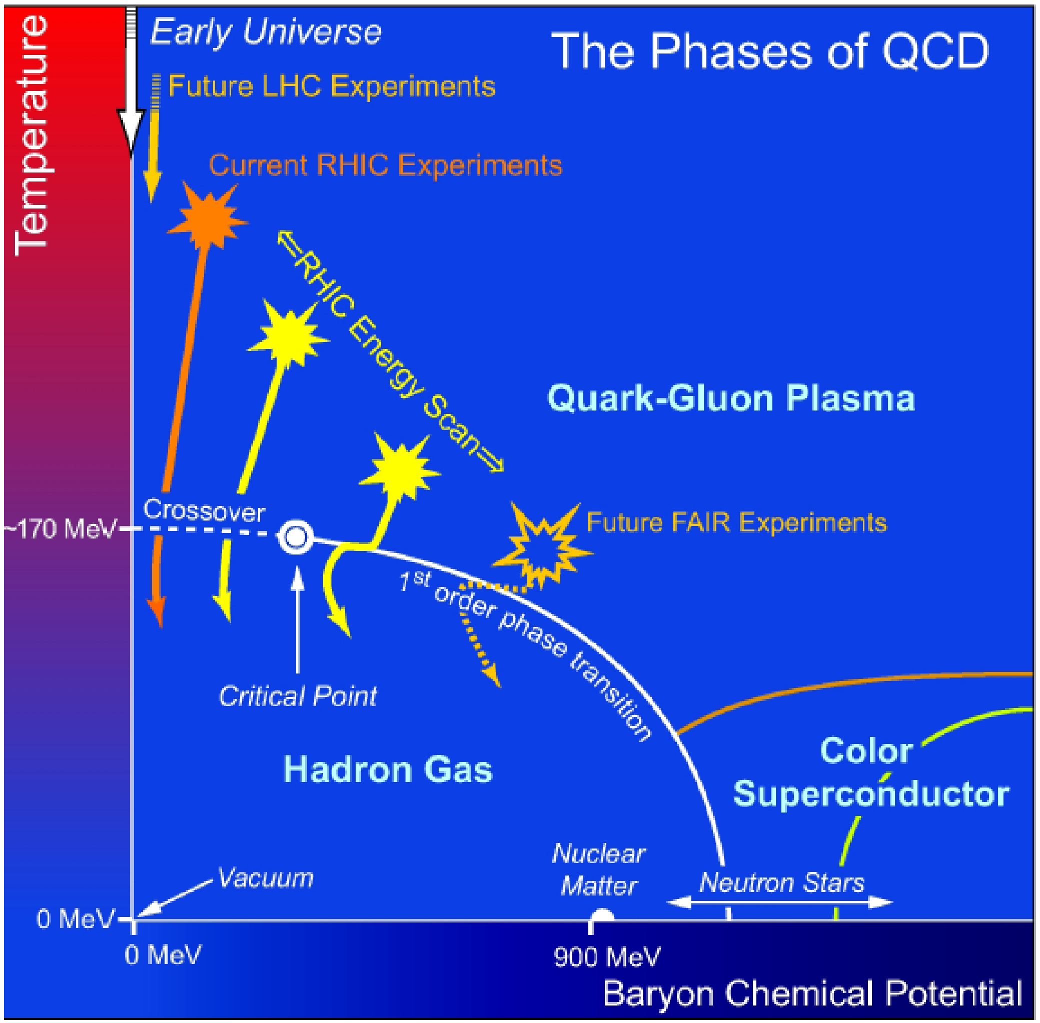
Probability distribution of conserved charges and the QCD phase transition



OUTLINE:

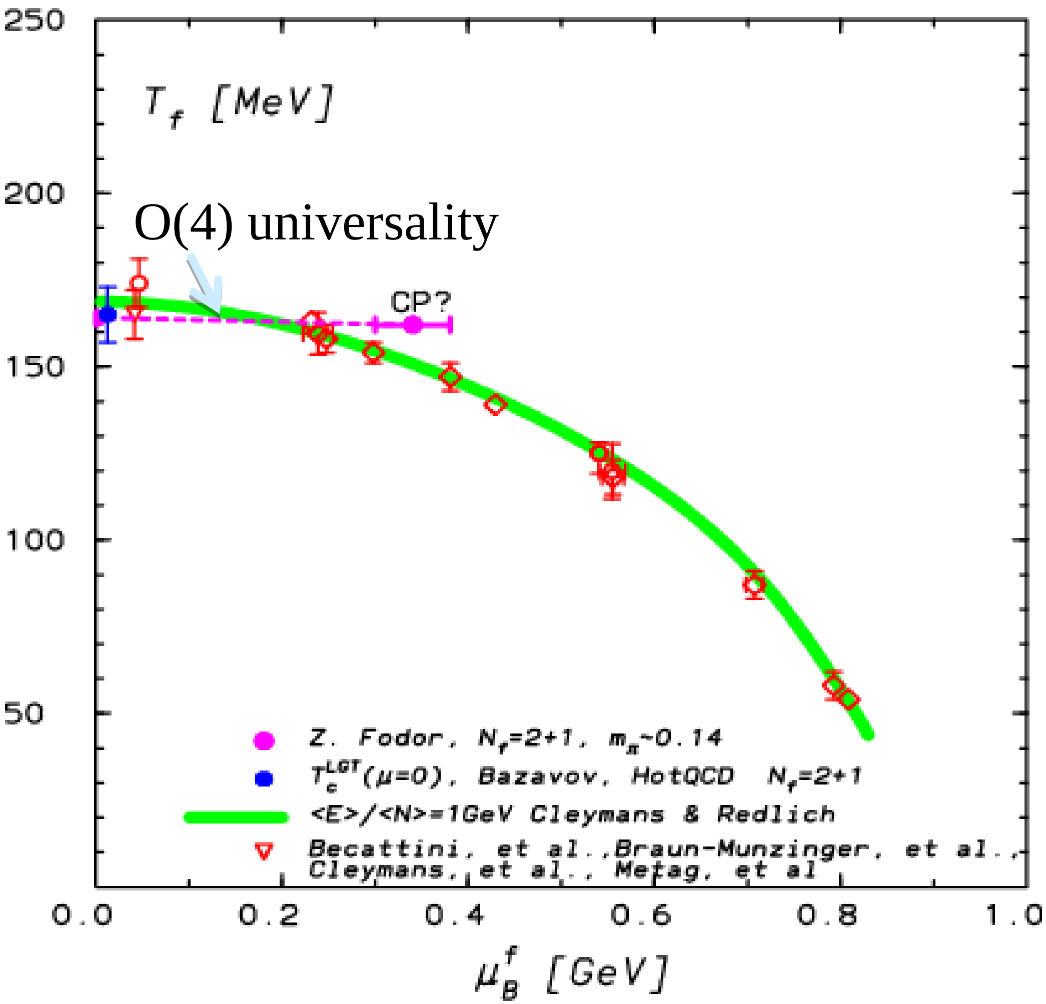
- QCD phase boundary, its O(4) scaling & relation to freezeout in HIC
- Moments and probablility distributions of conserved charges as probes for proximity to criticality
- STAR data & expectiations

