Sigma and pion excitations in nonequilibrium chiral fluid dynamics

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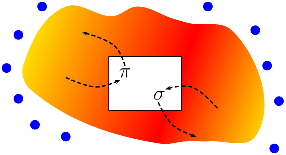
SUBATECH, Nantes & FIAS, Frankfurt



Non-equilibrium dynamics, Crete, 2011

FIAS Frankfurt Institute for Advanced Studies

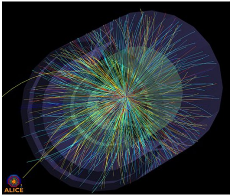
How to study the QCD phase diagram...

... be brave and solve

$$Z(T,\mu_B) = \int \mathcal{D}(A,q,q^\dagger)\mathrm{e}^{-S^E_{\mathrm{QCD}}}$$

ab initio and nonperturbatively,

... be strong and collide heavy ions at ultrarelativistic energies,



... be creative and study effective models of QCD.

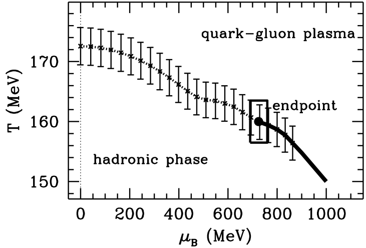
$$\mathcal{L}_{\mathrm{eff}}$$

Being brave

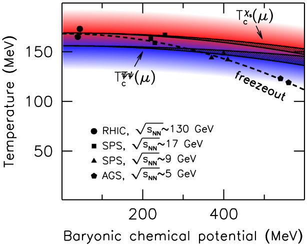
The critical point in lattice QCD

Methods to explore the $T - \mu_B$–plane

- reweighting



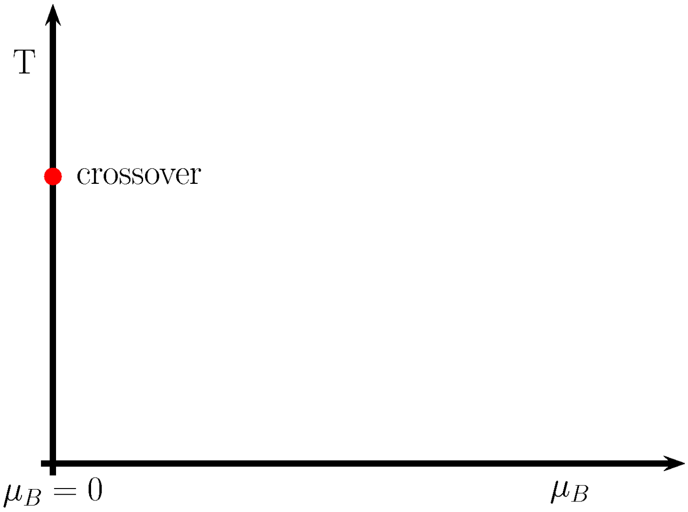
(Z. Fodor, S.D. Katz, JHEP **0203** (2002))



(G. Endrodi, Z. Fodor, S. D. Katz, K. K. Szabo, JHEP **1104** (2011))

- imaginäres μ_B (de Forcrand, Philipsen): $\mu_B^c > 500$ MeV

Being brave

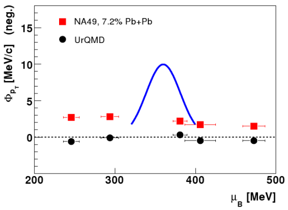


Being strong

The critical point in heavy-ion collisions

non-monotonic fluctuations in pion and proton multiplicities by coupling to the order parameter of chiral symmetry

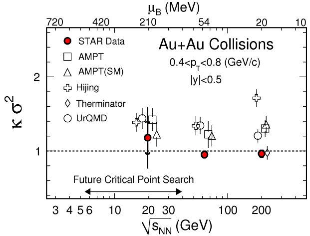
$$\langle \Delta n_p \Delta n_k \rangle = v_p^2 \delta_{pk} + \frac{1}{m_\sigma^2} \frac{G^2}{T} \frac{v_p^2 v_k^2}{\omega_p \omega_k}$$



(M. A. Stephanov, K. Rajagopal and E. V. Shuryak, PRD **60** (1999), NA49 collaboration J. Phys. G **35** (2008))

