A bayesian analysis of hybrid star properties with the NJL model

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Outline

- Modelization
  - The NJL model for quark matter
  - The nuclear meta-model
  - Hybrid EoS with Maxwell's construction
  - Bayesian method
- Posterior results
  - Quark parameters
  - Nuclear parameters
  - Phase transition characteristics
  - Hybrid star properties
  - EoS and speed of sound
- Conclusion

Introduction : the NJL model

- Effective quantum field theory (relativistic)
- Approximation of QCD in the non-perturbative domain
- Quark degrees of freedom (no gluons)
- Reproduces the flavor symmetries of QCD:

$$SU(N_f)_V \times SU(N_f)_A \times U(1)_B$$

Spontaneous breaking of $SU(N_f)_A$

Dynamical generation of fermion masses

Goldstone mechanism

- (T, μ) ↗ : symmetry restoration ⟹ phase transition(s)

Here we take the $T = 0$ limit (applicable to astrophysics)

Model parameters

- 4 coupling constants : G_S , G_ω , G_ρ , K
- 3 bare masses for the quarks : $m_{u\,0}$, $m_{d\,0}$, $m_{s\,0}$
- 1 momentum cutoff Λ

} 8 (7) parameters

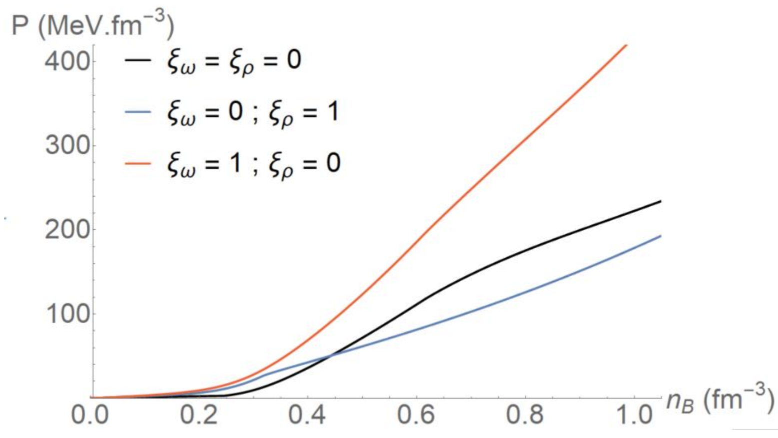
- Fitted to mesonic data in the vacuum :

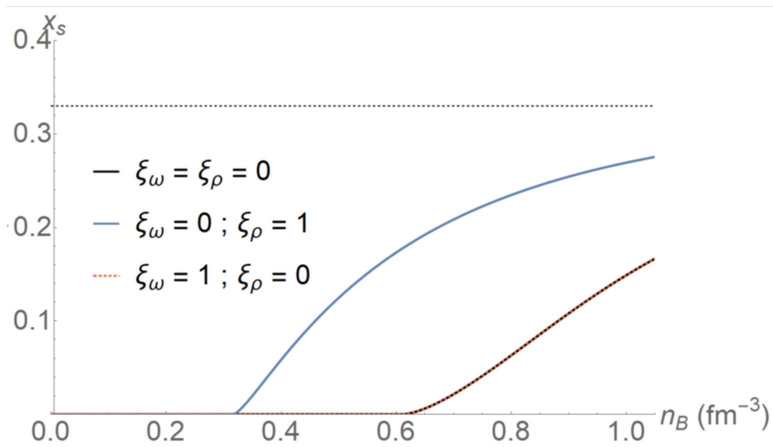
$m_\pi, f_\pi, m_K, m_{\eta'}, -\langle\bar{\psi}\psi\rangle^{\frac{1}{3}}$ ⟶ 5 experimental input

- We keep 2 free parameters: $\xi_\omega = \frac{G_\omega}{G_S}$, $\xi_\rho = \frac{G_\rho}{G_S}$

The NJL model for quark matter

- Vector interactions ξ_ω and ξ_ρ play crucial role at finite density





- The ω channel make the EOS stiffer (higher maximum mass)
- The ρ modifies the flavor content of the system, increases the pressure at moderate density and decreases the Fermi pressure at high density.

The nuclear EoS

- Meta-model of Margueron *et al* (2018)

$$e_N(x,\delta) = t_N(x,\delta) + \sum_\alpha \frac{1}{\alpha!}(v_\alpha^{sat} + \delta^2 v_\alpha^{sym})x^\alpha u_\alpha(x) \quad \text{with } x = \frac{n_B - n_{sat}}{3n_{sat}},\ \delta = \frac{n_n - n_p}{n_B}$$

- Simple, flexible ⟶ explores all possible nuclear EoS
- Directly parametrized by the nuclear empirical parameters:

X	E_{sat}	E_{sym}	n_{sat}	L_{sym}	K_{sat}	K_{sym}	Q_{sat}	Q_{sym}	Z_{sat}	Z_{sym}	$m^\star$	$\Delta m^\star$
Order	0	0	1	1	2	2	3	3	4	4		
Unit	MeV	MeV	fm^{-3}	MeV	MeV	MeV	MeV	MeV	MeV	MeV		
X_{min}	-17.5	27	0.15	20	190	-400	-1200	-2000	-4000	-5000	0.6	-0.1
X_{max}	-14.5	37	0.17	80	300	300	1000	5000	5000	5000	0.8	0.2

- Unified extension in the crust, using a CLD model for the inhomogeneous phases (Carreau *et al*)
- Finite size parameters are fitted to the masses of known nuclei

*J. Margueron, R. Homann Casali, and F. Gulminelli, Physical Review C **97**, 025805 (2018)*
T. Carreau, F. Gulminelli, and J. Margueron, The European Physical Journal A, vol. 55, (2019)

Hybrid EoS

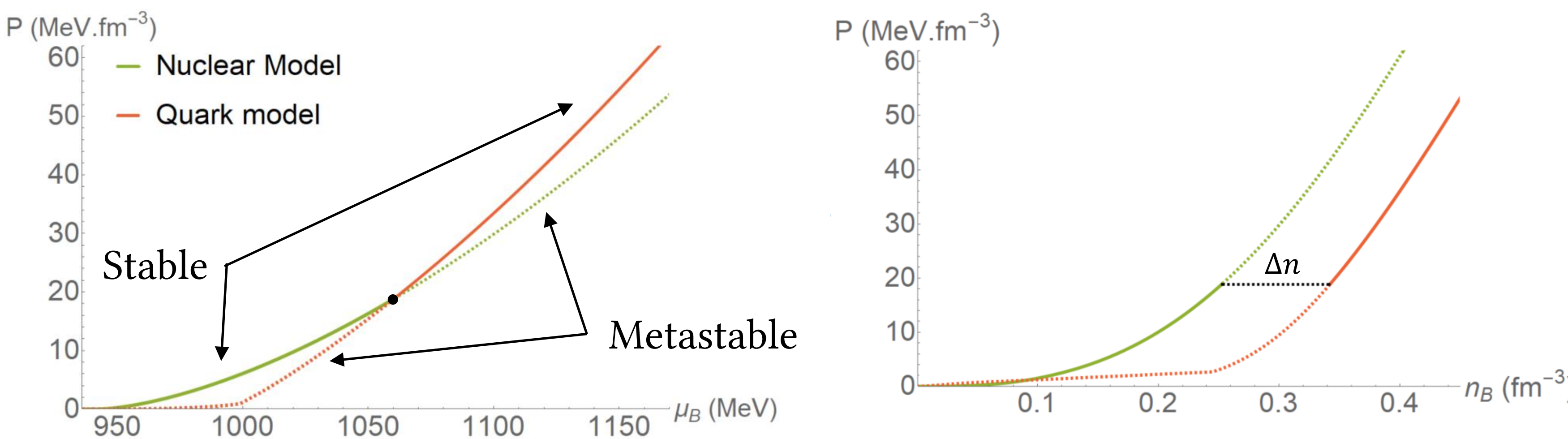
- Deconfinement phase transition:

