Non-equilibrium studies of the chiral phase transition in a quark-meson model

C. Greiner

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in collaboration with:

A. Meistrenko (phd thesis) and H. van Hees

A. Meistrenko, H.v. Hees, CG,
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GOETHE UNIVERSITÄT FRANKFURT AM MAIN

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Quantum Chromo Dynamics
Phase Diagram, Symmetries

$$\mathcal{L}_{QCD} = \mathcal{L}_{\psi,A_\mu} + \mathcal{L}_G := \sum_{i\in\{u,d,s,c,b,t\}} \bar{\psi}_{i,j} \left(i\gamma^\mu \left(D_\mu\right)^j_k - m_i \delta^j_k \right) \psi^k_i - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

$$\mathcal{L}_{\psi,A_\mu} \sim \sum_{i\in\{u,d\}} \left[\bar{\psi}_{i,R} \left(i\gamma^\mu D_\mu\right) \psi_{i,R} + \bar{\psi}_{i,L} \left(i\gamma^\mu D_\mu\right) \psi_{i,L} \right] - \sum_{i\in\{u,d\}} m_i \left[\bar{\psi}_{i,R} \psi_{i,L} + \bar{\psi}_{i,L} \psi_{i,R} \right] , \qquad \mathrm{SU_L}(2) \times \mathrm{SU_R}(2)$$

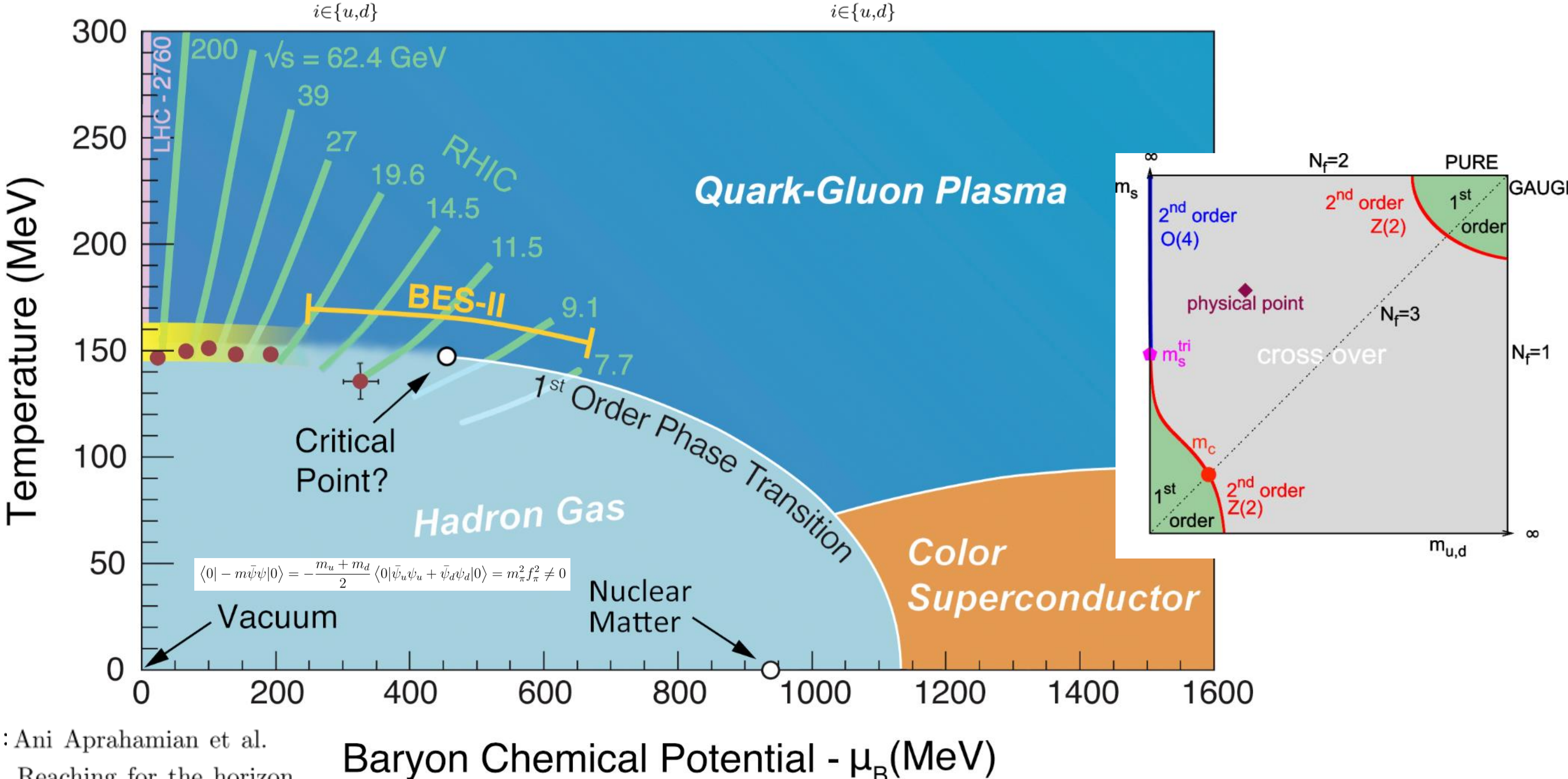


Fig.: Ani Aprahamian et al. Reaching for the horizon

Quantum Chromo Dynamics
Phase Diagram, Observables

$$\kappa_4(\sigma_V) := \langle(\delta\sigma_V)^4\rangle - 3\langle(\delta\sigma_V)^2\rangle^2,$$

$$\kappa_4(N_p) = \kappa_4(N_p^{stat.}) + \kappa_4(\sigma_V)\left(gd\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{\partial f_p^{eq.}}{\partial m}\right)^4 + \ldots$$
$$= \langle N_p\rangle + \kappa_4(\sigma_V)\left(\frac{gd}{T}\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{\partial E_p}{\partial m}f_p^{eq.}\right)^4 + \ldots,$$