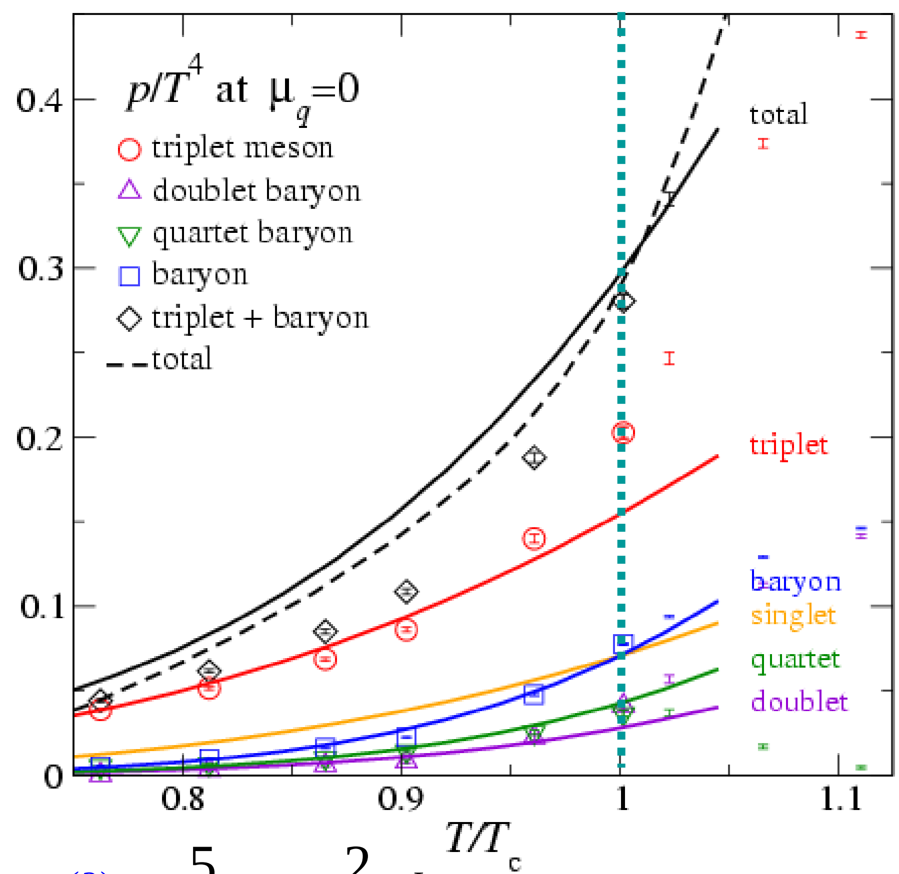
With: P. Braun-Munzinger, F. Karsch, B. Friman & V. Skokov

- Particle yields and their ratio, as well as LGT results are well described by the Hadron Resonance Gas Partition Function .

S. Ejiri, F. Karsch & K.R.

$$P = G_M^{(1)} + G_M^{(3)} + F_B^{(2)} + F_B^{(4)}$$





$$F_B^{(2)} = \frac{5}{18} c_2^q - \frac{2}{3} c_4^I$$

O(4) scaling and critical behavior

- Near T_c critical properties obtained from the singular part of the free energy density

$$F = F_{reg} + F_S$$

with $F_S(t,h) = b^{-d} F(b^{1/\nu} t, b^{\beta\delta/\nu} h)$

h: external field and

$$t = \frac{T - T_c}{T_c} + \kappa \left(\frac{\mu}{T_c} \right)^2$$

- Phase transition encoded in the "equation of state"

$$< \sigma > = -\frac{\partial F_s}{\partial h} \Rightarrow$$

$$< \sigma > = h^{1/\delta} F_h(z) \ , \quad z = t h^{-1/\beta\delta}$$

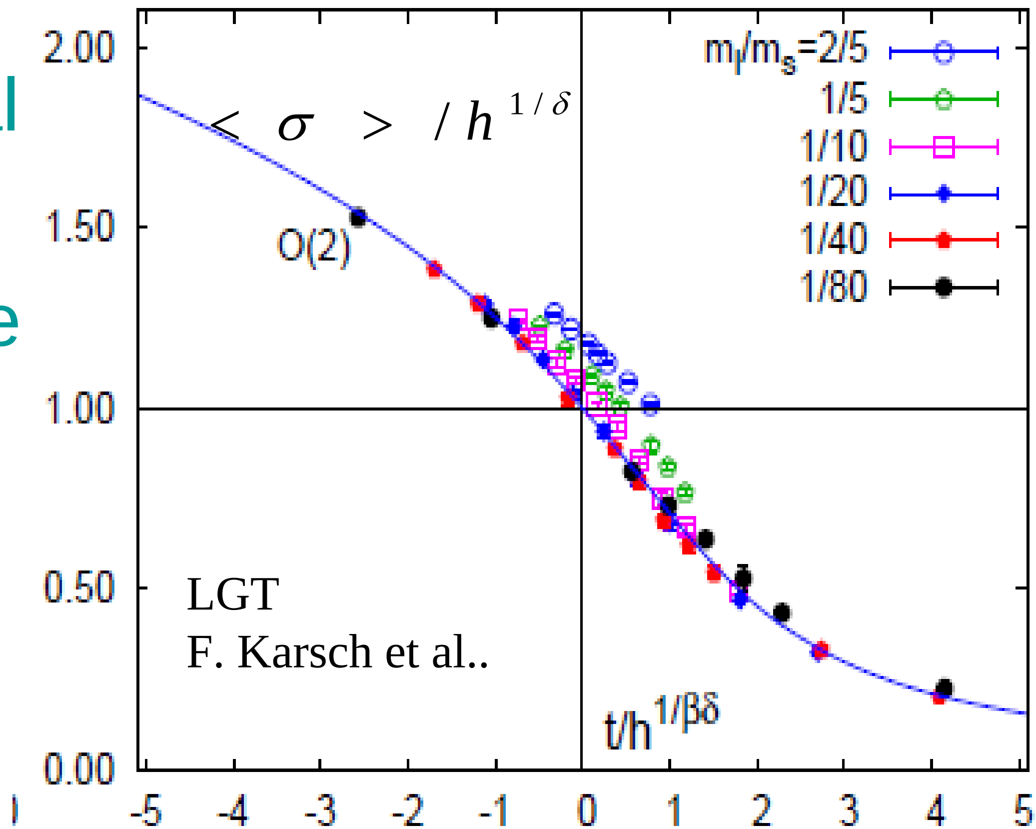
$$< \sigma > = | t |^{\beta} \ F_s^{'}(h | t |^{-\beta\delta})$$

- Resulting in the well known scaling behavior of $< \sigma >$

$$< \sigma > = \begin{cases} B(-t)^{\beta}, h = 0, \ t < 0 & \text{coexistence line} \\ B h^{1/\delta} \ , \ t = 0, \ h > 0 & \text{pseudo-critical point} \end{cases}$$

2+1 Flavor QCD with physical quark masses is sensitive to O(4) chiral dynamics expected in gauge theory with only two u and d quarks

- Change of the order chiral order parameter follows magnetic equation of state with scaling function of the O(2)/O(4) universality class



O(4) scaling of net-baryon number fluctuations

- The fluctuations are quantified by susceptibilities

$$\chi_B^{(n)} = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} := c_n \qquad \text{with} \qquad P = P_{reg} + P_{Singular}$$

- From free energy and scaling function one gets

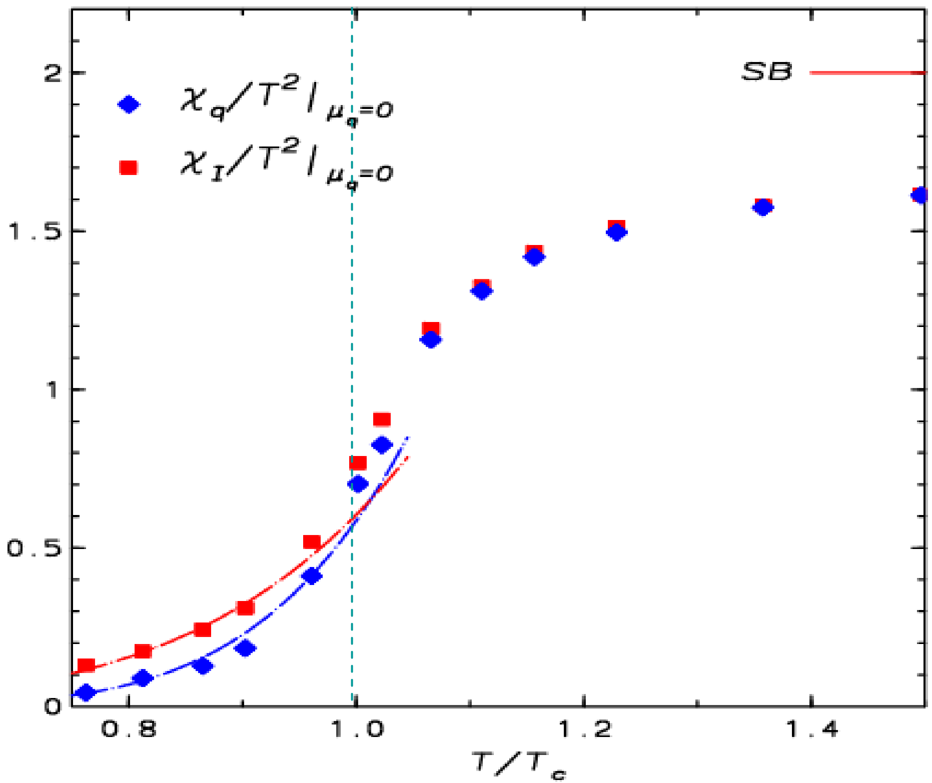
$$\chi_B^{(n)} \approx \chi_r^{(n)} + c\, h^{2-\alpha-n/2} f_{\pm}^{(n/2)}(z) \quad for \;\; \mu = 0 \;\; and \;\; n \;\; even$$

$$\chi_B^{(n)} \approx \chi_r^{(n)} + c_\mu\, h^{2-\alpha-n} f_{\pm}^{(n)}(z) \quad for \;\; \mu \neq 0$$

