Impact Parameter Dependent nPDFs Based on EKS98 and EPS09 Parametrizations

TORIC

Ilkka Helenius

In collaboration with

Kari J. Eskola, Heli Honkanen

University of Jyväskylä
Department of Physics

8.9.2011

Outline

1. Introduction and Motivation
   - Nuclear Geometry in A+A collisions
2. Model
   - Assumptions
   - Fitting Procedure
   - Outcome
3. Application
   - Central-to-Peripheral Ratio
4. Summary & Outlook

Outline

Outline

1. Introduction and Motivation
   - Nuclear Geometry in A+A collisions
2. Model
   - Assumptions
   - Fitting Procedure
   - Outcome
3. Application
   - Central-to-Peripheral Ratio
4. Summary & Outlook

Outline

1. Introduction and Motivation
   - Nuclear Geometry in A+A collisions
2. Model
   - Assumptions
   - Fitting Procedure
   - Outcome
3. Application
   - Central-to-Peripheral Ratio
4. Summary & Outlook

Nuclear Thickness Function

Amount of nuclear matter in beam direction

Thickness function

Woods-Saxon density profile:

$$
\begin{aligned}
T_A(\mathbf{s}) &= \int\limits_{-\infty}^{\infty} \mathrm{d}z \frac{n_0}{1+\exp[\frac{\sqrt{\mathbf{s}^2+z^2}-R_A}{d}]} \\
d &= 0.54 \text{ fm} \\
R_A &= 1.12A^{1/3} - 0.86A^{-1/3} \text{ fm} \\
n_0 &= \frac{3}{4}\frac{A}{\pi R_A^3}\frac{1}{(1+(\frac{\pi d}{R_A})^2)} \\
A &= \int \mathrm{d}^2\mathbf{s}\, T_A(\mathbf{s})
\end{aligned}
$$

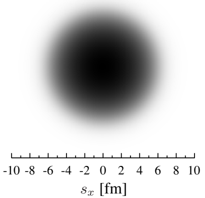
Nuclear Thickness Function

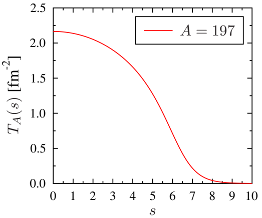
Amount of nuclear matter in beam direction

Thickness function

Woods-Saxon density profile:

$$
\begin{aligned}
T_A(\mathbf{s}) &= \int_{-\infty}^{\infty} \mathrm{d}z \frac{n_0}{1+\exp[\frac{\sqrt{\mathbf{s}^2+z^2}-R_A}{d}]} \\
d &= 0.54 \text{ fm} \\
R_A &= 1.12A^{1/3} - 0.86A^{-1/3} \text{ fm} \\
n_0 &= \frac{3}{4}\frac{A}{\pi R_A^3}\frac{1}{(1+(\frac{\pi d}{R_A})^2)} \\
A &= \int \mathrm{d}^2\mathbf{s}\, T_A(\mathbf{s})
\end{aligned}
$$





A+A collision

From $T_A(\mathbf{s})$ we can construct the nuclear overlap function

$$T_{AB}(\mathbf{b}) = \int \mathrm{d}^2\mathbf{s}\, T_A(\mathbf{s}) T_B(\mathbf{s}+\mathbf{b}),$$

Amount of the interacting matter at impact parameter $\mathbf{b}$.

In the absence of nuclear effects the hard cross section for given centrality class in $A+A$ collisions

$$\mathrm{d}\sigma^{AB\to k+X} = \int_{b_1}^{b_2} \mathrm{d}^2\mathbf{b}\, T_{AB}(\mathbf{b}) \mathrm{d}\sigma^{NN}, \quad (1)$$

where

$$\mathrm{d}\sigma^{NN} = \sum_{i,j} f_{i/N} \otimes f_{j/N} \otimes \mathrm{d}\hat{\sigma}^{ij\to k+X}$$

A+A collision

From $T_A(\mathbf{s})$ we can construct the nuclear overlap function

$$T_{AB}(\mathbf{b}) = \int \mathrm{d}^2\mathbf{s}\, T_A(\mathbf{s}) T_B(\mathbf{s}+\mathbf{b}),$$

Amount of the interacting matter at impact parameter $\mathbf{b}$.

