Hadrons in the NJL model

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Outline

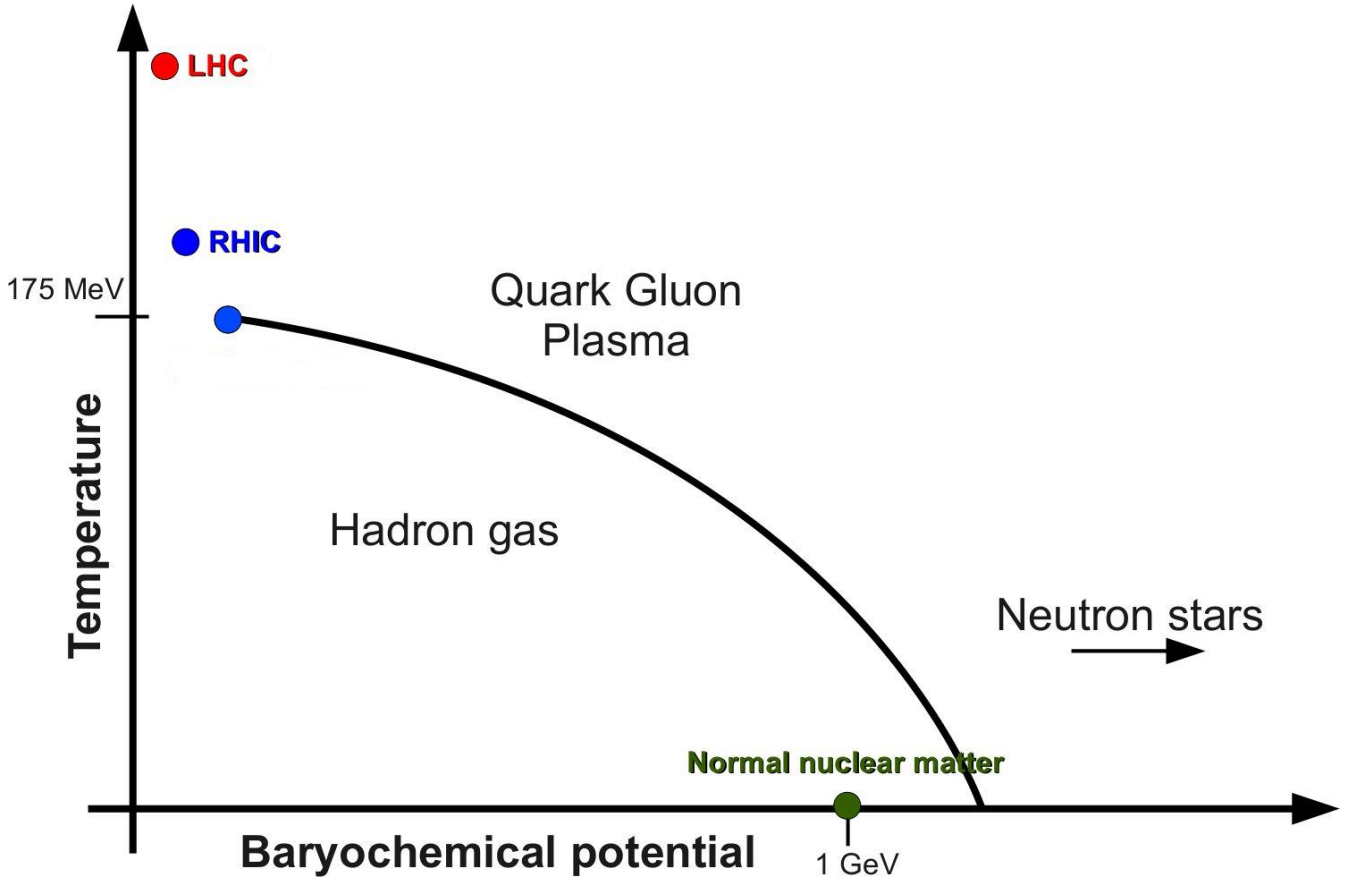
Quantum ChromoDynamics

Confinement

- Quarks are confined in hadronic matter, baryons or mesons, and are never observed separately.

- Perturbative QCD (pQCD) can be used for high energy physics.
- The lattice QCD (lQCD) : is used to solve numerically the QCD Lagrangian on a lattice of points in space and time
- Low energy models

Phase diagramm of QCD



The Nambu and Jona-Lasinio model

- Originaly a theory of nucleons similar to the BCS theory of superconductivity.
- We only use quarks as degrees of freedom because we assume gluon degrees of freedom are frozen in the low energy limit.
- Construct to have the same symmetries as QCD.

QCD symmetries

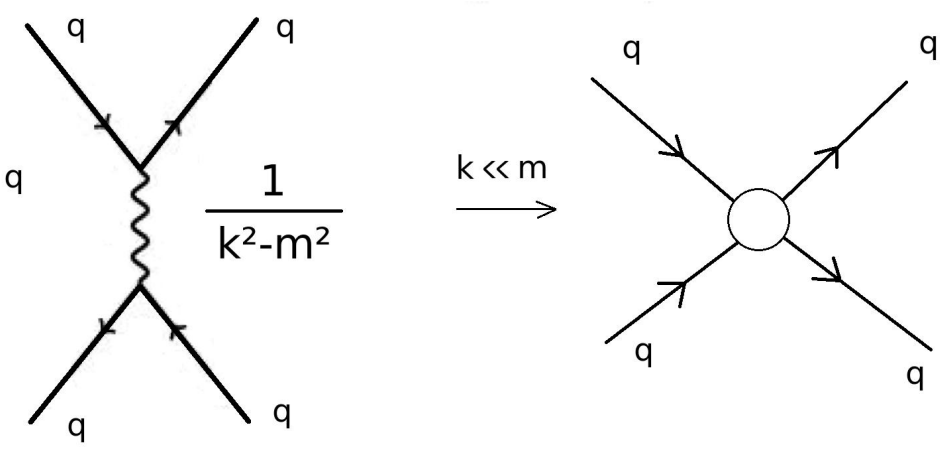
- $L_{QCD} = \bar{\psi}(iD - m_o)\psi - \frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu}$
- A symmetry in the Lagrangian implies a conserved current.

Symmetry	Name	Current
$U_v(1)$	Baryonic	$\bar{\psi}\gamma_\mu\psi$
$U_A(1)$	Axial	$\bar{\psi}\gamma_\mu\gamma_5\psi$
$SU_V(3)$	Vector	$\bar{\psi}\gamma_\mu\lambda_a\psi$
$SU_A(3)$	Chiral	$\bar{\psi}\gamma_\mu\gamma_5\lambda_a\psi$

- Isospin symmetry in parameters set.

Lagrangian NJL

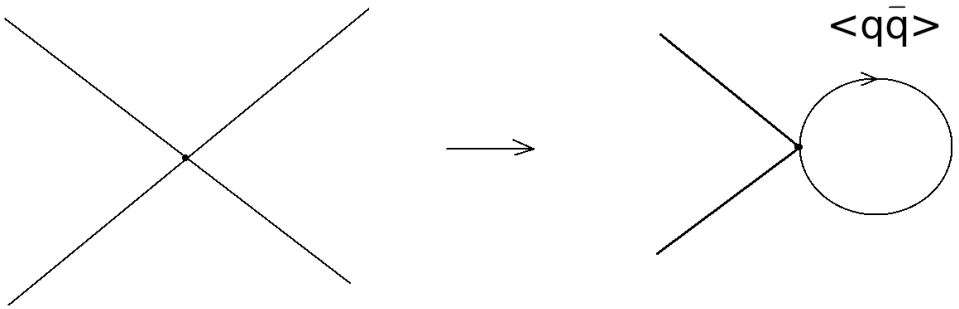
- Lagrangian : $L_{NJL} = \bar{\psi}(i\partial - m_o)\psi + G\sum_a[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}\gamma_5\lambda^a\psi)^2] - K[det\bar{\psi}(1+\gamma_5)\psi + det\bar{\psi}(1-\gamma_5)\psi]$
- Static approximation : Interaction between two quark currents by the exchange of a pointlike gluon.



- Non-renormalizable theory, we need to apply a cut-off.
- Don't include confinement

Gap equation

- The Hartree approximation reduces the N-body interaction to an interaction with a mean field.
- $(\bar{\psi}\lambda_a\psi)^2 \rightarrow 2\bar{\psi}\lambda_a\psi. < \bar{\psi}\lambda_a\psi >$
- The linearization of the interaction in the mean field approximation is like closing the quark loop.



- This defines a dynamical fermion mass :
  $m_i = mo - 2G < q_i\bar{q}_i > -2K < q_j\bar{q}_j >< q_k\bar{q}_k >$
- Breaking of the chiral symmetry

Quarks condensates

Matsubara Formalism

- The Matsubara Green's function is antiperiodic over an imaginary time

$S(x,x',\tau,\tau') = \frac{1}{\beta}\sum_n e^{i\omega_n(\tau-\tau')}\int \frac{d^3p}{(2\pi)^3} S(p,\omega_n) e^{ip(x-x')}$

Which allows us to introduce the Fermi-Dirac distribution :

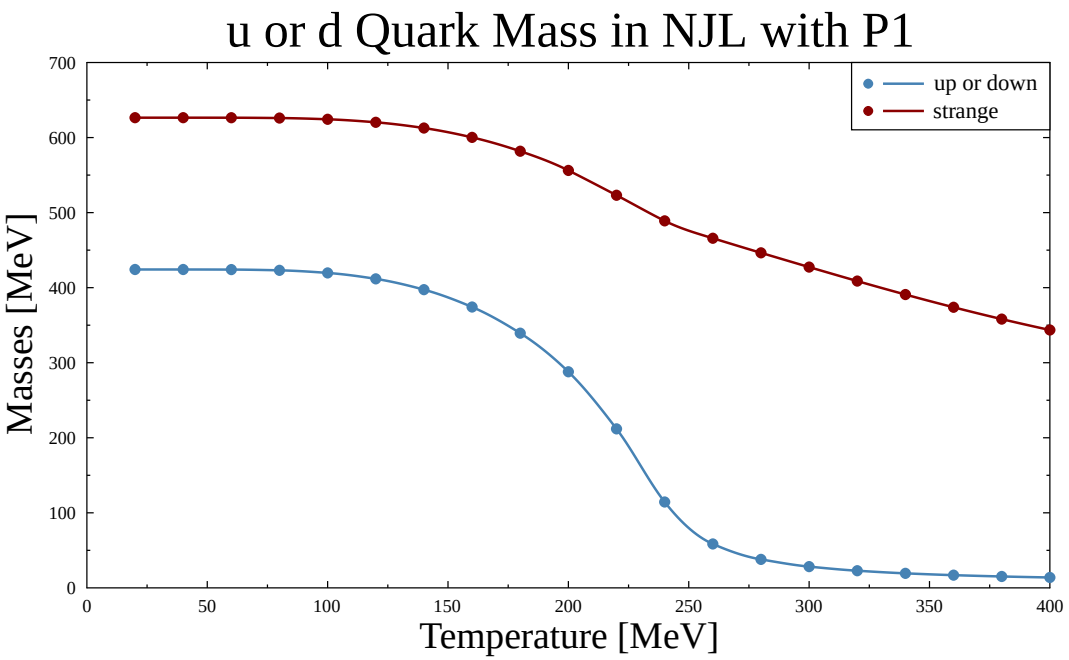
$\frac{1}{\beta}\sum_n e^{i\omega_n\eta}\frac{1}{i\omega_n \mp E_p} = f(\mp E_p)$

