Transport Coefficients of Deconfined Strongly Interacting Matter:

From Weak to Strong Coupling

Marcus Bluhm
SUBATECH, Nantes, France

with Burkhard Kämpfer and Krzysztof Redlich

NeD2011
Heraklion, Crete, Greece - 31/8/2011

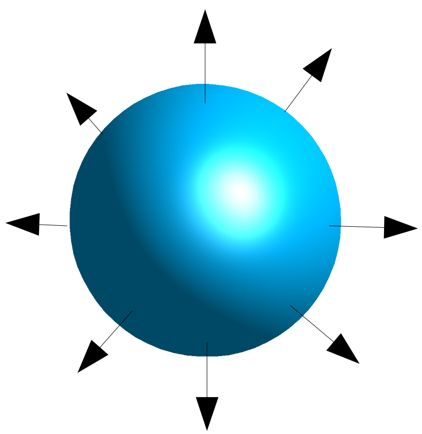
Motivation – Bulk and Shear Viscosities

system w/o conserved charge number density or particle-antiparticle symmetric systems

$$\partial_\mu T^{\mu\nu} = 0$$

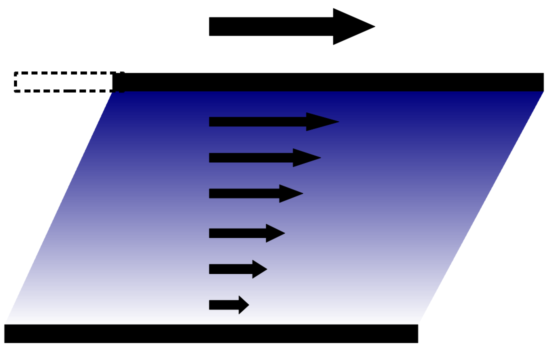
$$T^{\mu\nu} = T^{\mu\nu}_{(0)}(\epsilon, P; u^\mu) + T^{\mu\nu}_{(1)}(\zeta, \eta; u^\mu, \partial_\nu u^\mu)$$

ζ : bulk (volume) viscosity



change in volume **BUT** constant shape/form

η : shear viscosity



change in shape **BUT** constant volume

Viscosity of the QGP?

water as reference: $\eta \sim 10^{-3} Pa \cdot s$

honey: $\eta \sim 2 - 3\, Pa \cdot s$

pitch: $\eta \sim 2.3 \cdot 10^{8}\, Pa \cdot s$

specific holographic models: $\eta / s = 1/4\pi$ (cf. water at minimum: $\eta / s = 2 - 3$)

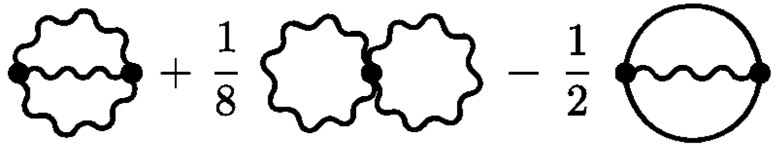
➡ application of ideal hydrodynamics modelling heavy-ion collisions at RHIC and LHC suggests at most small dissipative effects; viscous calculations confirm this

Quasiparticle Modell (QPM)

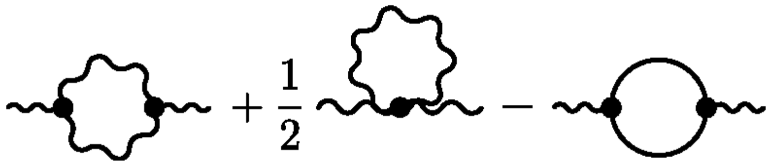
QPM based on Φ - functional approach to QCD:

$$\frac{\Omega[D,S]}{T} = \frac{1}{2}\mathrm{Tr}\left[\ln D^{-1} - \Pi D\right] - \mathrm{Tr}\left[\ln S^{-1} - \Sigma S\right] + \Phi[D,S]$$

$\Phi = \frac{1}{12}$ … $+ \frac{1}{8}$ … $- \frac{1}{2}$ …



$\Pi = \frac{1}{2}$ … $+ \frac{1}{2}$ … $-$ … $, \quad \Pi = 2\frac{\delta\Phi}{\delta D}$



$\Sigma =$ … $, \quad \Sigma = -\frac{\delta\Phi}{\delta S}$

- modell for equilibrium thermodynamics ► corresponding energy-momentum tensor:

$$T^{\mu\nu}_{(0)}(T) = \sum_i \int \frac{d^3\vec{p}}{(2\pi)^3 E_i(T)} p^\mu p^\nu f_i^{(0)} + g^{\mu\nu} B[\{\Pi_j(T)\}]$$