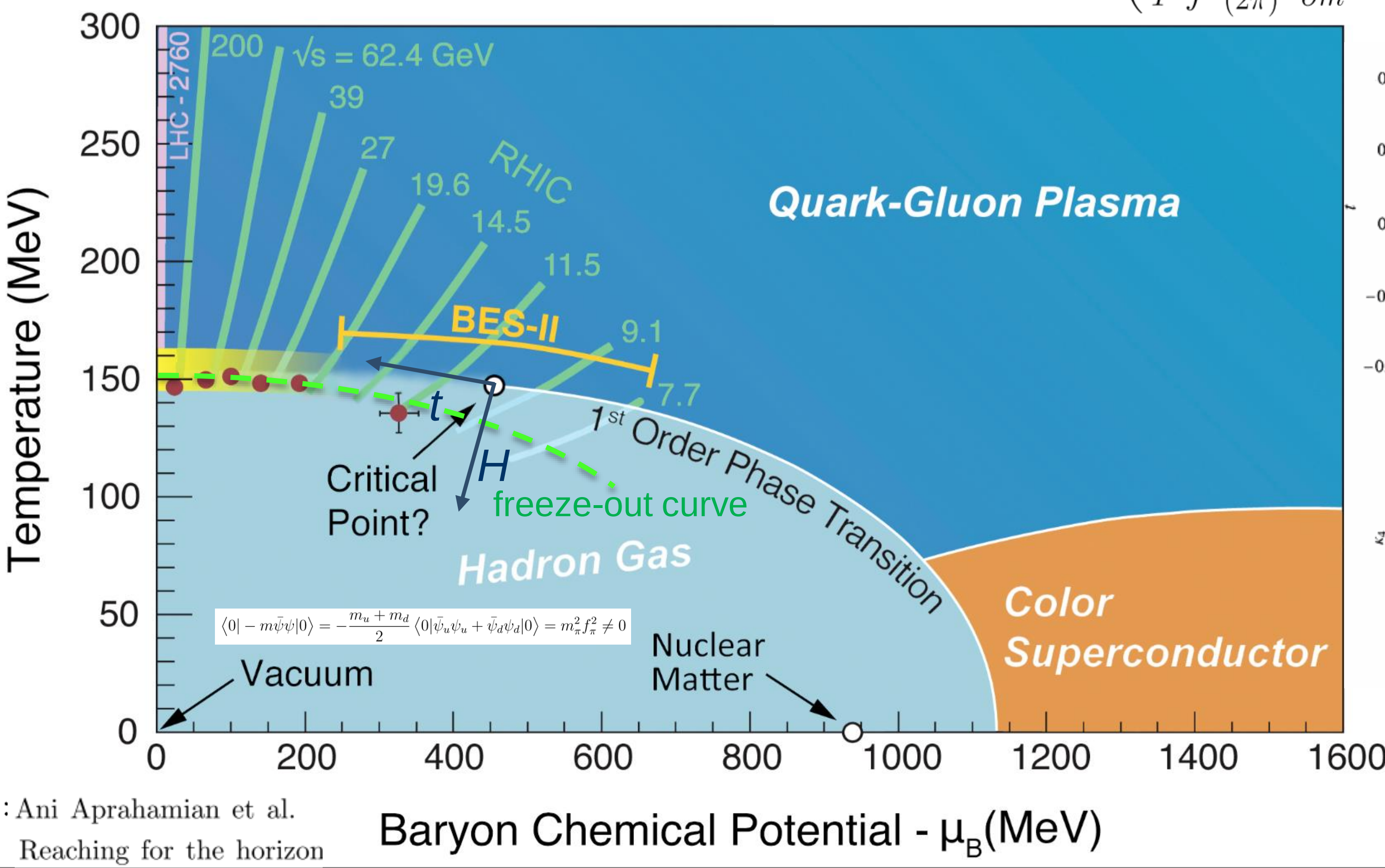


Fig.: Ani Aprahamian et al. Reaching for the horizon

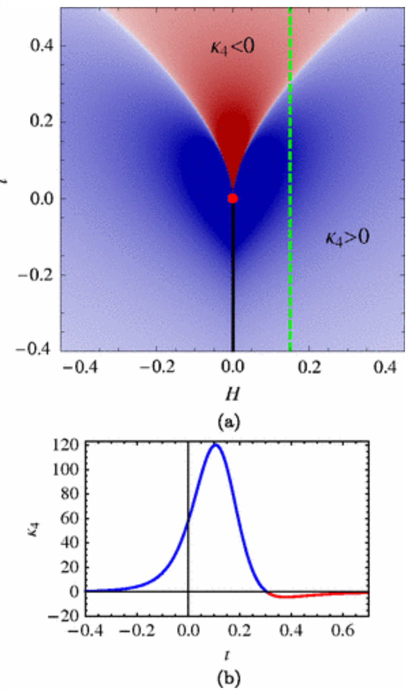
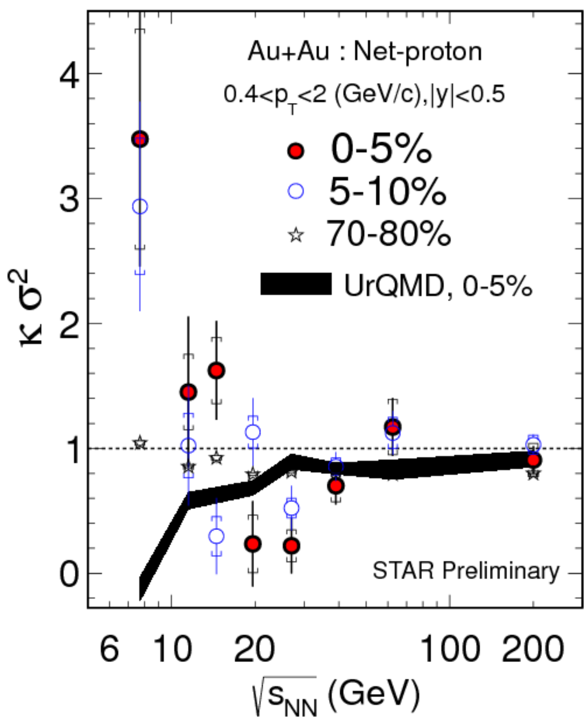


