QGP phase transition with a local equilibrium molecular dynamics with the NJL model

Rudy Marty

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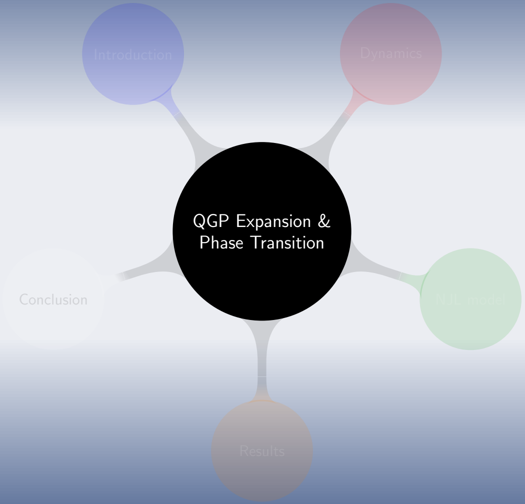
in collaboration with : J. Aichelin

Outline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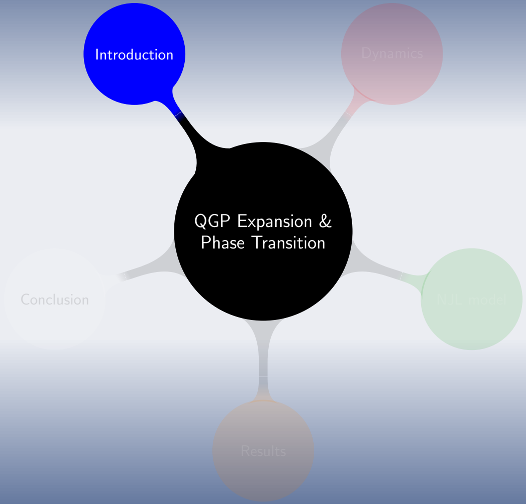
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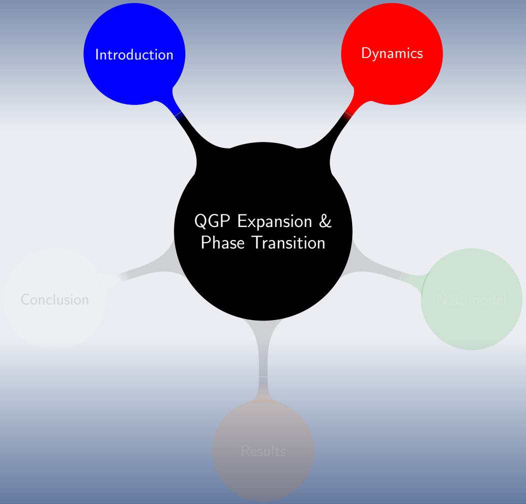
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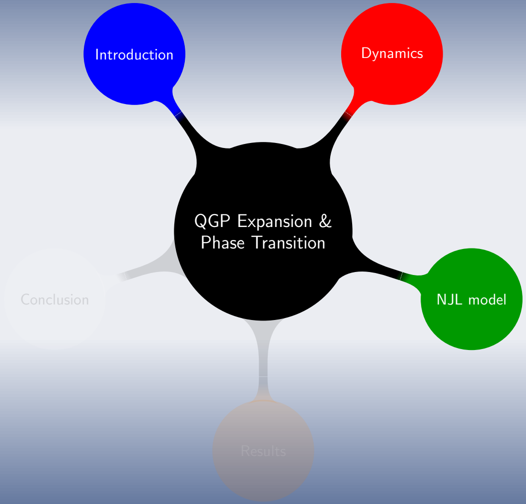
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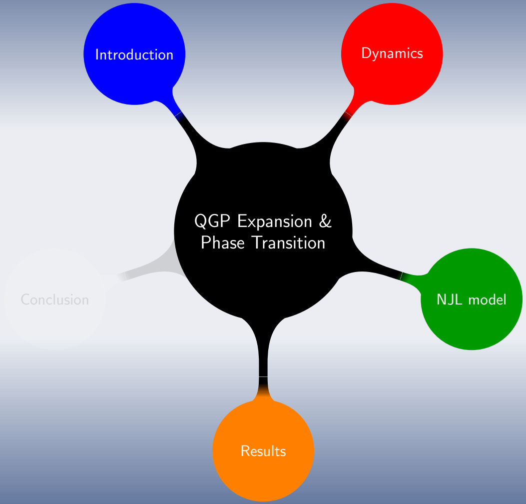
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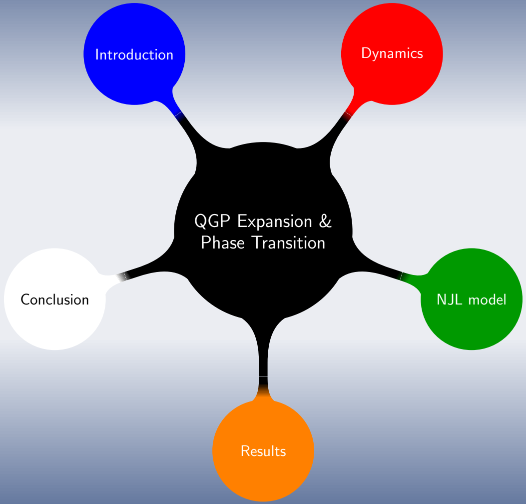
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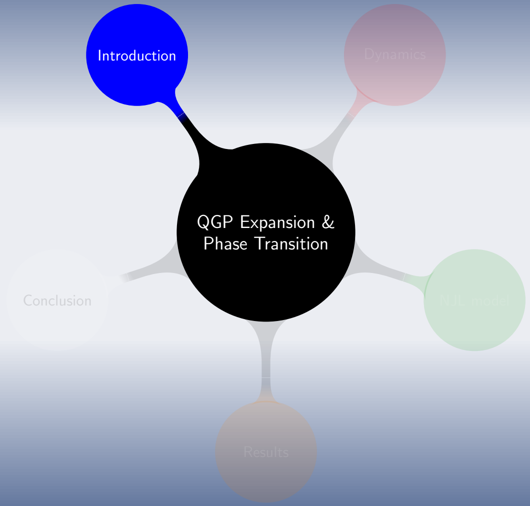
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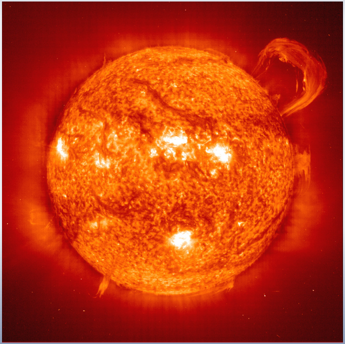
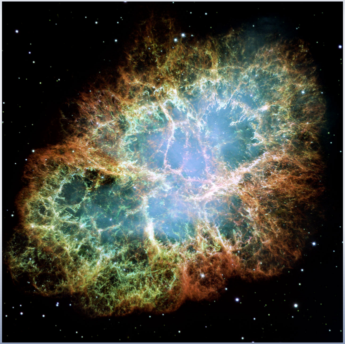
Non-equilibrium Dynamics

Sometimes we need to study systems
which can be suddenly **out of equilibrium**.