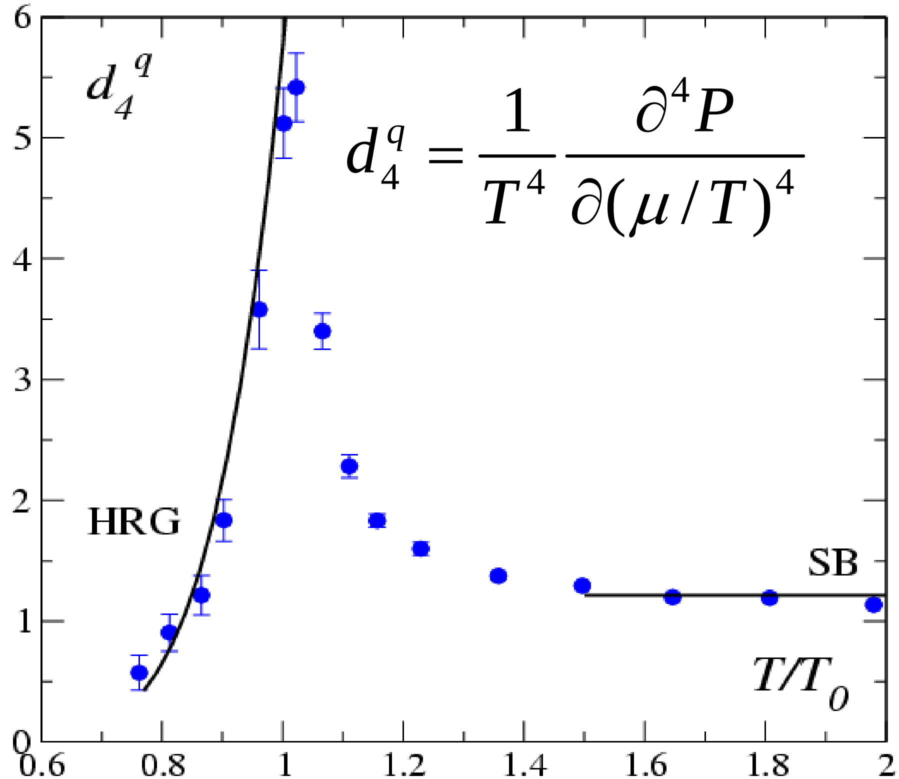
- Resulting in singular structures in n-th order moments which appear for $n \geq 6 \;\; at \;\; \mu = 0$ and for $n \geq 3 \;\; at \;\; \mu \neq 0$ since $\alpha \approx -0.2$ in O(4) univ. class

LGT and phenomenological HRG model

C. Allton et al.,



S. Ejiri, F. Karsch & K.R.



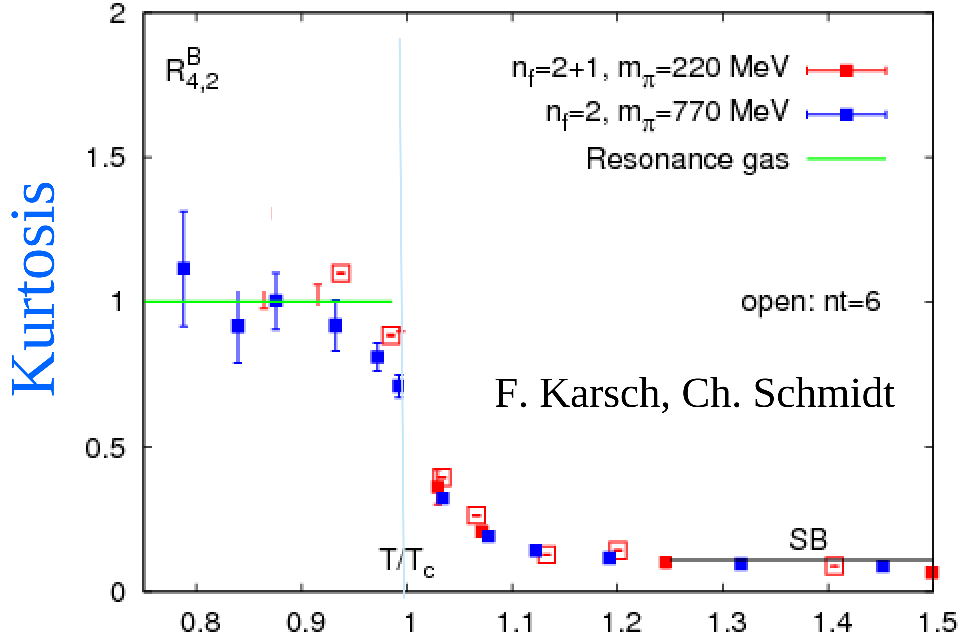
- Smooth change of $\chi_{q(I)}$ and peak in d_q^4 at T_c expected from O(4) universality argument and HRG

- For $T < T_c$ fluctuations as expected in the Hadron Resonance Gas

Kurtosis as an excellent probe of deconfinement

S. Ejiri, F. Karsch & K.R.

$$R_{4,2}^{B} = \frac{1}{9}\frac{c_4}{c_2}$$



The $R_{4,2}^{B}$ measures the quark content of particles carrying baryon number

- HRG factorization of pressure:

$$P^B(T,\mu_q) = F(T)\cosh(3\mu_q / T)$$

consequently: $c_4 / c_2 = 9$ in HRG

- In QGP, $SB = 6/\pi^2$
- Kurtosis=Ratio of cumulants

$$c_4^q / c_2^q = \frac{<(\delta N_q)^4>}{<(\delta N_q)^2>} - 3<(\delta N_q)^2>$$

excellent probe of deconfinement

Effective chiral model and is non-perturbative thermodynamics: Renormalisation Group Approach

$$S = \int_0^{\beta=1/T} d\tau \int_V d^3x [i\bar{q}(\gamma_\mu \partial_\mu - A_\mu \delta_{\mu 4})q - V^{\mathrm{int}}(q,\bar{q}) + \mu_q q^+ q - U(L,L^*)]$$

$U(L,L^*) -$ the $Z(3)$ invariant Polyakov loop potential

$V^{\mathrm{int}}(q,\bar{q}) -$ the SU(2)xSU(2) $\chi -$invariant quark interactions described through:

- Nambu-Jona-Lasinio model ⟹ PNJL chiral model
  K. Fukushima; C. Ratti & W. Weise; B. Friman , C. Sasaki ., ….
- coupling with meson fileds ⟹ PQM chiral model
  B.-J. Schaefer, J.M. Pawlowski & J. Wambach; B. Friman, V. Skokov, ...
- FRG thermodynamics :
  B. Friman, V. Skokov, B. Stokic & K.R.

Kurtosis of net quark number density in PQM model

V. Skokov, B. Friman &K.R.

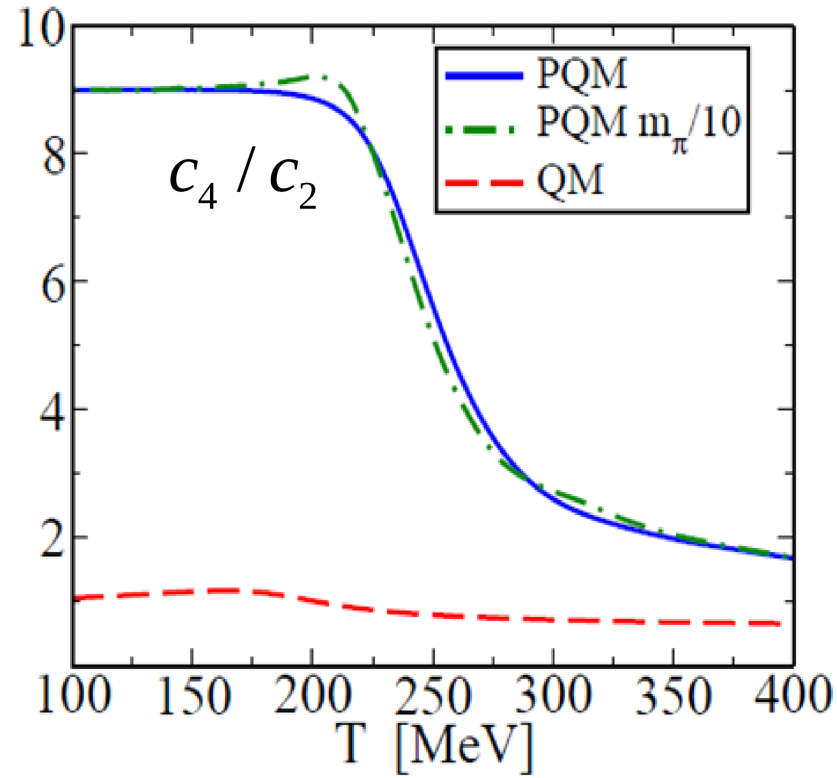
- For $T < T_c$

  the assymptotic value →

  due to „confinement" properties

$$\frac{P_{q\bar{q}}(T)}{T^4} \approx \frac{2N_f}{N_c^2}\left(\frac{3m_q}{T}\right)^2 K_2\left(\frac{3m_q}{T}\right)\cosh\frac{3\mu_q}{T}$$

