The global $U(2)_L \times U(2)_R$ Linear σ-Model with Vector and Axialvector Fields and Electroweak Interaction

Anja Habersetzer

in collaboration with F. Giacosa, D. H. Rischke

Heraklion, TORIC Workshop September 8th, 2011

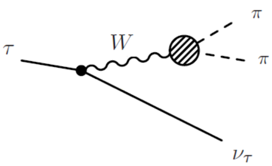
Motivation

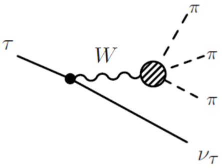
- τ spectral functions from ALEPH collaboration

  ALEPH Collaboration, S. Schael et al., Branching Ratios and Spectral Functions of Tau Decays: Final ALEPH measurements and physics implications, Phys.Rept. 421 (2005) 191284, [hep-ex/0506072]
- determine electroweak interactions for hadronic degrees of freedom in the vacuum
- results can be used to perfom calculations at nonzero temperature and density (e.g. dilepton decay rate)
- understand the nature of resonances like a_1, e.g. $\bar{q}q$ or $\rho\pi$-state?

τ-Decay

- weak semileptonic decay:
  decay rate $\Gamma_{\tau \to W \nu_\tau}$ well known from Standard Model of Particle Physics





- a hadronic model with electroweak interaction is needed

- Linear σ Model with global $U(2)_L \times U(2)_R$ chiral symmetry
- hadronic degrees of freedom are the light $N_f = 2$ meson multiplets, represented by the matrix-valued fields $\Phi\,,\ L^\mu\,,\ R^\mu$

$$\begin{aligned} \Phi : &\quad \text{scalar S } (\sigma, \vec{a_0})\,, && \text{pseudoscalar P } (\eta, \vec{\pi})\,, \\ L^\mu(R^\mu) : &\quad \text{vector } V^\mu\ (\omega^\mu, \vec{\rho^\mu})\,, && \text{axialvector } A^\mu\ (f_1^\mu, \vec{a_1^\mu})\,. \end{aligned}$$

- Strong isospin algebra with generators

$$t_0 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\,, \quad t_i = \frac{\sigma_i}{2}\,,\ \ i = 1, 2, 3$$

- τ-decay can be calculated within linear σ-model after including electroweak interactions

Representation of the Fields

S scalar, P pseudoscalar:

$$\Phi = \sum_{a=0}^{3}(S_a + iP_a)t_a \left(\sim \sqrt{2}\sum_{a=0}^{3}(\bar{q}t_a q + \bar{q}t_a\gamma_5 q)t_a\right)$$
$$= \frac{1}{2}\begin{pmatrix} \sigma + a_0^0 + i(\eta + \pi^0) & a_0^+ + i\pi^+ \\ a_0^- + i\pi^- & \sigma - a_0^0 + i(\eta - \pi^0) \end{pmatrix}$$

V_μ vector, A_μ axialvector:

$$(L,R)_\mu = \sum_{a=0}^{3}(V_\mu^a \pm A_\mu^a)t_a \left(\sim \sqrt{2}\sum_{a=0}^{3}(\bar{q}\gamma_\mu t_a q \pm \bar{q}t_a\gamma_\mu\gamma_5 q)t_a\right)$$
$$= \frac{1}{2}\begin{pmatrix} (\omega_\mu + \rho_\mu^0) \pm (f_{1\mu} + a_{1\mu}^0) & \rho_\mu^+ \pm a_{1\mu}^+ \\ \rho_\mu^- \pm a_{1\mu}^- & (\omega_\mu - \rho_\mu^0) \pm (f_{1\mu} - a_{1\mu}^0) \end{pmatrix}$$

