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Lattice QCD based equation of state at finite baryon density

Pasi Huovinen

J. W. Goethe Universität

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in collaboration with Peter Petreczky @ BNL

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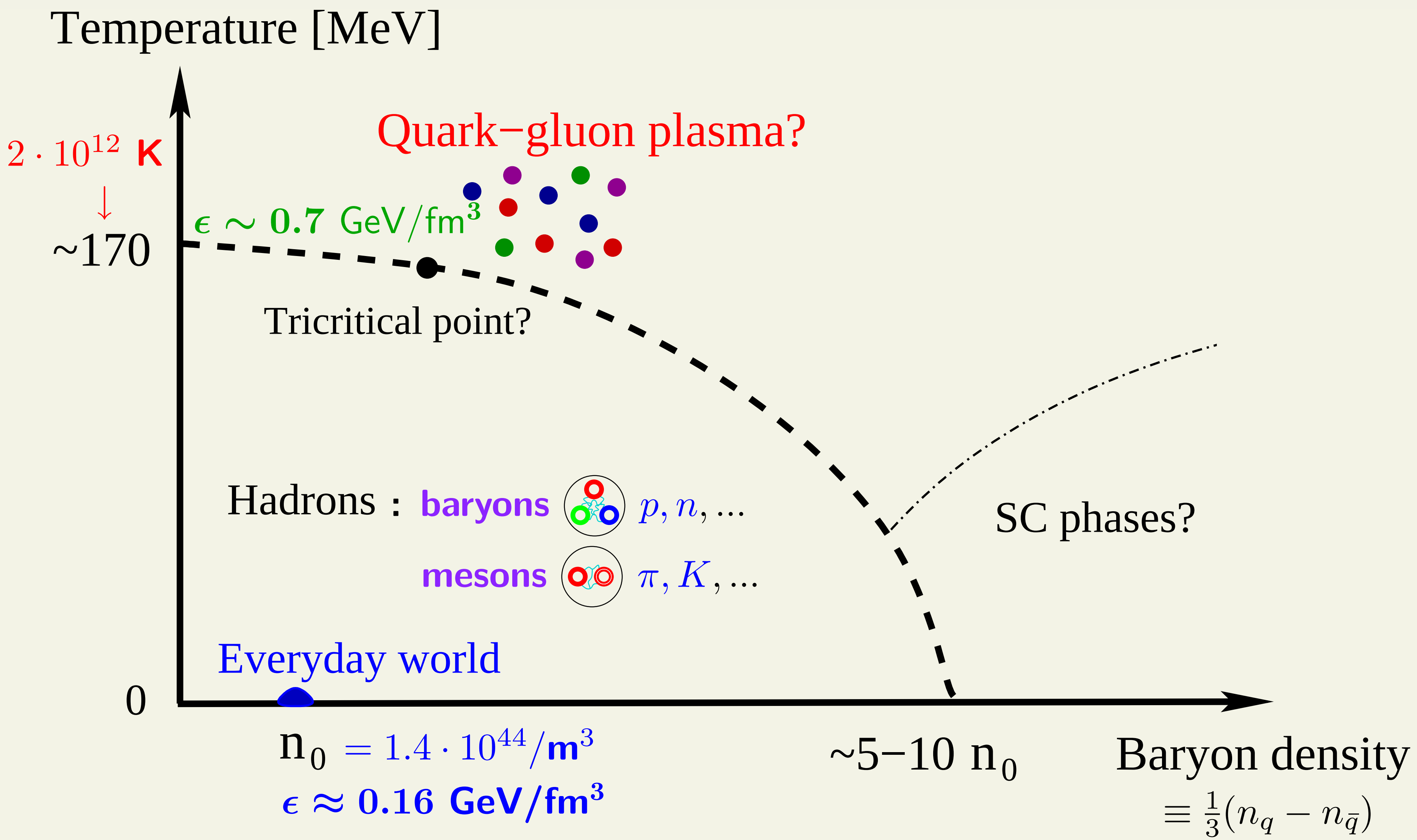
Nuclear phase diagram



Taylor expansion for pressure

$$\frac{P}{T^4} = \Sigma_{i,j}\, c_{ij}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_S}{T}\right)^j,$$

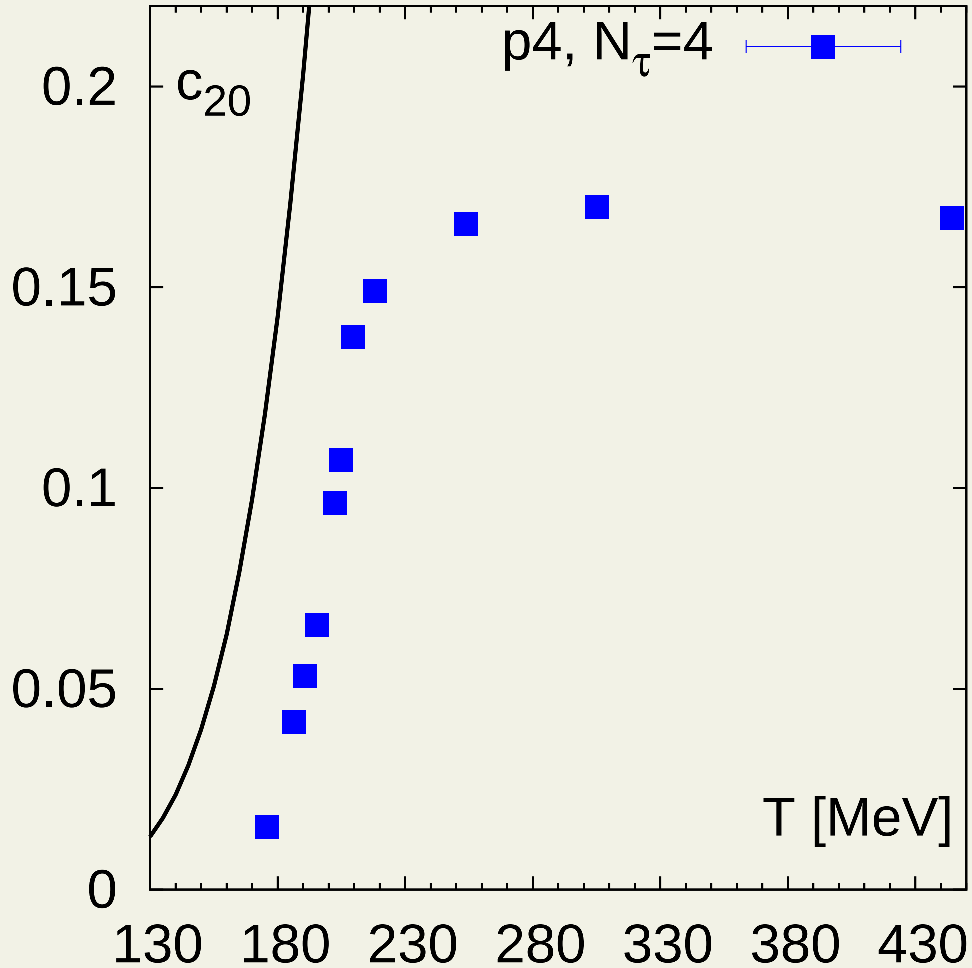
where

$$c_{ij}(T) = \frac{1}{i!j!}\frac{\partial^i}{\partial(\mu_B/T)^i}\frac{\partial^j}{\partial(\mu_S/T)^j}\frac{P}{T^4}$$

But: **Most extensive study using p4 action with** $N_\tau = 4$

$\Rightarrow$ **large discretization effects?**

Baryon number fluctuation



Hadrons on lattice

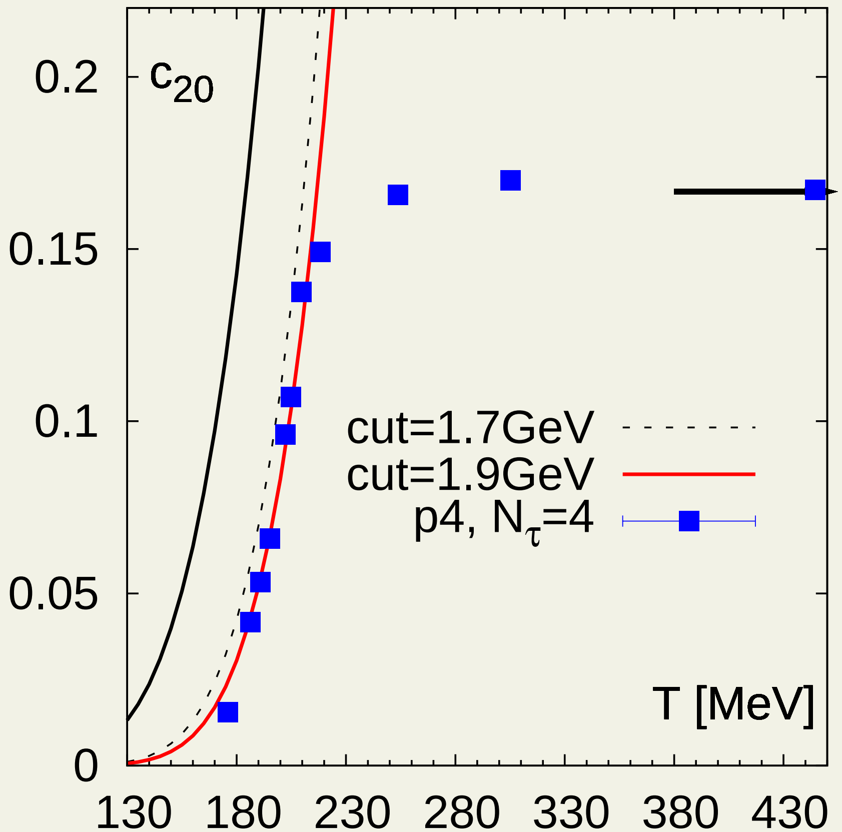
- 16 pseudoscalar mesons on lattice

- Hadron masses depend on lattice cutoff

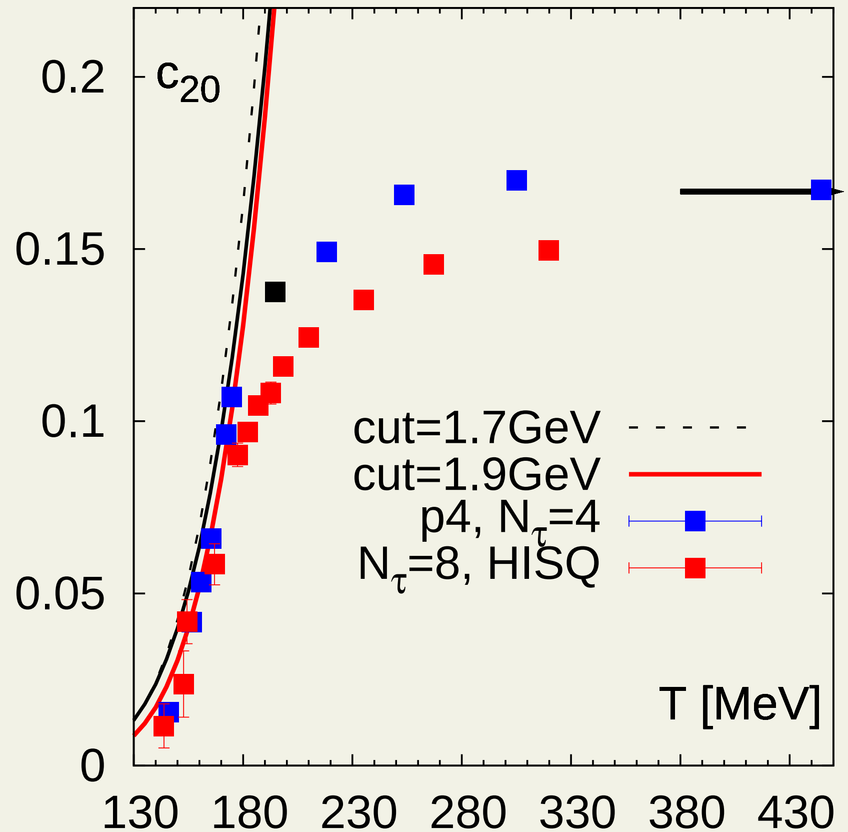
⇒ i.e. on temperature:
E.g. for pseudoscalar mesons on asqtad calculations

