

Quarkonia Production in Heavy Ion Collisions

Che-Ming Ko, Texas A&M University

- Introduction
 - Quarkonia in vacuum
 - J/ψ in QGP
- Quarkonia production mechanisms in HIC
 - Statistical model (regeneration)
 - Two-component model (primordial + regeneration)
- Nuclear modification factor for J/ψ
- Nuclear modification factor for $Y(1S)$
- J/ψ elliptic flow
- Summary

Based on work with Taesoo Song and Kyongchol Han: arXiv:1103.6197 [nucl-th]; PRC in press; and PRC 83, 014914 (10)

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Quarkonia in vacuum

State	$J/\psi(1S)$	$\chi_c(1P)$	$\psi'(2S)$
m (GeV/c ²)	3.10	3.53	3.68
r (fm)	0.5	0.72	0.90
Contribution to J/ψ @RHIC (%)	60	30	10

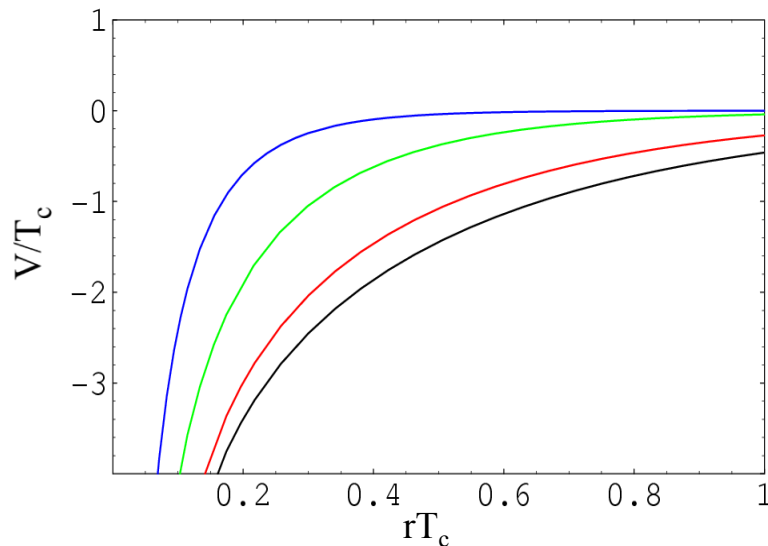
State	$\Upsilon(1S)$	$\chi_b(1P)$	$\Upsilon'(2S)$	$\chi'_b(2P)$	$\Upsilon''(3S)$
m (GeV/c ²)	9.46	9.99	10.02	10.26	10.36
r (fm)	0.28	0.44	0.56	0.68	0.78
Contribution to $\Upsilon(1S)$ @RHIC (%)	51	27	11	10	1

J/ψ properties in QGP

- Perturbative QCD → screening mass

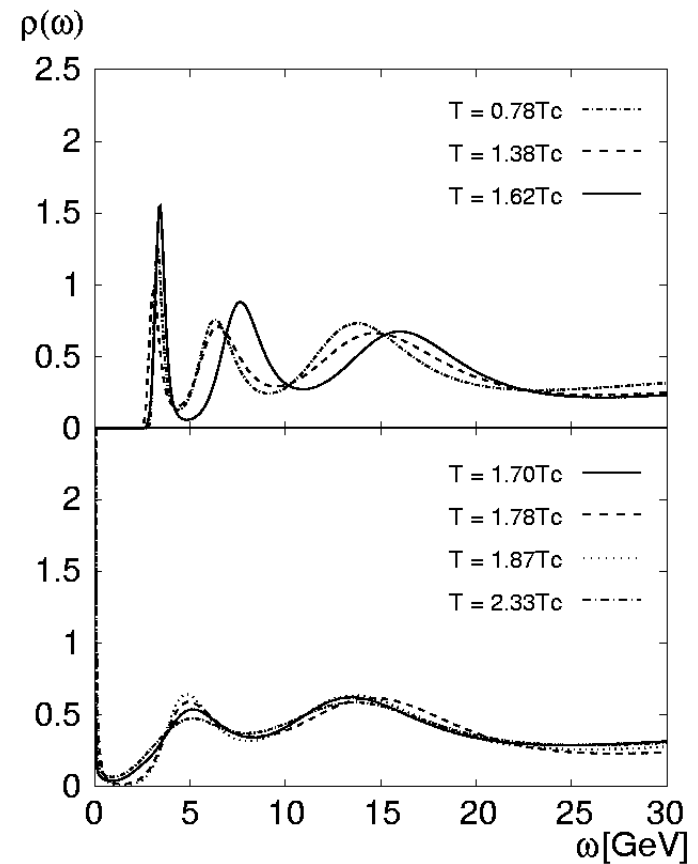
$$V = -\frac{\alpha_s}{r} \rightarrow V = -\frac{\alpha_s}{r} e^{-r/\lambda_D}$$

$$\lambda_D = \left(\frac{N_c}{3} + \frac{N_f}{6} \right)^{-\frac{1}{2}} (gT)^{-1} \approx \sqrt{\frac{2}{3}} (gT)^{-1}$$



→ J/ψ suppression in HIC
(Matsui & Satz, PLB 178, 416 (1986))

- Lattice QCD (Asakawa & Hatuda, Karsch et al.)



→ J/ψ survives below 1.62~1.70T_c

Dissociation temperature in potential model

Free energy F for a pair of $Q\bar{Q}$ from LQCD
(Kaczmarek, EJP 61, 811 (2009))

Two limits of the potential:

$$V(r, T) = \begin{cases} F, & \text{slow dissociation} \\ U = F + TS, & \text{rapid dissociation} \end{cases}$$

Schroedinger equation at finite T :

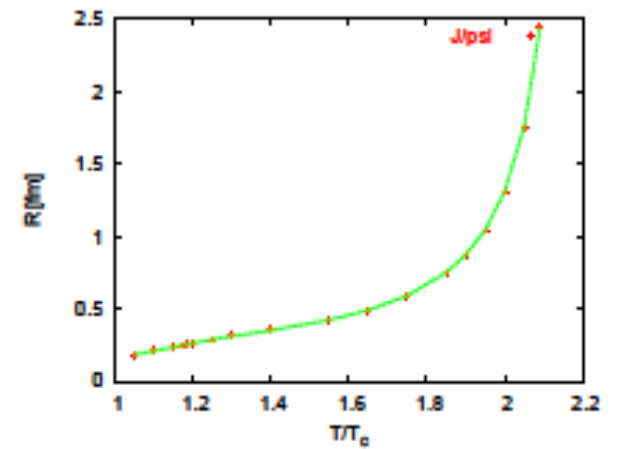
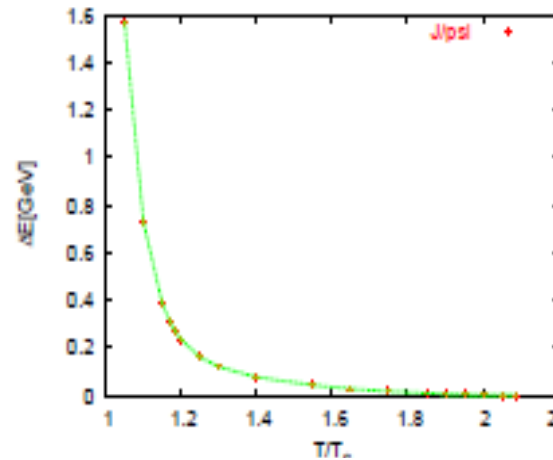
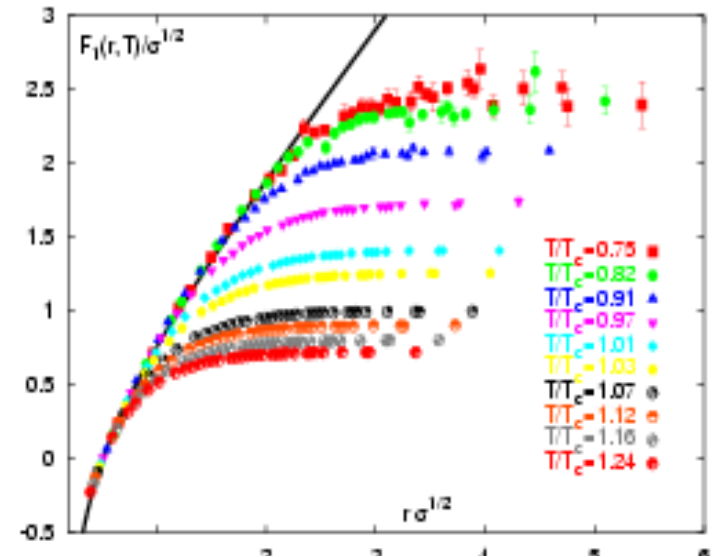
binding energy $\varepsilon(T)$

radius $R(T)$

Dissociation temperature:

$$\varepsilon(T_D) \rightarrow \infty, R(T_D) \rightarrow \infty$$

For $V=U$ (Satz et al.)



state	$J/\psi(1S)$	$\chi_c(1P)$	$\psi'(2S)$	$\Upsilon(1S)$	$\chi_b(1P)$	$\Upsilon(2S)$	$\chi_b(2P)$	$\Upsilon(3S)$
T_d/T_c	2.10	1.16	1.12	> 4.0	1.76	1.60	1.19	1.17

