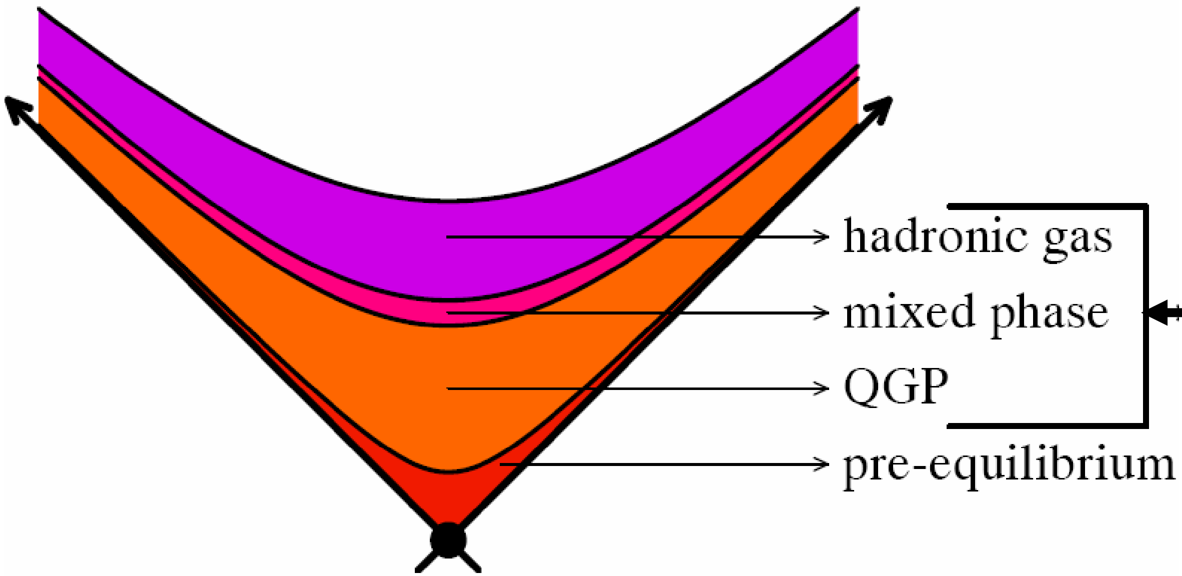
Helmholtz-Zentrum Dresden-Rossendorf

B. Kämpfer

# Transient Hydrodynamics

Mitglied der Helmholtz-Gemeinschaft

# Fluid Dynamics for urHICs




present day standard tool:

3. kinetics (transport model)
2. hydrodynamics
1. kinetics (transport model)

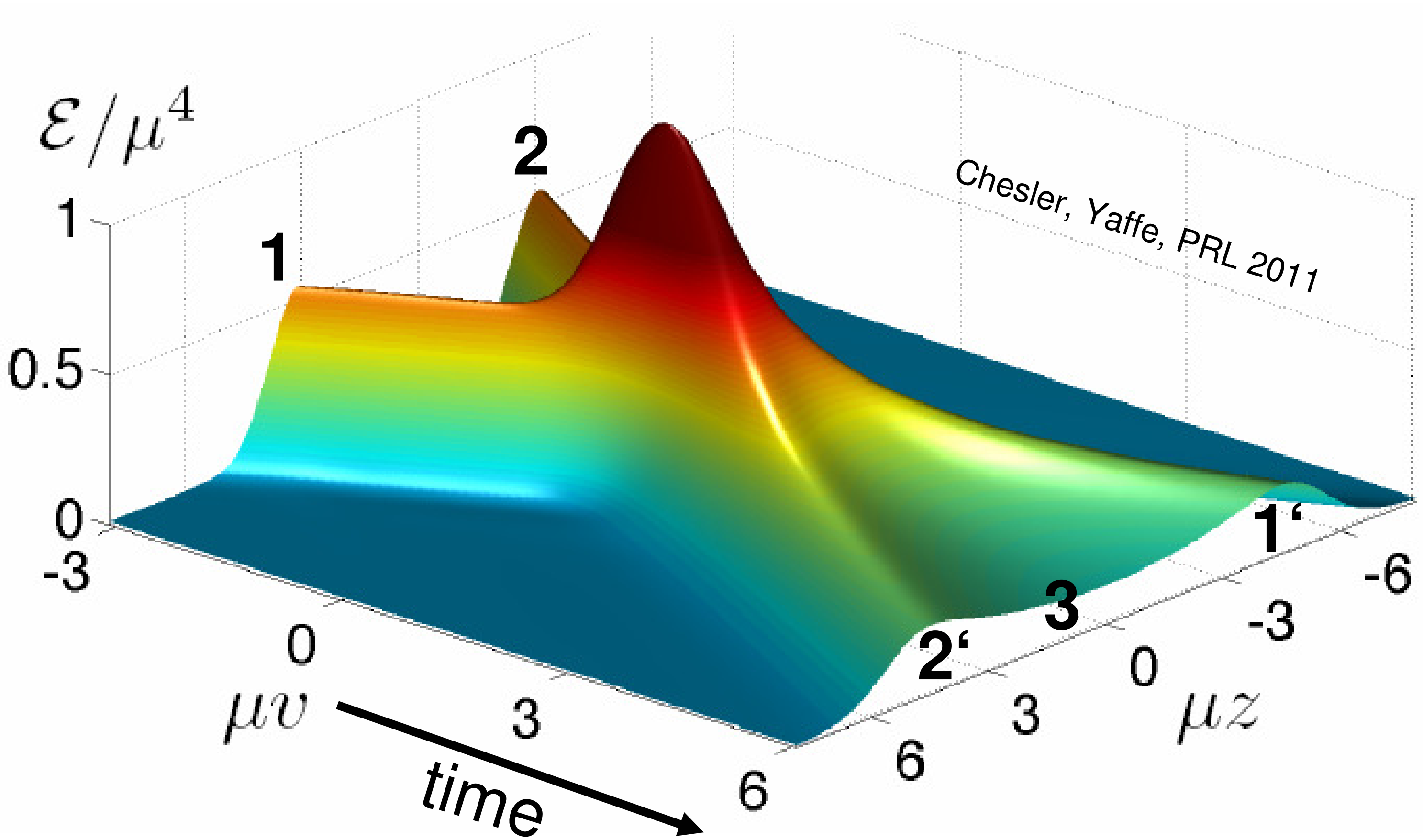
hydro needs external information: EoS, transport coeff.