Higher moments, e.g. Kurtosis $\propto \xi^7$

(M. A. Stephanov, PRL **102** (2009))



(STAR collaboration, PRL **105** (2010))

Being strong

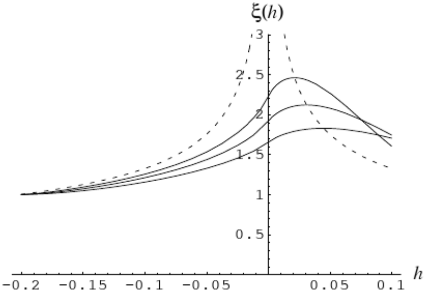
The critical point in dynamic systems

long relaxation times near a critical point $\Rightarrow$ critical slowing down
$\Rightarrow$ the system is driven out of equilibrium

$$\frac{d}{dt} m_\sigma(t) = -\Gamma[m_\sigma(t)](m_\sigma(t) - \frac{1}{\xi_{eq}(t)})$$

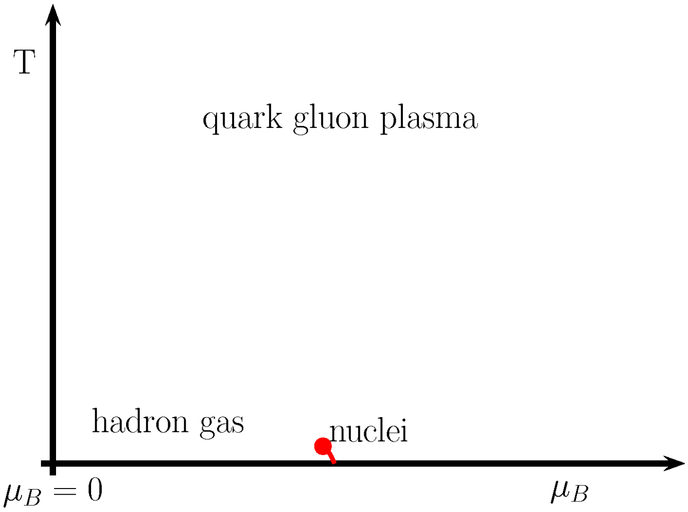
with $\Gamma(m_\sigma) = \frac{A}{\xi_0}(m_\sigma \xi_0)^z$
$z = 3$
(dynamic) critical exponent

$\Rightarrow \xi \sim 1.5 - 2$ fm



(B. Berdnikov and K. Rajagopal, PRD **61** (2000)); D.T.Son, M.Stephanov, PRD **70** (2004); M.Asakawa, C.Nonaka, Nucl. Phys. A774 (2006))

Being strong



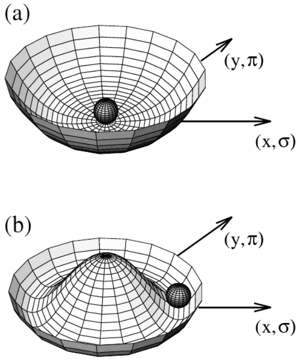
Being creative

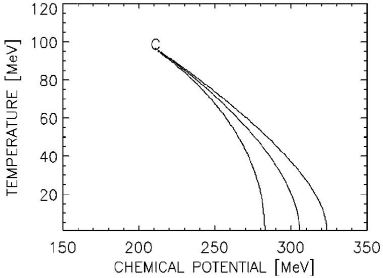
The linear sigma model with constituent quarks

$$\mathcal{L} = \overline{q}\left[i\gamma^{\mu}\partial_{\mu} - g\left(\sigma + i\gamma_5\tau\vec{\pi}\right)\right]q + 1/2\left(\partial_{\mu}\sigma\right)^2 + 1/2\left(\partial_{\mu}\vec{\pi}\right)^2 - U\left(\sigma,\vec{\pi}\right)$$

$$U\left(\sigma,\vec{\pi}\right) = \frac{\lambda^2}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 - h_q\sigma - U_0$$

(M.Gell-Mann, M.Levy, Nuovo Cim. 16, (1960))