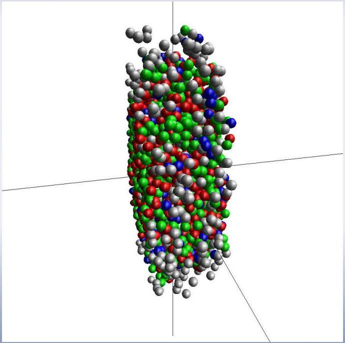
Equilibrium



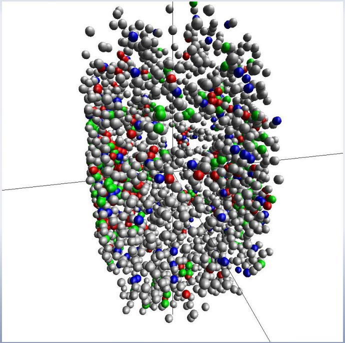
Non-equilibrium

Quark Gluon Plasma

This is the case of the **Quark Gluon Plasma**.
We want to study the expansion and the phase transition



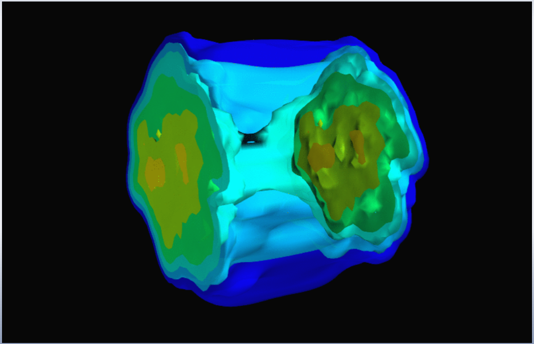
(unstable) Equilibrium



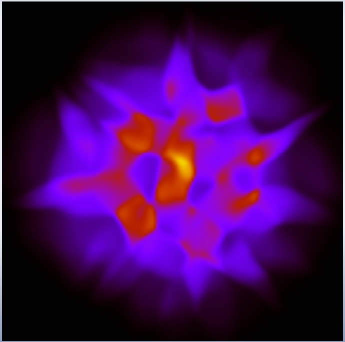
Non-equilibrium

Simulations

Now the question is : how to simulate such a system ?
Hydrodynamics comes to the rescue !



Expansion



Freeze out surface

Hydrodynamics

E-by-E Viscous Hydro 3+1D is not the only possible model !

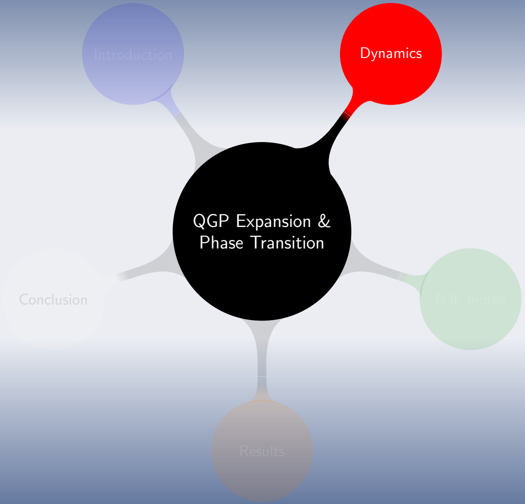
What can we do

- Describe big/dense systems with low viscosity (equilibrium),
- Without hard processes (high p_T, jets),
- With a hard transition to the hadron world (Cooper-Frye).

What do we want

- Describe all kind of systems,
- Include soft and hard processes together,
- Have a mixed phase for the transition (cross over).

Outline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Parton Dynamics

Now we are interested in **partons** themselves instead of energy cells.

We cannot simulate such a physics with Hydrodynamics, but we can do it using **Molecular Dynamics**.

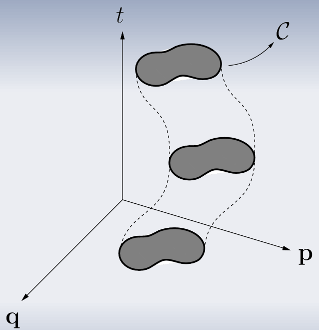
Starting from a particle distribution function we can formulate our dynamics with the help of **equations of motion**.

Molecular Dynamics

Equations of motion

$$\frac{\partial \mathbf{q}_i}{\partial t} = \{\mathbf{q}_i, \mathscr{H}\} = \frac{\mathbf{p}_i}{E_i}$$

$$\frac{\partial \mathbf{p}_i}{\partial t} = \{\mathbf{p}_i, \mathscr{H}\} = \sum_k \frac{1}{2E_k}\frac{\partial V_k}{\partial \mathbf{q}_i} + \langle \text{ coll. } \rangle$$



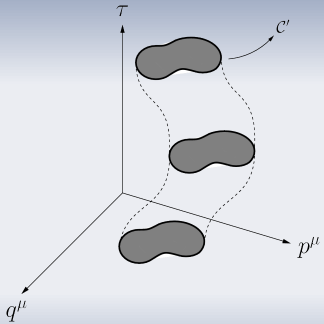
Classical dynamics is fine to describe particles with **low energy** in the **classical phase space** $(\mathbf{q}, \mathbf{p})$ but for relativistic particles we need to go to the **Minkowski phase space** (q^μ, p^μ).

Relativistic Molecular Dynamics

Relativistic e.o.m.

$$\frac{\partial q_i^\mu}{\partial \tau} = \{q_i^\mu, \mathcal{H}\}_D = 2\lambda_i p_i^\mu$$

$$\frac{\partial p_i^\mu}{\partial \tau} = \{p_i^\mu, \mathcal{H}\}_D = \sum_k \lambda_k \frac{\partial V_k}{\partial q_i^\mu} + \langle \text{ coll. } \rangle$$



Here we have a new definition of equation of motion. Indeed we live in a **constrained phase space** where λ play the role of a **relativistic factor**.