  Nuclear matter (n, p, e, μ) ⟶ Quark matter (u, d, s, e)

- Gibbs thermodynamical equilibrium conditions :

$$P_N = P_Q \qquad \mu_N = \mu_Q \qquad T_N = T_Q$$

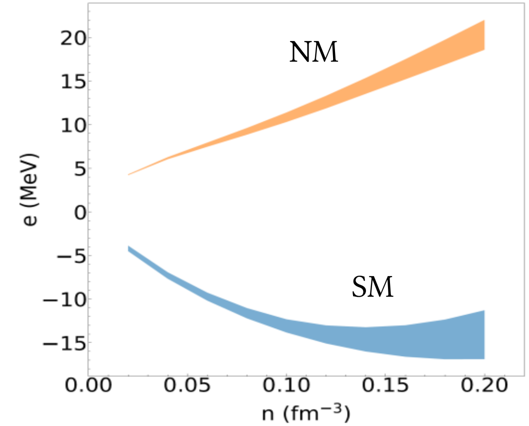
- Grand canonical ensemble : $\Omega = -P$



Maxwell's construction (first order)

Bayesian method

- Generate a large number (~ 10^8) of hybrid models with flat prior on:
  - The nuclear empirical parameters on the nuclear side
  - The 3 free parameter of our NJL parametrization on the quark side: $\xi_\omega, \quad \xi_\rho, B^*$
- Impose the model to be "reasonable":
  - Thermodynamic consistency (nuclear model):
    - $0 < c_s < 1$
    - $\frac{dP}{dn_B} > 0$
    - $e_{sym} = \frac{1}{n_B}\frac{\partial^2 \rho}{\partial \delta^2} > 0$
  - Compatibility with the ab initio χEFT energy bands (NM + SM) of Drischler *et al* (2016)
  - Possibility of a PT to quark matter before reaching the TOV mass



C. Drischler, K. Hebeler, and A. Schwenk, Phys. Rev. C, vol. 93, p. 054314, May 2016.

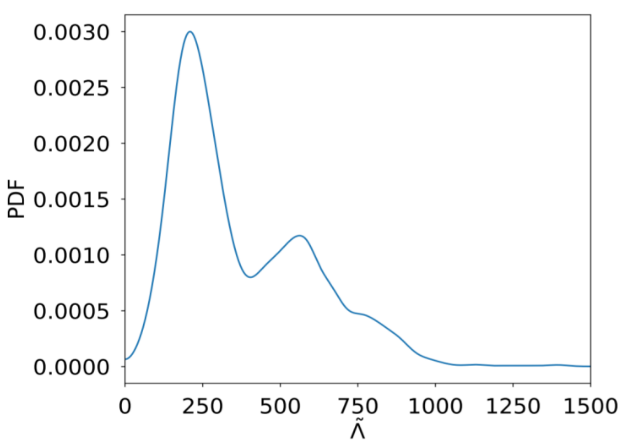
Bayesian method

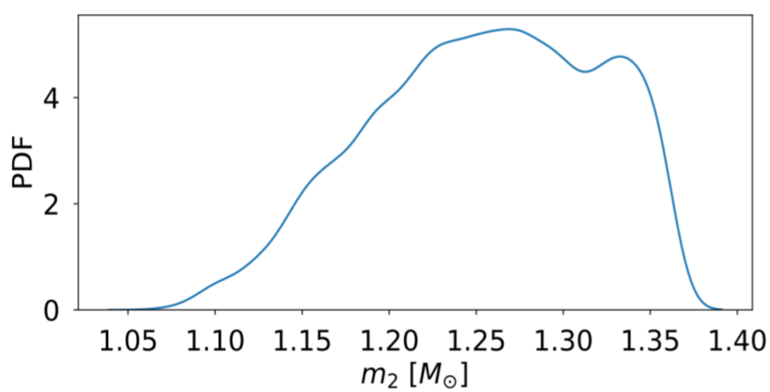
- Attribute weights to models based on their reproduction of experimental results:
  - Reaching a large enough TOV mass (J0740+6620 : $M = 2.08 \pm 0.07 M_{\odot}$)
  - Goodness of the fit to the experimental masses of nuclei:

$$w_{AME} = \frac{1}{N}\exp(-\frac{\chi^2}{2}) \quad \text{with } \chi^2 = \frac{1}{\nu}\sum_i (\frac{M^i - M^i{}_{AME}}{\sigma^i})^2$$

  - Reproduction of the PDF of $\widetilde{\Lambda}$ from GW170817 (Abbott *et al*, 2019)

$$w_{GW170817} = \frac{1}{N}\sum_j p_{LVC}\,(m_2{}^j) \times p_{LVC}\,(\widetilde{\Lambda}^j) \qquad \widetilde{\Lambda} = \frac{16}{13}\frac{(m_1 + 12m_2)m_1{}^4\Lambda_1 + (m_2 + 12m_1)m_2{}^4\Lambda_2}{(m_1 + m_2)^5}$$





$$\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \approx 1.188\ M_{\odot}$$

Abbott et al., Physical Review X 9, 011001 (2019)