- Quarks condensates also depends on quarks masses

$$\ll \psi\bar{\psi} \gg = -m\frac{N_c N_f}{\pi^2}\int dp \frac{p^2}{E_p}[1 - f^+(p,\mu) - f^-(p,\mu)]$$

- We need to solve a system of equations

Quark masses



- Quark condensates are the order parameter of the transition phase.

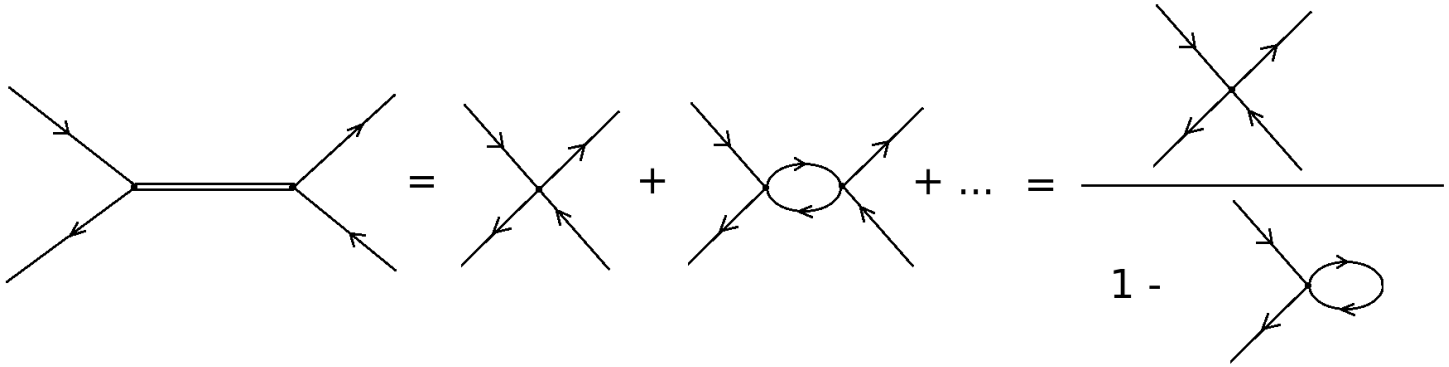
Parameter set

	$P1$	$R.K$-Costa
$m_0 u$	4.75	5.5
$m_0 s$	147	140.7
$G.\Lambda^2$	1.922	1.835
$K\Lambda^5$	10.0	12.36
G_V	$0.31G$	0.295
Λ	708	602.3

- One parameter will be added for the diquark sector

Bethe Salpether

- In the Random Phase Approximation :



- $T(q^2) = G + G\Pi(q^2)G + G\Pi(q^2)G\Pi(q^2)G + ... = \frac{G}{1-G\Pi(q^2)}$
- $T(q^2) = K_1 . \frac{i . g^2_{\pi q\bar{q}}}{q^2 - m^2} . K_2$
- The mass of the pion mode is determined by the pole.
- The coupling constant is given by an expansion around the pole : $g^2_{\pi q\bar{q}} = (\frac{\partial \Pi}{\partial q^2})^{-1}|_{q^2-m^2}$

Polarisation Function

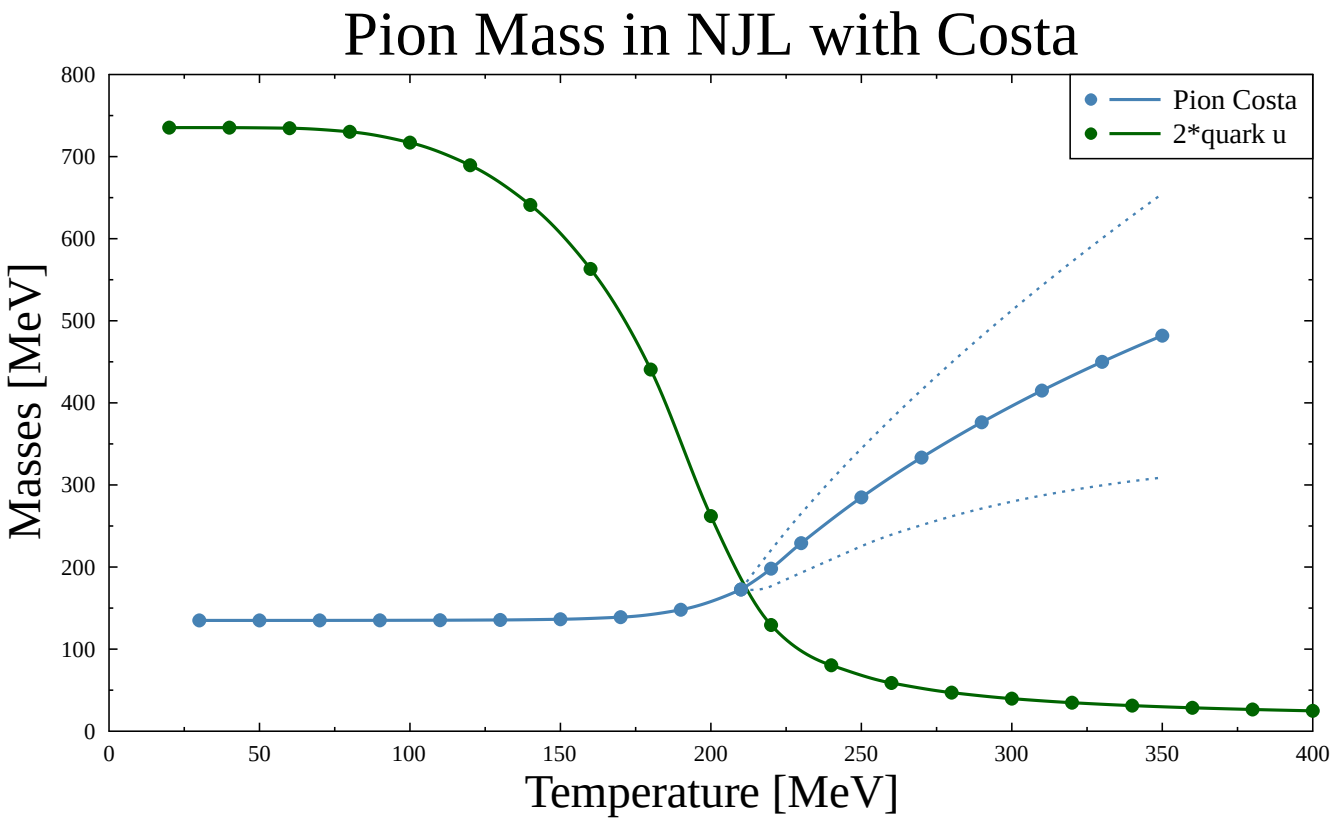
- The loop is obtained by integrating over the quarks propagators

$$\Pi(q^2) = N_c T \sum \int \frac{d^3p}{(2\pi)^3} Tr[iS^f(i\omega_n, \overrightarrow{p}).\Gamma.iS^f(i\omega_n - i\nu_m, \overrightarrow{p} - \overrightarrow{k}).\Gamma]$$

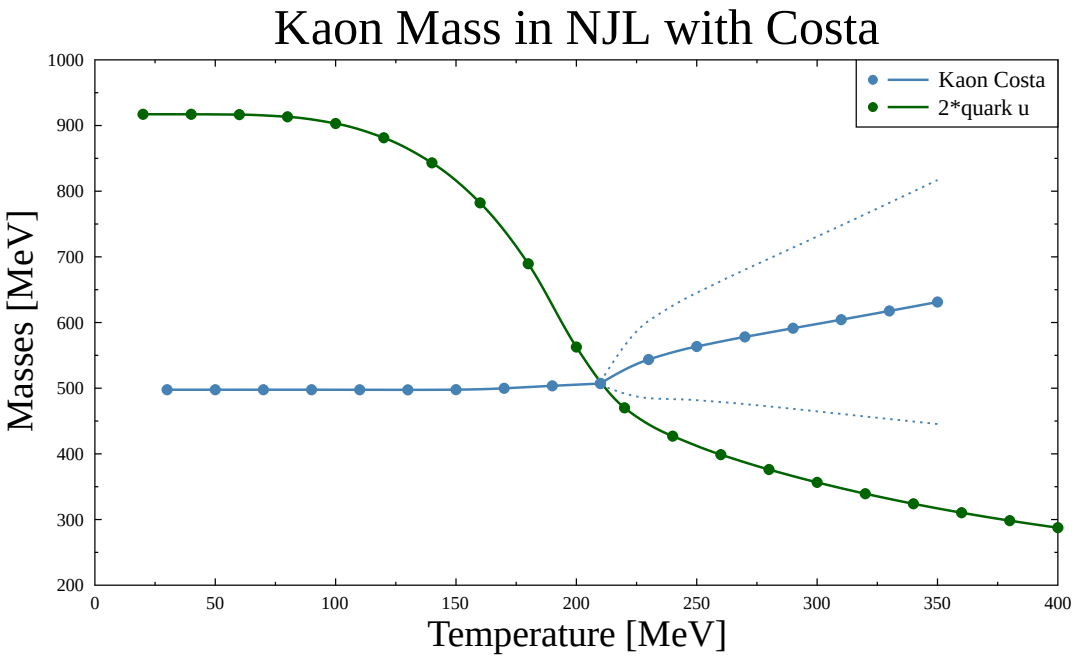
$\Gamma = i\gamma_5$ for pseudoscalar mesons

- Trace as to be performed in color and flavor spaces.

