

GENERATION OF HIGHER FLOW HARMONICS IN HYDRODYNAMICS WITH JETS IN RELATIVISTIC HIC'S

E. Zabrodin

in collaboration with

**L. Bravina, H. Brusheim Johansson, J. Crkovska,
G. Eyyubova, V. Korotkikh, I. Lokhtin, L. Malinina,
S. Petrushanko, and A. Snigirev**

University of Oslo and Moscow State University

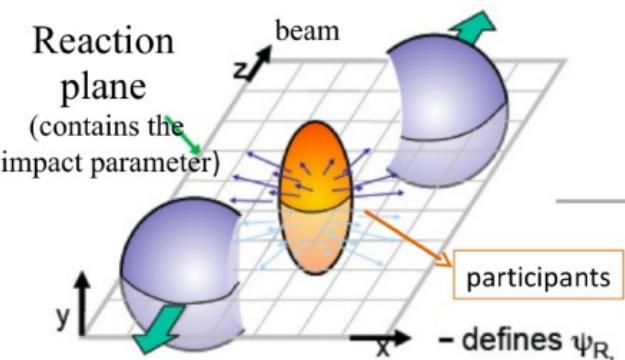
NeD/TURIC-2014

Hersonissos, Crete, Greece (9.06-14.06.2014)

OUTLINE

- I. Description of anisotropic flow in relativistic heavy ion collisions:**
 - (a) elliptic and triangular flows**
 - (b) initial fluctuations and higher harmonics**
- II. HYDJET++ model (hydro + jets)**
- III. Quadrangular flow and pentagonal flow**
- IV. Hexagonal flow**

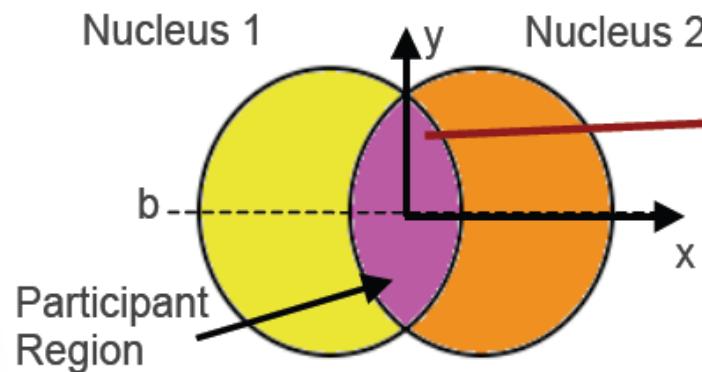
I. DESCRIPTION OF ANISOTROPIC FLOW IN RELATIVISTIC HEAVY ION COLLISIONS



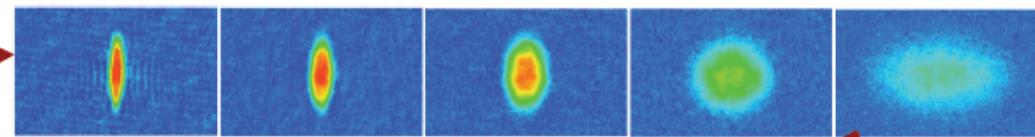
ELLIPTIC FLOW

Initial spatial anisotropy is converted to anisotropy in momentum space

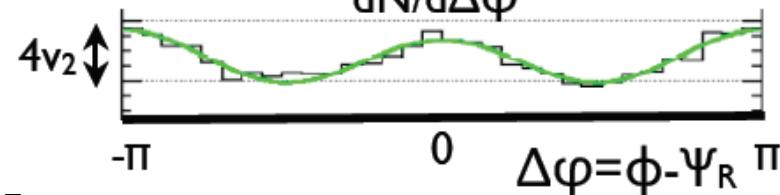
Initial anisotropy



Pressure driven expansion



Final anisotropy



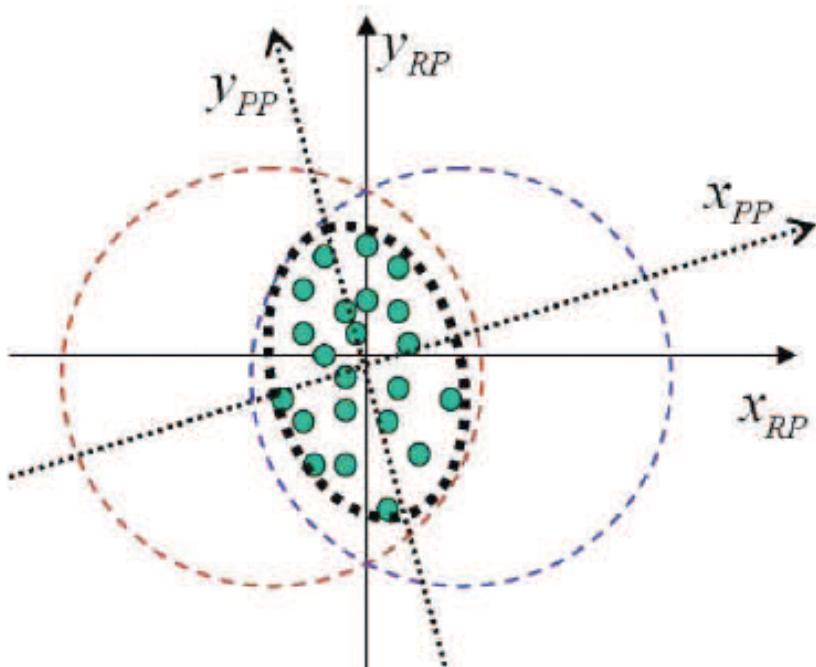
$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left(1 + \sum 2v_n \cos(n(\phi - \psi_R)) \right)$$

S.Voloshin, Y.Zhang, Z.Phys.C70 (1996) 665

$$v_2 = \langle \cos(2(\phi - \psi_R)) \rangle \propto \epsilon$$

Elliptic flow is quantified by the second Fourier coefficient (v_2) of the observed particle distribution

Eccentricity fluctuations



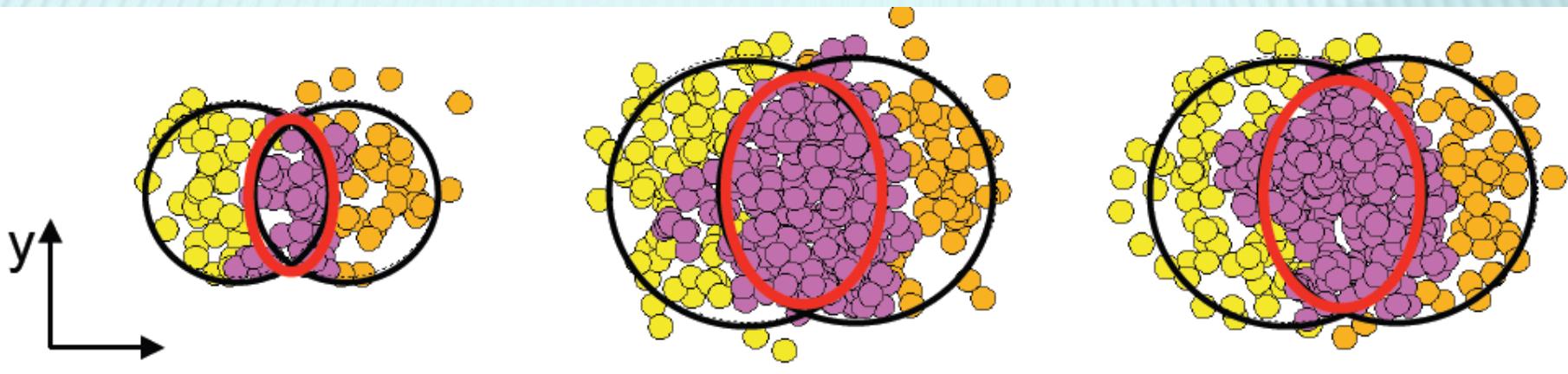
Depending on where the participant nucleons are located within the nucleus at the time of the collision, the orientation and eccentricity of the ellipse defined by participants fluctuates.

Assuming that v_2 scales like the eccentricity, eccentricity fluctuations translate into v_2 fluctuations

Eccentricity fluctuations can be computed in MC Glauber model or derived from experiment by comparing different methods for flow calculation.

ECCENTRICITY

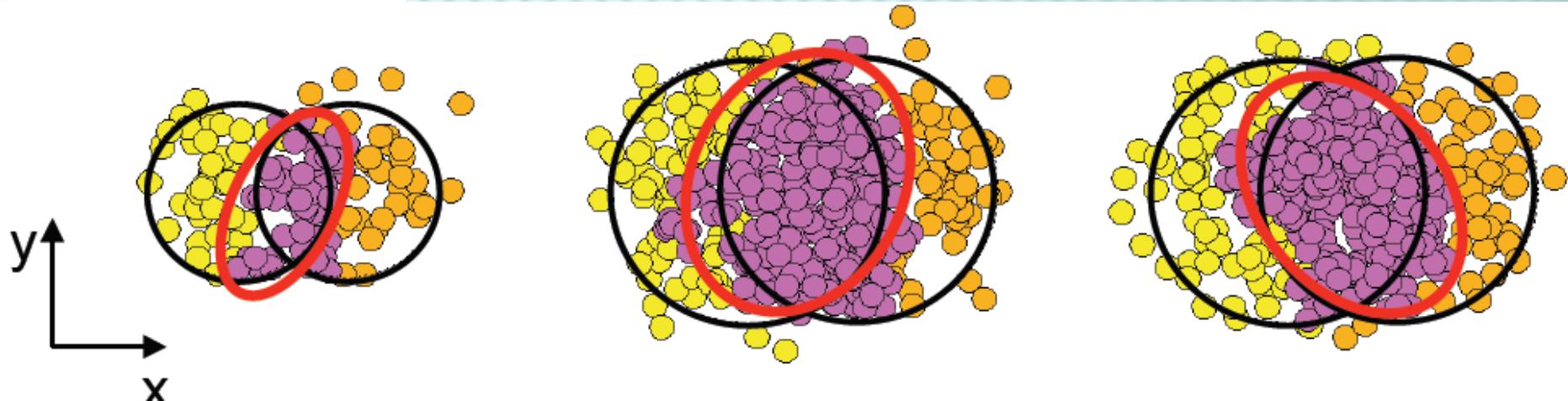
STANDARD



$$\varepsilon_{RP} = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle}$$

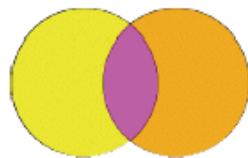
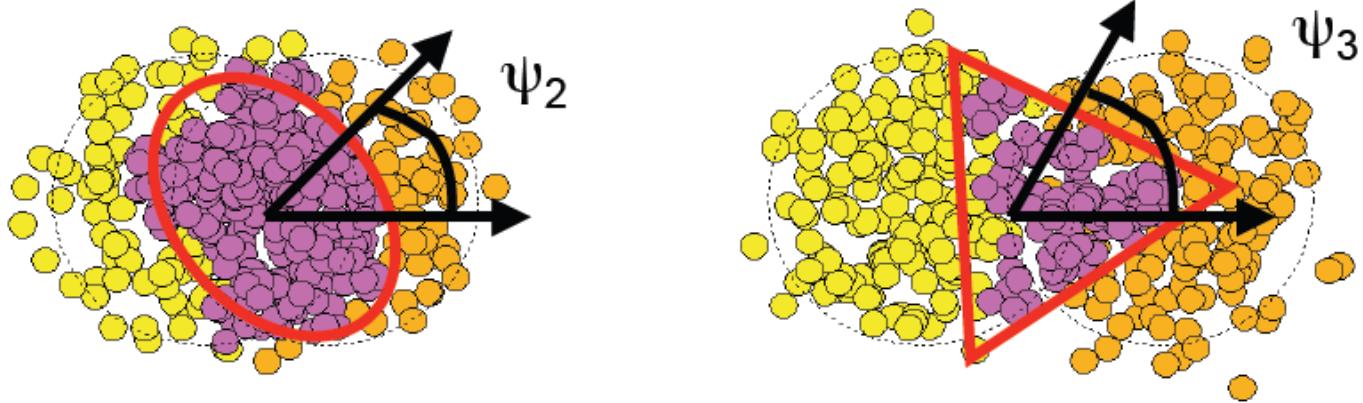
PARTICIPANT

$$\varepsilon_{part} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2}$$



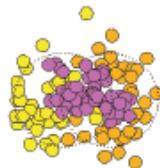
TRIANGULAR FLOW

B. Alver and G.Roland, PRC 81 (2010) 054905



$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left(1 + \sum 2v_n \cos(n(\phi - \psi_R)) \right)$$