Local equilibrium

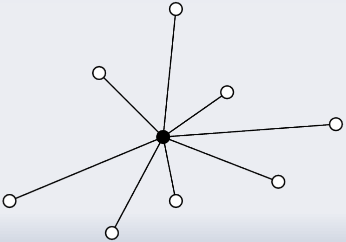
Local density

$$R_{ij} = \exp\left(-\frac{q_{ij}^2}{L^2}\right)$$

we *define*

$$\rho_{F\,i} = \sum_{i \neq j} R_{ij}$$

$$\rho_{B\,i} = \sum_{i \neq j} R_{ij}\ \mathrm{Sign}(j)$$



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Local potential

Thermodynamics gives :

$$T_i = (\hbar c)(\rho_{F\,i})^{1/3}\left(\frac{\pi^2}{g}\right)^{1/3}$$

$$(\text{for } \mu \approx 0)$$

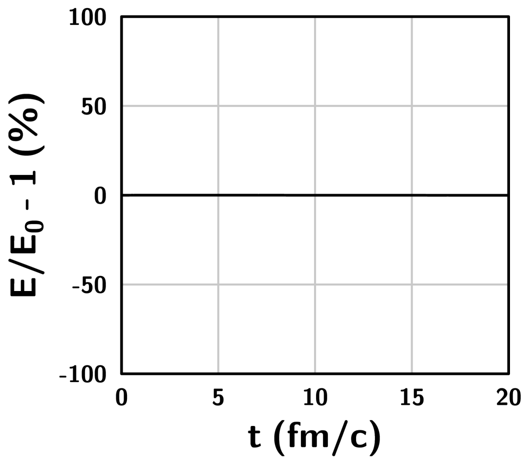
$$\mu_i = (\hbar c)(\rho_{B\,i})^{1/3}\left(\frac{6\pi^2}{g}\right)^{1/3}$$

$$(\text{for } T \approx 0)$$

Energy conservation

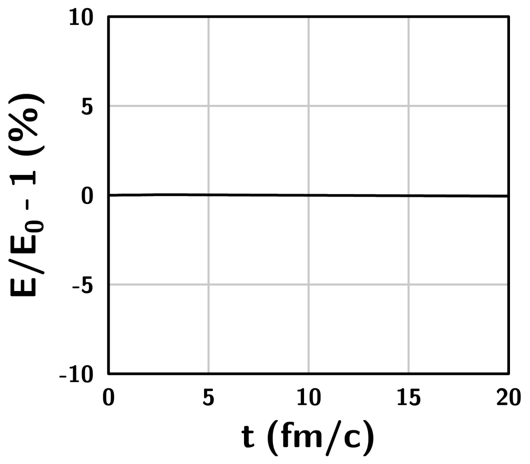
What about the quality of such a dynamics ?
We can look at the **energy conservation** !

Energy conservation



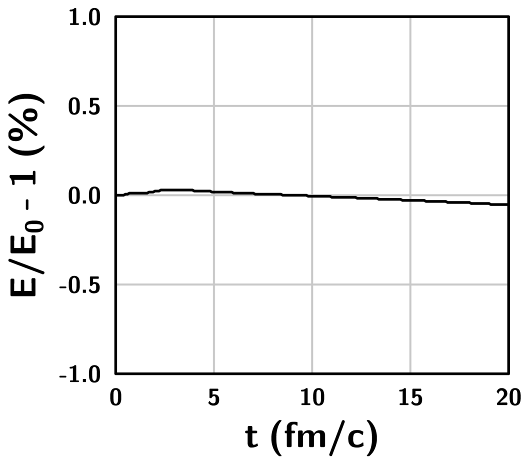
$\pm 100\%$?

Energy conservation



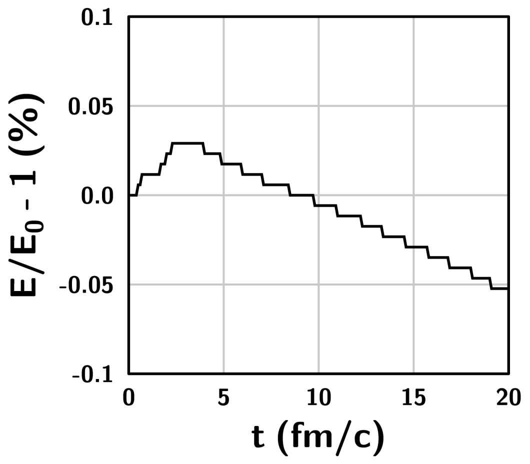
$\pm 10\%$?

Energy conservation



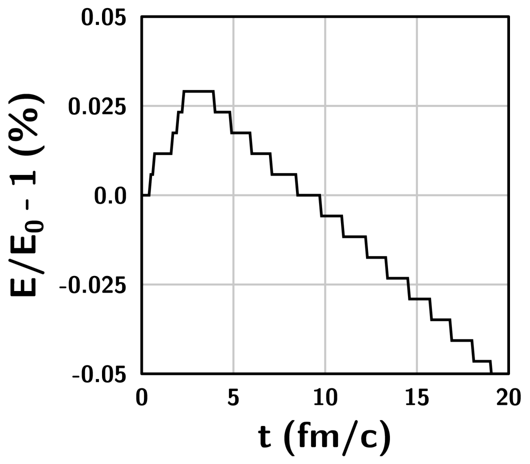
$\pm 1\%$?

Energy conservation



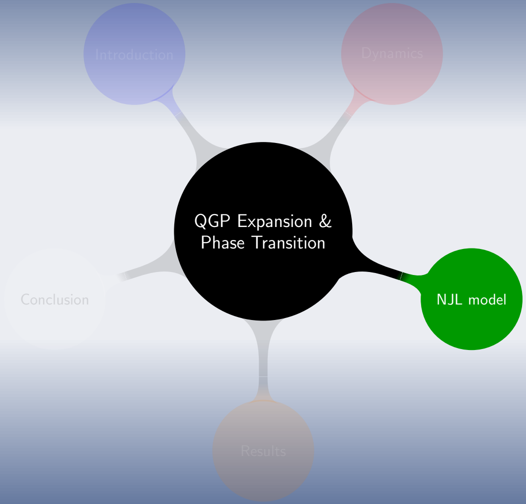
$\pm 0.1\%$?

Energy conservation



Here we have density break effect (meson decay) !

Outline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Basis of model I

Nambu–Jona-Lasinio : model for nuclear matter (1961). Now : a model for **quarks and hadrons** used to describe the nuclear phase transition. **Gluons are hidden** in the coupling constant because we are interested by the quark d.o.f.

This model only use two real data :
the pion coupling constant to get the right **pion mass**,
and the vacuum energy density from lQCD to get the **cut off**.

So in the end this is an effective model,
but without real *free* parameters.

Basis of model II

NJL Lagrangian

$$\mathscr{L}_{NJL} = \mathscr{L}_2 + \mathscr{L}_4 + \mathscr{L}_6$$

$\mathscr{L}_2 = \bar{q}_f \left(i\partial\!\!\!/ - m_{0f} \right) q_f$

(kinetic term, break explicitly chiral sym.)

$$\mathscr{L}_4 = G_S \sum_{a=0}^{8} \left[\left(\bar{q}_f \lambda^a q_f \right)^2 + \left(\bar{q}_f i \gamma_5 \lambda^a q_f \right)^2 \right] + G_V \sum_{a=0}^{8} \left[\left(\bar{q}_f \gamma_\mu \lambda^a q_f \right)^2 + \left(\bar{q}_f i \gamma_\mu \gamma_5 \lambda^a q_f \right)^2 \right]$$

(4-fermions term, respect chiral sym.)