In the absence of nuclear effects the hard cross section for given centrality class in $A + A$ collisions

$$\mathrm{d}\sigma^{AB\to k+X} = \int_{b_1}^{b_2} \mathrm{d}^2\mathbf{b}\, T_{AB}(\mathbf{b}) \mathrm{d}\sigma^{NN}, \tag{1}$$

where

$$\mathrm{d}\sigma^{NN} = \sum_{i,j} f_{i/N} \otimes f_{j/N} \otimes \mathrm{d}\hat{\sigma}^{ij\to k+X}$$

A+A collision

Nucleons in nucleus $\neq$ free nucleons

$\Rightarrow$ Modification to PDFs

$$f_{i/N}(x,Q^2) \rightarrow f_{i/A}(x,Q^2) = R_{i/A}(x,Q^2) \cdot f_{i/N}(x,Q^2)$$

Integrating the cross section formula (1) over the whole $\mathbf{b}$ –space, we have

$$\mathrm{d}\sigma^{AB\rightarrow k+X} = \sum_{i,j} \int \mathrm{d}^2\mathbf{b} \int \mathrm{d}^2\mathbf{s_1}\, T_A(\mathbf{s_1}) R_{i/A}(x_1,Q^2) f_{i/N}(x_1,Q^2) \otimes$$

$$\int \mathrm{d}^2\mathbf{s_2} T_B(\mathbf{s_2}) R_{j/B}(x_2,Q^2) f_{j/N}(x_2,Q^2) \otimes \delta(\mathbf{s_2}-\mathbf{s_1}-\mathbf{b}) \mathrm{d}\hat{\sigma}^{ij\rightarrow k+X}$$

Now we want to replace

$$\int \mathrm{d}^2\mathbf{s_1}\, T_A(\mathbf{s_1}) R_{i/A}(x_1,Q^2) \rightarrow \int \mathrm{d}^2\mathbf{s_1}\, T_A(\mathbf{s_1}) r_{i/A}(x_1,Q^2,\mathbf{s_1})$$

A+A collision

Nucleons in nucleus $\neq$ free nucleons

$\Rightarrow$ Modification to PDFs

$$f_{i/N}(x,Q^2) \rightarrow f_{i/A}(x,Q^2) = R_{i/A}(x,Q^2) \cdot f_{i/N}(x,Q^2)$$

Integrating the cross section formula (1) over the whole $\mathbf{b}$ –space, we have

$$\mathrm{d}\sigma^{AB\rightarrow k+X} = \sum_{i,j} \int \mathrm{d}^2\mathbf{b} \int \mathrm{d}^2\mathbf{s_1}\, T_A(\mathbf{s_1}) R_{i/A}(x_1,Q^2) f_{i/N}(x_1,Q^2) \otimes$$
$$\int \mathrm{d}^2\mathbf{s_2} T_B(\mathbf{s_2}) R_{j/B}(x_2,Q^2) f_{j/N}(x_2,Q^2) \otimes \delta(\mathbf{s_2}-\mathbf{s_1}-\mathbf{b}) \mathrm{d}\hat{\sigma}^{ij\rightarrow k+X}$$

Now we want to replace

$$\int \mathrm{d}^2\mathbf{s_1}\, T_A(\mathbf{s_1}) R_{i/A}(x_1,Q^2) \rightarrow \int \mathrm{d}^2\mathbf{s_1}\, T_A(\mathbf{s_1}) r_{i/A}(x_1,Q^2,\mathbf{s_1})$$

A+A collision

Nucleons in nucleus $\neq$ free nucleons

$\Rightarrow$ Modification to PDFs

$$f_{i/N}(x,Q^2) \rightarrow f_{i/A}(x,Q^2) = R_{i/A}(x,Q^2) \cdot f_{i/N}(x,Q^2)$$

Integrating the cross section formula (1) over the whole $\mathbf{b}$ –space, we have

$$\mathrm{d}\sigma^{AB\rightarrow k+X} = \sum_{i,j} \int \mathrm{d}^2\mathbf{b} \int \mathrm{d}^2\mathbf{s_1}\, T_A(\mathbf{s_1}) R_{i/A}(x_1,Q^2) f_{i/N}(x_1,Q^2) \otimes$$

$$\int \mathrm{d}^2\mathbf{s_2} T_B(\mathbf{s_2}) R_{j/B}(x_2,Q^2) f_{j/N}(x_2,Q^2) \otimes \delta(\mathbf{s_2} - \mathbf{s_1} - \mathbf{b}) \mathrm{d}\hat{\sigma}^{ij\rightarrow k+X}$$

