Towards a first principles description of quarkonium in medium

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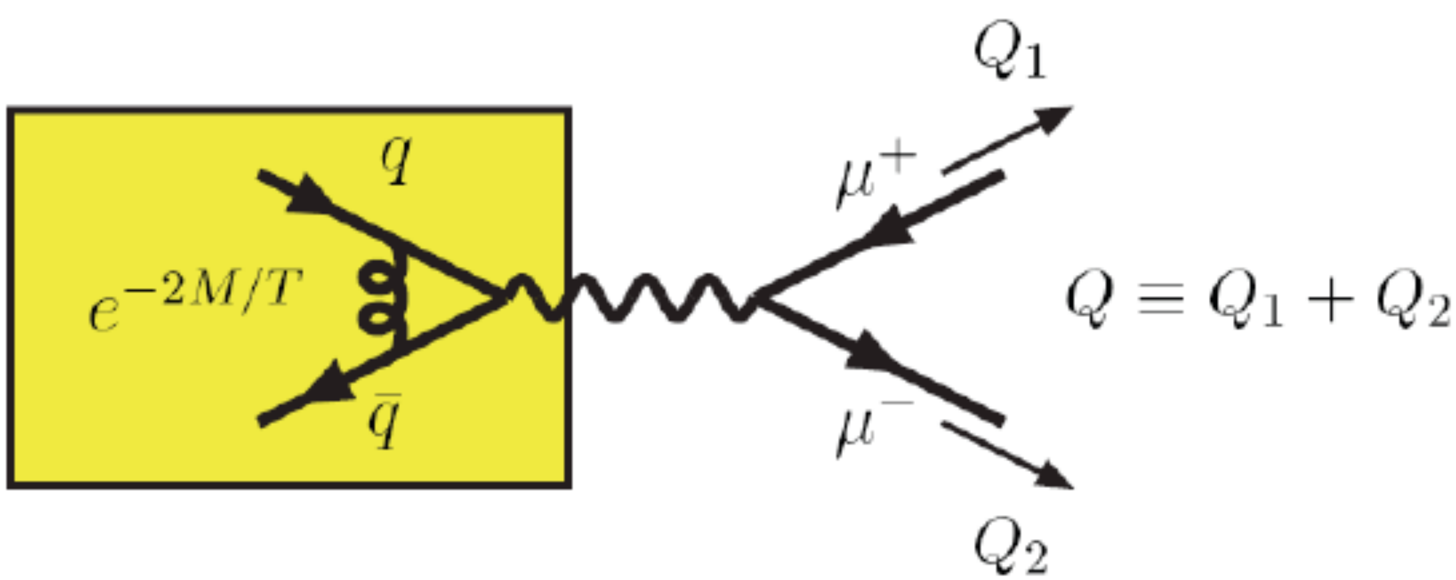
- Introduction
- Equilibrium: perturbative and non-perturbative evaluation
- Non-equilibrium: anisotropic plasmas
- Here: field theoretical aspects, not much phenomenology

collaborators: O. Jahn (MIT), M. Laine (Bielefeld), M. Tassler (McGill), P. Romatschke (Frankfurt)

Preparation of this talk....

....by finding inner equilibrium!

A real time problem: quarkonia in the QCD plasma



$$\frac{dN_{\mu^+\mu^-}}{d^4x d^4Q} \sim \frac{\rho_V(\omega)}{\omega^2} \mathrm{e}^{-\frac{\omega}{T}}$$

Bound states of heavy quarks: c (~1GeV), b (~4 GeV) suppressed in plasma?

Dilepton production proportional to spectral function; Matsui, Satz

QFT: need to know real time correlation function

$$\rho(Q) = \frac{1}{2}\left(1 - \mathrm{e}^{-\frac{q^0}{T}}\right) \tilde{C}_>(Q)$$

$$\tilde{C}_>(Q) \equiv \int dt \int d^3x \, \mathrm{e}^{iQx} \langle J^\mu(x) J_\mu(0) \rangle_T, \quad J^\mu(x) = e_q \bar{\psi}(x) \gamma^\mu \psi(x)$$

I. Direct lattice approach: euclidean correlator

$$G_\Gamma(\tau,\vec{p}) = \frac{1}{2\pi}\int_0^\infty \rho_\Gamma(\omega,\vec{p})K(\tau,\omega)d\omega\,,$$

$$K(\tau,\omega) = \frac{\cosh[\omega(\tau-1/2T)]}{\sinh(\omega/2T)} = e^{\omega\tau}n_B(\omega) + e^{-\omega\tau}[1+n_B(\omega)]$$

$$G_\Gamma(\tau) = \sum_{\vec{x},\vec{y},t}\langle\overline{\psi}(\vec{x},t)\Gamma\psi(\vec{x},t)\overline{\psi}(\vec{y},t+\tau)\Gamma\psi(\vec{y},t+\tau)\rangle$$

trivial continuation, but numerical inversion ill-defined!

→ measure euclidean correlator on anisotropic lattices, $(a_t \neq a_s)$
"fit" spectral function with Maximum Entropy Methods

Bielefeld
Riken/Columbia
Swansea
JLQCD

- Ultimately: with sufficient compute power (t-resolution!) might give the answers
- Currently: (strongly) prior-dependent; unable to resolve correlators from different potential models

Mocsy, Petreczky, PRD 08

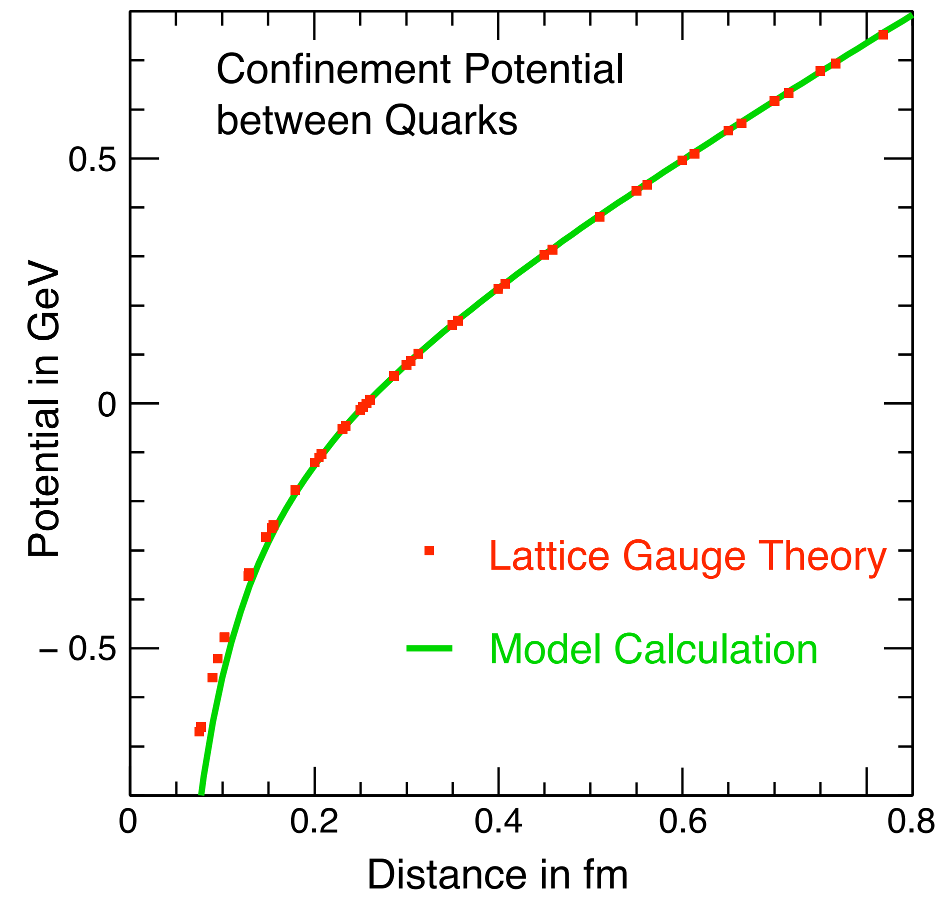
II. Potential models, effective theories