$\mathscr{L}_6 = G_D \left[\det \bar{q}_f \left(1 + \gamma_5 \right) q_f + \det \bar{q}_f \left(1 - \gamma_5 \right) q_f \right]$

('t Hooft term, break $U_A(1)$ anomaly)

(**Klevansky, Rev. Mod. Phys. 64(1992)**)

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$$+ G_V \sum_{a=0}^{8} \left[\left(\bar{q}_f \gamma_\mu \lambda^a q_f \right)^2 + \left(\bar{q}_f i \gamma_\mu \gamma_5 \lambda^a q_f \right)^2 \right]$$

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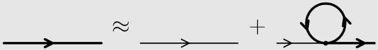
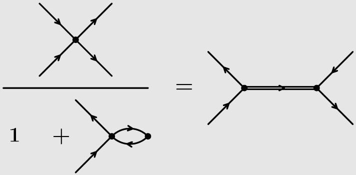
Quark mass



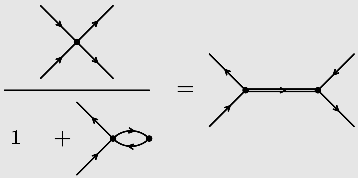
Meson mass



(Klevansky, Rev. Mod. Phys. 64(1992))

Hadrons

Meson propagator



Pion mass

$$M(k_0,\vec{k}) = \frac{2K_1^+}{1-4K_1^+\Pi^P_{q\bar{q}}(k_0,\vec{k})}$$

at the pole :

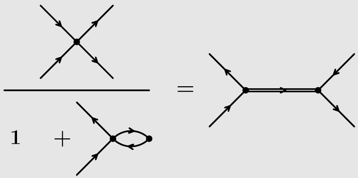
$$1 - 4K_1^+\Pi^P_{q\bar{q}}(m_\pi,\vec{0}) = 0$$

that gives

$$M(k_0,\vec{k}) = \frac{-g^2_{\pi q\bar{q}}}{k^2-m^2_\pi}$$

Hadrons

Meson propagator



Pion mass

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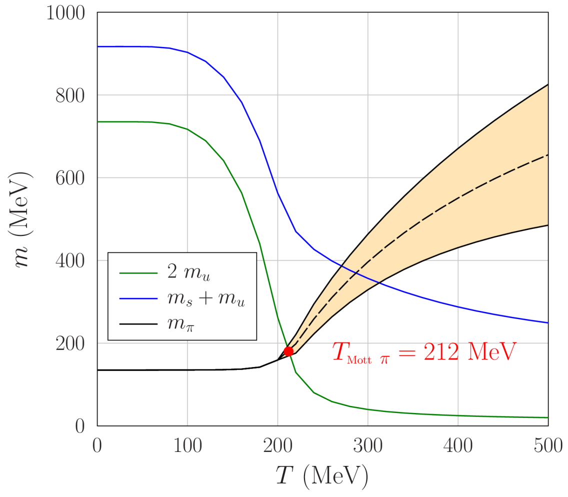
that gives

$$M(k_0, \vec{k}) = \frac{-g^2_{\pi q\bar{q}}}{k^2 - m_\pi^2}$$

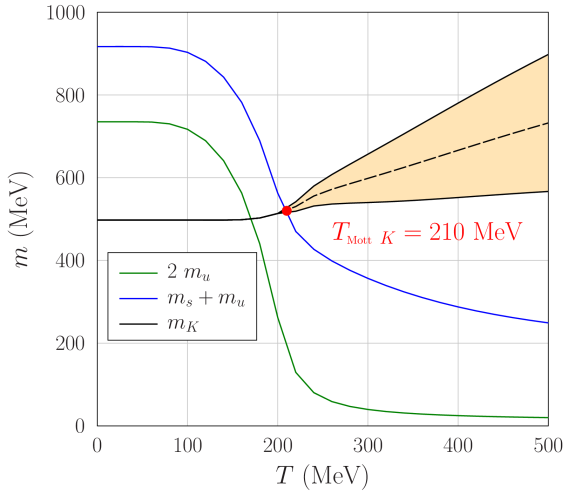
We note that mesons have an imaginary part in the mass above T_c (they are **unstable**).

Masses



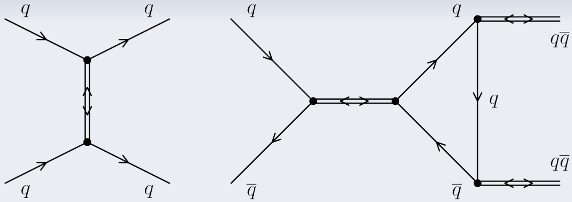
1000
800
600
400
200
0
0
100
200
300
400
500
m (MeV)
T (MeV)
$2\ m_u$
$m_s + m_u$
m_π
$T_{\text{Mott}\ \pi} = 212$ MeV

Masses


1000
800
600
400
200
0
m (MeV)
0
100
200
300
400
500
T (MeV)
$2\ m_u$
$m_s + m_u$
m_K
$T_{\text{Mott}\ K} = 210$ MeV

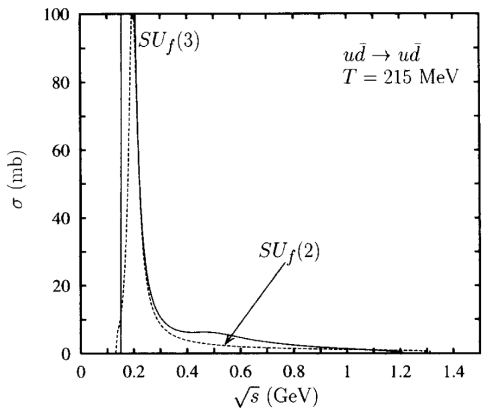
Hadronization

NJL cross sections look like this :



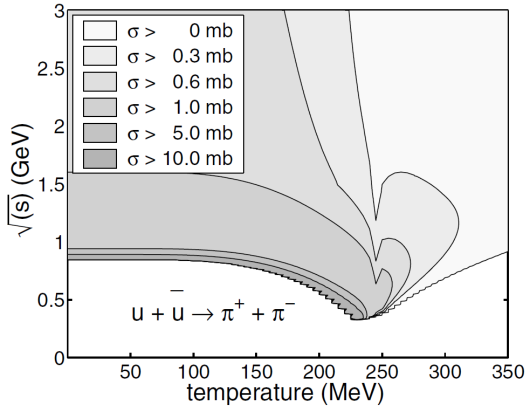
For the moment I didn't have included all processes (with diquarks and baryons) but all pseudo-scalar mesons cross sections are implemented.

