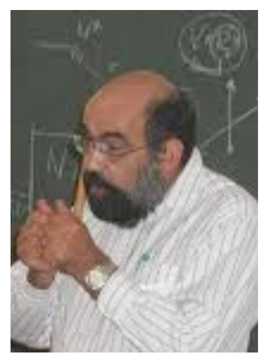
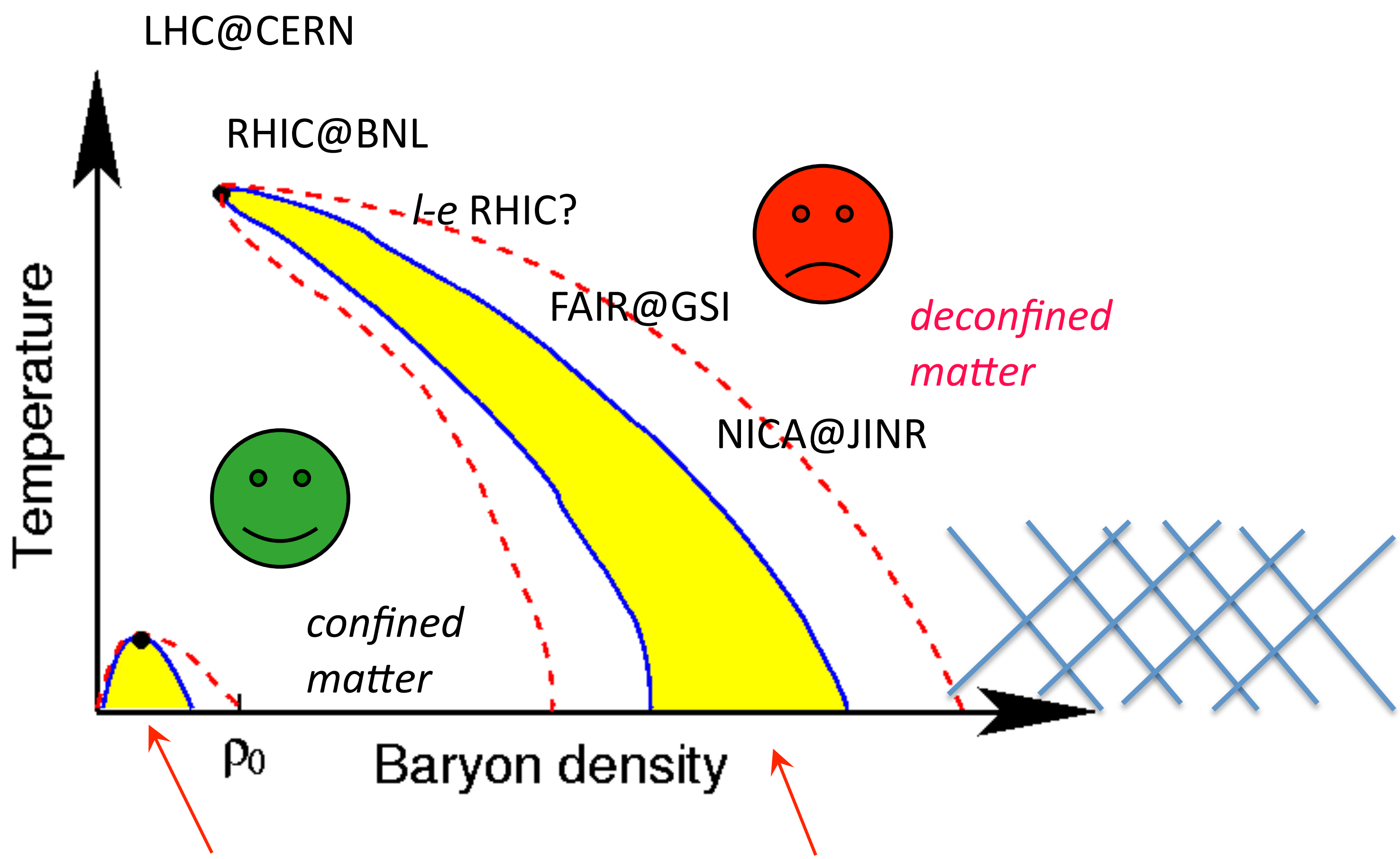
Dynamics of density instabilities associated with the first-order confinement phase transition

Jørgen Randrup, LBNL, Berkeley






Schematic and simplified
phase diagram of strongly interacting matter



Unstable medium hadronic

Unstable medium ?

Dynamics of collective modes in evolving many-body systems

Amplitude evolution: $\omega_\nu = \epsilon_\nu + i\gamma_\nu$

$$\frac{d}{dt}A_\nu(t) \;=\; -i\omega_\nu A_\nu(t) \;+\; B_\nu(t)$$

Markovian noise:

$$\prec B_\nu(t)\, B_\mu(t')^* \succ \;=\; 2\mathcal{D}_{\nu\mu}\,\delta(t-t')$$

Correlation function:

$$\sigma_{\nu\mu}(t_1,t_2) \;\equiv\; \prec A_\nu(t_1)\, A_\mu(t_2)^* \succ$$

Evolution:

$$\frac{d}{dt}\sigma_{\nu\mu}(t) \;=\; \underbrace{2\mathcal{D}_{\nu\mu}}_{\text{seed}} \;-\; \underbrace{i(\omega_\nu - \omega^*_\mu)\sigma_{\nu\mu}}_{\text{feedback}}$$