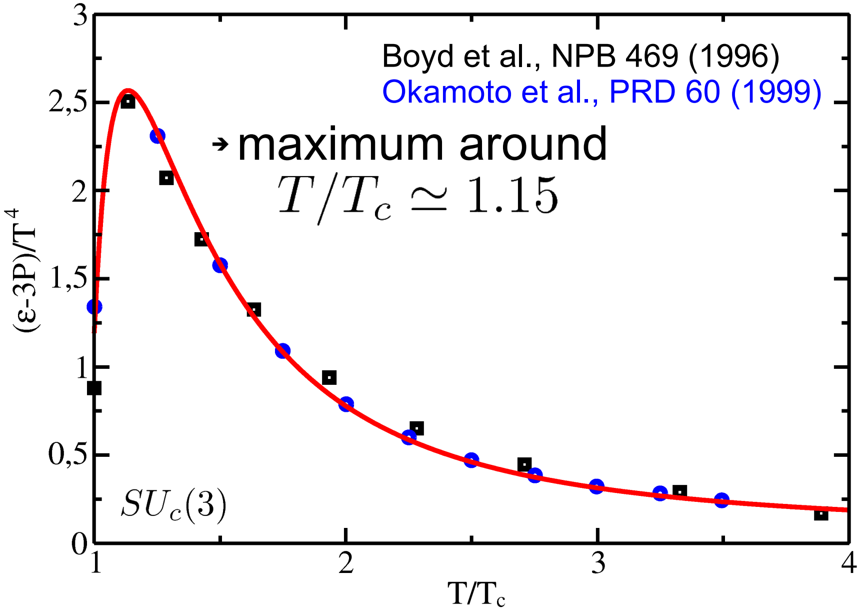
for excitations with medium-modified dispersion relations (thermal mass) $E_i^2(T) = \vec{p}^{\,2} + \Pi_i(T)$

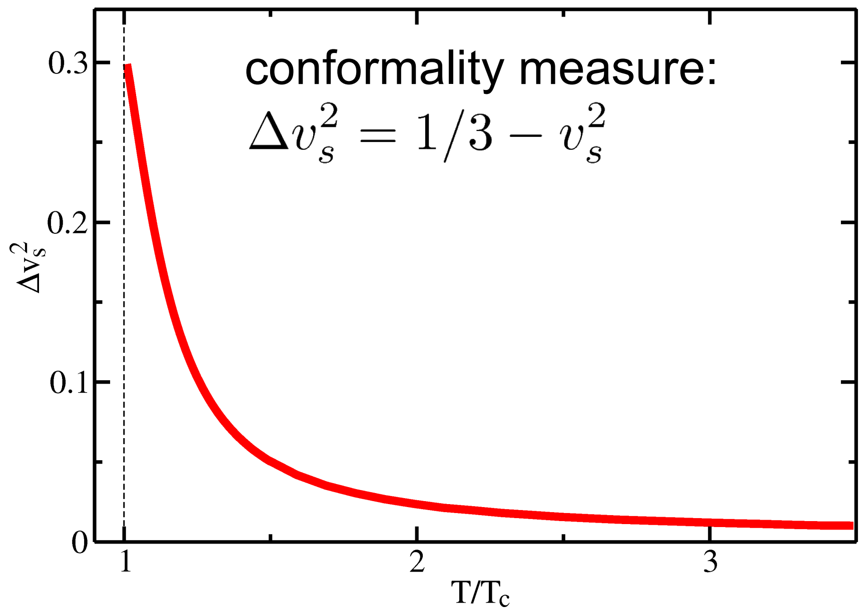
Thermal Equilibrium – Example: Gluodynamics

$$\Pi_g(T) = \frac{1}{2} T^2 G^2(T)\,, \text{where } G^2(T) = 16\pi^2 / \left(11 \log\left[\lambda(T - T_s)/T_c\right]^2\right)$$

energy density: $\epsilon = T^{\mu\nu}_{(0)} u_\mu u_\nu$

pressure: $P = T^{\mu\nu}_{(0)} (u_\mu u_\nu - g_{\mu\nu})/3$





Effective Kinetic Theory

- self-consistent generalization of $T^{\mu\nu}_{(0)}$ to non-equilibrium systems:
- space-time dependence of $T(x)$ implies $E = E(x)$ is a functional of the distribution function $f(x,p)$
- to assure **basic relations**:
  - $\partial_\mu T^{\mu\nu}(x) = 0$
  - $\delta \langle T^{00} \rangle / \delta f(x,p) = E$ (Fermi liquids)
  - in thermal equilibrium: $\epsilon + P = T \dfrac{\partial P}{\partial T}$

one generalizes (in case of a one-component system) to

$$T^{\mu\nu}(x) = \int \frac{d^3\vec{p}}{(2\pi)^3 E(x)} p^\mu p^\nu f(x,p) + g^{\mu\nu} B[\Pi(x)]$$

kinetic term **potential term**

cf. Jeon (1996)

$T^{\mu\nu}$ closely related to **effective kinetic equation of Boltzmann-Vlasov type** for the single-particle distribution function $f(x,p)$: $(\mathcal{L} + \mathcal{V}) f = \mathcal{C}[f]$

- above conditions satisfied **if** $\dfrac{\partial B}{\partial \Pi} = -\dfrac{1}{2} \displaystyle\int \dfrac{d^3\vec{p}}{(2\pi)^3 E(x)} f(x,p)$ related to form of Vlasov-term