Fig.: M. A. Stephanov Phys. Rev. Lett., 107 : 052301, 2011

$$\kappa_4\left(\sigma_V\right) := \left\langle\left(\delta\sigma_V\right)^4\right\rangle - 3\left\langle\left(\delta\sigma_V\right)^2\right\rangle^2,$$

$$\kappa_4\left(N_p\right) = \kappa_4\left(N_p^{stat.}\right) + \kappa_4\left(\sigma_V\right)\left(gd\int\frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\frac{\partial f_p^{eq.}}{\partial m}\right)^4 + \dots$$
$$= \left\langle N_p\right\rangle + \kappa_4\left(\sigma_V\right)\left(\frac{gd}{T}\int\frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\frac{\partial E_p}{\partial m}f_p^{eq.}\right)^4 + \dots,$$



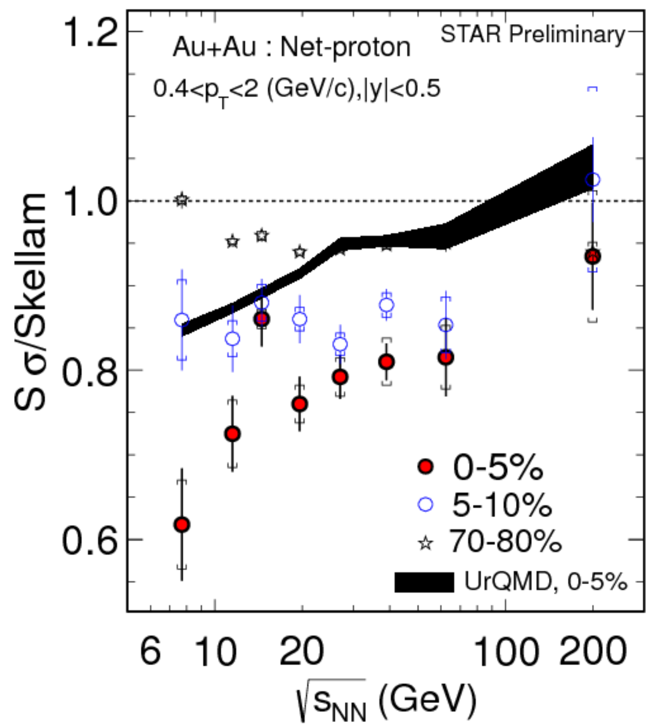


Fig.: Xiaofeng Luo, STAR collaboration
Nuclear Physics A, 956 : 75–82, 2016

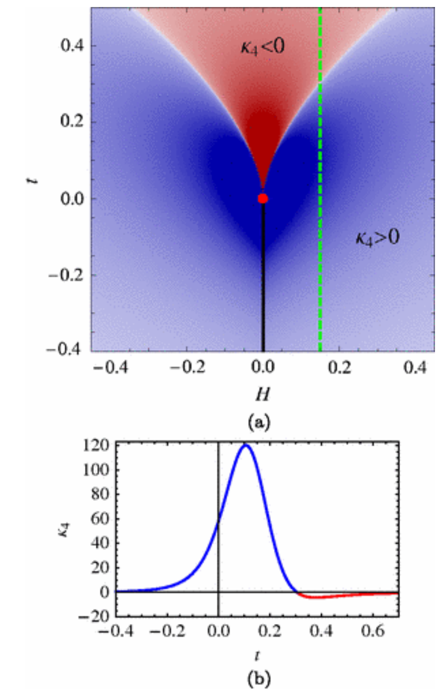
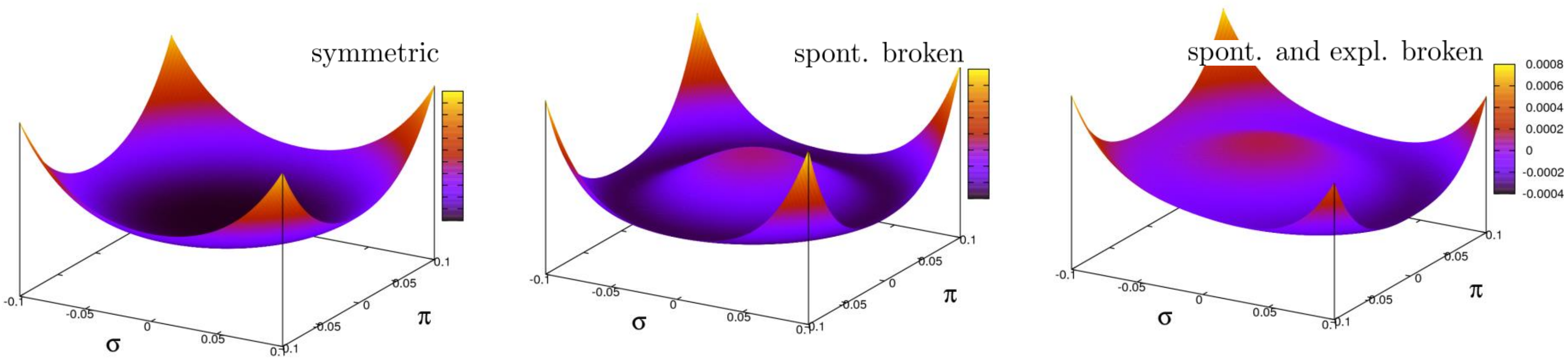


