Properties of $K^*$ in a medium

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Contents

1. Introduction and motivation
2. Framework
3. Dense matter
4. Hot matter
5. Summary and outlook
Introduction and motivation

- **In-medium properties of kaons** (K, anti-K, K* and anti-K*)

- First results were obtained using *chiral perturbation theory* (Kaplan, Nelson, *PLB* 175 (1986) 57) and *relativistic mean field models* (Schaffner, Gal, Mishustin, Stöcker, Greiner, *PLB* 334 (1994) 268)

- **Dirac-Brueckner Hartree-Fock** approximation (Brueckner, PR 97 (1955) 1353; Hjorth-Jensen et al., PR 261 (1995) 125) applied to *KN system*

- DBHF goes beyond a mean fields and uses realistic KN interactions for the calculations
Introduction and motivation

- Experimentally strangeness has been studied since the 1980s
- In-medium properties are studied e.g. in heavy-ion collisions
- For baryonic matter at SIS (and in the future at FAIR) etc. energies
- For hot nuclear matter at RHIC, LHC etc. energies
- Later KaoS collaboration published results which agreed with theoretical predictions when including K / anti-K in-medium effects

**Figs. by E. Bratkovskaya et al.**
Introduction and motivation

- Goal: study in-medium strange pseudoscalar and vector mesons within Breit-Wigner approach in a consistent way, for a convenient implementation in transport models of strangeness production in HICs

1. **Dense nuclear matter (FAIR):** self-consistent coupled-channel approach ("G-matrix")
   - $K^*$ and anti-$K^*$ modified from $K^*N$ and $K^* \to K\pi$ [*different behaviour!*]
   - $K^*$ self-energy within the unitarised chiral perturbation theory [*NEW!*]

2. **Hot nuclear matter (RHIC/LHC):** results from Chiral Perturbation Theory in hot meson gas
   - $K^*$ and anti-$K^*$ in-medium effects from $K^* \to K\pi$ coupling [behave similarly]
   - Estimation of the real part of the $K^*$ self-energy [*mass shift!*]
Framework

- **G-matrix** results are approximated by the **Breit-Wigner** spectral function
- Evaluation of in-medium **widths** and **masses**, which are connected to **imaginary** and **real** part of the **self-energy** of strange mesons
- Approximation implicitly neglects momentum dependence of self-energy
The meson propagator \((i = K, \text{anti-}K; K^*, \text{anti-}K^*)\)

\[
D_i(\omega, \vec{q}, \rho) = \frac{1}{\omega^2 - \vec{q}^2 - m_i^2 - \Pi_i(\omega, \vec{q}, \rho)}
\]

Spectral function

\[
S_i(\omega, \vec{q}, \rho) = -\frac{1}{\pi} \Im(D_i(\omega, \vec{q}, \rho)) = -\frac{1}{\pi} \frac{\Im(\Pi_i(\omega, \vec{q}, \rho))}{\left| \omega^2 - \vec{q}^2 - m_i^2 - \Pi_i(\omega, \vec{q}, \rho) \right|^2}
\]

Spectral function rewritten in a way more similar to the Cauchy-Lorentz distribution

\[
S_i(\omega, \vec{q}, \rho) = -\frac{1}{\pi} \frac{\Im(\Pi_i(\omega, \vec{q}, \rho))}{\left[ \omega^2 - \vec{q}^2 - (m_i^2 + \Re(\Pi_i(\omega, \vec{q}, \rho))) \right]^2 + \left[ \Im(\Pi_i(\omega, \vec{q}, \rho)) \right]^2}
\]
Spectral function in the Breit-Wigner approach

\[ A_i(M, \rho) = C_1 \frac{2}{\pi} \frac{M^2 \Gamma_i(M, \rho)}{(M^2 - M_0^*(\rho))^2 + (M \Gamma_i(M, \rho))^2} \]

For \( q = 0 \) a connection between the imaginary part of the meson propagator and the Breit-Wigner spectral can be established.

\[ A_i(M, \rho) = 2 \cdot C_1 \cdot M S_i(M, \rho) \]

The following relations follow from that connection.

\[ M_0^{2*} = m_i^2 + \Re \left( \Pi_i(M, \rho) \right) \quad \Gamma_i(M, \rho) = -\frac{\Im \left( \Pi_i(M, \rho) \right)}{M} \]

Spectral function is normalised

\[ \int_0^\infty A_i(M, \rho) dM = 1 \]
Dense matter

- Dense matter scenario: results from meson-baryon T (or G)-matrix in Dirac-Brueckner Hartree-Fock.
- Self-consistency, coupled-channels and unitarity.
- The Bethe-Salpeter equation in coupled channels is solved for the in-medium scattering amplitude.

\[
T_{ij}(\rho) = V_{ij} + V_{il} G_l(\rho) T_{lj}(\rho)
\]

Medium: [Diagram of the Bethe-Salpeter equation]

- V: LO interaction from chiral Lagrangian
  - KN: meson baryon ChPT (coupling of octet of pseudoscalar mesons to the octet of $J^P=1/2^+$ baryons)
  - K*N: Hidden Local Gauge Symmetry Lagrangian (vector-meson octet)
- G: dressed in-medium meson-baryon propagator, including Pauli blocking on nucleon states, baryon potentials and meson self-energies.