T=0: Schrödinger eqn., confining V(r) from lattice

very successful spectroscopy ~1%

eff. theory derived from QCD: pNRQCD

small expansion parameter $(E-2M)/M$

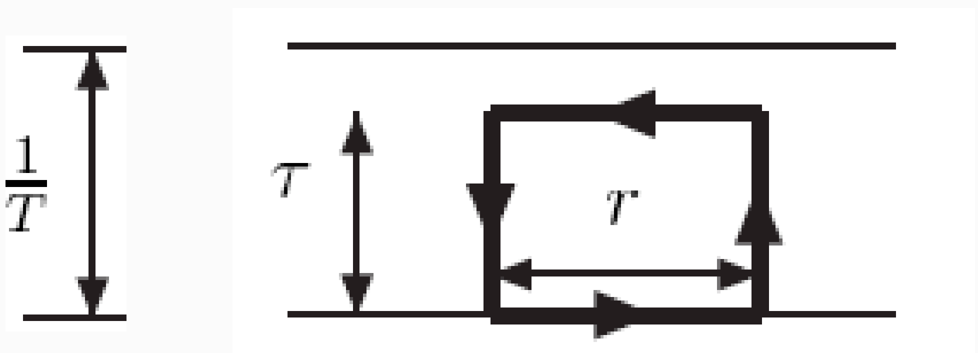


finite T: use 'finite T potential' $$V(r,T) \approx -\frac{g^2 C_F}{4\pi}\frac{\exp(-m_D(T)r)}{r}$$ Matsui, Satz

The static potential at T=0: Wilson loop

- Euclidean correlator of gauge invariant meson operator
- integrate out quarks in the limit $M \to \infty$

$$\langle \bar{\psi}(\mathbf{x},\tau)U(\mathbf{x},\mathbf{y};\tau)\psi(\mathbf{y},\tau)\bar{\psi}(\mathbf{y},0)U^{\dagger}(\mathbf{x},\mathbf{y};0)\psi(\mathbf{x},0)\rangle \longrightarrow \mathrm{e}^{-2M\tau}W_E(|\mathbf{x}-\mathbf{y}|,\tau)$$



$U(\mathbf{x},\mathbf{y};0) \equiv U_r(0)$

$$r = |\mathbf{x}-\mathbf{y}|$$

- spectral decomposition, $T \to 0$ $\quad W_E(r,\tau\to\infty) \longrightarrow c_{01}^2[U(\mathbf{x},\mathbf{y};0)]\,\mathrm{e}^{-V(r)\tau}$
- lattice generalization to finite T not clear, $N_\tau = \dfrac{1}{aT}$ on lattices short

Static 'potentials' at finite T, free energies: the Polyakov loop

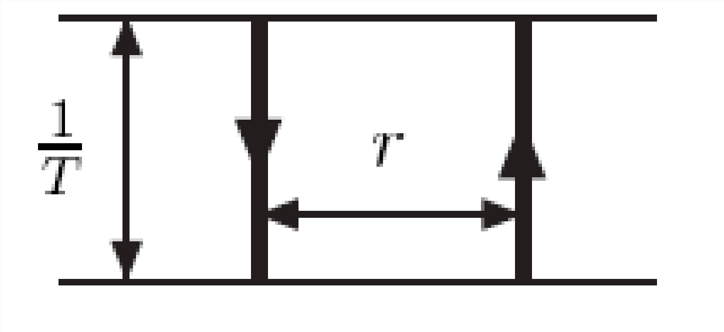
McLerran, Svetitsky PRD 81

- Static quarks propagate through periodic boundary
- finite T: sum over Boltzmann weighted excited states

➡ free energy of a static quark in a plasma: $\langle \mathrm{Tr}\, L_{\mathbf{x}} \rangle \sim \mathrm{e}^{-F_q/T}$

order parameter for confinement: $\langle L \rangle \begin{cases} = 0 \Leftrightarrow \text{confined phase}, & T < T_c \\ > 0 \Leftrightarrow \text{deconfined phase}, & T > T_c \end{cases}$

- free energy of static quark anti-quark pair:



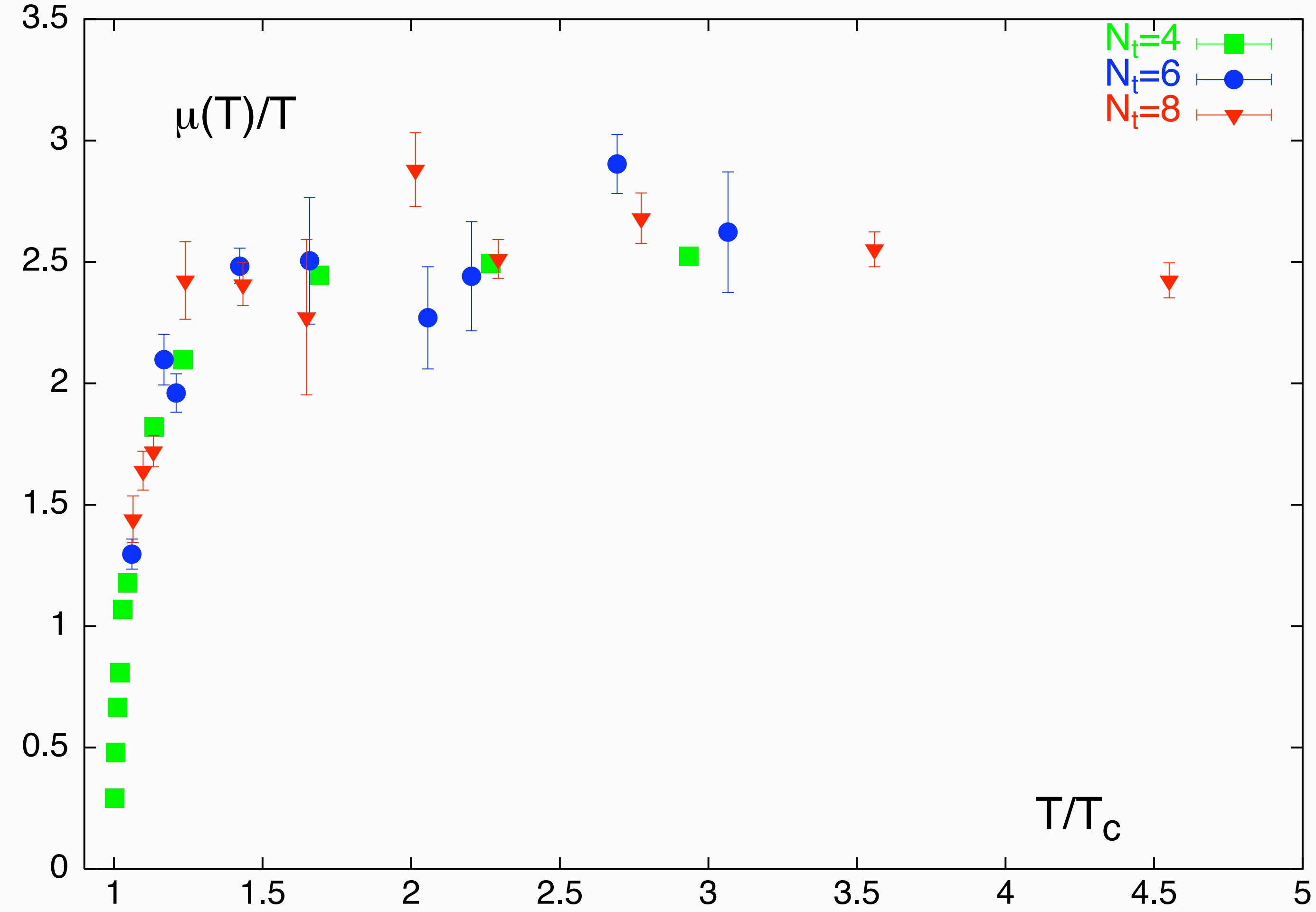
$$\mathrm{e}^{-F_{\bar{q}q}(r,T)/T} = \frac{1}{N^2} \langle \mathrm{Tr}\, L^\dagger(\mathbf{x})\, \mathrm{Tr}\, L(\mathbf{y}) \rangle, \quad r = |\mathbf{x} - \mathbf{y}|$$