Bulk and Shear Viscosity Coefficients

for quasiparticle systems in relaxation time approximation

Bulk and Shear Viscosities

- decompose $T^{\mu\nu}$ and compare w/ definition: $T^{\mu\nu}_{(1)} = \zeta\,\Delta^{\mu\nu}\partial_\alpha u^\alpha + \eta\,S^{\mu\nu}{}_{\alpha\beta}\partial^\alpha u^\beta$

bulk viscosity:

$$\zeta = \frac{1}{T}\int\frac{d^3\vec{p}}{(2\pi)^3 E}f^0(1+d^{-1}f^0)\frac{\tau}{E} \times\left\{\left[\left(\frac{pu}{T}\right)^2 - \frac{1}{2T}\frac{\partial\Pi}{\partial T}\right]T^2 v_s^2 + \frac{1}{3}[p^2-(pu)^2]\right\}^2$$

shear viscosity:

$$\eta = \frac{1}{15T}\int\frac{d^3\vec{p}}{(2\pi)^3 E}f^0(1+d^{-1}f^0)\frac{\tau}{E}[p^2-(pu)^2]^2$$

cf. Chakraborty, Kapusta (2010)
& MB, Kämpfer, Redlich (2009,'10,'11)

differences: Excitations with constant vs. thermal mass

bulk viscosity:

$$\zeta = \frac{1}{T}\int \frac{d^3\vec{p}}{(2\pi)^3 E} f^0 (1 + d^{-1} f^0) \frac{\tau}{E} \times \left\{ \left[\left(\frac{pu}{T}\right)^2 - \frac{1}{2T}\frac{\partial \Pi}{\partial T} \right] T^2 v_s^2 + \frac{1}{3}[p^2 - (pu)^2] \right\}^2$$

shear viscosity:

$$\eta = \frac{1}{15T}\int \frac{d^3\vec{p}}{(2\pi)^3 E} f^0 (1 + d^{-1} f^0) \frac{\tau}{E} [p^2 - (pu)^2]^2$$

cf. Gavin (1985)

Relaxation Time

- collision processes relevant for **shear** and **bulk viscosities** different; **assumption**: same τ, independent of $|\vec{p}|$