Variance of a single mode:

$$\frac{d}{dt}\sigma^2_\nu \;=\; 2\mathcal{D}_\nu \;+\; 2\gamma_\nu\sigma^2_\nu$$

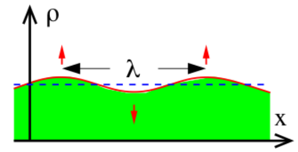
Lalime Equation

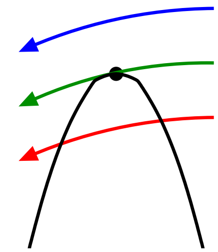
$$\sigma^2_\nu(t) \;=\; \left[2\mathcal{D}_\nu \int_0^t \mathrm{e}^{-2\Gamma_\nu(t')}dt' + \sigma^2_0\right]\mathrm{e}^{2\Gamma_\nu(t)}$$

$$\Gamma_\nu(t) \equiv \int_0^t \gamma_\nu(t')dt'$$

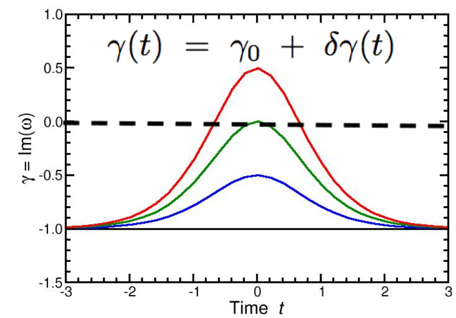
Amplification coefficient

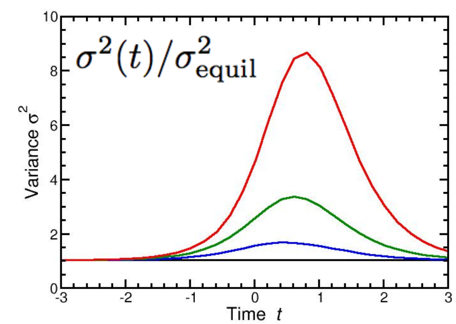
$$\gamma_\nu < 0: \quad \sigma^2_\nu(t) \;\to\; -\mathcal{D}_\nu/\gamma_\nu$$
$$\gamma_\nu = 0: \quad \sigma^2_\nu(t) \;=\; 2\mathcal{D}_\nu t$$
$$\gamma_\nu > 0: \quad \sigma^2_\nu(t) \;\sim\; \mathrm{e}^{2\Gamma_\nu(t)}$$





Colonna, Chomaz, Randrup,
Nucl Phys A 567 (1994) 637

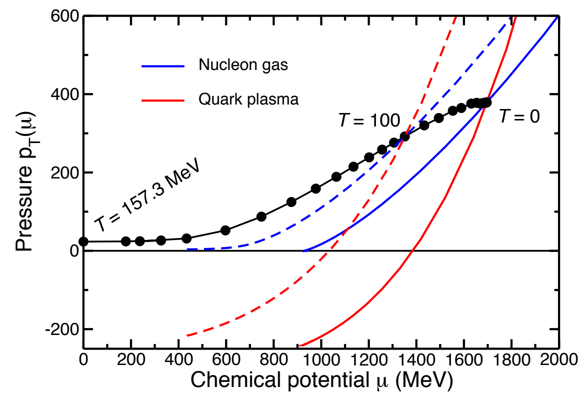




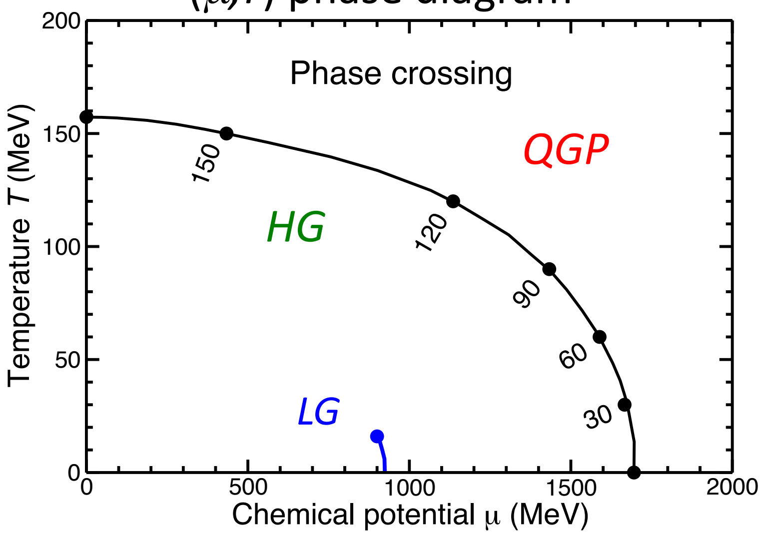
Phase crossing: *Two different media have the same T, μ, p*