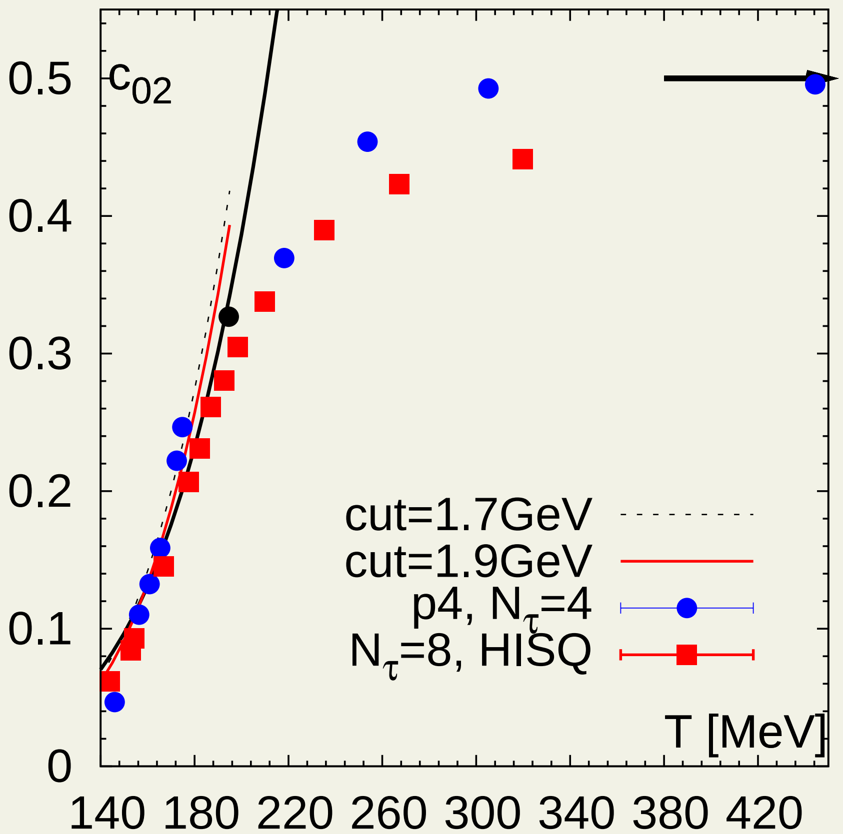
$$
\begin{aligned}
m^2_{\mathrm{ps}_i} &= m^2_{\mathrm{ps}_0} + \frac{1}{r_1^2}\frac{a^i_{\mathrm{ps}}x + b^i_{\mathrm{ps}}x^2}{(1+c^i_{\mathrm{ps}}x)^{\beta_i}} \\
x &= (a/r_1)^2 \\
a &= \frac{1}{N_\tau T}
\end{aligned}
$$

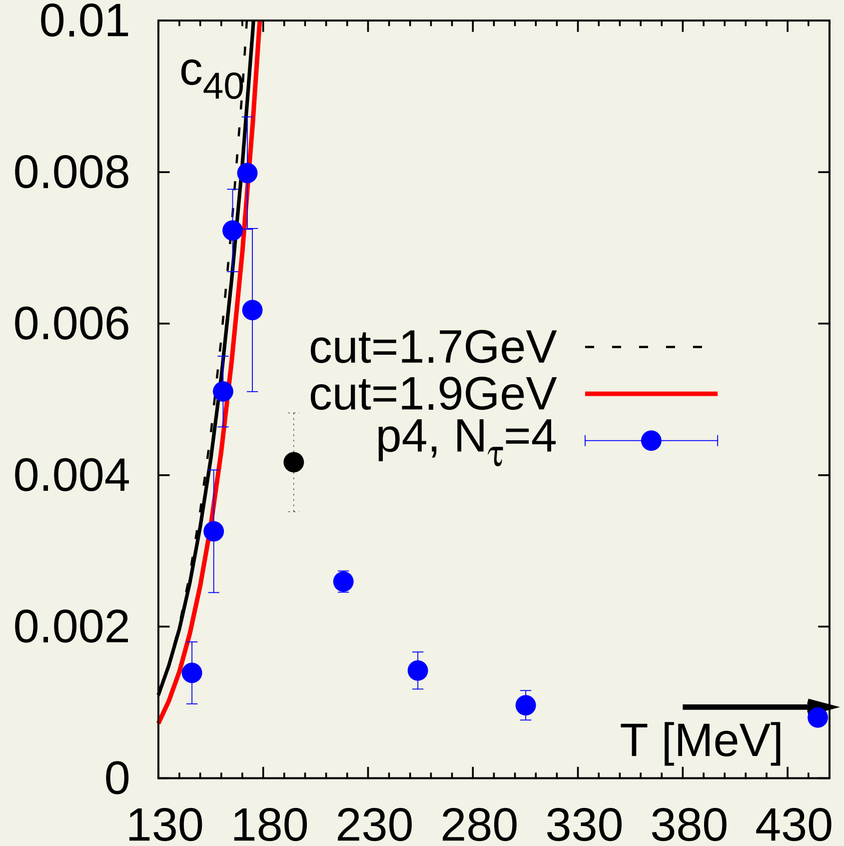
30 MeV shift



$\Rightarrow$




c02
0.5
0.4
0.3
0.2
0.1
0
140
180
220
260
300
340
380
420
T [MeV]
cut=1.7GeV
cut=1.9GeV
p4, Nτ=4
Nτ=8, HISQ


c40
0.01
0.008
0.006
0.004
0.002
0
130
180
230
280
330
380
430
T [MeV]
cut=1.7GeV
cut=1.9GeV
p4, Nτ=4

Parametrization

$$c_{ij}(T) = \frac{a_{1ij}}{T^{n_{1ij}}} + \frac{a_{2ij}}{T^{n_{2ij}}} + \frac{a_{3ij}}{T^{n_{3ij}}} + \frac{a_{4ij}}{T^{n_{4ij}}} + \frac{a_{5ij}}{T^{n_{5ij}}} + SB_{ij},$$

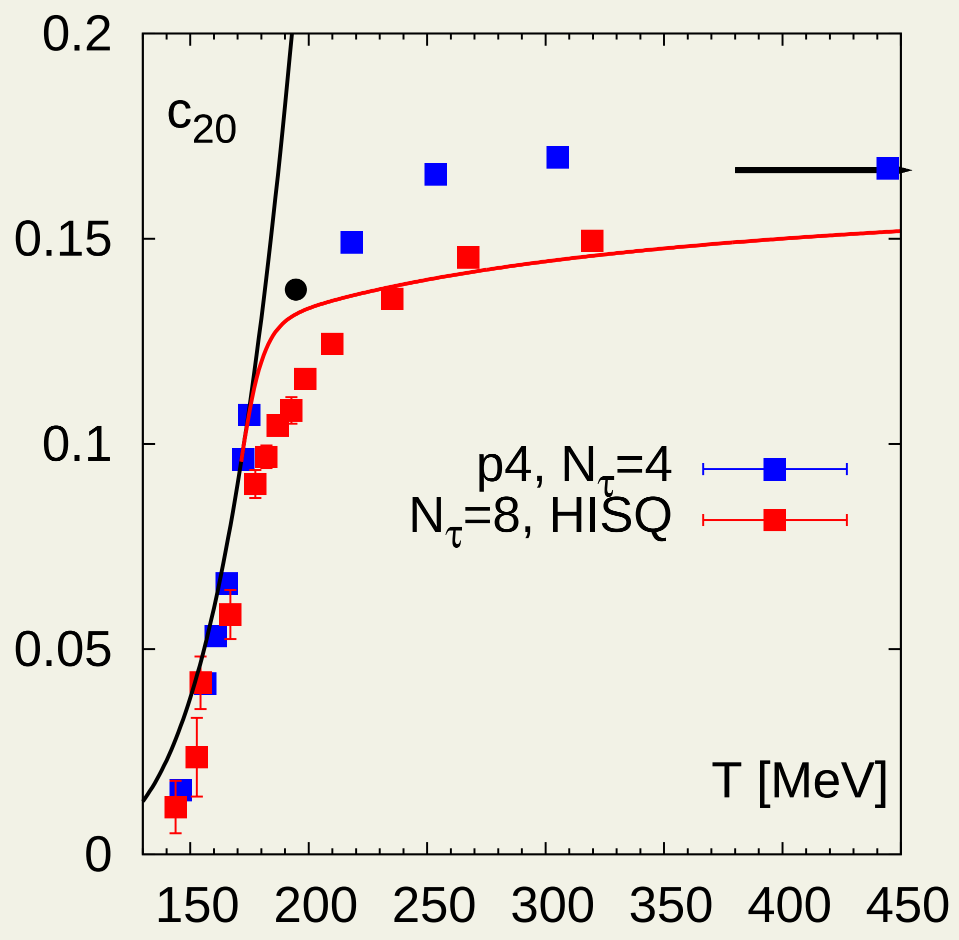
where n_{kij} are integers with $1 < n_{kij} < 42$.