- concentrate on SU(3): 2 ◄► 2 gluon-gluon scatterings

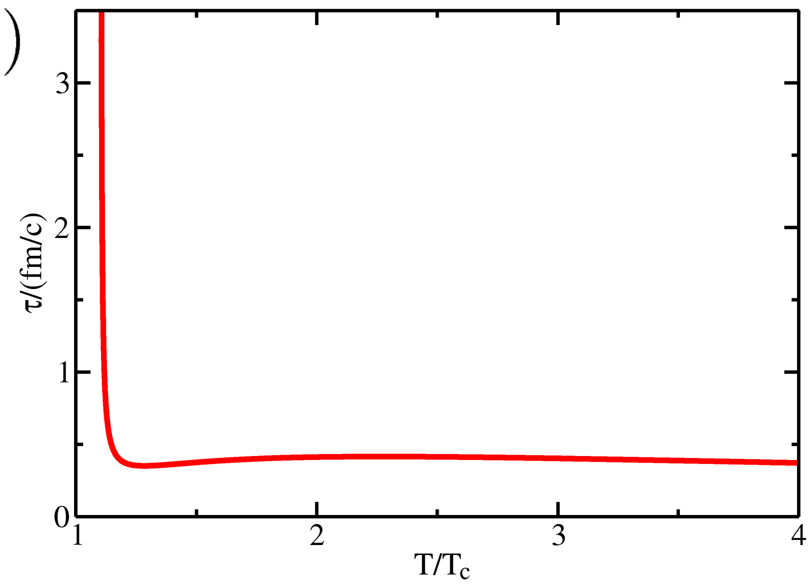
parametrically

$$\tau^{-1} \sim T G^4(T) \ln(a/G^2(T))$$

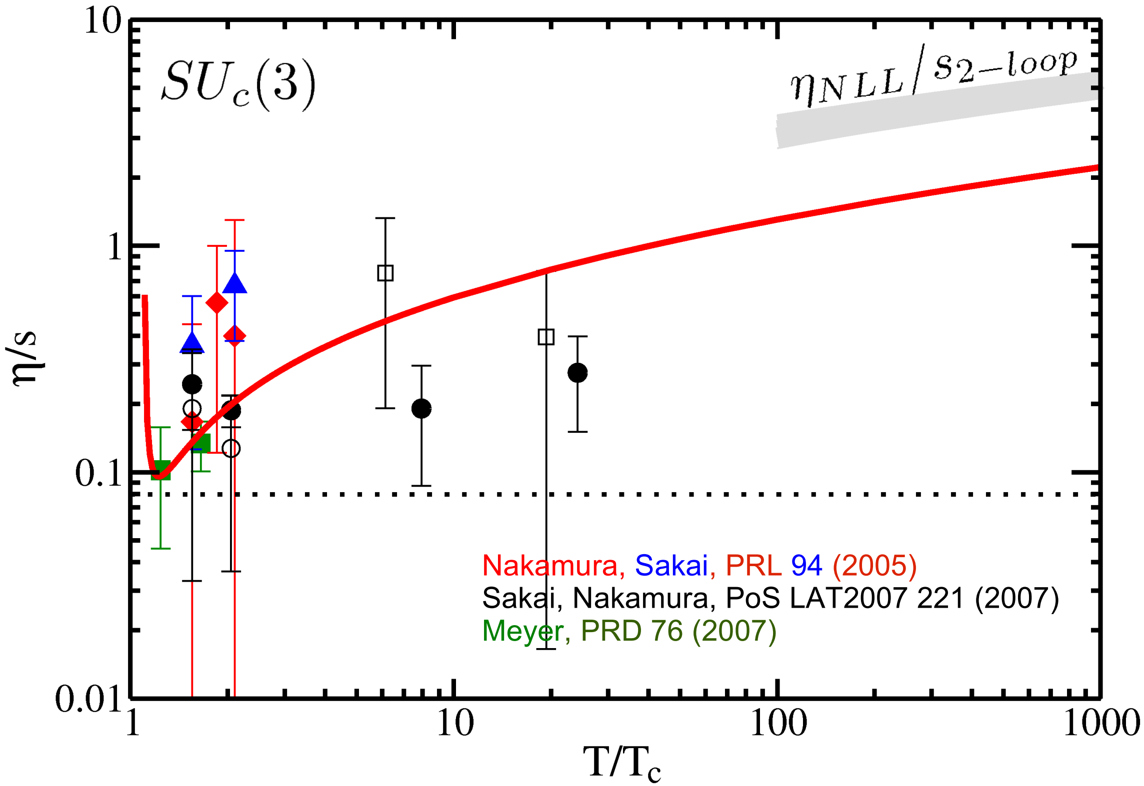
based on perturbative considerations

cross section depends crucially on ratio of maximum to minimum momentum transfer $\sim a$

cf. Heiselberg (1993)



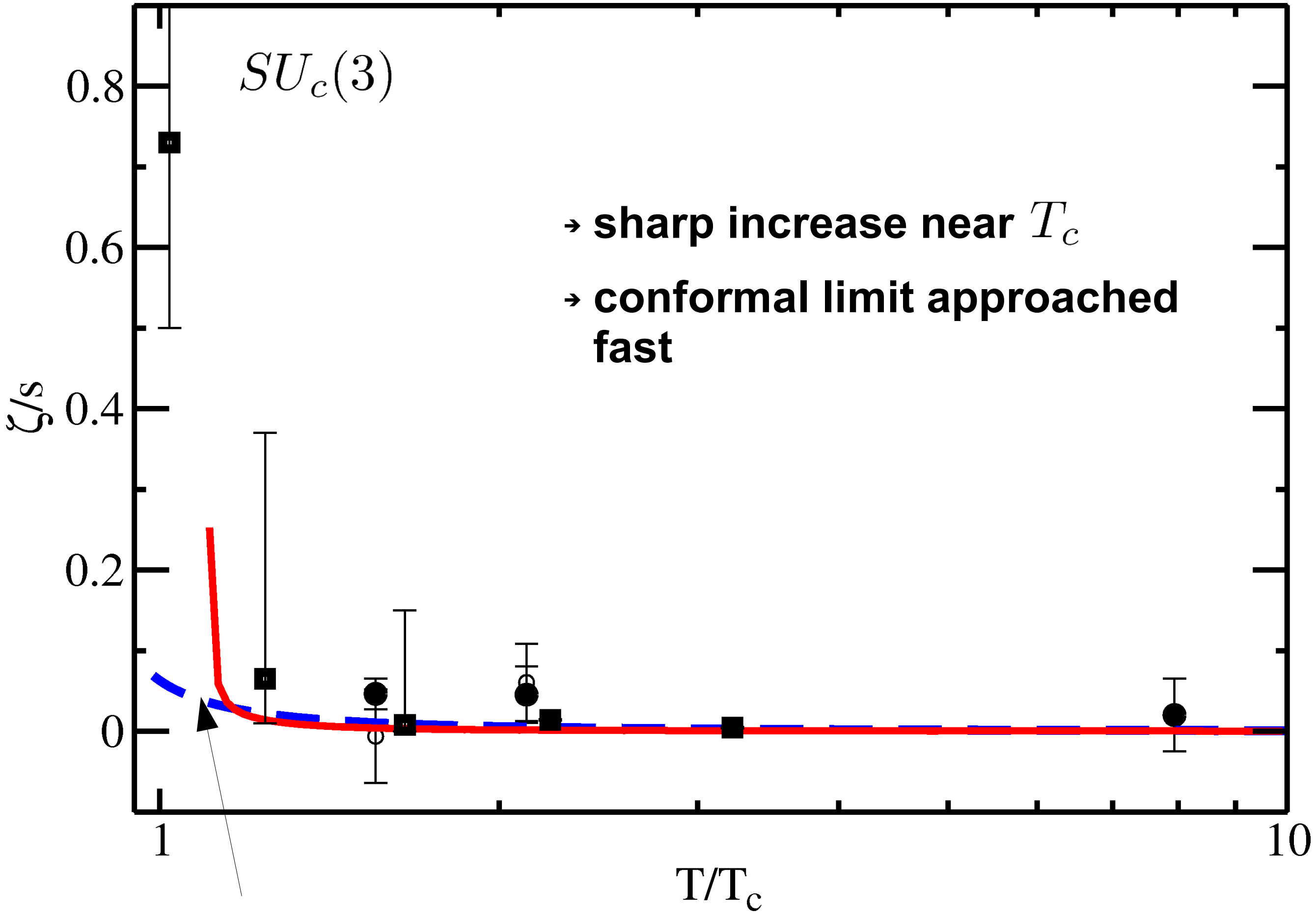
- parametric dependencies of pQCD results for ζ and η on coupling and temperature reproduced at large T