Screening of static quark free energy

...in pure gauge theory

Kaczmarek et al., PRD 00

$$\frac{F_{q\bar{q}}(r,T)}{T} = -\frac{c(T)}{(rT)^d}\mathrm{e}^{-\mu(T)r}$$



numerically: $d \approx 3/2, \mu \approx M_{0_+^{++}}$ lightest gauge inv. screening mass

LO perturbation theory: $d = 2, \mu = 2m_D^0$ exchange of two A_0

➡ $F_{\bar{q}q}$ does not correspond to the Debye-screened static potential!

Decomposition in different colour channels

McLerran, Svetitsky PRD 81

$$\begin{aligned}
\mathrm{e}^{-F_{\bar{q}q}(r,T)/T} &= \frac{1}{N^2}\langle \mathrm{Tr}\, L^\dagger(\mathbf{x})\, \mathrm{Tr}\, L(\mathbf{y})\rangle = \frac{1}{N^2}\mathrm{e}^{-F_1(r,T)/T} + \frac{N^2-1}{N^2}\mathrm{e}^{-F_8(r,T)/T} \\
\mathrm{e}^{-F_1(r,T)/T} &= \frac{1}{N}\langle \mathrm{Tr}\, L^\dagger(\mathbf{x}) L(\mathbf{y})\rangle, \\
\mathrm{e}^{-F_8(r,T)/T} &= \frac{1}{N^2-1}\langle \mathrm{Tr}\, L^\dagger(\mathbf{x})\, \mathrm{Tr}\, L(\mathbf{y})\rangle - \frac{1}{N(N^2-1)}\langle \mathrm{Tr}\, L^\dagger(\mathbf{x}) L(\mathbf{y})\rangle.
\end{aligned}$$

- correlators in ‘singlet’ and ‘octet’ channels gauge dependent, non-pert. meaning?
- $$F_1(r,T) \sim \frac{\mathrm{e}^{-m_D(T)r}}{4\pi r}$$ Nadkarni PRD 86

Spectral analysis of Polyakov loop correlators

Jahn, O.P., PRD 05

$\hat{T}_0 = \mathrm{e}^{-a\hat{H}_0}$ with Kogut-Susskind Hamiltonian in temporal gauge

$$\mathrm{e}^{-F_{\bar{q}q}/T} = \frac{1}{ZN^4}\hat{\mathrm{Tr}}\,[\hat{T}_0^{N_t}\ \hat{P}^{\mathrm{F}\otimes\bar{\mathrm{F}}}] = \frac{1}{ZN^2}\sum_n \langle n_{\alpha\beta}|n_{\beta\alpha}\rangle\ \mathrm{e}^{-E_n/T}$$

$$\mathrm{e}^{-F_1/T} = \frac{1}{ZN^2}\sum_n \langle n_{\delta\gamma}|\hat{U}_{\gamma\delta}(\mathbf{x},\mathbf{y})\hat{U}^{\dagger}_{\alpha\beta}(\mathbf{x},\mathbf{y})|n_{\beta\alpha}\rangle\ \mathrm{e}^{-E_n/T}$$

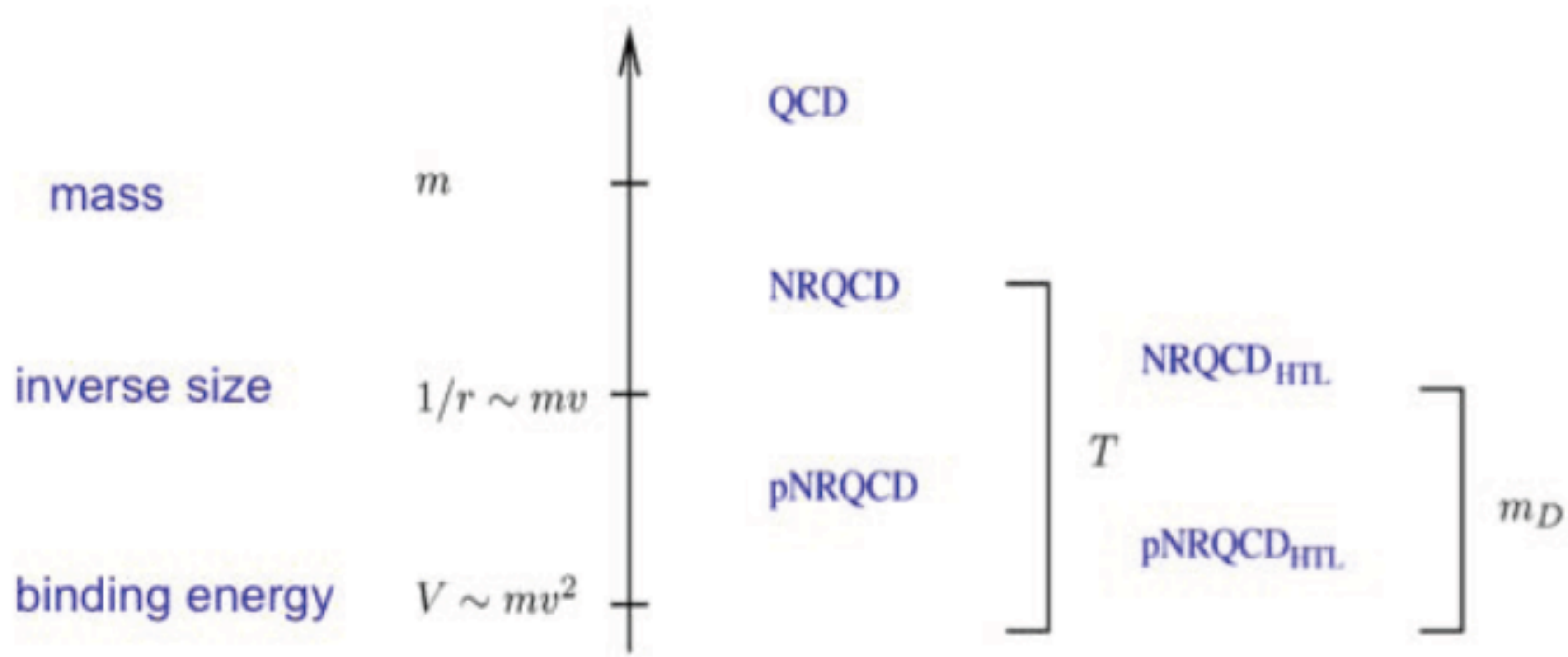
$$\mathrm{e}^{-F_8/T} = \frac{1}{ZN^2}\sum_n \langle n_{\delta\gamma}|\hat{U}^{a}_{\gamma\delta}(\mathbf{x},\mathbf{y})\hat{U}^{\dagger a}_{\alpha\beta}(\mathbf{x},\mathbf{y})|n_{\beta\alpha}\rangle\ \mathrm{e}^{-E_n/T}$$

- energies: usual (T=0) colour singlet potential + excit. in all three channels
- non-vanishing matrix elements in singlet and octet channel
- matrix elements path/gauge dependent but contribute , unphysical!