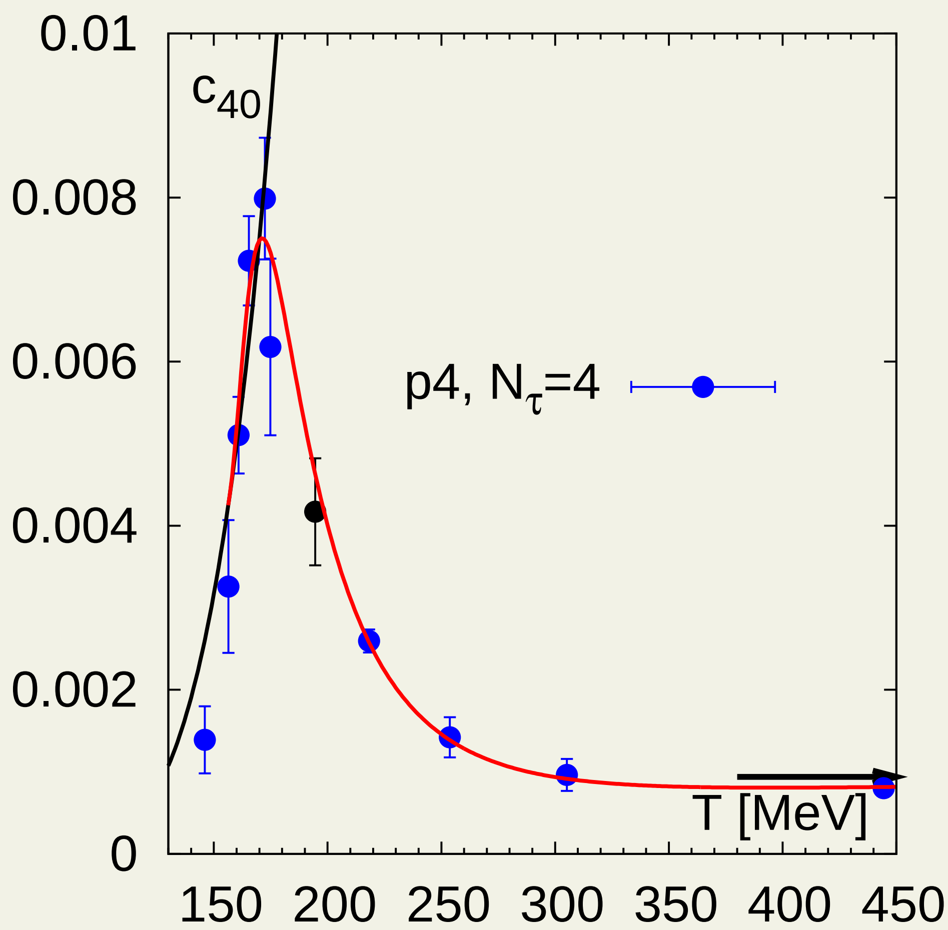
Require that

$$\begin{aligned} c_{ij}(T_{\mathrm{sw}}) &= c_{ij}^{\mathrm{HRG}}(T_{\mathrm{sw}}) \\ \frac{\mathrm{d}}{\mathrm{d}T} c_{ij}(T_{\mathrm{sw}}) &= \frac{\mathrm{d}}{\mathrm{d}T} c_{ij}^{\mathrm{HRG}}(T_{\mathrm{sw}}) \\ \frac{\mathrm{d}^2}{\mathrm{d}T^2} c_{ij}(T_{\mathrm{sw}}) &= \frac{\mathrm{d}^2}{\mathrm{d}T^2} c_{ij}^{\mathrm{HRG}}(T_{\mathrm{sw}}) \end{aligned}$$