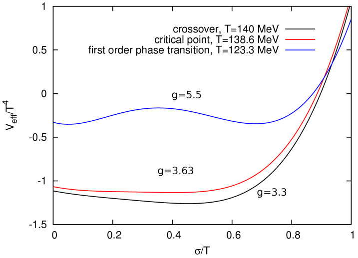


(O. Scavenius, A. Mocsy, I.N. Mishustin, D.H. Rischke, PRC **64** (2001))

The linear sigma model with constituent quarks

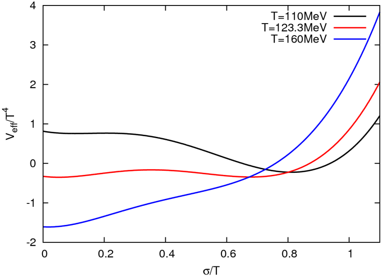
The effective potential at $\mu_B = 0$

$$V_{\mathrm{eff}} = -\frac{T}{V}\ln Z = -d_q T \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \ln\left(1 + \exp(-\frac{E}{T})\right) + U(\sigma, \vec{\pi})$$



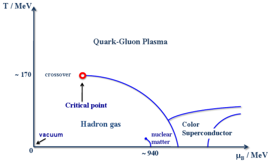
Tune the strength of the phase transition via the coupling g.

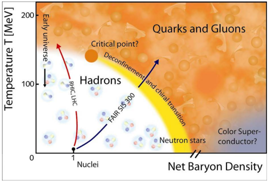
first order phase transition

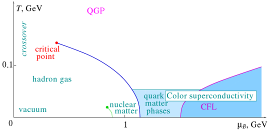


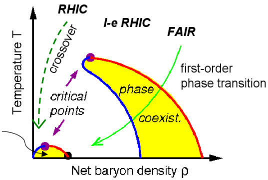
dynamic symmetry breaking

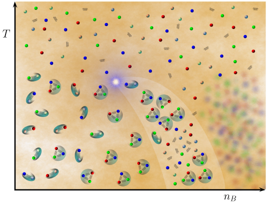
Being creative

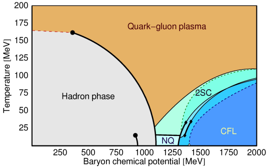


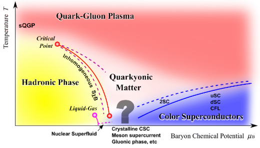


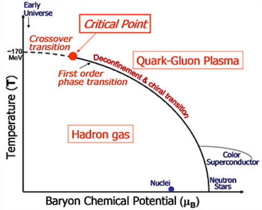


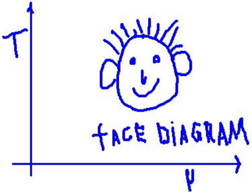










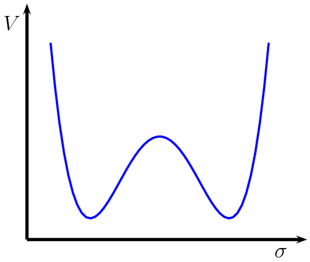


Phase transitions - thermodynamically

first order phase transition

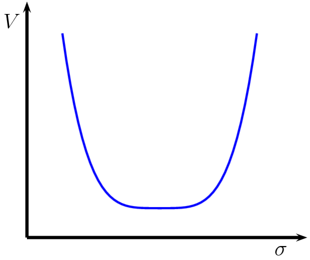
- two degenerate minima separated by a barrier
- nucleation
- spinodal decomposition

(I.N.Mishustin, PRL **82** (1999); Ph.Chomaz, M.Colonna, J.Randrup, Physics Reports **389** (2004))



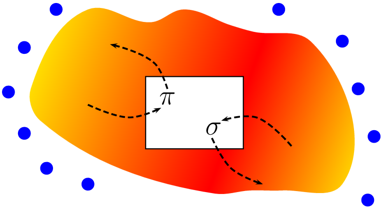
critical point

- $m_\sigma^2 = \frac{\partial^2 V}{\partial \sigma^2} \to 0$
- correlation length diverges $\xi = \frac{1}{m_\sigma} \to \infty$
- universality classes (for QCD: 3d Ising model) $\Rightarrow \langle \sigma^2 \rangle \propto \xi^2$
- critical opalescence

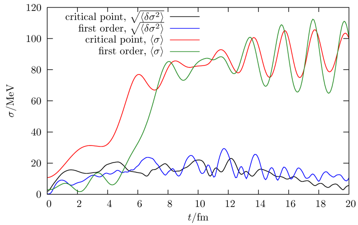


Chiral fluid dynamics

- Fluid dynamic description of a heavy-ion collision
- Model with a chiral phase transition
- Dynamics of the order parameter



(I. N. Mishustin and O. Scavenius, PRL **83** (1999); K. Paech, H. Stöcker and A. Dumitru, PRC **68** (2003))



Full nonequilibrium description with relaxational dynamics by including damping and noise!