- **behaviour close to T_c driven by τ:** minimum near T_c , can be as small as $1/4\pi$
- **perturbative limit approached slowly**

cf. MB, Kämpfer & Redlich Phys. Rev. C 84 (2011)

Quantitative Results – Specific Bulk Viscosity



- sharp increase near T_c
- conformal limit approached fast

holographic QCD,
cf. Gürsoy et al. (2009)

cf. MB, Kämpfer & Redlich Phys. Rev. C 84 (2011)

Ratio of Bulk to Shear Viscosities

Big Theoretical Motivation: Viscosity coefficients in strongly interacting Quantum Field Theories can be deduced from Black Hole Physics

- Kovtun-Son-Starinets bound: $\eta/s \geq 1/(4\pi)$

Bulk to Shear Viscosity Ratio

Big Theoretical Motivation: Viscosity coefficients in strongly interacting Quantum Field Theories can be deduced from Black Hole Physics

- Kovtun-Son-Starinets bound: $\eta/s \geq 1/(4\pi)$

- similar *universal* bounds for other transport coefficients are unknown **BUT** in some special classes of theories with holographically dual supergravity description there exists a lower bound for the ratio

Buchel bound: $(\zeta/\eta)_B \geq 2\left(\frac{1}{k} - v_s^2\right)$

Bulk to Shear Viscosity Ratio

Big Theoretical Motivation: Viscosity coefficients in strongly interacting Quantum Field Theories can be deduced from Black Hole Physics

- Kovtun-Son-Starinets bound: $\eta/s \geq 1/(4\pi)$

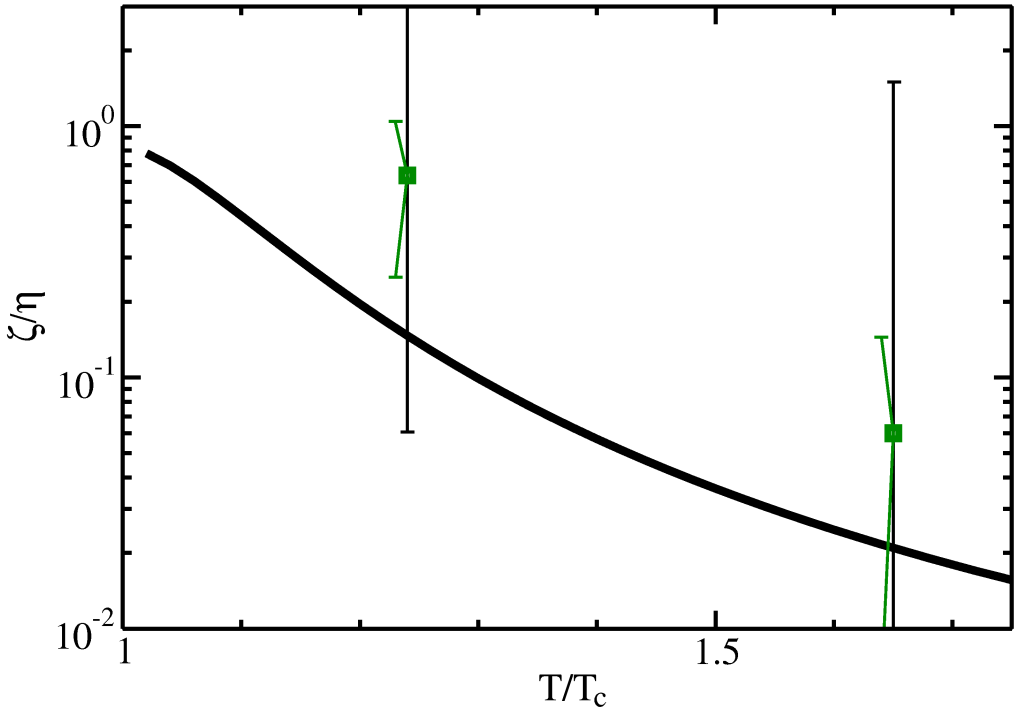
- similar *universal* bounds for other transport coefficients are unknown **BUT** in some special classes of theories with holographically dual supergravity description there exists a lower bound for the ratio