Lagrangian

$$
\begin{aligned}
\mathscr{L} &= Tr[(D^{\mu}\Phi)^{\dagger}(D^{\mu}\Phi)] - m^2 Tr(\Phi^{\dagger}\Phi) - \lambda_1 [Tr(\Phi^{\dagger}\Phi)]^2 - \lambda_2 Tr(\Phi^{\dagger}\Phi)^2 \\
&- \frac{1}{4} Tr[(L_0^{\mu\nu})^2 + (R_0^{\mu\nu})^2] + \frac{m_1^2}{2} Tr[(L^{\mu})^2 + (R^{\mu})^2] + Tr[H(\Phi + \Phi^{\dagger})] \\
&+ c(\det\Phi + \det\Phi^{\dagger}) + \text{higher order couplings of } \Phi\,,\ L^{\mu}\,,\ R^{\mu}
\end{aligned}
$$

$$
\begin{aligned}
D_{\mu}\Phi &= \partial_{\mu}\Phi - ig_1(L_{\mu}\Phi - \Phi R_{\mu})\ , \\
(L,R)_0^{\mu\nu} &= \partial^{\mu}(L,R)^{\nu} - \partial^{\nu}(L,R)^{\mu}
\end{aligned}
$$

1 invariant under global $U(2)_L \times U(2)_R$ transformations

$$
\begin{aligned}
\Phi &\rightarrow \Phi' = U_L \Phi U_R^{\dagger} \\
L^{\mu} &\rightarrow L^{\mu\,\prime} = U_L L^{\mu} U_L^{\dagger} \\
R^{\mu} &\rightarrow R^{\mu\,\prime} = U_R R^{\mu} U_R^{\dagger}
\end{aligned}
$$

2 symmetry is broken to $U(1)_V$, baryon number conservation

$SU(2)_L \times U(1)_y$ Transformation

- it turns out that $SU(2)_L \times U(1)_Y$ is simply a subgroup of $U(2)_L \times U(2)_R$
- transformation laws

$$
\begin{array}{ccccl}
\Phi & \longrightarrow & \Phi' & = & U_L \Phi U_Y^\dagger \\
L^\mu & \longrightarrow & L^{\mu\,\prime} & = & U_L L^\mu U_L^\dagger \quad , \\
R^\mu & \longrightarrow & R^{\mu\,\prime} & = & U_Y R^\mu U_Y^\dagger \ , \\
B^\mu & \longrightarrow & B^{\mu\,\prime} & = & U_Y B^\mu U_Y^\dagger + \frac{i}{g'} U_Y \partial^\mu U_Y^\dagger \ , \\
W^\mu & \longrightarrow & W^{\mu\,\prime} & = & U_L W^\mu U_L^\dagger + \frac{i}{g} U_L \partial^\mu U_L^\dagger \ .
\end{array}
$$

- covariant derivative

$$D^\mu \Phi = \partial^\mu \Phi - i g_1 (L^\mu \Phi - \Phi R^\mu) + i g' \Phi B^\mu - i g W^\mu \Phi \ .$$

Weinberg Mixing and Cabibbo Mixing

- neutral bare $SU(2)_L \times U(1)_Y$ gauge fields B^μ, W_3^μ are related to the physical fields A^μ, Z^μ by

$$\begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}$$

and

$$e = g' \cos\theta_W = g \sin\theta_W$$

- strong isospin eigenstates of d, s, b are related to the weak eigenstates by the CKM matrix
- Cabibbo mixing

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

- covariant derivative in terms of physical weak interaction fields and weak eigenstates

$$\boxed{\begin{aligned} D^\mu \Phi &= \partial^\mu \Phi - ig_1(L^\mu \Phi - \Phi R^\mu) - ig\cos\theta_C (W_1^\mu t_1 + W_2^\mu t_2)\Phi \\ &\quad - ieA^\mu[t_3, \Phi] - ig\cos\theta_W \left(Z^\mu \Phi + \tan^2\theta_W \Phi Z^\mu \right) \end{aligned}}$$

- also redefinition of the field strength tensors for left- and righthanded fields