Chiral fluid dynamics

- Langevin equation for the sigma field: damping and noise from the interaction with the quark fluid = heat bath

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\rho_s + \eta \partial_t \sigma = \xi$$

- Fluid dynamic expansion of the quark fluid = heat bath

$$T_{\mathrm{q}}^{\mu\nu} = (e+p)u^\mu u^\nu - pg^{\mu\nu}$$

- Energy and momentum exchange

$$\partial_\mu T_{\mathrm{q}}^{\mu\nu} = S^\nu = -\partial_\mu T_\sigma^{\mu\nu}$$

$\Longrightarrow$ Selfconsistent approach within the two-particle irreducible effective action!

(MN, S. Leupold, C. Herold, M. Bleicher, PRC **84** (2011))

The two-particle irreducible (2PI) effective action

for the σ mean field and the quark propagators S^{ab}

$$\Gamma[\sigma, S] = S_{\mathrm{cl}}[\sigma] - i\mathrm{Tr}\ln S^{-1} - i\mathrm{Tr} S_0^{-1} S + \Gamma_2[\sigma, S]\,,$$

equation of motion for σ and S^{ab}

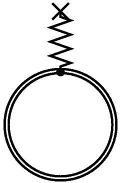
$$\frac{\delta\Gamma[\sigma, S]}{\delta\sigma^a} = 0 \quad \text{and} \quad \frac{\delta\Gamma[\sigma, S]}{\delta S^{ab}} = 0$$

give conserving transport equations if the self-energy is given by

$$-i\Sigma^{ab}(x, y) = -\frac{\delta\Gamma_2[\sigma, S]}{\delta S^{ab}(x, y)}\,.$$

(J. M. Luttinger, J. C. Ward, Phys. Rev. **118** (1960); G. Baym, L. P. Kadanoff, Phys. Rev. **124** (1961); G. Baym, Phys. Rev. **127** (1962))

The two-particle irreducible effective action

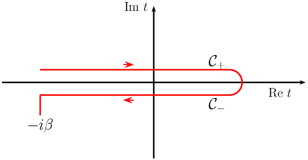


$$\Gamma_2[\sigma, S] = g \int_{\mathcal{C}} \mathrm{d}^4 x \mathrm{tr}(S^{++}(x,x)\sigma^+(x) + S^{--}(x,x)\sigma^-(x))$$

equation of motion for the σ mean field

$$-\frac{\delta S_{\mathrm{cl}}[\sigma]}{\delta \sigma^a} = \frac{\delta \Gamma_2[\sigma, S]}{\delta \sigma^a} = g \mathrm{tr} S^{aa}(x,x)$$

the effective action along the contour



$$\begin{aligned}\Gamma[\sigma, S] =& g \mathrm{tr} S_{\mathrm{th}}^{++}(x,x)\Delta\sigma(x) - \frac{T}{V}\ln Z_{\mathrm{th}} \\ &+ \int \mathrm{d}^4 x D[\bar{\sigma}](x)\Delta\sigma(x) \\ &+ \frac{i}{2}\int \mathrm{d}^4 x \int \mathrm{d}^4 y \Delta\sigma(x)\mathcal{I}[\bar{\sigma}](x,y)\Delta\sigma(y)\end{aligned}$$

with $\Delta\sigma = \sigma^+ - \sigma^-$ and $\bar{\sigma} = 1/2(\sigma^+ + \sigma^-)$ on the contour.