HG: $p^H = p_\pi + p_N + p_{\bar{N}} + p_w$

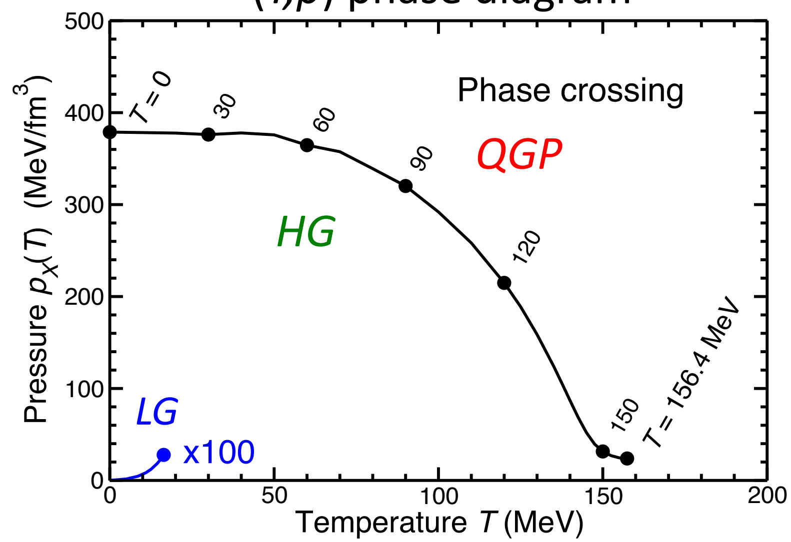
QGP: $p^Q = p_g + p_q + p_{\bar{q}} - B$



(μ,T) phase diagram



(T,p) phase diagram



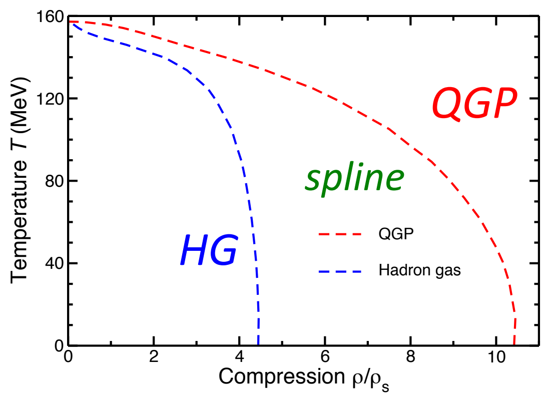
- qualitatively similar to LG

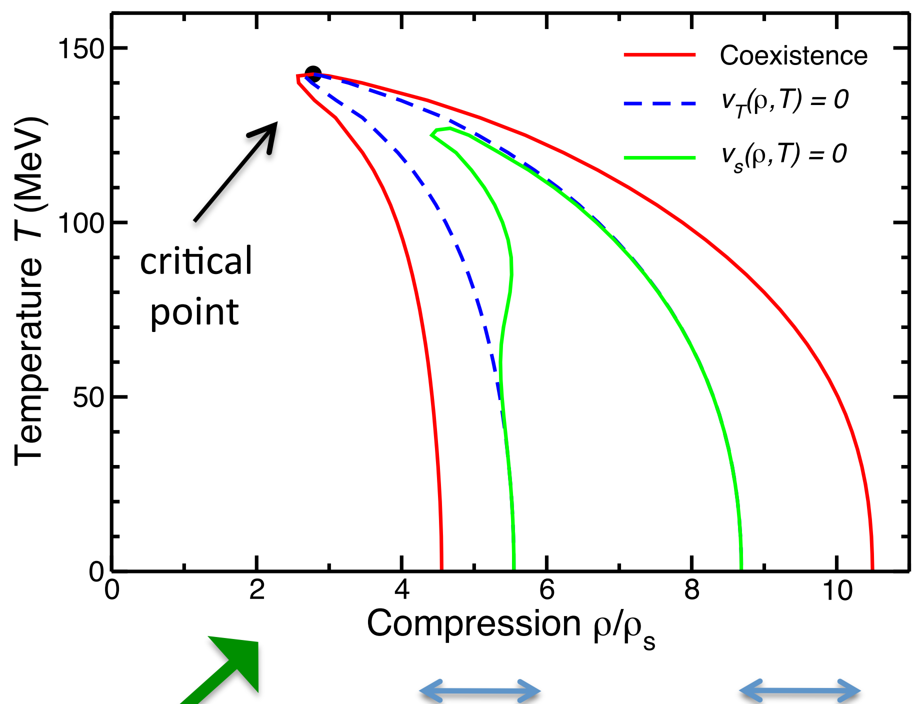
- qualitatively different from LG!

[Igor Iosilevsky, 2010]

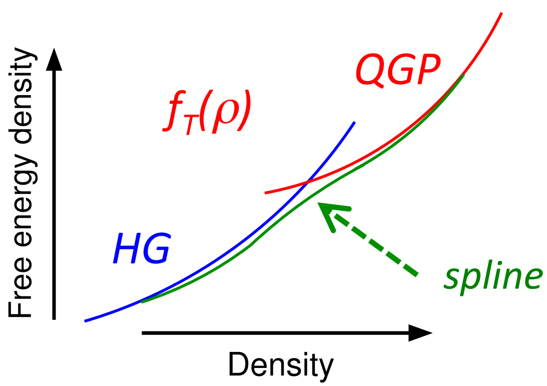
Construct a two-phase equation of state
by splining between the HG and the QGP:

HG & QGP phase crossing in the (ρ,T) plane:



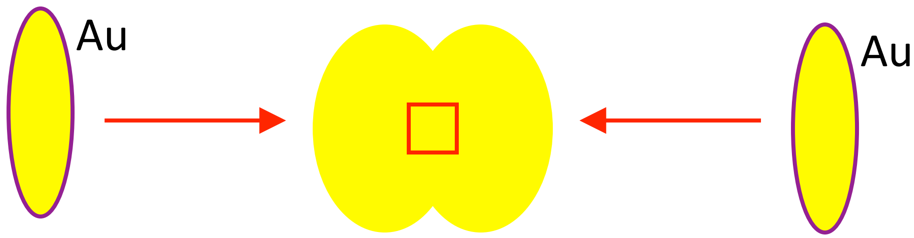


Spline between HG & QGP:



Horizontal scale is rather uncertain!