$$v_2 = \langle \cos(2(\phi - \psi_R)) \rangle$$
$$v_3 = 0$$



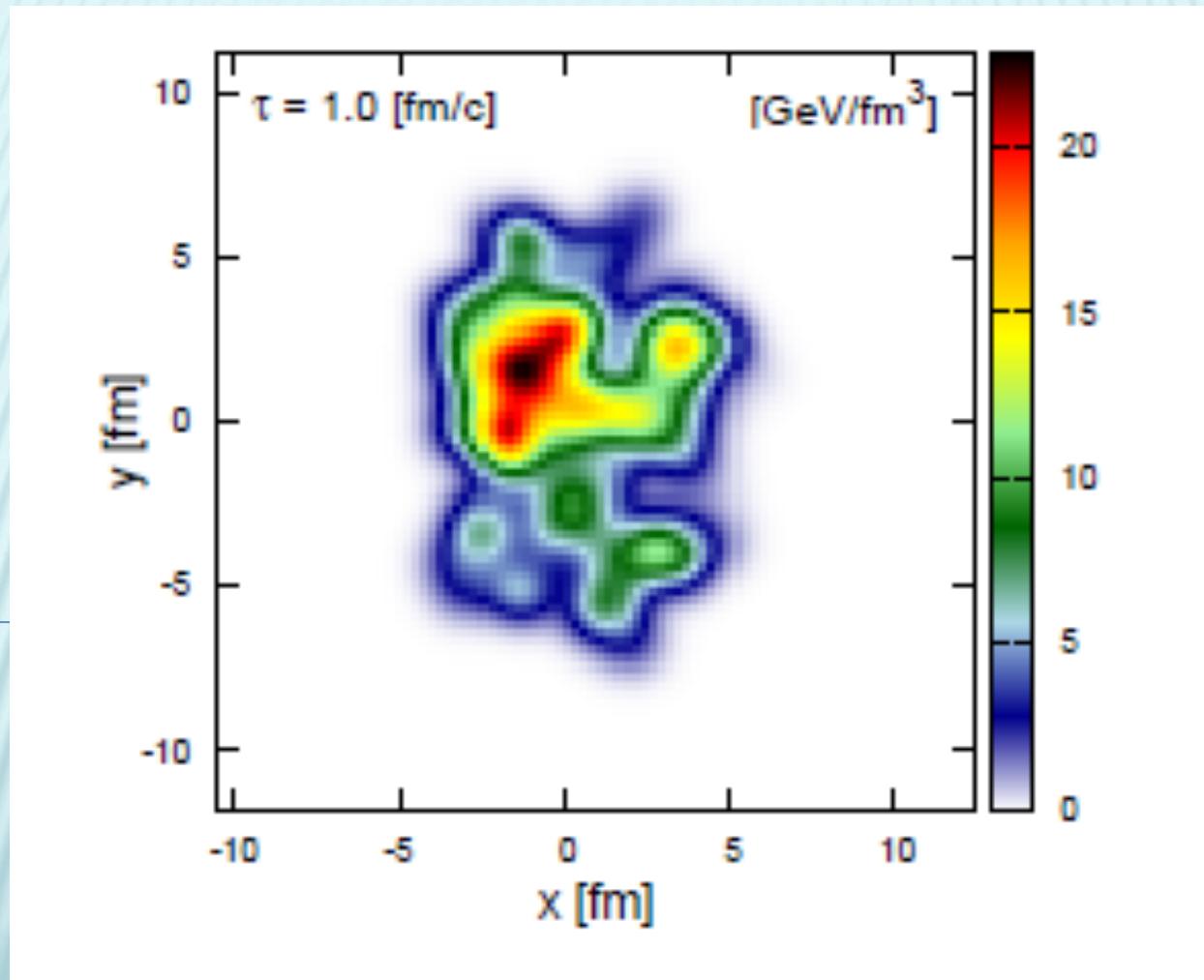
$$\frac{dN}{d\phi} = \frac{N}{2\pi} \left(1 + \sum 2v_n \cos(n(\phi - \psi_n)) \right)$$

$$v_2 = \langle \cos(2(\phi - \psi_2)) \rangle$$
$$v_3 = \langle \cos(3(\phi - \psi_3)) \rangle$$

The triangular initial shape leads to triangular hydrodynamic flow

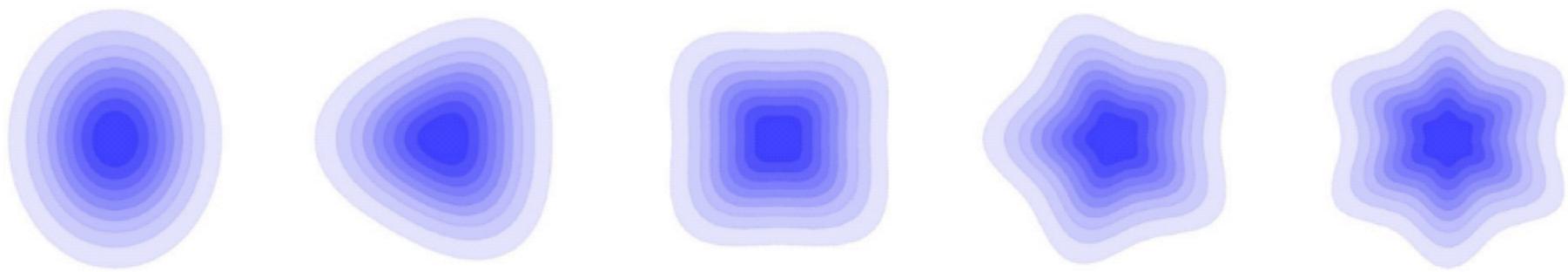
INITIAL-STATE FLUCTUATIONS (example)

W.-L. Qian et al., JPG 41 (2013) 015103



Energy distribution of a random NeXuS event

HIGHER FLOW HARMONICS



$n = 2$

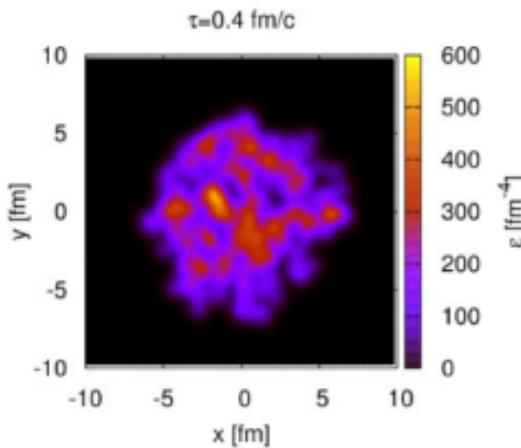
$n = 3$

$n = 4$

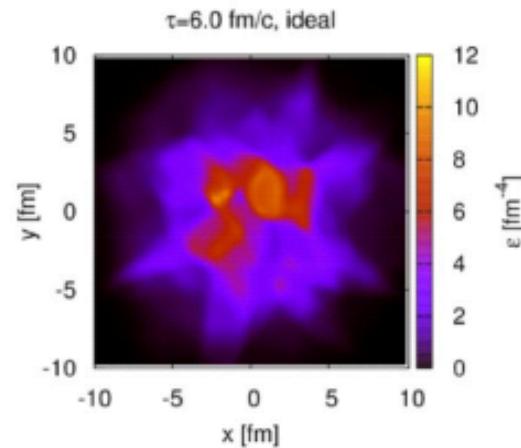
$n = 5$

$n = 6$

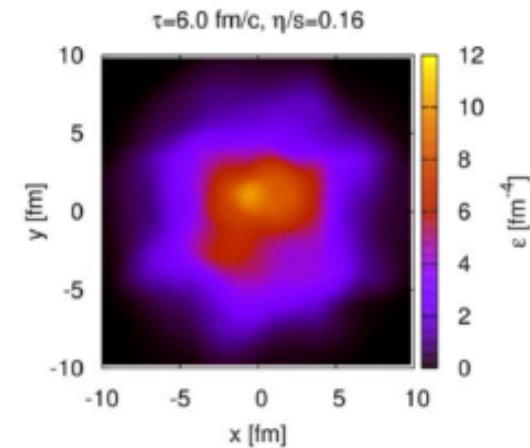
initial



ideal

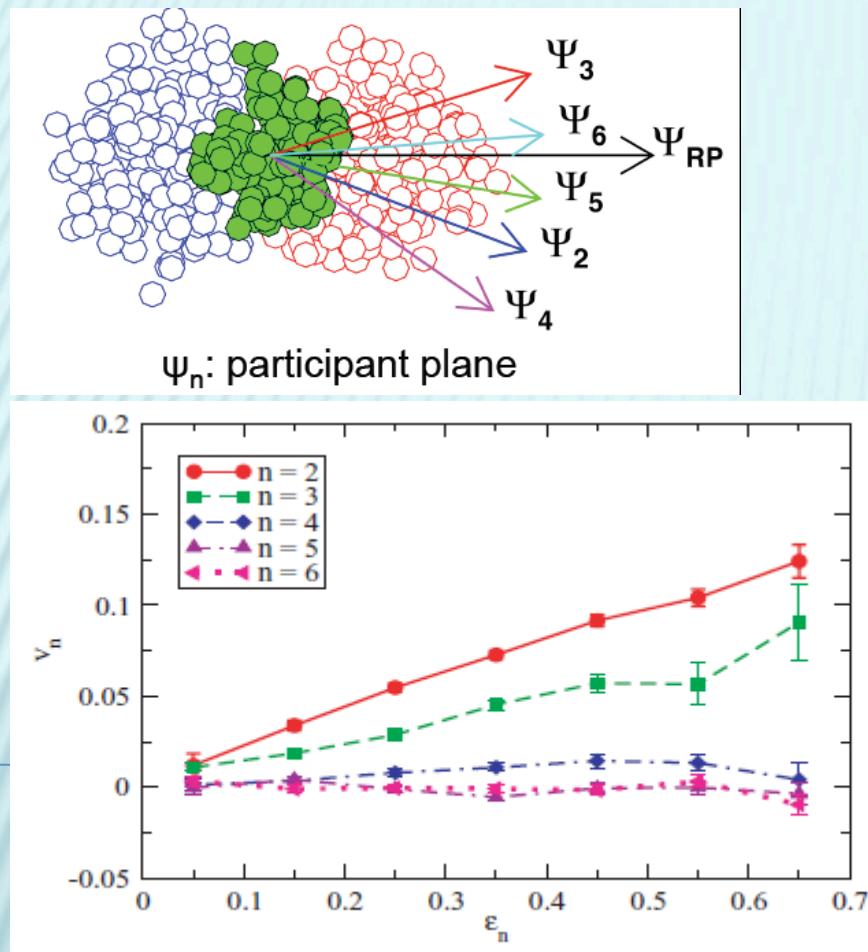


viscous

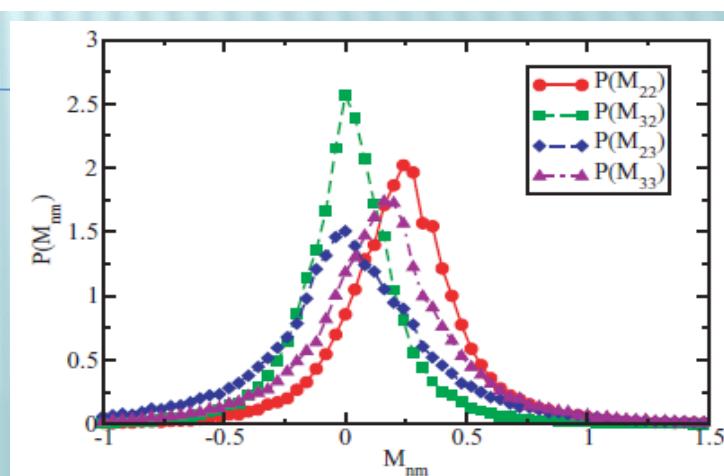
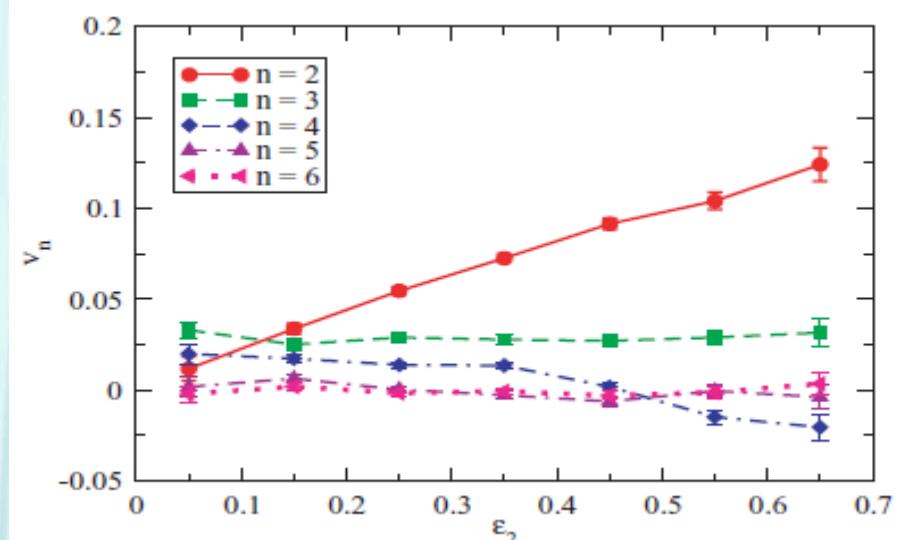