Effective field theory approach

Soto et al. 08,
Beraudo, Blaizot, Ratti NPA 08

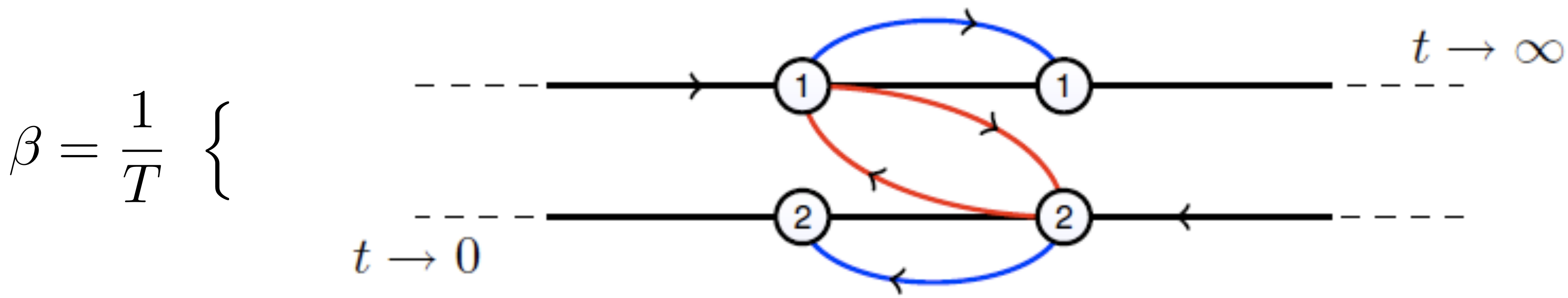
Laine, O.P., Romatschke, Tassler JHEP 07
Brambilla, Ghiglieri, Petreczky, Vairo PRD 08



Quarkonium correlator, infinite quark mass limit, real time, finite T:

$$\langle\bar{\psi}(\mathbf{x},\tau)U(\mathbf{x},\mathbf{y};\tau)\psi(\mathbf{y},\tau)\bar{\psi}(\mathbf{y},0)U^{\dagger}(\mathbf{x},\mathbf{y};0)\psi(\mathbf{x},0)\rangle_{T} \longrightarrow e^{-2M\tau}W_E(|\mathbf{x}-\mathbf{y}|,\tau) \qquad \tau = it$$

Real-time potential: Schwinger-Keldysh



Real-Time Correlators

$$iG = i\begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} = \begin{pmatrix} \langle T\hat{\psi}(t')\hat{\varphi}(t)\rangle & -\langle\hat{\varphi}(t)\hat{\psi}(t')\rangle \\ \langle\hat{\psi}(t')\hat{\varphi}(t)\rangle & \langle\tilde{T}\hat{\psi}(t')\hat{\varphi}(t)\rangle \end{pmatrix}$$

Retarded, advanced and symmetric correlators are defined via:

$$\boldsymbol{R}^{-1}\cdot\boldsymbol{G}\cdot\boldsymbol{R} = \begin{pmatrix} 0 & G_A \\ G_R & G_S \end{pmatrix} \quad \text{where} \quad \mathbf{R} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}.$$

Our correlator is $C_{21}(t,\mathbf{r}) = C_{>}(t,\mathbf{r}) \propto W_E(it,\mathbf{r})$

We need the large time limit (non-rel.) $C_{21}(t\to\infty) \approx C_{11}(t)$

evolution equation for the correlator:

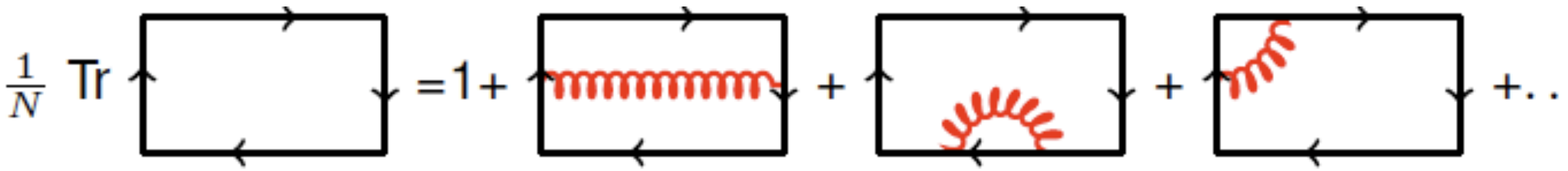
$$\left\{ i\partial_t - \left[2M + V_>(t,r) - \frac{\nabla_{\mathbf{r}}^2}{M} + O\left(\frac{1}{M^2}\right) \right] \right\} C_>(t,\mathbf{r}) = 0$$

potential = coefficient scaling as $O(M^0)$ in t-derivative of correlator
= matching coefficient in EFT

required scale hierarchy: $g^2 M < T < gM$

absorb heavy mass by rescaling, leading behaviour:

$$i\partial_t W_E(it,\mathbf{r}) = V_>(t,r) W_E(it,\mathbf{r})$$



Diagrams contributing to the Wilson-Loop

The Real-Time static potential to $\mathcal{O}(g^2)$

$$V(\boldsymbol{r}) = g^2 \mathrm{C_F} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} (1 - \cos \boldsymbol{k}\cdot\boldsymbol{r}) \tilde{G}_{11}^{00}(\omega = 0, \boldsymbol{k})$$

Here $\tilde{G}_{11}^{00}$ is the longitudinal component of the time ordered gluon propagator which can be decomposed as: $\tilde{G}_{11} = Re\, \tilde{G}_R + \frac{1}{2}\tilde{G}_S$.

In the special case of ***thermal equilibrium*** the potential takes the form:

$$V(r) = \underbrace{-\frac{g^2 C_F}{4\pi}\left[m_D + \frac{exp(-m_D r)}{r}\right]}_{\text{Re(V): Retarded contribution}} \underbrace{-i\frac{g^2 T C_F}{2\pi}\phi(m_D r)}_{\text{Im(V): Symmetric contribution}}$$

where $\phi(x) = 2\int_0^\infty \frac{dz\, z}{(z^2+1)^2}\left[1 - \frac{sin(zx)}{zx}\right]$.

- Real part: Debye screened as expected
- Imaginary part: Landau damping due to emission/absorption of soft gluons

Non-perturbative effects?

- Wilson loop in Minkowski time not directly calculable on the lattice
- non-perturbative corrections are due to infrared modes ~ g^2T

high occupation numbers, semi-classical physics!
cf. electroweak sphaleron rate: Bödeker, Moore, Rummukainen

$$G_S \sim n_B(\omega) + \frac{1}{2} = \frac{T}{\hbar\omega} + \frac{1}{12}\frac{\hbar\omega}{T} + \ldots$$

➡ $$\lim_{\hbar\to 0} V_>(\infty, r) = -\frac{ig^2TC_F}{4\pi}\phi(m_D r)$$

➡ non-perturbative check of Landau damping in classical lattice simulation!