Collision energy <==> Phase region explored



- Dynamical transport simulations (Au + Au at b=0)

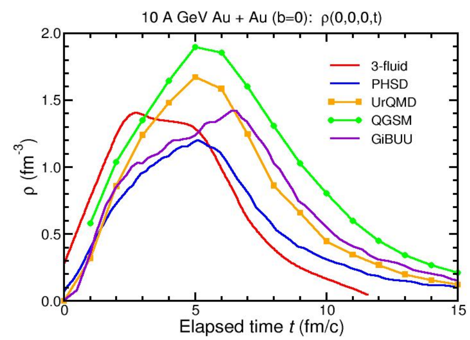
  3-fluid: Yu.B. Ivanov, V.N. Ruuskikh, V.D. Toneev

  PHSD: W. Cassing

  UrQMD: I.C. Arsene, L.V. Bravina

  QGSM: V.D. Toneev

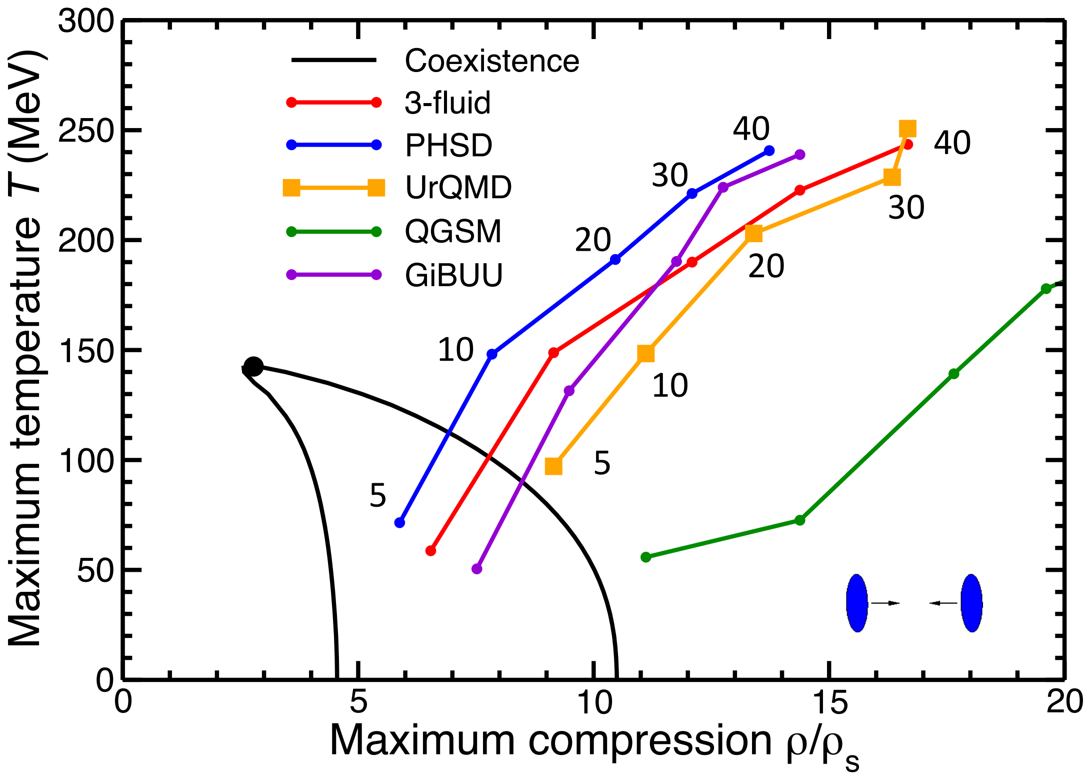
  GiBUU: A. Larionov



- Extract $\rho(t)$ and $\varepsilon(t)$ at the center

Arsene, Bravina, Cassing, Ivanov, Larionov, Randrup, Ruuskikh, Toneev, Zeeb, Zschiesche: PRC75 (2007) 034902

Maximum density and temperature achieved



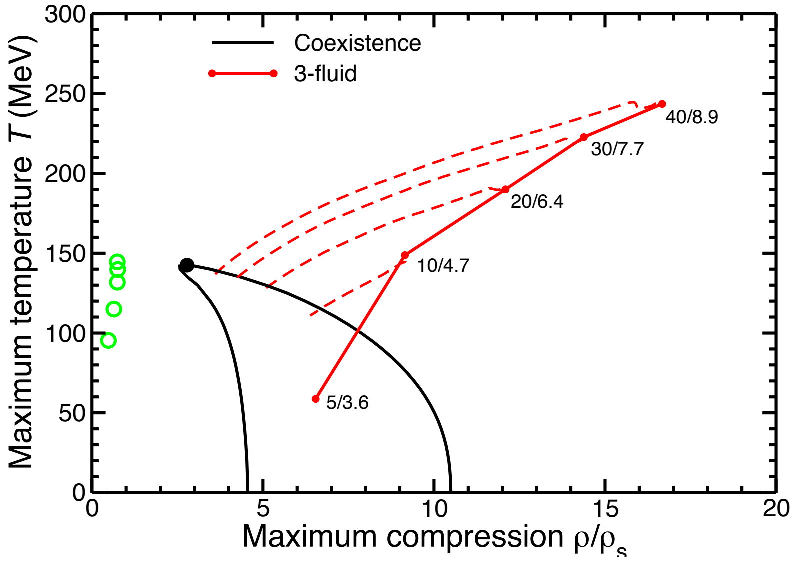
QGP:

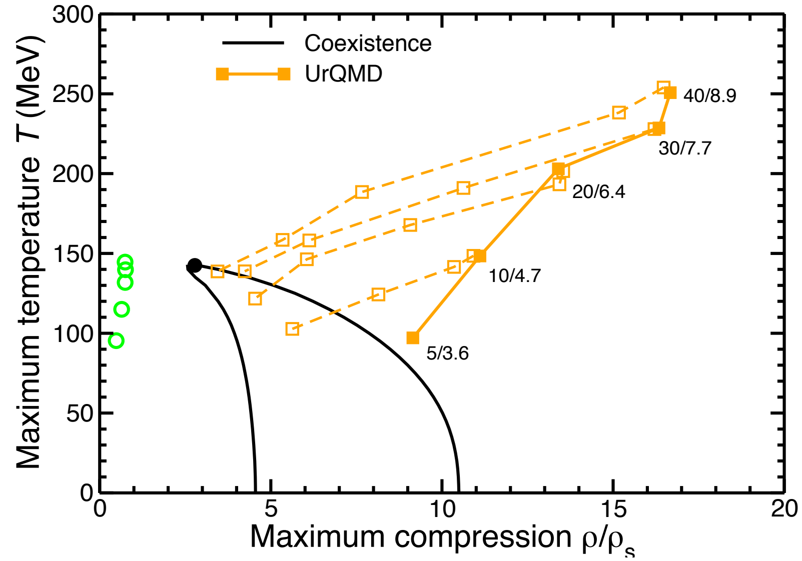
$$\rho = 2\mu_q T^2 + \frac{2}{\pi^2}\mu_q^3$$

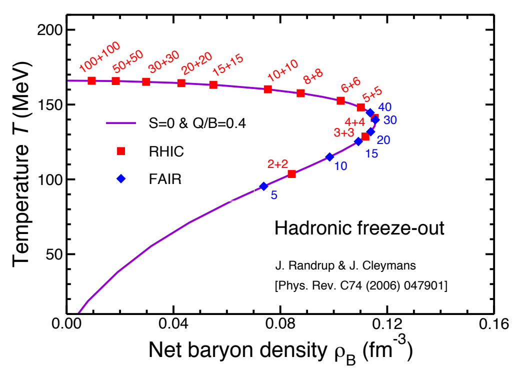
$$\varepsilon = \frac{37\pi^2}{30}T^4 + 3\mu_q^2 T^2 + \frac{3}{2\pi^2}\mu_q^4 + B$$

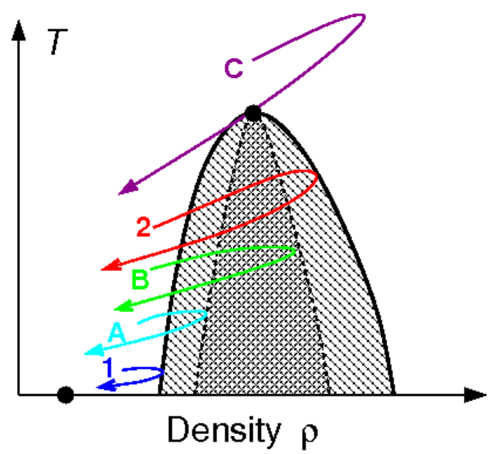
=> *(ρ,T)*

Expansion (ρ,T) phase trajectories

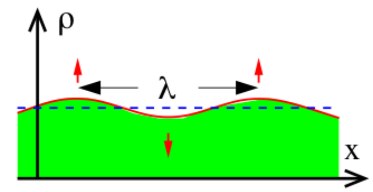








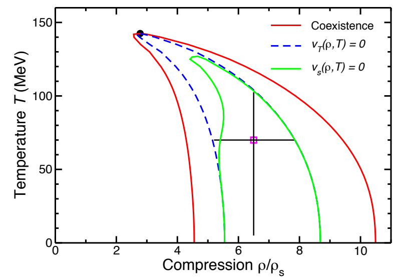
Evolution of density fluctuations with dissipative fluid dynamics

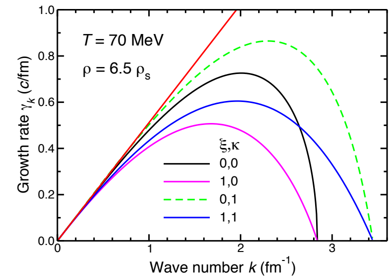


$$\omega_k = \epsilon_k + i\gamma_k$$

A_k ~ exp(-iω_kt)

Dispersion relations:





Ideal fluid dynamics: $$\omega^2 \doteq v_s^2 k^2$$

+ gradient term: $$\omega^2 \doteq v_s^2 k^2 + C\frac{\rho_0^2}{h_0}k^4$$ <= $$p_k \to p_k + C\rho_0 k^2 \rho_k$$

+ shear & bulk viscosity: $$\omega^2 \doteq v_s^2 k^2 + C\frac{\rho_0^2}{h_0}k^4 - i\xi\frac{\omega}{h_0}k^2$$ <= $$\xi \equiv \tfrac{4}{3}\eta + \zeta$$