Pion


Pion Mass in NJL with Costa
Masses [MeV]
Temperature [MeV]
Pion Costa
2*quark u
0
100
200
300
400
500
600
700
800
0
50
100
150
200
250
300
350
400

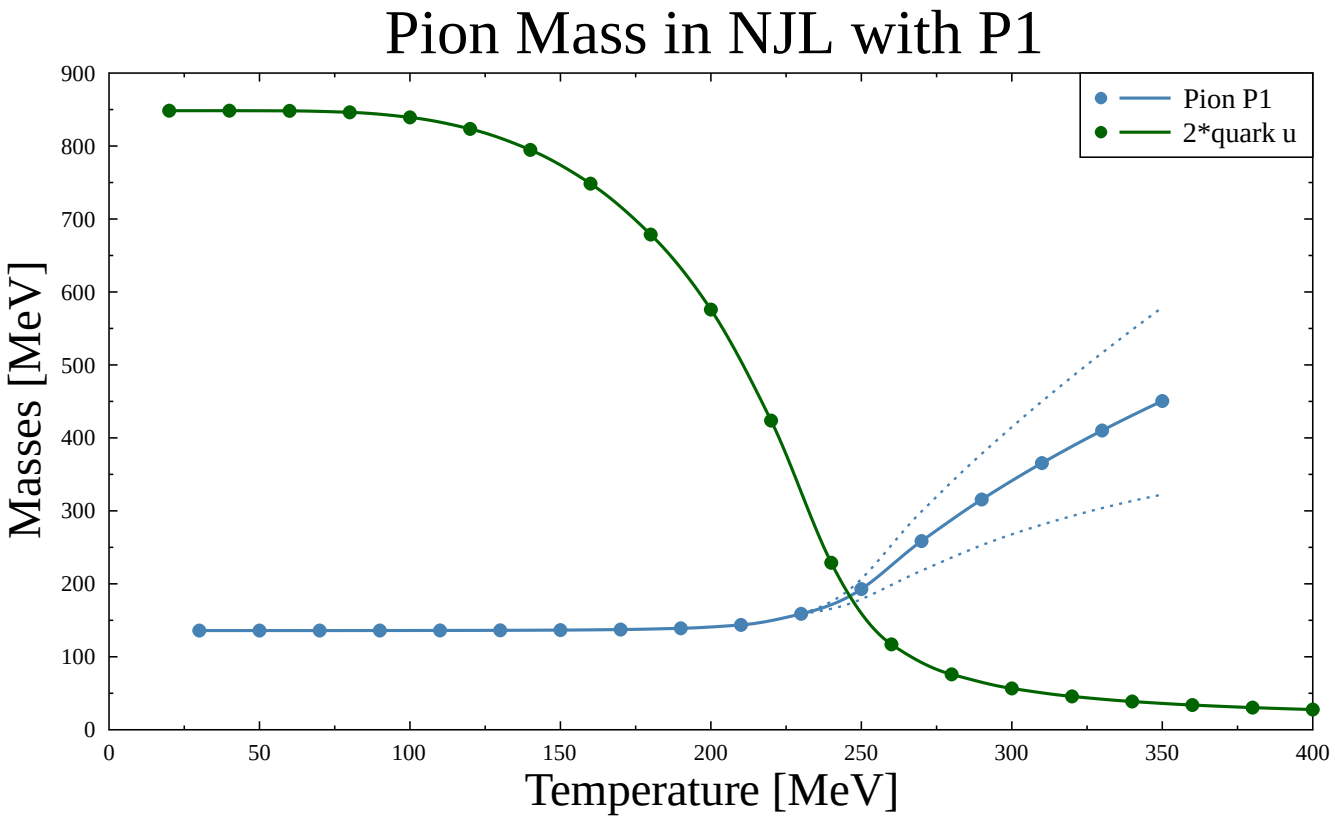
Kaon



Critical Temperature

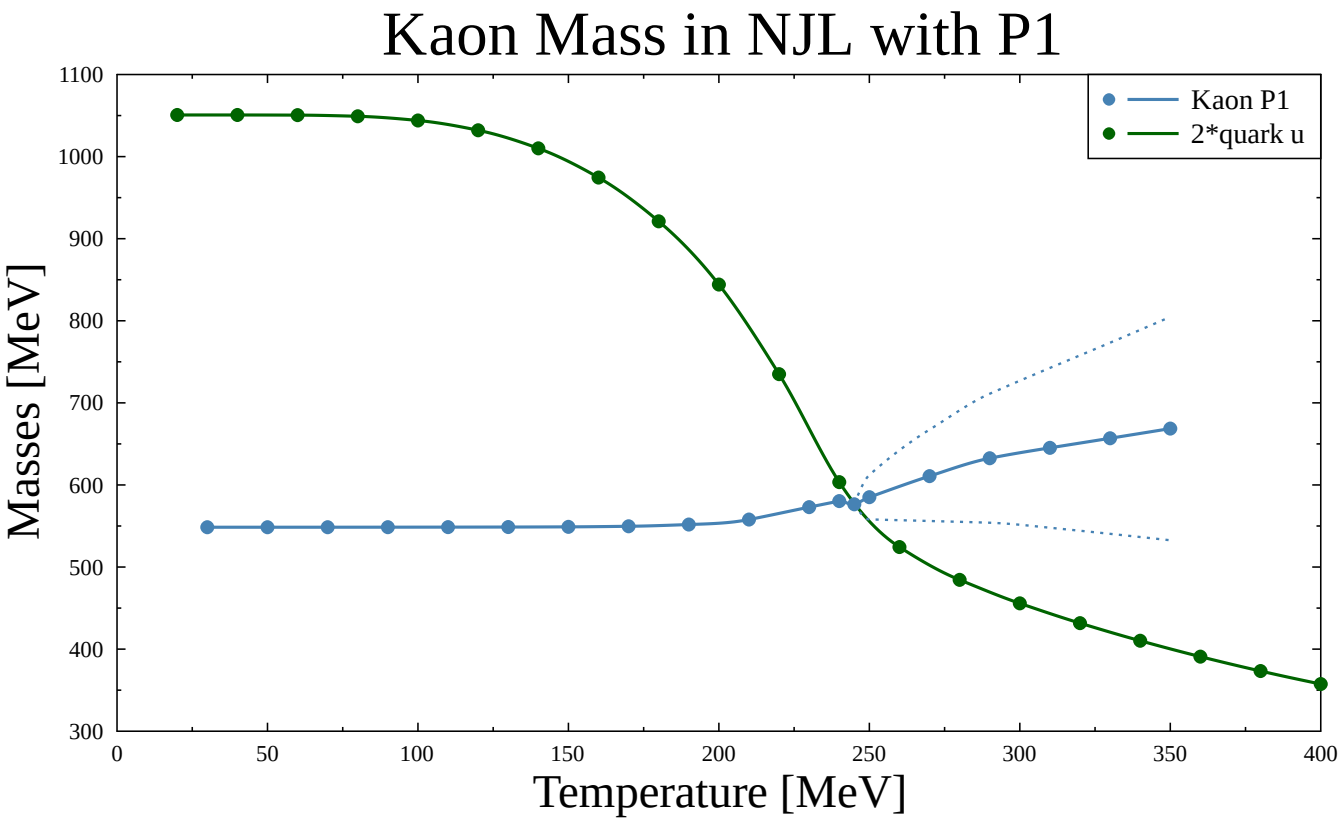
- Transition phase Tc around 210 MeV

From another model : P1



- Tc around 240 MeV

Kaon with P1


Kaon Mass in NJL with P1
Masses [MeV]
Temperature [MeV]
Kaon P1
2*quark u
1100
1000
900
800
700
600
500
400
300
0
50
100
150
200
250
300
350
400

Mesons

Mesons[T=0]	Pion	Kaon	a_0	K_0	a_1	K_1	ρ	K_0^*
Mass[MeV]	136	548	979	1178	1171	1388	746	912
Width[MeV]	-	-	194	200	434	464	-	-

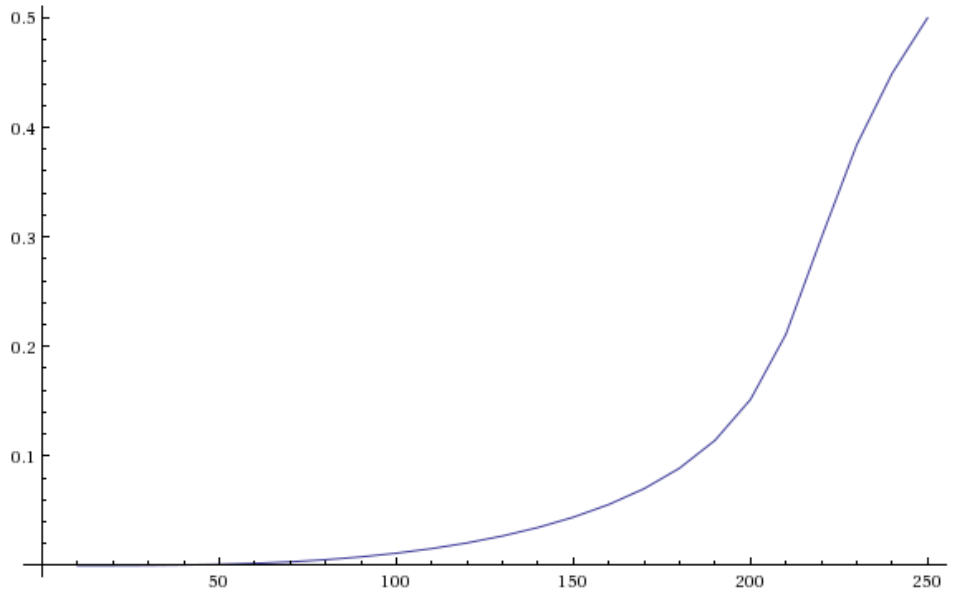
PNJL

- The Polyakov loop serves as an order parameter for the confinement in a pure gauge theory
- Parameters are from pure-gauge lattice data and some thermodynamic quantities
- The expectation value of the Polyakov loop is related to the change of free energy
- We add a potential to our lagrangian