Hadronization



(Rehberg, Nucl. Phys. A 608(1996))

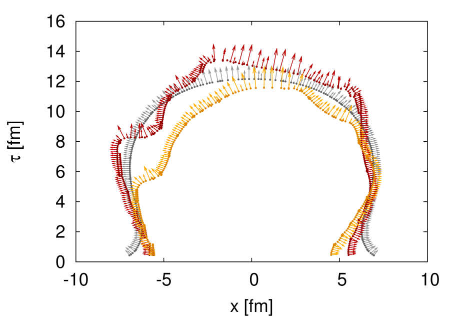
Hadronization



(Gastineau (2003))

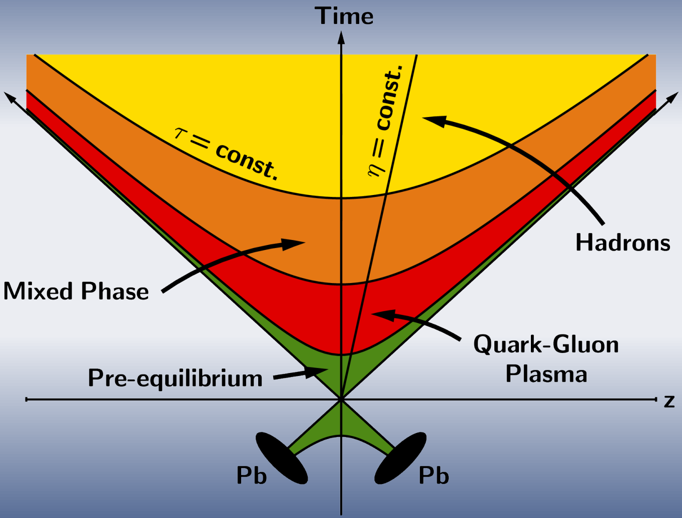
Mixed Phase

Using the NJL masses and cross sections within a molecular dynamics will help us to have a **mixed phase** instead of a **freeze-out barrier**.

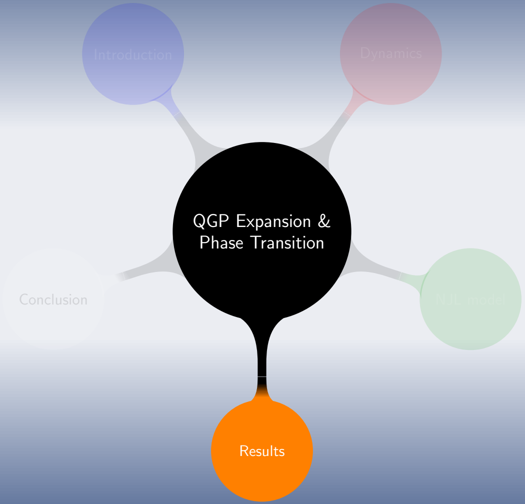


(**Schenke, 2010, arXiv:1009.3244v2**)

Mixed Phase


Time
$\tau = $ const.
$\eta = $ const.
Hadrons
Mixed Phase
Quark-Gluon
Plasma
Pre-equilibrium
z
Pb
Pb

Outline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Description of simulations

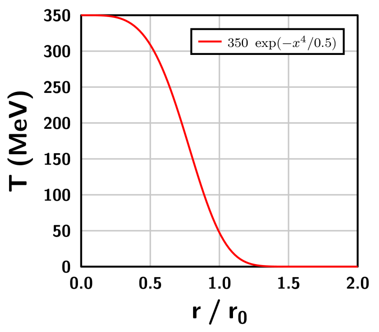
In practice, we need to check some results before going further. We present results for a given set of parameters, and varying the centrality b.

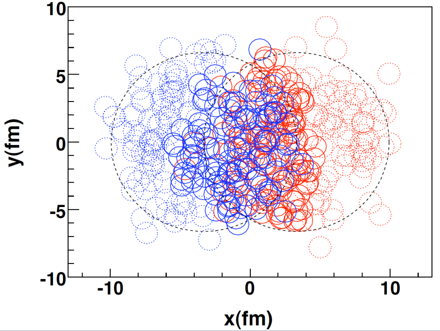
Parameters of the simulations

- $A = 208$, $b = \text{variable}$, $\sqrt{s_{NN}} = 200$ GeV,
- $T_0 = 350$ MeV, $\mu_0 = 0$ MeV,
- $L = 0.5$ fm (interaction length from R_{ij}),
- $t = 20$ fm/c.

Initial conditions

For the initial state, we use a **toy model** based on a **Monte-Carlo** with temperature distribution.

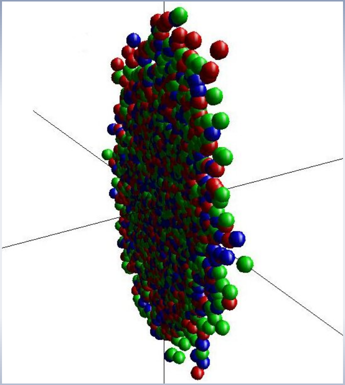




$$(T, \mu = 0) \rightarrow (N, m) \rightarrow (\mathbf{q}, \mathbf{p})$$

(**Alver, 2008, arXiv:0805.4411v1**)

Initial conditions

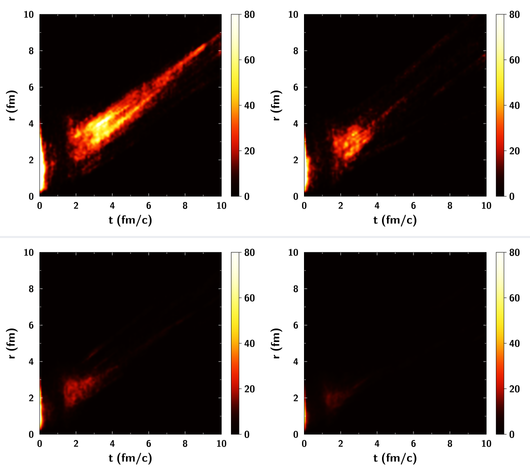


Our toy model in practice ($b = 6.5$ fm)

Results part I

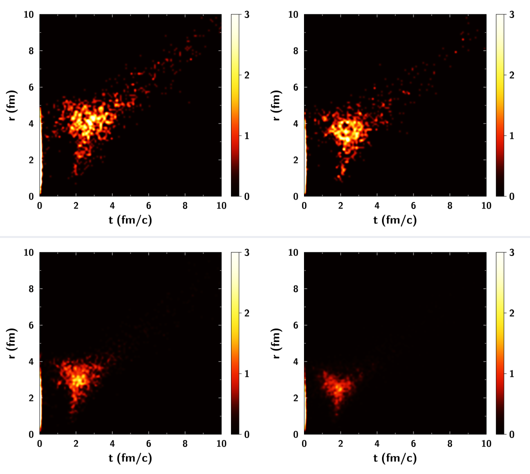
Before results, let's start with a **movie** !