Non-zero higher Fourier coefficients can carry important information about the space-time evolution of the QCD-matter and initial fluctuations

CROSS-TALK BETWEEN FLOW HARMONICS



G.-Y. Qin et al, PRC 82 (2010) 064903



$$\begin{pmatrix} v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} M_{22} & M_{23} \\ M_{32} & M_{33} \end{pmatrix} \begin{pmatrix} \epsilon_2 \\ \epsilon_3 \end{pmatrix}$$

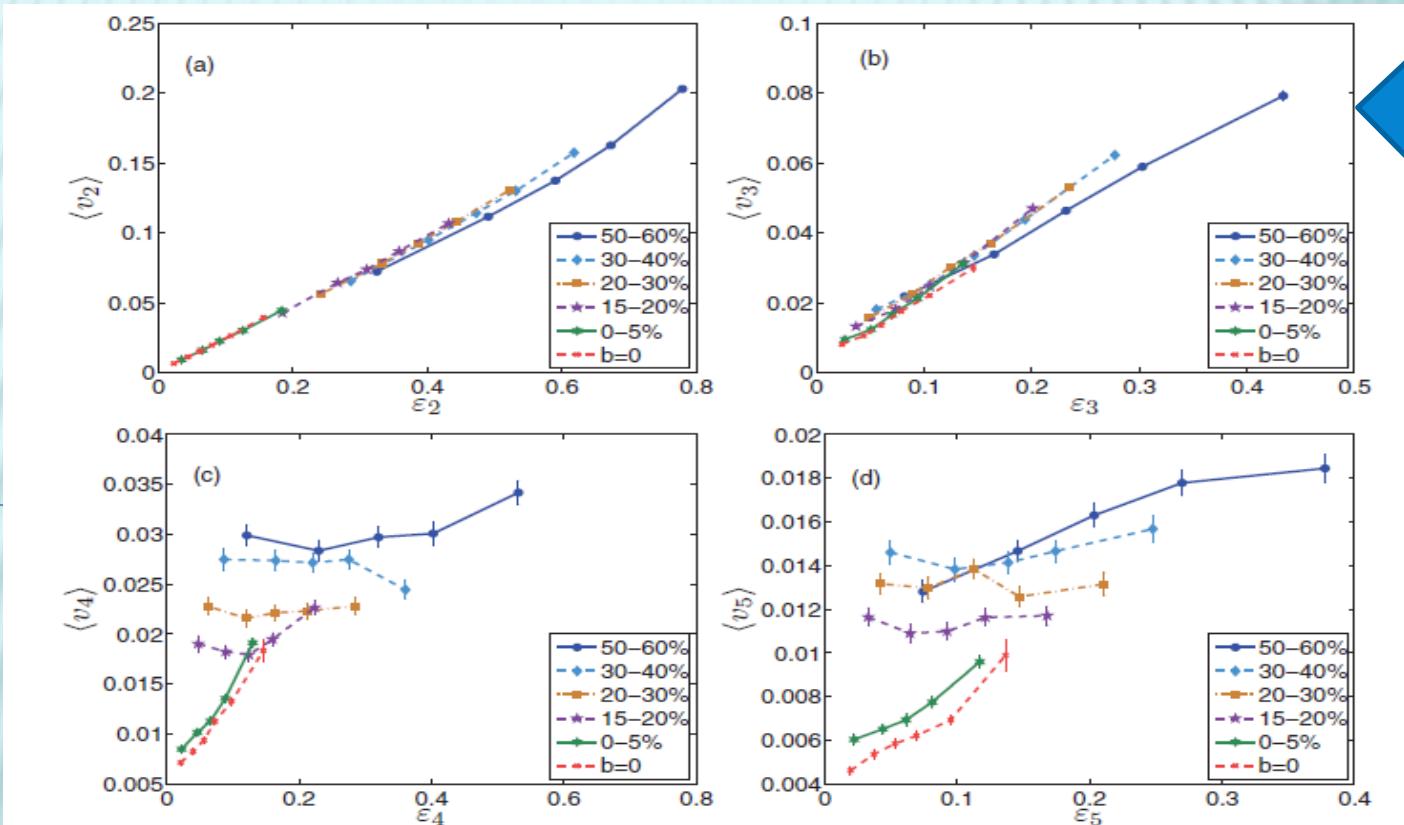
Only the first few flow harmonics of final-state hadrons survive after hydrodynamic evolution

CROSS-TALK BETWEEN FLOW HARMONICS

Z. Qiu and U. Heinz, PRC 84 (2011) 024911

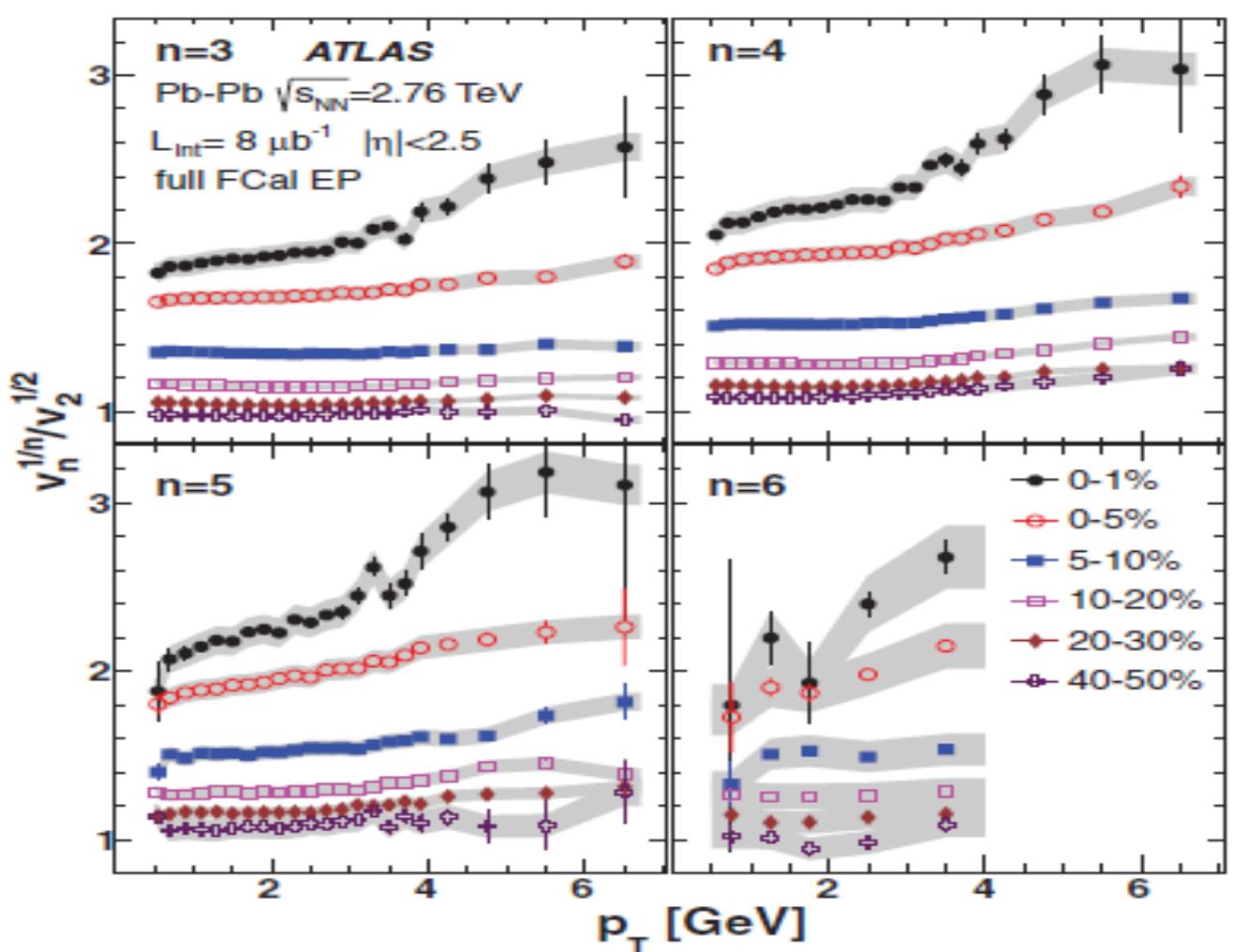
$$\varepsilon_n e^{in\psi_n^{\text{pp}}} = - \frac{\int dx dy r^2 e^{in\phi} e(x, y)}{\int dx dy r^2 e(x, y)}$$

$$v_n(y, p_T) e^{in\psi_n^{\text{EP}}(y, p_T)} = \frac{\int d\phi_p e^{in\phi_p} \frac{dN}{dy p_T dp_T d\phi_p}}{\frac{dN}{dy p_T dp_T}}$$



- (1) The basic response of v_2 and v_3 to eccentricities is approx. linear
- (2) Higher flow coefficients show poor correlation with the eccentricities of the same order

SCALING OF HIGHER ORDER FLOW HARMONICS



J.-Y. Ollitrault :

$$V \downarrow n \frac{1}{n} / n \propto V \downarrow 2 \frac{1}{2} / 2$$

Ratios are almost insensitive to P_T from 1 GeV/c to 4 GeV/c

Results for $n=3$ are lower than that for higher harmonics

**II. HYDJET++ =
FASTMS + HYDJET**



HYDJE++

event generator to simulate heavy ion event
as merging of two independent components
(**soft** hydro-type part + **hard** multi-partonic state)

<http://cern.ch/lokhtin/hydjet++>

*I.Lokhtin, L.Malinina, S.Petrushanko, A.Snigirev, I.Arsene, K.Tywoniuk,
Comp.Phys.Comm. 180 (2009) 779*

Soft

the “thermal” hadronic state generated on the chemical and thermal freeze-out hypersurfaces obtained from the parametrization of relativistic hydrodynamics with present freeze-out conditions (the adapted event generator **FAST MC**).

Hard

the multiple scattering of hard partons is based on accumulated energy loss via gluon radiation which is associated with each parton scattering in expanding quark-gluon fluid (**PYQUEN** jet quenching model).

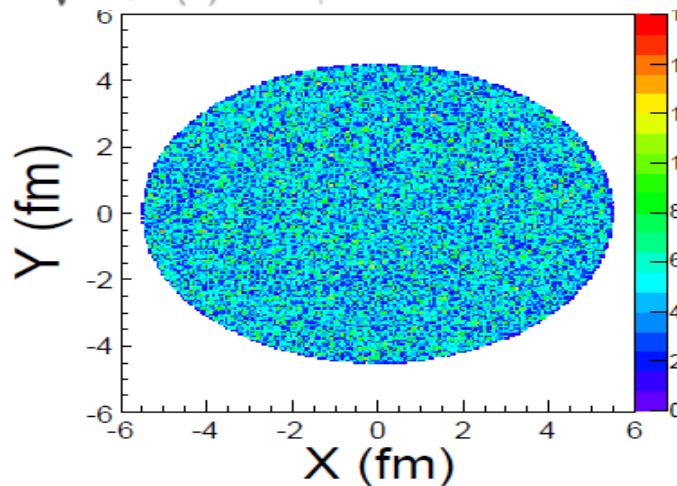
We apply HYDJET++ with tuned input parameters to reproduce the LHC data from PbPb collisions, and to estimate an influence of the hard production mechanism on physics observables.