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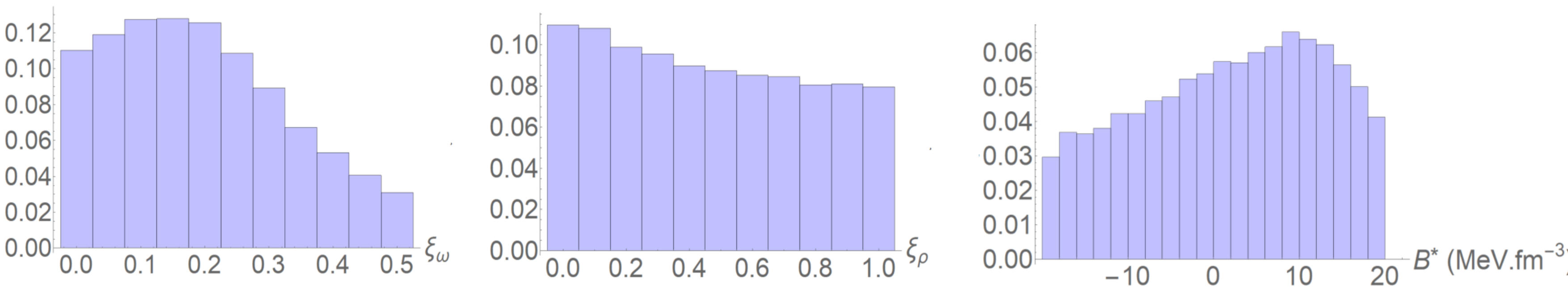
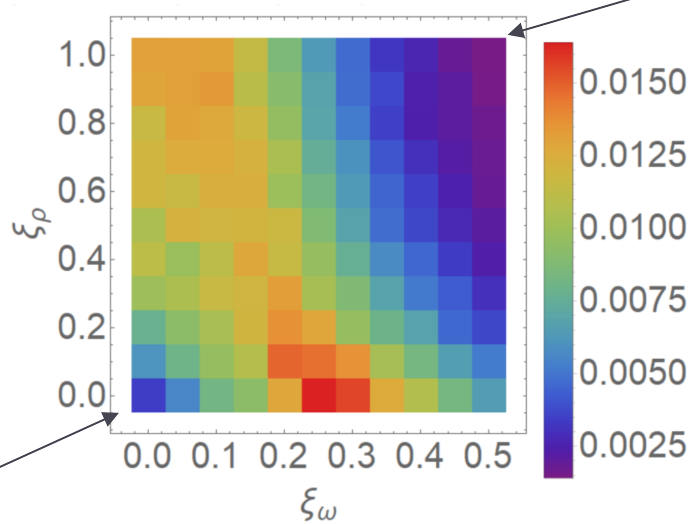
Posterior results – quark parameters