hydro eqs.: Romatschke ... Rischke & co ...

link to kinetics: ... Greiner & co ...

hydro applicants: Heinz ... Eskola ... Grassi ...

# Recovery of the 3-Fluid Model?




AdS/CFT: 1. solve 5d Einstein vacuum eqs. (with symmetries) with negative cosmological constant

2. obtain 4d energy-momentum tensor from holographic renormalization (boundary theory)

# Intriguing Questions

entropy production
approach to loc. equilibrium
isotropization of pressure
resummed hydrodynamics
e(tau) from AdS/CFT
origin of Bjorken flow

Gubser ... Schäfer ... Janik ... Kovchegov ...

# Transient Thermodynamics

entropy as thermodynamic potential: s(e,n,q)

↳ local anisotropy

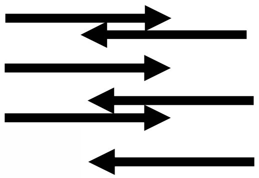
Gibbs-Duham: $e + p - Ts - \mu n - vq = 0$

Euler: $\frac{1}{T} = \frac{\partial s}{\partial e}, \ \frac{\mu}{T} = -\frac{\partial s}{\partial n}, \ \frac{v}{T} = -\frac{\partial s}{\partial q}$

construction of s: $s(e, n, q = 0) = s_0(e, n) =$ known

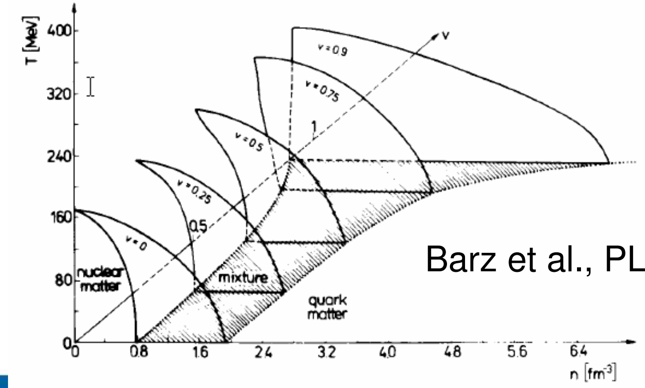
$$s = s_0(e - k, n) \qquad k(n, q)$$

relative kin. energy density



gen. Gibbs conds.:
equality of $T, \mu, p, v$

constructed phase trans.




Barz et al., PLB 1987

# Transient (Anisotropic) Hydro

$$\text{diag}(T^{\mu\nu}) = (e, p+q, p, p)$$

$$T^{\mu\nu} = e u^\mu u^\nu + p(\eta^{\mu\nu} + u^\mu u^\nu) + q t^{-2} t^\mu t^\nu$$

$u \cdot t = 0$ → t is spacelike: not a 2-fluid model

$s^\mu = s u^\mu + \gamma t^\mu : \quad s^\mu{}_{;\mu} \geq 0$

→requirement for $\dot{q}$
(cancel indef. terms
+ relax. time approx.)

1d flow:

$$\dot{n} + n\Theta = 0$$

$$\dot{u}^\mu + \frac{[p+q]u^\mu - [p+q]_{,\nu}\eta^{\mu\nu}}{e+p+q} = 0$$

$$\dot{e} + (e+p+q)\Theta = 0$$

$$\dot{q} + (q + qv^{-1})\Theta + \tau_{rel}^{-1} nq = 0$$

$$\dot{} \equiv u^\mu \partial_\mu$$

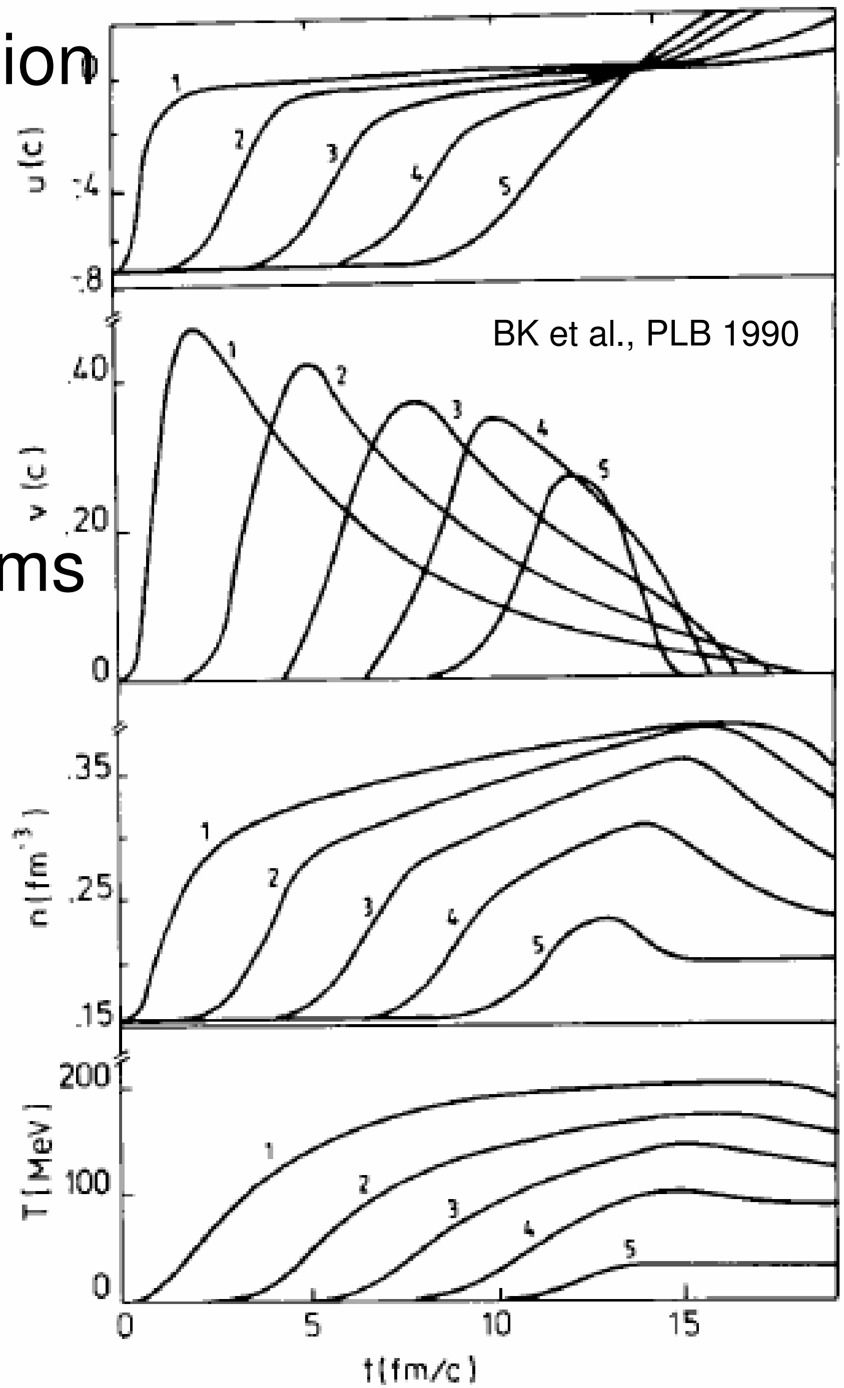
$$\Theta \equiv u^\mu{}_{;\mu}$$

intention:
partial transparency & local isotropization
→ no momentum transport
(viscosity, gradients)

limiting cases
small rel. time→ fluid behavior
large rel. time→ interpenetrating streams

Bonus:
no shock waves as in 1-fluid model

Malus:
very phenomenological setting



# Comparison of $\square$ with Frankfurt

third-order rel. diss. hydro,
extension of Israel-Stewart

El, Xu, Greiner, PRC 2010
Denicol, Koide, Rischke, PRL 2010

shear tensor:

$$\mathrm{diag}(\pi^{\mu\nu}) = (0, -\pi, \tfrac{1}{2}\pi, \tfrac{1}{2}\pi)$$ for Bjorken flow & symm.

$$\dot{p} + \frac{4p - \pi}{3\tau} = 0$$

$$\dot{\pi} + \frac{\lambda\pi - \frac{4}{3}\beta_\pi}{\tau} + \frac{\pi}{\tau_\pi} = 0$$

$$\beta_\pi = \frac{4}{5}p, \quad \lambda = \frac{124}{63}, \quad \tau_\pi = 5\frac{\eta/s}{T}$$

Boltzmann
m = 0

transient dynamics ($\tau_\pi$)