Now we want to replace

$$\int \mathrm{d}^2\mathbf{s_1}\, T_A(\mathbf{s_1}) R_{i/A}(x_1,Q^2) \rightarrow \int \mathrm{d}^2\mathbf{s_1}\, T_A(\mathbf{s_1}) r_{i/A}(x_1,Q^2,\mathbf{s_1})$$

Assumptions

Where $r_A(x, Q^2, \mathbf{s})$ is the nuclear modification depending on the transverse position of the nucleon $\mathbf{s}$.

For $r_A(x, Q^2, \mathrm{s})$ we now assume

- Normalization:

$$R_A(x, Q^2) = \frac{1}{A} \int \mathrm{d}^2\mathrm{s}\, T_A(\mathrm{s}) r_A(x, Q^2, \mathrm{s}),$$

  where $R_A(x, Q^2)$ from
  - EKS98 [*Eur.Phys.J.*, C9:61-68, 1999]
  - EPS09 [*JHEP*, 04:065, 2009]
- Spatial dependence trough $T_A(\mathrm{s})$

Assumptions

Where $r_A(x, Q^2, \mathbf{s})$ is the nuclear modification depending on the transverse position of the nucleon s.

For $r_A(x, Q^2, \mathrm{s})$ we now assume

- Normalization:

$$R_A(x, Q^2) = \frac{1}{A} \int \mathrm{d}^2\mathbf{s}\, T_A(\mathbf{s}) r_A(x, Q^2, \mathbf{s}),$$

  where $R_A(x, Q^2)$ from
  - EKS98 [*Eur.Phys.J.*, C9:61-68, 1999]
  - EPS09 [*JHEP*, 04:065, 2009]
- Spatial dependence trough $T_A(\mathrm{s})$

Assumptions

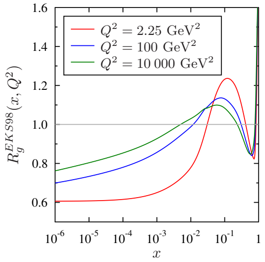
Where $r_A(x, Q^2, \mathbf{s})$ is the nuclear modification depending on the transverse position of the nucleon s.

For $r_A(x, Q^2, \mathrm{s})$ we now assume

- Normalization:

$$R_A(x, Q^2) = \frac{1}{A} \int \mathrm{d}^2\mathbf{s}\, T_A(\mathbf{s}) r_A(x, Q^2, \mathbf{s}),$$

  where $R_A(x, Q^2)$ from
  - EKS98 [*Eur.Phys.J.*, C9:61-68, 1999]
  - EPS09 [*JHEP*, 04:065, 2009]
- Spatial dependence trough $T_A(\mathrm{s})$



Assumptions

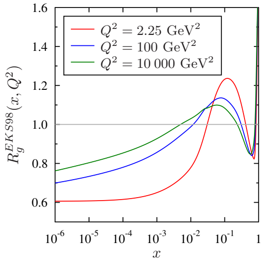
Where $r_A(x, Q^2, \mathbf{s})$ is the nuclear modification depending on the transverse position of the nucleon $\mathbf{s}$.

For $r_A(x, Q^2, \mathbf{s})$ we now assume

- Normalization:

$$R_A(x, Q^2) = \frac{1}{A} \int \mathrm{d}^2\mathbf{s}\, T_A(\mathbf{s}) r_A(x, Q^2, \mathbf{s}),$$

  where $R_A(x, Q^2)$ from
  - EKS98 [*Eur.Phys.J.*, C9:61-68, 1999]
  - EPS09 [*JHEP*, 04:065, 2009]
- Spatial dependence trough $T_A(\mathbf{s})$



Fitting Procedure

Functional form used for EKS98

$$r_A(x,Q^2,s) = 1 + c_1(x,Q^2)[T_A(s)] + c_2(x,Q^2)[T_A(s)]^2 + c_3(x,Q^2)[T_A(s)]^3 + c_4(x,Q^2)[T_A(s)]^4$$

Important: no A dependence in fit parameters $c_i(x,Q^2)$!