+ heat conduction: $$\omega^2 \doteq v_T^2 k^2 + C\frac{\rho_0^2}{h_0}k^4 - i\xi\frac{\omega}{h_0}k^2 + \frac{v_s^2 - v_T^2}{1 + i\kappa k^2/\omega c_v}k^2$$ <= κ

Transport coefficients

$$\eta_0 \geq 1$$
$$\kappa_0 \geq 1$$

1) Bulk viscosity ζ: Ignore

$$\zeta \ll \eta \;\Rightarrow\; \xi \equiv \tfrac{4}{3}\eta + \zeta \approx \tfrac{4}{3}\eta$$

2) Shear viscosity η:

$$\rho = 0: \quad h \equiv p + \varepsilon = T\sigma$$

*) $$\rho = 0: \quad \eta \geq \frac{\hbar}{4\pi}\sigma = \frac{\hbar}{4\pi}\frac{h}{T}$$

$$\eta(\rho, T) = \eta_0 \frac{c_0}{c} d(\rho, T)\, h(\rho, T)$$

$$\rho > 0,\; T \ll mc^2: \; h \asymp mc^2 n \gg T\sigma$$

$$\rho = 0: \quad n \sim T^3 \;\Rightarrow\; \frac{\hbar c}{T} = 4\pi c_0\, d$$

$$d \equiv n^{1/3}$$

$$\lambda_{\text{visc}} \equiv \frac{1}{c}\frac{\xi(\rho, T)}{h(\rho, T)/c^2} \approx \tfrac{4}{3}\eta_0\, c_0\, d(\rho, T)$$

3) Heat conductivity κ:

$$\eta \approx \tfrac{1}{3} n\bar{p}\ell$$
$$\kappa \approx \tfrac{1}{3}\bar{v}\ell c_v$$

$$\frac{\kappa}{\eta} \approx \frac{c_v}{h/c^2}$$

$$\bar{p} = m\bar{v} \qquad h \asymp mc^2 n$$
$$c_v \equiv \partial_T \varepsilon_T(\rho) \qquad c_v \asymp \tfrac{3}{2} n$$

$$\kappa(\rho, T) = \kappa_0\, c_0\, c\, d(\rho, T)\, c_v(\rho, T)$$

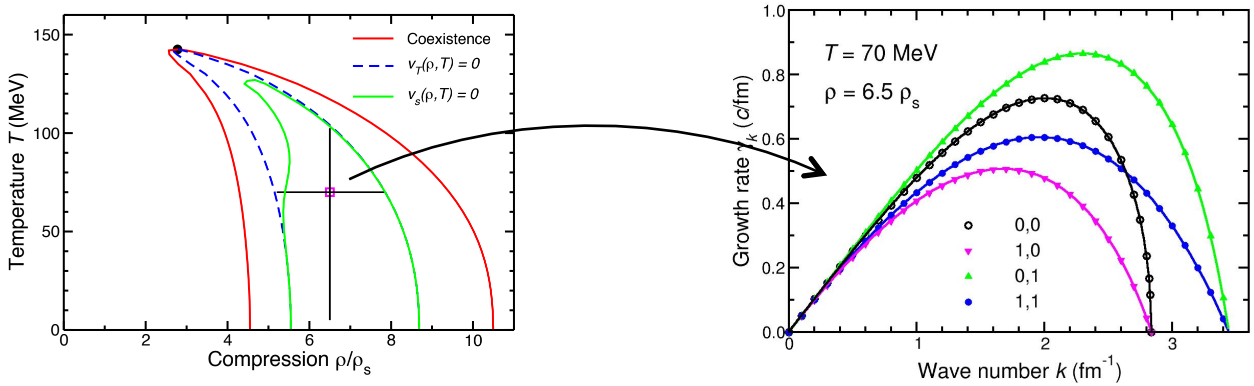
$$\lambda_{\text{heat}} \equiv \frac{1}{c}\frac{\kappa(\rho, T)}{c_v(\rho, T)} = \kappa_0\, c_0\, d(\rho, T)$$

$$c_0 = \frac{1}{4\pi}\left[(g_g + \tfrac{3}{3}g_q)\frac{\zeta(3)}{\pi^2}\right]^{\frac{1}{3}} \approx 0.12779$$

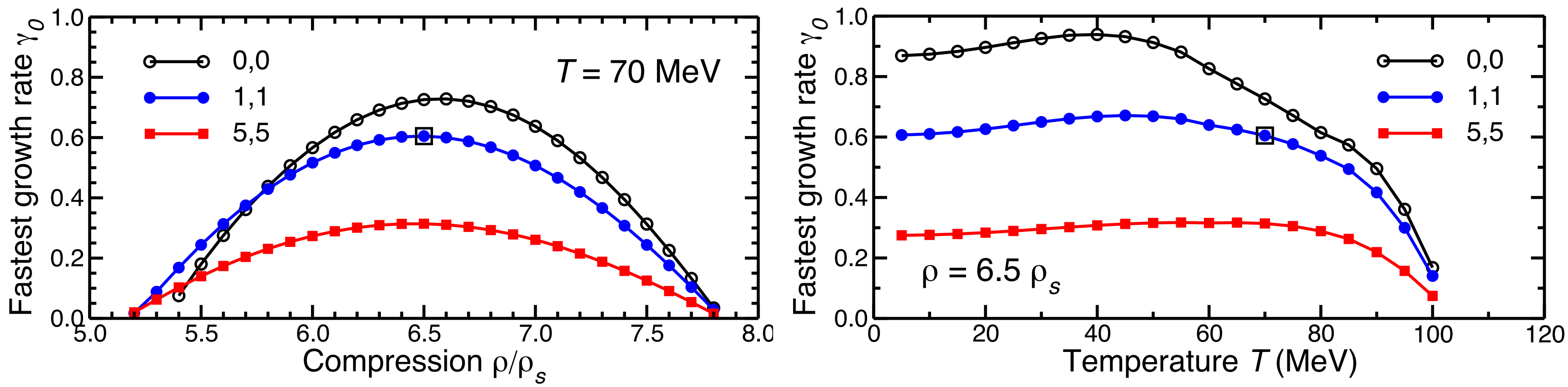
*) V Koch & J Liao, PRC81, 014902 (2010)