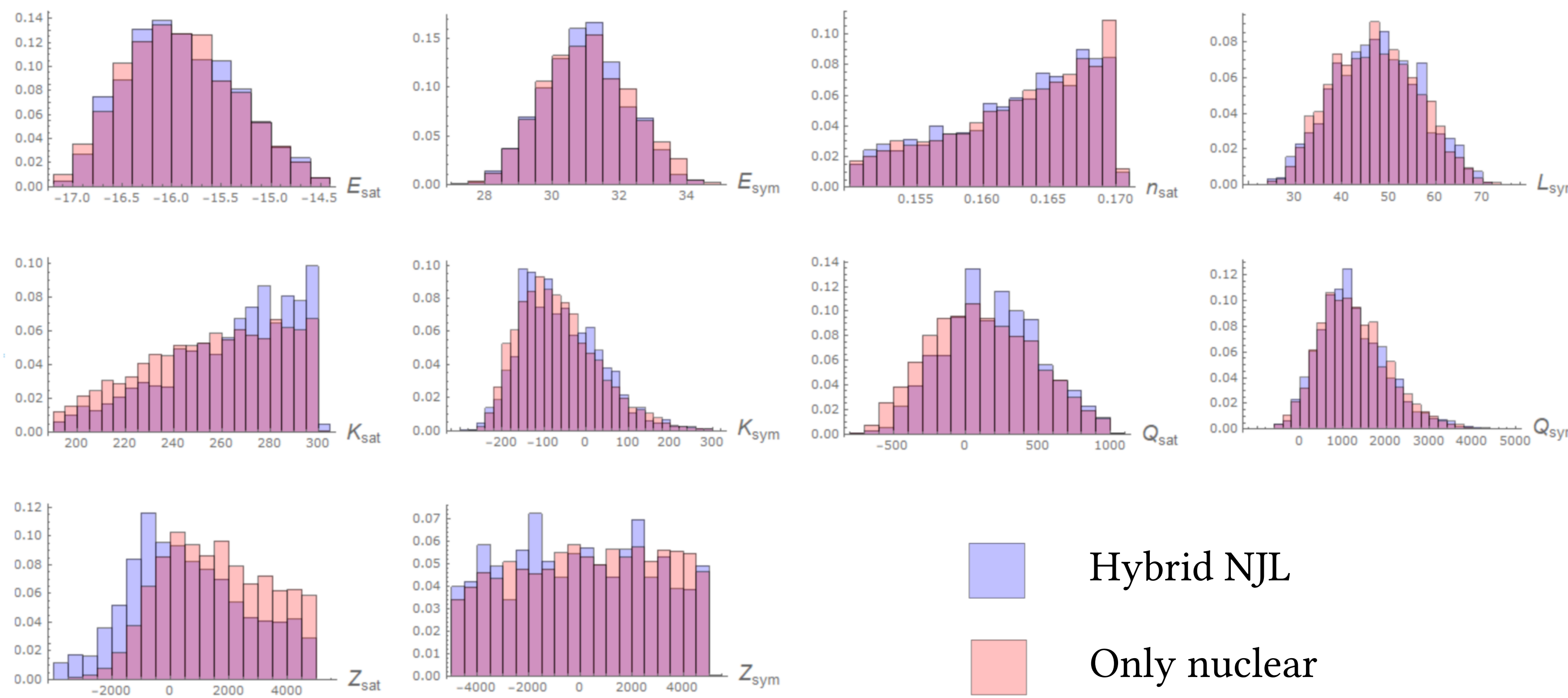


Quark EoS too stiff

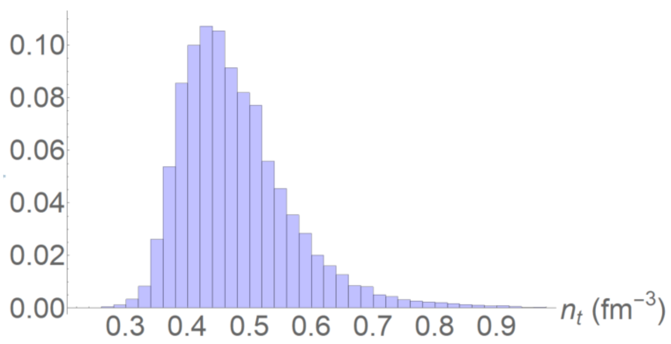


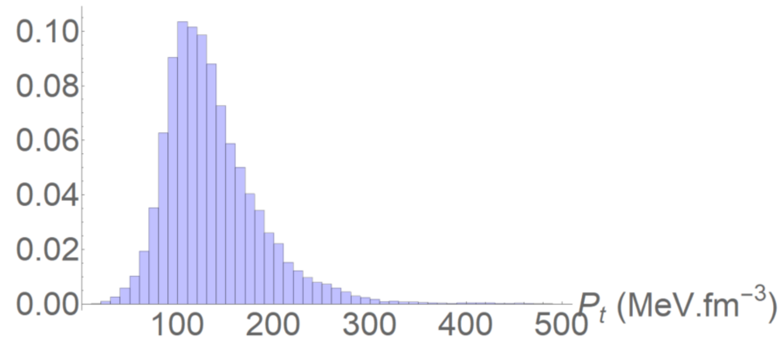
Quark EoS too soft

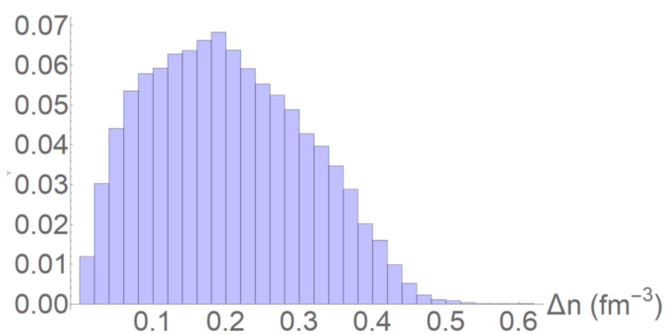
Posterior results – nuclear parameters



Posterior results – phase transition characteristics

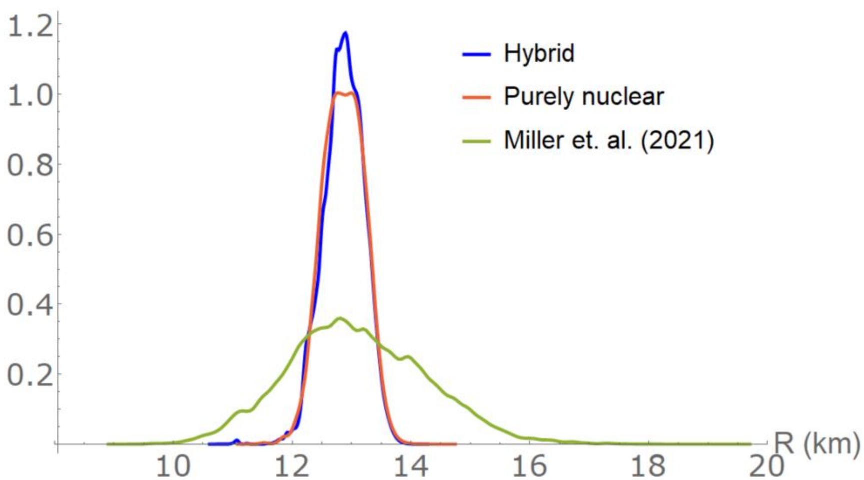




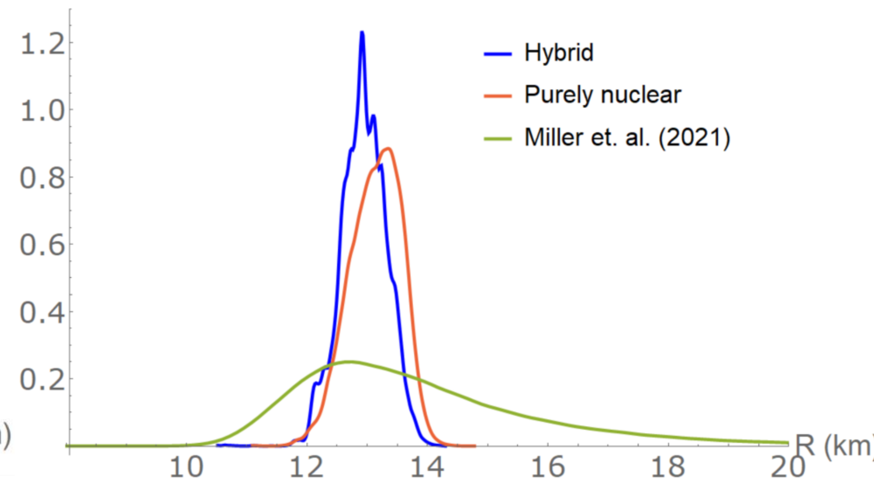


Neutron star radii

J0030+0451 : $M = 1.44 \pm 0.14 M_{\odot}$



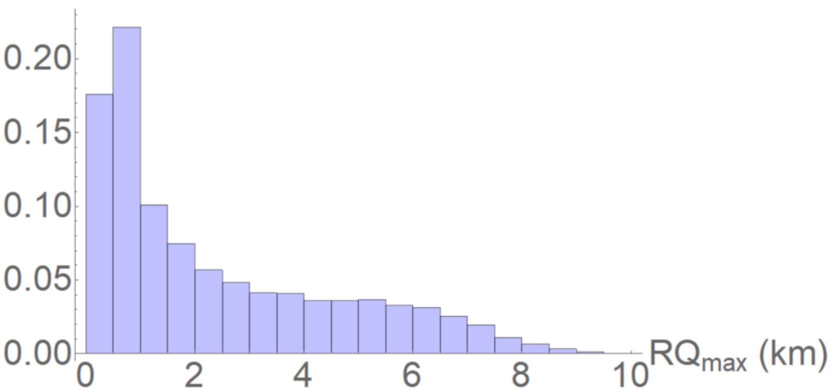
J0740+6620 : $M = 2.08 \pm 0.07 M_{\odot}$

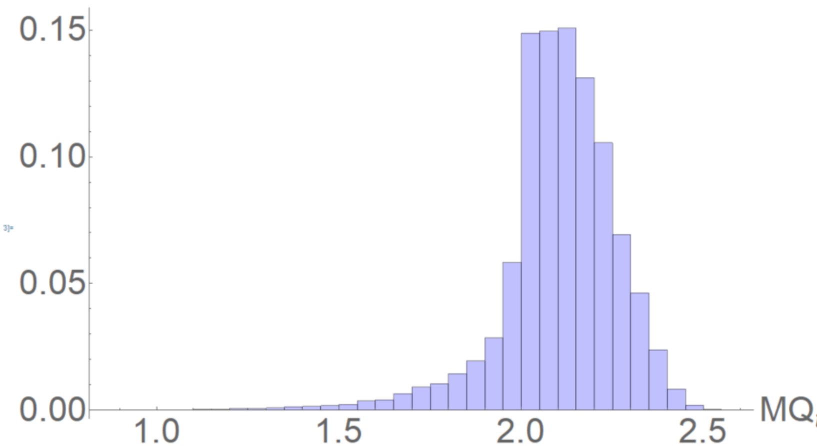


→ Both model assumptions are compatible with NICER data

Inclusion of a quark PT has weak influence on the radii

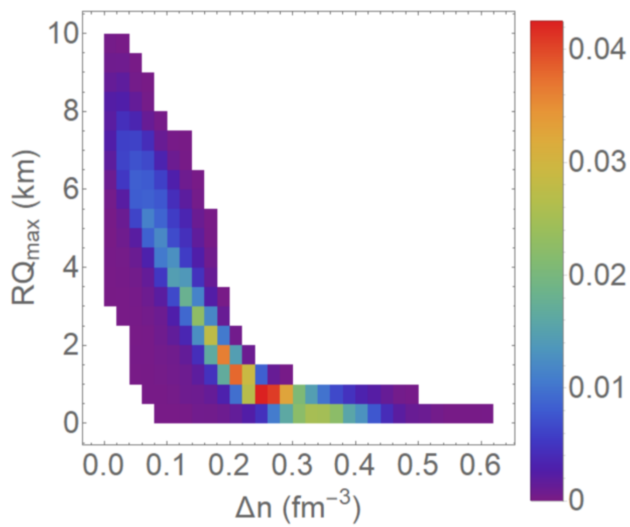
Posterior results – Hybrid star properties