⇨ $c_4 / c_2 = 9$



- For $T >> T_c$

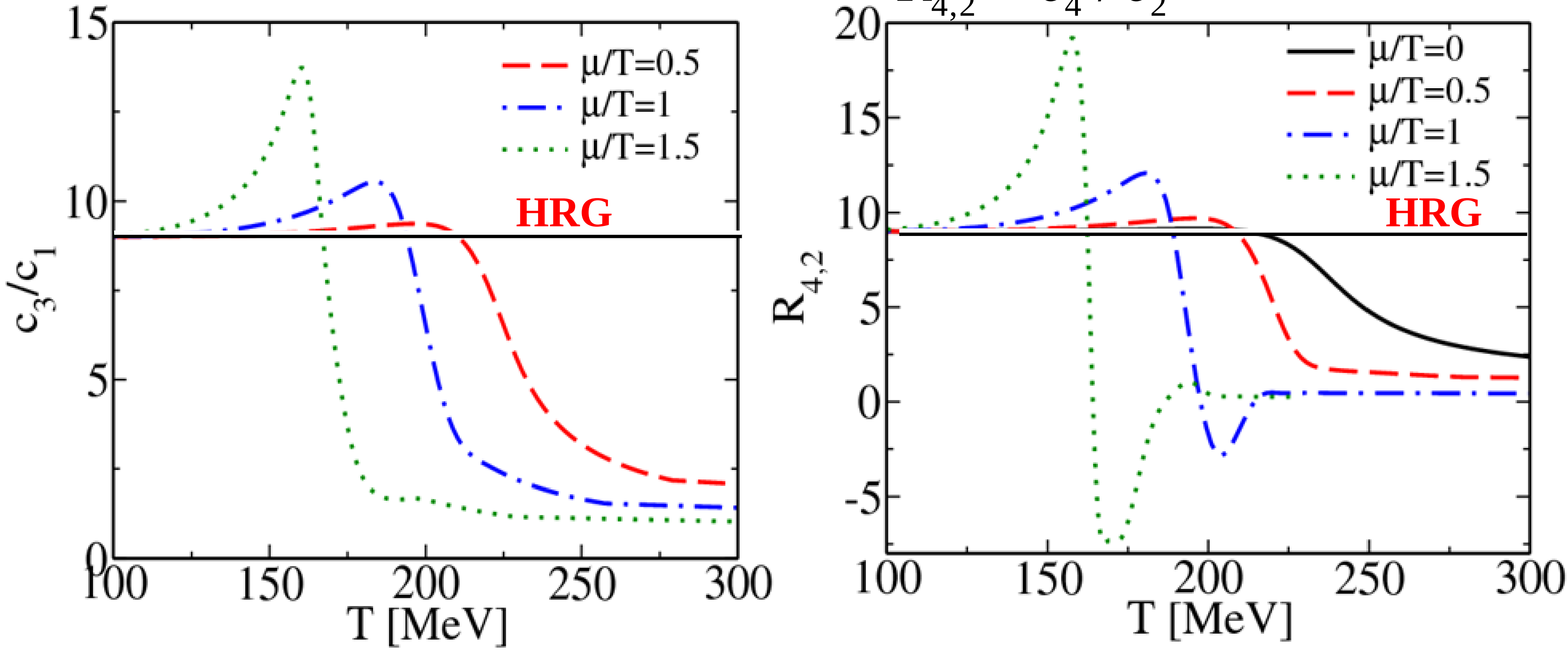
$$\frac{P_{q\bar{q}}(T)}{T^4} = N_f N_c \left[\frac{1}{12\pi^2}\left(\frac{\mu}{T}\right)^4 + \frac{1}{6}\left(\frac{\mu}{T}\right)^2 + \frac{7\pi^2}{180}\right]$$

⇨ $c_4 / c_2 = 6/\pi^2$

- Smooth change with a rather weak dependen-ce on the pion mass

Ratio of cumulants at finite density

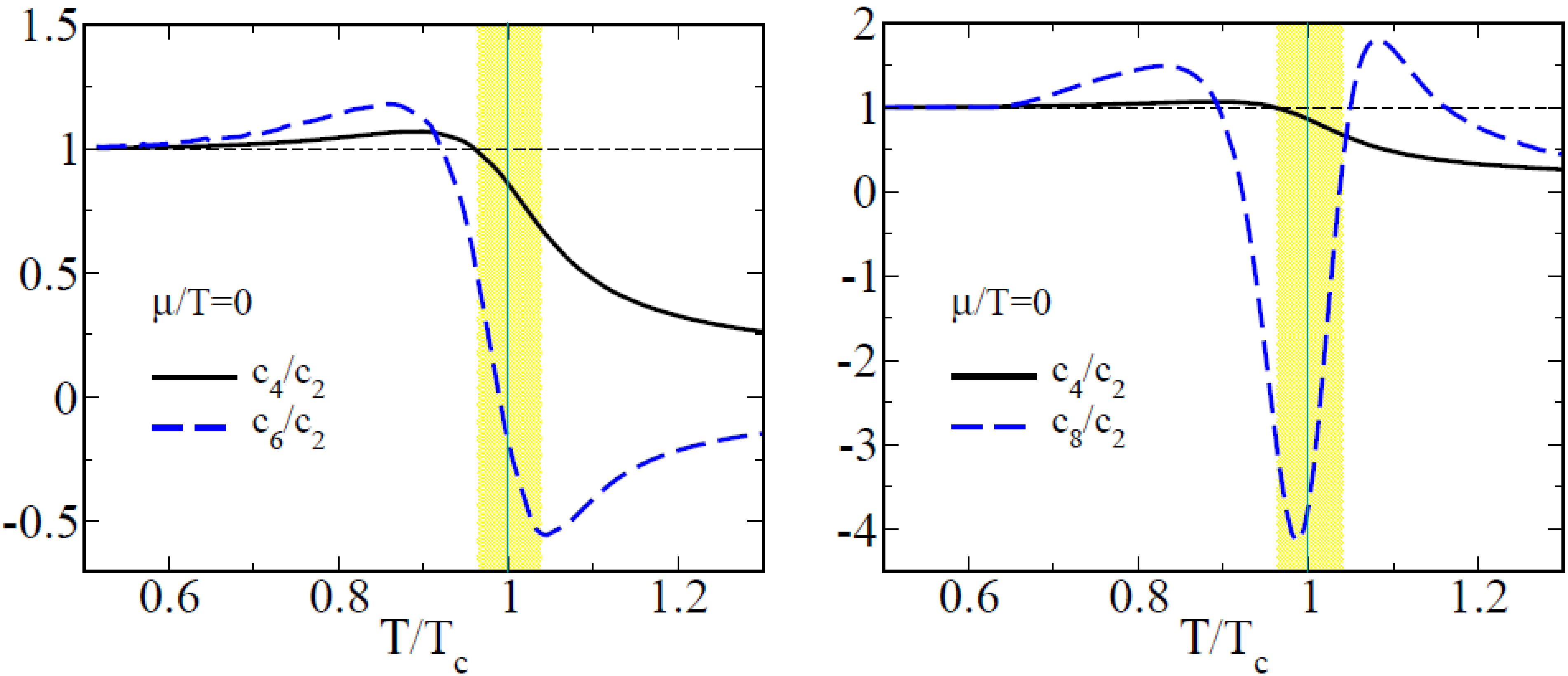
$R_{4,2} = c_4 / c_2$



Deviations of the ratios of odd and even order cumulants from their asymptotic, low T-value, $c_4 / c_2 = c_3 / c_1 = 9$ **are increasing with** μ / T **and the cumulant order**

Properties essential in HIC to discriminate the phase change by measuring baryon number fluctuations !