Charm and anticharm quark potential in QGP

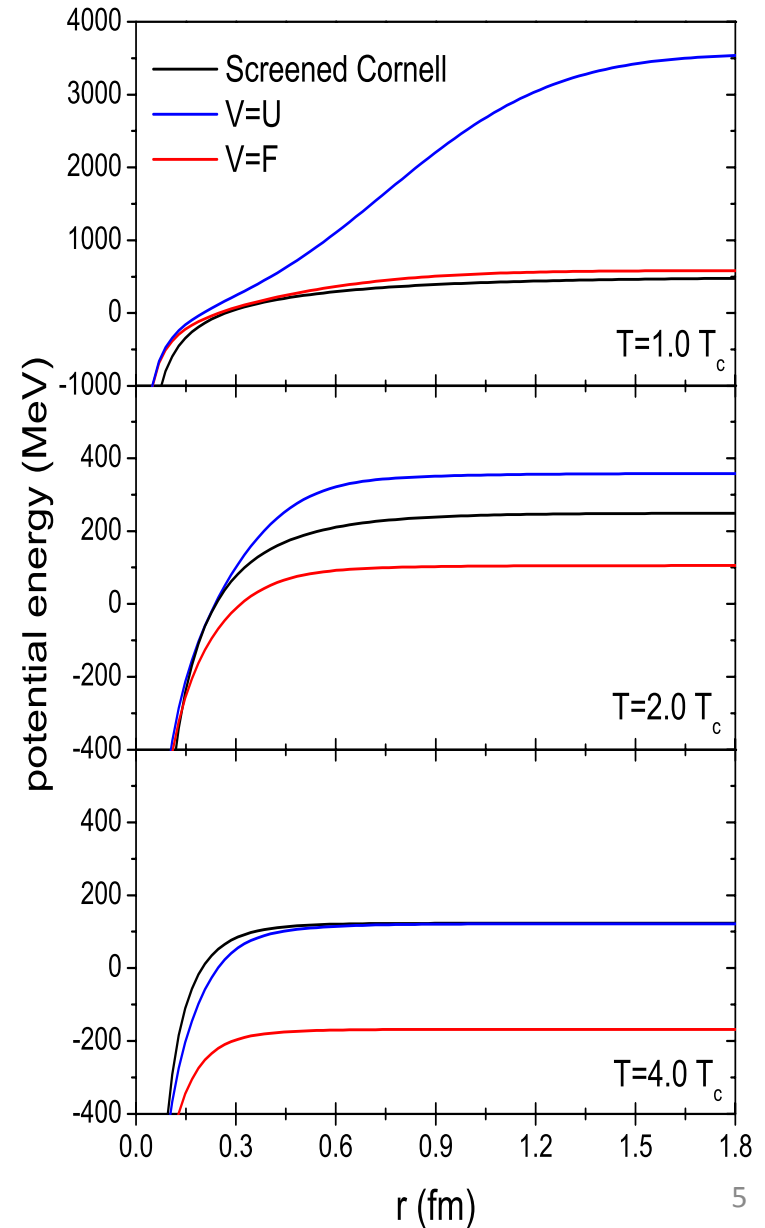
- Screened Cornell potential between charm and anticharm quarks

$$V(r,T) = \frac{\sigma}{\mu(T)} \left[1 - e^{-\mu(T)r} \right] - \frac{\alpha}{r} e^{-\mu(T)r}$$

with string tension $\sigma = 0.192 \text{ GeV}^2$
and screening mass

$$\mu(T) = \sqrt{\frac{N_c}{3} + \frac{N_f}{6}} gT$$

- Its strength is between the internal energy (U) and free energy (F) of heavy quark and antiquark from LQCD; similar to F at T_c and to U at $4T_c$.



Thermal properties of charmonia

- Binding energy

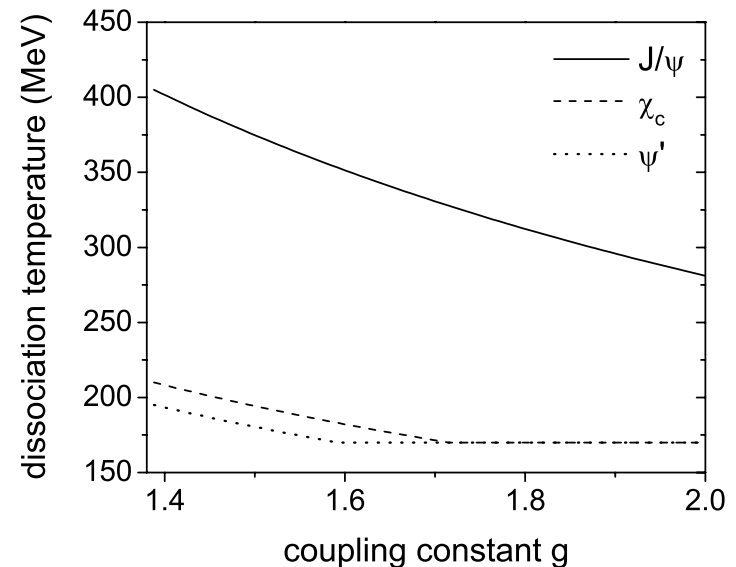
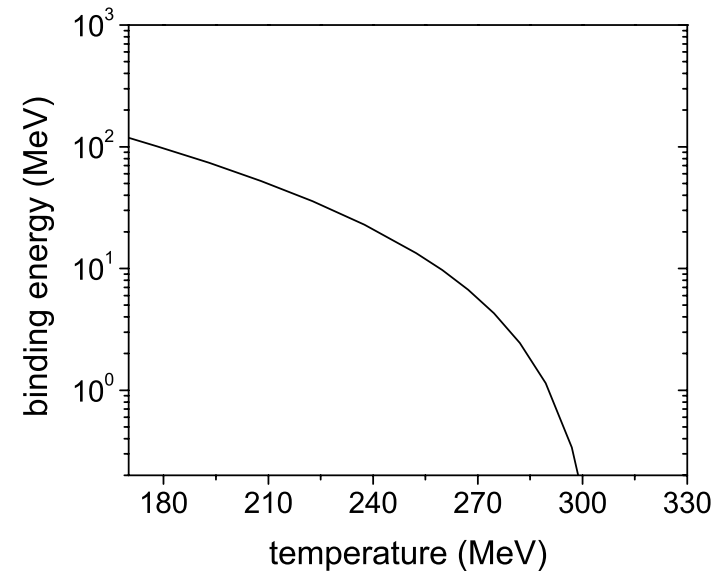
$$\varepsilon_0 = 2m_c + \frac{\sigma}{\mu(T)} - E$$

Charm quark mass $m_c = 1.32 \text{ GeV}$

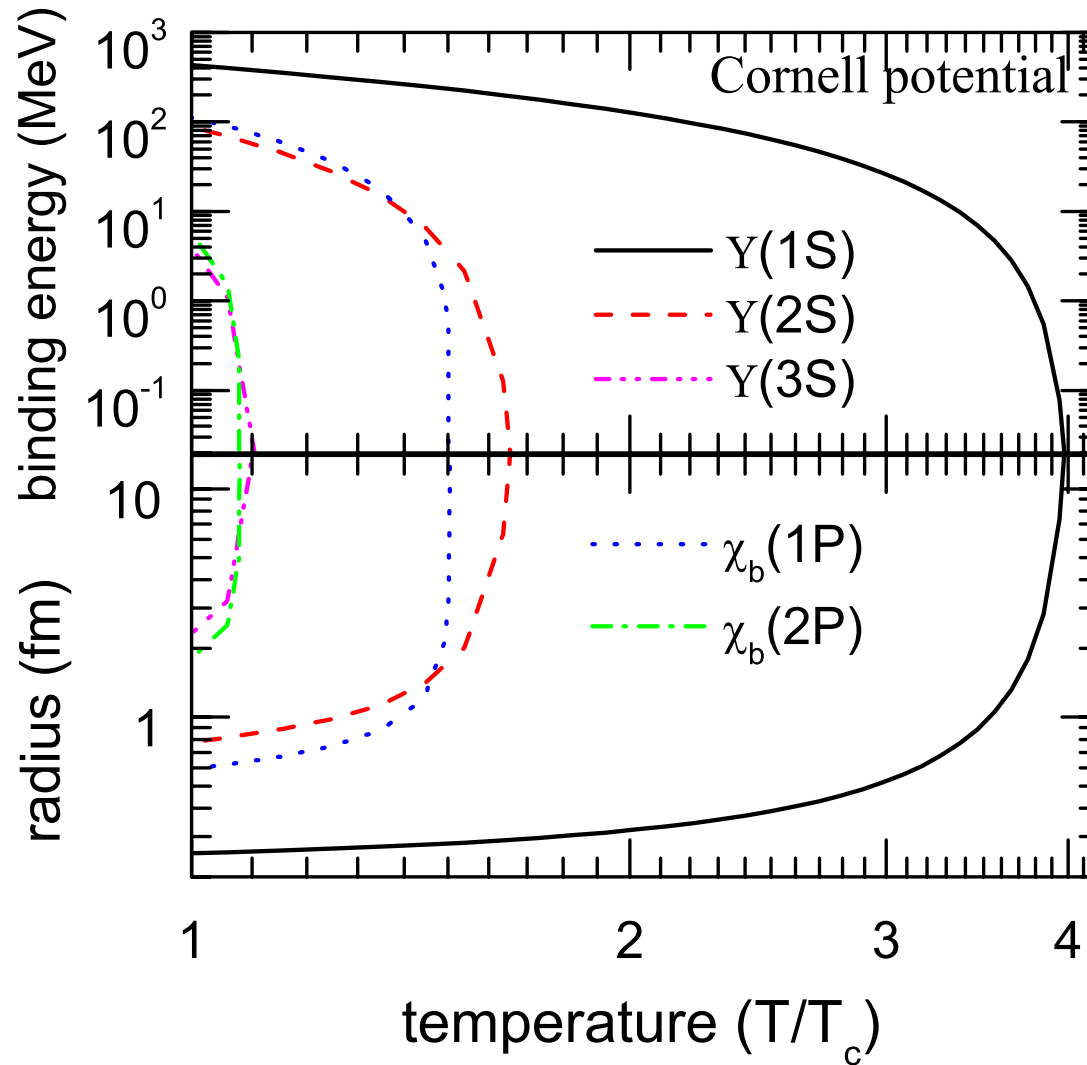
E : eigenvalues of Cornell potential

- Dissociation temperature T_D :
corresponding to $\varepsilon_0 = 0$

For $g = 1.87$, $T_D \sim 300 \text{ MeV}$ for J/ψ
and $\sim T_c = 175 \text{ MeV}$ for ψ' and χ_c