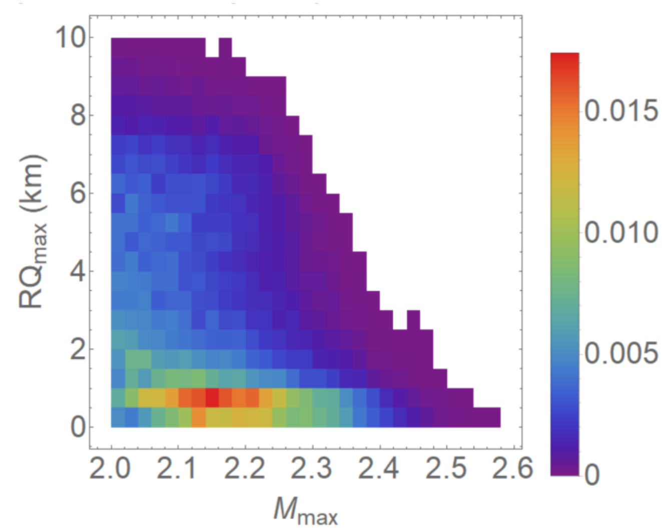




RQ_{max} ≡ Radius of the quark core in the maximum mass configuration
MQ_t ≡ Total mass of the star as quarks start to appear

Posterior results – Hybrid star properties





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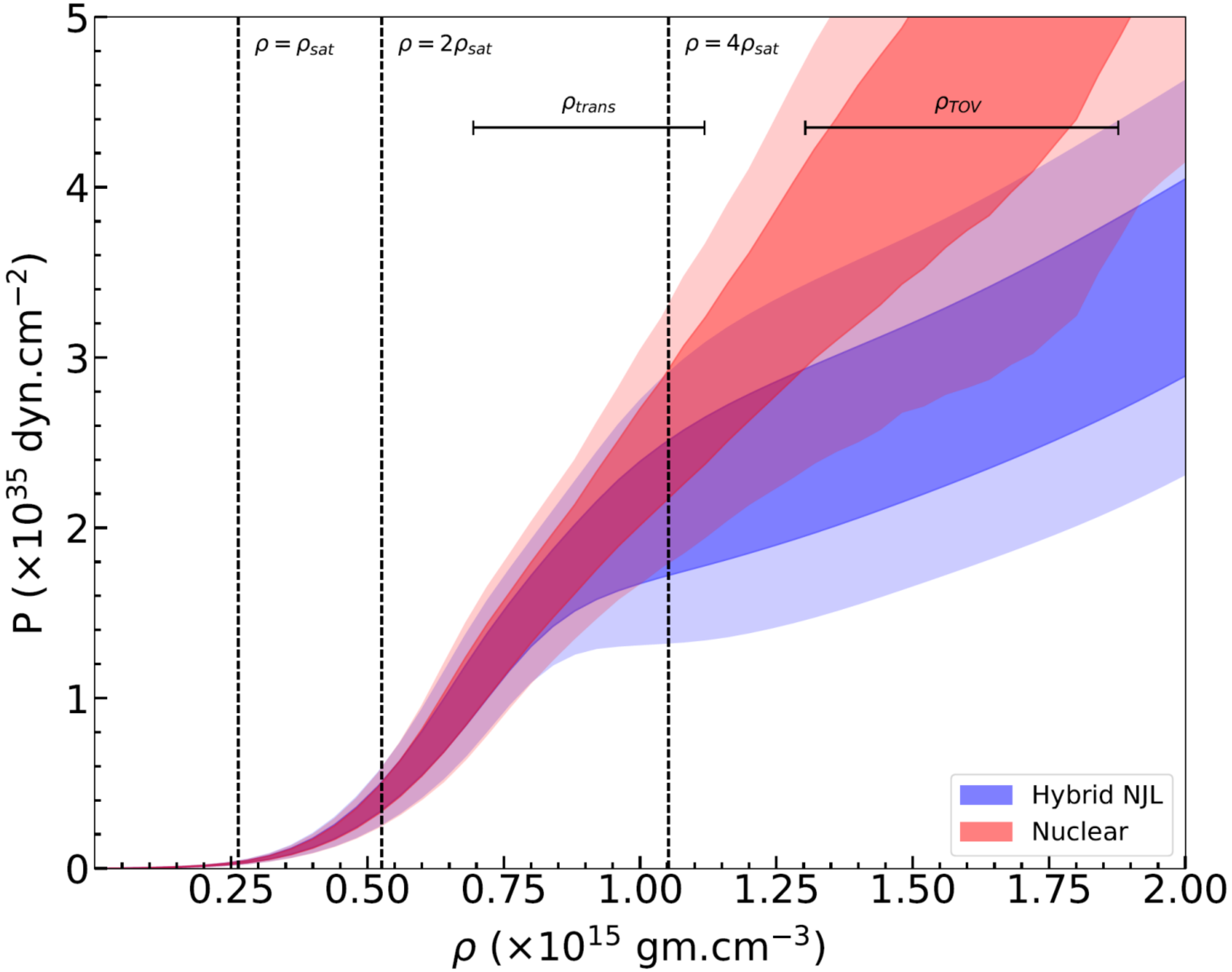
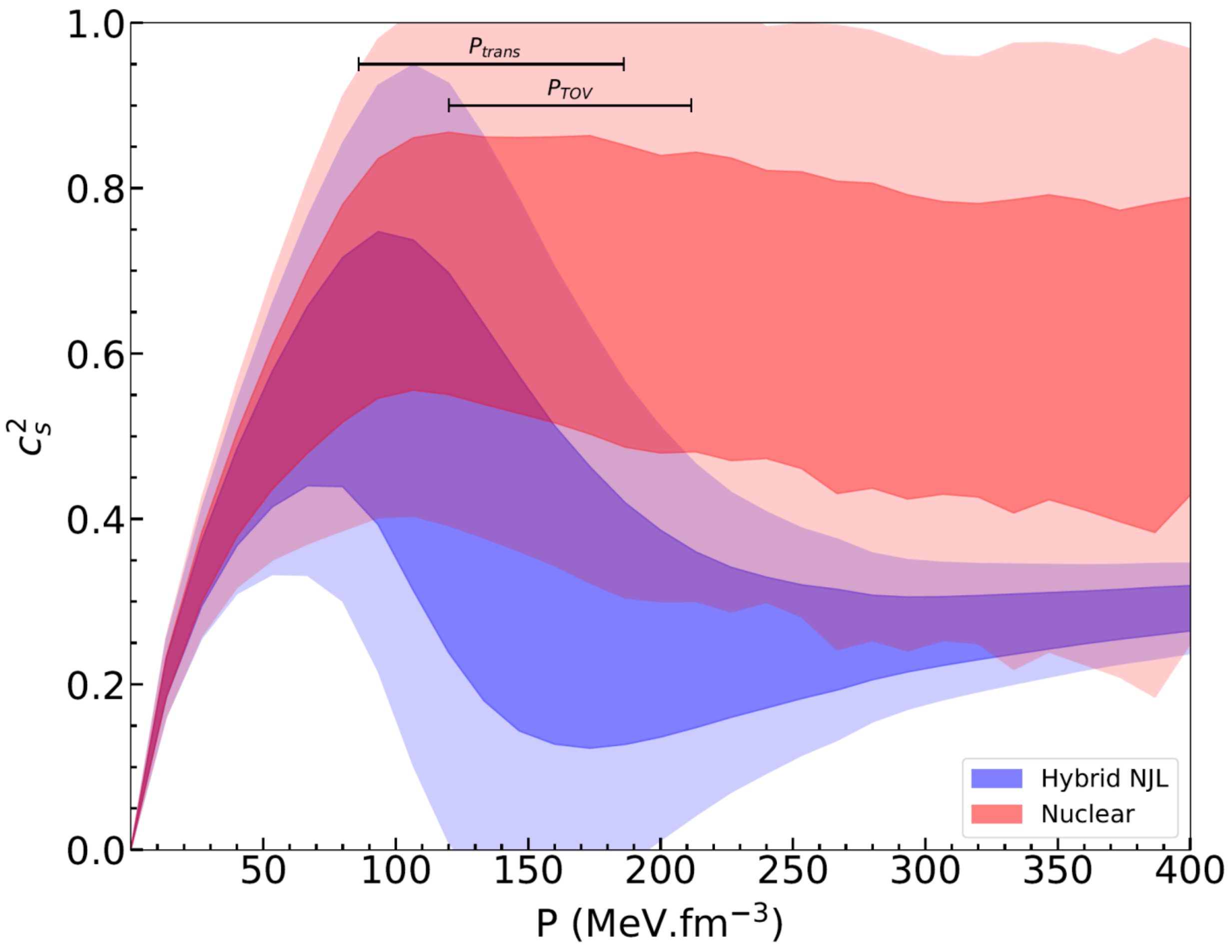
Global EoS properties