The 2PI effective action - term by term

equilibrium properties, equation of state:

$$-\frac{T}{V}\ln Z_{\mathrm{th}}$$

lowest order in the equation of motion for the sigma field:

$$g\,\mathrm{tr}S^{++}_{\mathrm{th}}(x,x)\Delta\sigma(x)$$

dissipative processes:

$$\int \mathrm{d}^4x D[\bar{\sigma}](x)\Delta\sigma(x)$$

origin of fluctuations:

$$\frac{i}{2}\int \mathrm{d}^4x \int \mathrm{d}^4y \Delta\sigma(x)\mathcal{I}[\bar{\sigma}](x,y)\Delta\sigma(y)$$

The origin of fluctuations

imaginary part of Γ is interpreted as stochastic fluctuations

$$\exp[-\frac{1}{2}\int \mathrm{d}^4x \int \mathrm{d}^4y \Delta\sigma(x)\mathcal{I}(x,y)\Delta\sigma(y)]$$
$$= \int \mathcal{D}\xi P[\xi] \exp[i \int \mathrm{d}^4x \xi(x)\Delta\sigma(x)]$$

$P[\xi]$ Gaussian measure with

$$\langle\xi\rangle = 0$$
$$\langle\xi(t)\xi(t')\rangle = \mathcal{I}^{-1}(t,\mathbf{x};t',\mathbf{y})$$

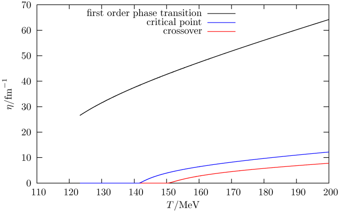
Semiclassical equation of motion for the sigma field

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\rho_s + \eta \partial_t \sigma = \xi$$

damping term η and noise ξ for $\mathbf{k} = 0$

$$\eta = g^2 \frac{d_q}{\pi} \left(1 - 2n_{\mathrm{F}}(\frac{m_\sigma}{2})\right) \frac{(\frac{m_\sigma^2}{4} - m_q^2)^{\frac{3}{2}}}{m_\sigma^2}$$

$$\langle \xi(t)\xi(t') \rangle = \frac{1}{V}\delta(t - t') m_\sigma \eta \coth\left(\frac{m_\sigma}{2T}\right)$$



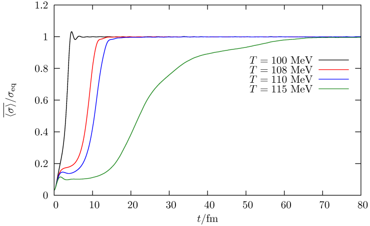
below T_c damping by the interaction with the hard pion modes, apply $\eta = 2.2/\mathrm{fm}$

(T. S. Biro and C. Greiner, PRL **79** (1997))

Relaxation for an isothermal heat bath

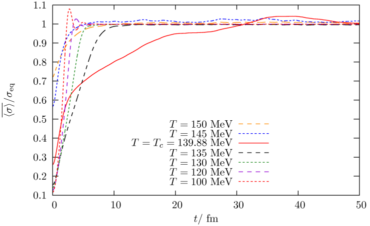
initialize the sigma field in equilibrium at $T = 160\mathrm{MeV} > T_c$, then quench the system (i.e. a sudden temperature drop from $T = 160$ MeV to $T < T_c$)

first order phase transition



long relaxation times at the phase transition due to phase coexistence

critical point

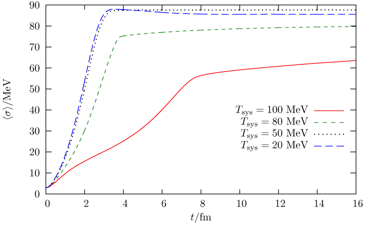


long relaxation times at the critical point due to critical slowing down

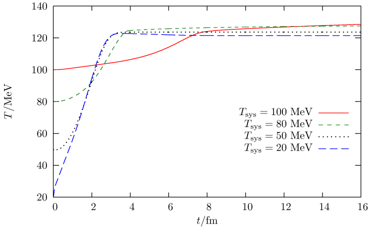
Equilibration for a heat bath with reheating

first order phase transition

relaxation of the σ field

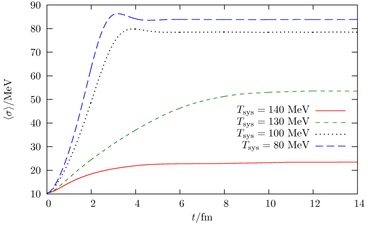


temperature

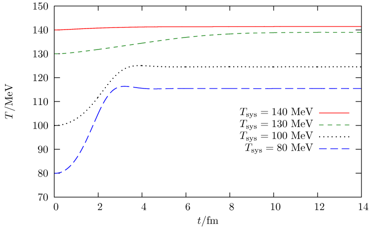


critical point

relaxation of the σ field



temperature



long relaxation times near the phase transition

(MN, S. Leupold, M. Bleicher, arXiv:1105.1396)