Parameters $c_i(x,Q^2)$ obtained by minimizing the χ^2

$$\chi^2(x,Q^2) = \sum_A \left[\frac{R_A(x,Q^2) - \frac{1}{A}\int \mathrm{d}^2 s\, T_A(s) r_A(x,Q^2,s)}{R_A(x,Q^2) - 1} \right]^2,$$

where $A = 16, 20, 24, \ldots, 300$.

Fitting Procedure

Functional form used for EKS98

$$r_A(x,Q^2,s) = 1 + c_1(x,Q^2)[T_A(s)] + c_2(x,Q^2)[T_A(s)]^2 + c_3(x,Q^2)[T_A(s)]^3 + c_4(x,Q^2)[T_A(s)]^4$$

Important: no A dependence in fit parameters $c_i(x,Q^2)$!

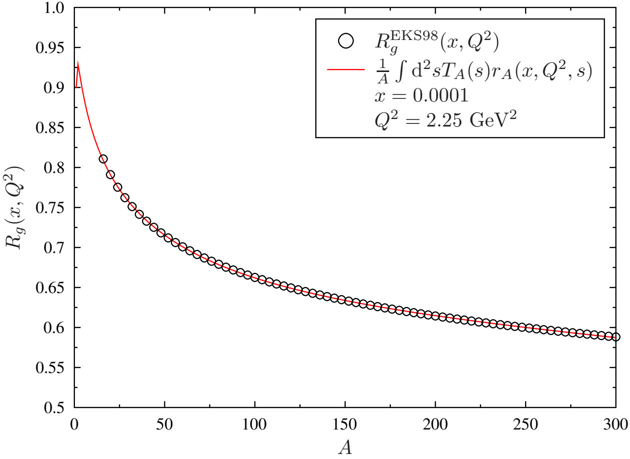
Parameters $c_i(x,Q^2)$ obtained by minimizing the χ^2

$$\chi^2(x,Q^2) = \sum_A \left[\frac{R_A(x,Q^2) - \frac{1}{A}\int \mathrm{d}^2s\, T_A(s) r_A(x,Q^2,s)}{R_A(x,Q^2) - 1} \right]^2,$$

where $A = 16, 20, 24, \ldots, 300$.

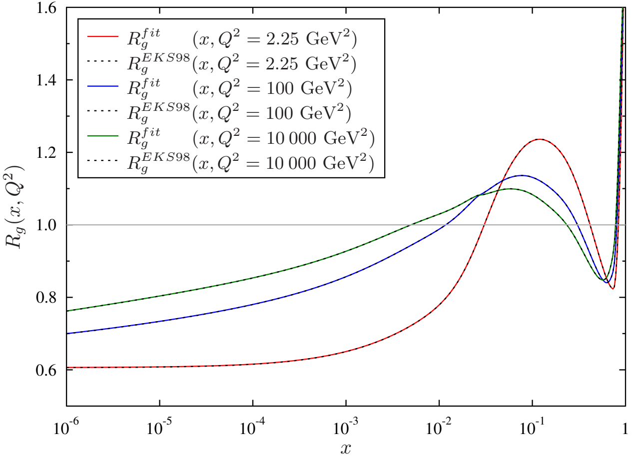
Fit Outcome

Example fit: gluon modification



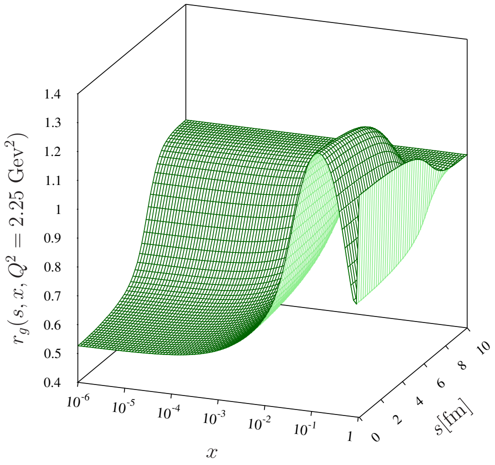
Fitted $R(x, Q^2)$ vs. old $R(x, Q^2)$

$$R^{fit}(x,Q^2) = \frac{1}{A}\int \mathrm{d}^2\mathbf{s}\, T_A(s)\left[1 + \sum_{i=1}^{4} c_i(x,Q^2)[T_A(s)]^i\right]$$