$$\dot{e} + (e + p + q)\ominus = 0$$

$$\dot{q} + (q + qv^{-1})\ominus + \frac{nq}{\tau_{rel}} = 0$$

$$\ominus = \frac{1}{\tau}$$

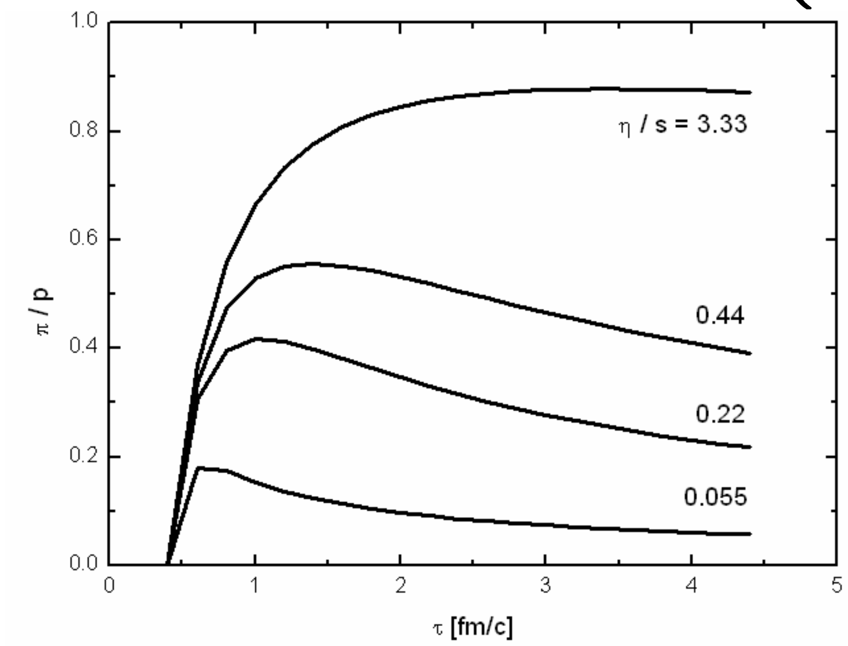
$$e = 3p$$

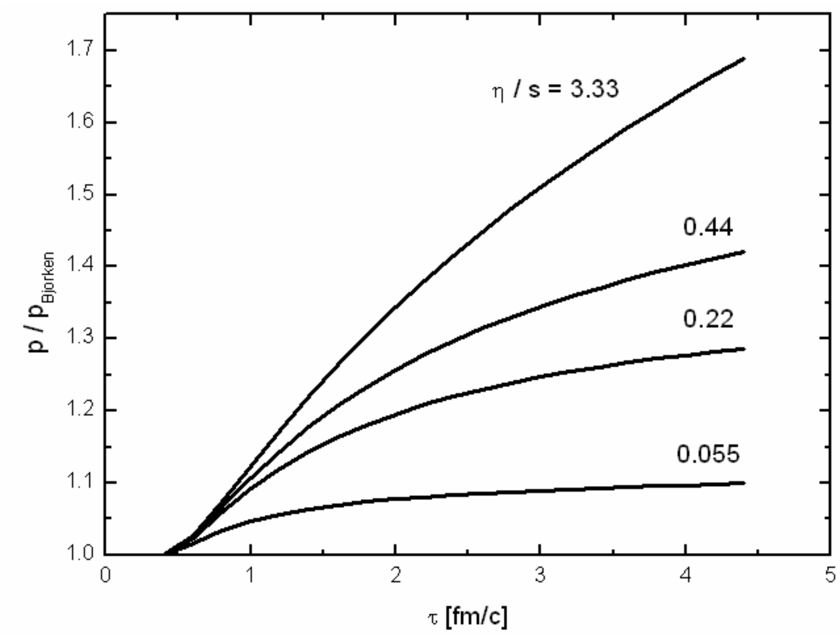
$$q \rightarrow -\pi$$

looks as gradient expansion

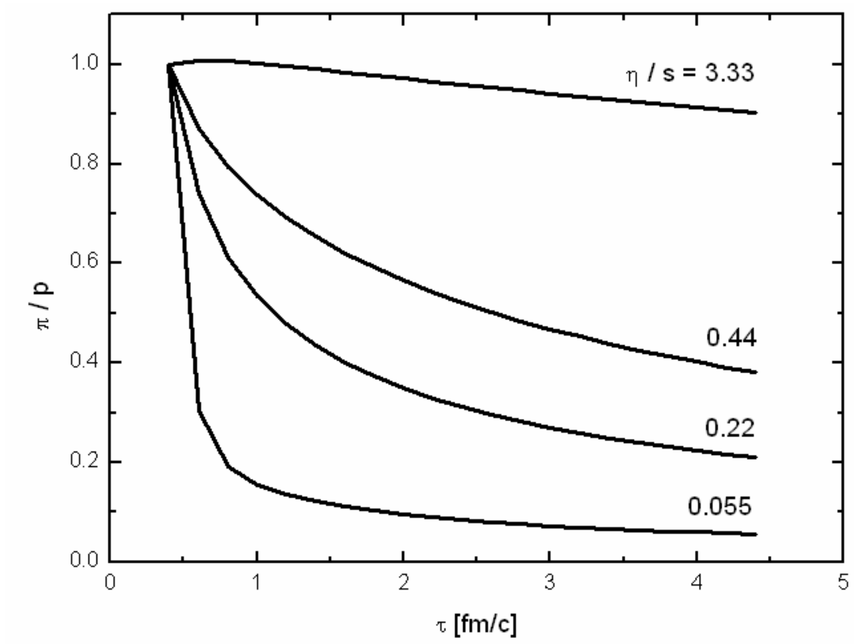
DRESDEN concept HZDR

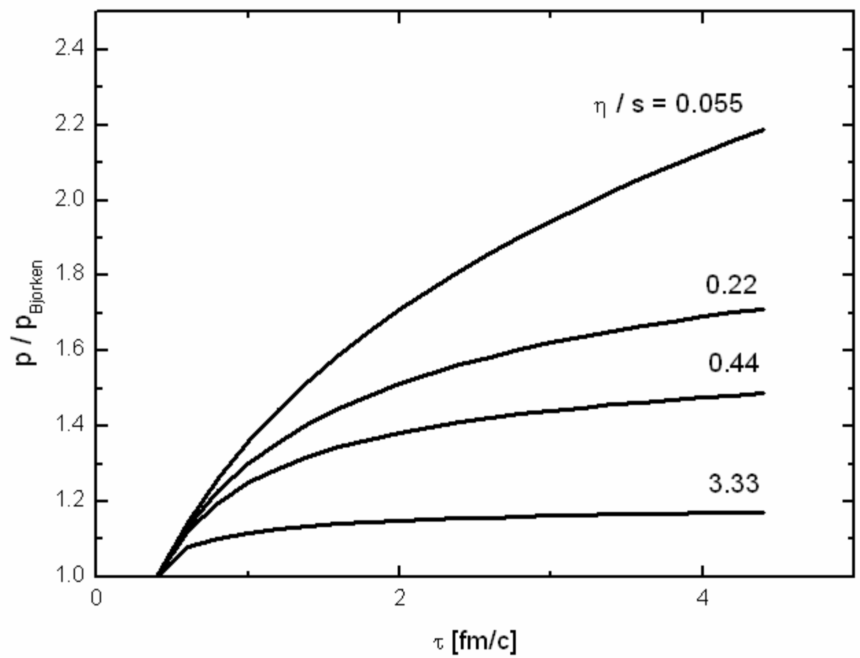
## start from equilibrium: $\pi(\tau_0) = 0$