Ratio of higher order cumulants



Deviations of the ratios from their asymptotic, low T-value, are increasing with the order of the cumulant

Properties essential in HIC to discriminate the phase change by measuring baryon number fluctuations !

Higher moments of charge fluctuations at RHIC and LHC

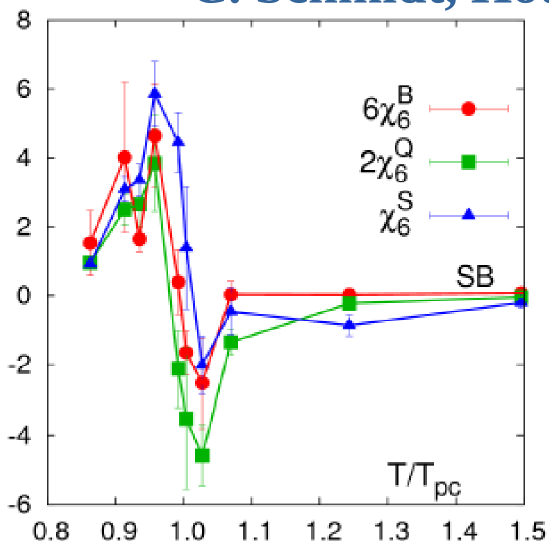
- higher moments (e.g. 6th order) are drastically different in QCD close to criticality and in a hadron resonance gas, e.g.

$\mu_B = 0$

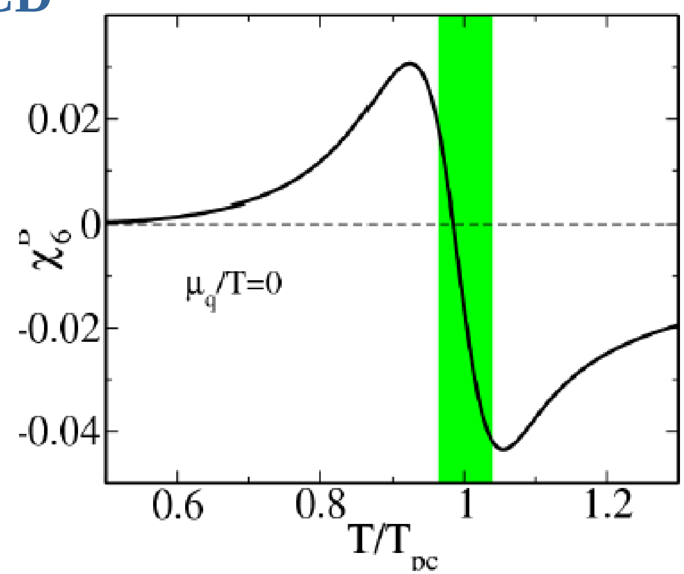
$$\frac{\chi_{B,0}^{(6)}}{\chi_{B,0}^{(2)}} = \begin{cases} = 1 & \text{, hadron resonance gas} \\ < 0 & \text{, QCD at the crossover transition} \end{cases}$$

LGT: $16^3 \times 4$ (p4)