Thermal properties of bottomonia

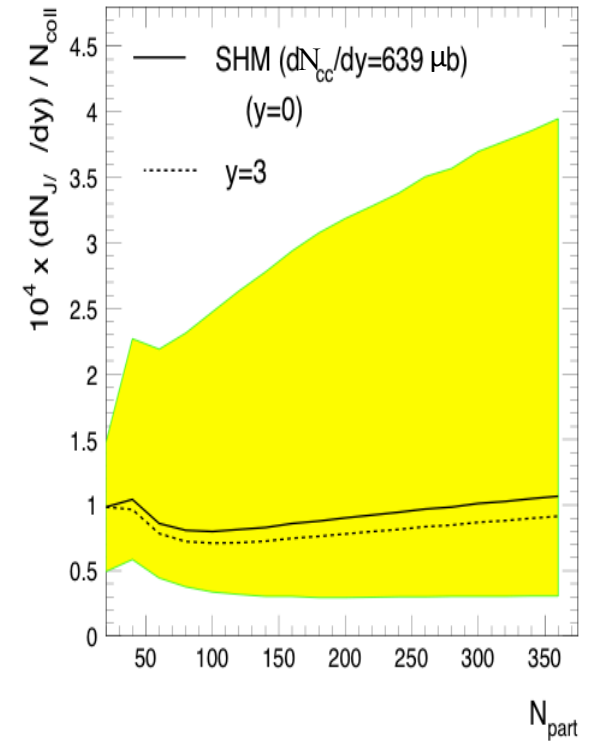
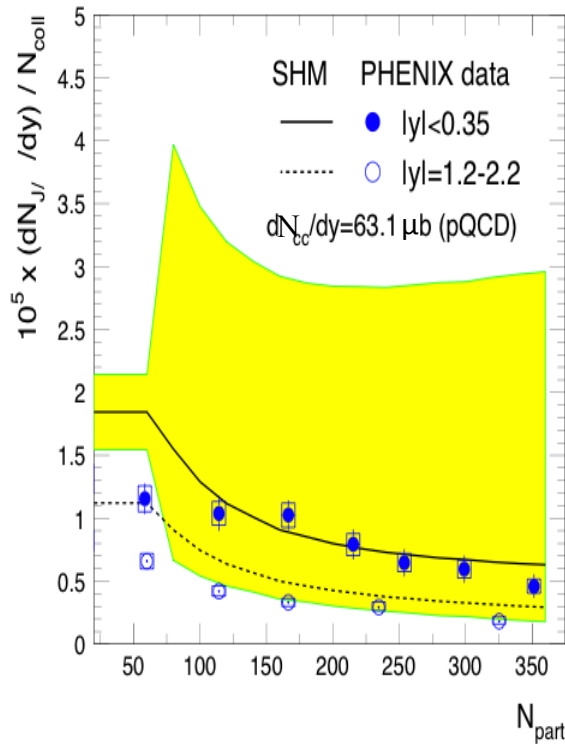
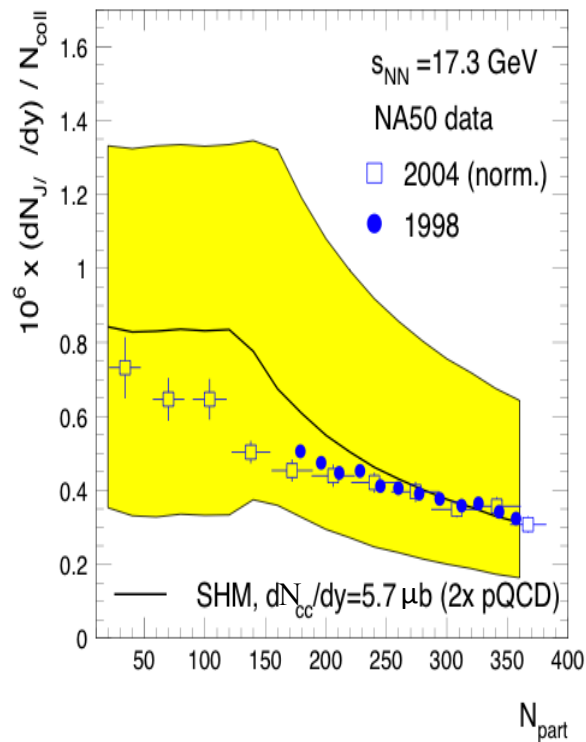


State	$\Upsilon(1S)$	$\chi_b(1P)$	$\Upsilon'(2S)$	$\chi_b'(2P)$	$\Upsilon''(3S)$
Dissociation temp (T_c)	4	1.51	1.67	1.09	1.12

Statistical hadronization model for J/ψ production

Andronic, Braun-Munzinger, Redlich & Stachel, NPA 789, 334 (2007)

$$N_{J/\psi} = \frac{g}{2p^2} \gamma_c^2 \int_0^\infty \frac{p^2 dp}{e^{\sqrt{m^2 + p^2}/T} + 1}$$



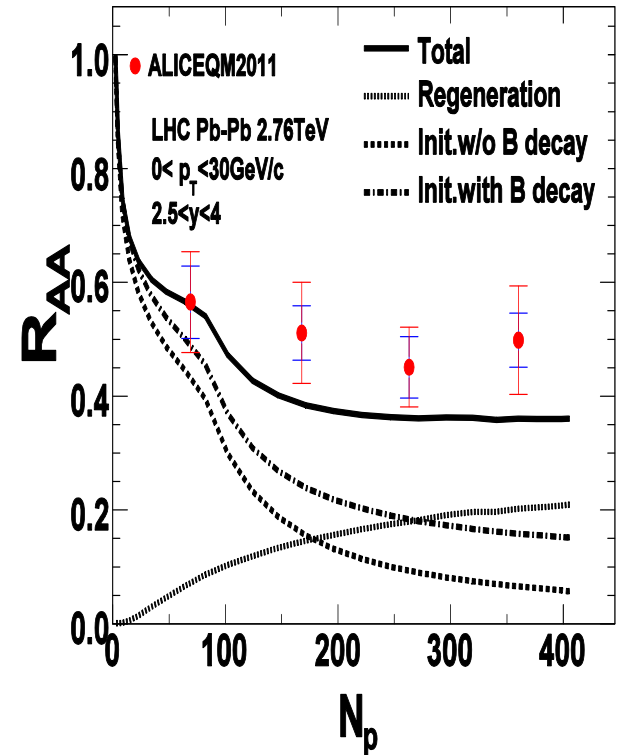
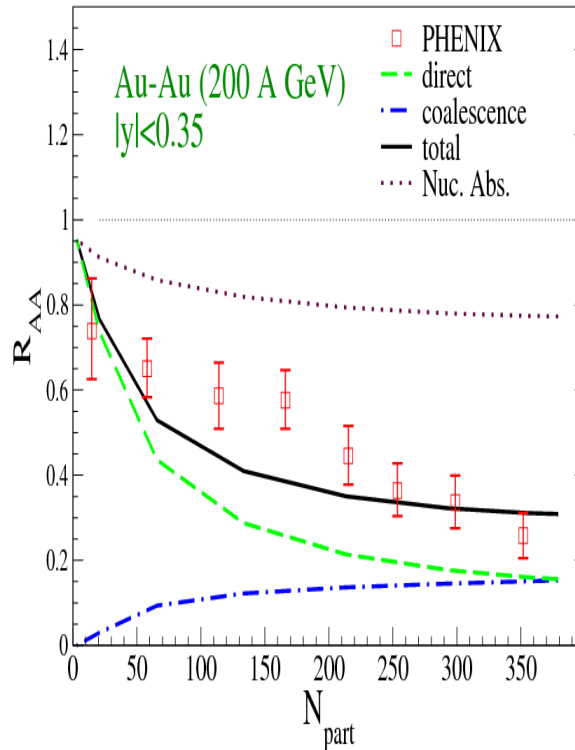
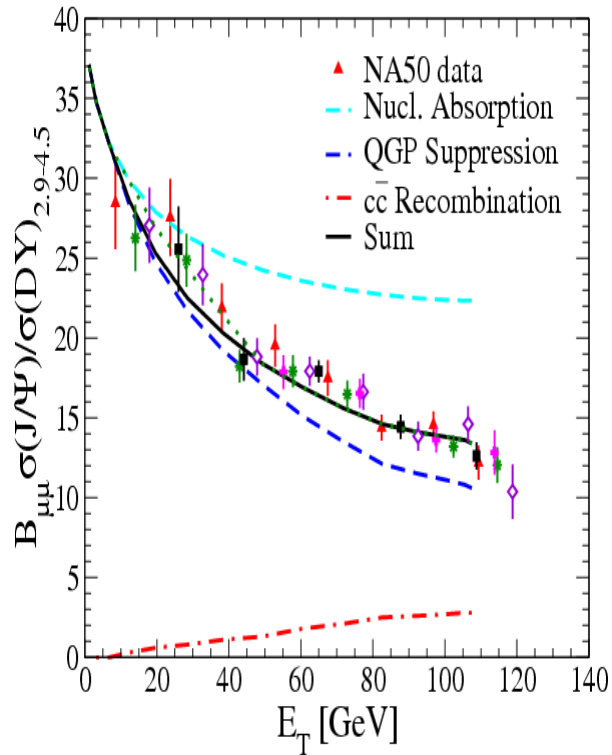
- Results are sensitive to the number of produced charm quark pairs.