Authors:
- Koch
- Kaiser, Waas, Weise
- Lutz, Kolomeitsev
- Schaffner-Bielich
- Ramos, Oset, Tolos
- Oller, Meissner
- Hosaka, Jido
- Nieves, Ruiz-Arriola
- Cassing, Bratkovskaya, Tolos, Ramos
• **Additional contribution for strange vector mesons:** $K^*$ decays into $K\pi$ (two-meson cloud effects)

• It is possible to account for the in-medium width of the $K^* \to K\pi$ mode by incorporating the $K$ spectral function

• *Pions* are also expected to experience in-medium modifications. Neglect this effect for simplicity (future work).
The \((p\text{-wave})\) decay width of the \(K^*\)

\[
\Gamma_{K^{*\text{ar}}} (\mu, \rho) = \Gamma_0 \left( \frac{\mu_0}{\mu} \right)^2 \frac{\int_{M_{\text{min}}}^{\mu - m_\pi} A_K (M, \rho) \cdot q(\mu, M)^3 \, dM}{\int_{M_{\text{min}}}^{\mu_0 - m_\pi} A_K (M, 0) \cdot q(\mu_0, M)^3 \, dM}
\]

\(M : \text{off-shell mass of the } K\)
\(\mu : \text{off-shell mass of the } K^*\)

with

\[
q(\mu, M) = \frac{\sqrt{\lambda(\mu, M, m_\pi)}}{2 \mu} \quad q(\mu_0, M) = \frac{\sqrt{\lambda(\mu_0, M, m_\pi)}}{2 \mu_0}
\]

This is a similar approach that was also used for the \(a_1\) decaying into \(\pi \rho\).
Dense matter: Kaons

- Neglect imaginary part of self-energy
- Re $\Pi \leftrightarrow$ mass shift from chiral Lagrangian dynamics in a $t$-$\rho$ approximation

![Graph showing $K^+$ mass shift vs $\rho/\rho_0$]
Dense matter: Kaons

- The K has almost no broadening and behaves like a stable particle when increasing nuclear density (the “full” calculation leads to a very small width)
- K experiences a repulsive potential → spectral function gains a positive mass shift
Dense matter: anti-Kaons

- Self-energy by Ramos et al. from G-matrix approach with ChPT, \( \text{anti-K} \ N \to \Lambda(1405) \! \). 
- Results were extrapolated to higher densities (i.e. beyond normal density), assuming some saturation.

Ramos, Oset, NPA 671 (2000) 481
Dense matter: anti-Kaons

- Spectral function for the anti-K
- In vacuum the particles are stable (Dirac delta)
- The anti-K experiences an **attractive potential** → negative mass shift
- And **considerable broadening**!
- (detailed structure of excitations not retained)

![K^- spectral function](image)

\[ A(M, \rho) \, [\text{GeV}^{-1}] \]

- \( \rho/\rho_0 = 1.0 \)
- \( \rho/\rho_0 = 2.0 \)

K^- in vacuum

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Ramos, Oset, NPA 671 (2000) 481
Dense matter: $K^*$

- **New contribution: $K^*$ self-energy from $K^*N$ interaction**
  - The $K^*$ in a dense nuclear medium can be treated just like the $K$ with respect to the self-consistent chiral coupled-channel calculations based on chiral dynamics ([Tolos et al PRC87 (2010) 045210; Oset et al EPJA44 (2010) 445](#)).
  - Same expression for the transition potential for $K^*N$ as for the $KN$, also for the self-energy

\[
\Pi(q^0,\vec{q},\rho) = 2 \int \frac{n(\vec{p})}{(2\pi)^3} [T_{K^*p}(p^0, \vec{p}, \rho) + T_{K^*n}(p^0, \vec{p}, \rho)] d^3p \approx \frac{1}{2} (T_{K^*p}(\rho) + T_{K^*n}(\rho)) \rho
\]

- $t-\rho$ approximation (no resonances in $K^*N$), $\Pi$ accounts for $K^*$ mass shift
- We have unitarised the interaction matrix using the Bethe-Salpeter equation
  \[
  T = V/(1 - VG)
  \]
Dense matter: $K^*$

- Real part obtained from self-energy expression (previous slide)
- Width of $K^*$ self-energy calculated through in-medium modification via $K^* \to K \pi$
- Width changes moderately (decreases)
Dense matter: $K^*$s

- $K^*$ experiences repulsive medium, spectral function shifted to higher masses.
- As density increases $\rightarrow$ $K^*$ width is actually lower due to the heavier Kaon in $K^* \rightarrow K\pi$
- The two effects compensate: shape of $K^*$ spectral function practically unchanged.
Dense matter: anti-K*s

