Polarization as a signature of local parity violation in hot QCD matter

Based on F.Becattini, M.Buzzegoli, A.P., G. Prokhorov, 2020 arXiV:2009.13449

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Motivations

Topological gauge configurations give rise to a fluctuating axial chemical potential: **local parity breaking!**

$$\mu_A \neq 0 \qquad \langle \langle \mu_A \rangle \rangle = 0$$

Experimental observable: chiral magnetic effect



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Drawbacks: magnetic field, difficult background, not observed even in isobar collisions (under discussion) (STAR,arXiv:2109.00131, 2021)

Using linear response theory we show that **the axial chemical potential contributes to polarization**.

Spin vector

In the presence of an **axial chemical potential** (Becattini, Buzzegoli, A.P. Prokhorov, arXiv:2009.13449, 2020)

 $S_{tot}^{\mu}(p) = S_{hyd}^{\mu}(p) + S_{\chi}^{\mu}(p)$

$$S_{\chi}^{\mu}(p) = \frac{g_h}{2} \frac{\int_{\Sigma} d\Sigma \cdot p \, \zeta_A n_F \, (1 - n_F)}{\int_{\Sigma} d\Sigma \cdot p \, n_F} \frac{\varepsilon p^{\mu} - m^2 \hat{t}^{\mu}}{m\varepsilon} \qquad \zeta_A = \frac{\mu_A}{T}$$

Independent of the magnetic field

Over many events:

$$\langle \langle S^{\mu} \rangle \rangle = \langle \langle S^{\mu}_{\text{hyd}} \rangle \rangle + \not \downarrow \langle S_{\chi} \not \downarrow \rangle$$

$$\text{Detectable:}$$

$$\langle \langle \zeta_A \rangle \rangle = 0 \qquad \langle \langle \zeta_A^2 \rangle \rangle \neq 0$$

$$\text{Detectable:}$$

$$\text{helicity-helicity}$$

$$\text{correlators}$$

Calculating polarization

Given a density matrix $\hat{\rho}$, we compute Wigner function of Dirac fermions (particle component)

$$W_{+}(x,p)_{AB} = \theta(p^{0})\theta(p^{2})\frac{1}{(2\pi)^{4}}\int d^{4}y \ e^{-ip \cdot y} \operatorname{Tr}(\widehat{\rho}:\overline{\Psi}_{B}(x+y/2)\Psi_{A}(x-y/2):)$$

The spin vector reads

$$S^{\mu}(p) = \frac{1}{2} \frac{\int_{\Sigma} d\Sigma \cdot p \operatorname{tr} \left[\gamma^{\mu} \gamma^{5} W_{+}(x, p)\right]}{\int_{\Sigma} d\Sigma \cdot p \operatorname{tr} \left[W_{+}(x, p)\right]}$$

In heavy ion collisions Σ is the **freeze-out hypersurface**.

$$\begin{aligned} & \mathcal{L} \text{ubarev's density operator} \\ & \widehat{\rho} = \frac{1}{Z} \exp \left[-\int_{\Sigma_{eq}} d\Sigma_{\mu} \left(\widehat{T}^{\mu\nu} \beta_{\nu} - \zeta_{A} \widehat{j}^{\mu}_{A} \right) \right] \\ & \beta^{\mu} = \frac{u^{\mu}}{T} \qquad \zeta_{A} = \frac{\mu_{A}}{T} \end{aligned}$$

$$\begin{aligned} & \text{Using the Gauss theorem} \\ & \int_{\Sigma_{eq}} d\Sigma_{\mu} \left(\widehat{T}^{\mu\nu} \beta_{\nu} - \zeta_{A} \widehat{j}^{\mu}_{A} \right) = \int_{\Sigma_{FO}} d\Sigma_{\mu} \left(\widehat{T}^{\mu\nu} \beta_{\nu} - \zeta_{A} \widehat{j}^{\mu}_{A} \right) + \\ & \int_{\Omega} d\Omega \left(\widehat{T}^{\mu\nu} \partial_{\mu} \beta_{\nu} - \widehat{j}^{\mu}_{A} \partial_{\mu} \zeta_{A} - \zeta_{A} \partial_{\mu} \widehat{j}^{\mu}_{A} \right) \end{aligned}$$