Energy-momentum conservation

Energy-momentum tensor of the coupled system is conserved for the full propagator:

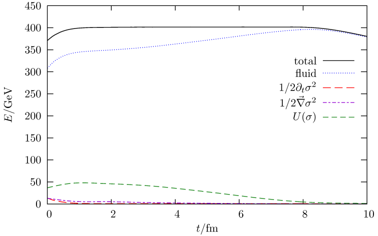
$$\partial_\mu T_{\mathrm{q}}^{\mu\nu} = g\mathrm{tr} S^{++}(x,x)$$
$$\partial_\mu T_{\sigma}^{\mu\nu} = -g\mathrm{tr} S^{++}(x,x)$$

HERE, approximation of an ideal fluid and the source term

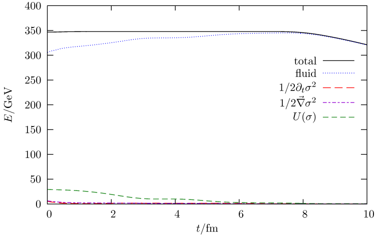
$$\partial_\mu T_{\mathrm{q}}^{\mu\nu} = g\mathrm{tr} S_{\mathrm{th}}^{++}(x,x) = -\partial_\mu T_{\sigma}^{\mu\nu} = S^\nu$$

(MN, M. Bleicher, S. Leupold, I. Mishustin, arXiv:1105.1962)

first order phase transition



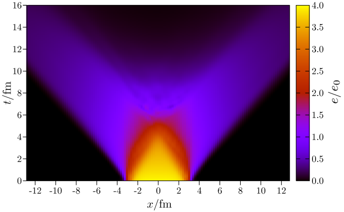
critical point



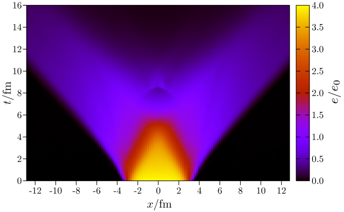
Time evolution

energy density

critical point

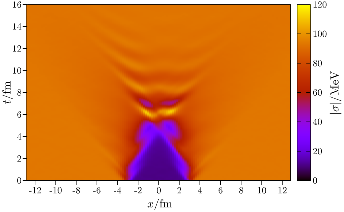


first order phase transition

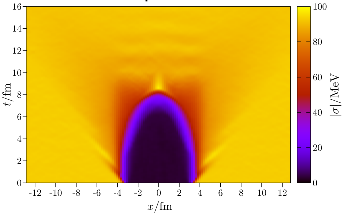


sigma field

critical point

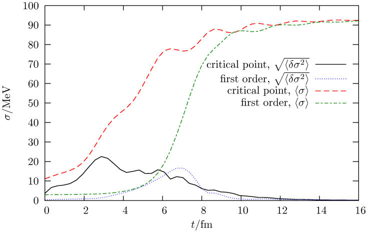


first order phase transition

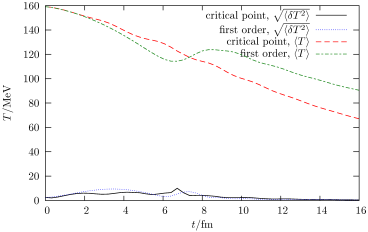


Reheating and supercooling

relaxation of the σ field



temperature



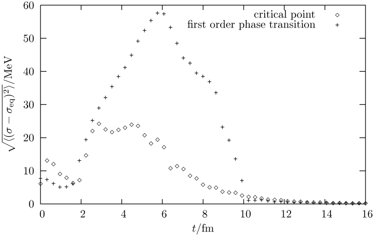
- oscillations at the critical point
- supercooling of the system at the first order phase transition
- reheating effect visible at the first order phase transition