Buchel bound: $(\zeta/\eta)_B \geq 2\left(\frac{1}{k} - v_s^2\right)$

- specific strongly coupled but nearly conformal theories (AdS/CFT) $\zeta/\eta \sim \Delta v_s^2 \equiv \left(\frac{1}{3} - v_s^2\right)$
- for scalar theory or photons in hot fluid $\zeta/\eta = 15\left(\Delta v_s^2\right)^2$ parametrically correct also in pQCD (weak coupling)

- might expect that there is a gradual change from one behaviour to the other as a function of temperature

Bulk to Shear Viscosity Ratio – Numerical Result



- temperature behaviour of viscosity ratio consistent with lattice QCD results
- Near T_c , bulk viscosity $\sim$ shear viscosity

Bulk to Shear Viscosity Ratio – Analytic Behaviour

$$\frac{\zeta}{\eta} = 15\left(\Delta v_s^2\right)^2\left[1-\mathcal{A}_0+\frac{1}{4}\mathcal{A}_2\right]+5\Delta v_s^2\left[\mathcal{A}_0-\frac{1}{2}\mathcal{A}_2\right]+\frac{5}{12}\mathcal{A}_2$$

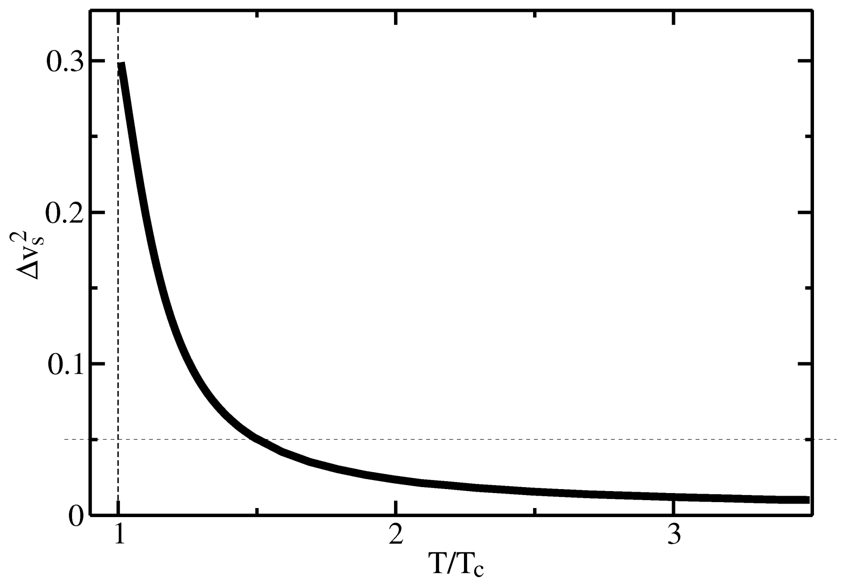
→ $\mathcal{A}_{0,2} = \mathcal{A}_{0,2}[dG^2/dT]$ non-perturbative

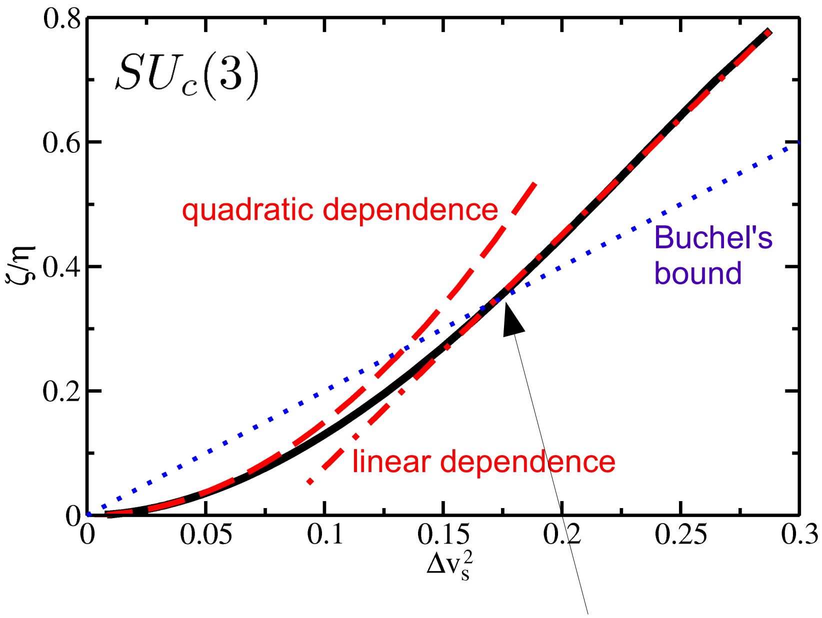
- large T: $\Delta v_s^2 \sim T\frac{dG^2}{dT} + \mathcal{O}\left(G^2 T\frac{dG^2}{dT}\right)$