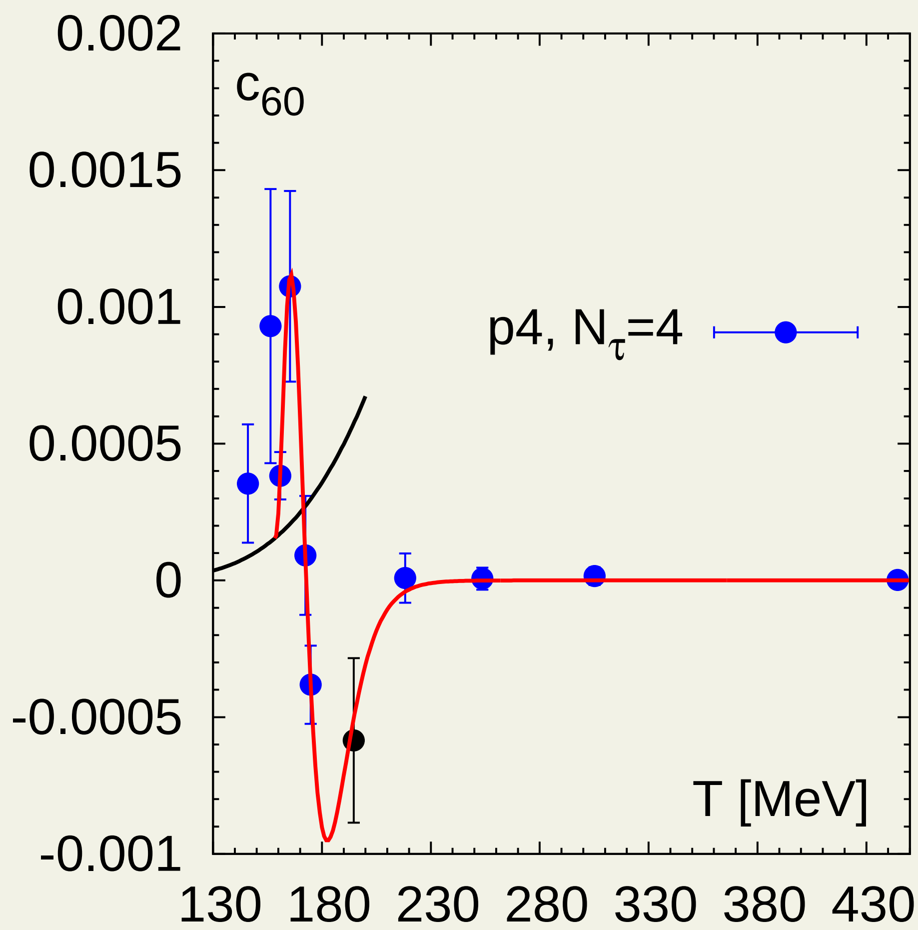
For second order terms also

$$c_{ij}(T = 800\,\mathrm{MeV}) = 0.95 \cdot SB_{ij}$$

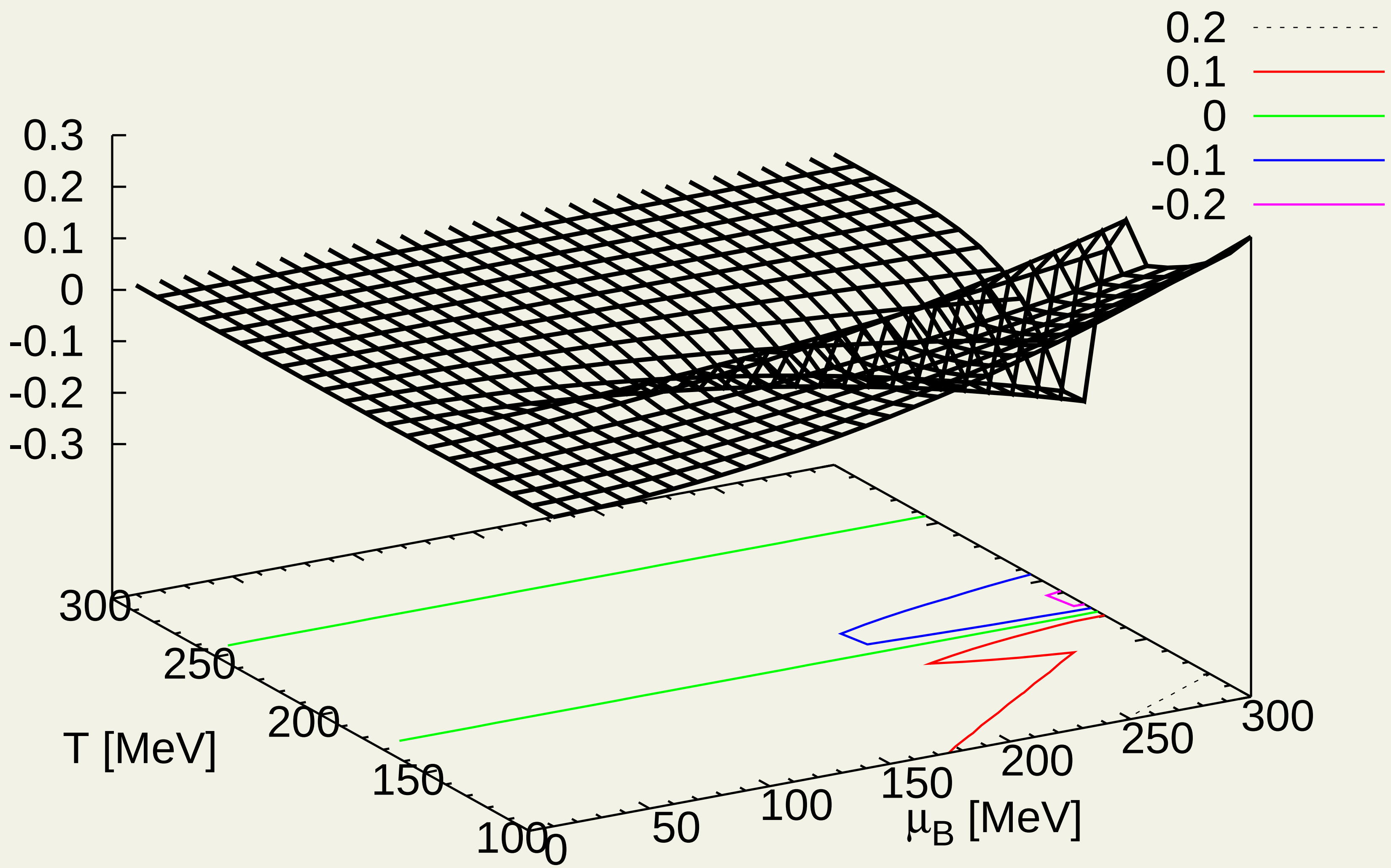

0.2
0.15
0.1
0.05
0
c20
p4, Nτ=4
Nτ=8, HISQ
T [MeV]
150
200
250
300
350
400
450



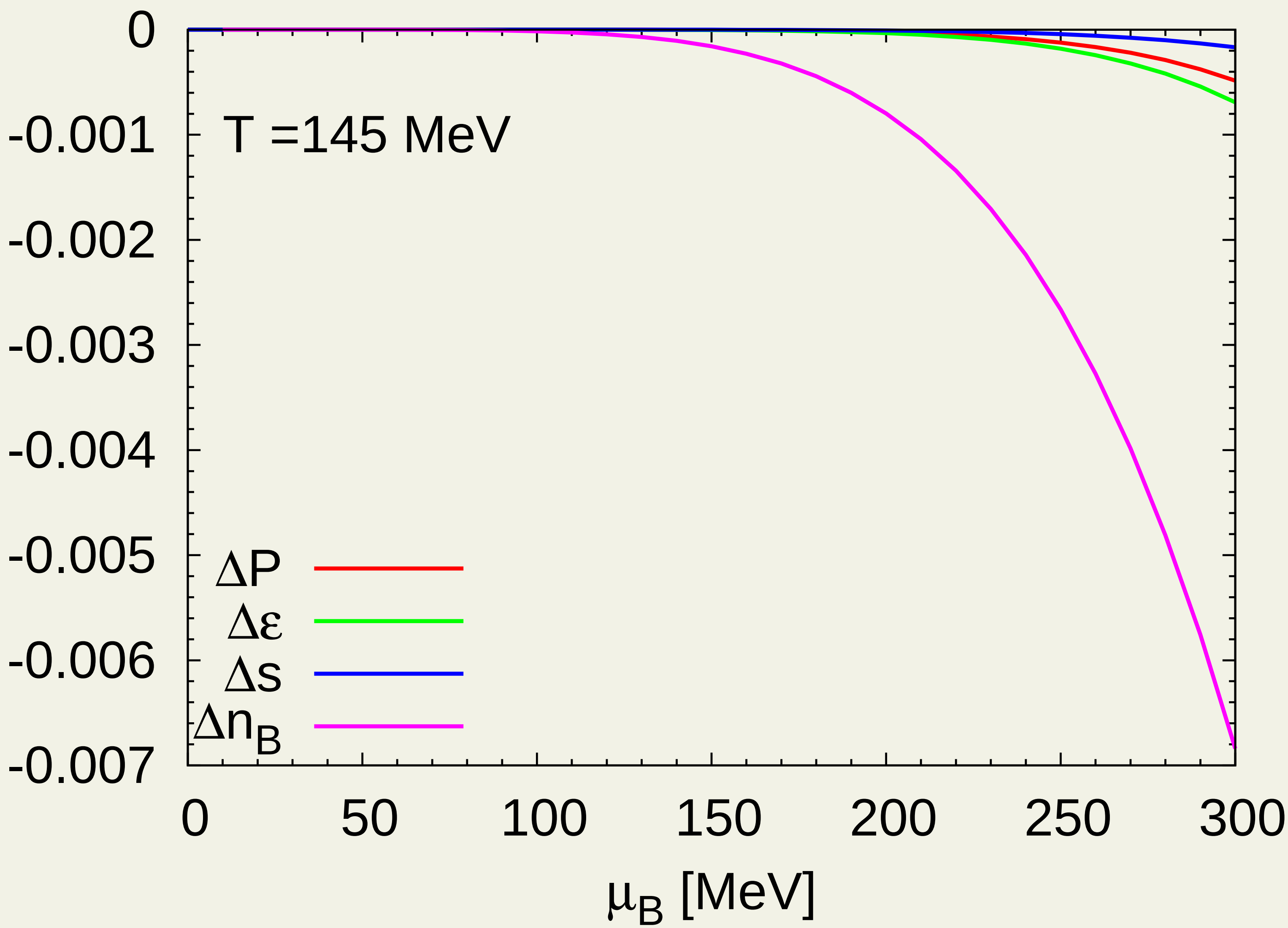
0.01
0.008
0.006
0.004
0.002
0
150
200
250
300
350
400
450
c40
p4, Nτ=4
T [MeV]


0.002
0.0015
0.001
0.0005
0
-0.0005
-0.001
130
180
230
280
330
380
430
c_{60}
p4, N_τ=4
T [MeV]