C. Schmidt, HotQCD



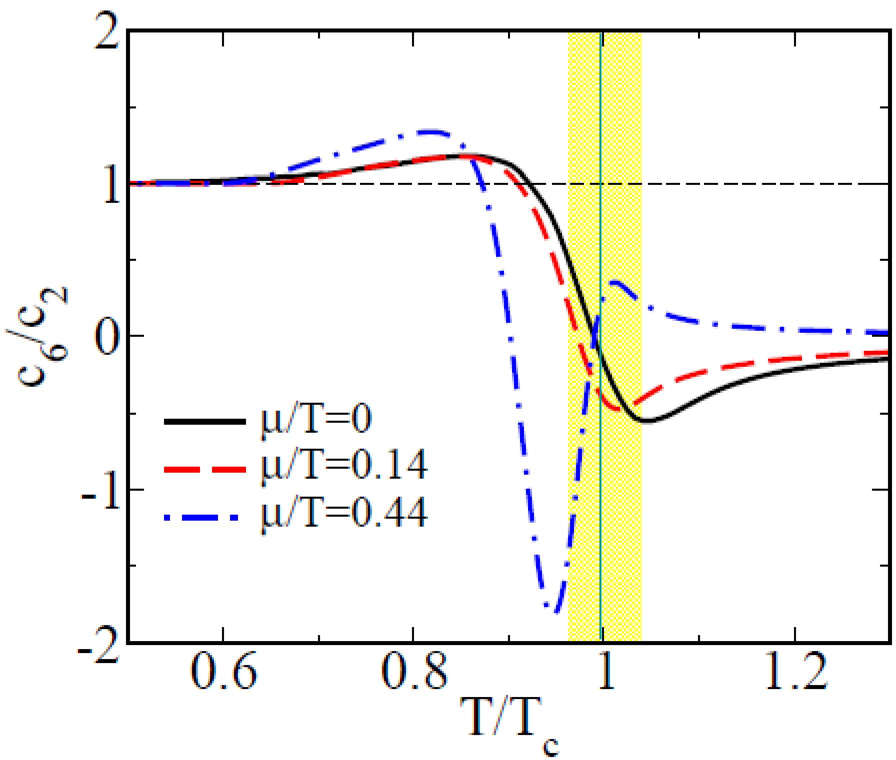
PQM model

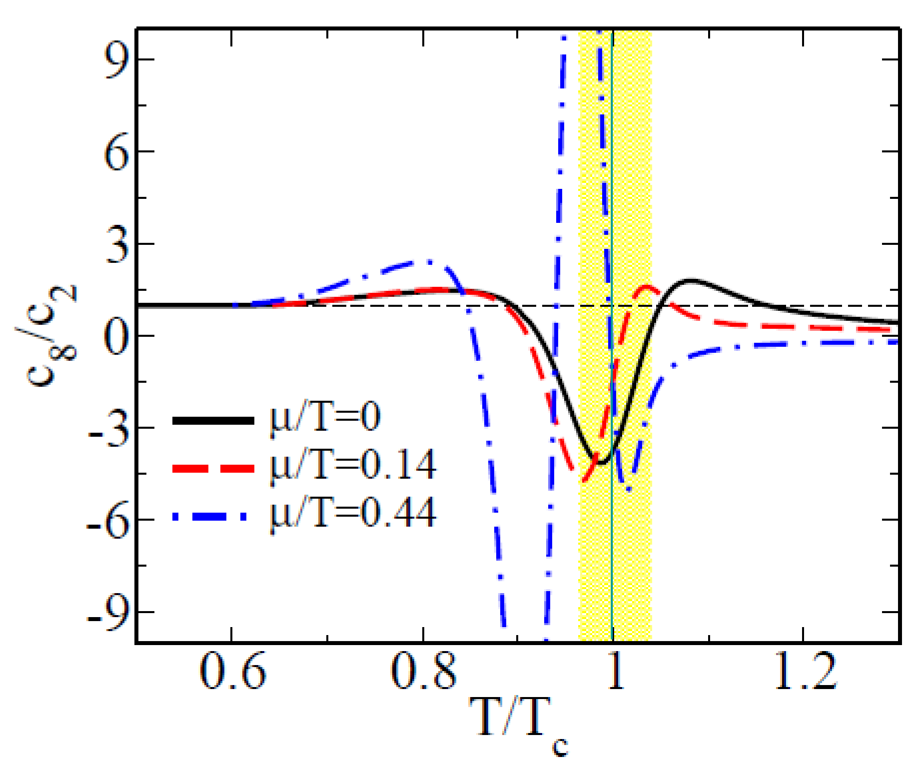


PQM model and LGT calculations reproduce expected O(4) scaling structure

B. Friman et al, arXiv:1103.3511

Ratio of higher order cumulants at finite density



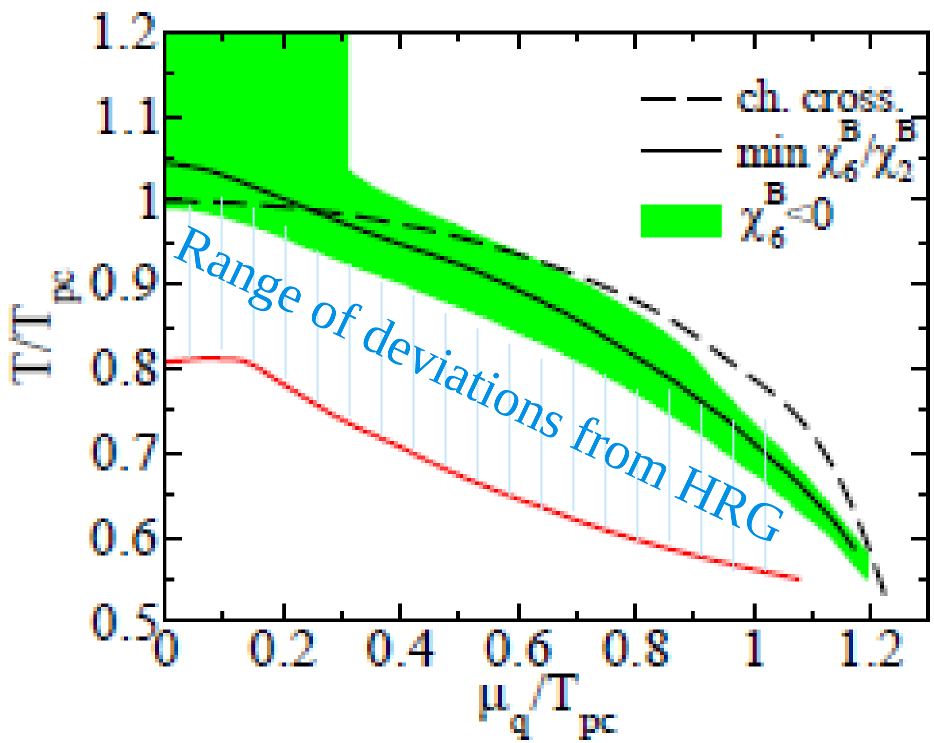


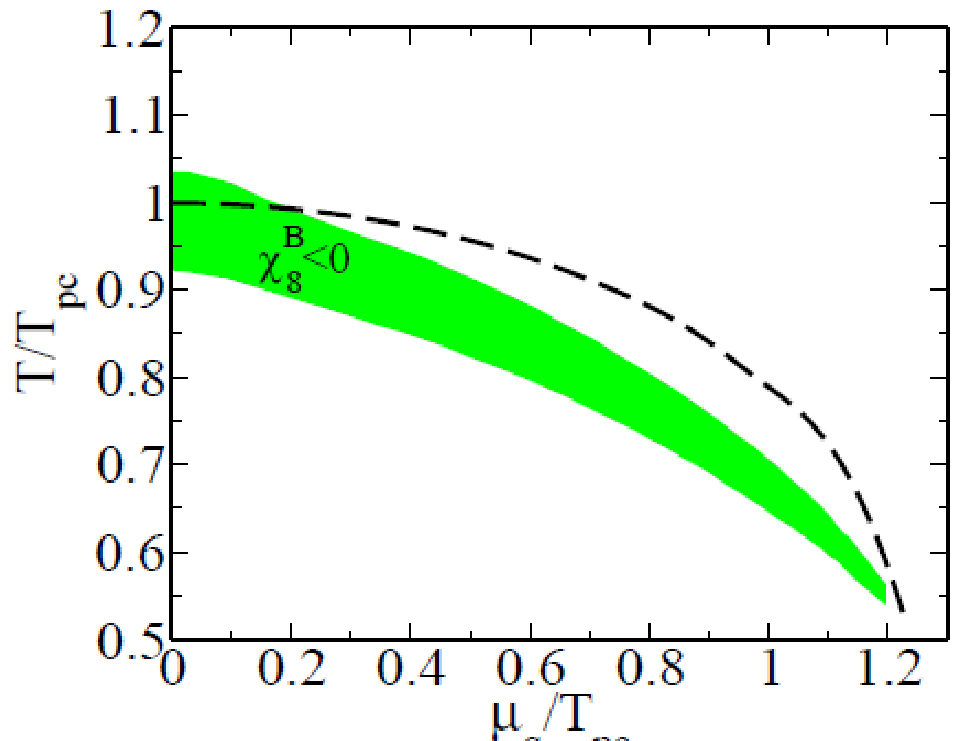
Deviations of the ratios from their asymptotic, low T-value, are increasing with the order of the cumulant order and with increasing chemical potential

Properties essential in HIC to discriminate the phase change by measuring baryon number fluctuations !

Fluctuations of 6th and 8th order moments exhibit strong variations from HRG results:

Their negative values near chiral transition to be seen in heavy ion collisions at LHC , RHIC & CBM



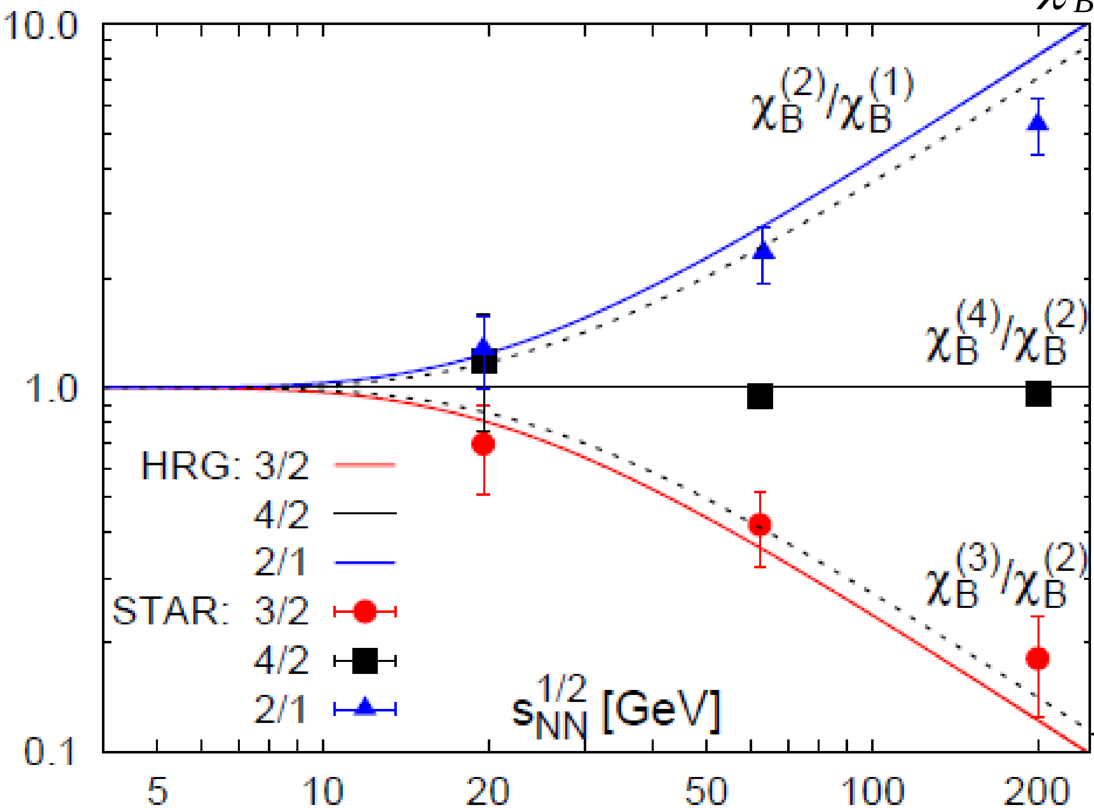


The range of negative fluctuations near chiral cross-over: PNJL model results with quantum fluctuations being included : These properties are due to O(4) scaling , thus should be also there in QCD.

Coparison of the Hadron Resonance Gas Model with STAR data

- Frithjof Karsch & K.R.

$$\frac{\chi_B^{(2)}}{\chi_B^{(1)}} \approx \coth(\mu_B / T)$$



$$\frac{\chi_B^{(4)}}{\chi_B^{(2)}} = 1$$

$$\frac{\chi_B^{(3)}}{\chi_B^{(2)}} \approx \tanh(\mu_B / T)$$

- RHIC data follow generic properties expected within HRG model for different ratios of the first four moments of baryon number fluctuations

deviations between HRG model and data for the variance ($\chi_B^{(2)}$)?