## start from off-equilibrium: $\pi(\tau_0) = p(\tau_0)$



# Fluid Dynamics from Gravity

FG coordinates

Bhattacharyya, Hubeny, Rangamani ... JHEP 2008

$$ds^2 = z^{-2}(dz^2 + [\eta_{\mu\nu} + \alpha z^d T_{\mu\nu}]dx^\mu dx^\nu)$$

bulk near z = 0, asymp. AdS metric & black brane

$$R_{NM} - \frac{1}{2}RG_{NM} + \Lambda_{(d+1)}G_{MN} = 0$$

strong coupling regime: universal sector in $\mathrm{AdS}_{d+1} \times S^{d+1}$

long-wavelength solutions, isotropization

→ assume $T(x),\ u^\mu(x)$ as relevant d.o.f.

epsilon expansion: z expansion equivalent?

$$G_{MN} = \sum_{k=0}^{\infty} \epsilon^k G_{MN}^{(k)},$$

$$\underbrace{T, u^\mu \text{ too}}$$

$$T^\mu_\mu = 0, T^{\mu\nu}{}_{;\mu} = 0$$

iterative solutions of Einstein eqs. → constitutive eqs.:

$$k = 0 : T^{\mu\nu}_{(0)} = \pi^4 T^4 (\eta^{\mu\nu} + 4u^\mu u^\nu)$$

$$k = 1 : T^{\mu\nu}_{(1)} = T^{\mu\nu}_{(0)} - 2\pi^3 T^3 \sigma^{\mu\nu} \longrightarrow \eta = 4\pi s$$

$$k = 2 : T^{\mu\nu}_{(2)} = T^{\mu\nu}_{(1)} - \pi^2 T^2 (\cdots)^{\mu\nu}$$

$$T^{\mu\nu} = -2 \lim_{r\to\infty} r^4 (K^\mu_\nu - \delta^\mu_\nu) \qquad z \to r$$

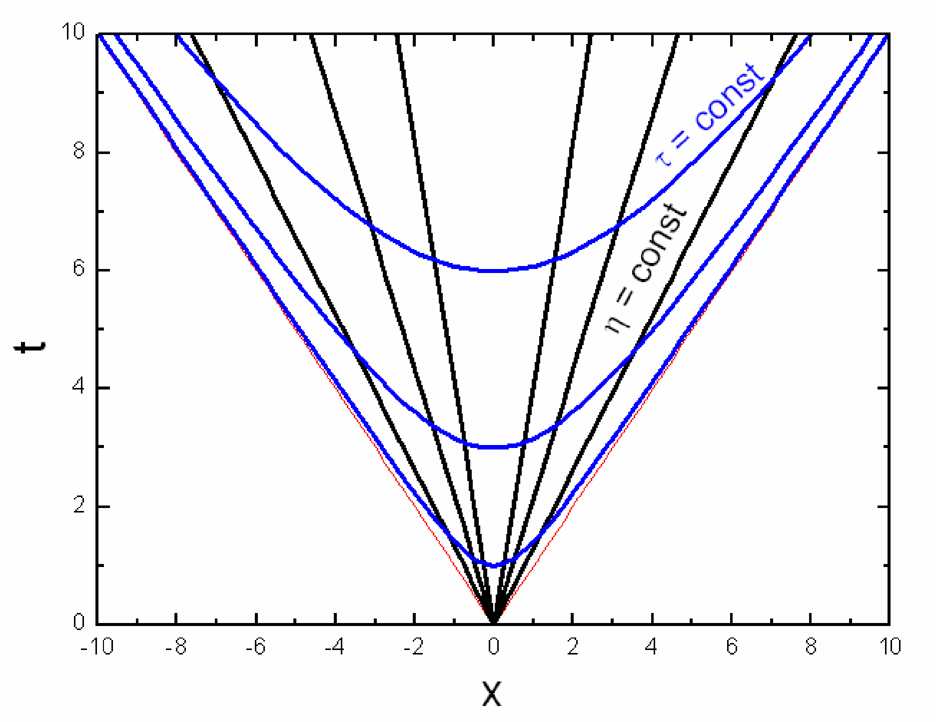
↑ extrinsic curvature on r = const from $G_{MN}$

Bonus: - anything (EoS, transport coeff., tensor structure) is determined
- strong coupling implied
- universality