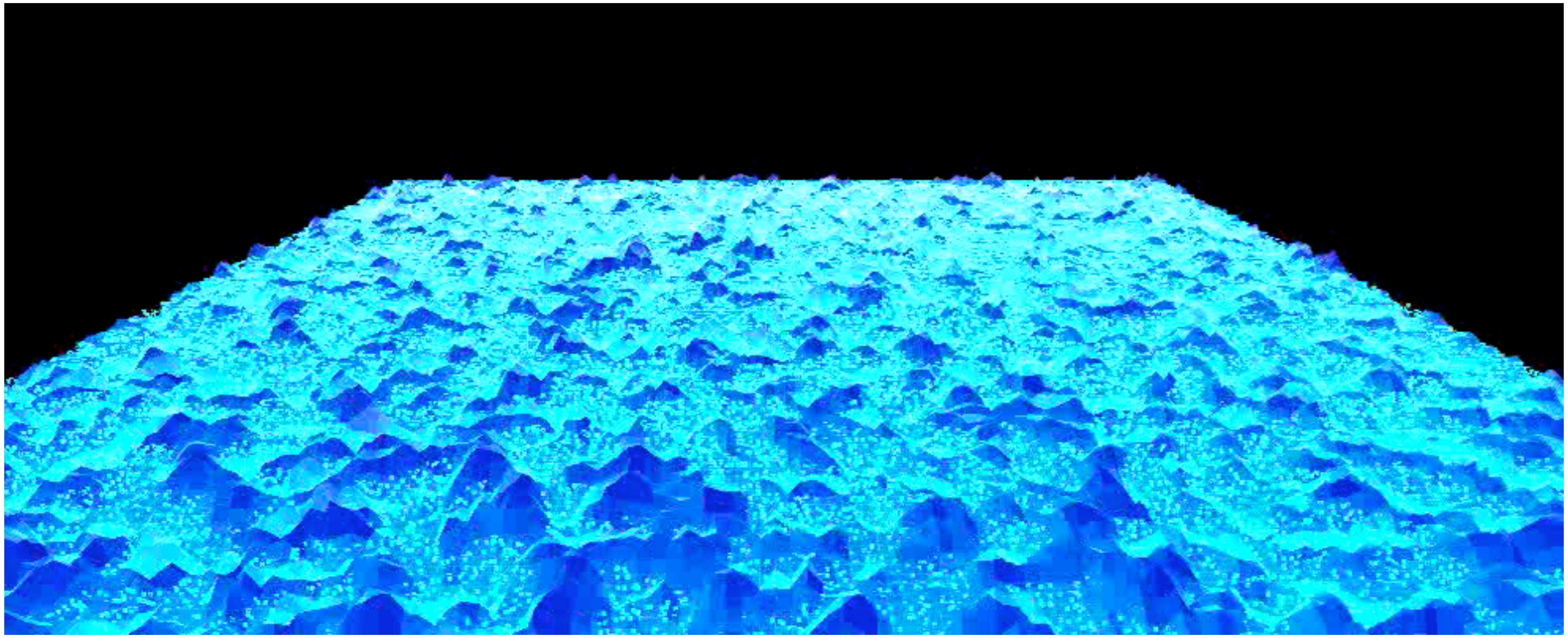
Classical lattice simulations

- Hamiltonian approach: t continuous, discretise on 3d lattice
- Inclusion of UV quantum effects via HTL effective theory possible

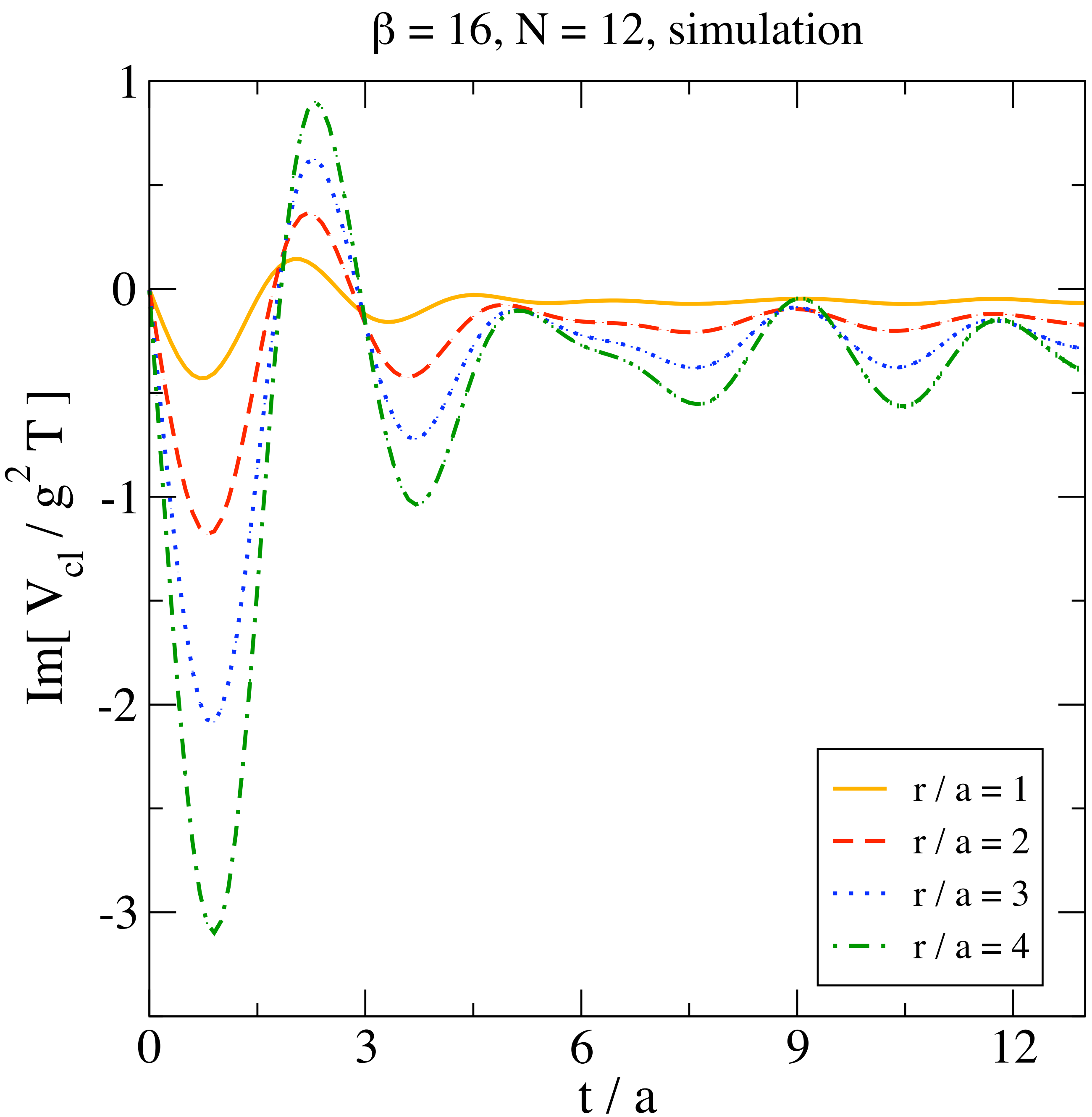
$$Z=\int \mathcal{D}U_i\,\mathcal{D}E_i\,\delta(G)e^{-\beta H},\quad \beta=\frac{2N}{g^2Ta},\quad H=\frac{1}{N}\sum_x\left[\sum_{i<j}\mathrm{Re\,Tr}\,(1-U_{ij})+\frac{1}{2}\mathrm{Tr}\,(E_i^2)\right]$$

real time evolution: $\dot{U}_i(x)=iE_i(x)U_i(x)\,,\quad \dot{E}_i^a(x)=-2\,\mathrm{Im\,Tr}\,[T^a\sum_{|j|\neq i}U_{ij}(x)]$