Two component model for J/ψ production

- Nuclear absorption: $J/\psi + N \rightarrow D + \Lambda_c$; p+A data $\rightarrow \sigma \sim 6$ mb
- Absorption and regeneration in QGP: $J/\psi + g \leftrightarrow c\bar{c}$, $J/\psi + g \leftrightarrow c\bar{c}g$
- Absorption and regeneration in hadronic matter: $J/\psi + \pi \leftrightarrow D\bar{D}$

Zhao & Rapp, EPJ 62, 109 (2009)

Zhuang et al.



- Regeneration from coalescence of charm quarks becomes increasingly important as the centrality and collision energy increase, as first pointed out by Thews et al.

The two-component model: directly produced J/ψ

Song, Park & Lee,
PRC 81, 034914 (10)

- Number of initially produced

$$N_{J/\psi}^{AA} = \sigma_{J/\psi}^{NN} A^2 T_{AA}(\vec{b})$$

- $\sigma_{J/\psi}^{NN}$: J/ψ production cross section in NN collision; $\sim 0.774 \mu\text{b}$ at $s^{1/2} = 200 \text{ GeV}$

- Overlap function

$$T_{AA}(\vec{b}) = \int d^2\vec{s} T_A(\vec{s}) T_A(\vec{b} - \vec{s})$$

- Thickness function

$$T_A(\vec{s}) = \int_{-\infty}^{\infty} dz \rho_A(\vec{s}, z)$$

- Normalized density distribution

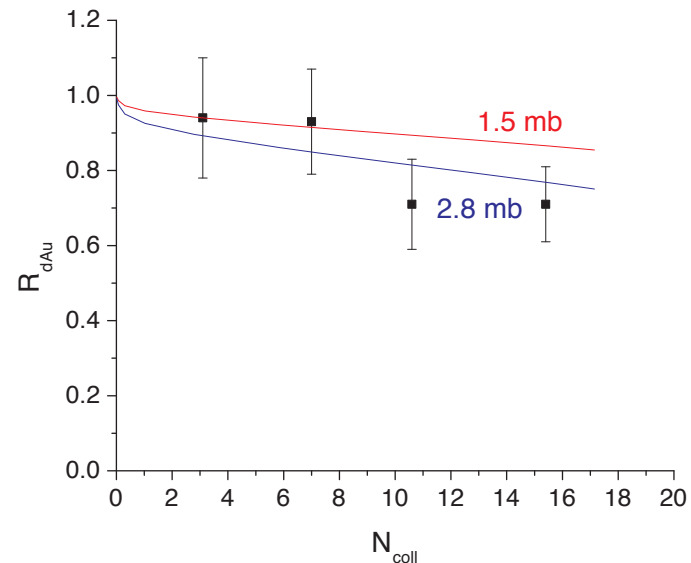
$$\rho(r) = \frac{\rho_0}{1 + e^{(r-r_0)/c}}$$

$r_0 = 6.38 \text{ fm}$, $c = 0.535 \text{ fm}$ for Au

- Nuclear absorption

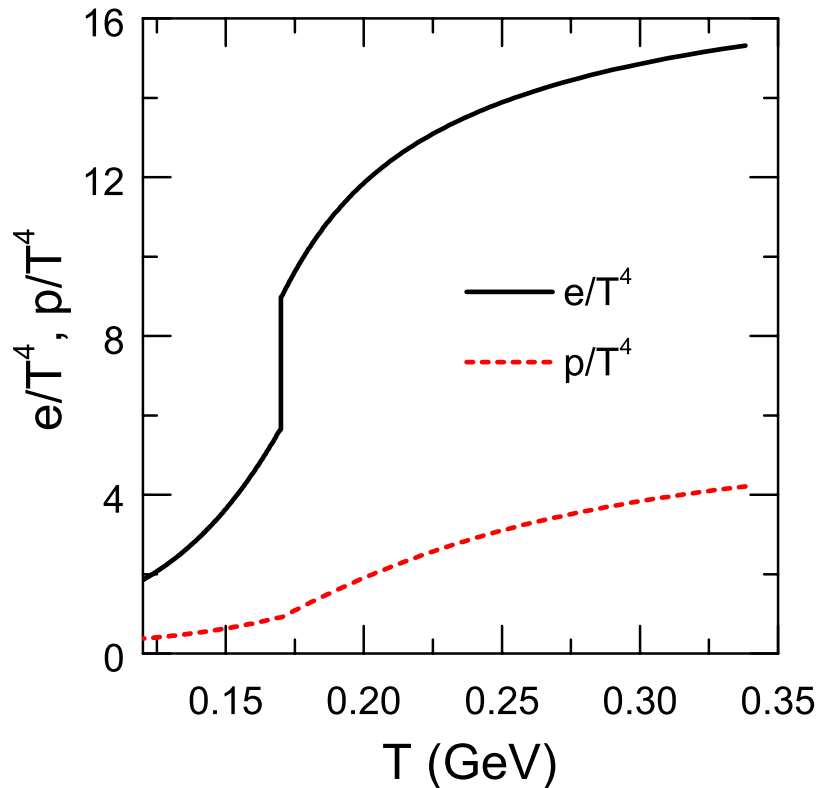
- Survival probability

$$S_{nuc}(\vec{b}, \vec{s}) = \frac{1}{T_{AB}} \int dz dz' \rho_A(\vec{s}, z) \rho_B(\vec{b} - \vec{s}, z) \times \exp \left\{ -(A-1) \int_z^\infty dz_A \rho_A(\vec{s}, z_A) \sigma_{nuc} \right\} \times \exp \left\{ -(B-1) \int_z^\infty dz_B \rho_B(\vec{s}, z_B) \sigma_{nuc} \right\}$$



Quasiparticle model for QGP

P. Levai and U. Heinz, PRC , 1879 (1998)



$$p(T) = \sum_{i=g,q,\bar{q}} \frac{g_i}{6\pi^2} \int_0^\infty dk f_i(T) \frac{k^4}{E_i} - B(T)$$

$$e(T) = \sum_{i=g,q,\bar{q}} \frac{g_i}{2\pi^2} \int_0^\infty dk k^2 f_i(T) E_i + B(T)$$

$$m_g^2 = \left(\frac{N_c}{3} + \frac{N_f}{6} \right) \frac{g^2(T) T^2}{2}$$

$$m_q^2 = \frac{g^2(T) T^2}{3}$$

$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f) \ln F^2(T, T_c, \Lambda)}$$

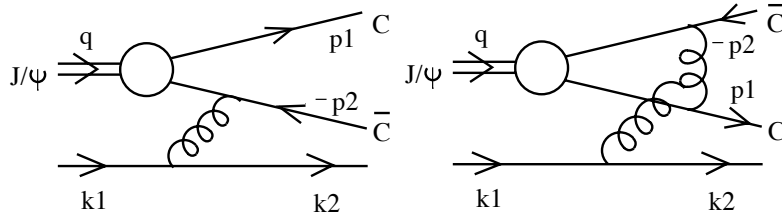
$$F(T, T_c, \Lambda) = \frac{18}{18.4 e^{-(T/T_c)^2/2} + 1} \frac{T}{T_c} \frac{T_c}{\Lambda}$$

- The model reproduces reasonably the QGP equation of state from LQCD

Thermal decay widths of quarkonia

Song, Park & Lee, PRC 81, 034914 (10)

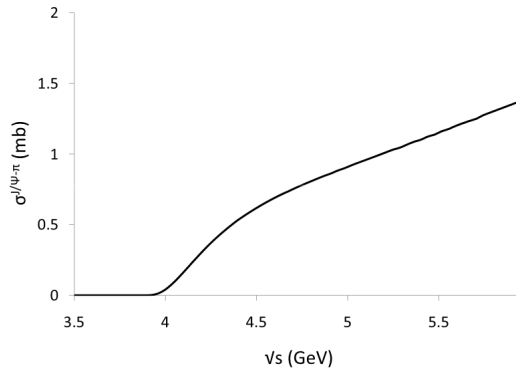
■ Dissociation by partons (NLO pQCD)



$$|\overline{M}|^2 = \frac{4}{3} g^4 m_c^2 m_{J/\psi} \left| \frac{\partial \psi(p)}{\partial p} \right|^2 \left\{ -\frac{1}{2} + \frac{(k_1^0)^2 + (k_2^0)^2}{2k_1 \cdot k_2} \right\}$$