GENERATION OF TRIANGULAR FLOW

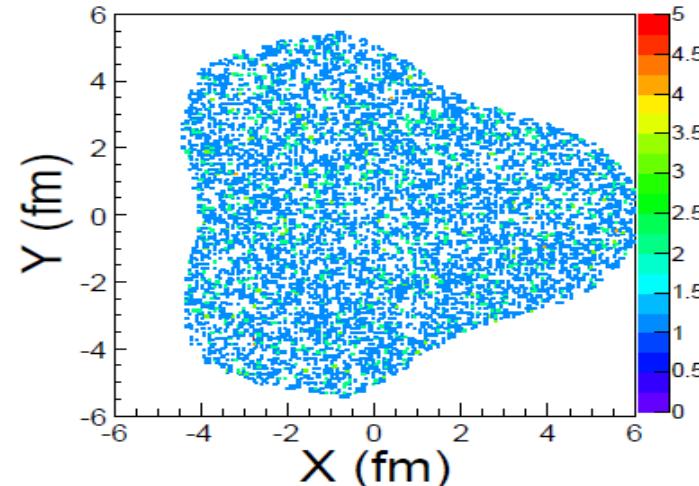
$$R(b, \phi) = R_f(b) \frac{\sqrt{1 - \epsilon^2(b)}}{\sqrt{1 + \epsilon(b) \cos 2\phi}} [1 + \epsilon_3(b) \cos 3(\phi + \Psi_3^{\text{RP}})]$$

Bravina et al., EPJC 74:2807 (2014)

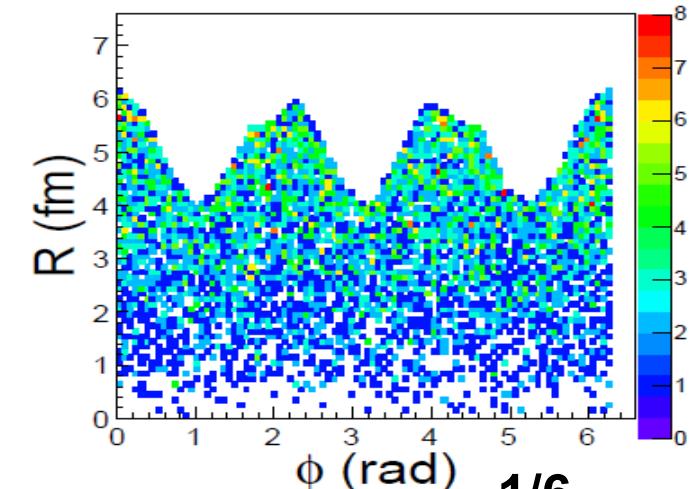
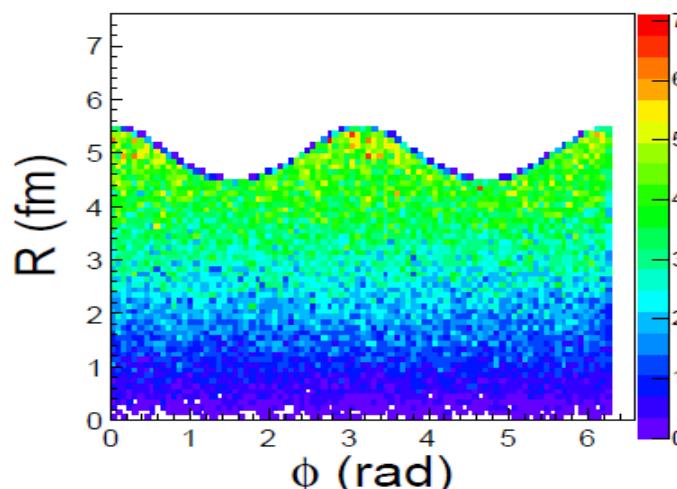
V2



V3



V2 and V3 are uncorrelated

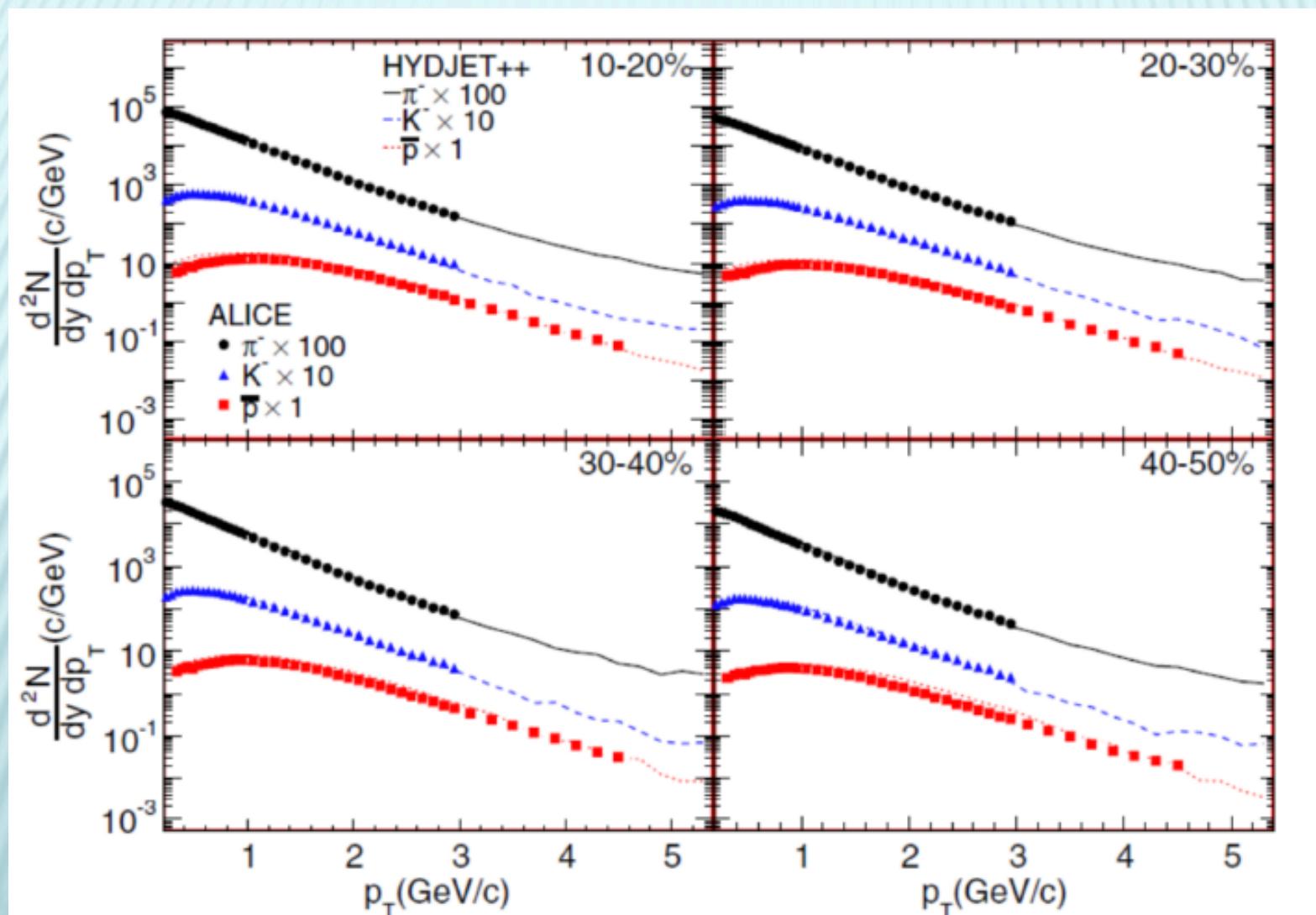


Hint (Teaney, Ollitrault, ...): $V_5 \propto V_2 V_3$ $V_6 \propto \alpha V_2^{1/6} + \beta V_3^{1/2}$

LHC DATA VS. HYDJET++ MODEL

Particle spectra

Pb+Pb @ 2.76 ATeV



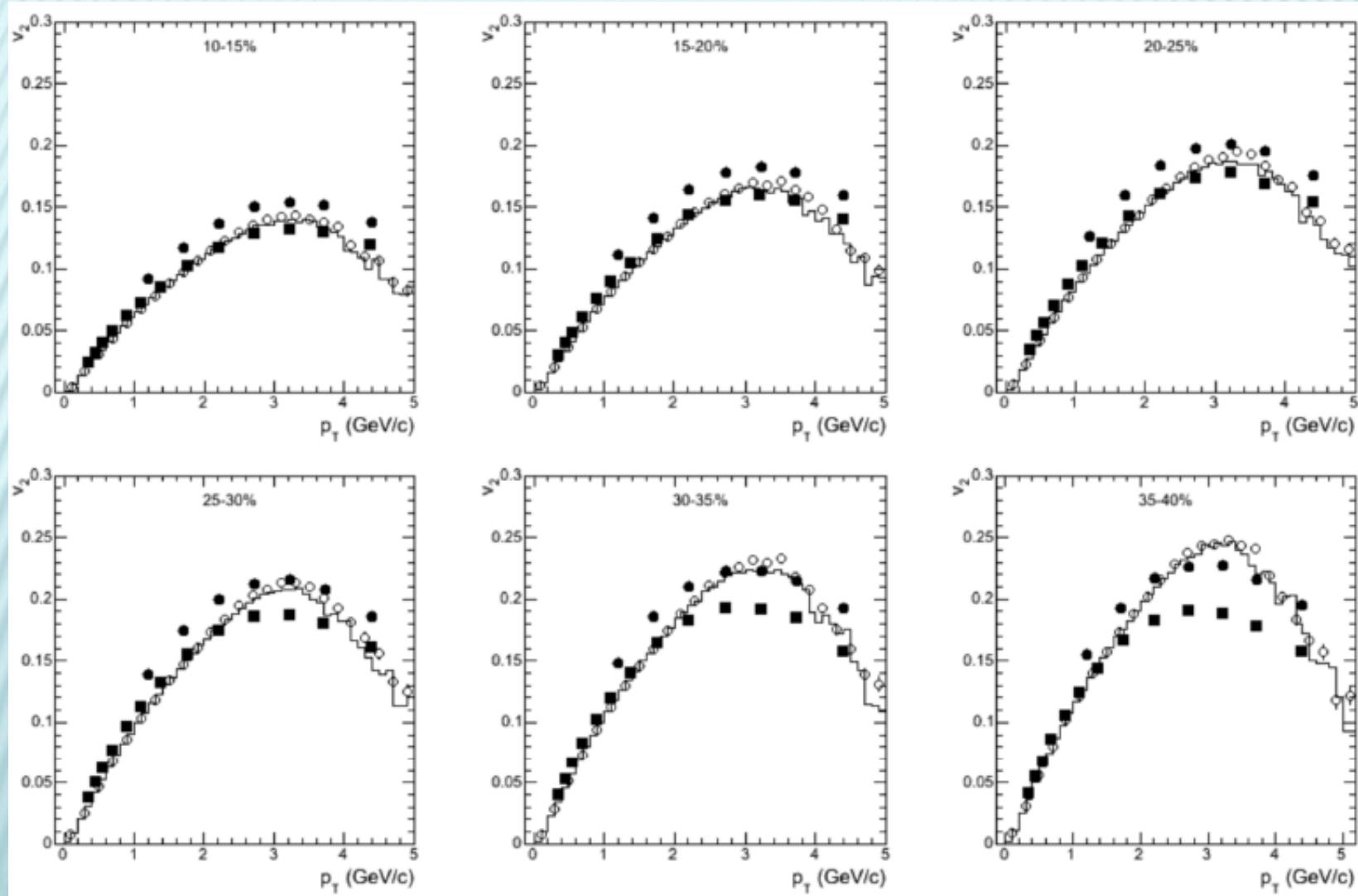
Closed symbols: ALICE data ; Lines: HYDJET++