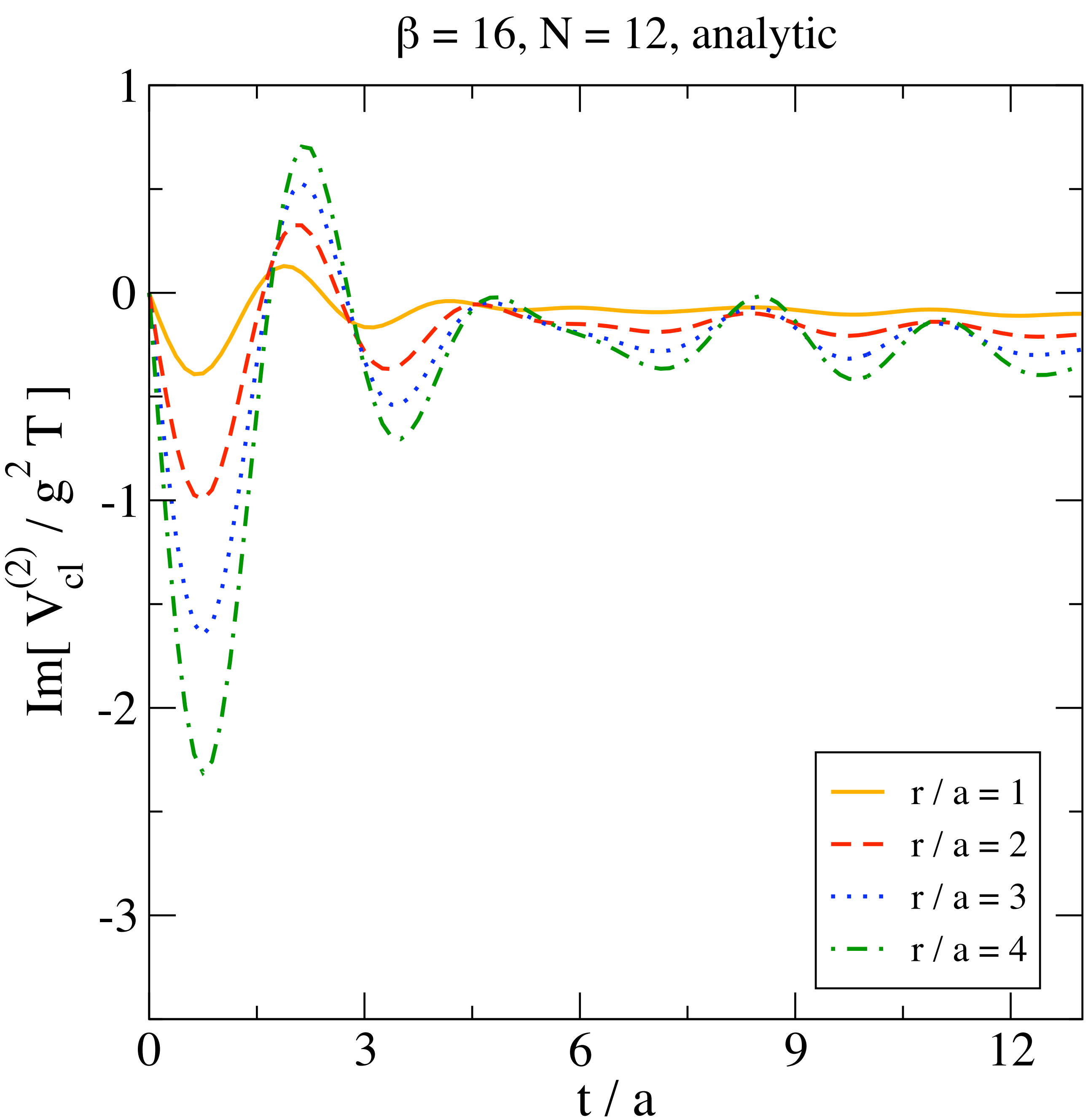
$$\langle \mathrm{Tr}U_{ij}\rangle = \langle \mathrm{Tr}F_{ij}^2\rangle$$



$$V_{\rm cl}(t,r) = \frac{i\partial_t C_{\rm cl}(t,r)}{C_{\rm cl}(t,r)}$$



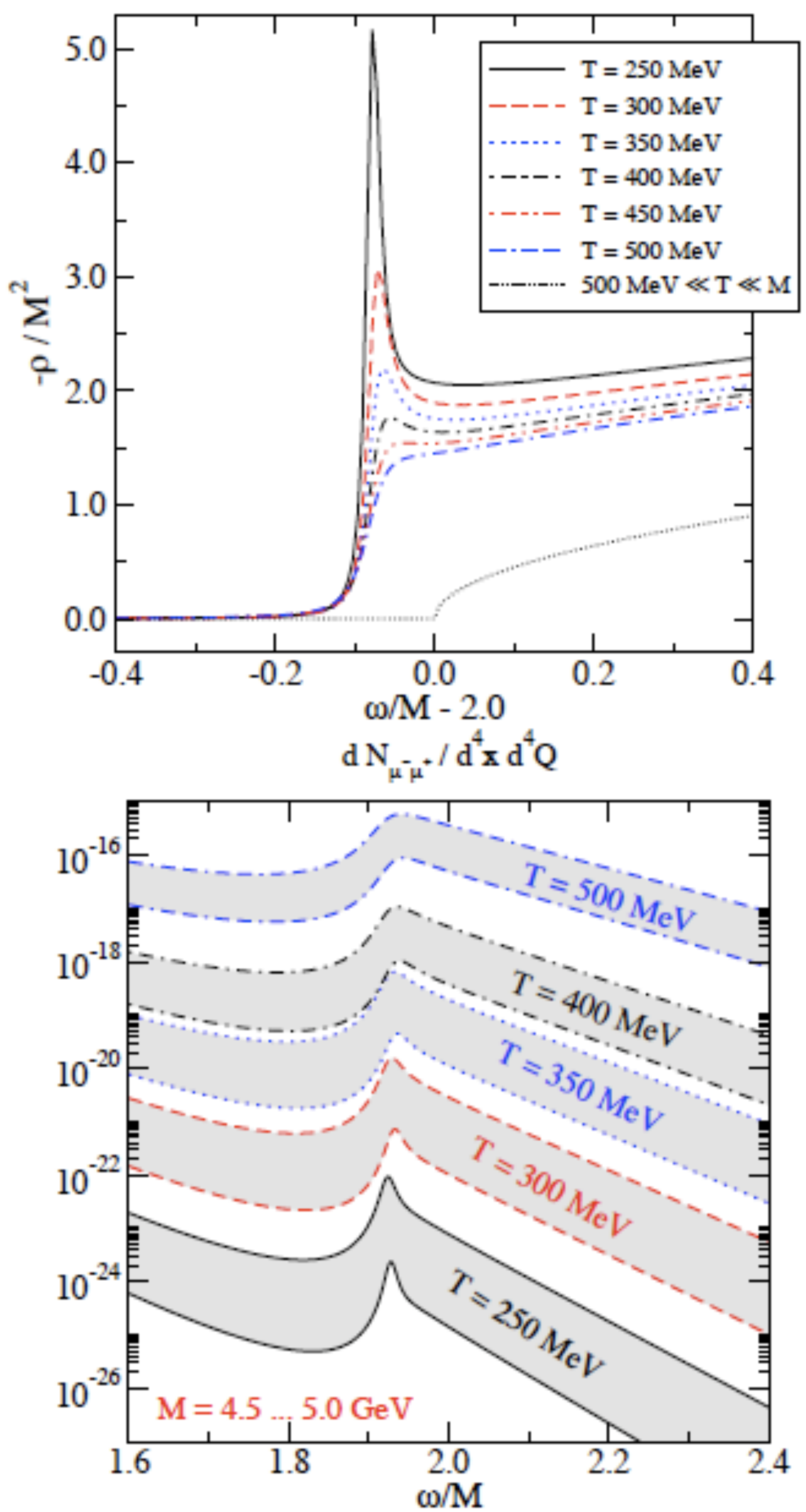
classical simulation



classical lattice perturbation theory

non-perturbative strengthening of damping

HTL perturbation theory good to ~ 25%



Physical signatures

Quarkonium signatures from the finite mass Schrödinger equation:

Spectral function Laine et al.,JHEP0801:043

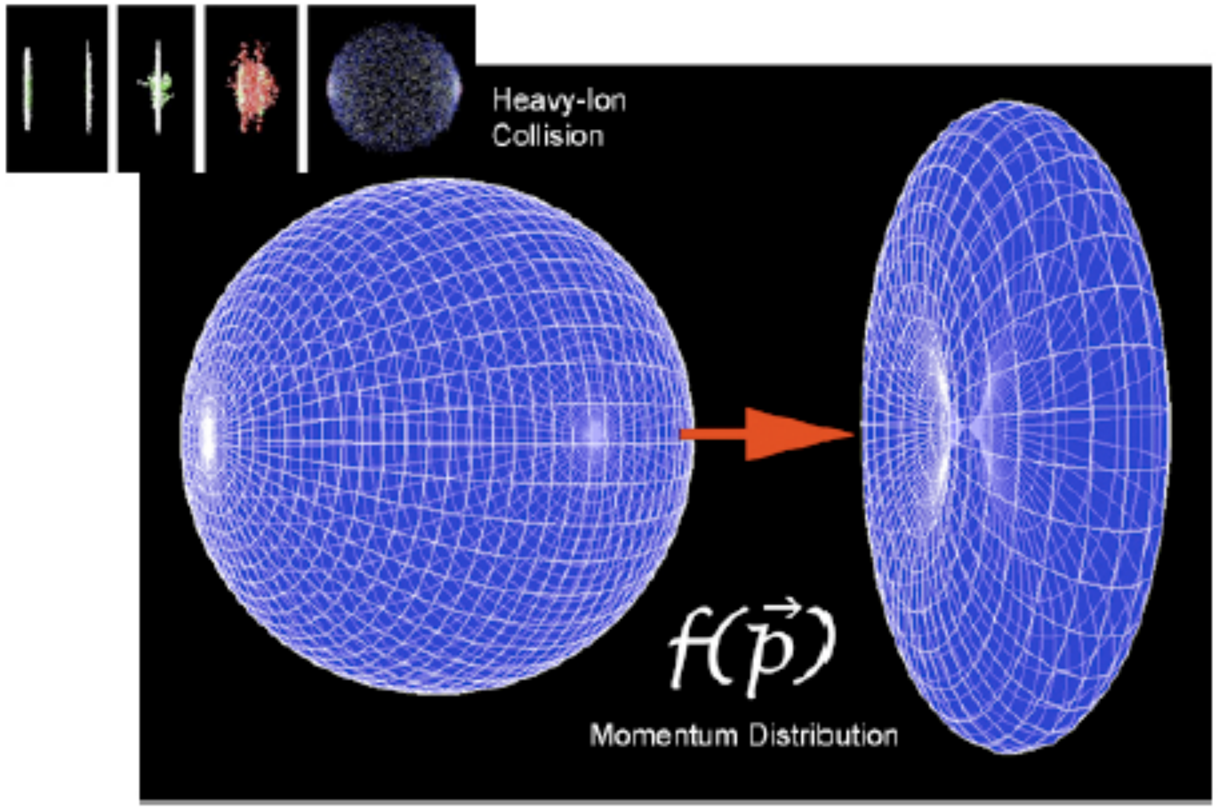
- The spectral function is depicted for Bottomonium.
- The imaginary part induces a finite width to the resonance peak (melting of the resonance).

Potential Laine, JHEP 0705:028

- The Dilepton rate is shown for Charmonium and Bottomonium.
- A softening of the resonance is seen for increased temperature.

N.B.: This is for an idealised plasma in complete equilibrium, not phenomenological (yet)

Non-equilibrium: anisotropic plasma



In the following the potential will be discussed for an anisotropic plasma characterized by the ***static*** momentum distribution:

$$f(\vec{p}) = n_B(p\sqrt{1+\xi(\vec{v}_p\cdot\vec{n})^2})$$

n_B: Thermal Bose Distribution, ξ: Anisotropy, $\vec{n}$: Collision axis

Romatschke, Strickland 03
Mrowczynski, Rebhan, Strickland 04

Effect on quarkonium: previous work

Dumitru, Guo, Strickland 08
Dumitru, Guo, Mocsy, Strickland 09
Burnier, Laine, Vepsälainen 09

Strategy: consider weak anisotropy,
expand to leading order in $\xi \leq 1$
evaluate effect on potential

General early findings: anisotropy weakens the effect of Landau damping

→ increase in dissociation temperature

Problem: normalisation issues

Observation:

Parametrisation removes particles with momentum in the anisotropy direction: dilution!

Particle density: $n = \int \frac{d^3p}{(2\pi)^3} f(\xi, \mathbf{p}) \quad = \{3.61, 2.55, 1.09, 0.3\} \quad \text{for} \quad \xi = \{0, 1, 5, 100\}$

Weak coupling limit: inconsistent, anisotropy in ideal gas does not mean dilution!

→ introduce normalisation and condition

$$n = \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} f_{\rm iso}(\boldsymbol{p}) \stackrel{!}{=} \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} f(\boldsymbol{p}, \xi)$$

More problems..... non-equilibrium

- Meaning of "T" in non-equilibrium
- Phenomenology: might want to keep energy density fixed, or entropy, or...
- Normalisation procedure not unique!

T-problem: small anisotropy = close to equilibrium;
soft modes (stongly coupled) thermalised with T
hard modes (weakly interacting) anisotropic, will relax to T

Let the parameter in $f(\xi, \mathbf{p})$ be T'

a) T'=T, i.e. parametrise by final equilibrium temperature — used a lot in literature

b) Decide which quantity to keep fixed; rescale $T' = T'(\xi, T)$

Here: keep particle density fixed

- ***Multiplicative Normalization***

  To keep the particle density of the medium fixed the distribution function $f(\vec{p})$ is multiplied by the prefactor

$$N(\xi) = \sqrt{1+\xi}.$$

- ***Landau matching***

  This matching procedure is often used in the context of hydrodynamic simulations. The particle density is kept fixed by rescaling T:

$$T(\xi) = TR^{-\frac{1}{4}}(\xi) \text{ where } R(\xi) = \frac{1}{2}\left(\frac{1}{1+\xi} + \frac{\arctan(\sqrt{\xi})}{\sqrt{\xi}}\right).$$

The results from both schemes are consistent for $\xi \ll 1$.

Note: when expanded to first order, Landau matching also keeps the energy density fixed!

How to obtain the gluon propagator $\tilde{G}^{00}_{11}(\omega = 0)$?

