I. HYDJET++ model (hydro + jets)

II. Model results for the ratio $v^4/(v^2)^2$ at RHIC and LHC

III. NCQ-scaling at RHIC and LHC
I. HYDJET++ = FASTMC + HYDJET
The soft part of HYDJET++ event represents the "thermal" hadronic state.
✓ multiplicities are determined assuming thermal equilibrium
✓ hadrons are produced on the hypersurface represented by a parameterization of relativistic hydrodynamics with given freeze-out conditions
✓ chemical and kinetic freeze-outs are separated
✓ decays of hadronic resonances are taken into account (360 particles from SHARE data table) with “home-made” decayer

the model reproduces soft hadroproduction features at RHIC (particle spectra, elliptic flow, HBT)

The hard, multi-partonic part of HYDJET++ event is identical to the hard part of Fortran written HYDJET (PYTHIA6.4xx + PYQUEN1.5) => now PYTHIA Perugia 2011 tune!!
PYQUEN event generator is used for simulation of rescattering, radiative and collisional energy loss of hard partons in expanding quark-gluon plasma created in ultrarelativistic heavy ion AA collisions. HYDJET++ includes nuclear shadowing correction for parton distributions (important at LHC!)
Model parameters.

1. Thermodynamic parameters at chemical freeze-out: $T_{ch}$, $\{\mu_B, \mu_S, \mu_Q\}$
2. If thermal freeze-out is considered: $T_{th}$, $\mu_\pi$-normalisation constant
3. Volume parameters: $T$, $\Delta T$, $R$
   1. $\rho^\max$ - maximal transverse flow rapidity for Bjorken-like parametrization
   5. $\eta^\max$ - maximal space-time longitudinal rapidity which determines the rapidity interval $[-\eta^\max, \eta^\max]$ in the collision center-of-mass system.
4. Impact parameter range: minimal $b_{\min}$ and maximal $b_{\max}$ impact parameters
5. Flow anisotropy parameters $\delta(b)$, $\epsilon(b)$

PYTHYA+PYQUEN obligatory parameters

9. Beam and target nuclear atomic weight $A$
10. $\sqrt{S_{NN}}$ - c.m.s. energy per nucleon pair (PYTHIA initialization at given energy)
11. $p_{t\min}$ - minimal pt of parton-parton scattering in PYTHIA event (ckin(3) in /pysubs/)
12. $nhsel$ flag to include jet production in hydro-type event:
   0 - jet production off (pure FASTMC event),
   1 - jet production on, jet quenching off (FASTMC+njet*PYTHIA events),
   2 - jet production & jet quenching on (FASTMC+njet*PYQUEN events),
   3 - jet production on, jet quenching off, FASTMC off (njet*PYTHIA events),
   4 - jet production & jet quenching on, FASTMC off (njet*PYQUEN events);
13. $ishad$ flag to switch on/off nuclear shadowing
Parameters of energy loss model in PYQUEN
(default, but can be changed from the default values by the user)

1. T0 - initial temperature of quark-gluon plasma for central Pb+Pb collisions at mid-rapidity (initial temperature for other centralities and atomic numbers will be calculated automatically)
at LHC: T0=1 GeV, at RHIC(200 AGeV) T0=0.300 GeV

2. tau0 - proper time of quark-gluon plasma formation
at LHC: tau0=0.1 fm/c, at RHIC(200 AGeV) tau0=0.4 fm/c

3. nf - number of active quark flavours in quark-gluon plasma
(nf=0, 1, 2 or 3) at LHC: nf=0, at RHIC(200 AGeV) nf=2

4. ienglu - flag to fix type of medium-induced partonic energy loss
(ienglu=0 - radiative and collisional loss, ienglu=1 - radiative loss only, ienglu=2 - collisional loss only, default value is ienglu=0);
iangлу - flag to fix type of angular distribution of emitted gluons
(iangлу=0 - small-angular, iangлу=1 - wide-angular, iangлу=2 - collinear, default value is iangлу=0).
ienglu=0
RHIC DATA VS. HYDJET++ MODEL

Au+Au @ 200 AGeV

Elliptic flow

G. Eyyubova et al., PRC 80 (2009) 064907; N.S. Amelin et al., PRC 77 (2008) 014903
$V_2$ in HYDJET++ for different particles (centrality 30%)

Hydrodynamics gives mass ordering of $v_2$. The model possesses crossing of baryon and meson branches.

Why?