If we neglect dissipation (supposedly small)

$$\widehat{\rho} \simeq \widehat{\rho}_{LE} = \frac{1}{Z_{LE}} \exp\left[-\int_{\Sigma_{FO}} \mathrm{d}\Sigma_{\mu} \left(\widehat{T}^{\mu\nu}\beta_{\nu} - \zeta_{A}\widehat{j}^{\mu}_{A}\right)\right]$$

The Wigner function is the expectation value of a Wigner operator.

$$\widehat{W}_{+}(x,p)_{AB} = \theta(p^{0})\theta(p^{2})\frac{1}{(2\pi)^{4}}\int d^{4}y \ e^{-ip \cdot y} : \overline{\Psi}_{B}(x+y/2)\Psi_{A}(x-y/2):$$
$$W_{+}(x,p)_{AB} = \operatorname{Tr}(\widehat{\rho}\ \widehat{W}_{+}(x,p)_{AB})$$

We use the **hydrodynamic approximation**: thermodynamic quantities vary slowly with respect to microscopic correlation lengths.

We can Taylor expand β about x.

$$\beta_{\mu}(y) \simeq \beta_{\mu}(x) + \partial_{\nu}\beta_{\mu}(x)(y-x)^{\nu}$$

$$\widehat{\rho}_{LE} \simeq \frac{1}{Z} \exp\left[-\beta(x) \cdot \widehat{P} + \partial_{\nu}\beta_{\mu}(x) \int d\Sigma_{\alpha} \widehat{T}^{\alpha\mu}(y-x)^{\mu} + \int d\Sigma_{\mu}\zeta_{A}(y) \widehat{j}_{A}^{\mu}(y)\right]$$

Hydrodynamic polarization

The density operator is approximated in **linear response theory.** The gradients of temperature yield the known results.

$$e^{\widehat{A}+\widehat{B}} = e^{\widehat{A}} + \int_0^1 dz \, e^{z\widehat{A}} \, \widehat{B} \, e^{-z\widehat{A}} \, e^{\widehat{A}} + \cdots$$
$$\widehat{A} = -\beta(x) \cdot \widehat{P}, \quad \widehat{B} = \int_{\Sigma} d\Sigma_\rho(y) \zeta_A(y) \widehat{j}_A^\rho(y)$$

The Wigner function reads:

$$\langle \widehat{W}_{+}(x,p) \rangle \simeq W_{+}(x,p)|_{\mu_{A}=0} + \Delta W_{+}(x,p)$$

$$\Delta W_{+}(x,p) = \int_{\Sigma} \mathrm{d}\Sigma_{\rho} \zeta_{A} \int_{0}^{1} \mathrm{d}z \langle \widehat{W}_{+}(x,p) \widehat{j}_{A}^{\rho}(y+iz\beta) \rangle_{c,\beta(x)} \qquad \langle \bullet \rangle_{\beta(x)} = \frac{1}{Z} \mathrm{Tr}(e^{-\beta(x) \cdot \widehat{P}} \bullet)$$

The correction to the Wigner function induces polarization

$$S_{tot}^{\mu} = S_{hyd}^{\mu} + S_{\chi}^{\mu}$$

$$S^{\mu}_{\chi}(p) = \frac{1}{2} \frac{\int_{\Sigma} d\Sigma \cdot p \operatorname{tr} \left[\gamma^{\mu} \gamma^{5} \Delta W_{+}(x, p) \right]}{\int_{\Sigma} d\Sigma \cdot p \operatorname{tr} \left[W_{+}(x, p) |_{\zeta_{A}=0} \right]}$$