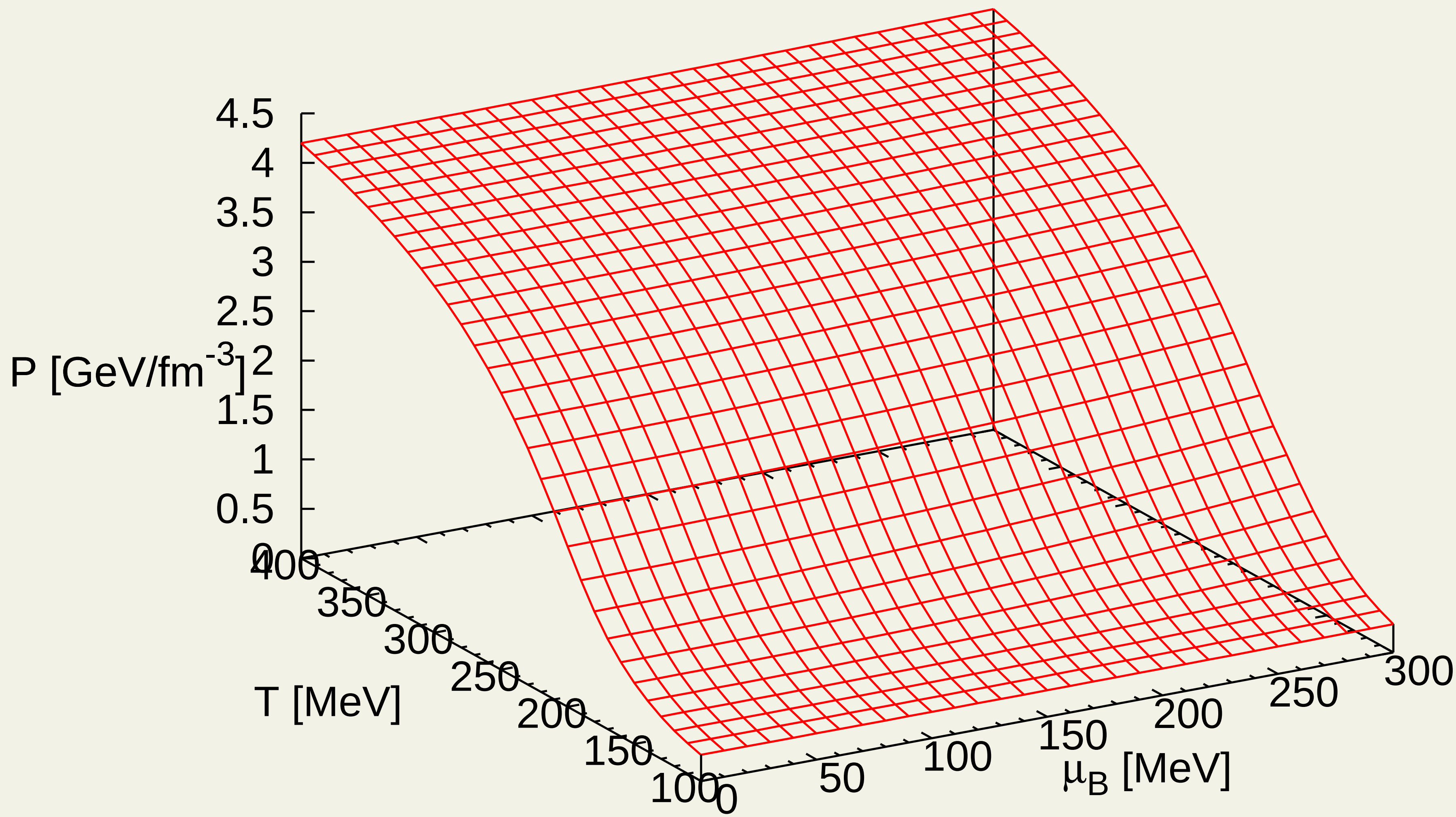
Ratio of sixth to fourth term



Taylor expanded hadron gas

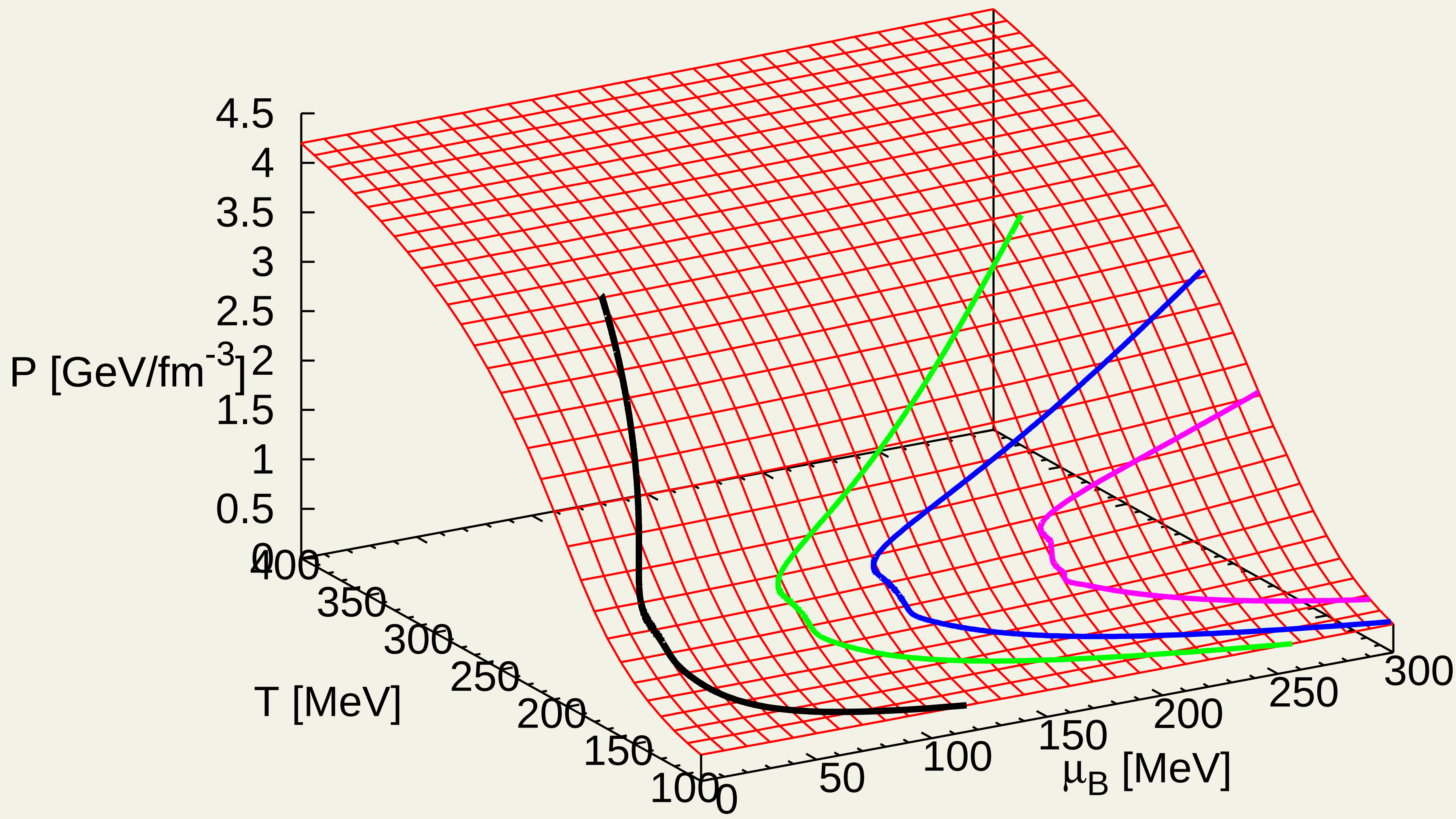


P/T^4



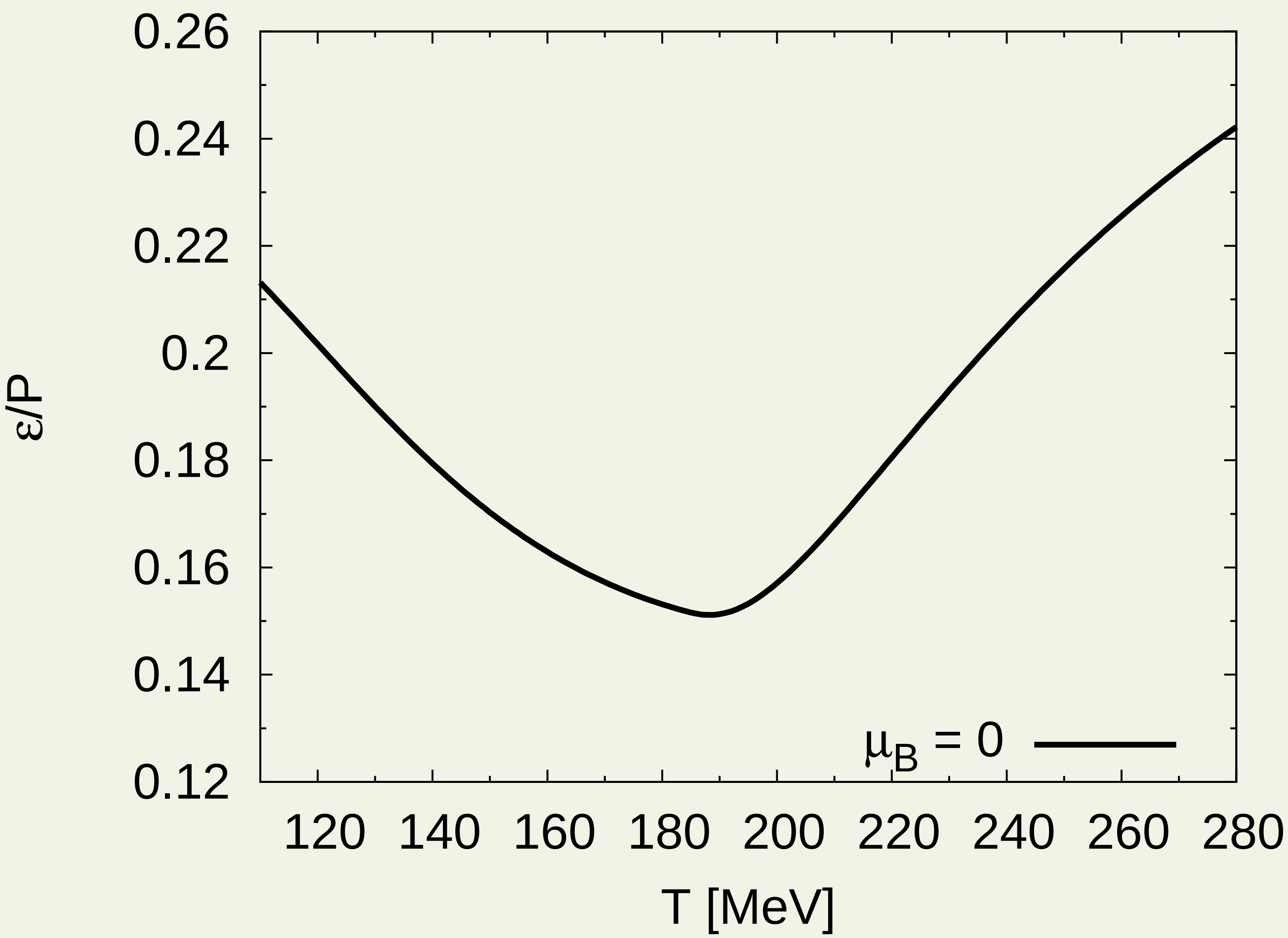
4.5
4
3.5
3
2.5
P [GeV/fm^{-3}]
2
1.5
1
0.5
0
400
350
300
250
200
150
100
T [MeV]
0
50
100
150
200
250
300
μ_B [MeV]

$$P/T^4$$

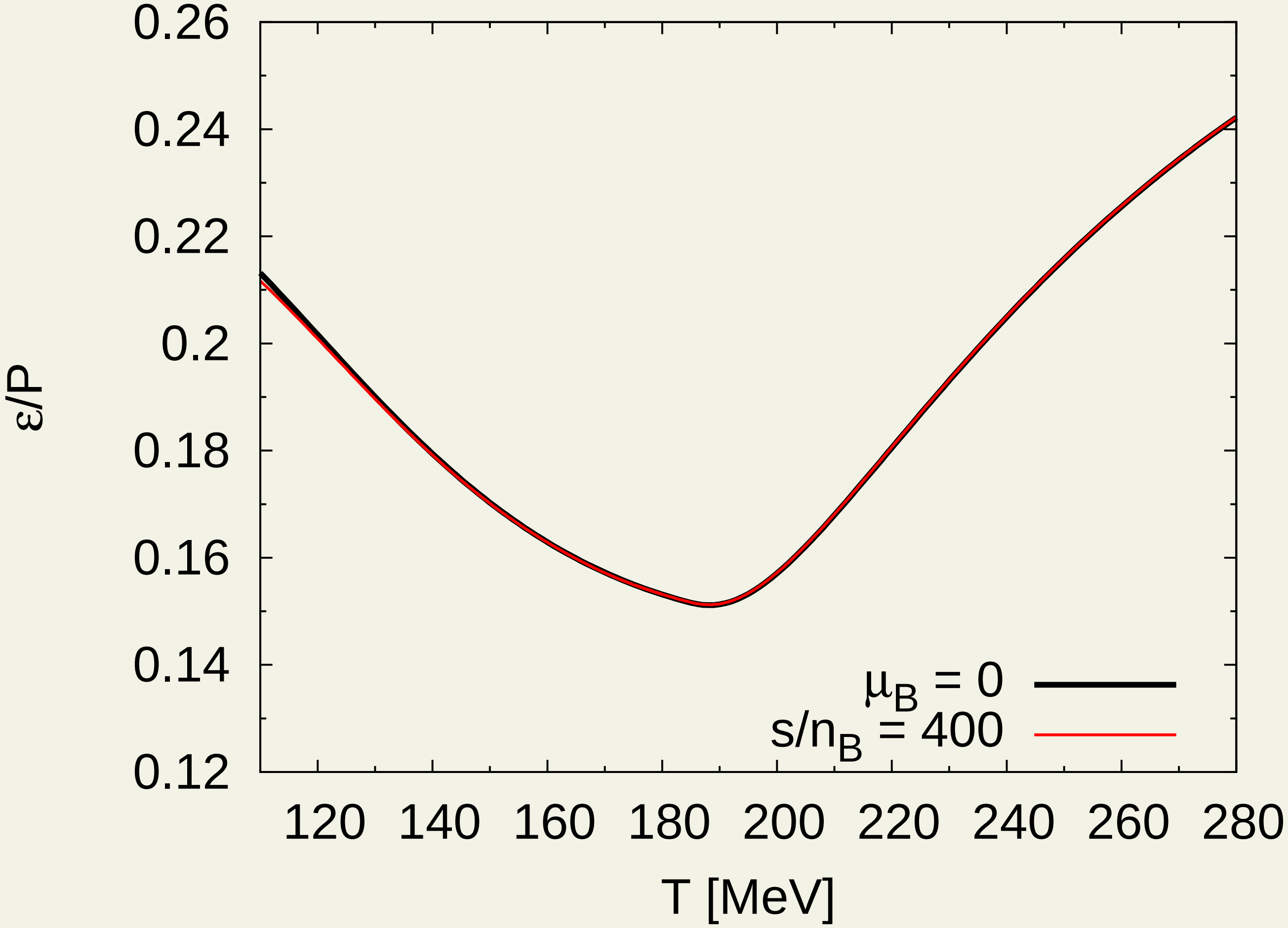


P [GeV/fm-3]
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
400
350
300
250
200
150
100
T [MeV]
0
50
100
150
200
250
300
μB [MeV]