PNJL Lagrangian

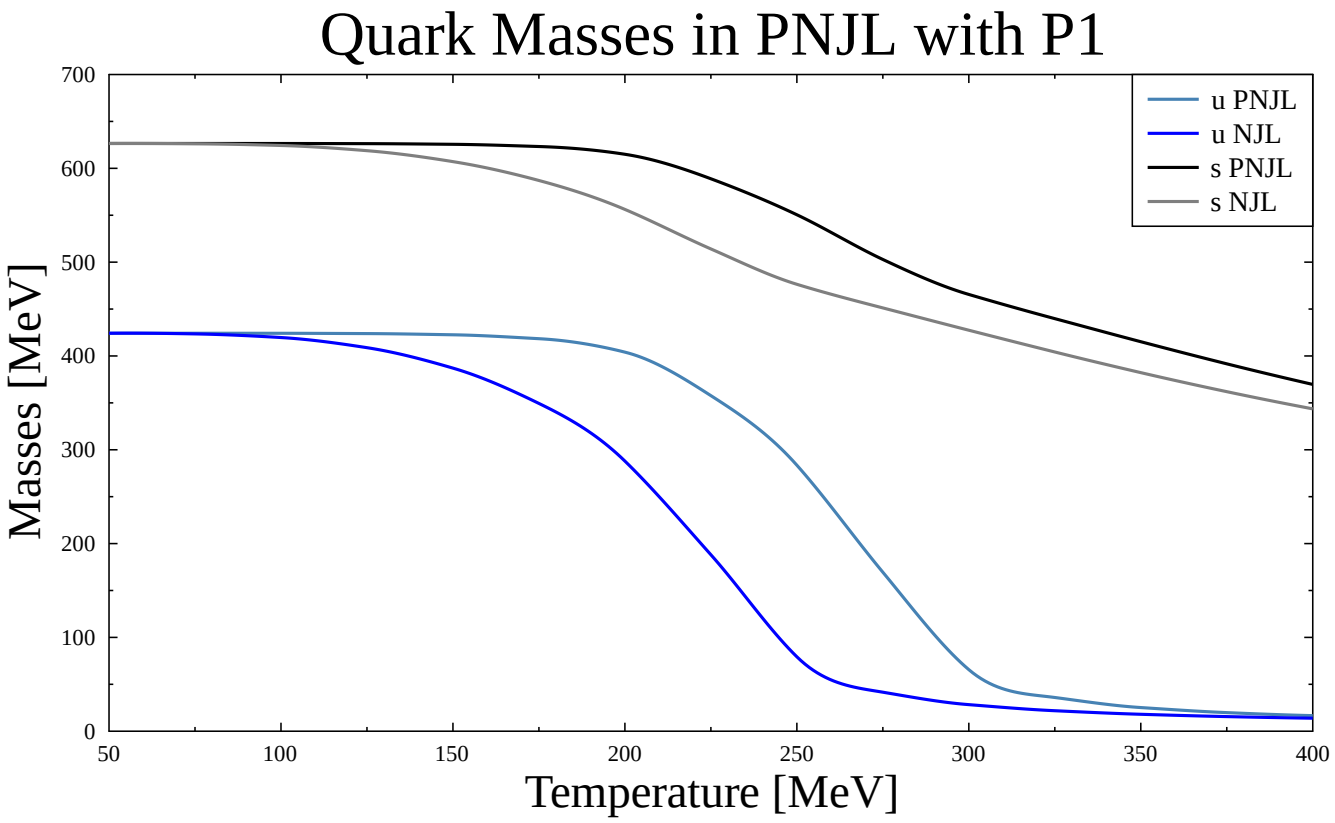
- $L_{NJL} = \bar{\psi}(i\partial - m_o)\psi + G\sum_a[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}\gamma_5\lambda^a\psi)^2] - K[det\bar{\psi}(1+\gamma_5)\psi + det\bar{\psi}(1-\gamma_5)\psi] - U(\phi, \bar{\phi}, T)$

Polyakov loop

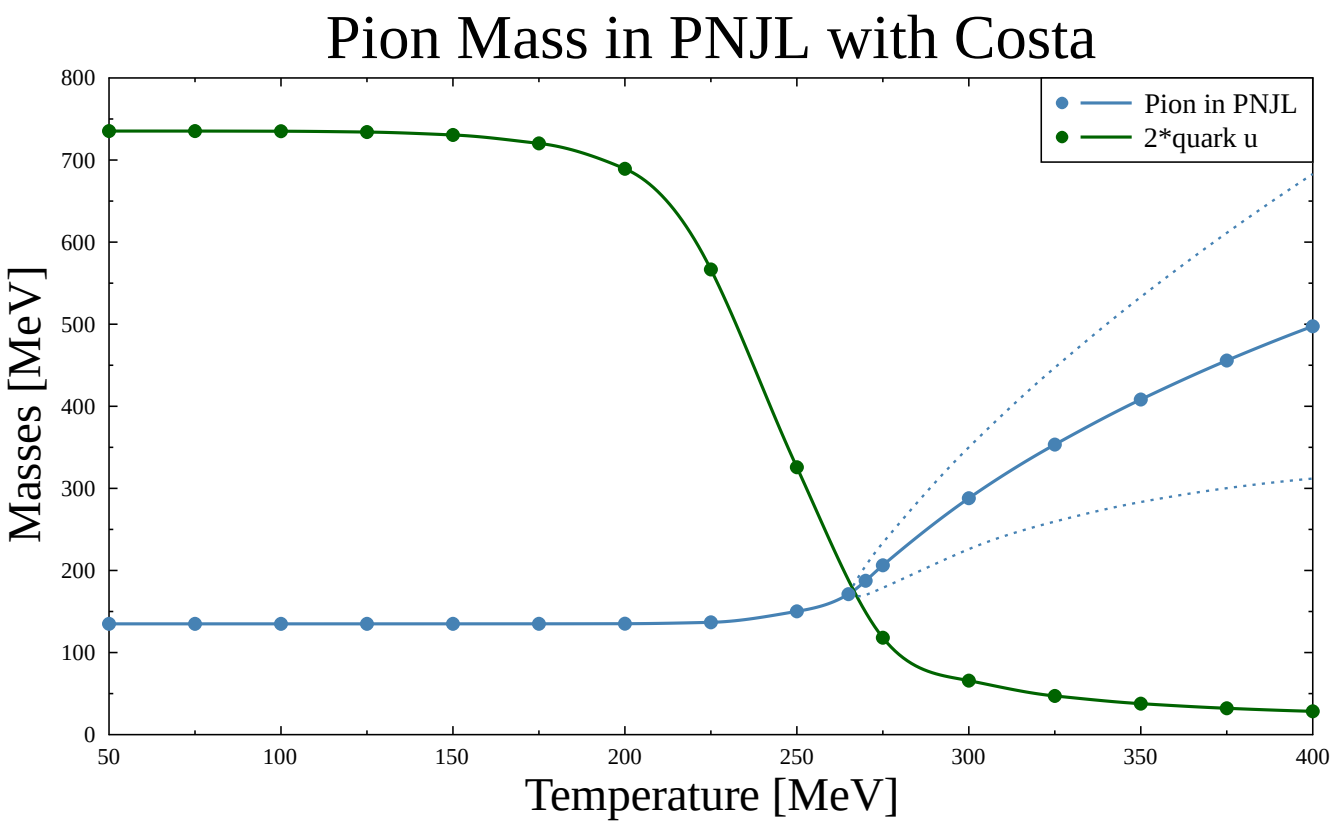


[2] Phase diagram and critical properties within an effective model of QCD: the Nambu-Jona-Lasinio model coupled to the Polyakov loop, P.Costa, M.C.Ruivo, C.A.de Sousa, H.Hansen

Quarks PNJL

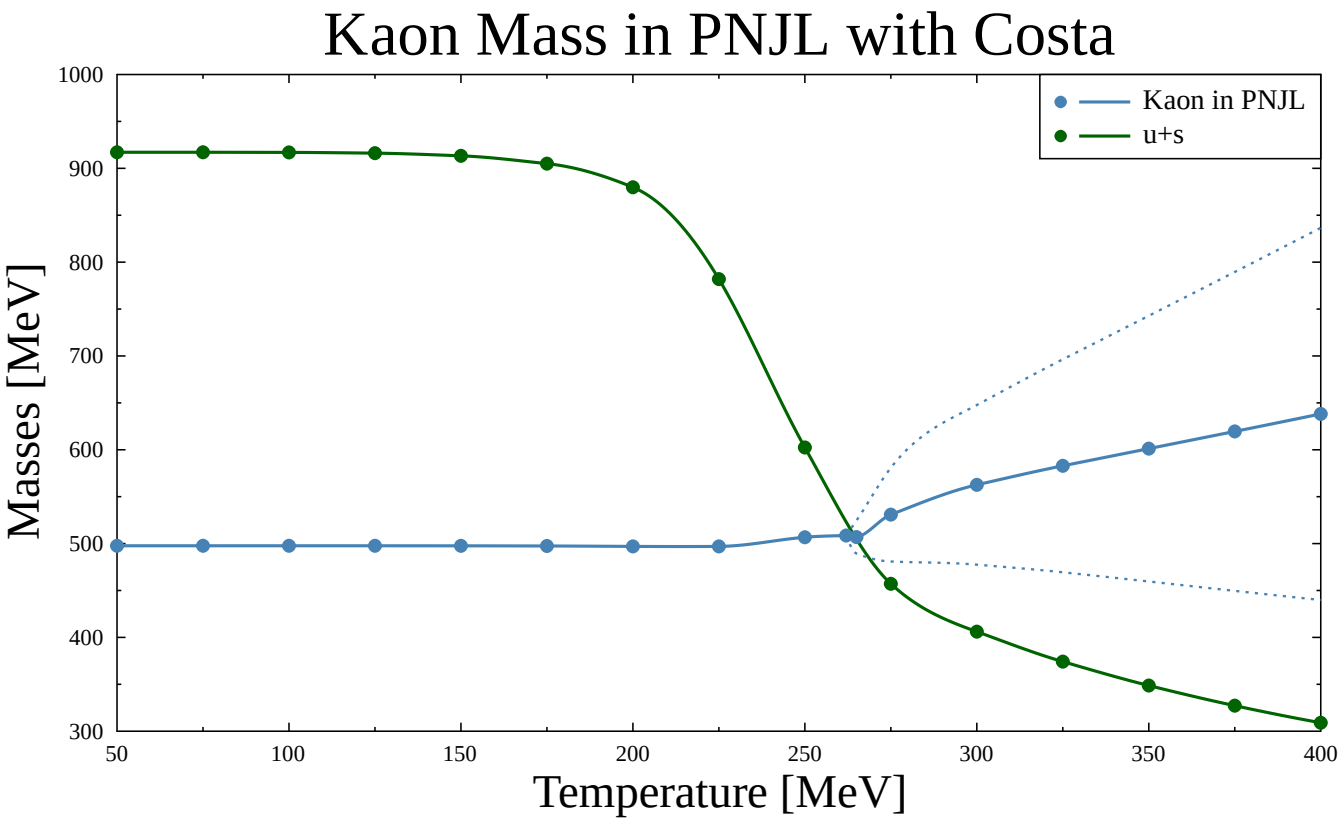

Quark Masses in PNJL with P1
Masses [MeV]
Temperature [MeV]
0
100
200
300
400
500
600
700
50
100
150
200
250
300
350
400
u PNJL
u NJL
s PNJL
s NJL