Results part I



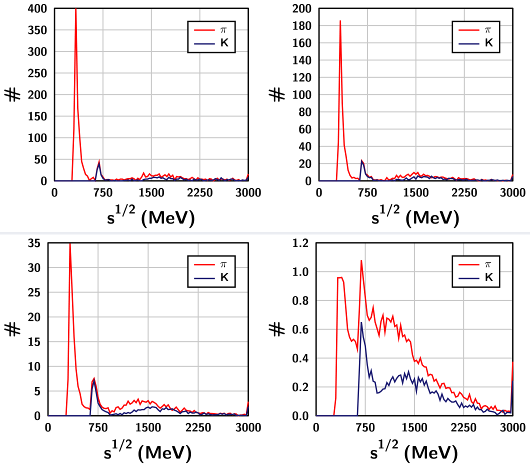
Elastic collisions ($b = 6.5/7.5/8.5/9.5$)

Results part I



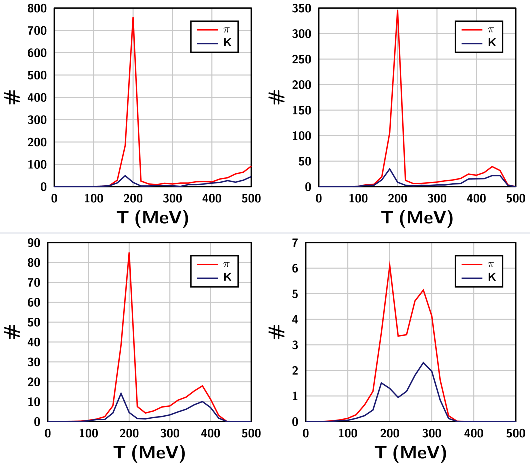
Hadronization collisions ($b = 6.5/7.5/8.5/9.5$)

Results part I



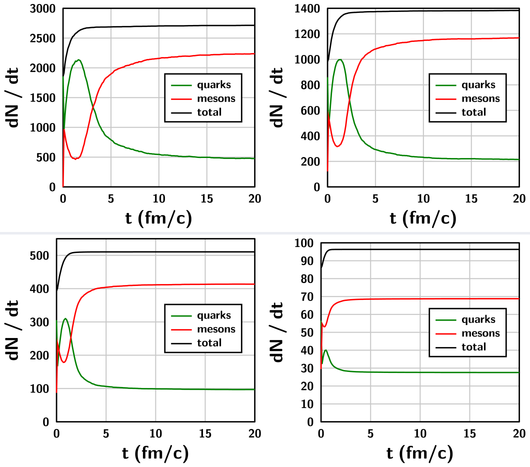
$\sqrt{s}$ of collisions ($b = 6.5/8.5/10.5/12.5$)

Results part I



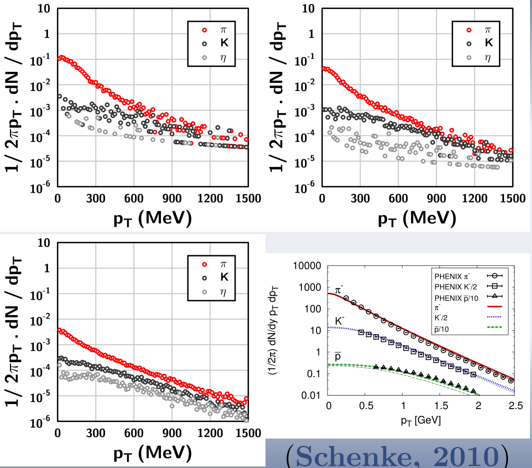
T of collisions ($b = 6.5/8.5/10.5/12.5$)

Results part I



Hadronization ($b = 6.5/8.5/10.5/12.5$)

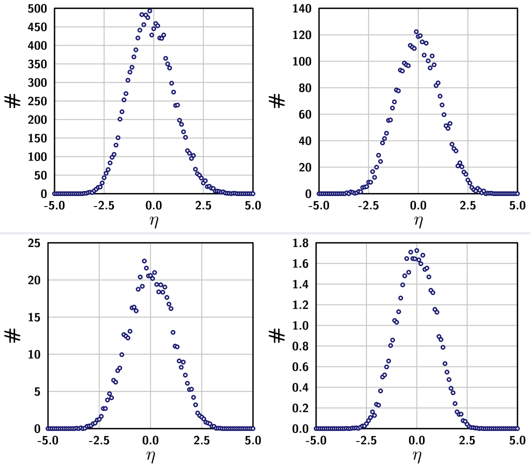
Results part I



(Schenke, 2010)

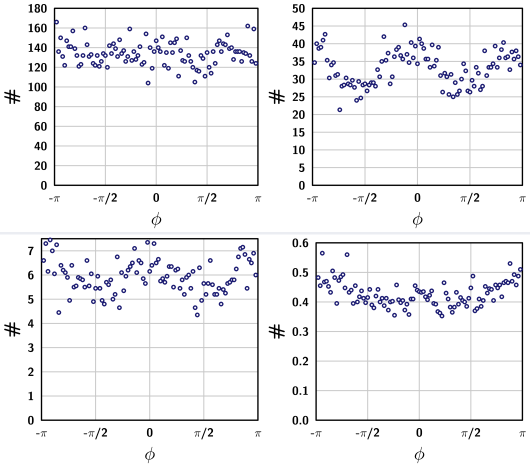
$$\mathrm{d}N/\mathrm{d}p_T \ (b = 8.5/10.5/12.5)$$

Results part I



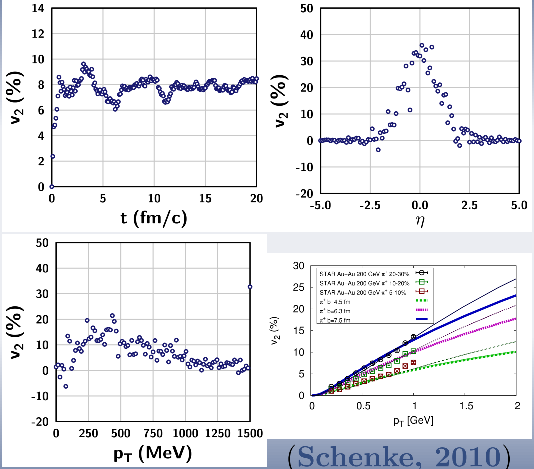
$$\mathrm{d}N/\mathrm{d}\Delta\eta \ (b = 6.5/8.5/10.5/12.5)$$