  $\zeta/\eta \sim \left(\Delta v_s^2\right)^2$

- for $T \to T_c^+$: $\Delta v_s^2 \to A$

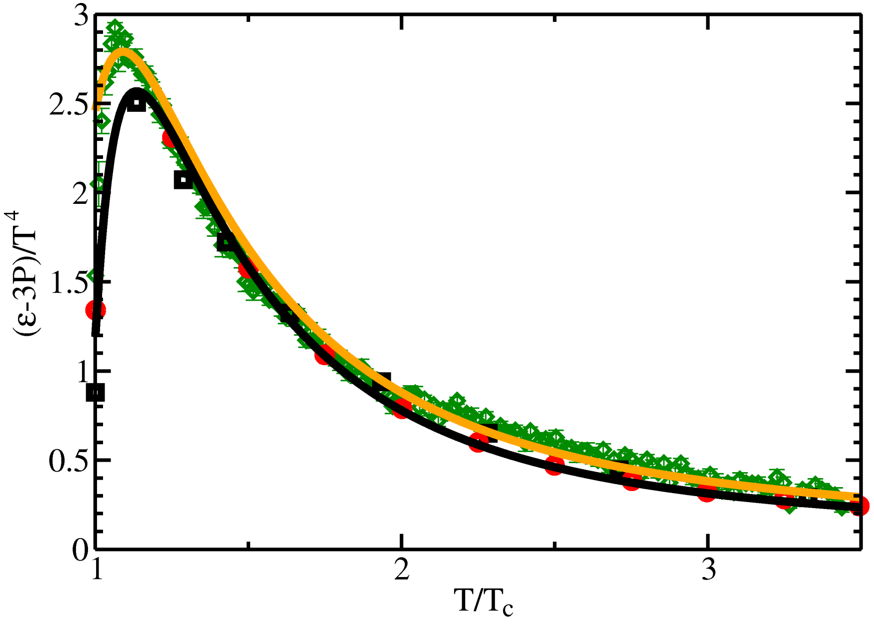
  $\zeta/\eta = \alpha\,\Delta v_s^2 + \beta$

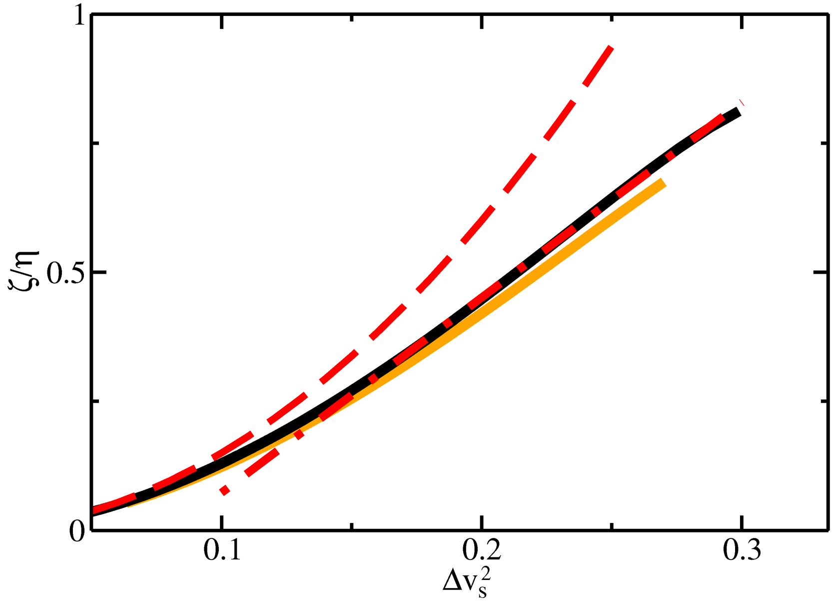




➡ linear dependence on Δv_s^2 and Buchel's bound satisfied for $T \leq 1.15\,T_c$