Pion PNJL

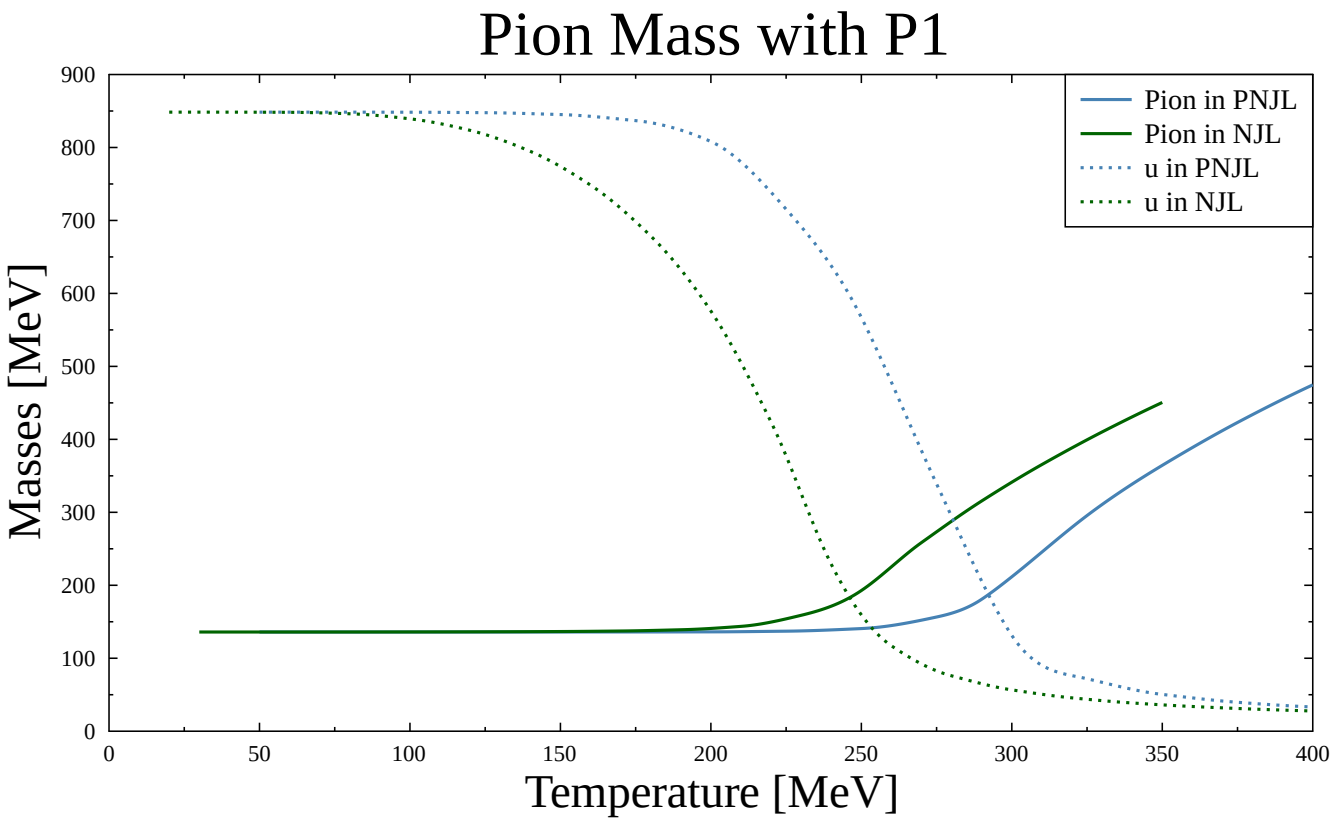


Transition phase around 270 MeV

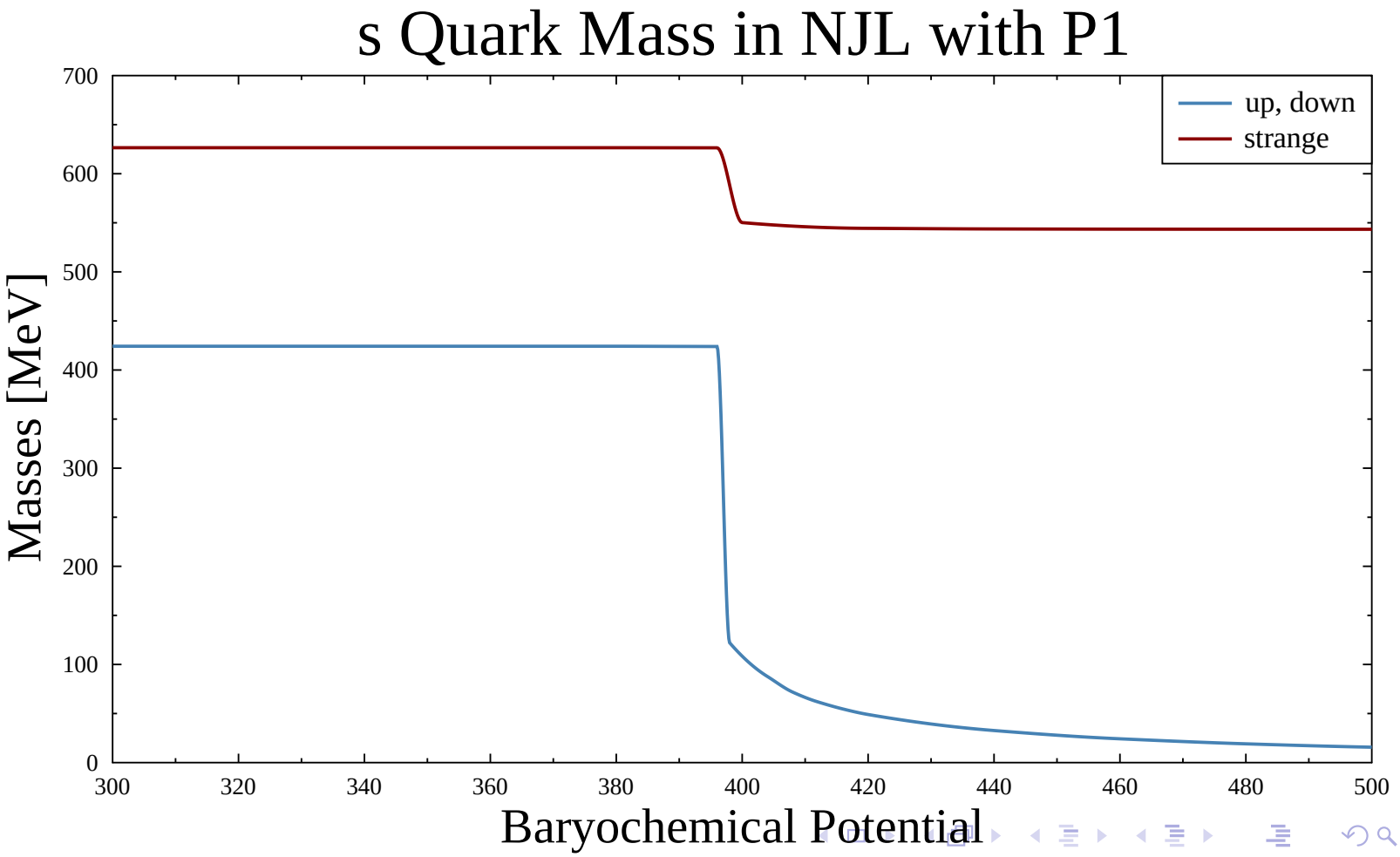
Kaon PNJL


Kaon Mass in PNJL with Costa
Kaon in PNJL
u+s
Masses [MeV]
Temperature [MeV]
1000
900
800
700
600
500
400
300
50
100
150
200
250
300
350
400