Intensity of sigma fluctuations

$$\frac{\mathrm{d}N_\sigma}{\mathrm{d}^3k} = \frac{(\omega_k^2|\sigma_k|^2 + |\partial_t\sigma_k|^2)}{(2\pi)^3 2\omega_k}$$

$$\omega_k = \sqrt{|k|^2 + m_\sigma^2}$$

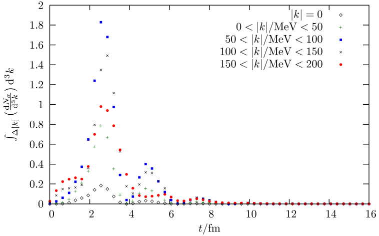
$$m_\sigma = \sqrt{\partial^2 V_{\mathrm{eff}} / \partial\sigma^2|_{\sigma=\sigma_{\mathrm{eq}}}}$$



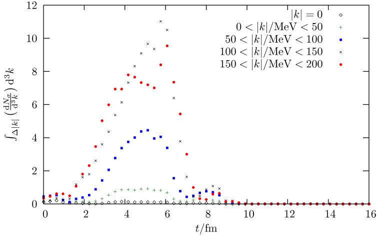
deviation from equilibrium

(MN, M. Bleicher, S. Leupold, I. Mishustin, arXiv:1105.1962)

critical point



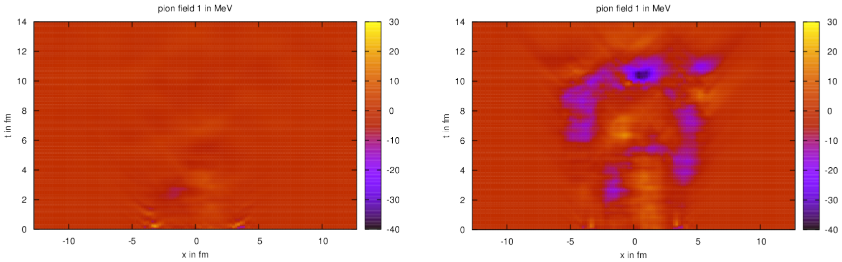
first order phase transition



Pion fluctuations

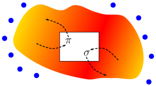
So far: pion fluctuations were not considered and $\vec{\pi} = \langle\vec{\pi}\rangle = 0$.
Now: extend the model to explicitly propagate pion fluctuations, too.

critical point | first order phase transition

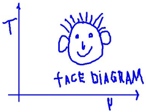


Larger fluctuations in the pionic field in a scenario with a first order phase transition $\Rightarrow$ potential formation of disoriented chiral condensates!

Summary



- chiral fluid dynamics including damping and noise
- energy-momentum conservation by the back reaction on the heat bath
- effects of supercooling, reheating, critical slowing down
- enhanced fluctuations at the first order phase transition



- Polyakov-loop extended chiral fluid dynamics ⇒ talk by Christoph Herold on Monday, TORIC meeting

In collaboration with: Marcus Bleicher, Stefan Leupold, Igor Mishustin, Carsten Greiner, Christoph Herold

Fluid dynamics

equation of state, pressure:

$$p(\sigma, T) = -V_{\text{eff}}(\sigma, T) + U(\sigma)$$

energy density:

$$e(\sigma, T) = T\frac{\partial p(\sigma, T)}{\partial T} - p(\sigma, T)$$

This relation is obtained from thermodynamic consistency, which is guaranteed by the 2PI effective action!

Initial conditions

temperature profile, $T_{\rm ini} = 160$ MeV:

$$T(\vec{x}, t=0) = \frac{T_{\rm ini}}{(1+\exp((\tilde{r}-\tilde{R})/\tilde{a}))(1+\exp(((|z|-l_z)/\tilde{a}))}$$

sigma field:

$$\sigma(\vec{x}, t=0) = \sigma_{\rm eq} + \delta\sigma(\vec{x})\,.$$

with

$$\langle \delta\sigma^2 \rangle = \frac{T}{V}\frac{1}{m_\sigma^2}\,.$$

energy density in units of e_0

$$e(\vec{x}, t=0) = e_{\rm eq}(T, \sigma)$$