Hydrodynamics and jets

Interplay between hydrodynamics and jets
The $p_T$ spectra of $\pi$, $K$, $p$, $\Lambda$ with HYDJET++ model, $\sqrt{s}=200\text{GeV}$

The slope for the hydro part depends strongly on mass:
- the heavier the particle -- the harder the spectrum

The hydro part dies out earlier for light particles than for heavy ones
LHC DATA VS. HYDJET++ MODEL

Transverse momentum

Pb+Pb @ 2.76 ATeV

Rapidity

Correlation radii (femtoscopy)

I. Lokhtin et al., arXiv:1204.4820
LHC DATA VS. HYDJET++ MODEL

Pb+Pb @ 2.76 ATeV

Elliptic flow

Model gives a fair description of various observables at both RHIC and LHC

I. Lokhtin et al., arXiv:1204.4820
II. $\frac{V_4}{(V_2*V_2)}$ RATIO
II. \( \frac{\nu^4}{(\nu^2)^2} \) ratio

Anisotropic flow

\[
\frac{dN}{d\varphi} = \frac{1}{2\pi} \left(1 + \sum_{n=1}^{\infty} 2\nu_n(p_t) \cos[n(\varphi - \psi_r)]\right)
\]
Predictions


• Within the approximation that the particle momentum $p$ and the fluid velocity $v$ are parallel (valid for large momentum $p_t$ and low freeze-out temperature $T$)

$$\frac{dN}{d\varphi} = \exp\left(2\varepsilon p_t \cos(2\varphi)/T\right)$$

• Expanding to order $\varepsilon$, the $\cos(2\varphi)$ term is

$$v_2 = \varepsilon p_t / T$$

• Expanding to order $\varepsilon^2$, the $\cos(4\varphi)$ term is

$$v_4 = \frac{1}{2} (v_2)^2$$

Hydrodynamics has a universal prediction for $v_4/(v_2)^2$!

Should be independent of equation of state, initial conditions, centrality, rapidity, particle type
The ratio is significantly larger than 0.5. Can this be explained by viscous corrections?
Viscosity lowers $v_4 / (v_2)^2$ for a realistic $T_f$. Initial profile has little effect although eccentricities differ. Results strongly depend on viscosity.
Eccentricity fluctuations can be computed in MC Glauber model or derived from experiment by comparing different methods for flow calculation.
Why $\varepsilon$ fluctuations change $v_4/v_2^2$

Experimentally, no direct measure of $v_2$ and $v_4$

$v_2$ and $v_4$ are measured via azimuthal correlations

$v_2$ from \[ \langle \cos(2\phi_1 - 2\phi_2) \rangle = \langle (v_2)^2 \rangle \]

$v_4$ from \[ \langle \cos(4\phi_1 - 2\phi_2 - 2\phi_3) \rangle = \langle v_4(v_2)^2 \rangle \]

Experimentally measured

\[
\frac{v_4}{v_2^2} = \frac{\langle v_4(v_2)^2 \rangle}{\langle (v_2)^2 \rangle^2} = \frac{1}{2} \frac{\langle (v_2)^4 \rangle}{\langle (v_2)^2 \rangle^2} > \frac{1}{2}
\]

Similar results obtained using Event Plane method
$v_4 / v_2^2(p_T)$ at mid-rapidity $|\eta| < 0.8$

Significantly higher than RHIC: experimental method dependent
HYDJET++

Effects to be studied: resonance decay and hard part influence
HYDJET++ RESULTS FOR RHIC

Jets increase the ratio
HYDJET++ RESULTS FOR LHC

The same tendency is observed in Pb+Pb at LHC

Still, the ratio is below 1
DECAYS OF RESONANCES PLAY MINOR ROLE
III. Number-of-constituent-quark (NCQ) scaling
The agreement seems to be good at $KE_{T/n_q} < 0.7$ GeV.
One of the explanations of $\text{KE}_T/n_q$ scaling is partonic origin of the elliptic flow. However, final state effects (such as resonance decays and jets) may also lead to appearance of the scaling.
NCQ scaling at LHC

No scaling for direct particles

Appearance of the approximate scaling for all particles

LHC: NCQ scaling will be only approximate (prediction, 2009)
Experimental results (LHC)

Semi-sentral collisions

Semi-peripheral collisions

The NCQ scaling is indeed only approximately (2011)
CONCLUSIONS

The HYDJET++ model allows to investigate flow of hydro and jet parts separately, to look at reconstruction of pure hydro flow and its modification due to jet part.

- **Jets result to increase by 25% - 30% of the ratio** $v_4/(v_2^2v_2)$

- **Eccentricity fluctuations can increase the ratio by factor 1.5**

- **Jets + eccentricity fluctuations are enough to explain RHIC data**

- **For LHC we can explain 75% of the signal. Other effects are needed**

- **The predicted violation of the NCQ scaling at LHC is observed**
Back-up Slides
Effects of flow fluctuations and partial thermalization


Stars: with fluctuations inferred from the difference between $v_2^2$ and $v_2^{LYZ}$.
Dotted line: eccentricity fluctuations from a Monte-Carlo Glauber
III. INFLUENCE OF RESONANCE DECAYS
Influence of resonance decay on $v_2$ value

The elliptic flow of directly produced particles is smaller than that for all particles.