LHC DATA VS. HYDJET++ MODEL

Elliptic flow

Pb+Pb @ 2.76 ATeV

Parameters are tuned



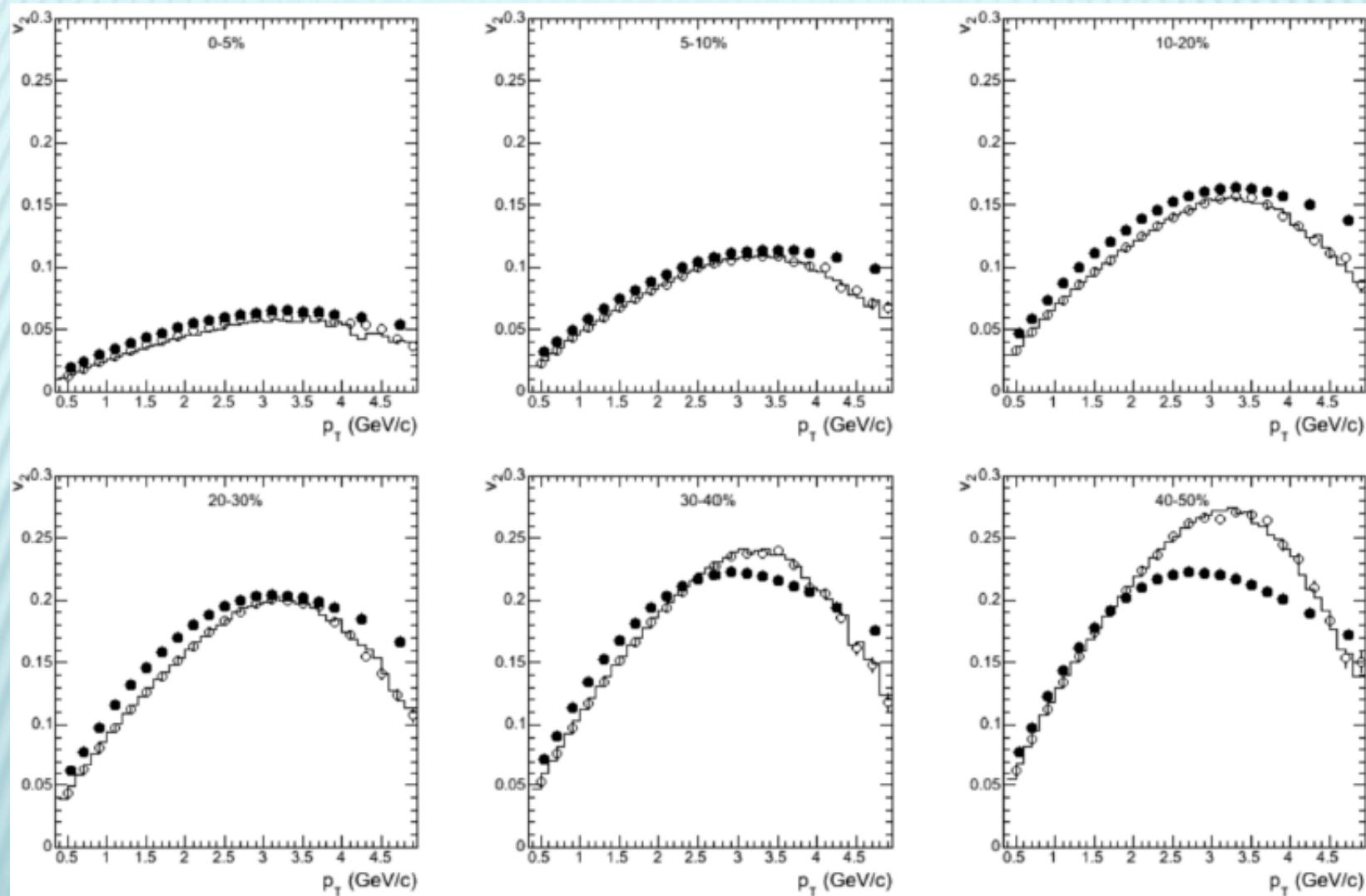
Closed symbols: CMS data $v_2\{2\text{Part} \& \text{LYZ}\}$;

Open symbols and histograms: HYDJET++ $v_2\{\text{EP} \& \text{Psi2}\}$

LHC DATA VS. HYDJET++ MODEL

Elliptic flow

Pb+Pb @ 2.76 ATeV



Closed symbols: ATLAS data $v_2\{\text{RP}\}$;

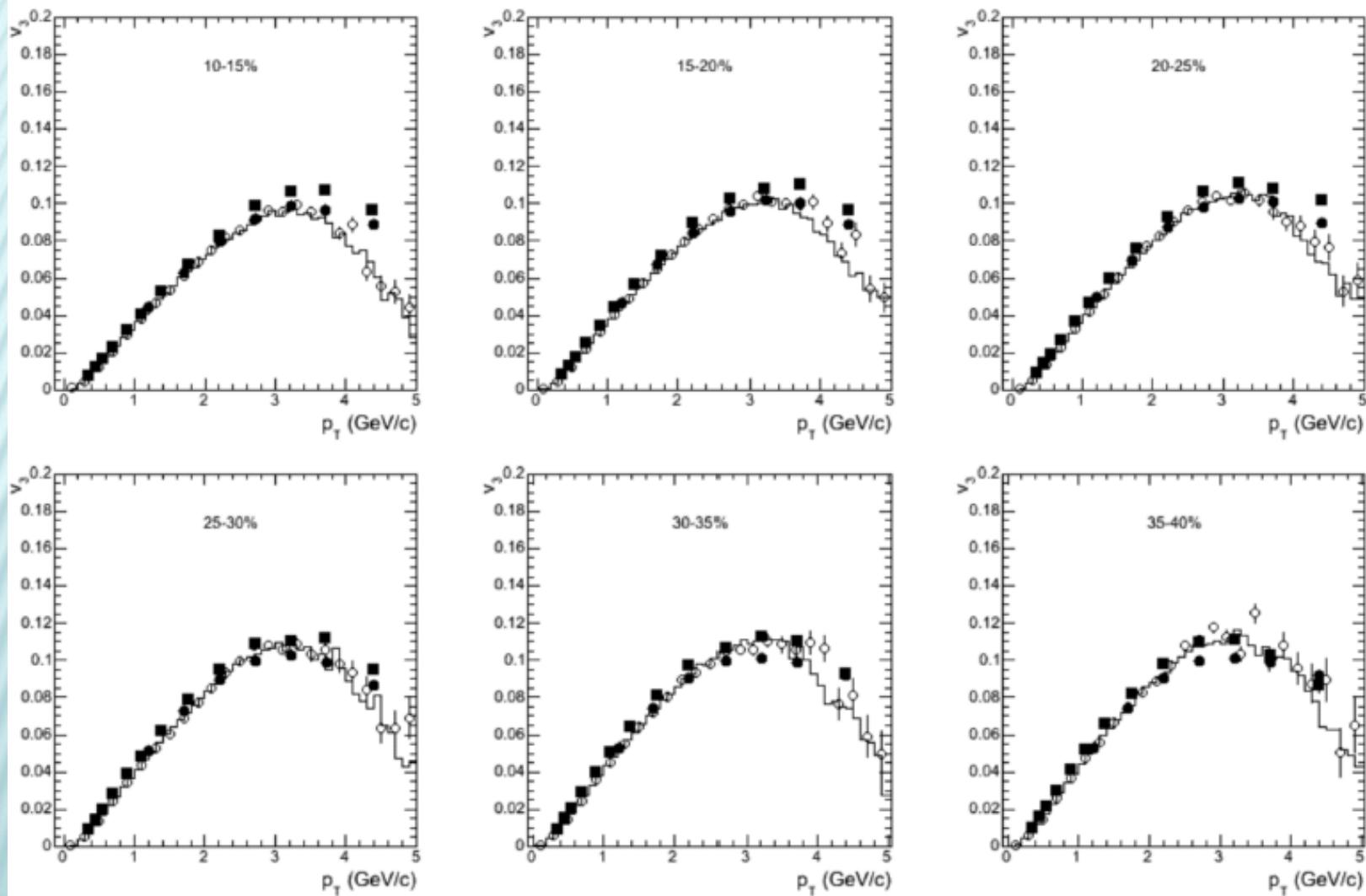
Open symbols and histograms: HYDJET++ $v_2\{\text{EP} \& \text{Psi2}\}$

LHC DATA VS. HYDJET++ MODEL

Triangular flow

Pb+Pb @ 2.76 ATeV

Parameters are tuned



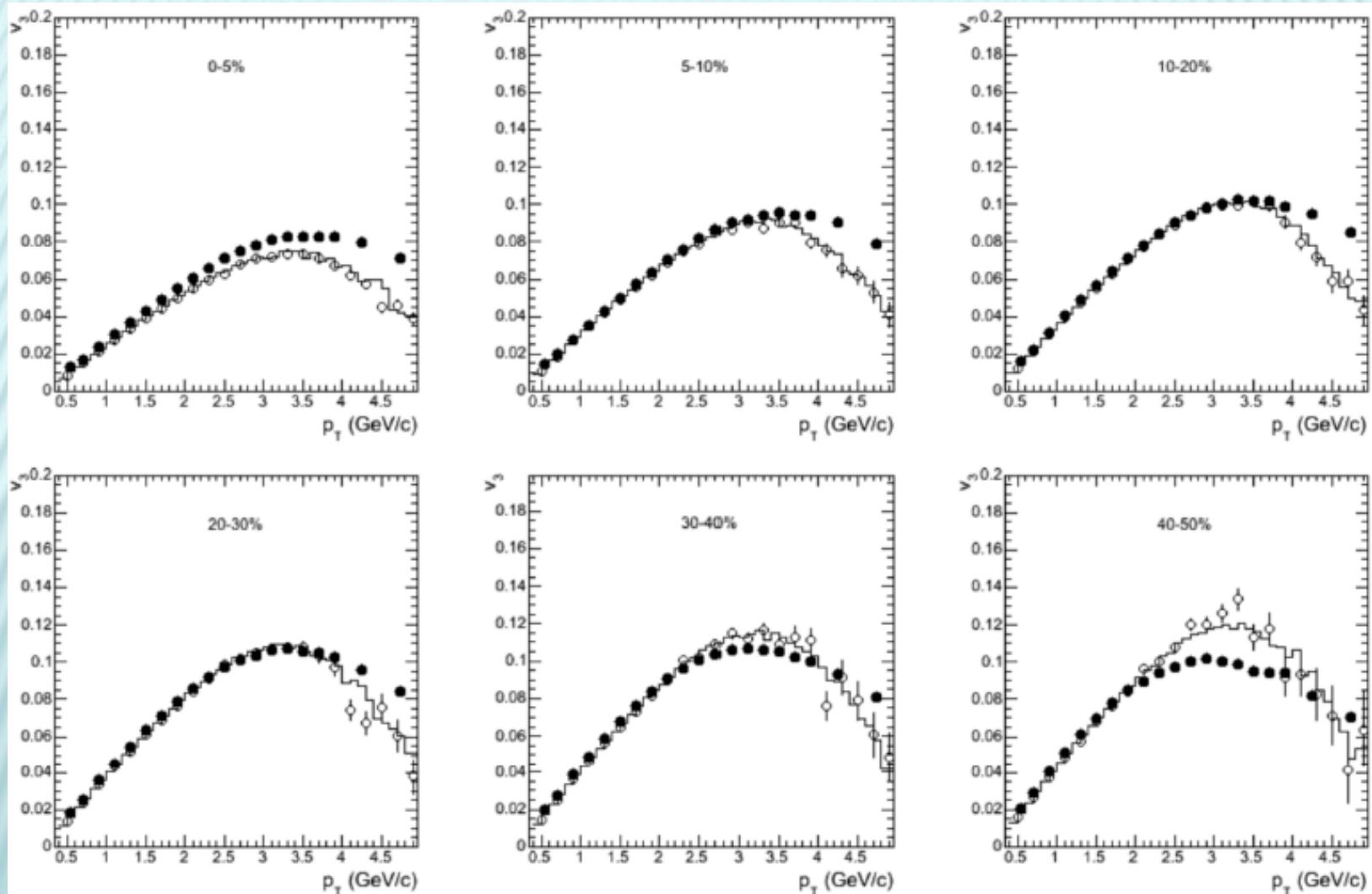
Closed symbols: CMS data $v_3\{2\text{Part} \& \text{LYZ}\}$;

Open symbols and histograms: HYDJET++ $v_3\{\text{EP} \& \text{Psi3}\}$

LHC DATA VS. HYDJET++ MODEL

Triangular flow

Pb+Pb @ 2.76 ATeV



Closed symbols: ATLAS data $v_2\{RP\}$;

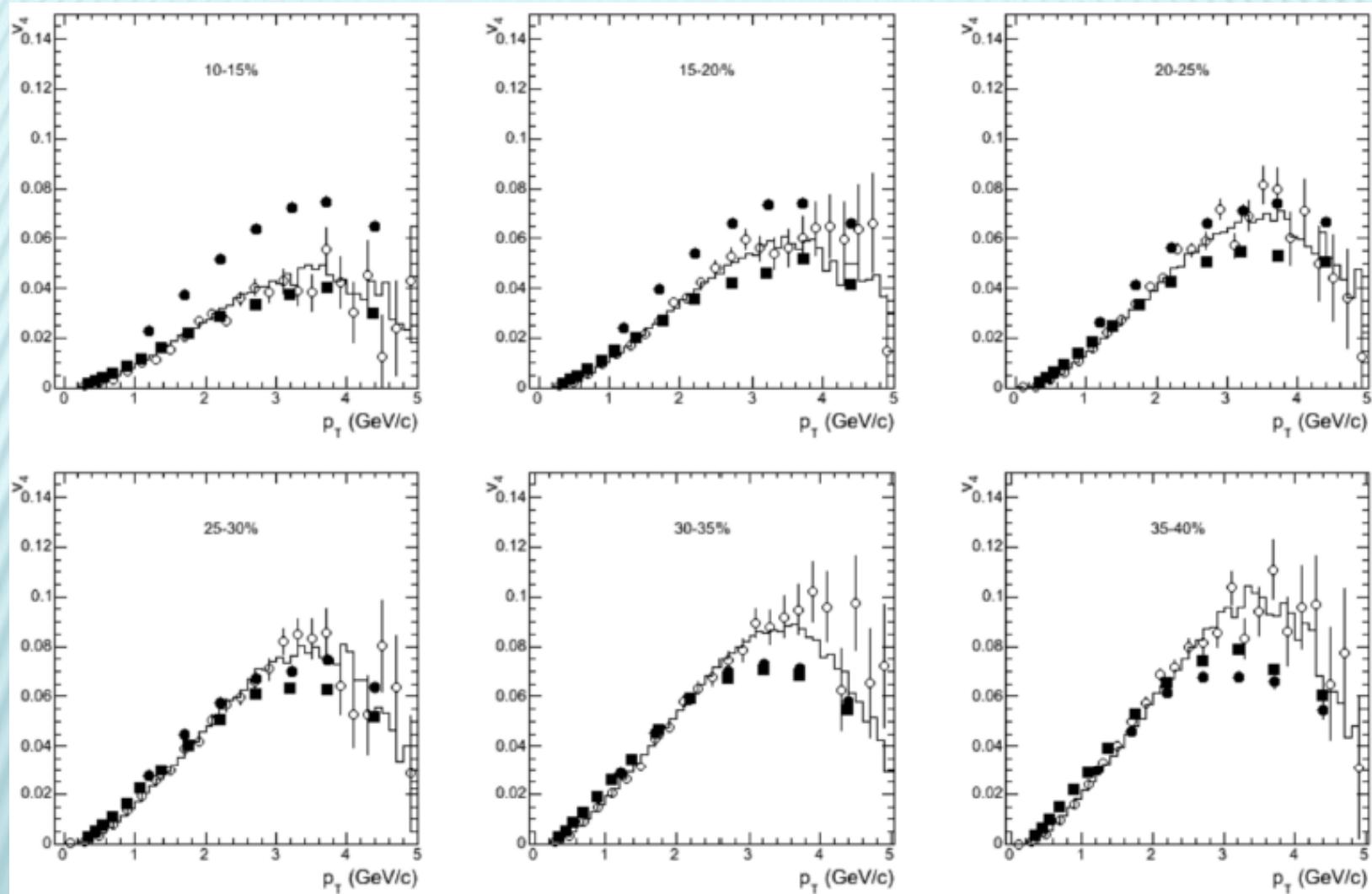
Open symbols and histograms: HYDJET++ $v_2\{EP \& Psi3\}$

