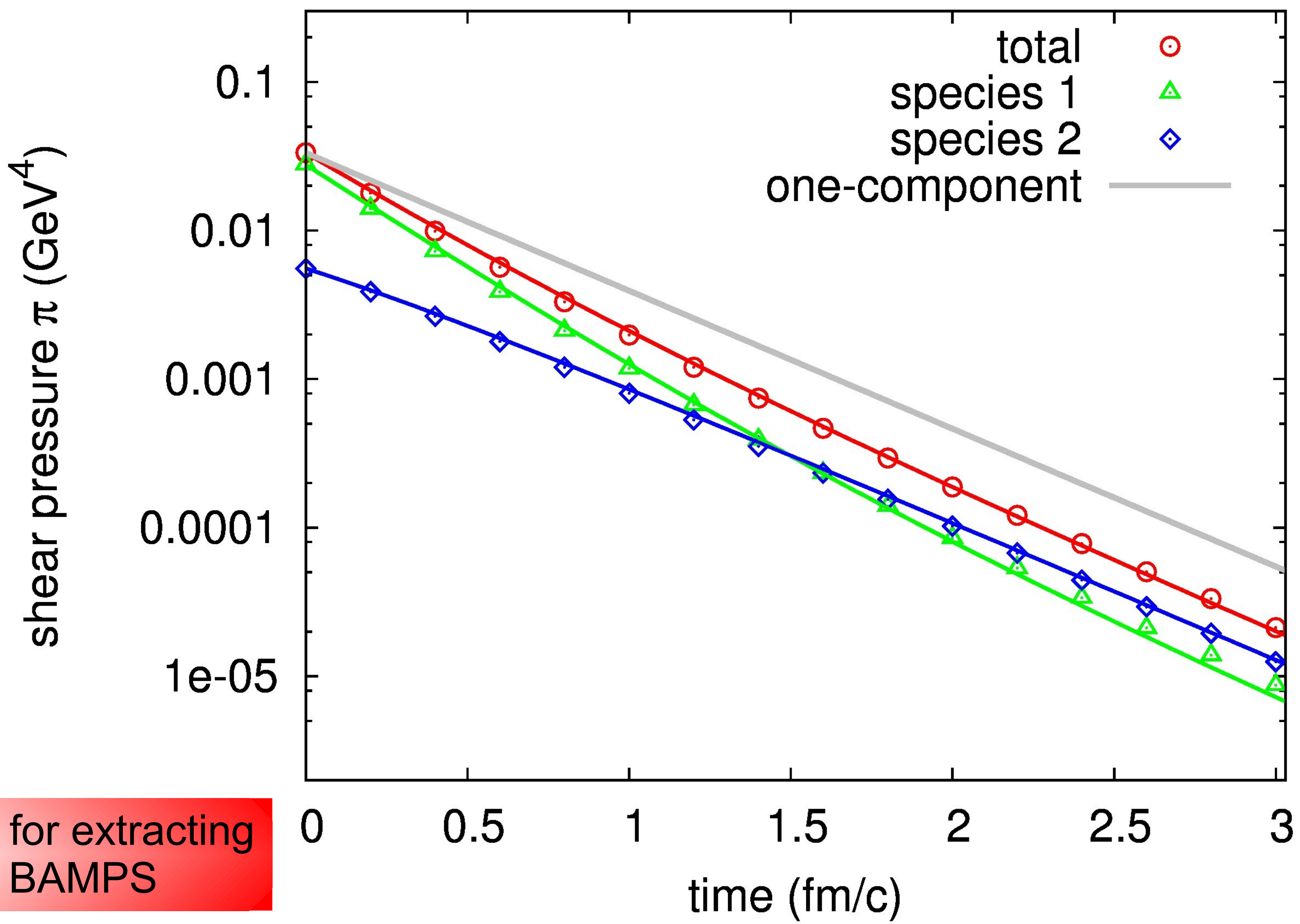
Comparison of kinetic theory to viscous hydrodynamics

Consider a mixture of *N* components → *arXiv:1103.4038v1 [hep-ph]*

$$\dot{\pi}_i = -\pi_i \cdot \left(\frac{5}{9} \sigma_{ii} n_i + \frac{7}{9} \sigma_{ij} n_j \right) + \pi_j \cdot \left(\frac{2}{9} \sigma_{ij} n_i \right)$$

Initial conditions:

$$T = 400\ MeV$$
$$n_1 / n_2 = 5$$
$$\pi_1 / \pi_2 = 5$$
$$\sigma_{11} = 3.8\ mb$$
$$\sigma_{12} = \sigma_{11} / 2$$
$$\sigma_{22} = \sigma_{11} / 4$$



See the talk of C. Wesp for extracting the shear viscosity from BAMPS