Moments obtained from probability distributions

- Moments obtained form probability distribution

$$< N^k >= \sum_N N^k P(N)$$

- Probability quantified by all moments

$$P(N) = \frac{1}{2\pi} \int_0^{2\pi} dy \exp[iyN - \chi(iy)]$$

Moments generating function: $\chi(y) = \beta V[p(T, y+\mu) - p(T, \mu)] = \sum_k \chi_k y^k$

P(N) in the Hadron Resonance Gas

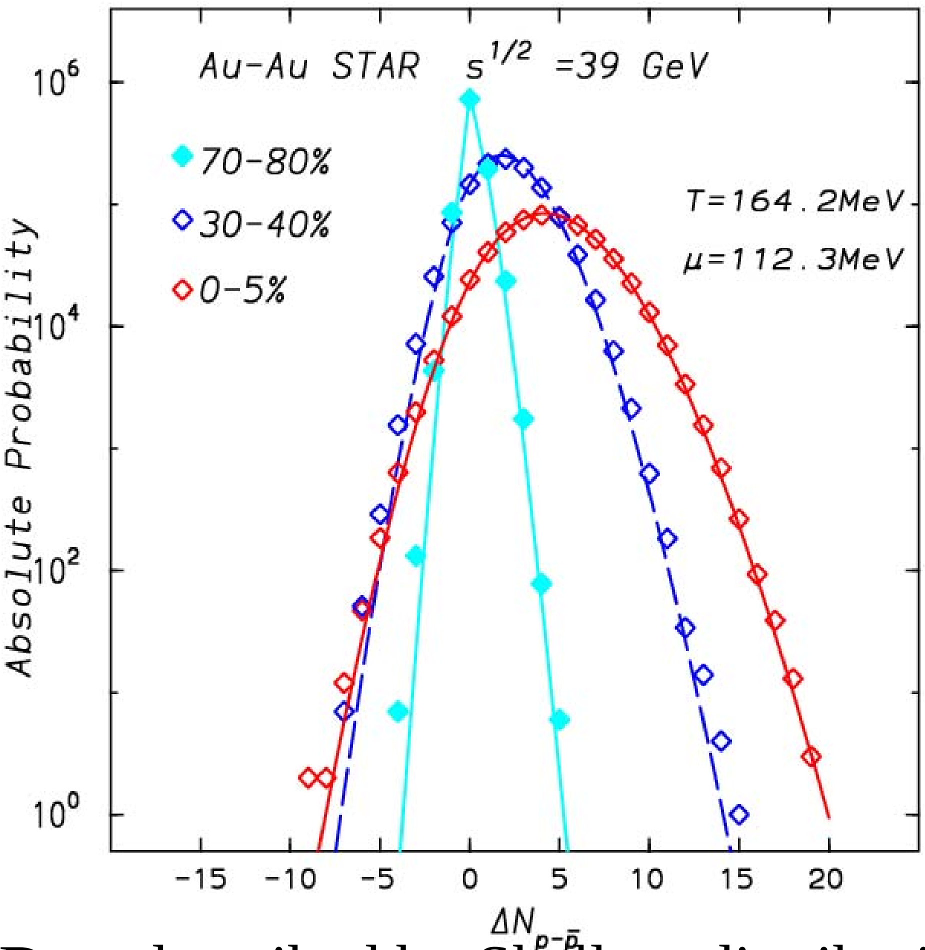
The probability distribution for net baryon number N is governed in HRG by the Skellam distribution

$$P(N) = \left(\frac{b}{\bar{b}}\right)^{N/2} I_N(2\sqrt{b\bar{b}}\,)\,\exp[-(b+\bar{b})]$$

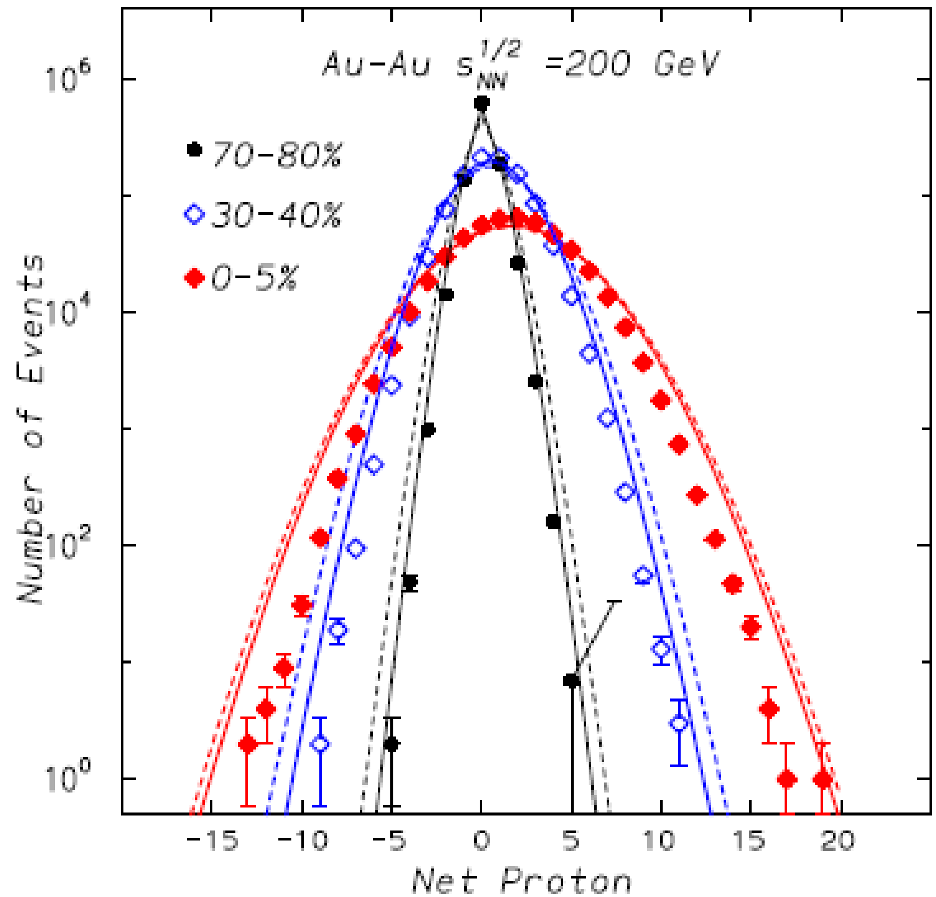
The probability distribution for net baryon number N is entirely given in term of (mesurable) mean number of baryons b and anti-baryons $\bar{b}$