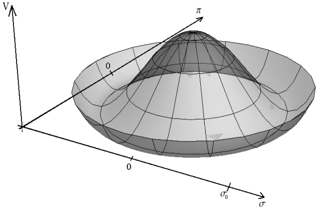
$$L^{\mu\nu} = \frac{i}{g}[D_L^\mu, D_L^\nu]\,, \qquad D_L^\mu = \partial^\mu - igW^\mu \cdot *$$

and

$$R^{\mu\nu} = \frac{i}{g'}[D_R^\mu, D_R^\nu]\,, \qquad D_R^\mu = \partial^\mu - ig * \cdot\, B^\mu$$

Spontaneous Symmetry Breaking

- for $m^2 < 0$ we obtain infinite number of degenerate groundstates



$\sigma \to \sigma + \phi_0$ generates mass shift between σ and π-triplet

π massless Goldstone bosons

- explicit symmetry breaking term $\mathscr{L}_{eSB} = Tr[H(\Phi + \Phi^\dagger)]$ renders π-triplet massive

- gives rise to vertices proportional to ϕ_0 , ϕ_0^2 and generates mixing terms between weak bosons and vector and axialvector fields
- we encounter two problems:
  - a_1-π-mixing term $-g_1\phi_0\vec{a_1}^\mu \cdot \partial_\mu\vec{\pi}$
    requires shifting $\vec{a_1}^\mu$ and renormalizing $\vec{\pi}$

$$\left.\begin{array}{ll} \vec{a}_{1\mu} & \rightarrow \vec{a}_{1\mu} + Zw\partial_\mu\vec{\pi} \\ \vec{\pi} & \rightarrow Z\vec{\pi} \end{array}\right\} \quad \text{eliminate } a_1\text{-}\pi\text{-mixing}$$

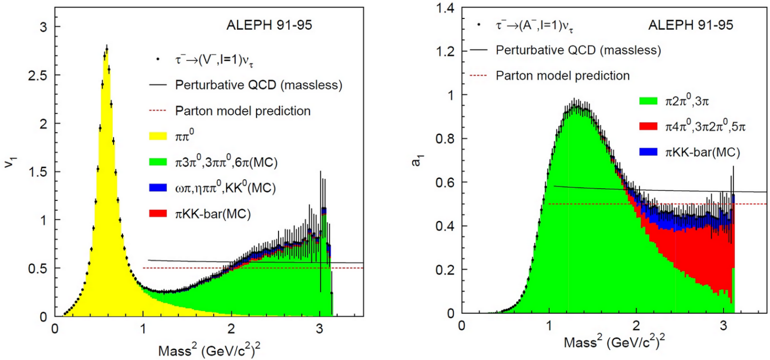
  - model does not contain $W\rho$ mixing terms, therefore introduce

$$\mathscr{L}_{W\rho} = \frac{\delta}{2} g \cos\theta_C \mathrm{Tr}[W_{\mu\nu}L^{\mu\nu}]$$

ALEPH Inclusive Spectral Functions

vector channel
$\tau \to \nu_\tau 2\pi\nu_\tau(4\pi\nu_\tau)$

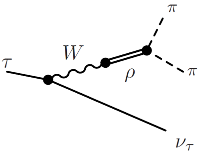
axialvector channel
$\tau \to \nu_\tau 3\pi\nu_\tau(5\pi\nu_\tau)$



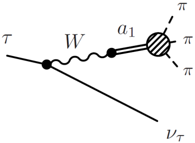
ALEPH Collaboration / Physics Reports 421 (2005) 191–284

dominant decay channels are 2π vector channel and 3π axialvector channel

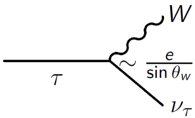
Components τ-decay



Vectorchannel



Axialvectorchannel



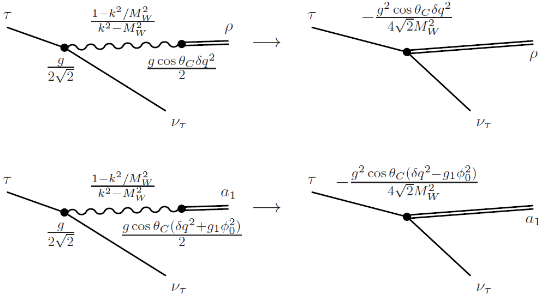
- $\mathscr{L}_{\tau\to W\nu_\tau} = \frac{g}{2\sqrt{2}}\bar{\nu}_\tau W_\mu^- \gamma^\mu(1-\gamma^5)\tau + h.c.$
- $\Gamma_{\tau\to W\nu_\tau}(s) = \frac{1}{8\pi}\left(\frac{g}{2\sqrt{2}}\right)^2 \frac{m_\tau^3}{M_w^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)$