III. HIGHER HARMONICS: V4 and V5

LHC DATA VS. HYDJET++ MODEL

Pb+Pb @ 2.76 ATeV

Quadrangular flow



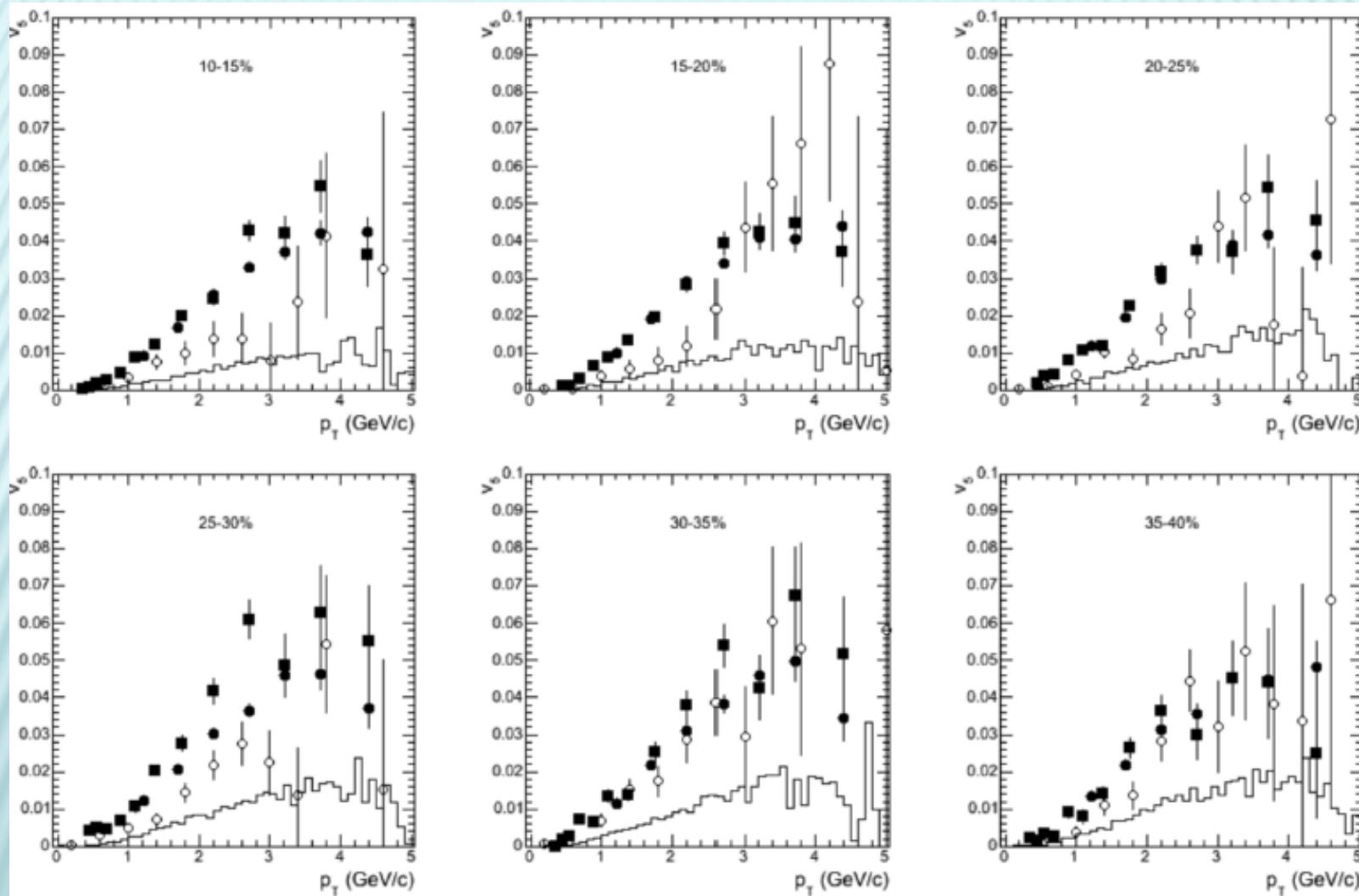
Closed symbols: CMS data $v_4\{2\text{Part} \& \text{LYZ}\}$;
 Open symbols and histograms: HYDJET++ $v_4\{\text{EP} \& \text{Psi2}\}$

**v4 is there even if
v3 is absent**

LHC DATA VS. HYDJET++ MODEL

Pb+Pb @ 2.76 ATeV

Pentagonal flow



Closed symbols: CMS data $v_5\{2\text{Part} \& \Psi\text{5}\}$;