Bulk to Shear Viscosity Ratio – Sensitivity on EoS





➡ qualitative behaviour rather insensitive to details in the EoS

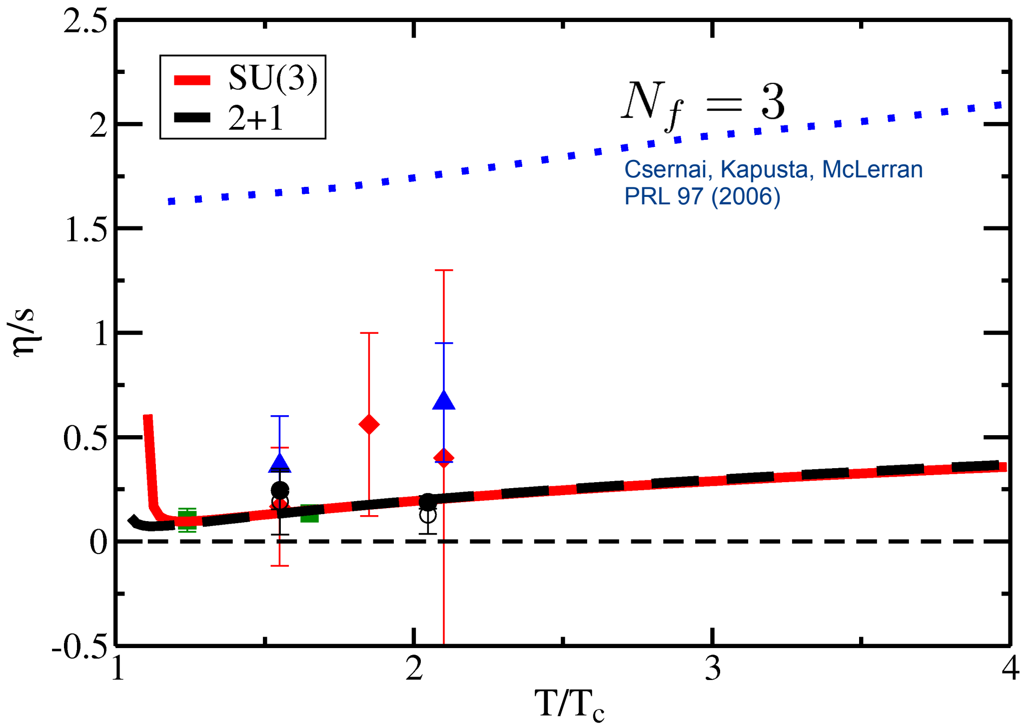
Estimating the QGP Specific Shear Viscosity

inclusion of quark degrees of freedom by assuming that relations between gluon and quark sector known from perturbative regime hold close to T_c

➡ **leading-order estimate:**

$$\eta = \eta_g + \eta_q \text{ (additive)}$$

$$\eta_q \simeq 2.2 \frac{(1 + 11N_f/48)}{(1 + 7N_f/33)} N_f \, \eta_g$$



→ **mild overall increase with T; still small at $3T_c$**

Energy Loss Parameter

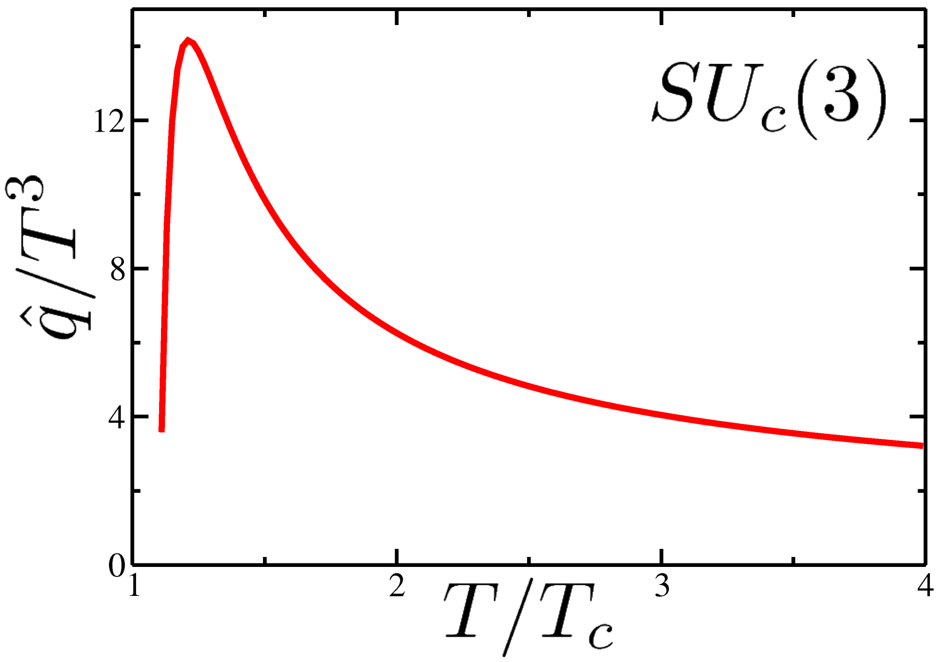
relation between η/s and averaged transverse momentum transfer squared per unit distance of an energetic parton $\hat{q}$

cf. Majumder, Müller, Wang (2007)

$$\eta \sim \frac{1}{3}\rho\langle p\rangle\lambda$$

$$\Rightarrow \quad \hat{q} \simeq \frac{1}{12}\frac{\rho}{s}\langle p\rangle\langle\hat{s}\rangle\left(\frac{\eta}{s}\right)^{-1}$$

underlying assumption:
interaction between energetic parton and medium is of same structure and strength as interaction among thermal excitations



➡ **minimum in η/s implies maximum in $\hat{q}/T^3$**

Conclusions

picture: excitations with effective thermal mass

- inclusion of mean field term in energy-momentum tensor necessary for self-consistency of the approach
- follows from kinetic equation of Boltzmann-Vlasov type

transport coefficients:

- fairly nice agreement w/ available lQCD data ($SU_c(3)$); specific shear viscosity as small as $1/4\pi$
- ratio of bulk to shear viscosities exhibits both quadratic and linear dependence on conformality measure; turning point located at the maximum in the scaled interaction measure
- pronounced temperature dependence in energy loss parameter