Centrality dependence at $\sqrt{s}_{NN} = 39$ and 200 GeV

Preliminary STAR data



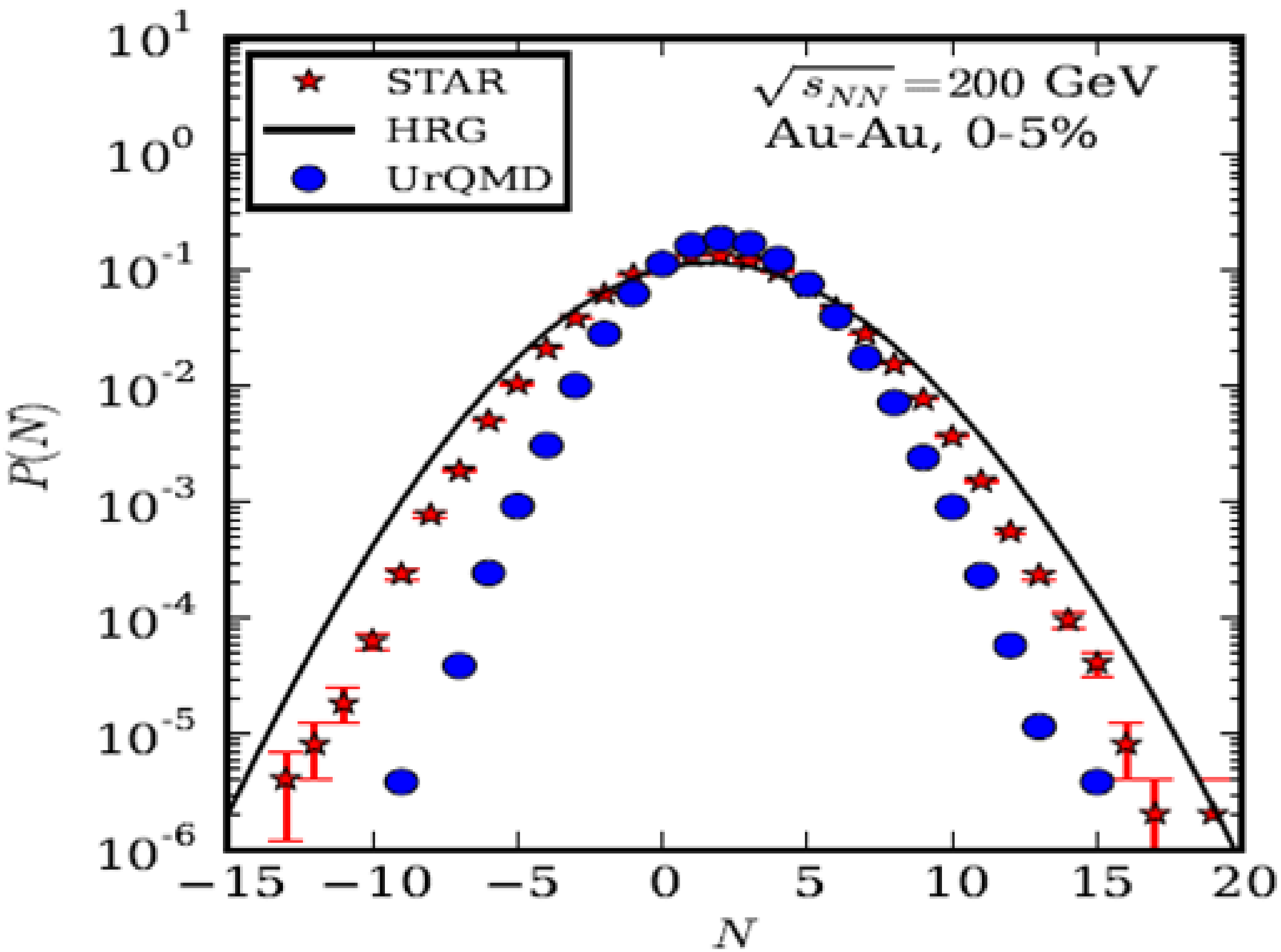
Published STAR data-efficiency uncorected



Data described by Skellam distribution

HRG shows broader distribution

Data versus UrQMD

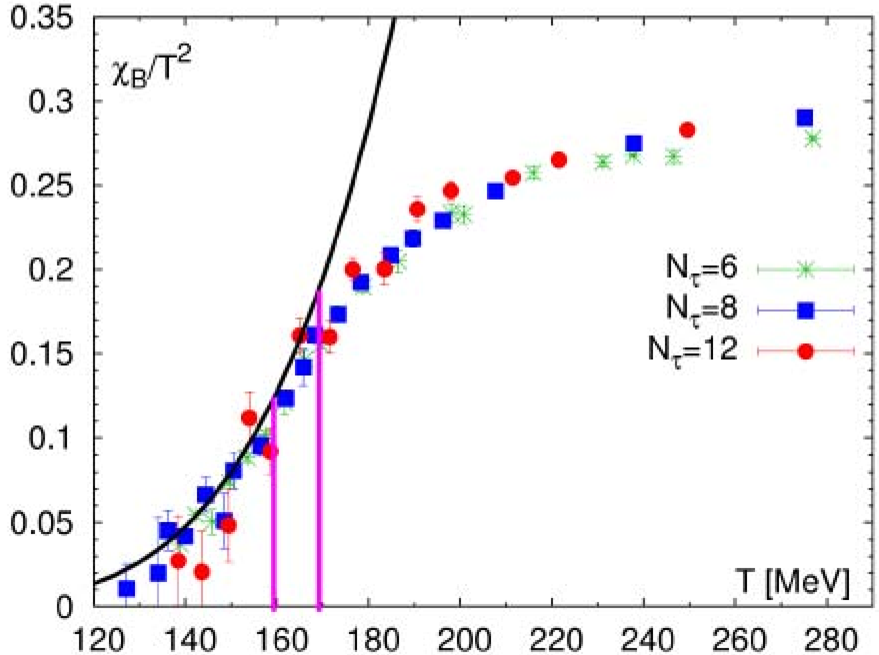


- UrQMD provides much narrower probability distribution of the net proton number

Net-baryon number susceptibility

P(N) may be expressed in terms of a cumulant expansion; leading order corresponds to Gaussian approximation:

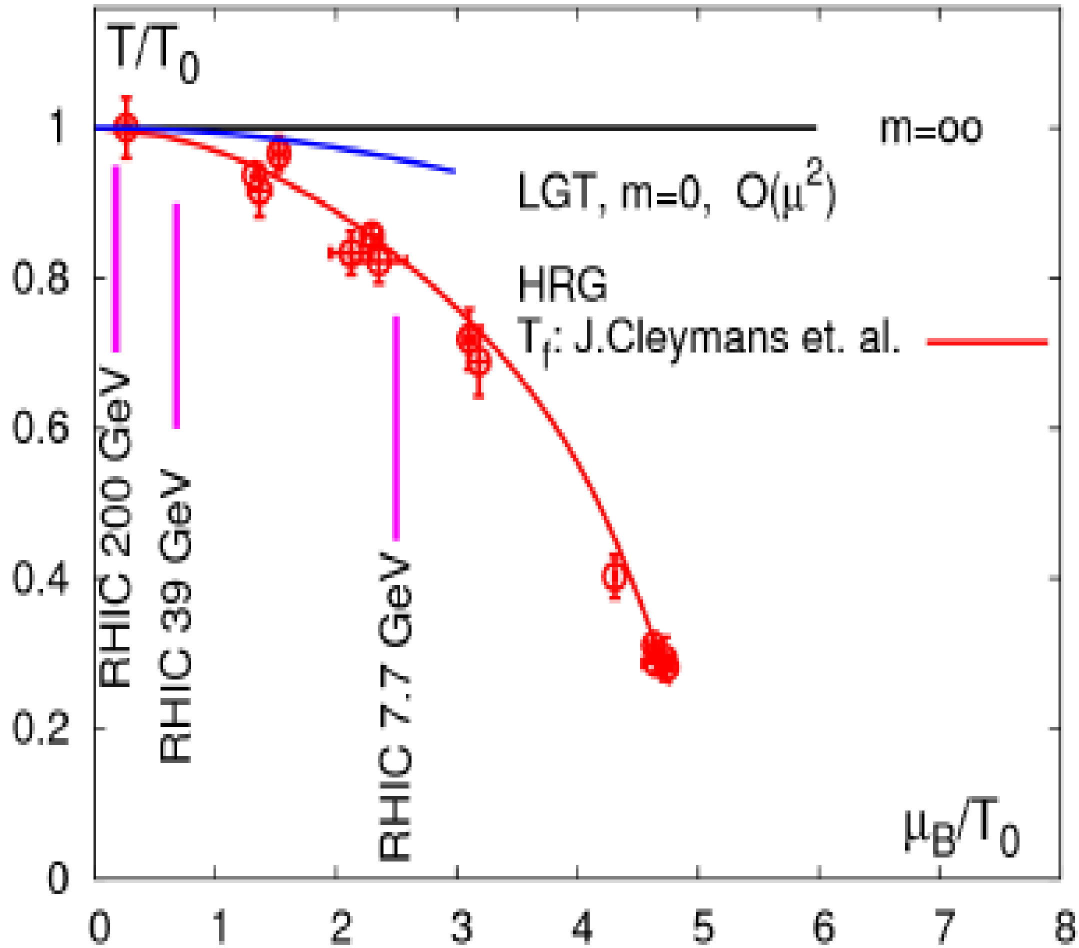
$$P(N) \sim \exp[-N^2/(2VT^3\chi_B^{(2)})]$$



HISQ action:
hotQCD, preliminary

in the transition region the probability distribution derived in leading order from QCD may become narrower than the HRG model result

Conclusions:



freeze-out line and chiral phase transition

J. Cleymans et al, PRC73, 034905 (2006)

BNL-Bielefeld-GSI, arXiv:1011.3130

- Higher order moments and probabilty distributions are excelent probes of criticality in HIC
- The 6th and 8th order moments cen be negative already at the LHC
- Deviation of P(N) sets in at $\sqrt{s} > 39 GeV$ and increses with centrality:

remnant of O(4) criticality

This might indicate that critical line decouples from freezeout line near this energy:?

The above need to be verified with efficiency corrected data !