The longitudinal part of $\tilde{G}_{11} = Re\,\tilde{G}_R + \frac{1}{2}\tilde{G}_S$ in the static limit is needed.
$\tilde{G}_R$ is known and $\tilde{G}_S$ is obtained from a Schwinger-Dyson relation.

- ***Retarded gluon propagator in covariant gauge*** Dumitru et al., Romatschke

$$\begin{aligned}\tilde{G}^{\mu\nu}_R(K) &= \Delta_G\left[(K^2-\alpha-\gamma)\frac{\omega^4}{K^4}B^{\mu\nu} + (\omega^2-\beta)C^{\mu\nu} + \delta\frac{\omega^2}{K^2}D^{\mu\nu}\right] \\ &\quad + \Delta_A[A^{\mu\nu} - C^{\mu\nu}] - \frac{\lambda}{K^4}K^\mu K^\nu\end{aligned}$$

with structure functions $\alpha(K) - \delta(K)$ and

$$\Delta_G^{-1} = (K^2-\alpha-\gamma)(\omega^2-\beta) - \delta^2[\boldsymbol{k}^2 - (n\cdot K)^2] \text{ and } \Delta_A^{-1} = K^2 - \alpha.$$

$A(K) - D(K)$ form a *tensor basis* for this system where Lorentz symmetry is broken by the plasma rest frame and the anisotropy vector.

- ***Schwinger-Dyson Relation*** Arnold, Moore, Yaffe

The needed Schwinger-Dyson relation [Π_S: symmetric self-energy] is:

$$\begin{aligned}\tilde{G}_R &= G_R + G_R\cdot\Pi_R\cdot\tilde{G}_R, \\ \tilde{G}_S &= G_S + G_R\cdot\Pi_R\cdot\tilde{G}_S + G_S\cdot\Pi_A\cdot\tilde{G}_A + G_R\cdot\Pi_S\cdot\tilde{G}_A \quad \longrightarrow \quad \tilde{G}_S = \tilde{G}_R\cdot\Pi_S\cdot\tilde{G}^*_R\end{aligned}$$

- ***Retarded Propagator***

It is straightforward to obtain the retarded propagator in the static limit:

$$\tilde{G}_{\mathrm{R}}^{00}(\omega = 0, \boldsymbol{k}) = \frac{k^2 + m_\alpha^2 + m_\gamma^2}{(k^2 + m_\alpha^2 + m_\gamma^2)(k^2 + m_\beta^2) - m_\delta^4}$$

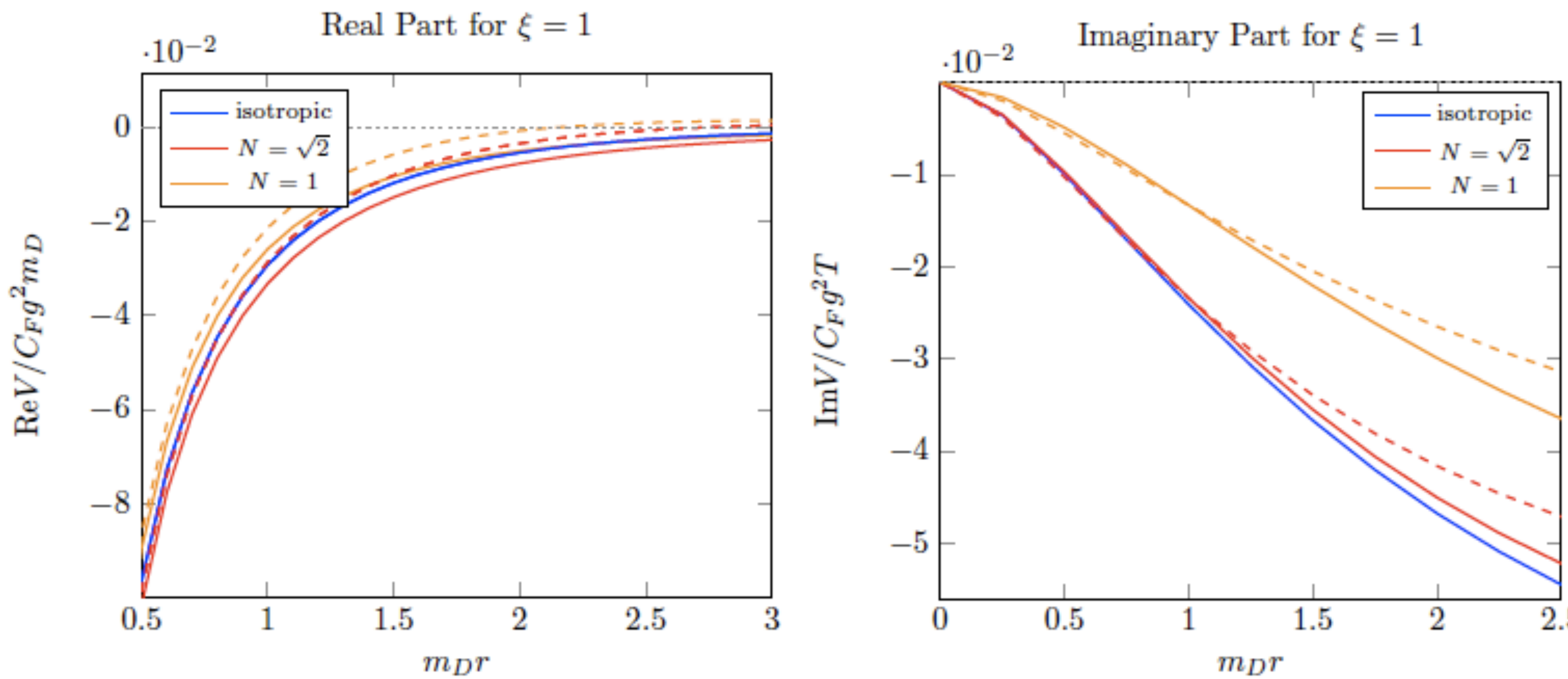
Effective masses [$\theta_k = \angle(\boldsymbol{n}, \boldsymbol{k})$]:

$$\hat{m}_\alpha^2 = -m_D^2 \frac{\xi}{3} \cos^2\theta_k \qquad \hat{m}_\beta^2 = 1 + \xi(\cos^2\theta_k - \frac{1}{6})$$

$$\hat{m}_\gamma^2 = \frac{\xi}{3} \sin^2\theta_k, \qquad \hat{m}_\delta^2 = -\xi \frac{\pi}{4} \sin\theta_k \cos\theta_k$$

Note that $\hat{m_x} = m_x / m_D$ where m_D is the isotropic Debye mass.

$$\mathrm{i}\Pi_{\mathrm{S}}^{\mu\nu} = 8\pi g^2 N \frac{1}{k} \int \frac{d^3p}{(2\pi)^3} v_p^\mu v_p^\nu f(\mathbf{p})(1 + f(\mathbf{p} + \boldsymbol{k}))\delta(\mathbf{v}_p \cdot \mathbf{v}_k)$$



- ***No Normalization***

  This case implies a specific relation between the particle density and the anisotropy. The medium is diluted quickly with increased anisotropy and the (perturbative) vacuum potential is approached.

- ***Fixed particle density***

  The change in the potential is very small compared to the isotropic result. The change observed in the upper case is a density effect.

Conclusions

- Quark anti-quark free energy is not a potential
- "Singlet" and "octet" free energies are unphysical
- Static potential + Schrödinger equation at finite T can be defined in effective field theory
- Landau damping effective before Debye screening
- Anisotropy: how to parametrise?
- Anisotropic distribution function alone has very small effect when energy or particle density are kept fixed, cf. Strickland 11