Results part I



$\mathrm{d}N/\mathrm{d}\Delta\phi$ $(b = 6.5/8.5/10.5/12.5)$

Results part I



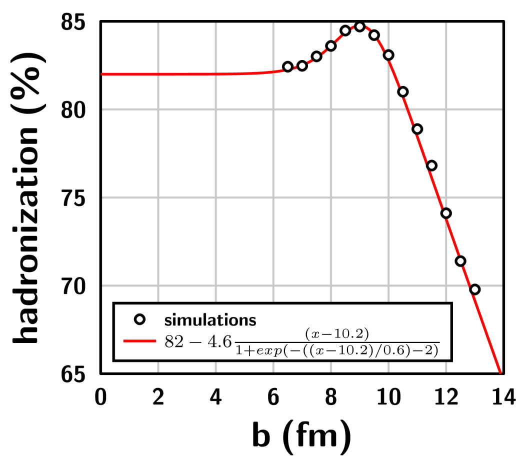
(Schenke, 2010)

Elliptic flow ($b = 6.5/12.5/12.5$)

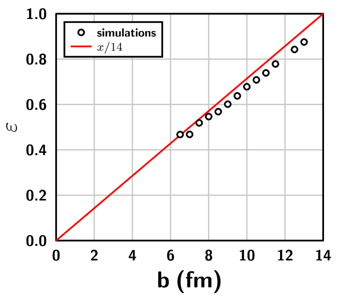
Results part II

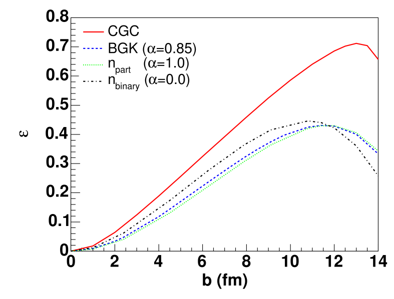
We can have a look at the global results that we summarize from the previous plots.

Results part II

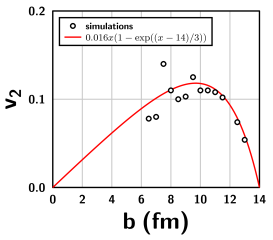


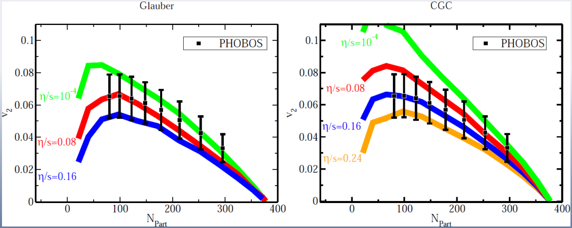
Results part II



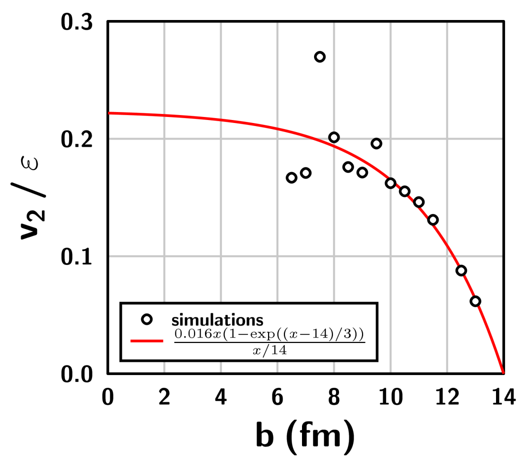


Results part II

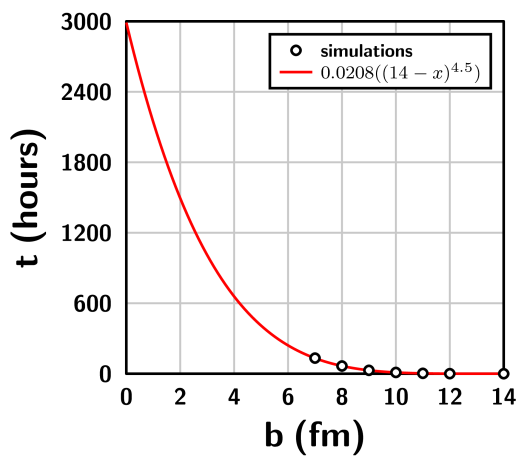




Results part II



Results part II



Conclusion

Well, as you can see, the game just begin :

- NJL is a good model to describe the phase transition **dynamically**,
- Simulation at **finite** (T, μ) **are possible** with local dependance,
- Clusterization of phase space allow a kind of **confinement** (with enough particles),
- **Huge cross sections** and **mixed phase** close to the phase transition are present,
- Relativistic dynamic which is **causal** and **fully invariant**.

Outlooks

- Improve the program speed : introduce parallelization with **OpenCL**,
- Make a lot of **bigger simulations**,
- Introduce **diquarks and Baryons** in the model,
- Try to use **PNJL and ePNJL** instead of NJL,
- Compare **different initial conditions** and from other event generators too.

Thanks for your attention

Hydrodynamics

The main ingredients of Hydrodynamics are (**cf. Muronga**) :

14 Field equations

(1) $\partial_\mu N^\mu = 0$ Net charge (e.g., baryon, strangeness, ...),

(4) $\partial_\nu T^{\mu\nu} = 0$ Energy-momentum ($T^{\mu\nu} = T^{\mu\nu}_{\mathrm{ideal}} + \Pi^{\mu\nu}$),

(9) $\partial_\lambda F^{\mu\nu\lambda} = P^{\mu\nu}$ Balance law of fluxes (+Equation of state).

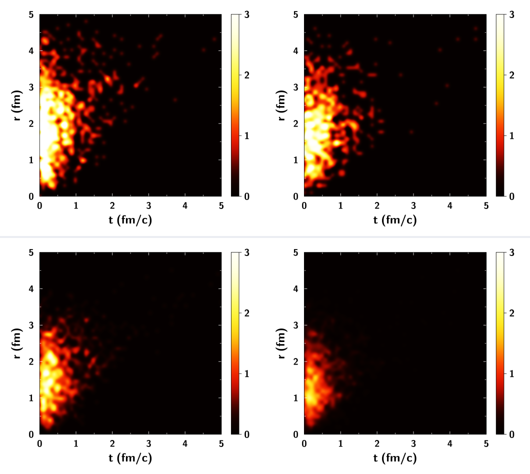
3 principles

- Principle of relativity (frame invariance of equations),
- Entropy principle : $\partial_\mu S^\mu \geq 0$,
- Hyperbolicity (finite wave speed = causality).

More results

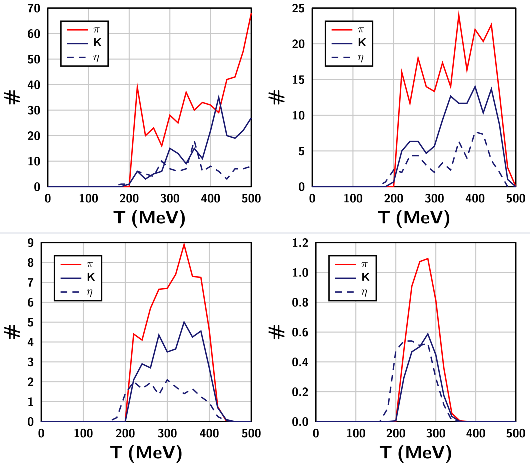
Simulations for b variable and L **fixed** at 0.5 fm.