Global EoS properties



Conclusions

- The bayesian framework allows us to consistently take into account all types of experimental knowledge.
- Hybrid NJL stars usually have a small QC that only appears in very heavy stars
- Relatively low impact on the radii

  ➡ Low observability of the PT with X-ray radii measurements

- Substantial effect on the susceptibilites (sound speed)

Thank you !

The NJL Lagrangian

$$\mathcal{L}_{QCD} = \underbrace{\bar{\psi}(i\gamma^\mu\partial_\mu - \hat{m} + \hat{\mu}\gamma_0)\psi}_{\text{Quarks}} + \overbrace{\bar{\psi}\gamma^\mu \frac{\lambda_a}{2} A_\mu{}^a\psi}^{\text{Quark/gluon interaction}} - \underbrace{\frac{1}{4} F_{\mu\nu}{}^a F^{\mu\nu}{}_a}_{\text{Gluons}}$$

$$\mathcal{L}_{NJL} = \bar{\psi}(i\gamma^\mu\,\partial_\mu - \hat{m} + \hat{\mu}\gamma_0)\psi + \sum_C G^C\left(\bar{\psi}\Gamma^C\psi\right)^2 + \mathcal{L}_{'t\ Hooft}$$

- Interaction parametrized by several coupling constants associated to the different channels considered: G_S, G_ρ, G_ω, K
- Total interaction must preserve the flavor symmetries
- $U(1)_A$ symmetry group broken by the 't Hooft term (anomaly)

The mean field approximation

- Assume small fluctuations of the fields around a mean value

$$\hat{O} = \langle \hat{O} \rangle + \delta \hat{O}$$

- For scalar interactions : $G_S(\bar{\psi}\psi)^2 \approx \underbrace{2G_S\langle\bar{\psi}\psi\rangle\bar{\psi}\psi}_{\text{Mass modification}} + G_S\langle\bar{\psi}\psi\rangle^2$

- For vector interactions $G_V(\bar{\psi}\gamma_0\psi)^2 \approx \underbrace{2G_V\langle\psi^\dagger\psi\rangle\psi^\dagger\psi}_{\text{Chemical potential modification}} + G_V\langle\psi^\dagger\psi\rangle^2$

- With correct flavor factors following $SU(N_f = 3)$ algebra:

$$m_i = m_{i\,0} - 4G_S\,\langle\bar{\psi}_i\psi_i\rangle + 2K\langle\bar{\psi}_j\psi_j\rangle\langle\bar{\psi}_k\psi_k\rangle$$

$$\tilde{\mu}_i = \mu_i - \frac{4}{3}G_\omega(n_i + n_j + n_k) - \frac{4}{3}G_\rho(2n_i - n_j - n_k)$$

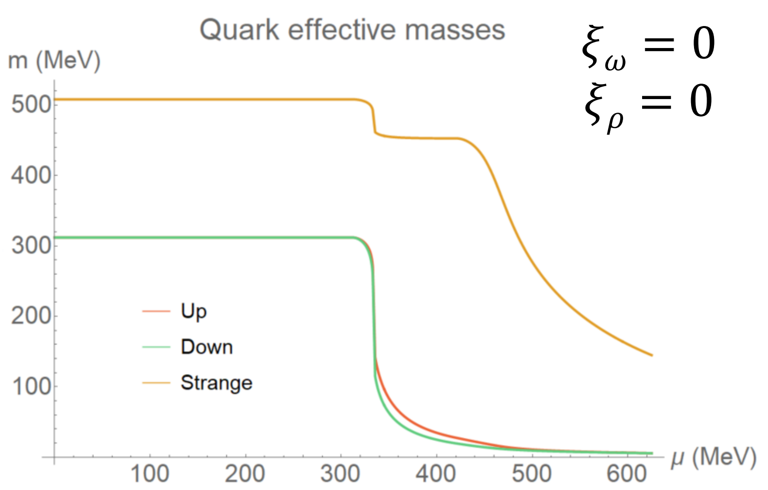
$i, j, k = u, d, s$

The phase transition

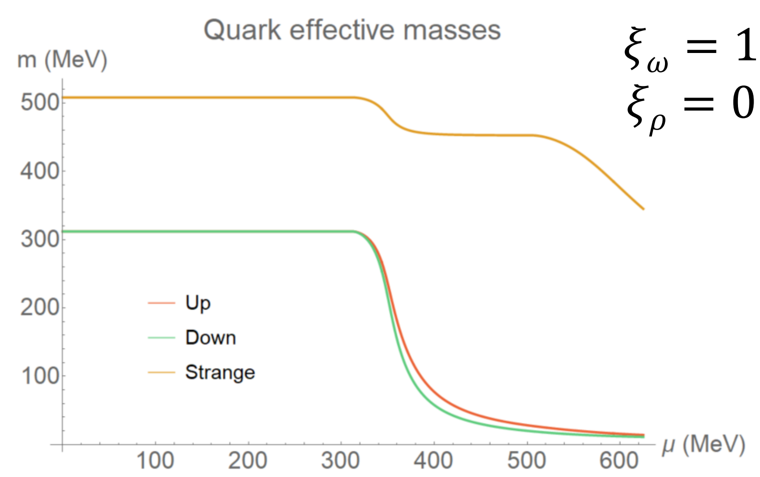
- Fix external conditions for NS at equilibrium:
  - Zero temperature
  - Charge neutrality
  - β-equilibrium

$$\Longrightarrow \begin{cases} \mu_u = \mu + \frac{2}{3}\mu_e \\ \mu_d = \mu_s = \mu - \frac{1}{3}\mu_e \end{cases}$$