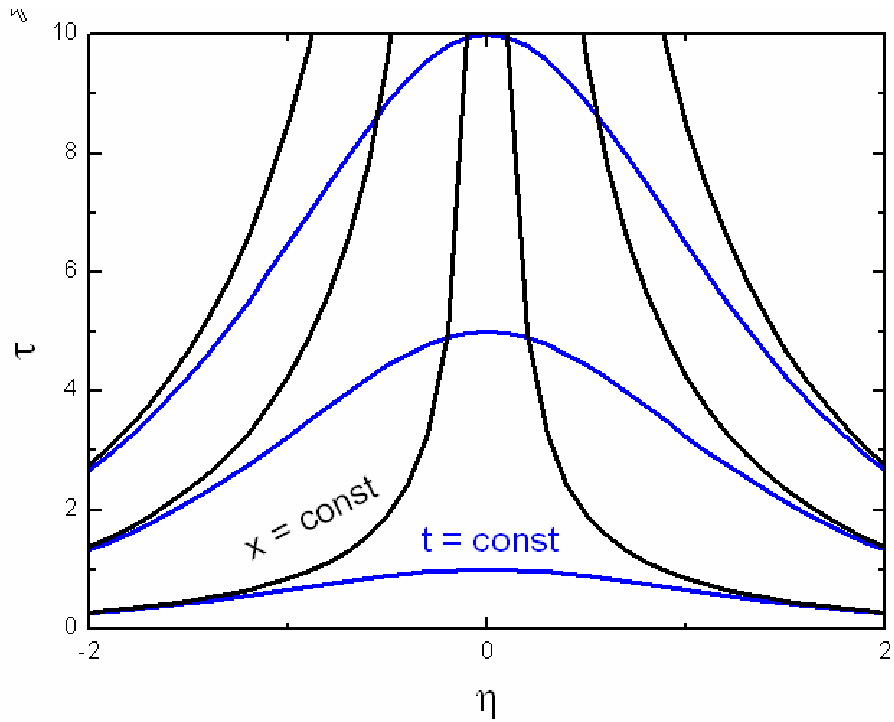
Malus: - existence of gravity dual is assumed
- this is not QCD
→ deformations, further fields ... soft/hard wall