- **anti-K* N**: dynamics ruled by S=-1 resonances (as for anti-K) → complicated **many-body structure** and **E-dependence** of self-energy

- Parametrise full G-matrix calculation [Tolos et al PRC87 (2010)]
  - 1. Solve dispersion relation: quasi-particle energy $\omega_{K^*}$
  - 2. Use $\omega_{K^*}$ to find width: $\Gamma_{K^*} = \text{Im} \frac{\Pi_{K^*}}{\omega_{K^*}}$

Tolos et al., PRC 82 (2010) 045210
Dense matter: anti-K*s

- Blue: Original G-matrix calculation
- Orange: Breit-Wigner spectral function
- Green: Breit-Wigner spectral function with corrected mass shift
- Middle plot: Magenta line denotes evaluation of K\pi width with in-medium kaons

Breit-Wigner does not retain multi-pole structure and overestimates the strength at low energies, but *keeps essential features*
Hot matter: kaons

- Consider hot, isotopically symmetric gas of pions: $K^*$ and anti-$K^*$ identical
- Medium effects tied to $K^* \rightarrow K \pi$ decay mode
- $K$ in hot pion gas: evaluated by Martemyanov et al. in meson-meson ChPT + phenomenological extension for higher energies
- (pions considered stable)

The quantities of interest in this case are the width $\Gamma_K$ and the mass shift $\delta M_K$ to build the $K^*$ selfenergy

Martemyanov et al., PRL 93 (2004) 052301
Hot matter: kaons

- Use Breit-Wigner to construct K spectral function at different temperatures
**Hot matter: K*s**

- Use K spectral function to obtain K* width

\[
\Gamma_{K^*}(\mu, \rho) = \Gamma_0 \left(\frac{\mu_0}{\mu}\right)^2 \cdot \frac{\int_{M_{\text{min}}}^{\mu-m_\pi} A_K(M, \rho) \cdot q(\mu, M)^3 dM}{\int_{M_{\text{min}}}^{\mu_0-m_\pi} A_K(M, 0) \cdot q(\mu_0, M)^3 dM}
\]

**K* in-medium width**

Graph showing the in-medium width of K* as a function of chemical potential \(\mu\) for different temperatures: vacuum, \(T = 0.09\) GeV, and \(T = 0.15\) GeV.
Hot matter: K*s

- Use dispersion relation to calculate the real part using the imaginary part of the self-energy!

\[ \Re(\Pi(\mu)) - \Re(\Pi_{\text{vac}}(\mu)) = -\frac{2}{\pi} \int_{m_\pi}^{\infty} \frac{\mu'}{\mu'^2 - \mu^2} \left[ \Gamma_{\text{dec}}(\mu', T) - \Gamma_{\text{vac}}(\mu') \right] d\mu' \]

- Unfortunately the emerging integral is divergent
- Use phenomenological form factor in K* -> K π vertex to regularise integral

\[ F(\Lambda, \mu) = \left( \frac{\Lambda^2 + q(\mu_0, M_0)^2}{\Lambda^2 + q(\mu, M)^2} \right)^2 \]

\[ q(\mu, M) = \frac{\sqrt{\lambda(\mu, M, m_\pi)}}{2\mu} \]

\[ q(\mu_0, M) = \frac{\sqrt{\lambda(\mu_0, M_0, m_\pi)}}{2\mu_0} \]

M: off-shell mass of the K
\mu: off-shell mass of the K*
\mu_0: pole mass of the K
M_0: pole mass of the K*
Hot matter: K*s

- This results in the following real part of the self-energy (↔ mass shift) 
  
  *(repulsive and VERY small)*
Hot matter: $K^*$s

- We obtain the following spectral function (for different temperatures)
- Spectral function is fairly stationary with respect to temperature
- Barely any broadening at the $qp$ peak (some strength below threshold)

$K^*$ spectral function
Summary and outlook

- The in-medium properties and the behaviour of the strange mesons K, anti-K, K* and anti-K* bar in a hot, pionic and dense, baryonic nuclear medium have been studied within a Breit-Wigner parametrisation of the spectral function.

- In dense nuclear matter, the S=+1 mesons keep their vacuum structure, and can be easily cast in BW form with mild changes in their masses and widths.

- In the S=-1 sector, only an approximate (“average”) description of the spectral function is achieved, retaining essential features as attractive potential and broadening.

- A new contribution for the K* self-energy in dense matter was calculated in ChPT, leading to a positive mass shift 40 MeV at normal matter density.

- In hot hadronic matter, the K* experiences a mild broadening and negligible mass shift only at very high temperatures, from changes in the kaon spectral function.

- These results can now be implemented into transport models and used in simulations to get more realistic behaviour for strange particles in HICs.
The end.

Thank you for your attention.