More results



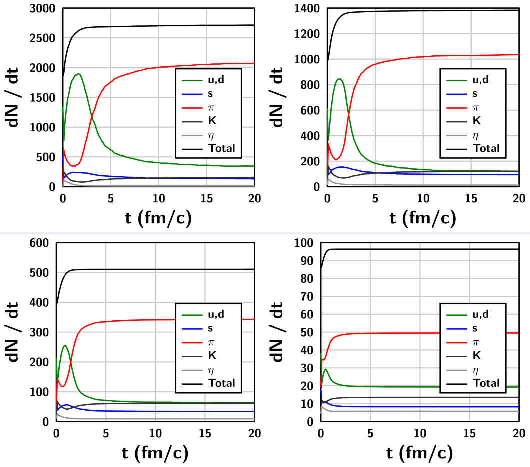
Decays ($b = 6.5/7.5/8.5/9.5$)

More results



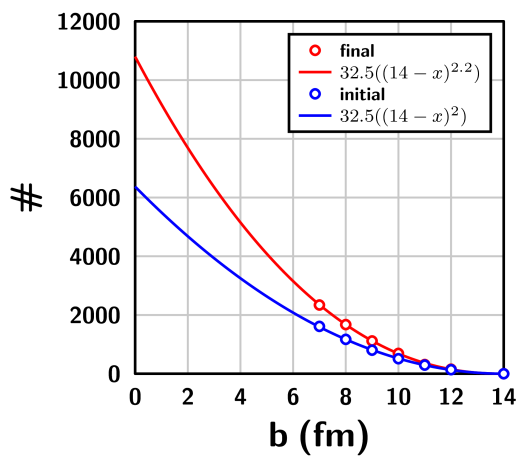
T of decays ($b = 6.5/8.5/10.5/12.5$)

More results

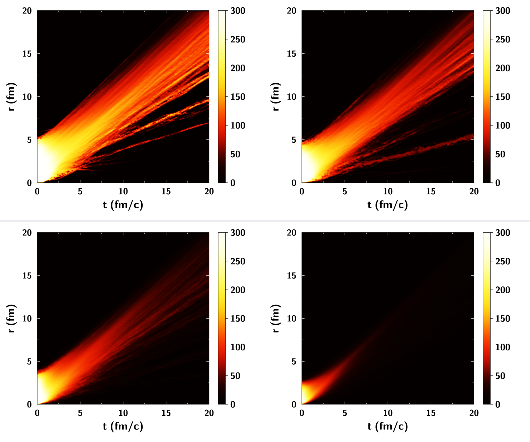


Multiplicity in time ($b = 6.5/8.5/10.5/12.5$)

More results

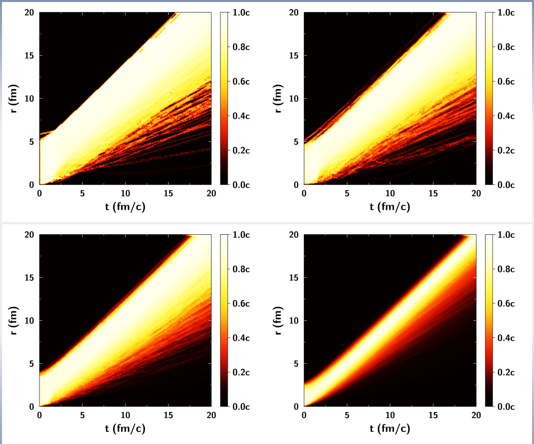


More results



Temperature ($b = 6.5/8.5/10.5/12.5$)

More results

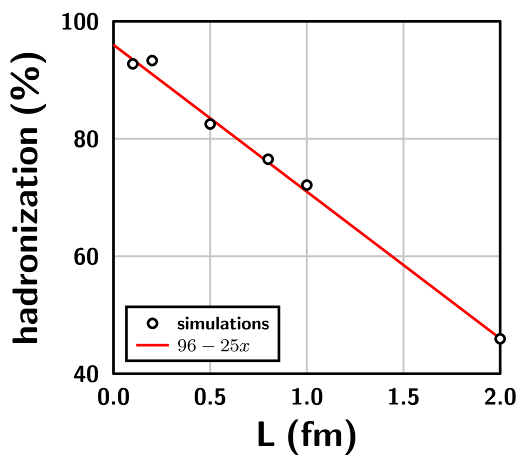


Velocity ($b = 6.5/8.5/10.5/12.5$)

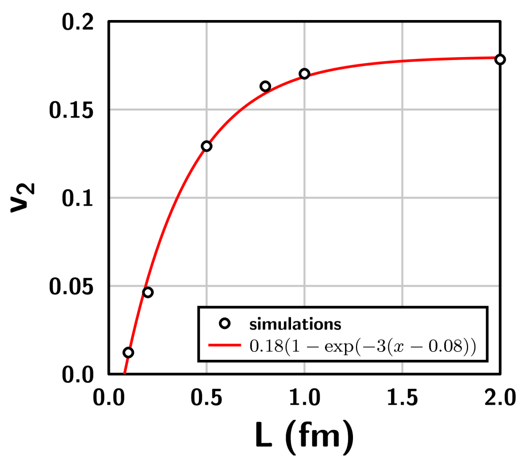
Impact of width L

The final results are the effect of different L on the simulations for a given impact parameter $b = 10$ fm.

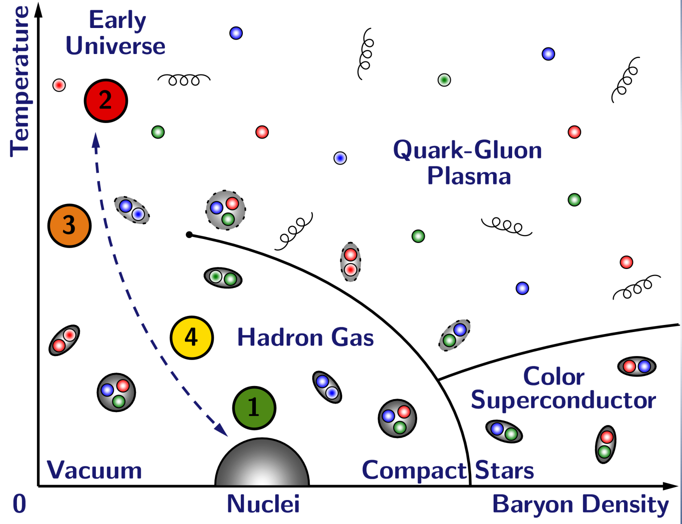
Impact of width L



Impact of width L



Mixed Phase


Temperature
Early Universe
2
3
4
1
Quark-Gluon Plasma
Hadron Gas
Color Superconductor
Vacuum
Compact Stars
0
Nuclei
Baryon Density