Fig.: M. A. Stephanov
Phys. Rev. Lett.,
107 : 052301, 2011

Quark-Meson Model
Chiral Symmetry

$$\mathcal{L} = \sum_i \bar{\psi}_i \left[i\partial\!\!\!/ - g\left(\sigma + i\gamma_5 \vec{\pi}\cdot\vec{\tau}\right)\right]\psi_i + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) \underbrace{- \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 + f_\pi m_\pi^2 \sigma + U_0}_{-U(\sigma,\vec{\pi})}$$

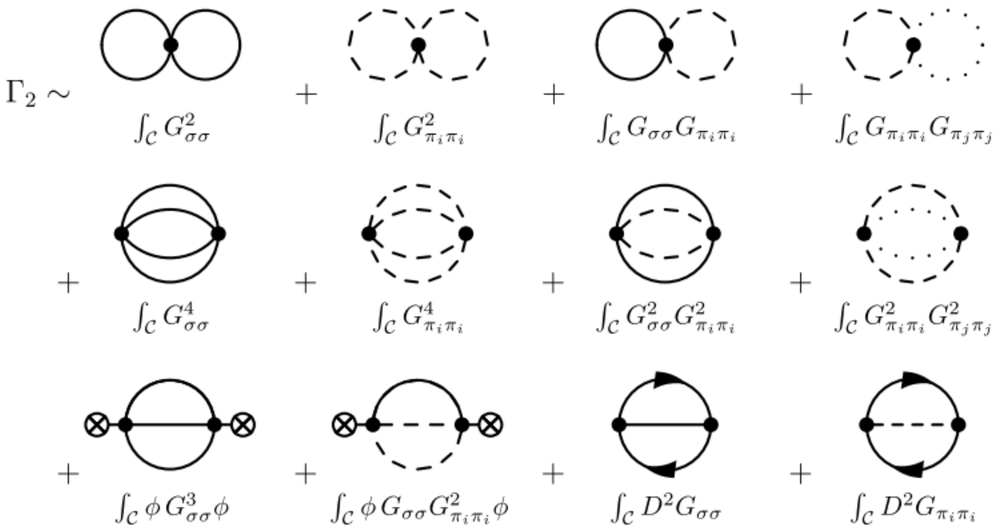


parameter	value	description
λ	20	coupling constant for σ and $\vec{\pi}$
g	$2-5$	coupling constant between $\sigma, \vec{\pi}$ and ψ_i
f_π	93 MeV	pion decay constant
m_π	138 MeV	pion mass
ν^2	$f_\pi^2 - m_\pi^2/\lambda$	field shift term
U_0	$m_\pi^4/(4\lambda) - f_\pi^2 m_\pi^2$	ground state

2PI Quantum Effective Action

$$\mathcal{L} = \sum_i \bar{\psi}_i \left[i\partial\!\!\!/ - g\left(\sigma + i\gamma_5 \vec{\pi}\cdot\vec{\tau}\right)\right]\psi_i + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\vec{\pi}\partial^\mu\vec{\pi}\right) - \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - \nu^2\right)^2 + f_\pi m_\pi^2 \sigma + U_0$$

$$\Gamma\left[\sigma, \vec{\pi}, G, D\right] = S\left[\sigma, \vec{\pi}\right] + \frac{i}{2}\mathrm{Tr}\log G^{-1} + \frac{i}{2}\mathrm{Tr}\, G_0^{-1}G - i\mathrm{Tr}\log D^{-1} - i\mathrm{Tr} D_0^{-1} D + \Gamma_2\left[\sigma, \vec{\pi}, G, D\right]$$

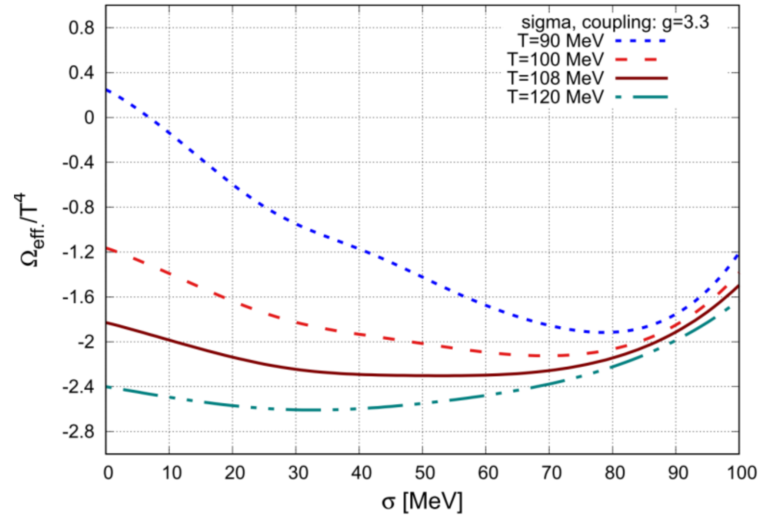


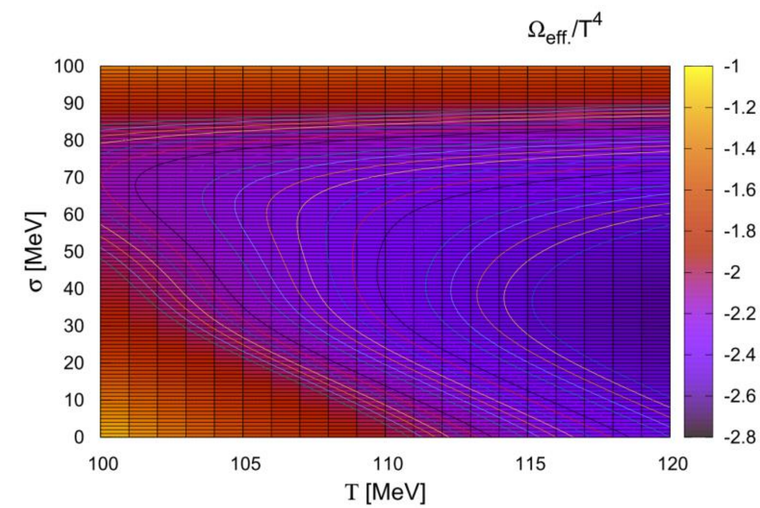
Quark-Meson Model
Thermodynamic Properties, Effective Potential