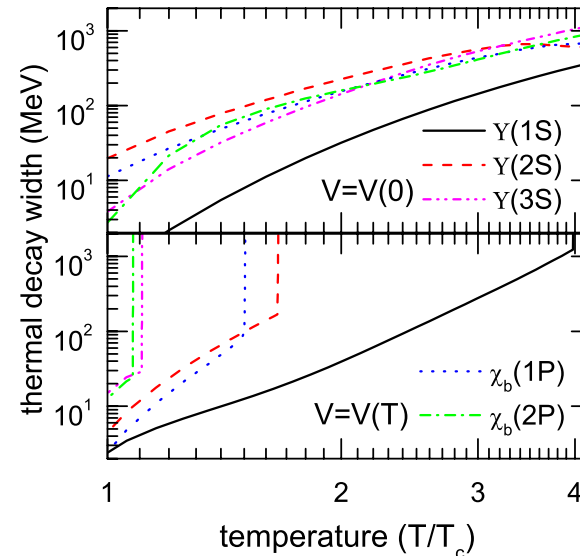
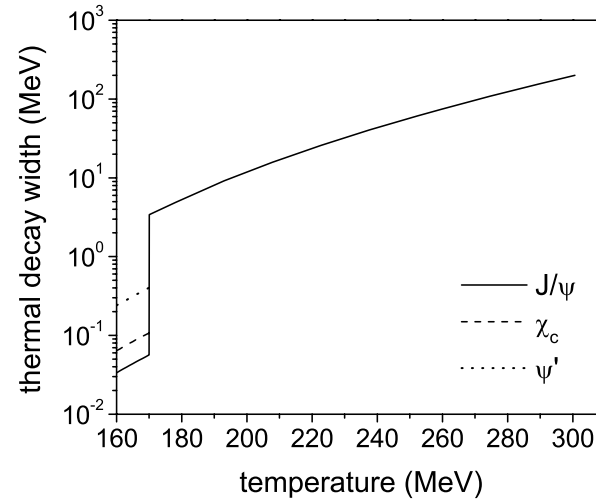
■ Dissociation by hadrons

$$\sigma(s) = \sum_i \int dx n_i(x, Q^2) \sigma_i(xs, Q^2)$$



■ Thermal dissociation width

$$\Gamma(T) = \sum_i \int \frac{d^3 k}{(2\pi)^3} v_{rel}(k) n_i(k, T) \sigma_i^{diss}(k, T)$$



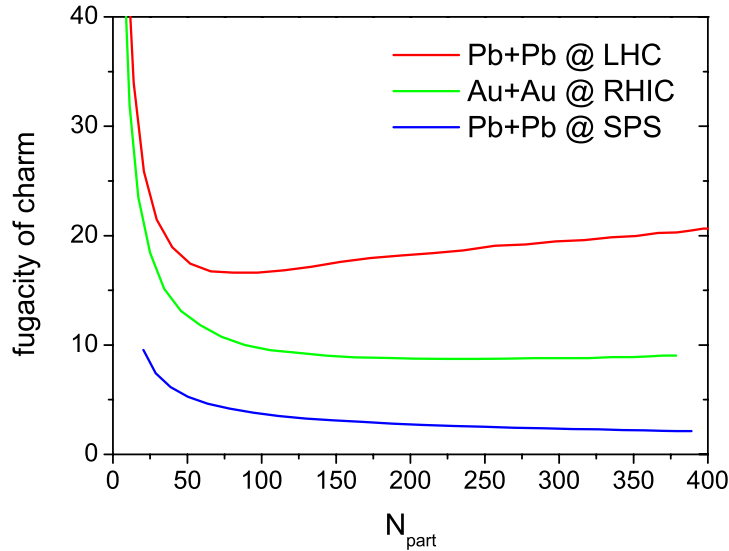
J/ψ production from regeneration

Rate equation for J/ψ production $\frac{dN_i}{d\tau} = -\Gamma_i(N_i - N_i^{\text{eq}}), \quad N_i^{\text{eq}} = \gamma^2 R n_i^{\text{GC}} V$

- Charm fugacity is determined by

$$N_{c\bar{c}}^{AA} = \left[\frac{1}{2} \gamma n_o \frac{I_1(\gamma n_o V)}{I_0(\gamma n_o V)} + \gamma^2 n_h \right] V = \sigma_{c\bar{c}}^{NN} A^2 T_{AA}(\vec{b})$$

- $\sigma_{c\bar{c}}^{NN}$: charm production cross section in NN collision; $\sim 63.7 \mu\text{b}$ at $s^{1/2} = 200 \text{ GeV}$

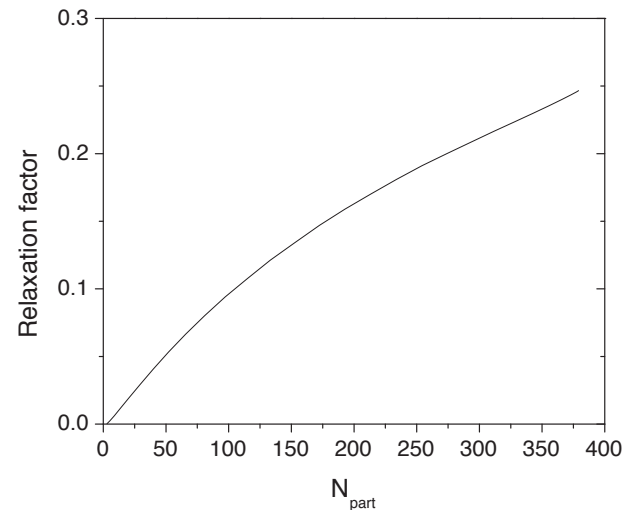


- Charm relaxation factor

$$R = 1 - \exp\left\{ - \int_{\tau_0}^{\tau_{QGP}} d\tau \Gamma_c(T(\tau)) \right\}$$

$$\Gamma(T) = \sum_i \int \frac{d^3k}{(2\pi)^3} v_{rel}(k) n_i(k, T) \sigma_i^{diss}(k, T)$$

as J/ψ is more likely to be formed if charm quarks are in thermal equilibrium



Viscous hydrodynamics

Heinz, Song & Chaudhuri, PRC 73, 034904 (06)

Hydrodynamic Equations

$$\partial_{\mu} T^{\mu\nu}(x) = 0 \quad \text{Energy-momentum conservation}$$

$$\partial_{\mu} n_j u^{\mu}(x) = 0 \quad \text{Charge conservations (baryon, strangeness,...)}$$

$$\pi_{\mu\nu} = \eta \left(\partial_{\mu} u_{\nu} + \partial_{\nu} u_{\mu} - \frac{2}{3} \Delta_{\mu\nu} \partial_{\alpha} u^{\alpha} \right) - \tau_{\pi} \left(\frac{4}{3} \pi_{\mu\nu} \partial_{\alpha} u^{\alpha} + \Delta_{\mu}^{\alpha} \Delta_{\nu}^{\beta} u^{\sigma} \partial_{\sigma} \pi_{\alpha\beta} \right) \quad (\text{Israel-Stewart})$$

with $T^{\mu\nu}(x) = [e(x) + p(x)] u^{\mu}(x) u^{\nu}(x) - p(x) g^{\mu\nu} + \pi_{\mu\nu}$

e: energy density, p(e): pressure, $\pi_{\mu\nu}$: shear stress tensor, u^{μ} : four velocity, τ_{π} : relaxation time

Cooper-Frye instantaneous freeze out

$$E \frac{dN_i}{d^3q} \approx \frac{g_i}{(2\pi)^3} \int q \cdot d\sigma \frac{1}{\exp(q \cdot u/T) \pm 1} \left[1 + \frac{q_{\mu} q_{\nu} \pi^{\mu\nu}}{2T^2(e+p)} \right]$$

Schematic viscous hydrodynamics

Song, Han & Ko, PRC 83, 014914 (11)

Assuming thermal quantities (energy density, temperature, entropy density, and pressures) and shear tensor are uniform along the transverse direction

$$\partial_\tau(A\tau\langle T^{\tau\tau}\rangle) = -(p + \pi_\eta)A,$$

$$\frac{T}{\tau}\partial_\tau(A\tau s\langle\gamma_r\rangle) = -A\left\langle\frac{\gamma_r v_r}{r}\right\rangle\pi_\phi - \frac{A\langle\gamma_r\rangle}{\tau}\pi_\eta + \left\{\partial_\tau(A\langle\gamma_r\rangle) - \frac{\gamma_R\dot{R}}{R}A\right\}(\pi_\phi + \pi_\eta),$$

$$\partial_\tau(A\langle\gamma_r\rangle\pi_\eta) - \left\{\partial_\tau(A\langle\gamma_r\rangle) + 2\frac{A\langle\gamma_r\rangle}{\tau}\right\}\pi_\eta = -\frac{A}{\tau_\pi}\left[\pi_\eta - 2\eta_s\left\{\frac{\langle\theta\rangle}{3} - \frac{\langle\gamma_r\rangle}{\tau}\right\}\right],$$

$$\partial_\tau(A\langle\gamma_r\rangle\pi_\phi) - \left\{\partial_\tau(A\langle\gamma_r\rangle) + 2A\left\langle\frac{\gamma_r v_r}{r}\right\rangle\right\}\pi_\phi = -\frac{A}{\tau_\pi}\left[\pi_\phi - 2\eta_s\left\{\frac{\langle\theta\rangle}{3} - \left\langle\frac{\gamma_r v_r}{r}\right\rangle\right\}\right],$$