Weak Interaction Vertices and Effective Couplings

Tree-level weak interaction vertices with ρ and a_1:

$W\rho$: $$\frac{g\cos\theta_C}{2}\delta s W_\mu^- \rho^{\mu +} + \text{ h.c.}$$

Wa_1: $$\frac{g\cos\theta_C}{2}(\delta s - g_1\phi_0^2) W_\mu^- a_1^{\mu\,+} + h.c.$$



- decay rates

$$\Gamma_{\tau\to\rho\nu_\tau}(s) = \frac{g_\rho^2 m_\tau^3}{8\pi}\frac{1}{s}\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right) ,$$
$$\Gamma_{\tau\to a_1\nu_\tau}(s) = \frac{g_{a_1}^2 m_\tau^3}{8\pi}\frac{1}{s}\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)$$

- spectral densities (in a form suitable to compare to ALEPH data)

$$v_1(s) = \frac{(2\pi)^2}{S_{EW}}(\delta s)^2\frac{1}{s}\frac{\rho_V(s)}{N}$$
$$a_1(s) = \frac{(2\pi)^2}{S_{EW}}(\delta s - g_1\phi_0^2)^2\frac{1}{s}\frac{\rho_A(s)}{N}$$

The s-dependent Decay Width $\Gamma_{\rho^- \to \pi^- \pi^0}(s)$

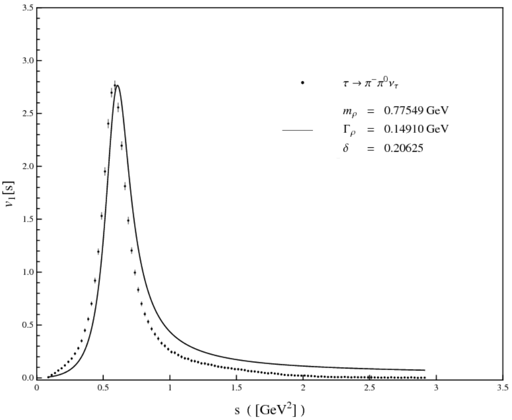
- $\Gamma_{\rho^- \to \pi^- \pi^0}(s) = \frac{m_\rho^5}{48\pi m_{a_1}^4} \left[1 - \left(\frac{2m_\pi}{\sqrt{s}}\right)^2\right]^{\frac{3}{2}} \left[g_1 Z^2 + \frac{g_2}{2}(1 - Z^2)\right]^2$
- $v_1(s) = \frac{(2\pi)^2}{N S_{EW}} (\delta s)^2 \, \frac{1}{s} \, \frac{\sqrt{s}\, \Gamma_{\rho^- \to \pi^- \pi^0}(s)}{(s - m_\rho^2)^2 + (\sqrt{s}\, \Gamma_{\rho^- \to \pi^- \pi^0}(s))^2}$
- decay width calculated from $U(2)_L \times U(2)_R$ Linear Sigma Model

  D. Parganlija, F. Giacosa, and D. H. Rischke, Vacuum Properties of Mesons in a Linear Sigma Model with Vector Mesons and Global Chiral Invariance, Phys. Rev. D82 (2010) 054024, [arXiv:1003.4934]