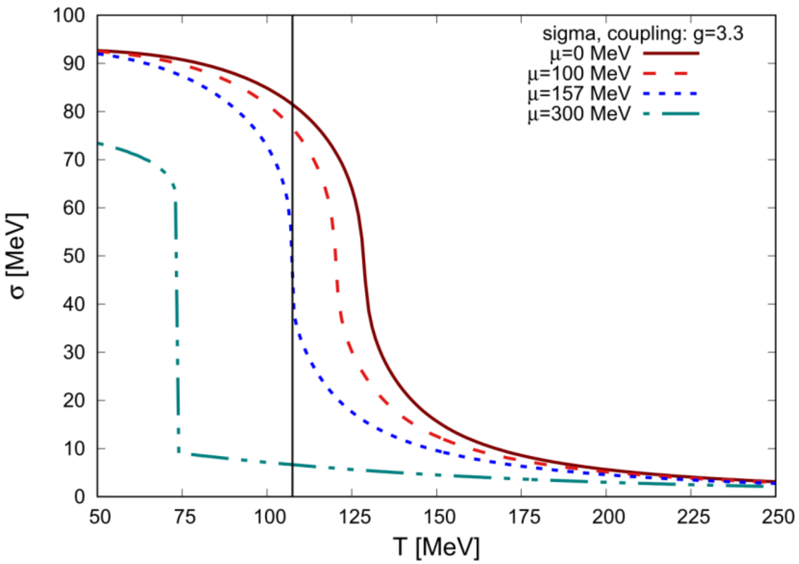
$$\Omega_{\text{eff.}}\left[\sigma,\vec{\pi},G,D\right] = -\frac{1}{\beta V} i\Gamma\left[\sigma,\vec{\pi},G,D\right]$$

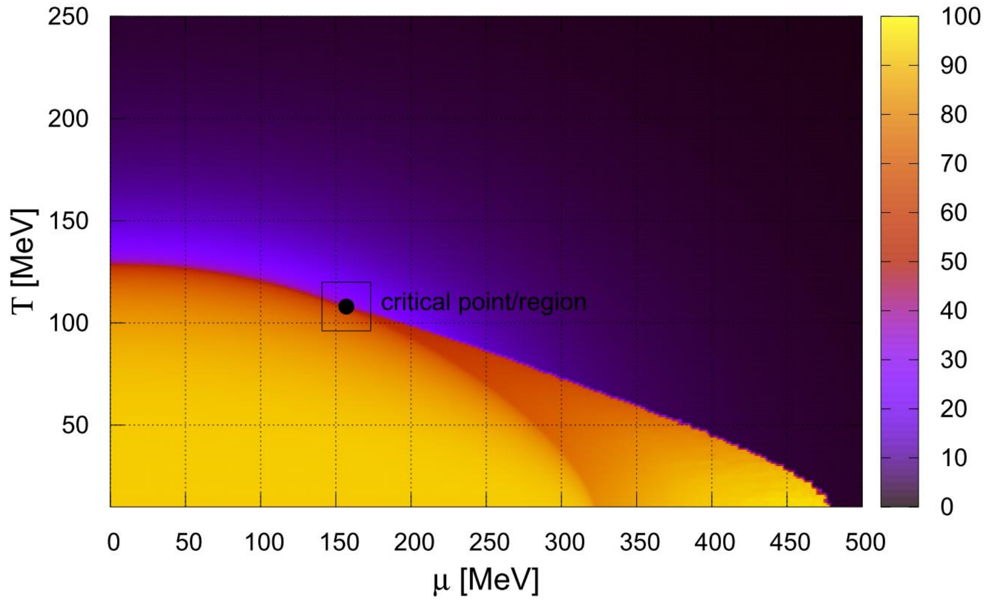
$$\Gamma_2 \sim \int_{\mathcal{C}} G_{\sigma\sigma}^2 + \int_{\mathcal{C}} G_{\pi_i\pi_i}^2 + \int_{\mathcal{C}} G_{\sigma\sigma}G_{\pi_i\pi_i} + \int_{\mathcal{C}} G_{\pi_i\pi_i}G_{\pi_j\pi_j}$$

$$\begin{aligned}
\Omega_{\text{eff.}} = \Omega_{\text{eff.}}^{\text{MF}} &+ \frac{1}{\beta}\int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\log\left(1-e^{-\beta E_\sigma}\right) + \sum_i \frac{1}{\beta}\int \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3}\log\left(1-e^{-\beta E_{\pi_i}}\right) \\
&- \frac{1}{2}\left[M_\sigma^2 - \lambda\left(3\sigma^2 + \sum_i \pi_i^2 - \nu^2\right)\right] Q\left(M_\sigma\right) \\
&- \frac{1}{2}\sum_i\left[M_{\pi_i}^2 - \lambda\left(\sigma^2 + 3\pi_i^2 + \sum_{j\neq i}\pi_j^2 - \nu^2\right)\right] Q\left(M_{\pi_i}\right) \\
&+ \frac{3}{4}\lambda\left[Q\left(M_\sigma\right)\right]^2 + \frac{3}{4}\lambda\sum_i\left[Q\left(M_{\pi_i}\right)\right]^2 \\
&+ \frac{1}{2}\lambda\sum_i\left[Q\left(M_\sigma\right)Q\left(M_{\pi_i}\right)\right] + \frac{1}{2}\lambda\sum_{i,j\neq i}\left[Q\left(M_{\pi_i}\right)Q\left(M_{\pi_j}\right)\right] + R_{\text{eff.}}
\end{aligned}$$