- Chiral symmetry restoration



$\xi_\omega = 0$
$\xi_\rho = 0$



$\xi_\omega = 1$
$\xi_\rho = 0$

The phase transition

- Fix external conditions for NS at equilibrium:
  - Zero temperature
  - Charge neutrality
  - β-equilibrium

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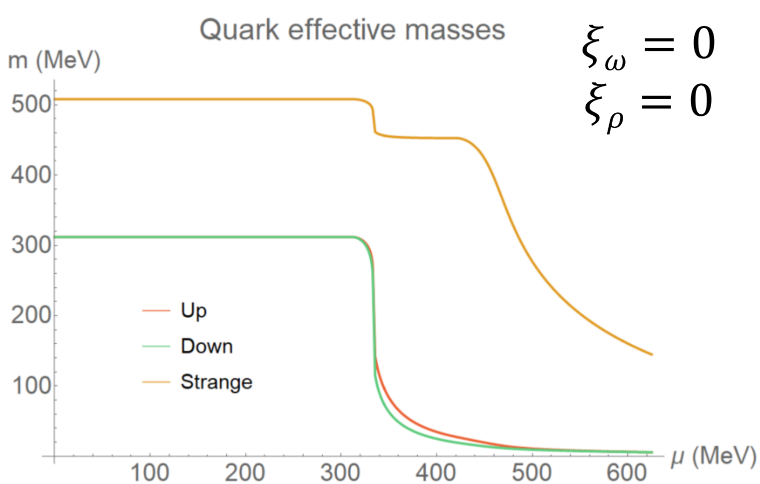

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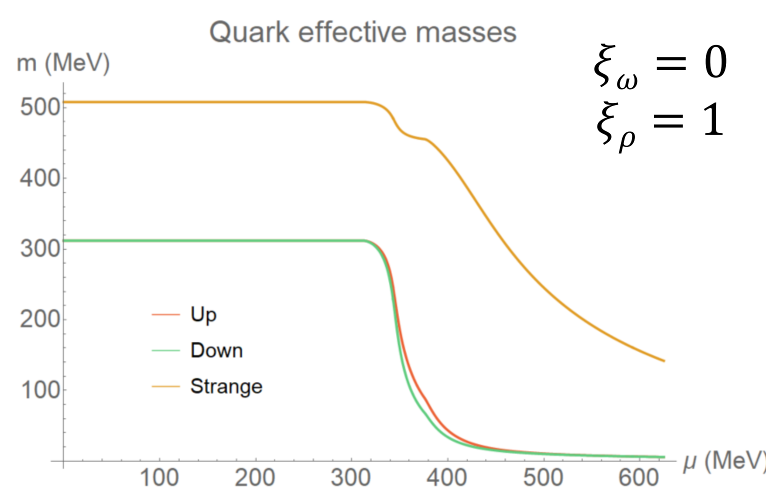


$\xi_\omega = 0$
$\xi_\rho = 1$

The equation of state

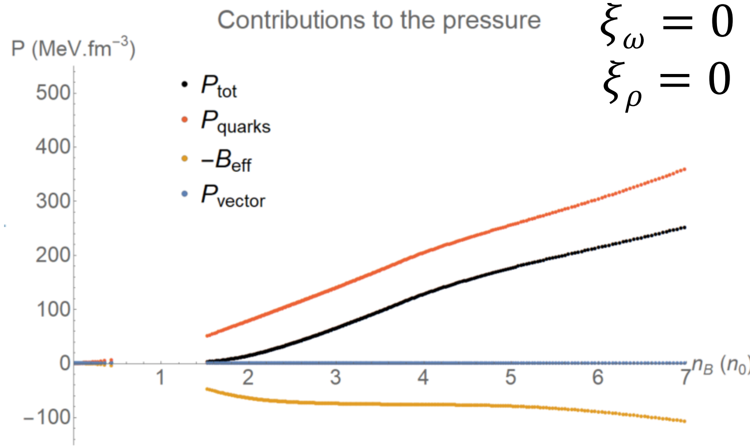
$$P_{tot} = P_{quarks} - B_{eff} + P_{vector} + P_{leptons}$$

- P_{quarks}: Fermi pressure of a free gas of quasi-quarks
- B_{eff}: Effective " bag " pressure
- P_{vector}: Additional contribution from vector interactions
- $P_{leptons}$: Fermi pressure of electrons (+ muons)

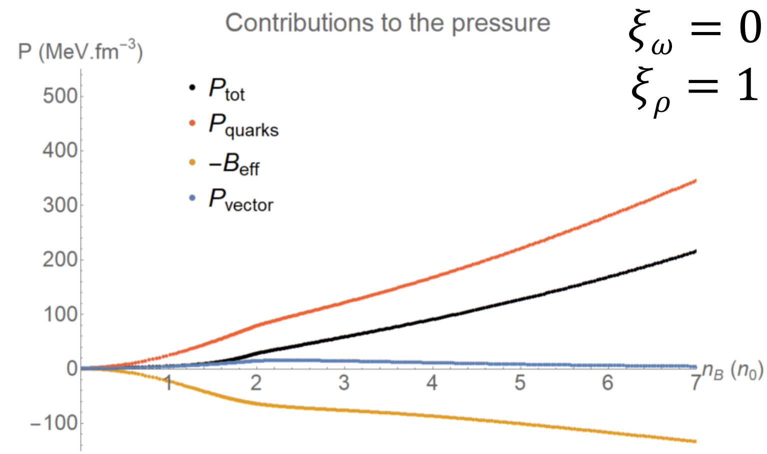
$$P_{vector} = \frac{2}{3}G_{\omega}(n_u + n_d + n_s)^2 + G_{\rho}(n_u - n_d)^2 + \frac{1}{3}G_{\rho}(n_u + n_d - 2n_s)^2$$

- ω interactions couple to the total baryonic density of the system (symmetric in flavor)
- ρ interactions couple to the isospin and flavor hypercharge densities (asymmetric in flavor)