Parameters from hadronic sector

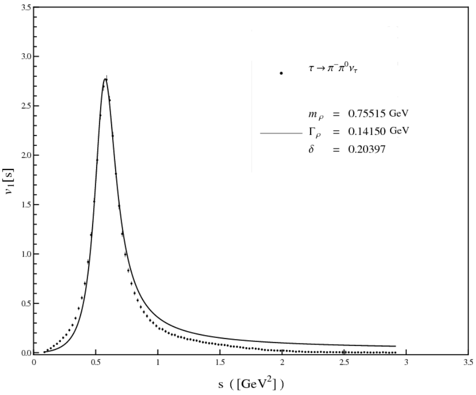
PDG: $m_\rho = 0.77549$ GeV , $\Gamma_{\rho^- \to \pi^- \pi^0}(m_\rho^2) = 0.1491$ GeV

one free parameter δ, fixed to obtain peak value

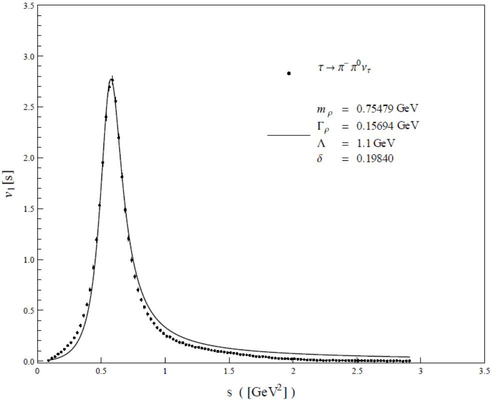


Fitted Parameters

improve agreement by refitting ρ mass and width:
$\delta = 0.20397\,,\ m_\rho = 0.75515\ \text{GeV}\,,\ \Gamma_{\rho^- \to \pi^- \pi^0}(m_\rho) = 0.1415\ \text{GeV}$



we can even further improve the result by removing the contributions from the 4 pion continuum by introducing a regularisation function $e^{\frac{4m_\pi^2 - s}{\Lambda^2}}$ with cut-off parameter $\Lambda = 1.1$ GeV

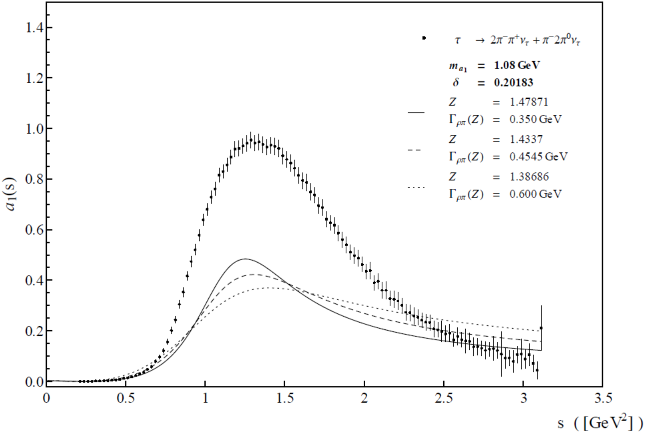


The s-dependent Decay Width $\Gamma_{a_1 \to \rho\pi}(s)$

- $\Gamma_{a_1 \to \rho\pi}(s) = \frac{k(\sqrt{s}, m_\rho, m_\pi)}{12\pi m_{a_1}^2} \left[h_{\mu\nu}^2 - \frac{(h_{\mu\nu} K_1^\nu)^2}{m_\rho^2} - \frac{(h_{\mu\nu} P^\mu)^2}{m_{a_1}^2} + \frac{(h_{\mu\nu} P^\mu K_1^\nu)^2}{m_\rho^2 m_{a_1}^2} \right]$
  also calculated by Parganlija, Giacosa and Rischke
- $a_1(s) = \frac{(2\pi)^2}{N S_{EW}} (\delta s + g_1 \phi_0^2)^2 \, \frac{1}{s} \, \frac{\sqrt{s}\, \Gamma_{a_1 \to \rho\pi}(s)}{(s - m_{a_1}^2)^2 + (\sqrt{s}\, \Gamma_{a_1 \to \rho\pi}(s))^2}$