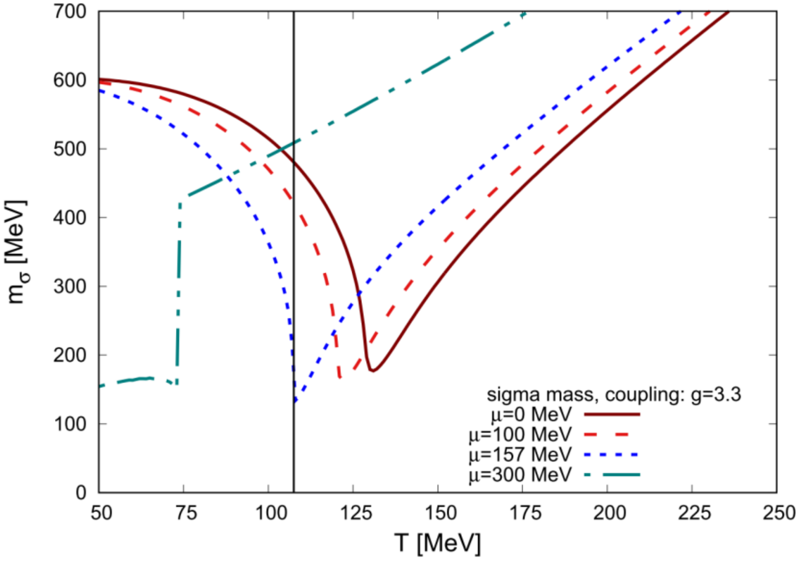


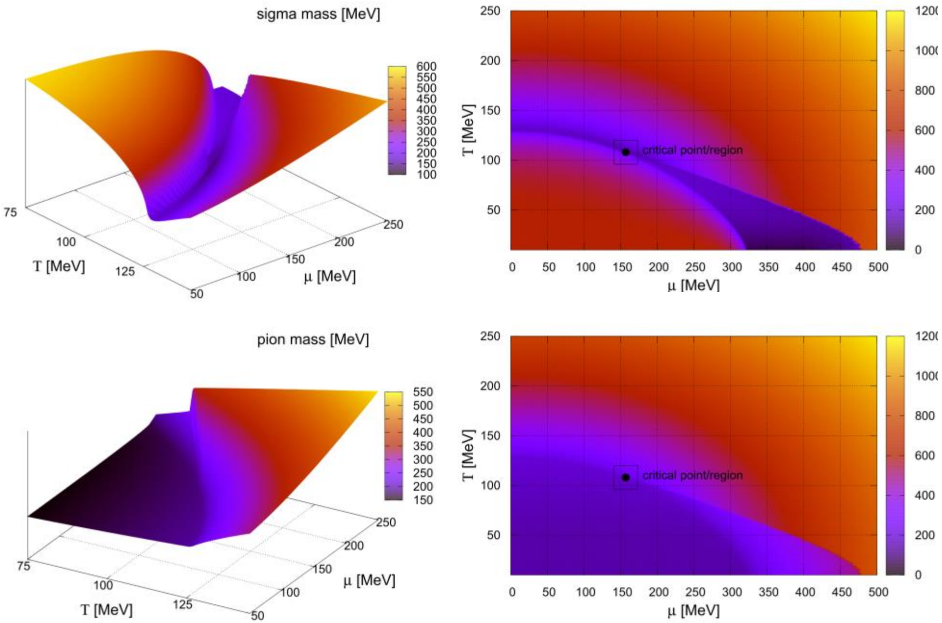
$$\frac{\partial\Omega_{\text{eff.}}}{\partial\sigma} = \lambda\left[\left(\sigma^2+\sum_i \pi_i^2-\nu^2\right)+3Q\left(M_\sigma\right)+\sum_i Q\left(M_{\pi_i}\right)\right]\sigma - f_\pi m_\pi^2 + g\left\langle\bar\psi\psi\right\rangle \stackrel{!}{=} 0$$

$$\frac{\partial^2\Omega_{\text{eff.}}}{\partial\sigma^2} = \lambda\left[\left(3\sigma^2+\sum_i \pi_i^2-\nu^2\right)+3Q\left(M_\sigma\right)+\sum_i Q\left(M_{\pi_i}\right)\right] + \frac{\partial}{\partial\sigma} g\left\langle\bar\psi\psi\right\rangle\Big|_{\sigma=\langle\sigma\rangle} =: M_\sigma^2$$

$$\frac{\partial^2\Omega_{\text{eff.}}}{\partial\pi_i^2} = \lambda\left[\left(\sigma^2+3\pi_i^2+\sum_{j\neq i} \pi_j^2-\nu^2\right)+3Q\left(M_{\pi_i}\right)+\sum_{j\neq i} Q\left(M_{\pi_j}\right)+Q\left(M_\sigma\right)\right] + \frac{\partial}{\partial\pi_i} g\left\langle\bar\psi i\gamma_5\tau_i\psi\right\rangle\Big|_{\pi_i=\langle\pi_i\rangle} =: M_{\pi_i}^2$$

Quark-Meson Model
Thermodynamic Properties, Effective Mass





$$\frac{\partial \Omega_{\text{eff.}}}{\partial \sigma} = \lambda \left[\left(\sigma^2 + \sum_i \pi_i^2 - \nu^2 \right) + 3Q\left(M_\sigma\right) + \sum_i Q\left(M_{\pi_i}\right) \right] \sigma - f_\pi m_\pi^2 + g \left\langle \bar{\psi}\psi \right\rangle \stackrel{!}{=} 0$$

$$\frac{\partial^2 \Omega_{\text{eff.}}}{\partial \sigma^2} = \lambda \left[\left(3\sigma^2 + \sum_i \pi_i^2 - \nu^2 \right) + 3Q\left(M_\sigma\right) + \sum_i Q\left(M_{\pi_i}\right) \right] + \frac{\partial}{\partial \sigma} g \left\langle \bar{\psi}\psi \right\rangle \Big|_{\sigma = \langle \sigma \rangle} =: M_\sigma^2$$