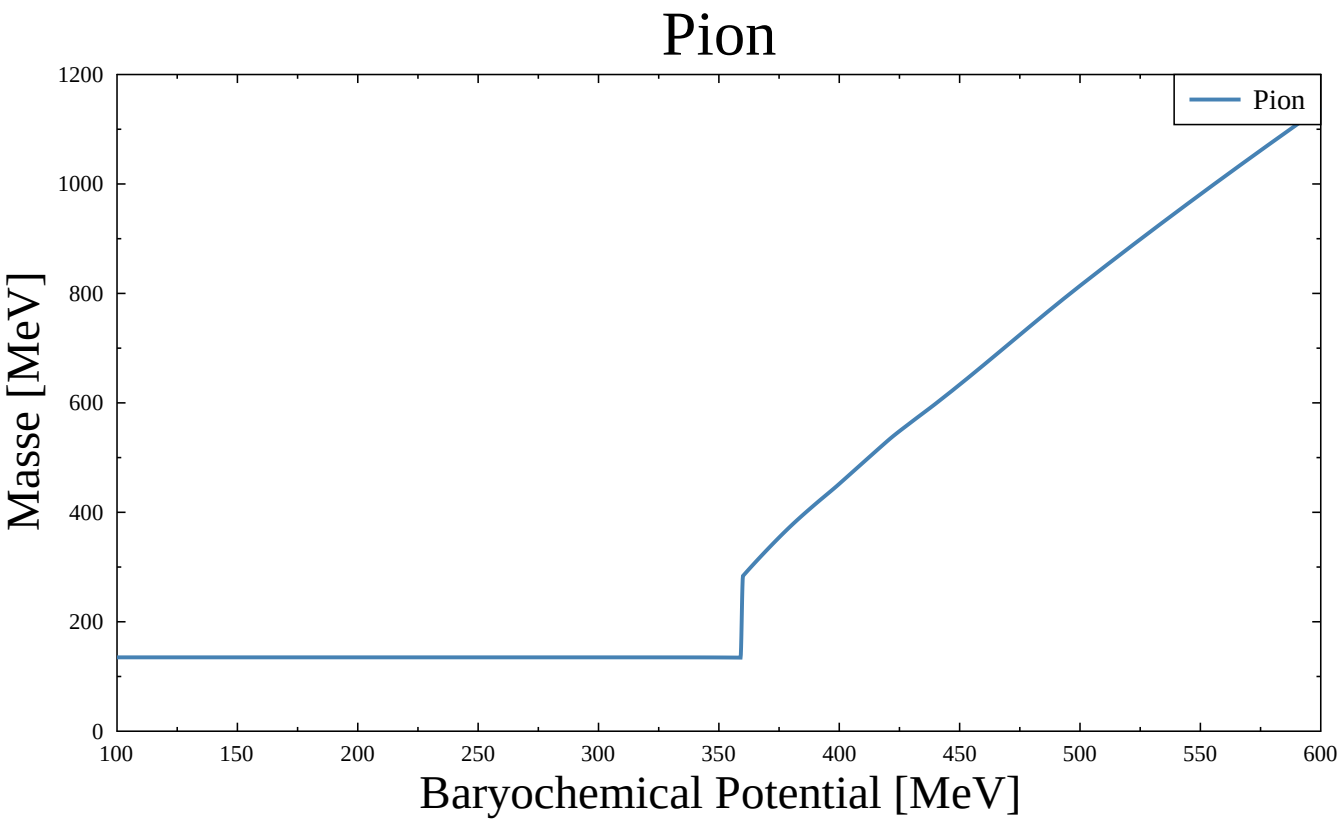
NJL vs PNJL


Pion Mass with P1
Masses [MeV]
Temperature [MeV]
0
100
200
300
400
500
600
700
800
900
0
50
100
150
200
250
300
350
400
Pion in PNJL
Pion in NJL
u in PNJL
u in NJL

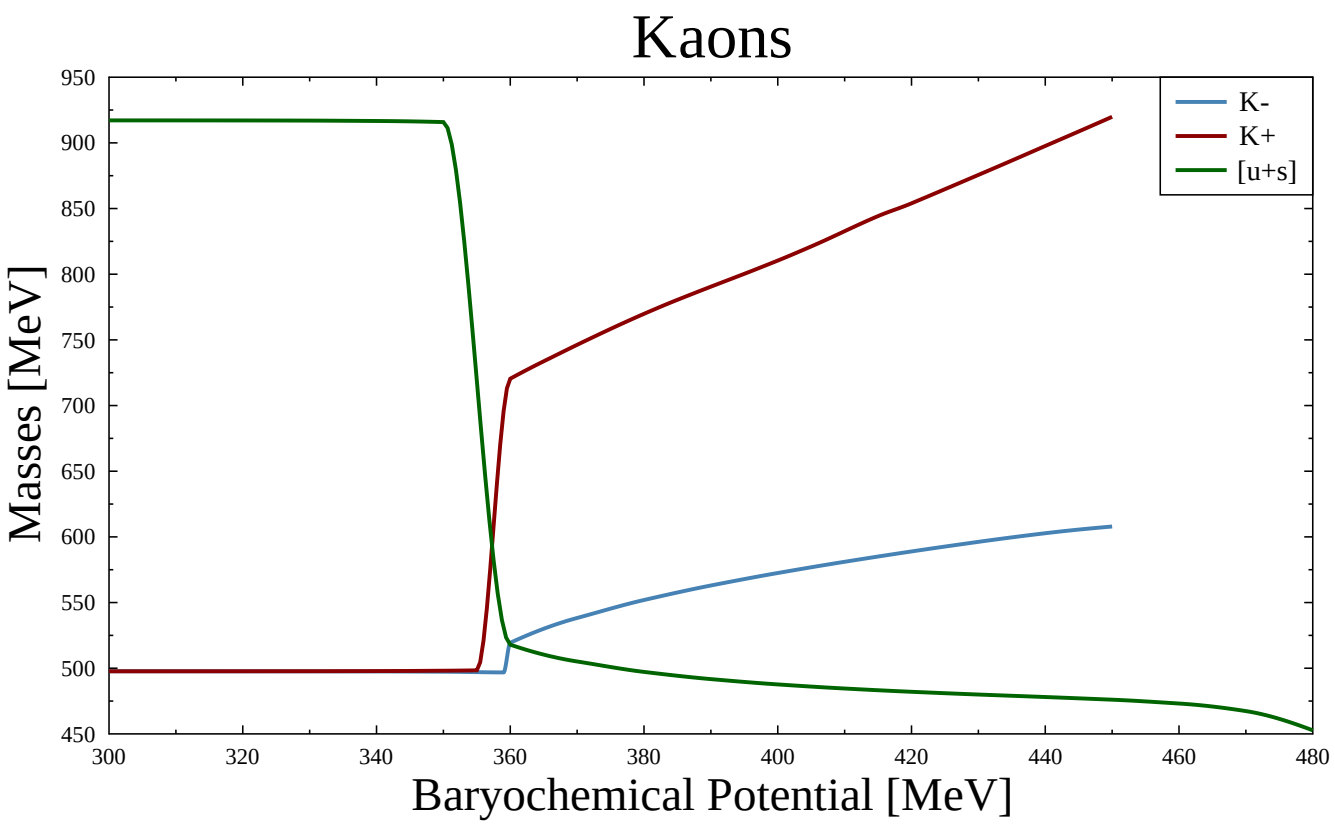
Baryochemical potential



Mesons

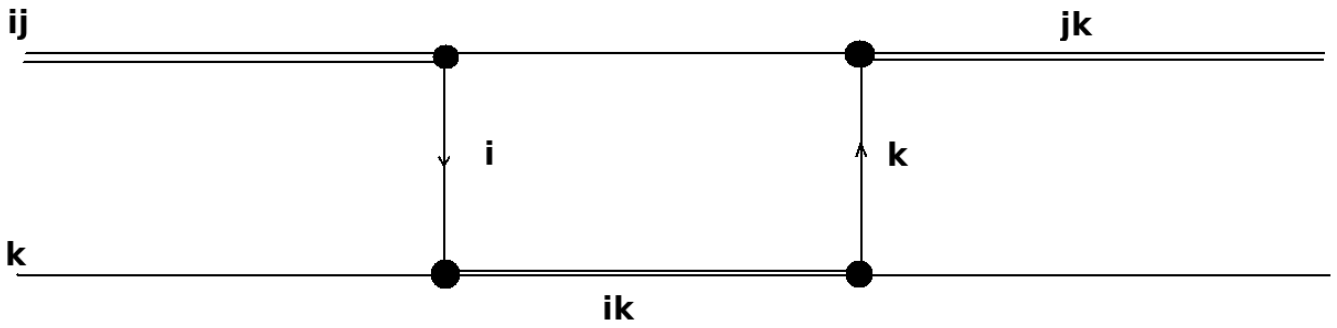

Pion
Pion
Masse [MeV]
Baryochemical Potential [MeV]
0
200
400
600
800
1000
1200
100
150
200
250
300
350
400
450
500
550
600

Kaons



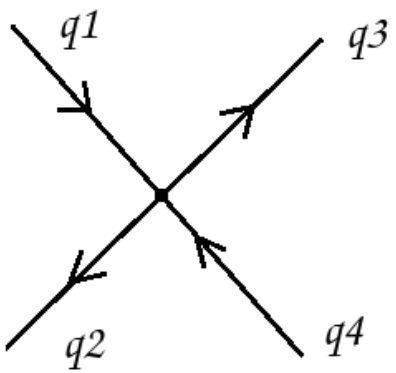
Here we use : $\mu_s = 0$

Baryons



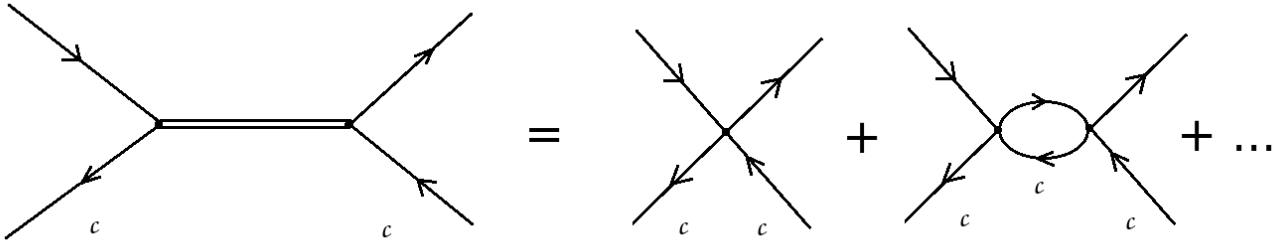
- System of a quark and a diquark
- We consider that the exchange quark is heavy enough to use the static approximation