with

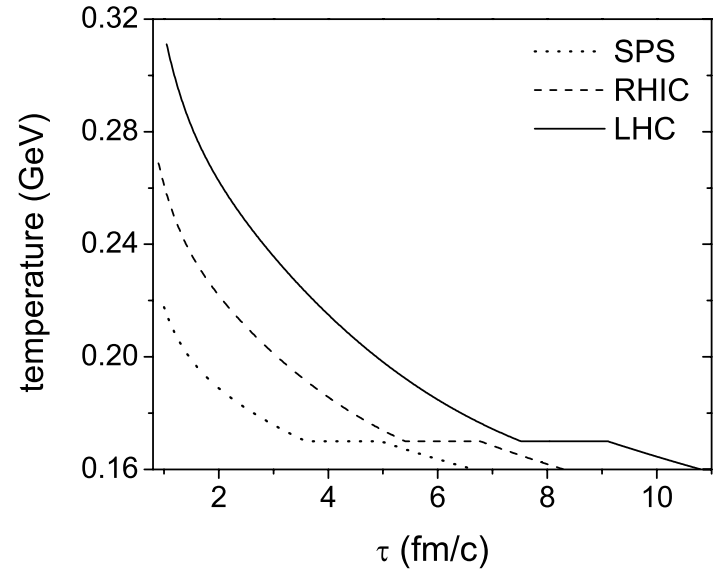
$$\langle\gamma_r\rangle = \frac{2}{3\gamma_R^2\dot{R}^2}(\gamma_R^3 - 1), \quad \left\langle\frac{\gamma_r v_r}{r}\right\rangle = \frac{\gamma_R\dot{R}^2}{R}$$

$$\langle\gamma_r^2\rangle = 1 + \frac{\gamma_R^2\dot{R}^2}{2}, \quad \langle\gamma_r^2 v_r^2\rangle = \frac{\gamma_R^2\dot{R}^2}{2}, \quad \gamma_R = \frac{1}{\sqrt{1 - \dot{R}^2}}$$

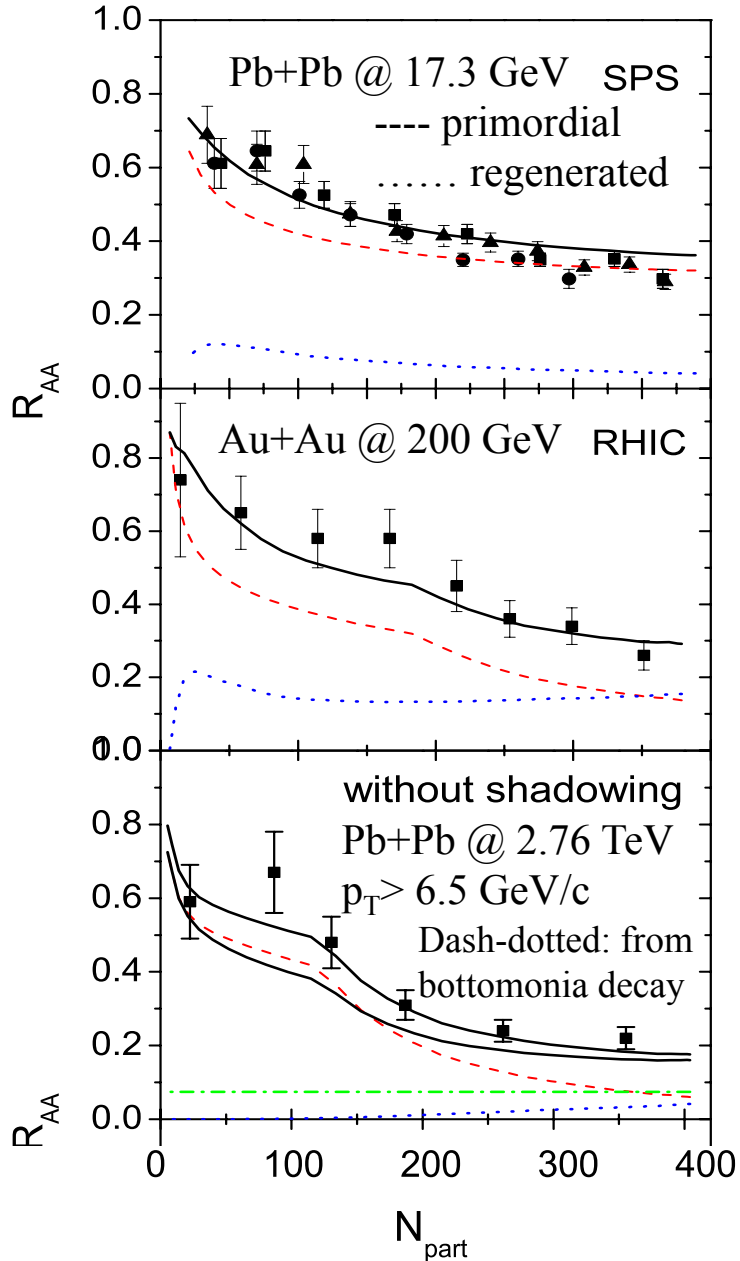
$$\theta = \frac{1}{\tau}\partial_\tau(\tau\gamma_r) + \frac{1}{r}\partial_r(rv_r\gamma_r), \quad A = \pi R^2$$

Taking initial thermalization time

$\tau_0=1.0, 0.9$ and 1.05 for SPS, RHIC and LC;
 $\eta/s=0.16$ for QGP at SPS and RHIC and 0.2 at
 LHC, and 0.8 for HG; and $\tau_\pi=3/T(\eta/s)$.



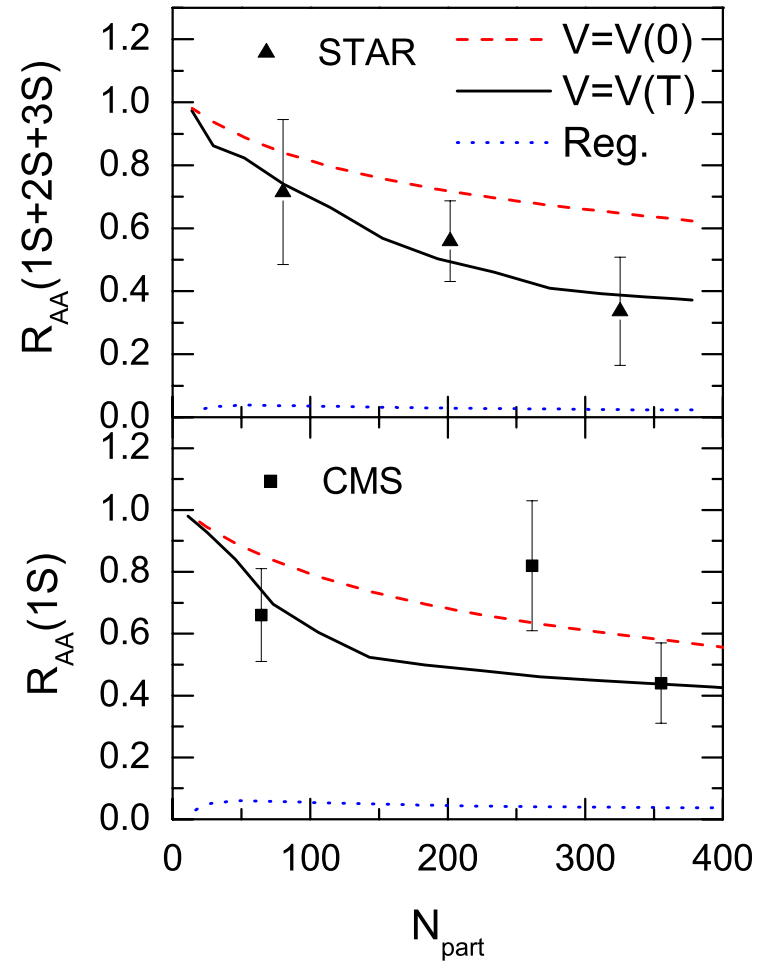
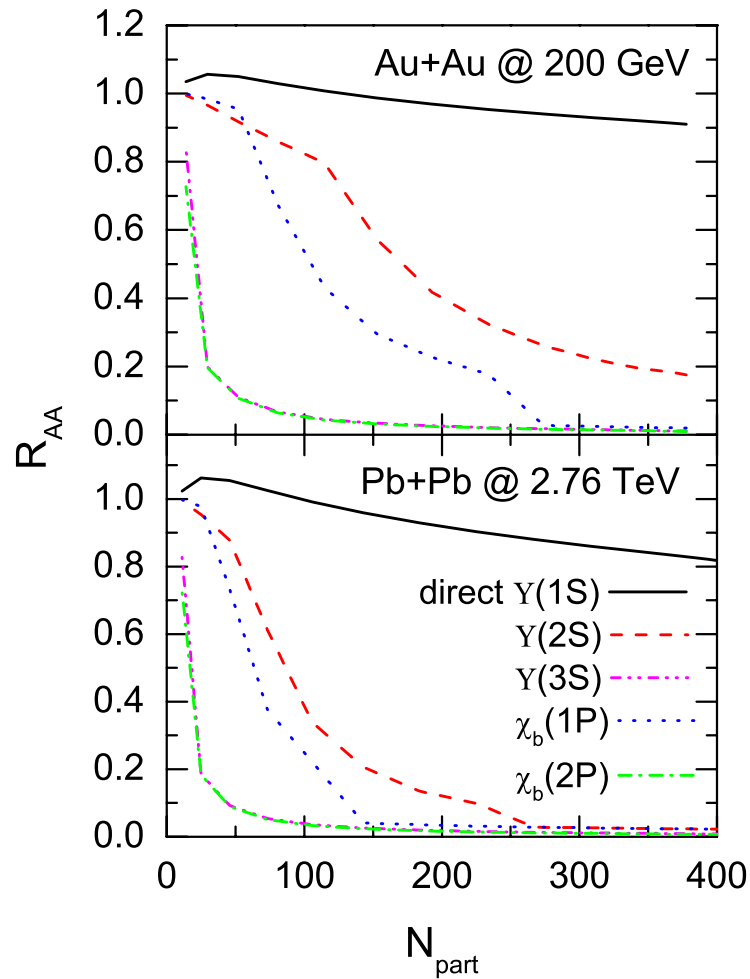
Nuclear modification factor for J/ψ



	SPS	RHIC	LHC	LHC
				$p_T > 6.5$ GeV
production (μb)				
$d\sigma_{J/\psi}^{pp}/dy$	0.05	0.774	4.0	
$d\sigma_{c\bar{c}}^{pp}/dy$	5.7	119	615	
feed-down (%)				
f_{χ_c}	25	32	26.4	23.5
$f_{\psi'(2S)}$	8	9.6	5.6	5
f_b			11	21
nuclear absorp.				
σ_{abs} (mb)	4.18	2.8	0 or 2.8	

- Most J/ψ are survivors from initially produced.
- The kink in R_{AA} is due to the onset of initial temperature above the J/ψ dissociation temperature in QGP.