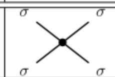
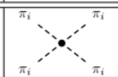
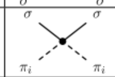
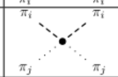
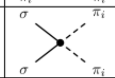
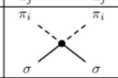
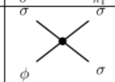
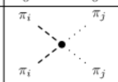
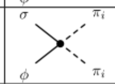
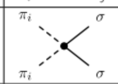
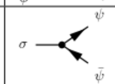
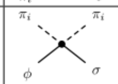
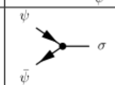
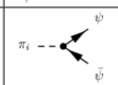
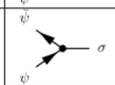
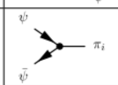
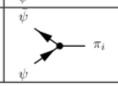
$$\frac{\partial^2 \Omega_{\text{eff.}}}{\partial \pi_i^2} = \lambda \left[\left(\sigma^2 + 3\pi_i^2 + \sum_{j \neq i} \pi_j^2 - \nu^2 \right) + 3Q\left(M_{\pi_i}\right) + \sum_{j \neq i} Q\left(M_{\pi_j}\right) + Q\left(M_\sigma\right) \right] + \frac{\partial}{\partial \pi_i} g \left\langle \bar{\psi} i \gamma_5 \tau_i \psi \right\rangle \Big|_{\pi_i = \langle \pi_i \rangle} =: M_{\pi_i}^2$$

mean-field equation without expansion:

$$\partial_t^2 \phi + D\,(t) + J\,(t) = 0\,,$$
$$J\,(t) := \lambda \left(\phi^2 - \nu^2 + 3G^{11}_{\sigma\sigma} + \sum_i G^{11}_{\pi_i \pi_i} \right) \phi - f_\pi m_\pi^2 + g \left\langle \bar{\psi} \psi \right\rangle$$

Boltzmann-like equations for mesons and quarks:

$$\begin{aligned}
\partial_t f^\sigma\,(t,\vec{p}) &= \mathcal{I}^{b.}_\sigma\,(t,\vec{p}_1) + \mathcal{I}^{b.s.}_\sigma\,(t,\vec{p}_1) + \mathcal{I}^{f.s.}_\sigma\,(t,\vec{p}_1)\\
&= \mathcal{C}^{b.}_{\sigma\sigma\leftrightarrow\sigma\sigma} + \sum_i \mathcal{C}^{b.}_{\sigma\pi_i\leftrightarrow\sigma\pi_i} + \sum_i \mathcal{C}^{b.}_{\sigma\sigma\leftrightarrow\pi_i\pi_i}\\
&\quad + \mathcal{C}^{b.s.}_{\sigma\phi\leftrightarrow\sigma\sigma} + \sum_i \mathcal{C}^{b.s.}_{\sigma\phi\leftrightarrow\pi_i\pi_i} + \mathcal{C}^{f.s.}_{\sigma\leftrightarrow\psi\bar{\psi}}\,,\\
\partial_t f^{\pi_i}\,(t,\vec{p}) &= \mathcal{I}^{b.}_{\pi_i}\,(t,\vec{p}_1) + \mathcal{I}^{b.s.}_{\pi_i}\,(t,\vec{p}_1) + \mathcal{I}^{f.s.}_{\pi_i}\,(t,\vec{p}_1)\\
&= \mathcal{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_i\pi_i} + \sum_{j\neq i} \mathcal{C}^{b.}_{\pi_i\pi_j\leftrightarrow\pi_i\pi_j} + \sum_{j\neq i} \mathcal{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_j\pi_j} + \mathcal{C}^{b.}_{\pi_i\sigma\leftrightarrow\pi_i\sigma} + \mathcal{C}^{b.}_{\pi_i\pi_i\leftrightarrow\sigma\sigma}\\
&\quad + \mathcal{C}^{b.s.}_{\pi_i\phi\leftrightarrow\pi_i\sigma} + \mathcal{C}^{f.s.}_{\pi_i\leftrightarrow\psi\bar{\psi}}\\
\partial_t f^{\psi}\,(t,\vec{p}_1) &= \mathcal{I}^{f.s.}_{\psi}\,(t,\vec{p}_1) = \mathcal{C}^{f.s.}_{\psi\bar{\psi}\leftrightarrow\sigma} + \sum_i \mathcal{C}^{f.s.}_{\psi\bar{\psi}\leftrightarrow\pi_i}\\
\partial_t f^{\bar{\psi}}\,(t,\vec{p}_1) &= \mathcal{I}^{f.s.}_{\bar{\psi}}\,(t,\vec{p}_1) = \mathcal{C}^{f.s.}_{\bar{\psi}\psi\leftrightarrow\sigma} + \sum_i \mathcal{C}^{f.s.}_{\bar{\psi}\psi\leftrightarrow\pi_i}\,,
\end{aligned}$$