Open symbols and histograms: HYDJET++ $v_5\{\text{EP} \& \Psi\text{3}\}$

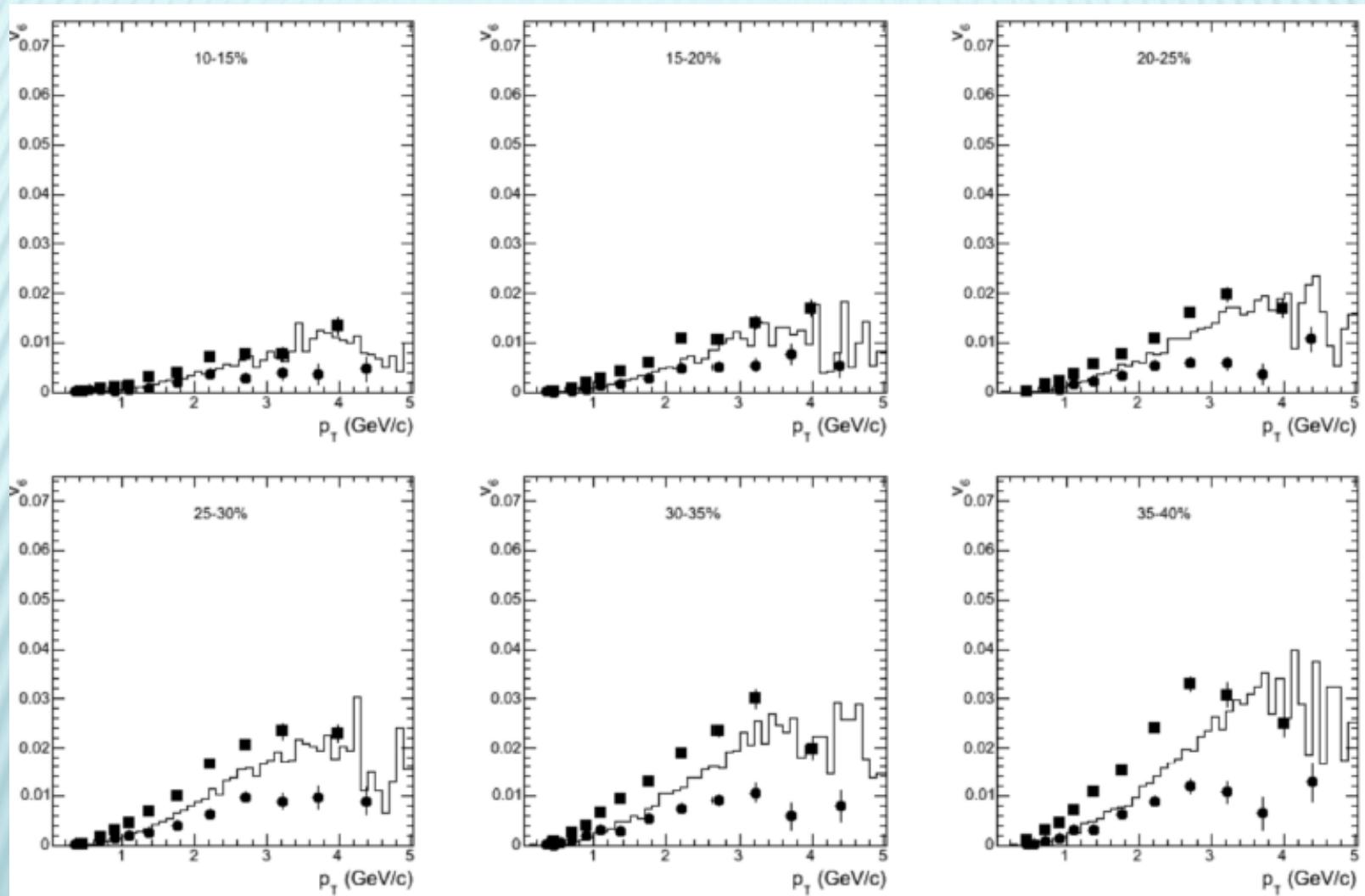
v5 is zero if either v2 or v3 is absent

IV. HIGHER HARMONICS: hexagonal flow v6

LHC DATA VS. HYDJET++ MODEL

Hexagonal flow

Pb+Pb @ 2.76 ATeV



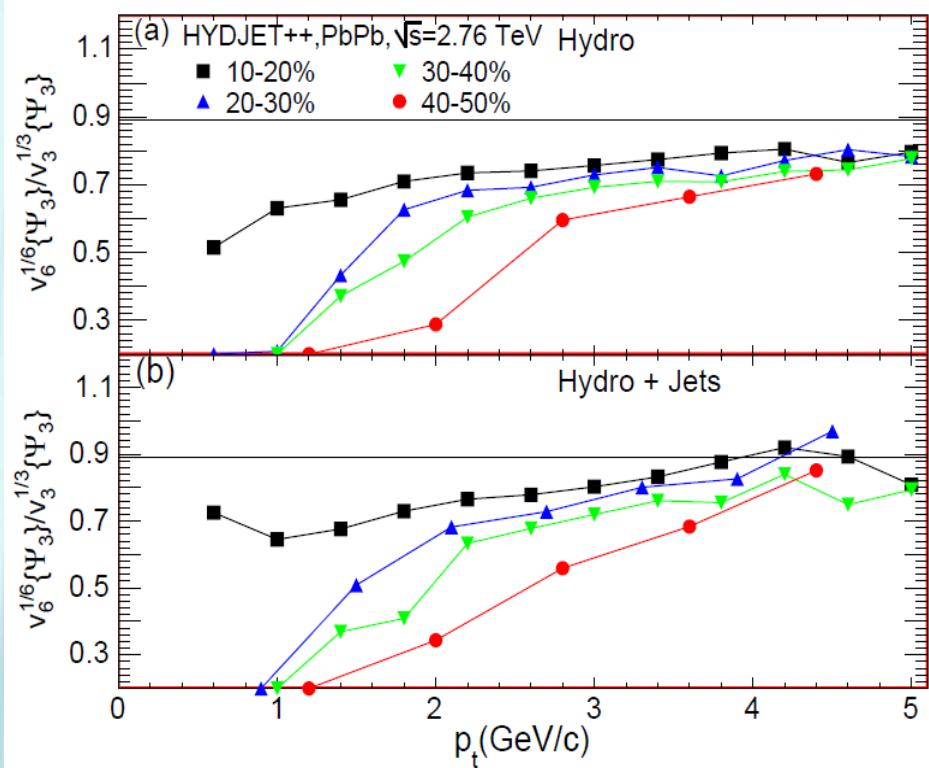
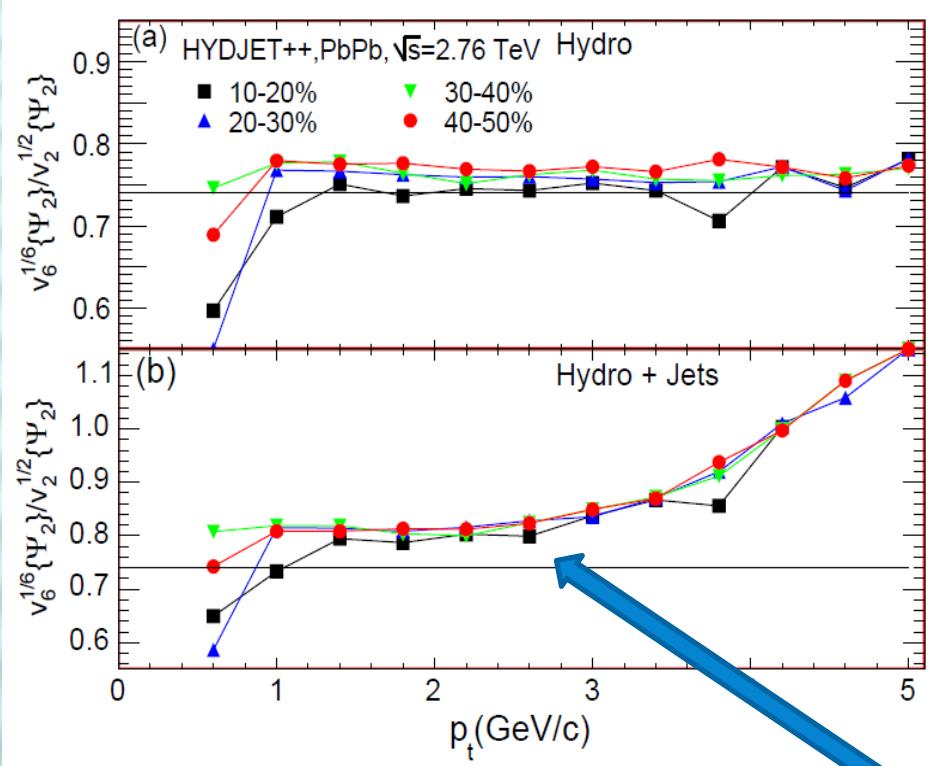
Closed symbols: CMS data $v_6\{\text{Psi2 \& LYZ}\}$;
 Open symbols and histograms: HYDJET++ $v_6\{\text{Psi2}\}$

**v6 is non-zero if either
v2 or v3 is absent**

Hexagonal flow:

$$V_6 \propto \alpha V_2^3 + \beta V_3^2$$

Bravina et al., PRC 89, 024909 (2014)



Ψ_2

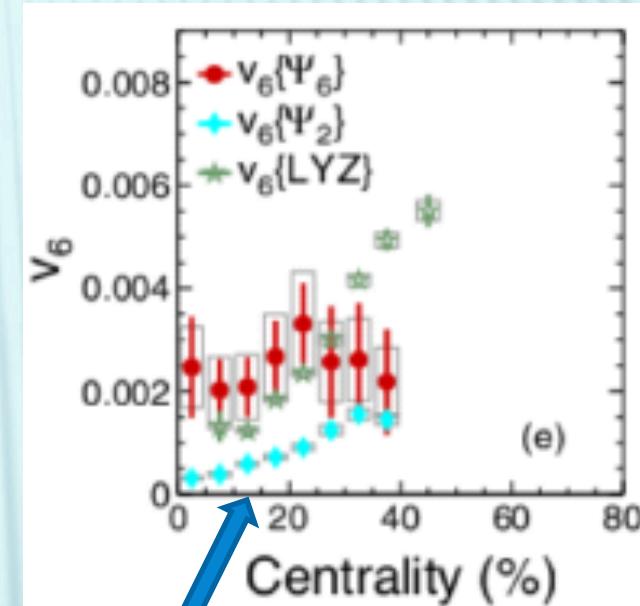
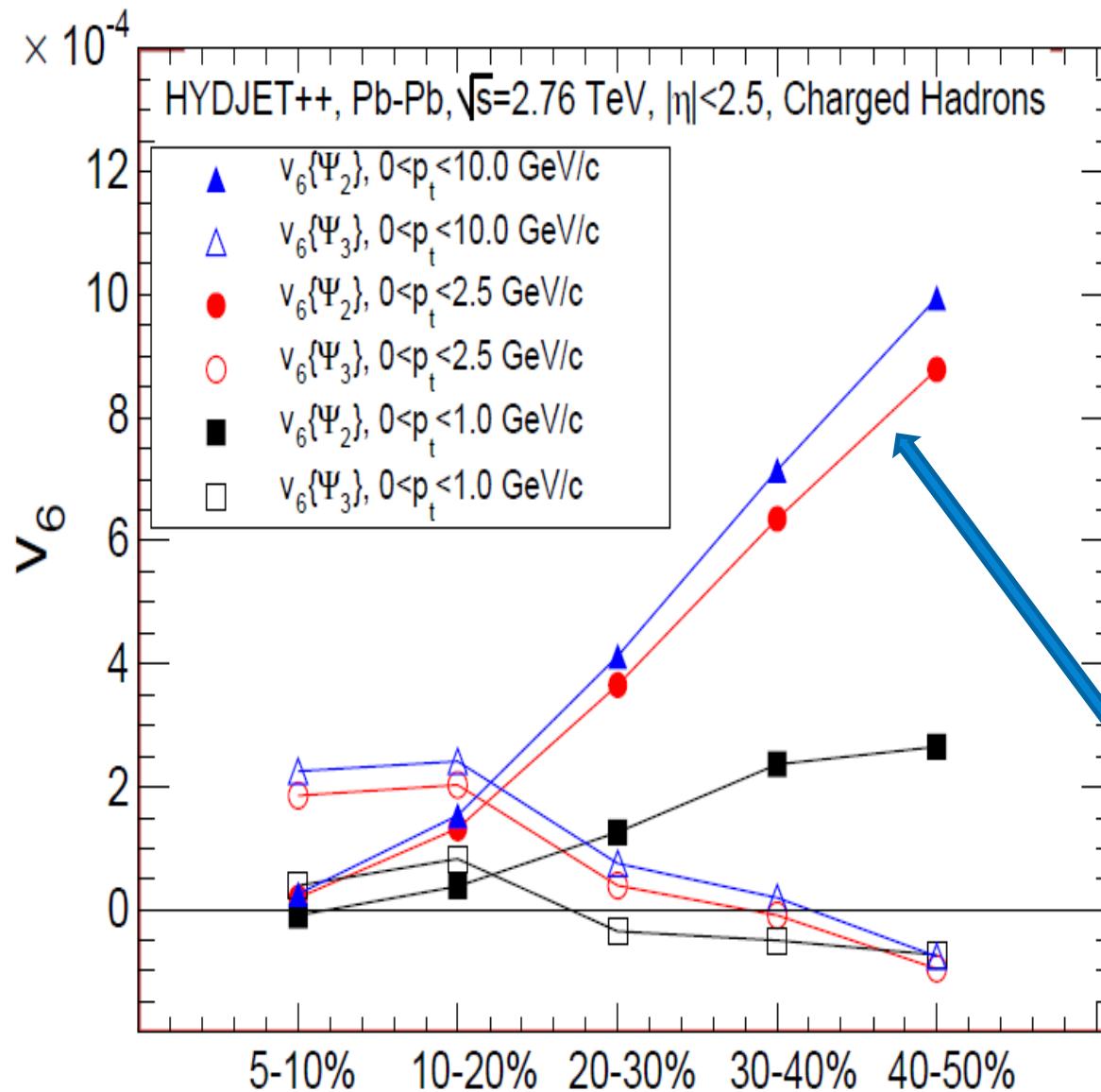
Scaling?

Ψ_3

It would be interesting to study $V_6(\Psi_2)$ and $V_6(\Psi_3)$ in experiment

Hexagonal flow: centrality dependence

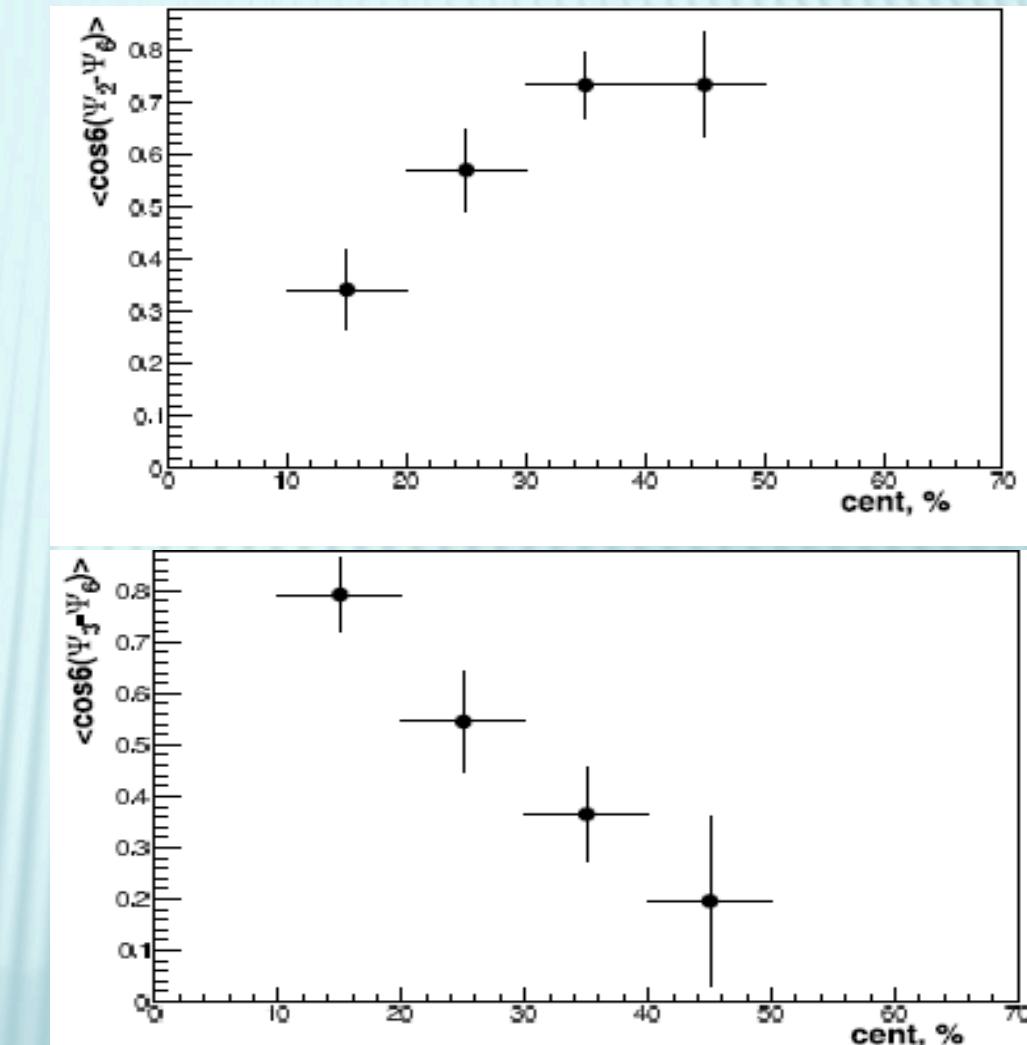
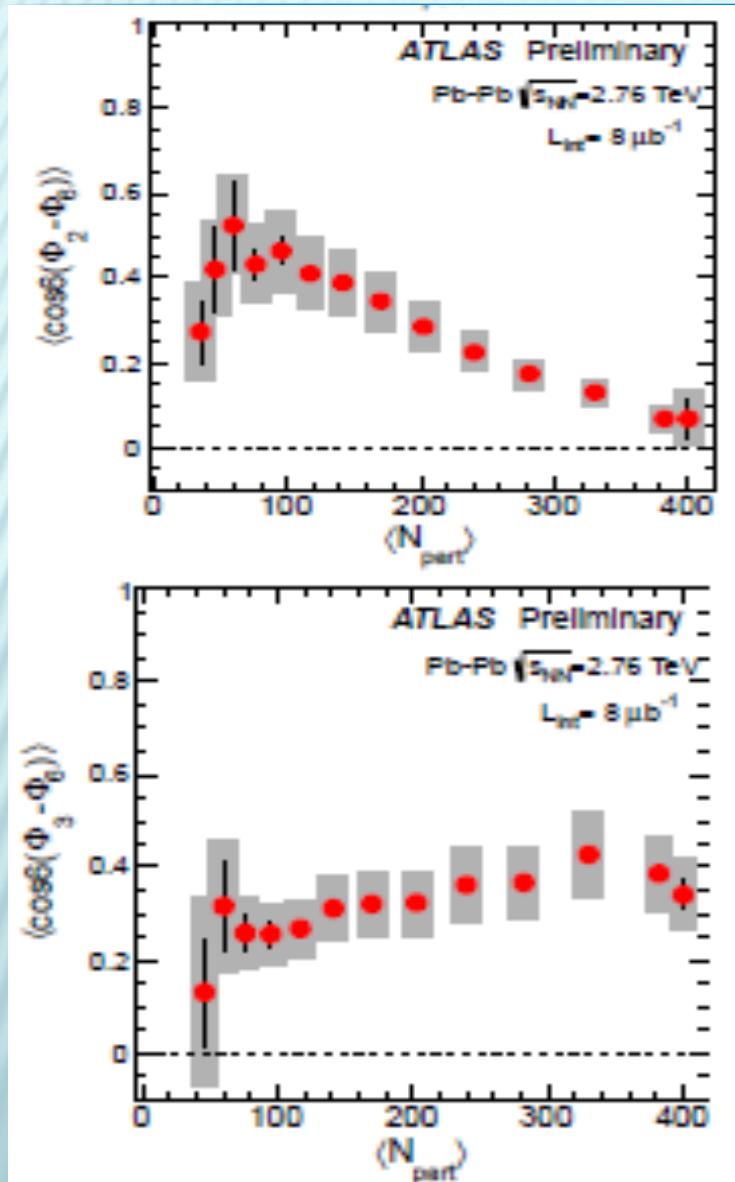
CMS Collab., PRC 89, 024909 (2014)