Spatial Dependence of Nuclear Modification for Au

$$r^{EKS98s}(x, Q^2, s) = 1 + \sum_{i=1}^{4} c_i(x, Q^2)[T_A(s)]^i$$



Observations

- The shape in x is similar to EKS98
- Effects are slightly stronger in small s compared to EKS98
- Nuclear effects die out when $s > R_A$

Nuclear Modification Factor R_{AA}^{1-jet}

- The 1-jet distribution for a centrality class with $b \in [b_1, b_2]$ can be calculated from

$$\left\langle \frac{\mathrm{d}^2 N_{AA}^{1-jet}}{\mathrm{d}p_T \mathrm{d}y} \right\rangle_{b_1,b_2} = \frac{\int_{b_1}^{b_2} \mathrm{d}^2 b \frac{\mathrm{d}^2 N_{AA}^{1-jet}(b)}{\mathrm{d}p_T \mathrm{d}y}}{\int_{b_1}^{b_2} \mathrm{d}^2 b \, p_{AA}^{inel}(b)}$$

- $p_{AA}^{inel}(b) = 1 - e^{-T_{AA}(\mathbf{b})\sigma_{inel}^{NN}}$ (optical Glauber model)
- Comparision with proton-proton (pp) collision gives the R_{AA}^{1-jet} for each centrality class

$$R_{AA}^{1-jet} = \frac{\left\langle \frac{\mathrm{d}^2 N_{AA}^{1-jet}}{\mathrm{d}p_T \mathrm{d}y} \right\rangle_{b_1,b_2}}{\frac{\langle N_{bin}\rangle_{b_1,b_2}}{\sigma_{inel}^{NN}} \frac{\mathrm{d}^2 \sigma_{pp}^{1-jet}}{\mathrm{d}p_T \mathrm{d}y}}$$

Nuclear Modification Factor R_{AA}^{1-jet}

- The 1-jet distribution for a centrality class with $b \in [b_1, b_2]$ can be calculated from

$$\left\langle \frac{\mathrm{d}^2 N_{AA}^{1-jet}}{\mathrm{d}p_T \mathrm{d}y} \right\rangle_{b_1,b_2} = \frac{\int_{b_1}^{b_2} \mathrm{d}^2 b \frac{\mathrm{d}^2 N_{AA}^{1-jet}(b)}{\mathrm{d}p_T \mathrm{d}y}}{\int_{b_1}^{b_2} \mathrm{d}^2 b\, p_{AA}^{inel}(b)}$$

- $p_{AA}^{inel}(b) = 1 - e^{-T_{AA}(\mathbf{b})\sigma_{inel}^{NN}}$ (optical Glauber model)
- Comparision with proton-proton (pp) collision gives the R_{AA}^{1-jet} for each centrality class

$$R_{AA}^{1-jet} = \frac{\left\langle \frac{\mathrm{d}^2 N_{AA}^{1-jet}}{\mathrm{d}p_T \mathrm{d}y} \right\rangle_{b_1,b_2}}{\frac{\langle N_{bin}\rangle_{b_1,b_2}}{\sigma_{inel}^{NN}} \frac{\mathrm{d}^2 \sigma_{pp}^{1-jet}}{\mathrm{d}p_T \mathrm{d}y}}$$

Central-to-Peripheral Ratio R_{CP}^{1-jet}

The 1-jet central-to-peripheral ratio

$$R_{CP}^{1-jet} = \frac{\left\langle \frac{\mathrm{d}^2 N_{AA}^{1-jet}}{\mathrm{d}p_T \mathrm{d}y} \right\rangle \frac{1}{\langle N_{bin} \rangle} (central)}{\left\langle \frac{\mathrm{d}^2 N_{AA}^{1-jet}}{\mathrm{d}p_T \mathrm{d}y} \right\rangle \frac{1}{\langle N_{bin} \rangle} (peripheral)}$$

Parameters from Optical Glauber Model

	$central = 0-5\%$			$peripheral = 60-80\%$		
	b_1 [fm]	b_2 [fm]	$\langle N_{bin} \rangle$	b_1 [fm]	b_2 [fm]	$\langle N_{bin} \rangle$
RHIC	0.0	3.41	1079	11.82	13.65	12.06
LHC	0.0	3.47	1717	12.03	13.89	18.81