Fierz Transform



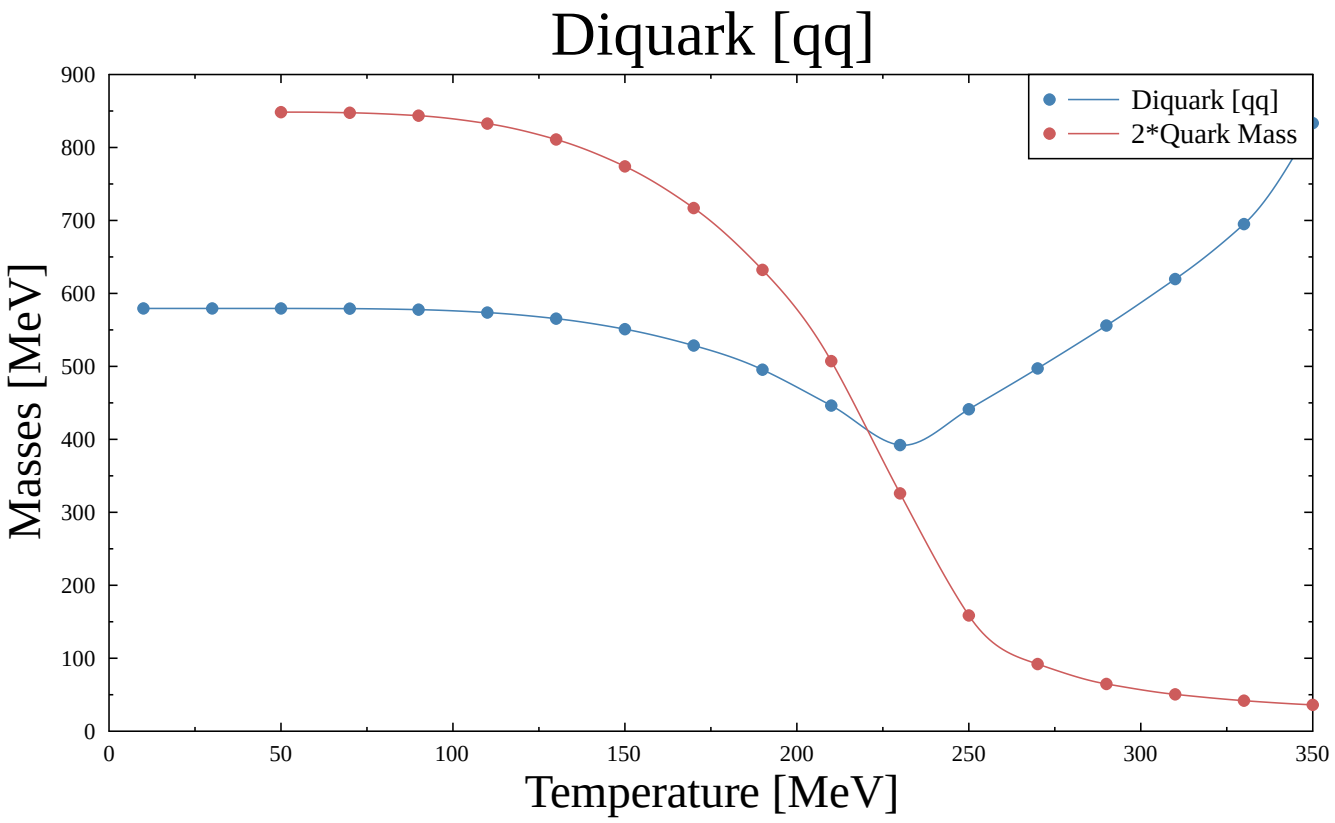
- A mathematic transformation allows to reorganize the indices
- $L_{int} = g(\overline{q}\Gamma q)^2 = g\Gamma_{ij}\Gamma_{kl}\bar{q}_i q_l \bar{q}_k q_j$
- $L_{int} = g\Gamma_{ij}\Gamma_{kl}\bar{q}_i \bar{q}_k q_l q_j = L_{qq}$

Diquarks

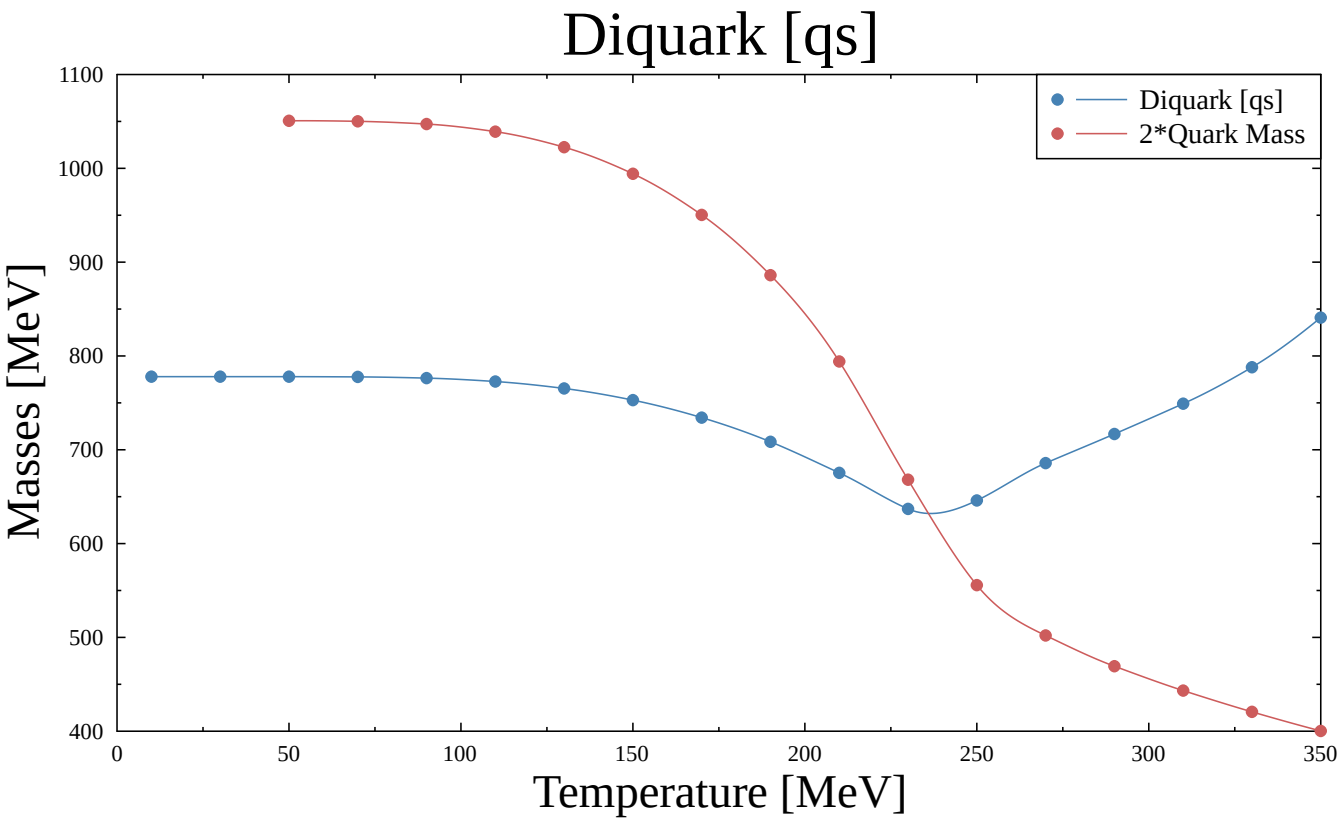


- Can be treated like a meson by replacing the antiquark by a charge conjugated quark
- $L_{qq} = G_{Diq}(\bar{q}\Gamma C\bar{q}^T)^2(q^T C\Gamma q)^2 q_j$
- $G_{Diq} = \frac{3}{4}G$

Diquark[qq]


Diquark [qq]
Diquark [qq]
2*Quark Mass
Masses [MeV]
Temperature [MeV]
0
100
200
300
400
500
600
700
800
900
0
50
100
150
200
250
300
350

Diquark[qs]


Diquark [qs]
Diquark [qs]
2*Quark Mass
Masses [MeV]
Temperature [MeV]
1100
1000
900
800
700
600
500
400
0
50
100
150
200
250
300
350

Other sector

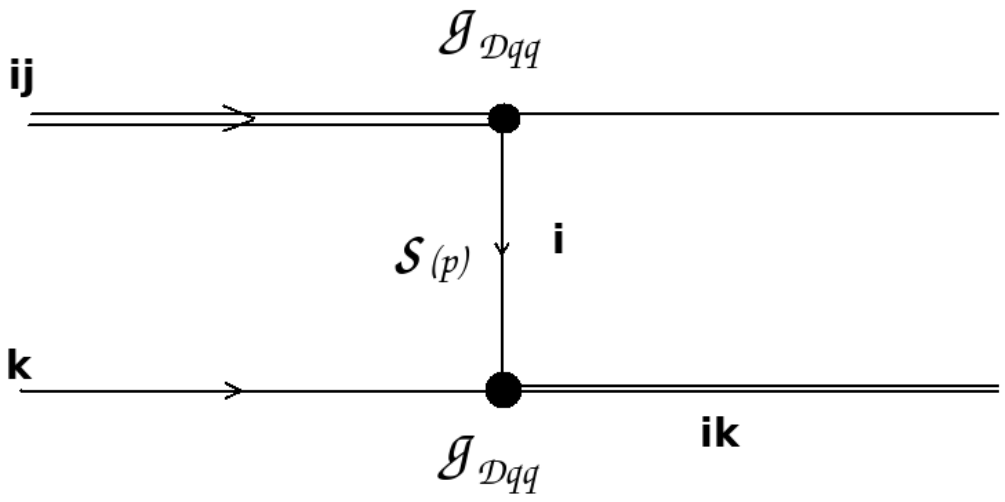
Diquark	$[ud]_S$	$[us]_S$	$[ud]_{PS}$	$[us]_{PS}$
Mass[MeV]	674	795	929	1146
Width[MeV]	-	-	91	126