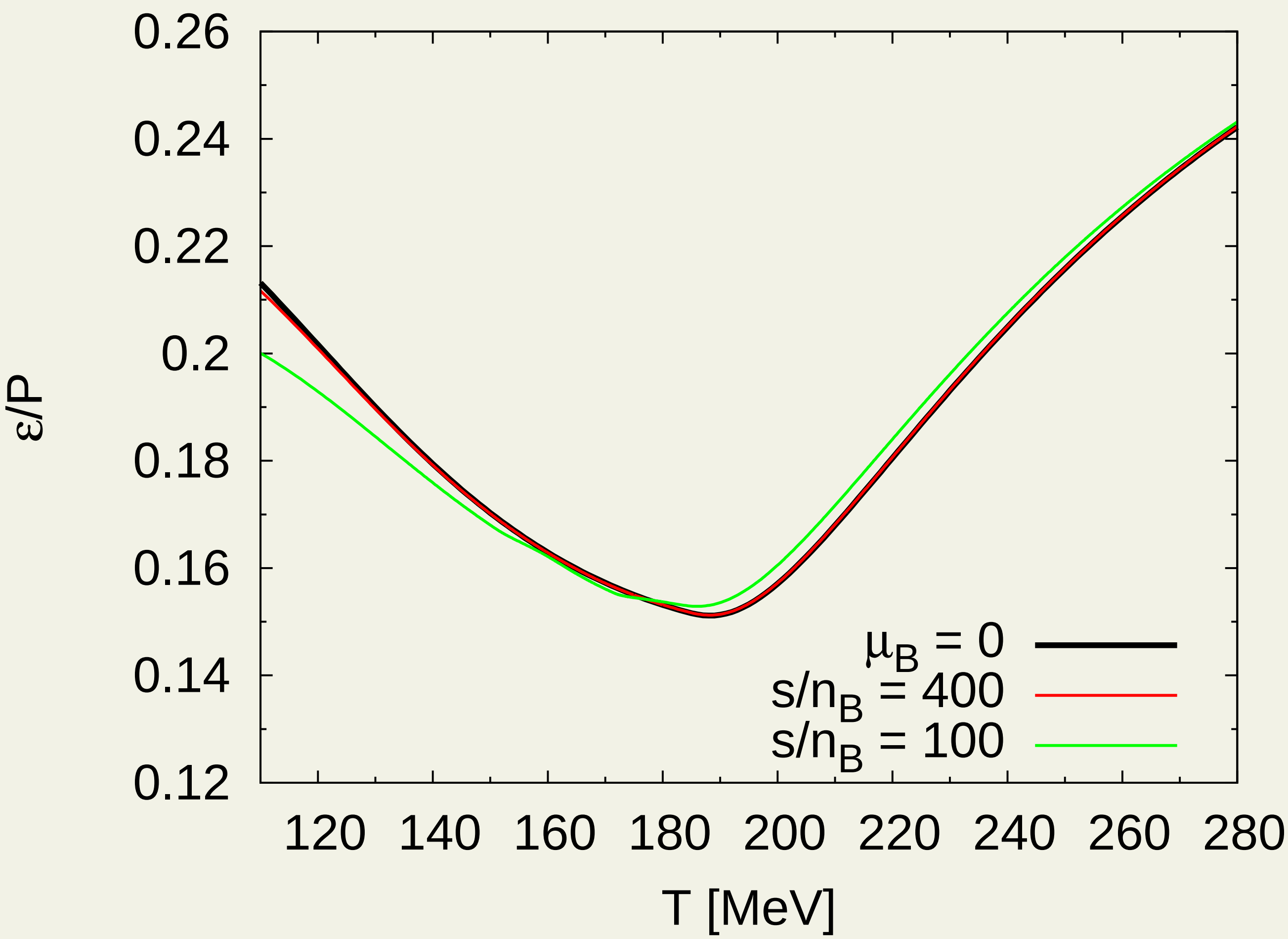
Softness of EoS



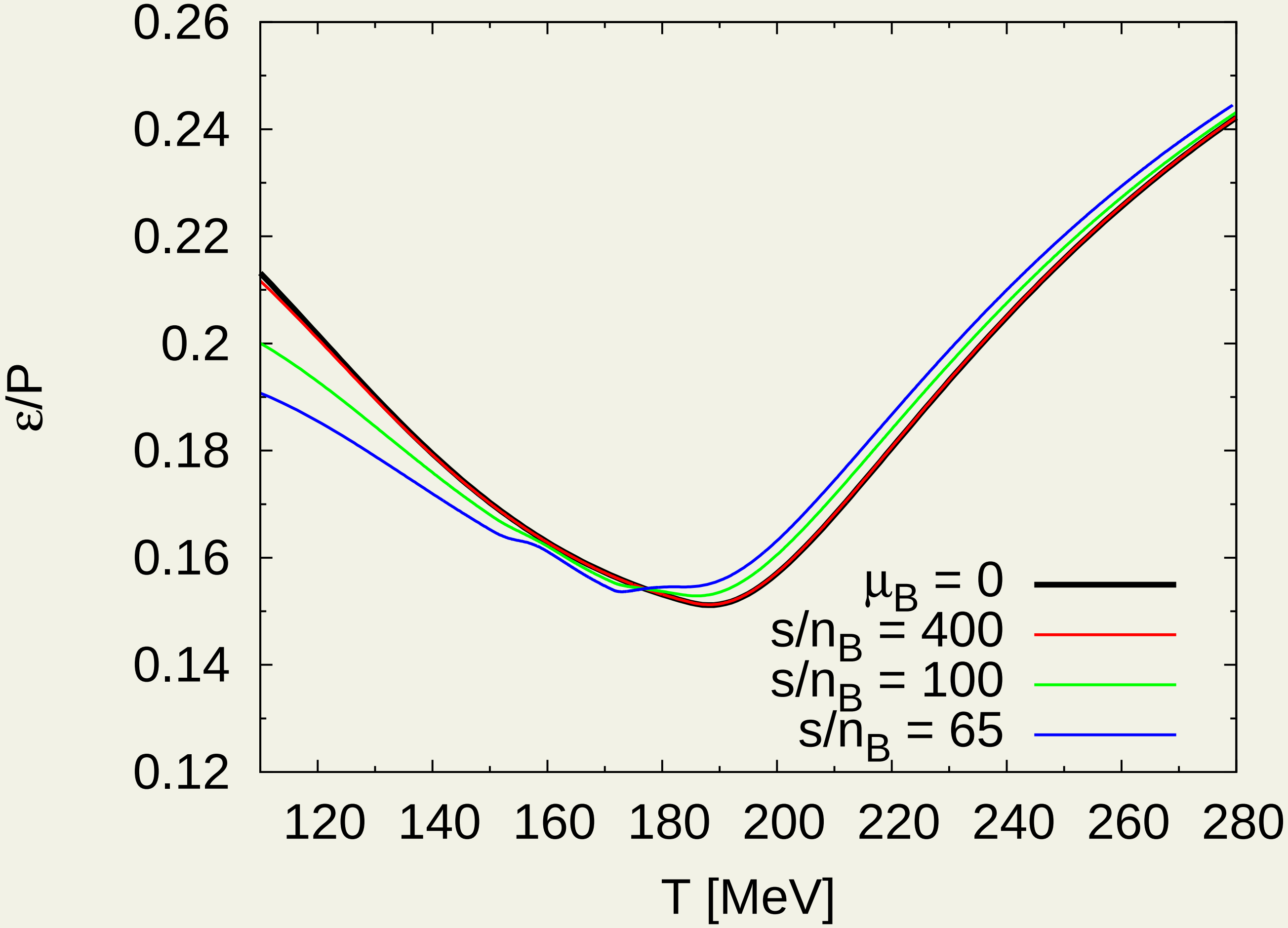
Softness of EoS



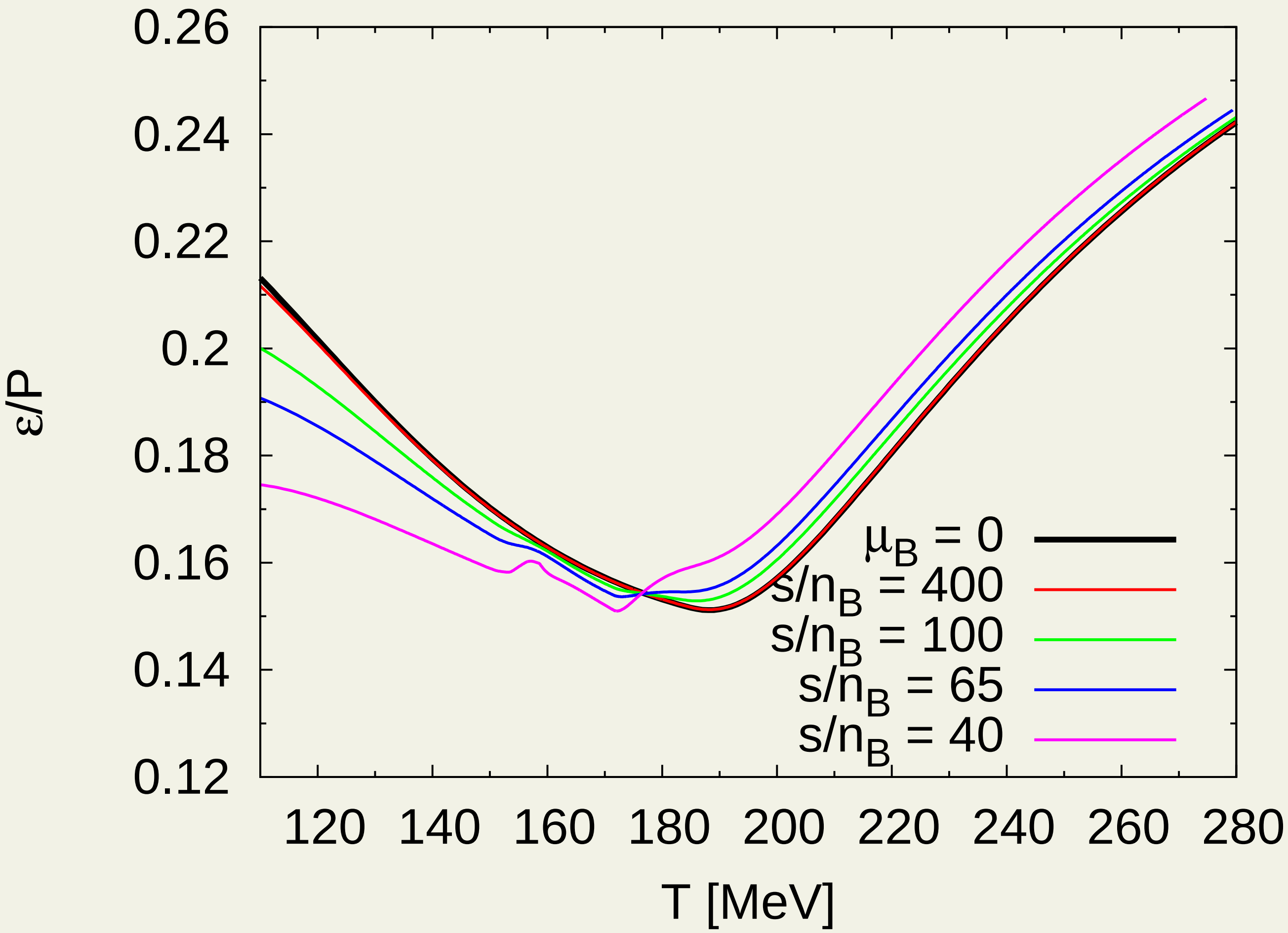
Softness of EoS



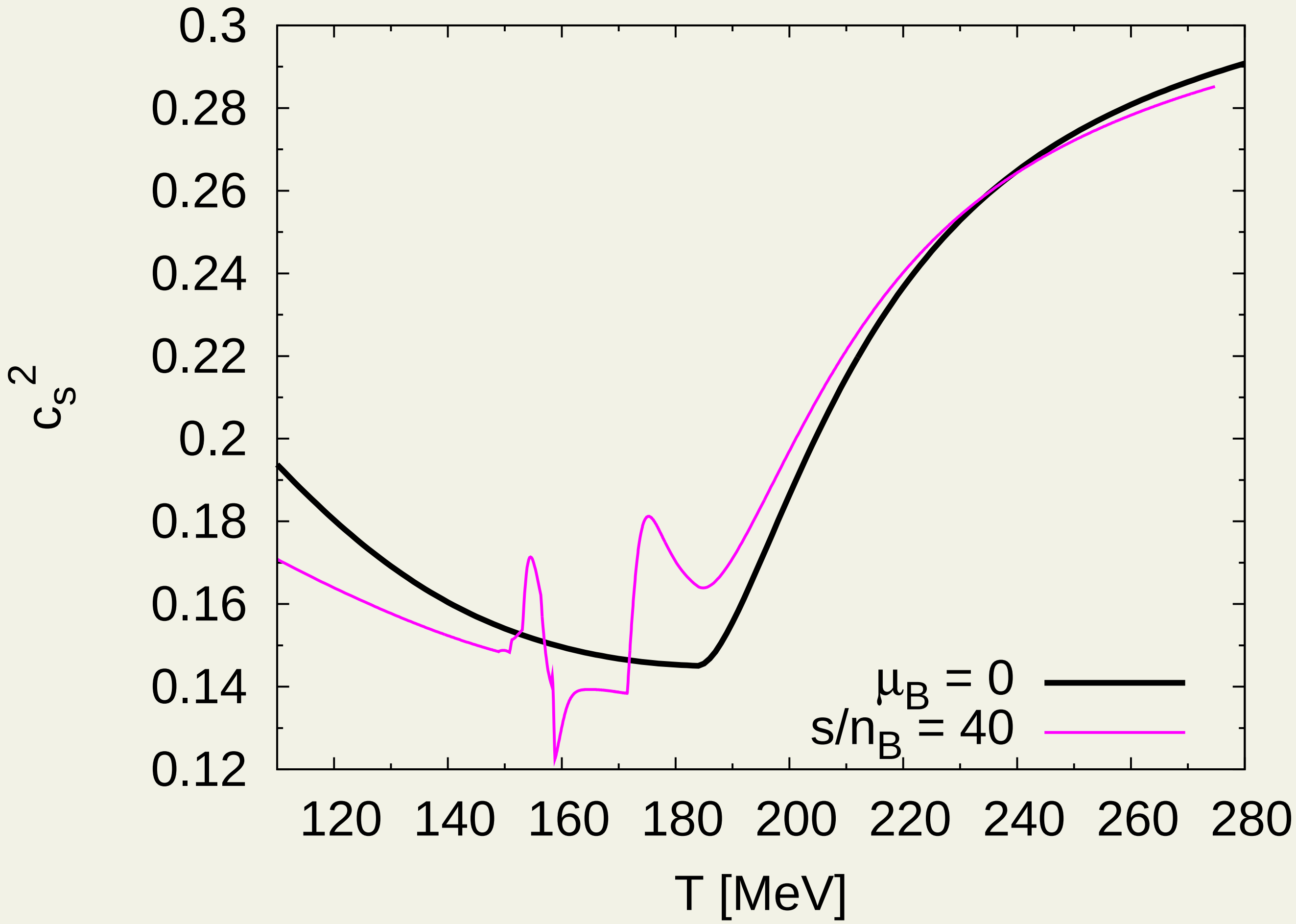