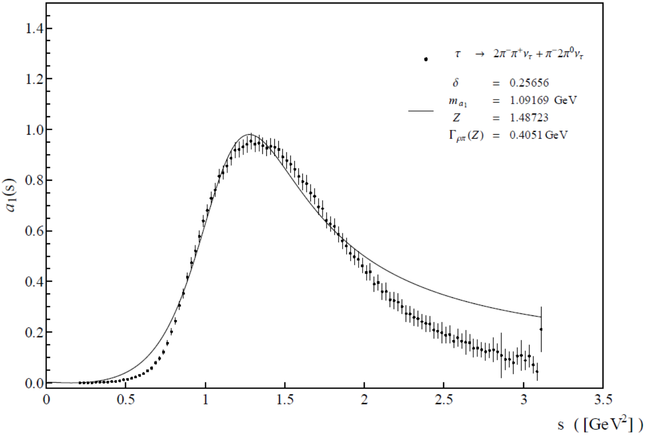
Axialvector Spectral Density

$m_{a_1} = 1.08 \text{ GeV}\,,\ \delta = 0.20183$



Axialvector Spectral Density

Fitted axialvector spectral density



Conclusion and Outlook

- Vector Spectral Density can be described well with weak hadron-coupling $\delta \approx 0.2$ within the global $U(2)_L \times U(2)_R$ Linear Sigma Model
- Axialvector Spectral Density is still causing problems, however

  δ may in fact be larger due to destructive interference effects from additional contributions in the vectorchannel, that will be considered

  there are also other contributions in the axialvector channel that have not been considered yet

$$v_1(s) = \frac{m_\tau^2}{6\cos\theta_C{}^2 S_{EW}} \frac{B(\tau^- \to V^- \nu_\tau)}{B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \times \frac{dN_V}{N_V ds} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \right]^{-1} \quad (1)$$

$$a_1(s) = \frac{m_\tau^2}{6\cos\theta_C{}^2 S_{EW}} \frac{B(\tau^- \to A^- \nu_\tau)}{B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \times \frac{dN_A}{N_A ds} \left[\left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \right]^{-1} \quad (2)$$

$$\frac{dN_V}{N_V ds} = \frac{\rho_V(s) \Gamma_{\tau^- \to V^- \nu_\tau}(s)}{\Gamma_{\tau \to V^- \nu_\tau}}, \quad \frac{dN_A}{N_A ds} = \frac{\rho_A(s) \Gamma_{\tau^- \to A^- \nu_\tau}(s)}{\Gamma_{\tau \to A^- \nu_\tau}} \quad (3)$$

where $\Gamma_{\tau \to V^- \nu_\tau}$ and $\Gamma_{\tau \to A^- \nu_\tau}$ are the partial widths in the corresponding channel and related to the branching fraction as

$$\Gamma_{\tau \to V^-(A^-)\nu_\tau} = B(\tau^- \to V^-(A^-)\nu_\tau) \cdot \Gamma_\tau^{\text{full}}$$

- First estimate for the parameter δ is taken from the convolution

$$\Gamma_{\tau\to 2\pi\nu_\tau} = \int_0^\infty ds \rho_{\rho\to 2\pi}(s)\Gamma_{\tau\to\rho\nu_\tau}(s) \tag{4}$$

- purely hypothetical assumption that ρ were stable

$$\Gamma_{\tau^-\to\pi^-\pi^0\nu_\tau} = \int_0^\infty ds\ \delta(s-m_\rho^2)\Gamma_{\tau^-\to\rho^-\nu_\tau}(s)$$

$$= \frac{m_\tau^3}{8\pi m_\rho^2}\left(\frac{g^2\cos\theta_C}{4\sqrt{2}M_W^2}\,\delta\, m_\rho\right)^2\left(1-\frac{m_\rho^2}{m_\tau^2}\right)^2\left(1+\frac{2m_\rho^2}{m_\tau^2}\right)$$

- from experiment we know

$$\Gamma^{\text{exp.}}_{\tau^-\to\pi\pi^0\nu_\tau}(m_\rho^2) = 5.7811\cdot 10^{-13}\ \text{GeV}$$

$$\Rightarrow\ \delta\approx 0.2$$