# Milne Coordinates

$$t = \tau\, ch\eta,\; x = \tau\, sh\eta$$




$$ds^2 = -dt^2 + dx^2 + d\vec{x}_\perp^2$$



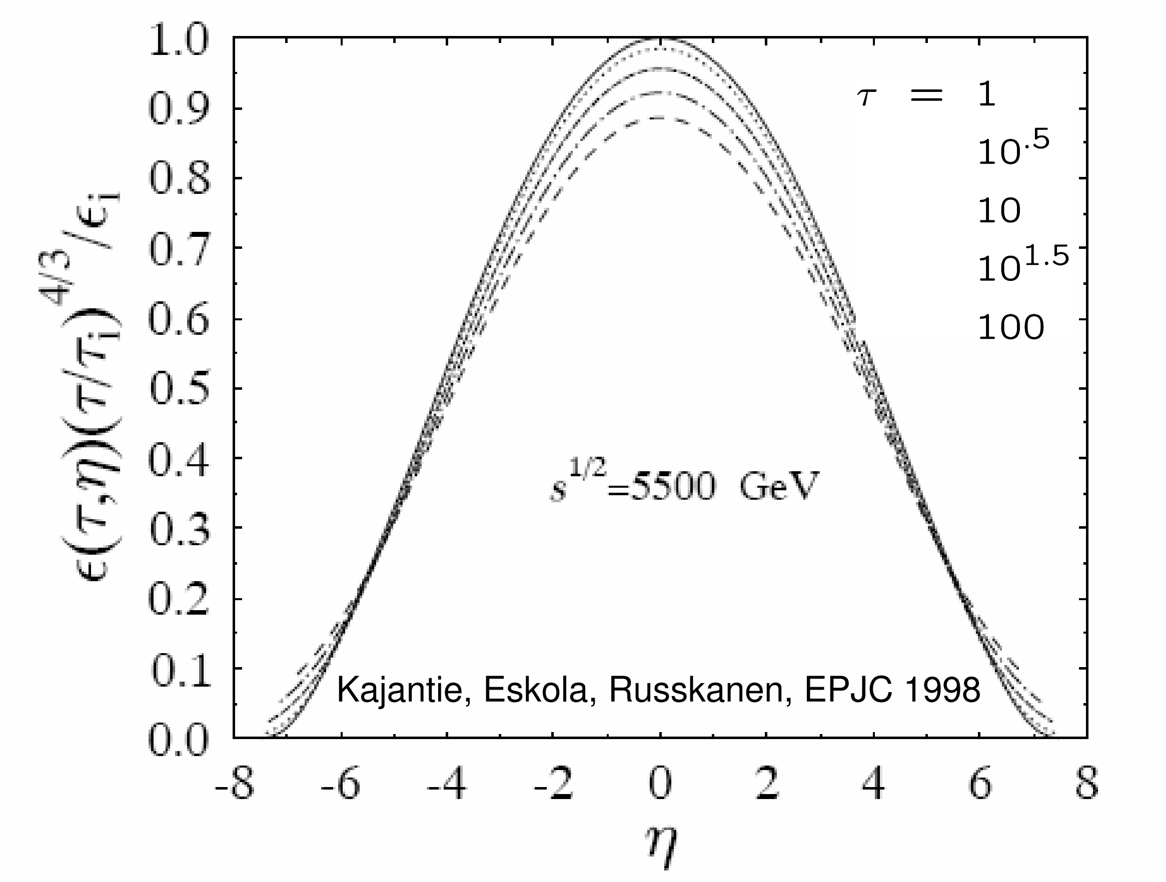

$$ds^2 = -d\tau^2 + \tau^2 d\eta^2 + d\vec{x}_\perp^2$$

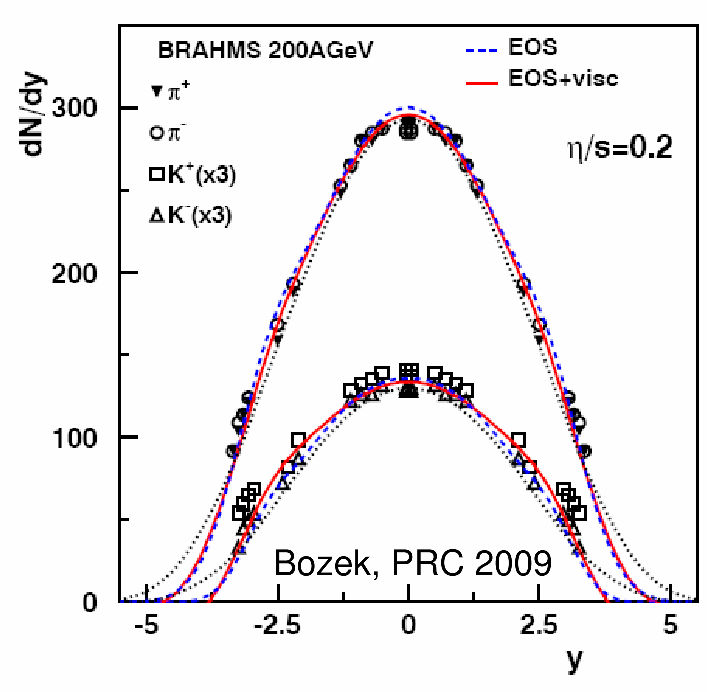
Bjorken flow: $y = \eta,\; \vec{v}_\perp = 0$, Bjorken symmetry: $T(\tau)$

Gubser flow: $y = \eta,\; v_\perp(\tau, x_\perp)$

# Flow with Longitudinal Pressure Gradient

initial conds: Bjorken flow




init. non-Bjorken flow: different evolution

→ it is hard to modify Bjorken‘s flow once it is there

→ origin of Bjorken flow?

cf. F. Wunderlich‘s talk

Janik, Lecture Notes Phys. 2011

# The Janik Route

Bjorken flow + symmetry, Milne coordinates

$$\mathrm{diag}(T^{\mu\nu}) = (\underbrace{T_{\tau\tau}}_{\text{e}}, T_{\eta\eta}, T_{\perp\perp}, T_{\perp\perp})$$

$T^{\mu}_{\mu} = 0: \quad -e + \tau^{-2} T_{\eta\eta} + 2T_{\perp\perp} = 0$

$T^{\mu\nu}{}_{;\mu} = 0: \quad \tau\dot{e} + e + \tau^{-2} T_{\eta\eta} = 0$

$$\mathrm{diag}(T^{\mu\nu}) = (e, \tau^2[p+q], p, p)$$

$q = -(\frac{3}{2}\tau\dot{e} + 2e)$

$p = \frac{1}{2}\tau\dot{e} + e$

isotropic pressure:

$q = 0: \ e = e_0(\tau_0/\tau)^{4/3}$ only $e(\tau)$ matters

# Get e(tau) from AdS/CFT

FG coordinates: $ds^2 = z^{-2}(dz^2 + A d\tau^2 + \tau^2 B d\eta^2 + C d\vec{x}_\perp^2)$

$$A(z,\tau),\ B(z,\tau),\ C(z,\tau)$$

boundary theory: z = 0 $\quad G_{MN} \sim z^{-2} \left[\begin{array}{c|c} g_{\mu\nu} & 0 \\ \hline 0 & 1 \end{array}\right]$

z expansion (indices suppressed):

$$g(z,\tau) = g^{(0)}(\tau) + z^2 g^{(2)}(\tau) + z^4 \overbrace{g^{(4)}(\tau)}^{T^{\mu\nu}} + z^6 g^{(6)}(\tau) + \dots$$

$$g^{(n)} = \mathcal{F}(g^{(n-1)},\ \text{derivs.})$$

→ iterative solution for n > 4

↑ Einstein eqs.

Skenderis et al., 2000

Definition: Kretschmann $K = R_{ABCD} R^{ABCD}$

ansatz: $e \propto \tau^{s_1} + \tau^{s_2} + \tau^{s_3} + \ldots$

## Large tau:

requirement: K = regular (no singularities in the bulk)

→ values of $s_{1,2,3}$

the only scale

$$T = T_0 \tau^{-1/3} (1 - \tilde{\eta} \tau^{-2/3} + \tilde{\lambda} \tau^{-4/3} + \# \tau^{-6/3} + \ldots)$$

Gyulassy, Danilewicz PRD 1985

shear viscosity = $s/4\pi$

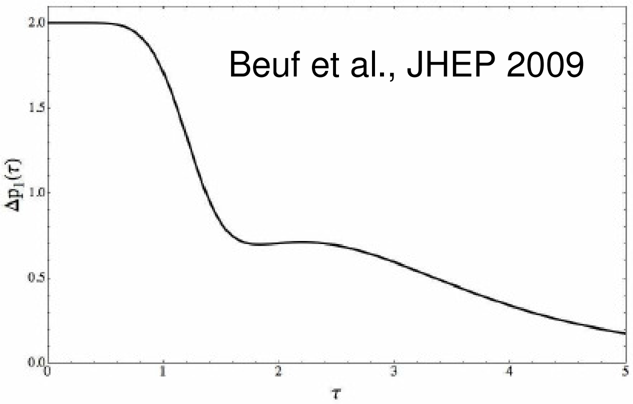
$\tilde{\eta}, \tilde{\lambda}, \#$: numbers

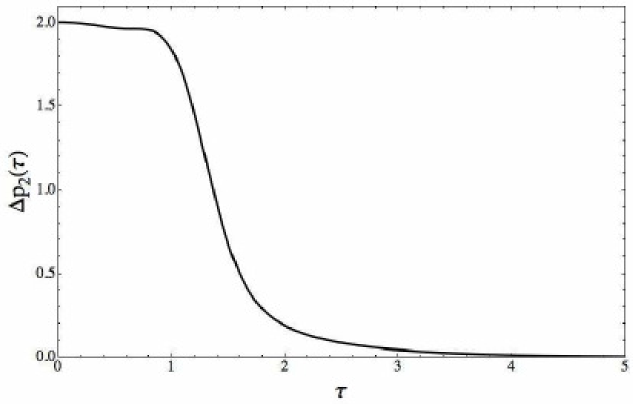
## Small tau:

Einstein eqs. → constraints for A, B, C → allowed init. conds.