The equation of state



$\xi_\omega = 0$
$\xi_\rho = 0$



$\xi_\omega = 0$
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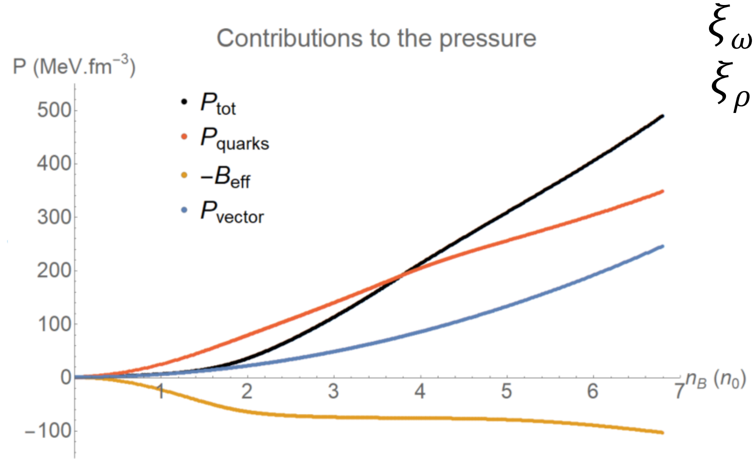


$\xi_\omega = 1$
$\xi_\rho = 0$

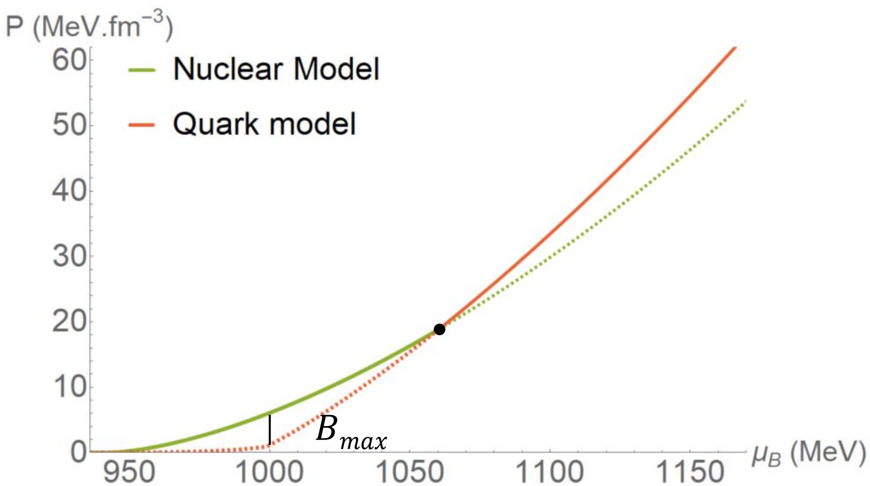
Hybrid EoS

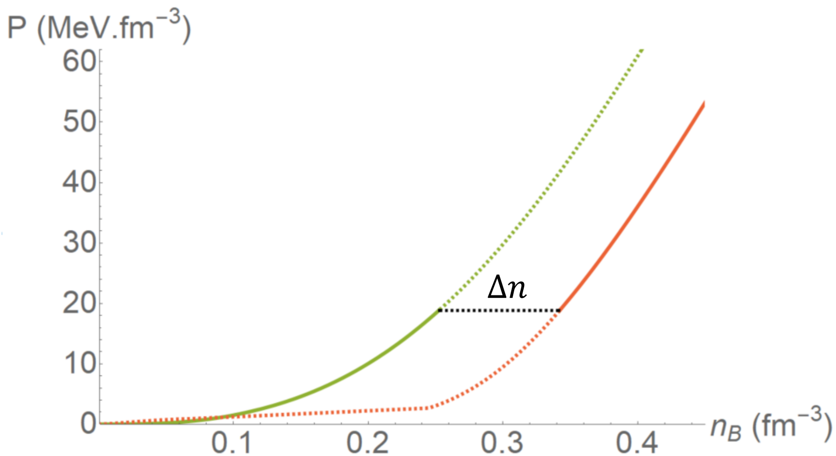
- NJL pressure defined up to constant; we allow the freedom:

$$P_Q \rightarrow P_Q + B^* \qquad\qquad \rho_Q \rightarrow \rho_Q - B^*$$

where B^* is a constant free parameter satisfying $B^* < B_{max}$

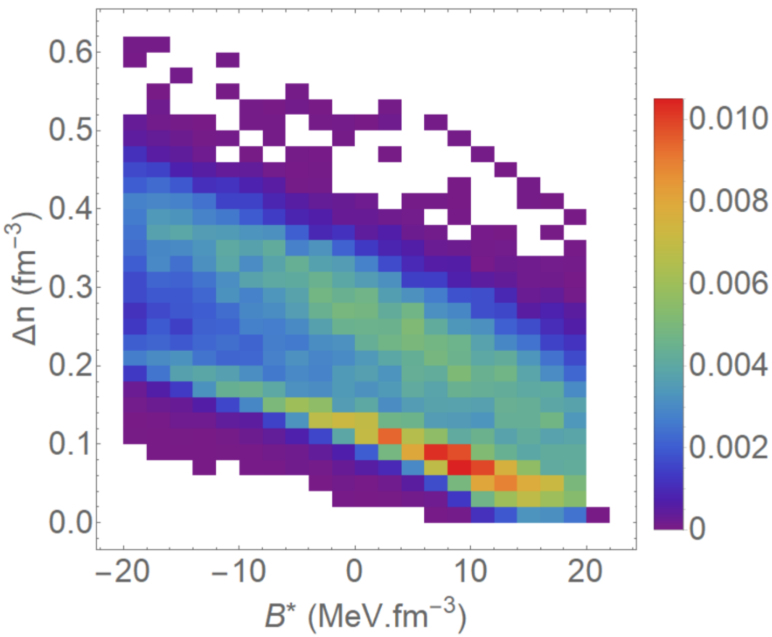
- Maxwell's construction not totally consistent : μ_e discontinuity

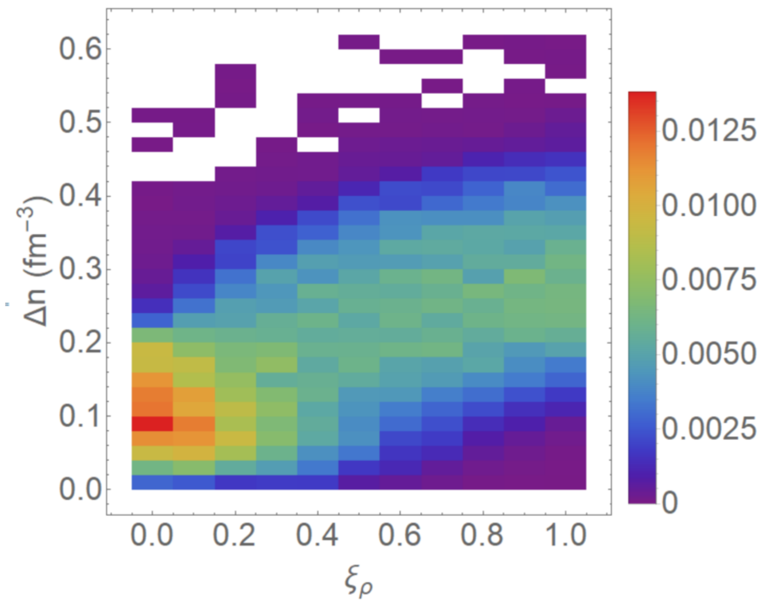




Maxwell's construction (first order)

Posterior results – phase transition characteristics





Posterior results – Hybrid star properties