$[ud]_V$	$[us]_V$	$[ud]_A$	$[us]_A$
1229	1430	847	1017
714	734	-	-

Results

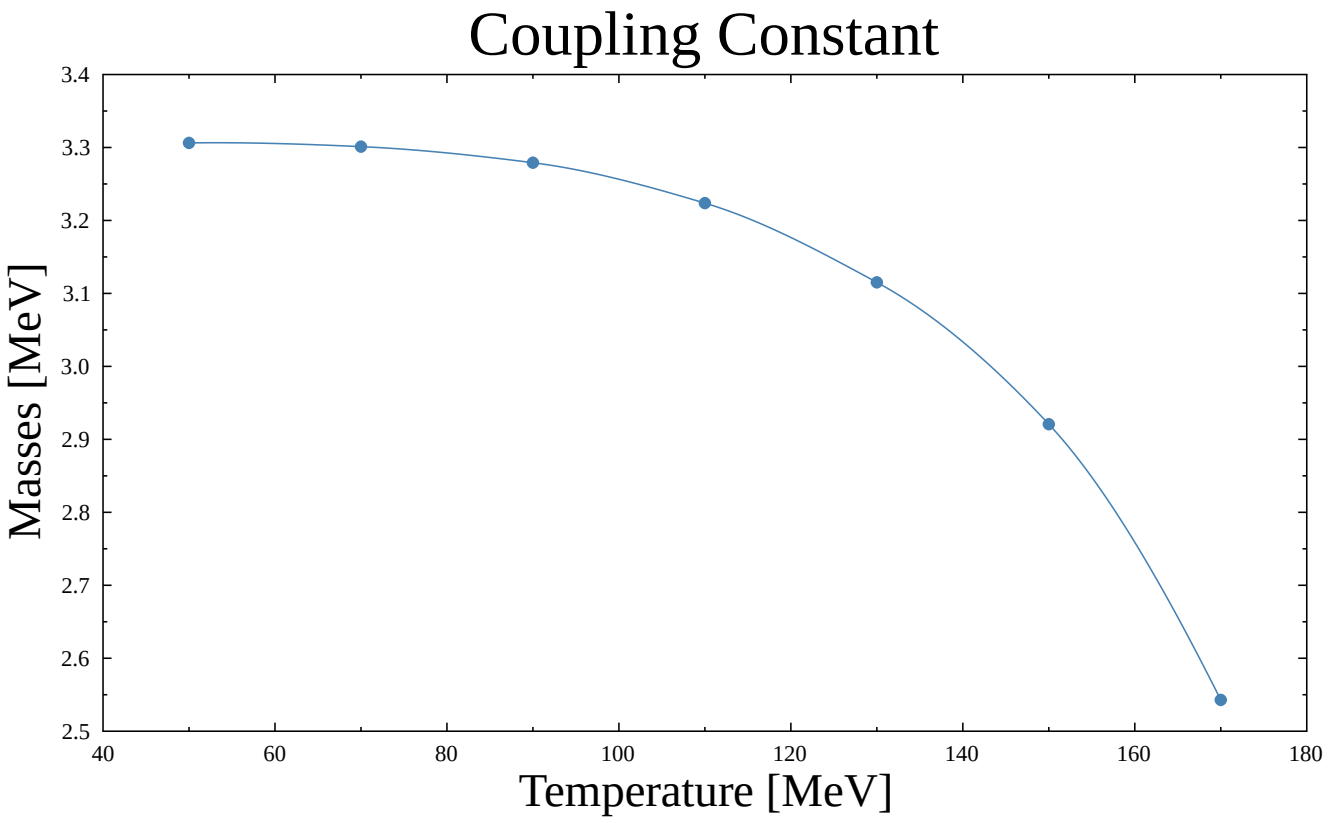
Masses (MeV)	Costa	P1	Experimental
u	367,6	424,2	
s	549,5	626,5	
Pion	135,0	135,9	135
Kaon	497,7	548,5	498
Diquark [ud]	525,6	599,1	623
Diquark [us]	700,9	794,8	-

Baryon Vertex

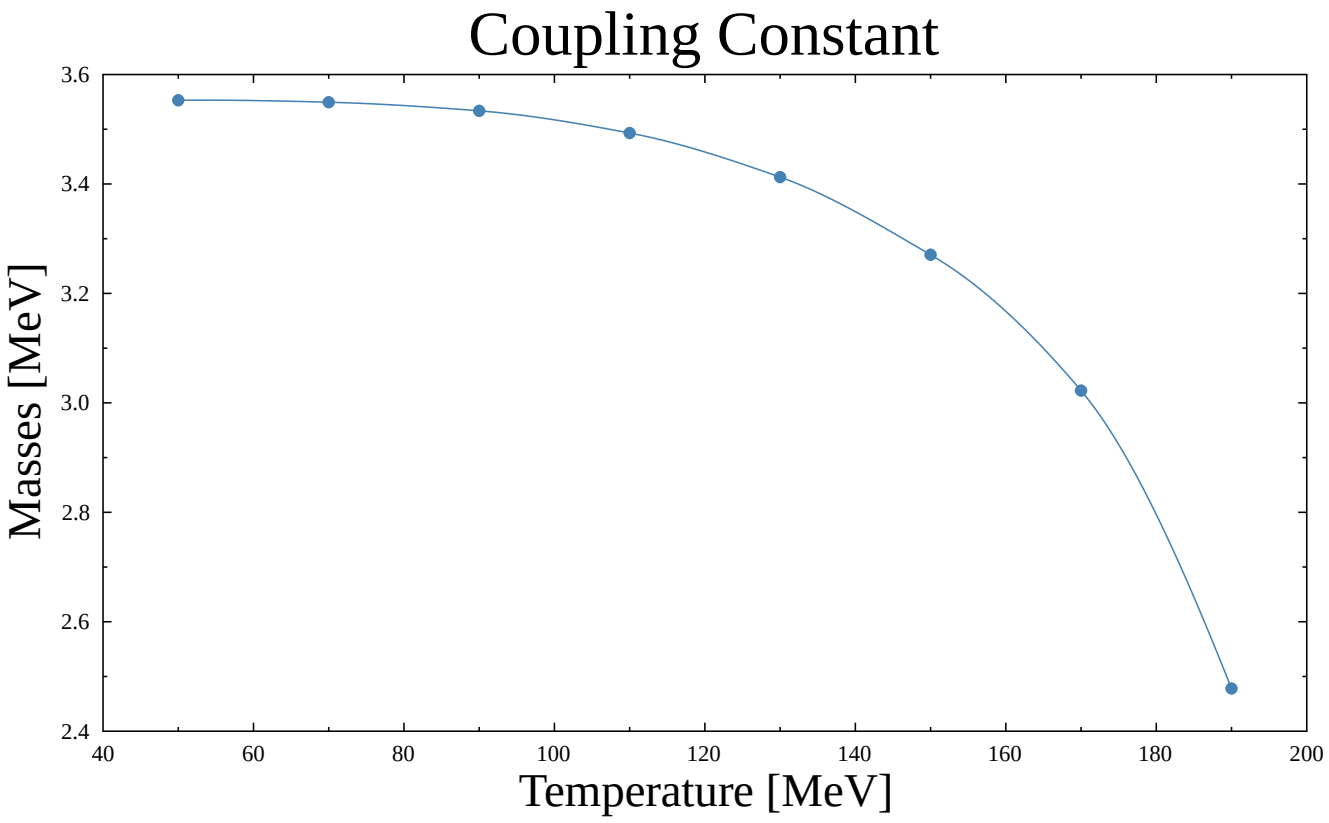


- All the possibles exchanges have to be projected on the wavefunction.
- We obtain a matrix equation, in Dirac and flavour spaces.

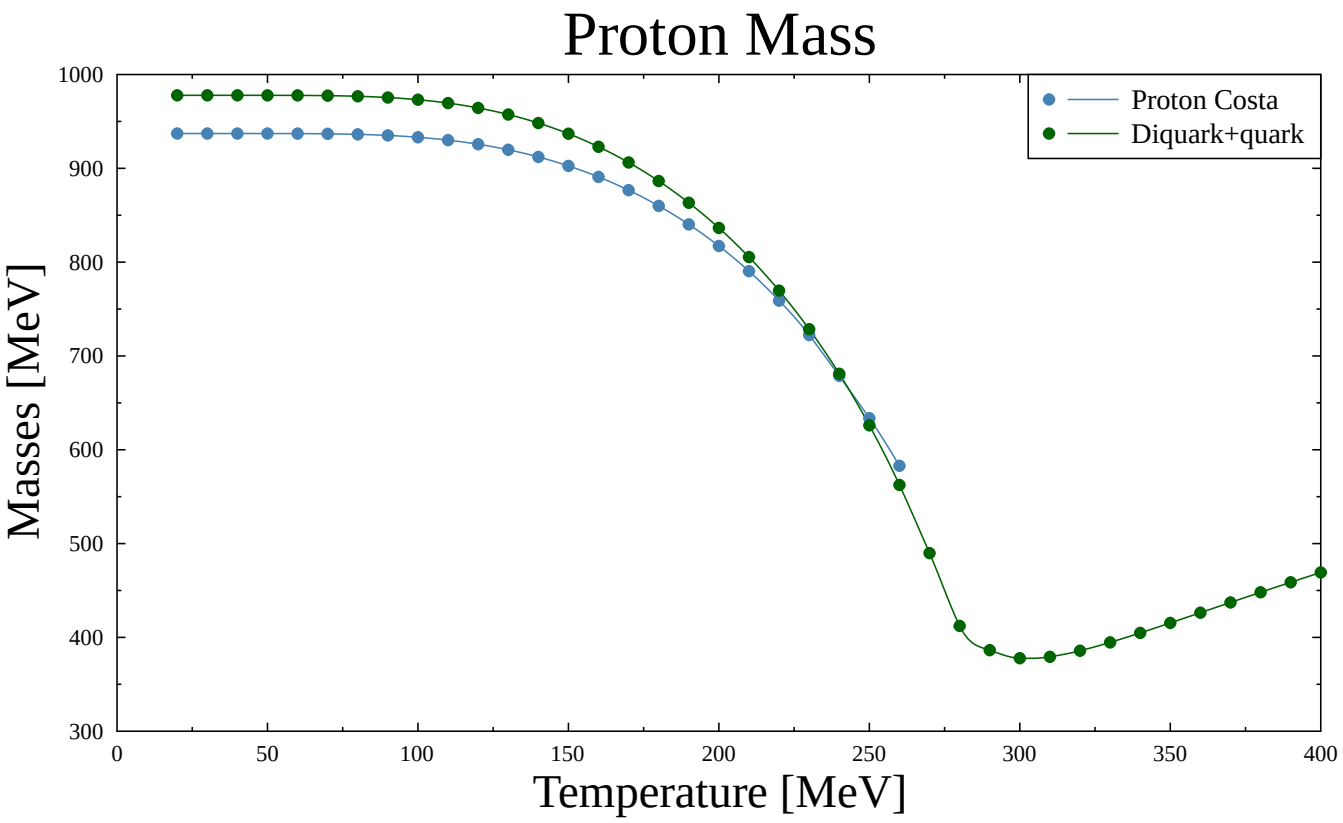
Coupling Constant



Coupling Constant



Proton


Proton Mass
Masses [MeV]
Temperature [MeV]
Proton Costa
Diquark+quark
1000
900
800
700
600
500
400
300
0
50
100
150
200
250
300
350
400

Summary

- We can reproduce mesons and diquarks masses or baryons with a parameter set.

- Outlook
  - We need to find a parametrization where baryons are bound
  - Cross sections need to be added