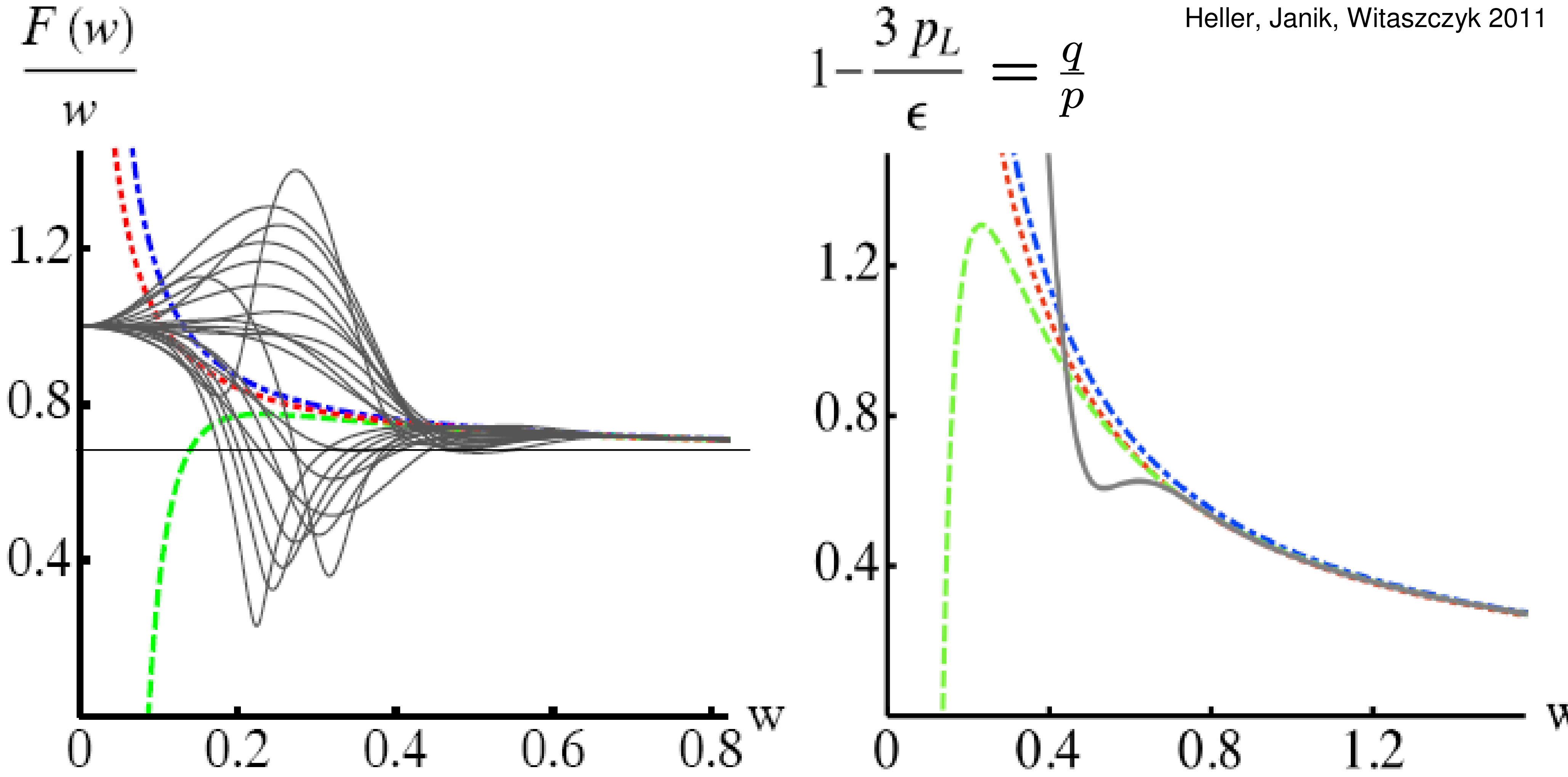
anisotropy measure

$$\Delta p = 1 - \frac{p_{||}}{p_{\perp}}$$



Heller, Janik, Witaszczyk 2011




$$w \propto e^{1/4}\tau,\ F = \frac{3}{4} w \left(1 - \frac{p+q}{9p}\right)$$

1st-order hydro stage: q = 0 → F/w = 2/3

# Summary/Outlook

Higher-order fluid dynamics from gravity: all coeff. are given
(resummation possible?)

Transient thermo/hydro dynamics: anisotropy dynamics
formally not a gradient expansion, but equivalent to
Greiner/Rischke 3rd order/extended IS hydro

Iterative hydrodynamics: tau expansion → F. Wunderlich
(in the spirit of z expansion in AdS/CFT)

AdS/CFT approach to transient thermo/hydro dynamics
possible? → explicit strong-coupling approach
with all coeff. given