Softness of EoS



Softness of EoS

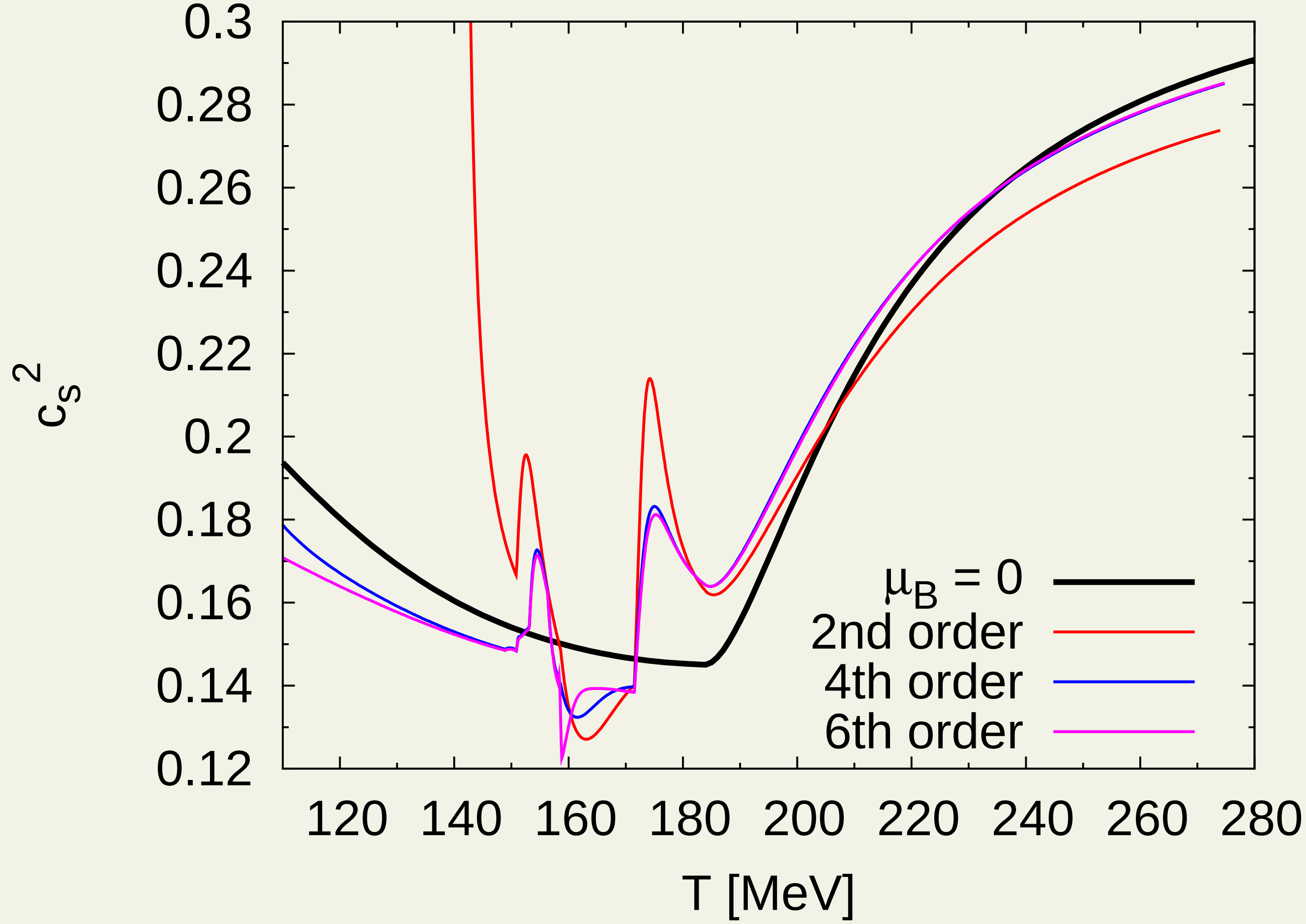


Speed of sound



- **Structure due to details in the fitting procedure**
- **Each kink corresponds to a $T_{\rm sw}$ of a particular coefficient**