Centrality dependence
in HYDJET++ is correct

Hexagonal flow: correlator analysis

ATLAS-CONF-2012-049



In line with experimental data

CONCLUSIONS

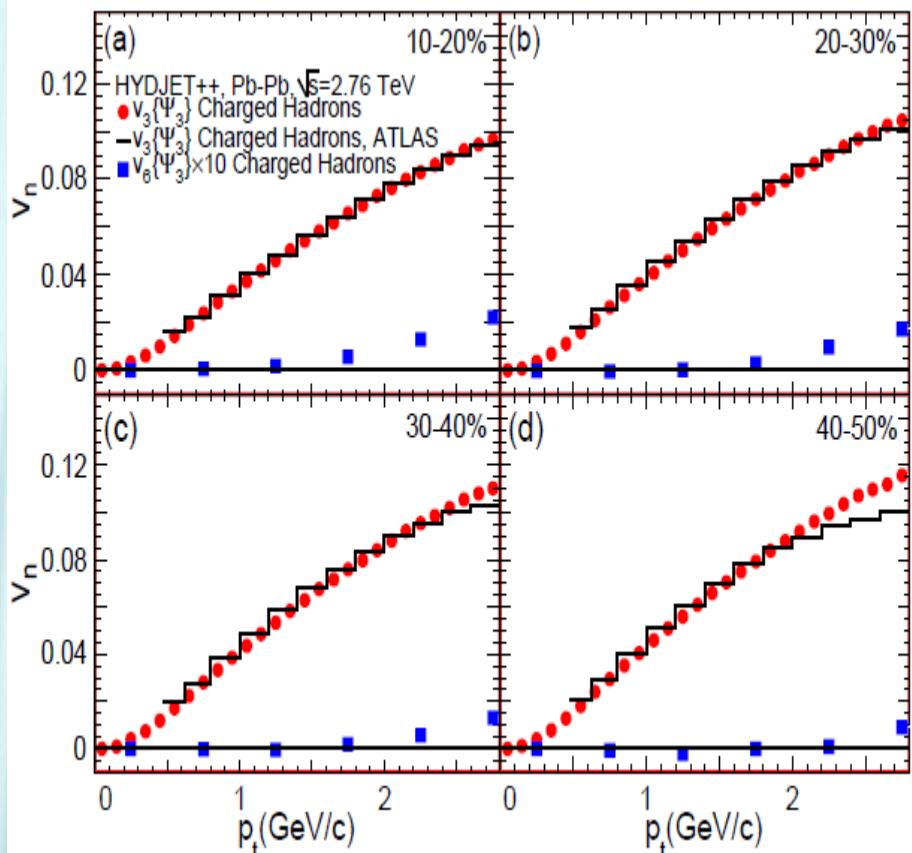
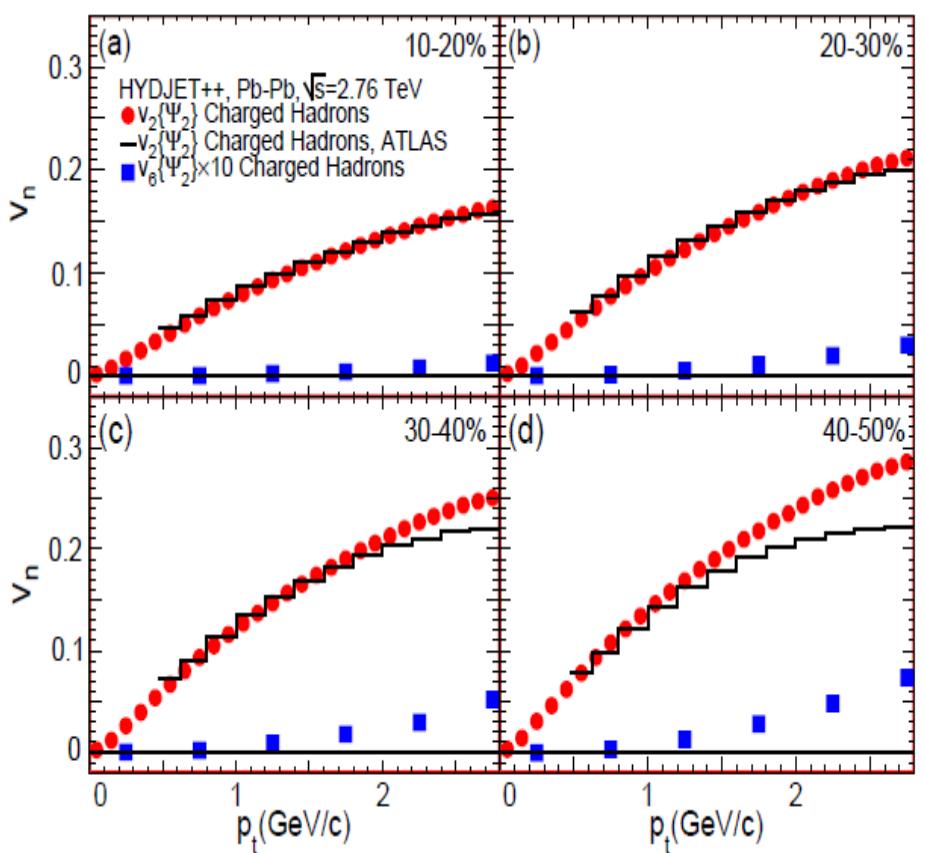
The HYDJET++ model allows us to investigate flow of hydro and jet parts separately, to look at reconstruction of pure hydro flow and its modification due to jet part.

- *HYDJET++ permits us to study cross-talk of v2 and v3, while other harmonics are absent*
- *If only v2 is present, only even harmonics appear; odd harmonics arise if v3 is included*
- *Scaling of $v_6^{1/6}(\psi_2)/v_2^{1/2}(\psi_2)$ is predicted*
- *Jets result to increase by 10%-15% of this ratio and lead to rise of its high-pT tail*
- *Significant part of hexagonal flow and other higher order harmonics comes from elliptic and triangular flows*

Back-up Slides

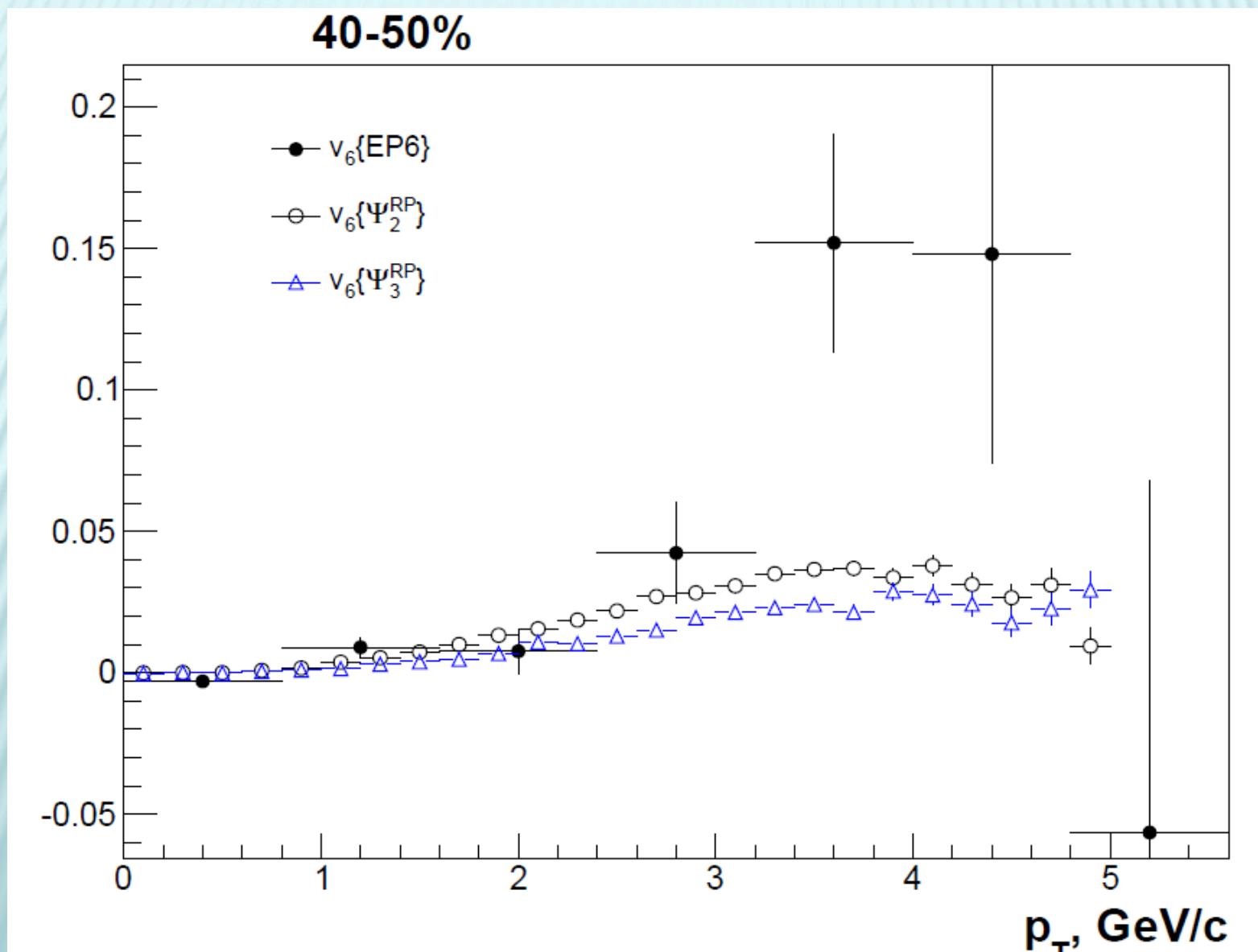
HEXAGONAL FLOW IN HYDJET++ AT LHC

Bravina et al., PRC 89, 024909 (2014)



- (1) V6 is weak
- (2) Its high-pt tail increases with rising pT

Extraction of V6 (Event Plane method)



Methods for v_2 calculation

(1) Event plane method

$$v_2^{obs} \{EP\} = \langle \cos 2(\varphi_i - \Psi_2) \rangle$$

Ψ_2 is the calculated reaction plane angle: $\tan n\psi_n = \frac{\sum_i \omega_i \sin n\varphi_i}{\sum_i \omega_i \cos n\varphi_i}$, $n \geq 1$, $0 \leq \psi_n < 2\pi/n$

$$v_2 \{EP\} = \frac{v_2^{obs} \{EP\}}{R} = \frac{v_2^{obs} \{EP\}}{\langle \cos 2(\Psi_2 - \Psi_R) \rangle}$$

(2) Two particle correlation method

$$v_2 \{2\} = \sqrt{\langle \cos 2(\varphi_i - \varphi_j) \rangle}$$

(3) Lee-Yang zero method

$$G(ir) = \langle e^{irQ} \rangle, Q = \sum \cos(2\varphi)$$

Integral v_2 is connected with the first minimum r_0 of the module of the $G(ir)$:

$$v_2 = \frac{j_0}{Nr_0}$$

Differential flow is calculated by the formula:

$$\frac{v_2(p_T)}{Nv_2} = \text{Re} \left(\frac{\langle \cos(2\varphi)e^{ir_0Q} \rangle}{\langle Qe^{ir_0Q} \rangle} \right)$$