Spinodal growth rates

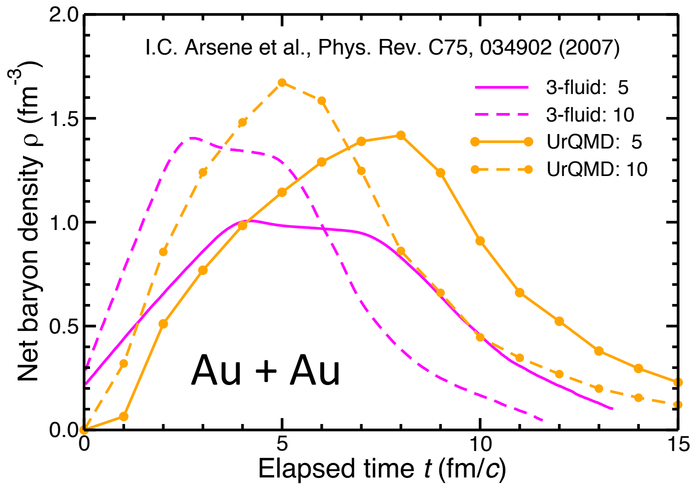
$$(\rho,T):\quad \omega^2 \doteq v_T^2 k^2 + C\frac{\rho_0^2}{h_0}k^4 - i\xi\frac{\omega}{h_0}k^2 + \frac{v_s^2 - v_T^2}{1 + i\kappa k^2/\omega c_v}k^2 \qquad \Rightarrow \quad \gamma_k(\rho,T)$$



Fastest growth rates:

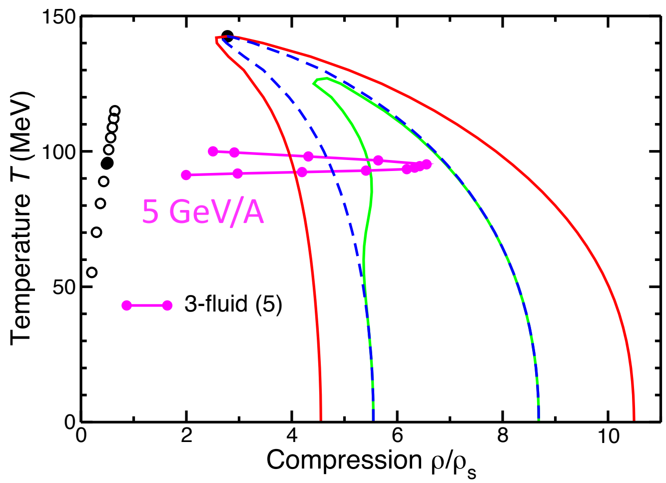


Dynamical phase trajectories ($\rho(t), T(t)$):