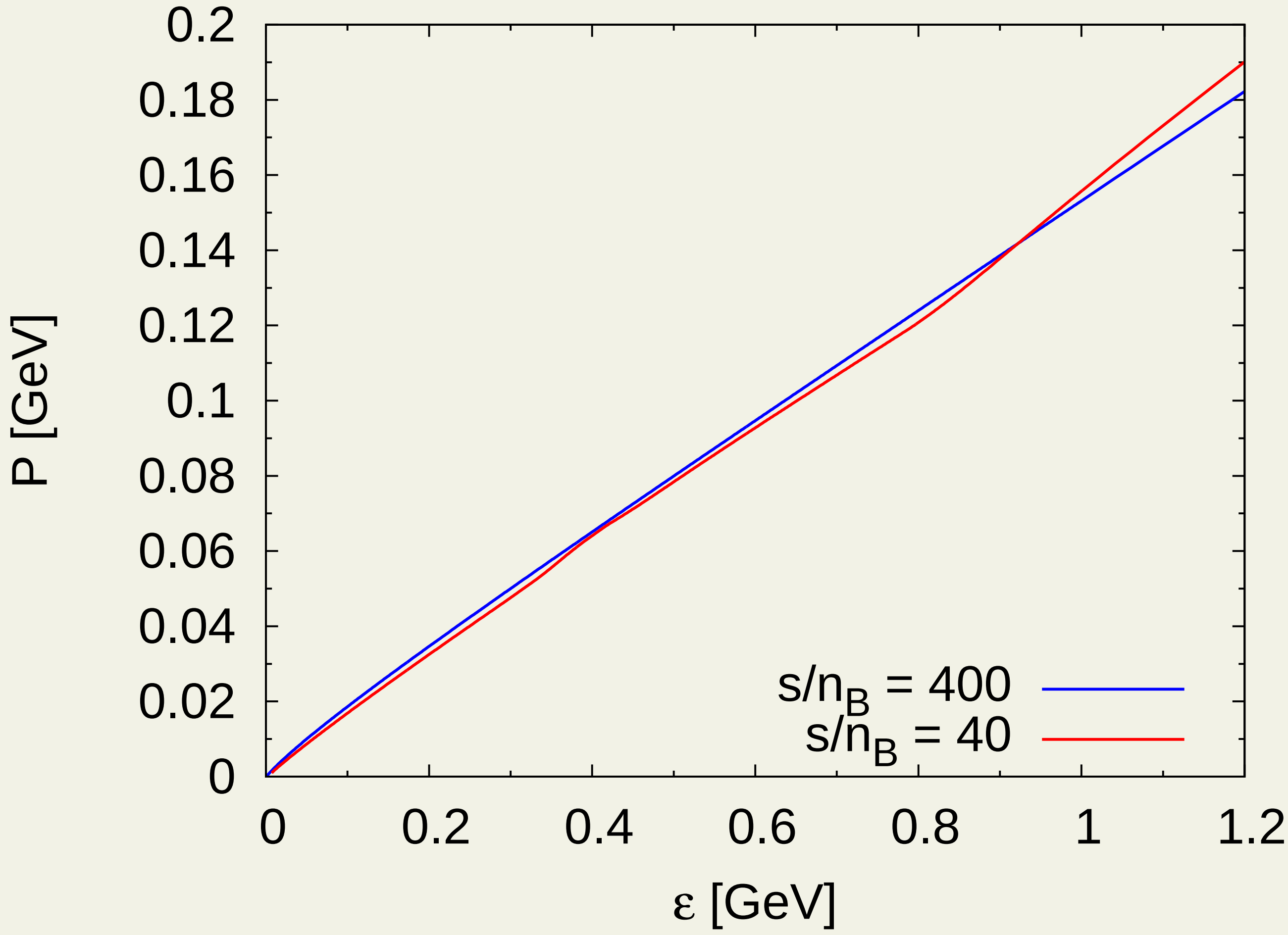
Speed of sound



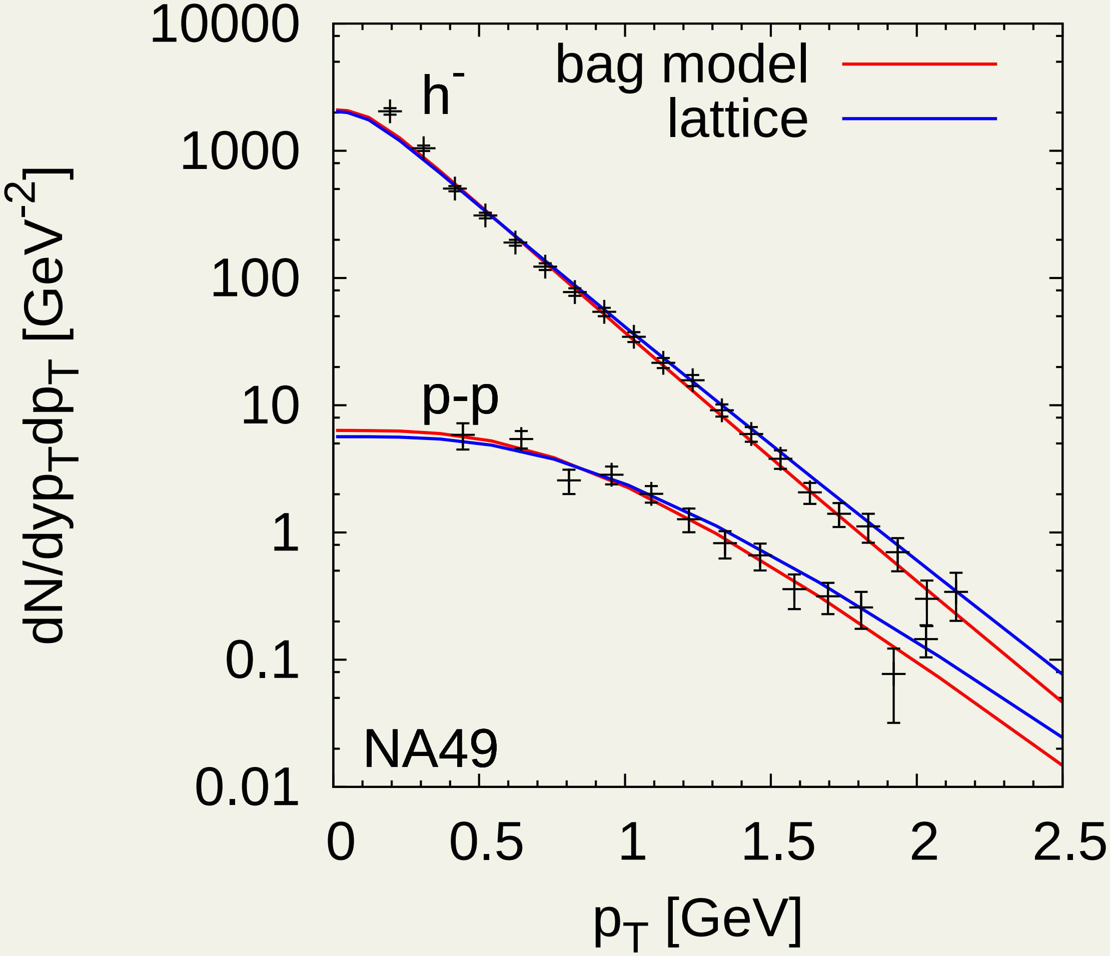
- 6th order correction smaller than 4th

⇒ expansion under control

Pressure vs. energy density

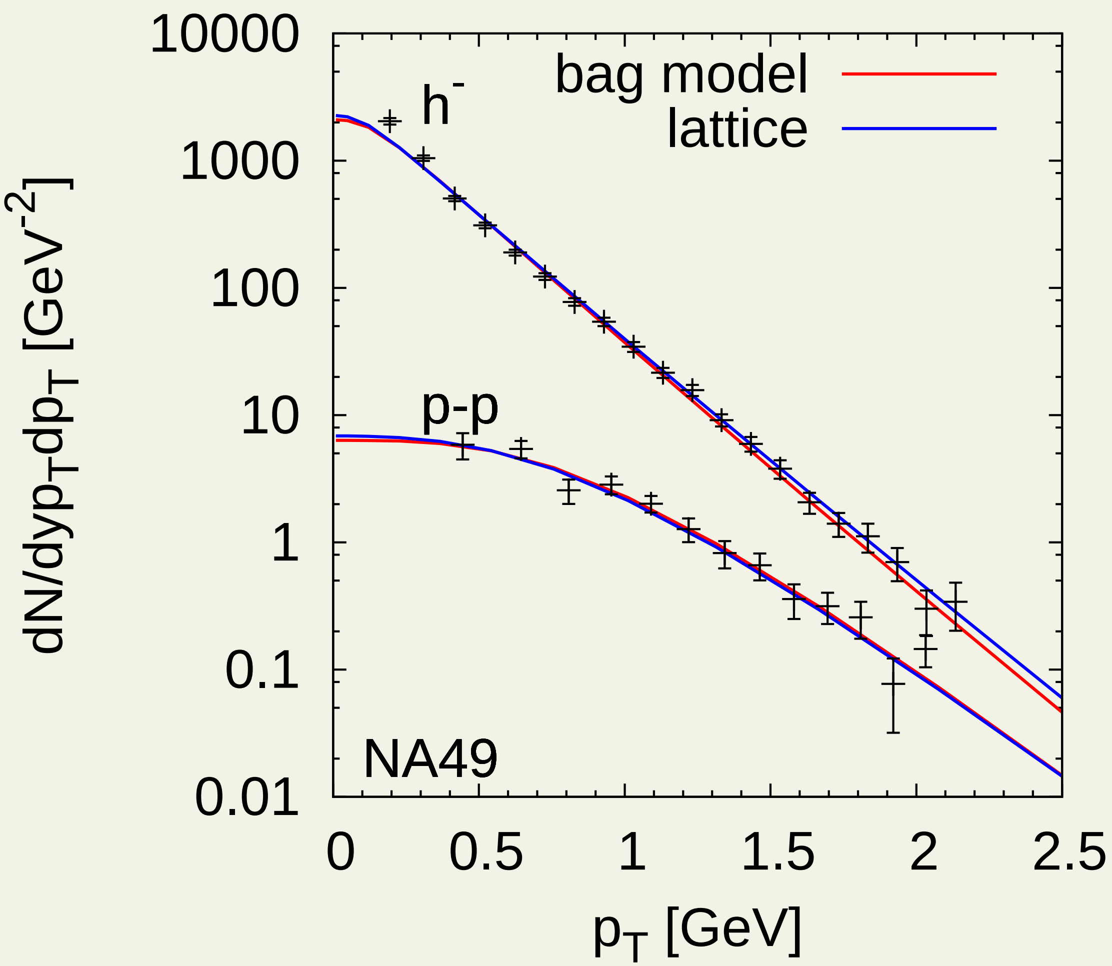


p_T-spectra at SPS



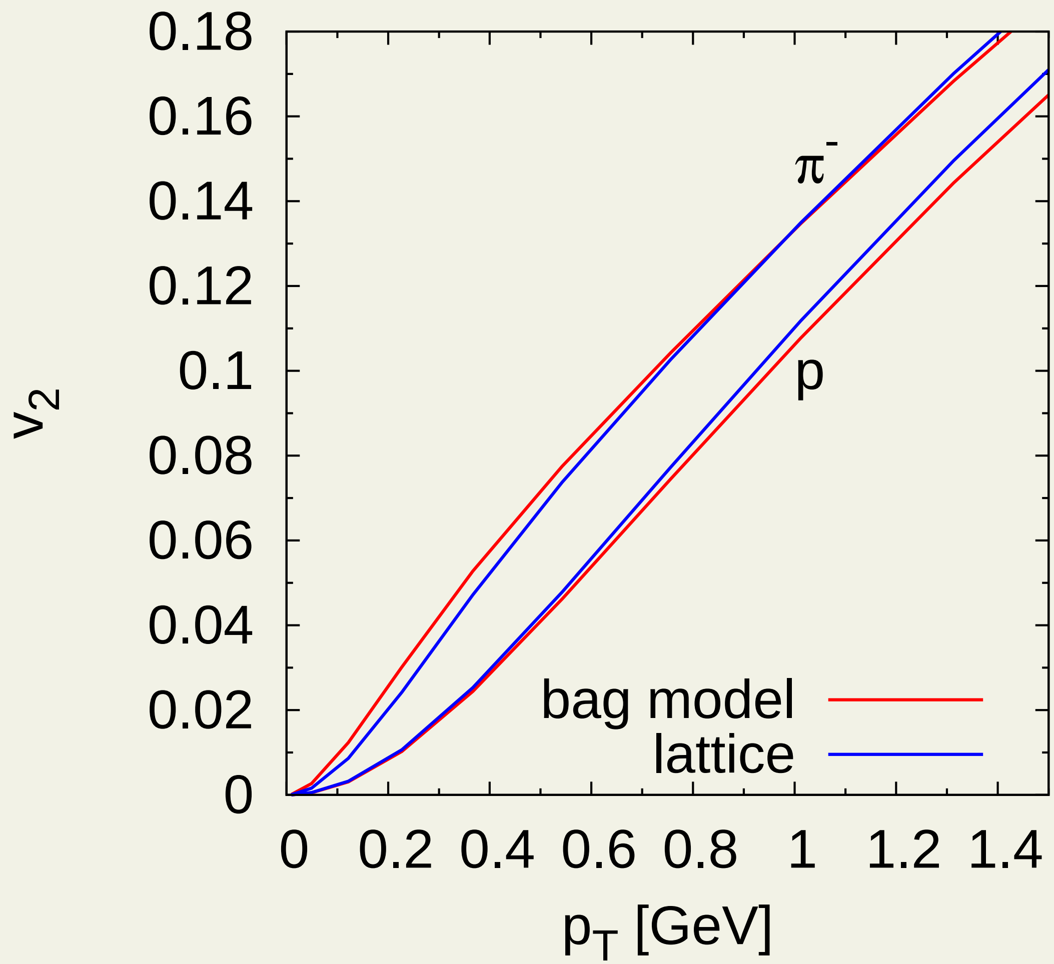
- **harder EoS, more transverse flow, flatter spectra**

p_T-spectra at SPS



- $T_{\rm fo} \approx 120$ **MeV (bag)** $\Rightarrow$ **130 MeV (lattice)**

v_2 at SPS ($b = 7$ fm)

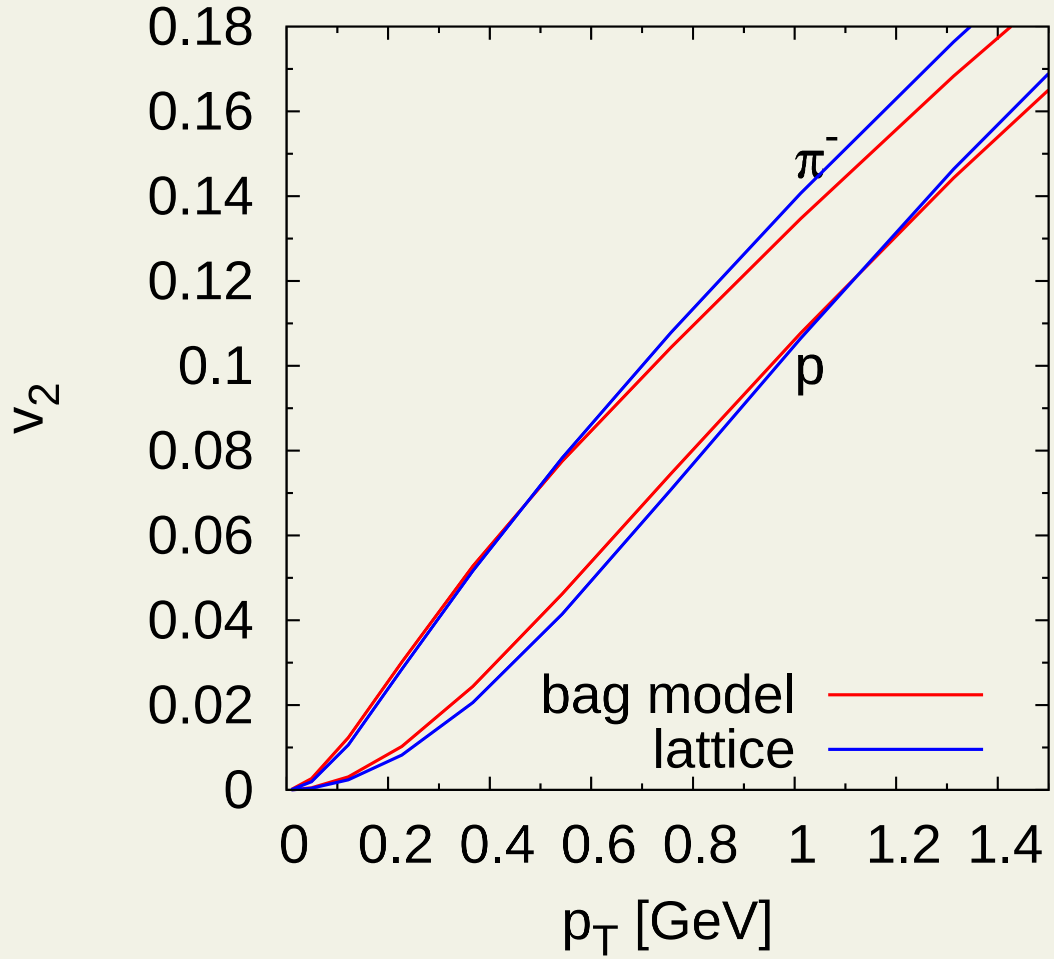


- $T_{\rm fo} \approx 120$ **MeV (bag)** $\Rightarrow$ **130 MeV (lattice)**

Conclusions

- lattice spacing dependence of hadron masses explains the difference between HRG and lattice QCD
  - 30 MeV shift in temperature
- EoS at finite baryon densities based on lattice QCD calculations of baryon number and strangeness fluctuations and correlations
  - $\sim$10% uncertainty in speed of sound around the transition
- effect on flow when compared to bag model EoS tiny at SPS and (some?) RHIC low energy scan energies

v_2 at SPS ($b = 7$ fm)



- $T_{\text{fo}} \approx 120$ **MeV (both)**