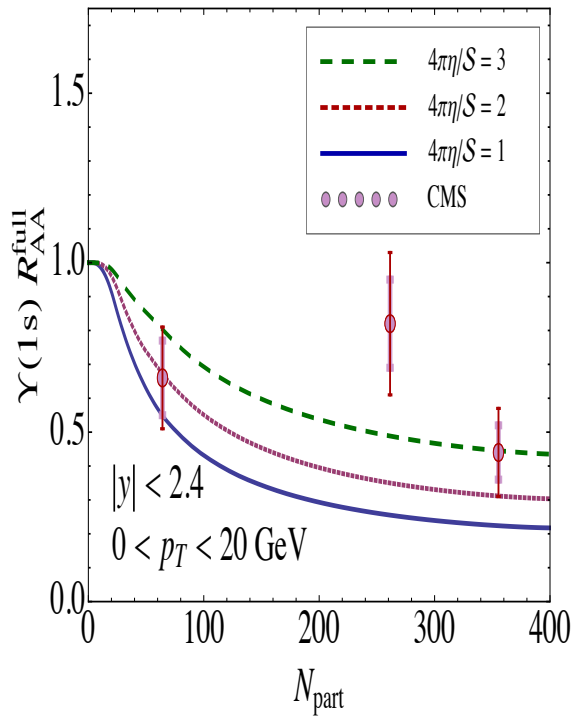
Nuclear modification factor for $\Upsilon(1S)$



- Regeneration contribution is negligible.
- Primordial excited bottomonia are largely dissociated.
- Medium effects on bottomonia reduce R_{AA} of $\Upsilon(1S)$.

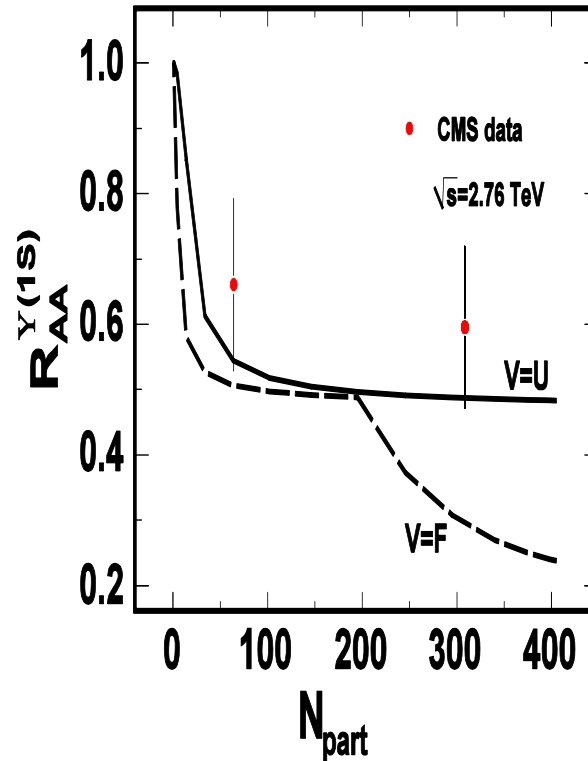
Y(1S) nuclear modification factor at LHC from various models

Strickland, arXiv:1106.2571



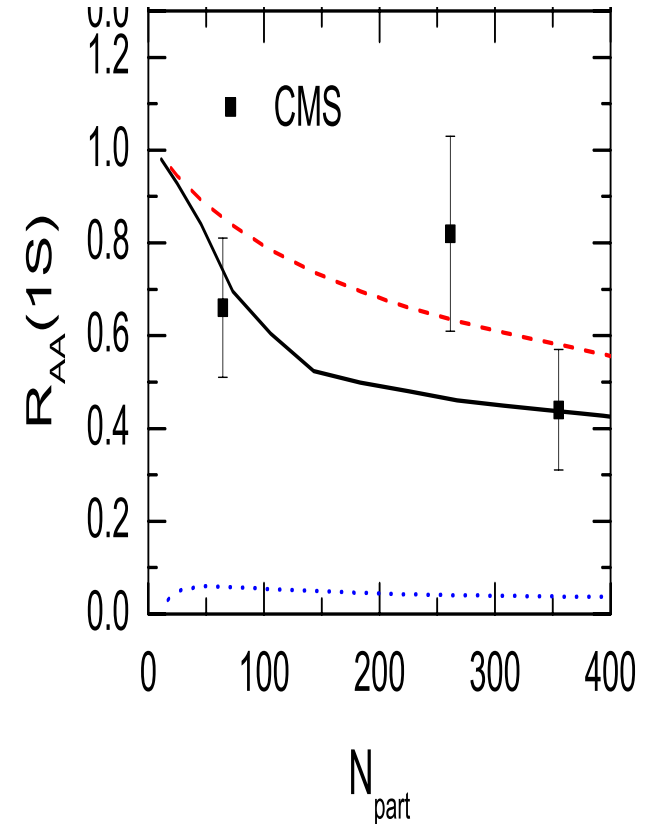
- Potential: U
- Decay: LO pQCD
- Dynamics: anisotropic hydro

Zhuang et al.



- Potential: U or F
- Decay: OPE
- Dynamics: ideal hydro

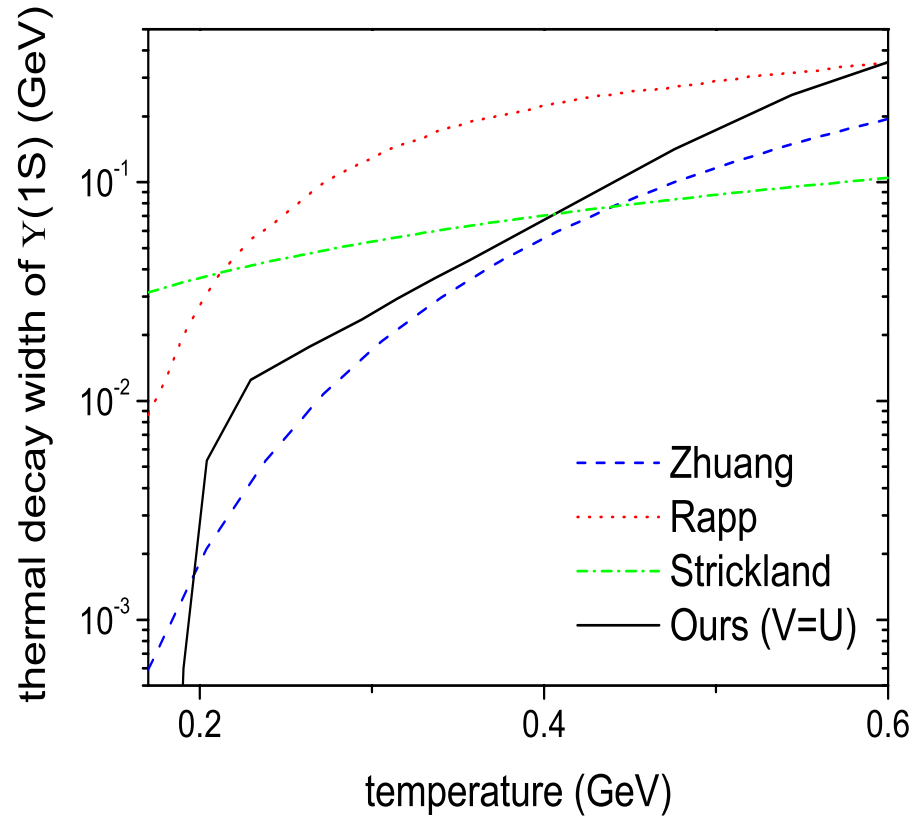
Song et al.