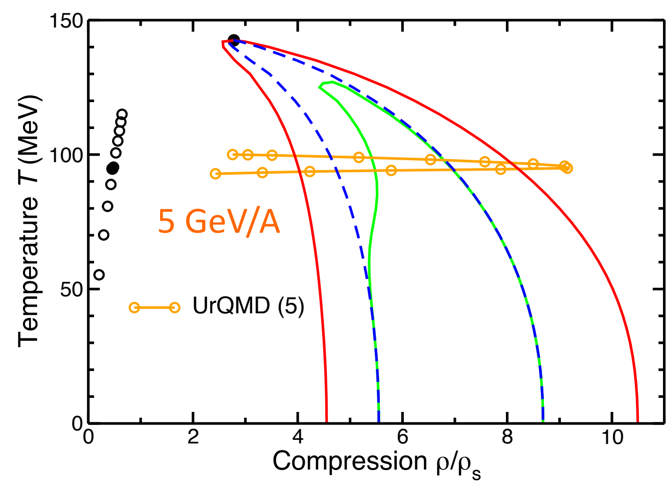


UrQMD
3-fluid
PHSD
QGSM
GiBUU

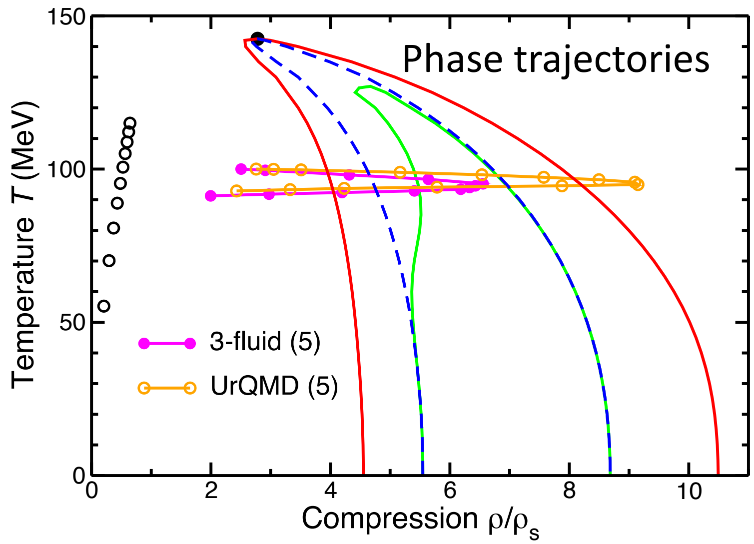
3-fluid



UrQMD



Spinodal amplification

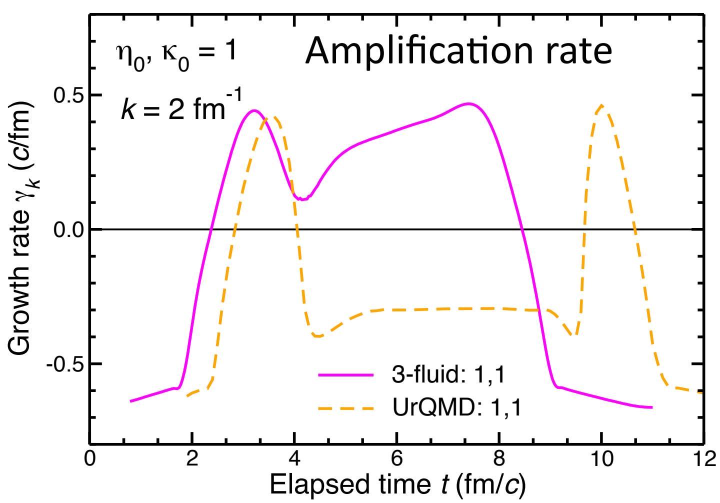


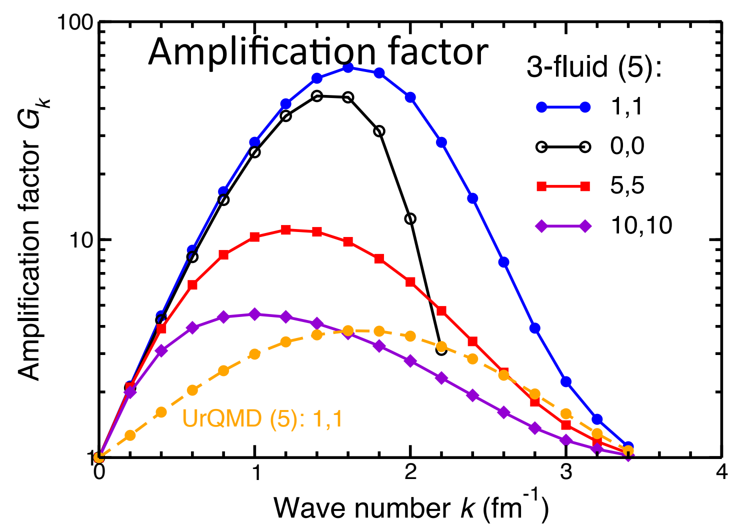
Amplification coefficient:

$$\Gamma_\nu(t) \equiv \int_0^t \gamma_\nu(t')dt'$$

Amplification factor:

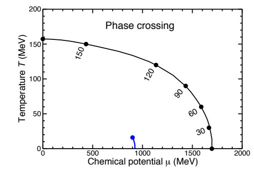
$$G_k = \mathrm{e}^{2\Gamma_k}$$

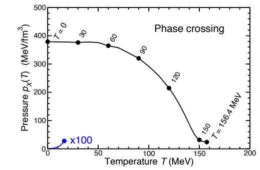




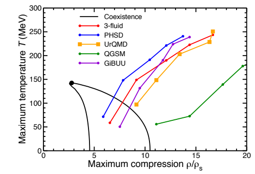
Summary & outlook

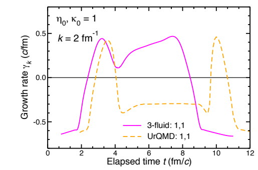
Elementary considerations suggest the existence of a deconfined QGP phase at high T and/or μ.





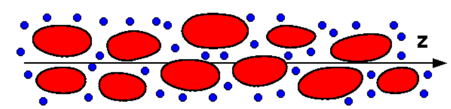
Dynamical model simulations suggest that this new phase is entered already at $E_{beam} \approx 10$ GeV/A





The energy range *below* this threshold is optimal for exploring the character of the phase transition

Spinodal decomposition (clumping) *may* occur in this range of collision energies



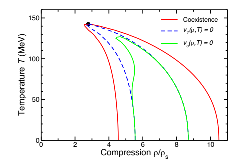
BUT: Full dynamical simulations are needed!

(SOME) MODEL REQUIREMENTS:

Two phases: hadron gas & quark-gluon plasma

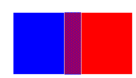
Equation of State with phase *transition*

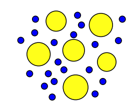
HG



QGP

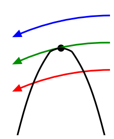
Phase *co*existence: interface & mixed phase*

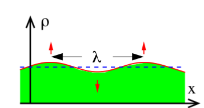


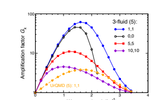


Dynamics with instabilities*

Spinodal modes*







* *Requires finite range*