- RHIC: $\sigma_{inel}^{NN} = 42$ mb
- LHC: $\sigma_{inel}^{NN} = 62$ mb

Central-to-Peripheral Ratio R_{CP}^{1-jet}

The 1-jet central-to-peripheral ratio

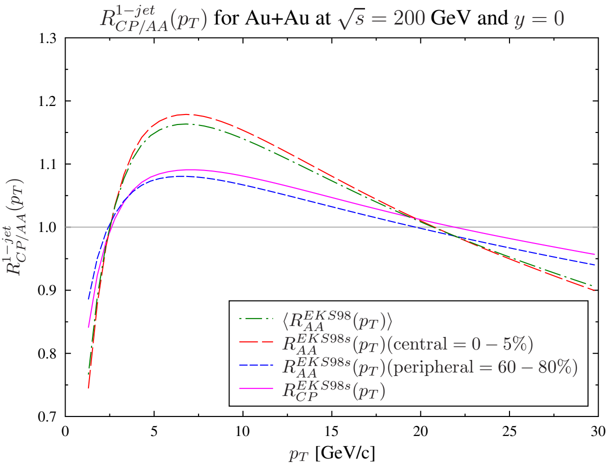
$$R_{CP}^{1-jet} = \frac{\left\langle \frac{\mathrm{d}^2 N_{AA}^{1-jet}}{\mathrm{d}p_T \mathrm{d}y} \right\rangle \frac{1}{\langle N_{bin} \rangle}(central)}{\left\langle \frac{\mathrm{d}^2 N_{AA}^{1-jet}}{\mathrm{d}p_T \mathrm{d}y} \right\rangle \frac{1}{\langle N_{bin} \rangle}(peripheral)}$$

Parameters from Optical Glauber Model

	$central = 0 - 5\%$			$peripheral = 60 - 80\%$		
	b_1 [fm]	b_2 [fm]	$\langle N_{bin} \rangle$	b_1 [fm]	b_2 [fm]	$\langle N_{bin} \rangle$
RHIC	0.0	3.41	1079	11.82	13.65	12.06
LHC	0.0	3.47	1717	12.03	13.89	18.81