- Potential: screened Cornell
- Decay: NLO pQCD
- Dynamics: schematic viscous hydro

Comparisons of different two-component models

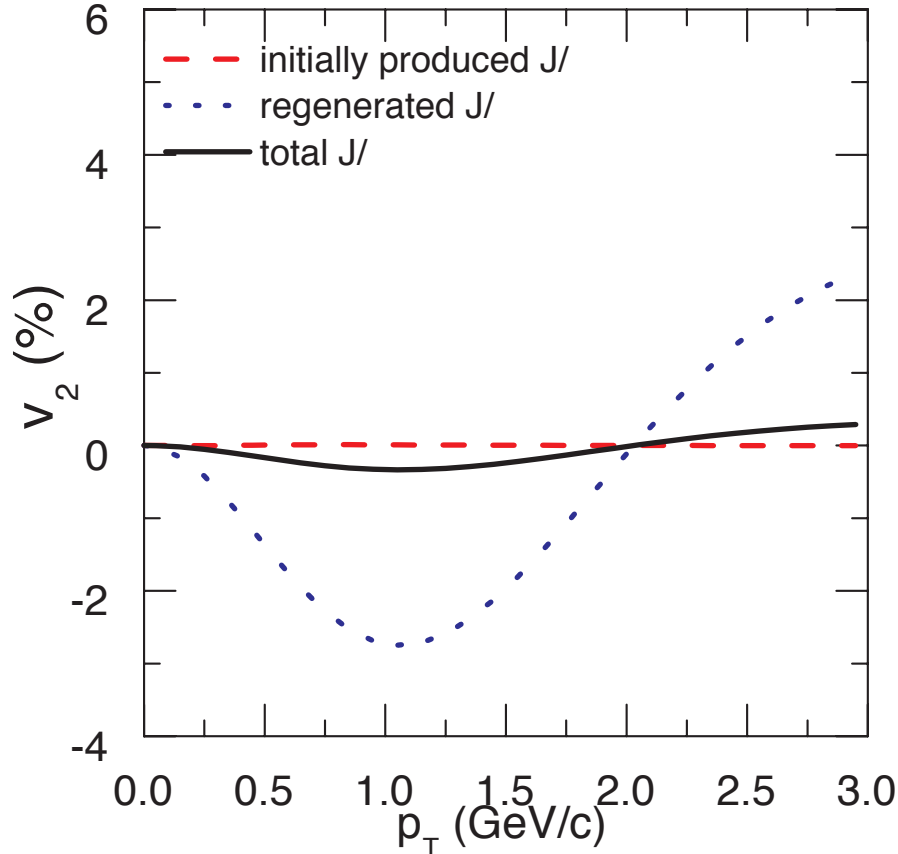
- Heavy quark potential
 - Rapp: lattice U or F
 - Zhuang: lattice U or F
 - Strickland: U
 - Song: Screened Cornell
- Thermal decay width
 - Rapp: Quasielastic scattering
 - Zhuang: OPE by Peskin
 - Strickland: LO pQCD
 - Song: NLO pQCD
- HIC dynamics
 - Rapp: expanding fireball
 - Zhuang: ideal hydro
 - Strickland: anisotropic hydro
 - Song: Schematic viscous hydro



- Very different thermal decay widths are used in different models.

J/ψ elliptic flow

Song, Lee, Xu & Ko, PRC 83, 014914 (11)



$$v_2 = \frac{\int d\varphi \cos(2\varphi) (dN / dy d^2 p_T)}{\int d\varphi (dN / dy d^2 p_T)}$$

$$= \frac{\int dA_T \cos(2\varphi) I_2(p_T \sinh \rho / T) K_1(m_T \cosh \rho / T)}{\int dA_T I_0(p_T \sinh \rho / T) K_1(m_T \cosh \rho / T)}$$

Introduced viscous effect at freeze out
T=125 MeV

$$\Delta v = (v_x - v_y) \exp[-C p_T / n]$$

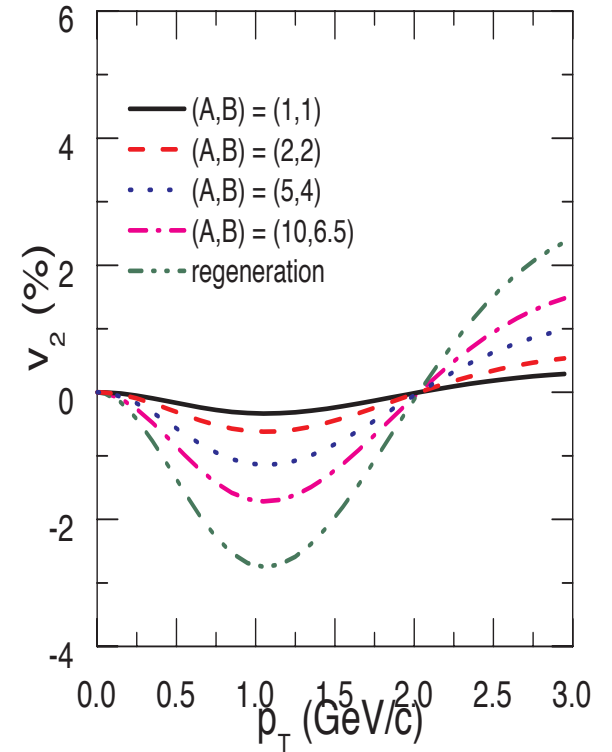
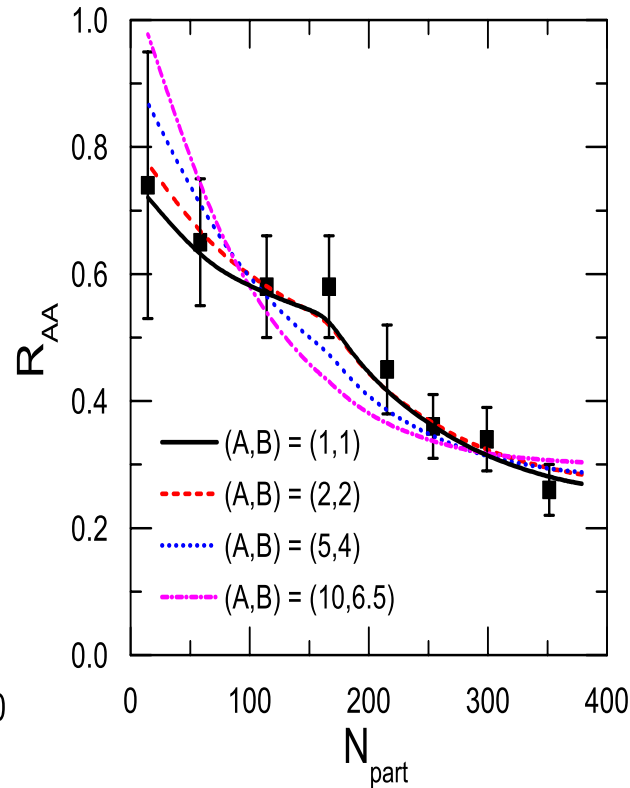
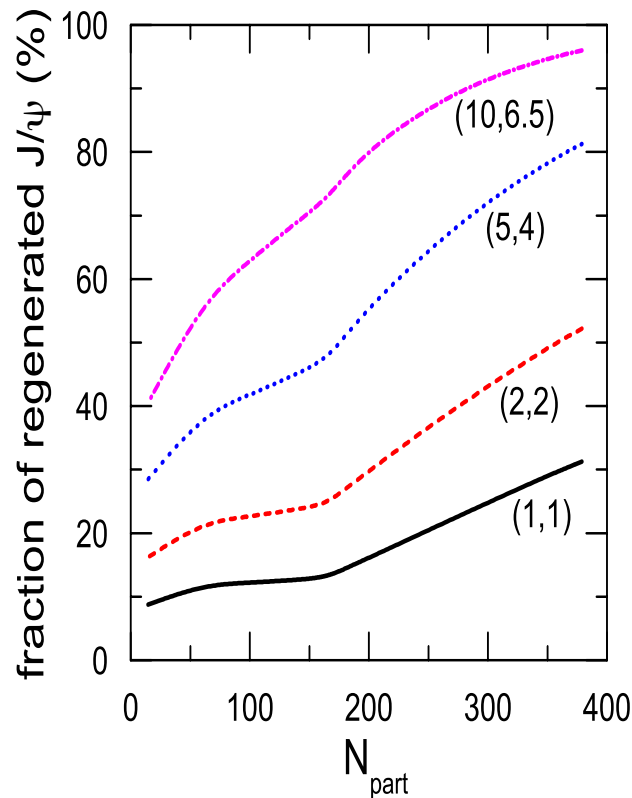
with $C=1.14 \text{ GeV}^{-1}$ and n = number of quarks in a hadron

- Initially produced J/ψ have essentially vanishing v_2 .
- Regenerated J/ψ have large v_2 .
- Final J/ψ v_2 is small as most are initially produced.

Effects of higher-order corrections

$$\sigma(J/\psi + q(g) \rightarrow c + \bar{c} + q(g)) = A\sigma(J/\psi + q(g) \rightarrow c + \bar{c} + q(g))$$

$$\sigma(c + q(g) \rightarrow c + q(g)) = B\sigma(c + q(g) \rightarrow c + q(g))$$



- Higher-order effects are small on J/ψ nuclear modification factor but large on their elliptic flow.

Summary

- J/ψ survives up to $1.7 T_c$ and $Y(1S)$ survives up to $4 T_c$.
- Most observed J/ψ and $Y(1S)$ are from primordially produced; contribution from regeneration is small at present HIC.
- Various models with different assumptions can describe experimental data.
- Nuclear modification factor R_{AA} of J/ψ is insensitive to the fraction of their production from regeneration \rightarrow both the statistical model and the two-component model can describe data.
- Elliptic flow of regenerated J/ψ is large, while that of directly produced ones is essentially zero. Studying v_2 of J/ψ is useful for distinguishing the mechanism for J/ψ production in HIC.