PbPb collisions, $c=30\%$

The elliptic flow of directly produced particles is smaller than that for all particles.

<table>
<thead>
<tr>
<th>TABLE I: Yields of the particles produced directly and with resonance decays, $5.6 \cdot 10^6$ events, $c=42%$, midrapidity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_T$, GeV/c</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>4</td>
</tr>
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<td>6</td>
</tr>
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<td>8</td>
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<tr>
<td>16</td>
</tr>
<tr>
<td>18</td>
</tr>
<tr>
<td>20</td>
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<table>
<thead>
<tr>
<th>Particle</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^\pm$</td>
<td>860</td>
</tr>
<tr>
<td>$K + \bar{K}$</td>
<td>185</td>
</tr>
<tr>
<td>$p + \bar{p}$</td>
<td>63.8</td>
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<tr>
<td>$\Lambda + \bar{\Lambda} + \Sigma + \bar{\Sigma}$</td>
<td>42.3</td>
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<tr>
<td>$\phi$</td>
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<tr>
<td>Direct</td>
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<tr>
<td>$\phi$</td>
<td>81.4</td>
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<tr>
<td>$p + \bar{p}$</td>
<td>18.6</td>
</tr>
<tr>
<td>$\Lambda + \bar{\Lambda} + \Sigma + \bar{\Sigma}$</td>
<td>14.2</td>
</tr>
<tr>
<td>$\phi$</td>
<td>6.5</td>
</tr>
<tr>
<td>Direct %</td>
<td>20 %</td>
</tr>
<tr>
<td>$\phi$</td>
<td>44 %</td>
</tr>
<tr>
<td>$p + \bar{p}$</td>
<td>30 %</td>
</tr>
<tr>
<td>$\Lambda + \bar{\Lambda} + \Sigma + \bar{\Sigma}$</td>
<td>39 %</td>
</tr>
<tr>
<td>$\phi$</td>
<td>99 %</td>
</tr>
</tbody>
</table>
Pions and kaons: the resulting flow is weaker at low-\( p_T \) and larger at high-\( p_T \)

Baryons: the resulting flow is stronger than the flow of direct particles
Influence of resonance decays for different type of particles at LHC

Pions: the resulting flow is weaker at low-\(p_T\) and larger at high-\(p_T\)

Kaons: both flows almost coincide

Baryons: the resulting flow is stronger than the flow of direct particles
The secondary pion spectrum is much softer than proton spectrum.
The heavier resonances have larger $v_2$ at high transverse momenta.
The decay kinematics keeps this high $v_2$ for products of resonance decays.
At low transverse momenta: pions from baryon resonances enhance the flow; pions from meson resonances reduce it
V. PARAMETERS OF THE MODEL
Methods for $v_2$ calculation

(1) Event plane method

$$v_2^{obs} \{ EP \} = \langle \cos 2(\varphi_i - \Psi_2) \rangle$$

$\Psi_2$ is the calculated reaction plane angle:

$$\tan n \psi_n = \frac{\sum \omega_i \sin n \varphi_i}{\sum \omega_i \cos n \varphi_i}, \quad n \geq 1, \quad 0 \leq \psi_n < 2\pi / n$$

$$v_2 \{ EP \} = \frac{v_2^{obs} \{ EP \}}{R} = \frac{v_2^{obs} \{ EP \}}{\langle \cos 2(\Psi_2 - \Psi_R) \rangle}$$

(2) Two particle correlation method

$$v_2 \{ 2 \} = \sqrt{\langle \cos 2(\varphi_i - \varphi_j) \rangle}$$

(3) Lee-Yang zero method

$$G(ir) = \langle e^{iQ} \rangle, \quad Q = \sum \cos(2\varphi)$$

Integral $v_2$ is connected with the first minimum $r_0$ of the module of the $G(ir)$:

$$v_2 = \frac{j_0}{N r_0}$$

Differential flow is calculated by the formula:

$$\frac{v_2(p_T)}{N v_2} = \text{Re} \left( \frac{\langle \cos(2\varphi)e^{i n_0 Q} \rangle}{\langle Q e^{i n_0 Q} \rangle} \right)$$
The better reconstruction is achieved in midcentral collision for the methods, while Lee-Yang zero method tends to reconstruct true value at more central and more peripheral collision.
Comparison of Event Plane and Lee-Yang zeroes methods (c=30%)

Event Plane method overestimates $v_2$ at high $p_t$ due to non-flow correlation (mostly because of jets).