- RHIC: $\sigma_{inel}^{NN} = 42$ mb
- LHC: $\sigma_{inel}^{NN} = 62$ mb

Central-to-Peripheral Ratio for RHIC



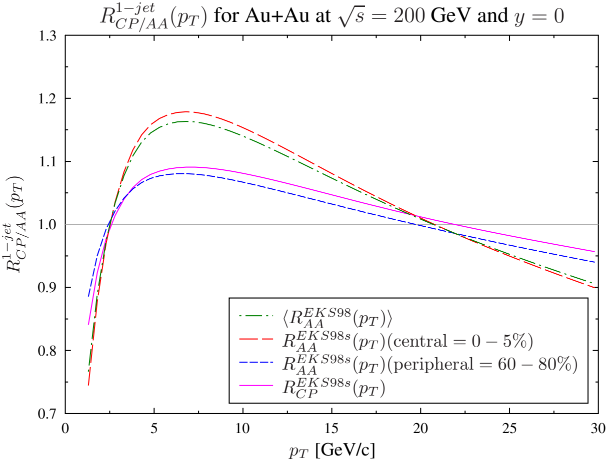
Observations

$R_{AA}(central) \approx \langle R_{AA}\rangle$

$R_{AA}(peripheral) \neq 1$

$R_{CP} \neq \langle R_{AA}\rangle$

Central-to-Peripheral Ratio for RHIC



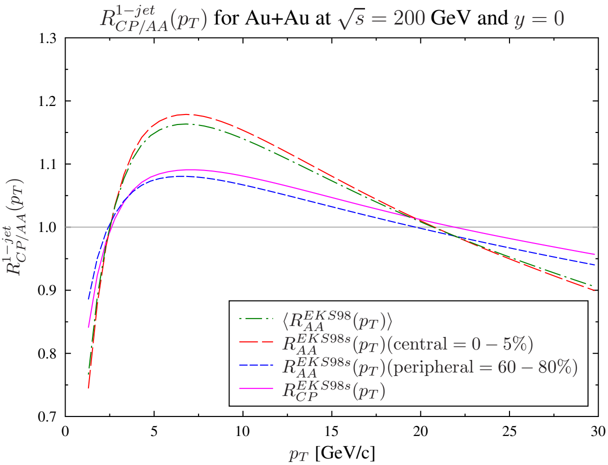
Observations

$$R_{AA}(central) \approx \langle R_{AA}\rangle$$

$$R_{AA}(peripheral) \neq 1$$

$$R_{CP} \neq \langle R_{AA}\rangle$$

Central-to-Peripheral Ratio for RHIC



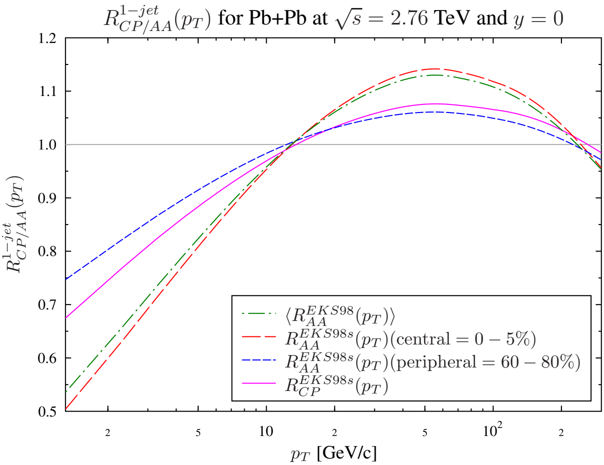
Observations

$R_{AA}(central) \approx \langle R_{AA}\rangle$

$R_{AA}(peripheral) \neq 1$

$R_{CP} \neq \langle R_{AA}\rangle$

Central-to-Peripheral Ratio for LHC



Observations

$R_{AA}(central) \approx \langle R_{AA} \rangle$

$R_{AA}(peripheral) \neq 1$

$R_{CP} \neq \langle R_{AA} \rangle$

Summary

We have

- Developed a model for spatial dependence of nuclear modification based on
  - the A dependence of the EKS98/EPS09 parametrizations (= data!)
  - the nuclear thickness function $T_A(s)$
- Program to calculate $r_A(x, Q^2, \mathbf{s})$
- Calculated R_{AA}^{1-jet} and R_{CP}^{1-jet} in LO
  - $R_{AA}(central) \approx \langle R_{AA} \rangle$
  - $R_{AA}(peripheral) \neq 1$
  - $R_{CP} \neq \langle R_{AA} \rangle$

Summary

We have

- Developed a model for spatial dependence of nuclear modification based on
  - the A dependence of the EKS98/EPS09 parametrizations $(=$ data!)
  - the nuclear thickness function $T_A(s)$
- Program to calculate $r_A(x, Q^2, \mathbf{s})$
- Calculated R_{AA}^{1-jet} and R_{CP}^{1-jet} in LO
  - $R_{AA}(central) \approx \langle R_{AA} \rangle$
  - $R_{AA}(peripheral) \neq 1$
  - $R_{CP} \neq \langle R_{AA} \rangle$

Summary

We have

- Developed a model for spatial dependence of nuclear modification based on
  - the A dependence of the EKS98/EPS09 parametrizations ($=$ data!)
  - the nuclear thickness function $T_A(s)$
- Program to calculate $r_A(x, Q^2, \mathbf{s})$
- Calculated R_{AA}^{1-jet} and R_{CP}^{1-jet} in LO
  - $R_{AA}(central) \approx \langle R_{AA} \rangle$
  - $R_{AA}(peripheral) \neq 1$
  - $R_{CP} \neq \langle R_{AA} \rangle$

Outlook

We will

- Fit the parameter values also for EPS09
- make the codes for $r_A(x, Q^2, \mathrm{s})$ calculation public (EKS98s and EPS09s)

$\Rightarrow$ Nuclear modfications of any hard process in different centrality classes can be computed!

Outlook

We will

- Fit the parameter values also for EPS09
- make the codes for $r_A(x, Q^2, \mathrm{s})$ calculation public (EKS98s and EPS09s)

$\Rightarrow$ Nuclear modfications of any hard process in different centrality classes can be computed!

Backup

Centrality Classes for RHIC