After some calculations (for details, see the paper):

$$S^{\mu}_{\chi}(p) = \frac{g_h}{2} \frac{\int_{\Sigma} d\Sigma \cdot p \, \zeta_A n_F \, (1 - n_F)}{\int_{\Sigma} d\Sigma \cdot p \, n_F} \frac{\varepsilon p^{\mu} - m^2 \hat{t}^{\mu}}{m\varepsilon} \qquad \begin{array}{c} \text{Polarization} \\ \text{By} \\ \text{Axial imbalance!} \end{array}$$

Helicity and local parity violations

Back boosting to the rest frame of the particle:

$$\boldsymbol{S}_{0,\chi}(p) = \frac{g_h}{2} \frac{|\boldsymbol{p}|}{\varepsilon} \frac{\int_{\Sigma} d\Sigma \cdot p \, \zeta_A n_F \, (1 - n_F)}{\int_{\Sigma} d\Sigma \cdot p \, n_F} \hat{\boldsymbol{p}} \equiv h_{\chi}(p) \hat{\boldsymbol{p}}$$

We define the helicity: $h_{\chi}(p) \equiv S_{0,\chi}(p) \cdot \hat{p}$

Over many events, the axial chemical potential averages to zero

$$\langle\langle\zeta_A\rangle\rangle = 0 \Rightarrow \langle\langle h_\chi(p)\rangle\rangle = 0$$

Geometrical symmetries in heavy ion collisions (**model independent**):

- Parity
- Rotation of π around global angular momentum



Local parity violation: parity is not a symmetry of the density operator.

Parity is a symmetry of ρ ParitNo parity violation: helicity is a pseudoscalarParity $h_P(-p) = -h_P(p)$ Parity

Parity is not a symmetry of ρ Parity violation: helicity is a scalar $h_S(-p) = h_S(p)$

In general:

$$h(p) = h_P(p) + h_S(p)$$

Consider particles at midrapidity: $p_z=0$, $p=(p_T,\phi)$

Rotational and parity symmetries imply

$$h_P(p_T, \phi) = \sum_k P_k(p_T) \sin[(2k+1)\phi]$$
$$h_S(p_T, \phi) = \sum_k S_k(p_T) \cos[2k\phi]$$

Contribution of the leading harmonics:

$$h(p) \simeq S_0(p_T) + P_0(p_T) \sin \phi + \dots$$

We have to devise a suitable observable to highlight scalar components of the helicity

$$\langle \langle S_0 \rangle \rangle = 0 \qquad \langle \langle S_0^2 \rangle \rangle \neq 0$$

Helicity-helicity correlators

Correlation between helicities in the same event, $\Delta \phi$ apart.

$$\langle h_1 h_2(\Delta \phi) \rangle = \frac{1}{N} \int d^2 \mathbf{p}_{T1} d^2 \mathbf{p}_{T2} \delta(\phi_2 - \phi_1 - \Delta \phi) h_1(\mathbf{p}_{T1}) h_2(\mathbf{p}_{T2}) n(\mathbf{p}_{T1}, \mathbf{p}_{T2})$$



Correlator at large angle reveal parity violations!

Parity violation: $\langle h_1 h_2(\pi) \rangle > 0$

No parity violation: $\langle h_1 h_2(\pi) \rangle < 0$

Using the leading harmonics (\bar{S}_0, \bar{P}_0 averaged over transverse momentum).

$$\langle h_1 h_2(\Delta \phi) \rangle = \bar{S}_0^2 + \frac{1}{2} \bar{P}_0^2 \cos \Delta \phi$$

Constant term in the correlator reveals local parity breaking

$$\frac{1}{\pi} \int_0^{\pi} \mathrm{d}\Delta\phi \, \langle h_1 h_2(\Delta\phi) \rangle = \bar{S}_0^2 \ge 0$$

This observable:

- **Avoids** the identification of the reaction plane.
- **Independent** of the magnetic field.



Conclusions

- Local parity violations can be detected by measuring polarization.
- We connected the mean spin vector to the axial chemical potential.
- Suitable observable: helicity-helicity correlator.
- Independent of the magnetic field.



Scan for more details! arXiv:2009.13449

Thank you for the attention!