collision integral	diagram	collision integral	diagram
$\mathcal{C}^{b.}_{\sigma\sigma\leftrightarrow\sigma\sigma}$		$\mathcal{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_i\pi_i}$	
$\mathcal{C}^{b.}_{\sigma\pi_i\leftrightarrow\sigma\pi_i}$		$\mathcal{C}^{b.}_{\pi_i\pi_j\leftrightarrow\pi_i\pi_j}$	
$\mathcal{C}^{b.}_{\sigma\sigma\leftrightarrow\pi_i\pi_i}$		$\mathcal{C}^{b.}_{\pi_i\sigma\leftrightarrow\pi_i\sigma}$	
$\mathcal{C}^{b.s.}_{\sigma\phi\leftrightarrow\sigma\sigma}$		$\mathcal{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_j\pi_j}$	
$\mathcal{C}^{b.s.}_{\sigma\phi\leftrightarrow\pi_i\pi_i}$		$\mathcal{C}^{b.}_{\pi_i\pi_i\leftrightarrow\sigma\sigma}$	
$\mathcal{C}^{f.s.}_{\sigma\leftrightarrow\psi\bar{\psi}}$		$\mathcal{C}^{b.s.}_{\pi_i\phi\leftrightarrow\pi_i\sigma}$	
$\mathcal{C}^{f.s.}_{\psi\bar{\psi}\leftrightarrow\sigma}$		$\mathcal{C}^{f.s.}_{\pi_i\leftrightarrow\psi\bar{\psi}}$	
$\mathcal{C}^{f.s.}_{\bar{\psi}\psi\leftrightarrow\sigma}$		$\mathcal{C}^{f.s.}_{\psi\bar{\psi}\leftrightarrow\pi_i}$	
		$\mathcal{C}^{f.s.}_{\bar{\psi}\psi\leftrightarrow\pi_i}$	

A. Meistrenko, H.v. Hees, CG,
Annals Phys. 431 (2021) 168555
2007.09929 [hep-ph]

memory kernel in mean-field equation:

$$D(t) = 6\lambda^2 \int_{t_0}^{t} \mathrm{d}t' \dot{\phi}(t') \, \Gamma(t, t - t')$$

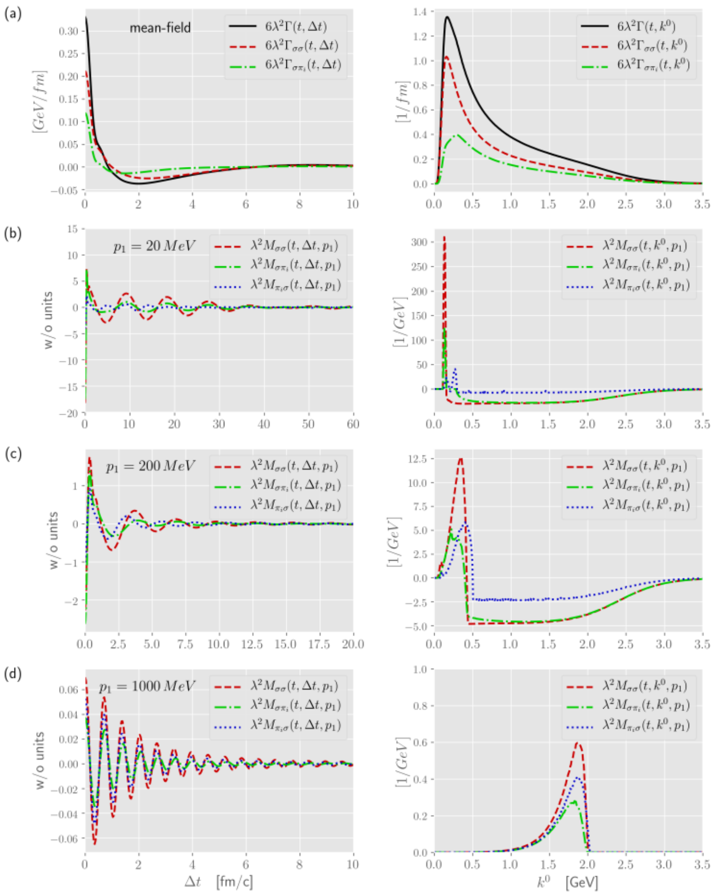
$$\Gamma(t, \Delta t) := \int \frac{\mathrm{d}k^0}{(2\pi)} e^{-ik^0 \Delta t} \frac{1}{k^0} \left[\tilde{\mathcal{M}}_{\sigma\sigma}(t, k^0) + \frac{1}{3} \sum_i \tilde{\mathcal{M}}_{\sigma\pi_i}(t, k^0) \right]$$

memory kernel for non-zero modes:

$$\mathcal{I}^{b.s.}_{\sigma}(t, \vec{p}_1) := \frac{9}{2}\pi\lambda^2 \phi(t) \int_0^{\infty} \mathrm{d}\Delta t \, \phi(t - \Delta t) \int \frac{\mathrm{d}k^0}{(2\pi)} e^{-ik^0 \Delta t} \mathcal{M}_{\sigma\sigma}(t, k^0, p_1) + \frac{1}{2}\pi\lambda^2 \phi(t) \sum_i \int_0^{\infty} \mathrm{d}\Delta t \, \phi(t - \Delta t) \int \frac{\mathrm{d}k^0}{(2\pi)} e^{-ik^0 \Delta t} \mathcal{M}_{\sigma\pi_i}(t, k^0, p_1)$$

$$\mathcal{I}^{b.s.}_{\pi_i}(t, \vec{p}_1) := \pi\lambda^2 \phi(t) \int_0^{\infty} \mathrm{d}\Delta t \, \phi(t - \Delta t) \int \frac{\mathrm{d}k^0}{(2\pi)} e^{-ik^0 \Delta t} \mathcal{M}_{\pi_i\sigma}(t, k^0, p_1)$$

$T = 105\,\mathrm{MeV},\ \mu = 165\,\mathrm{MeV}$



Quark-Meson Model
Evolution Equations, Expanding Geometry

mean-field equation with expansion:

$$\partial_t^2\phi + E(t) + D(t) + J(t) = 0 \quad \text{with} \quad E(t) := 3H\partial_t\phi$$

$$J(t) := \lambda\left(\phi^2 - \nu^2 + 3G^{11}_{\sigma\sigma} + \sum_i G^{11}_{\pi_i\pi_i}\right)\phi - f_\pi m_\pi^2 + g\left\langle\bar\psi\psi\right\rangle$$

FLRW metric:

$$ds^2 = dt^2 - R^2(t)\left(dx_1^2 + dx_2^2 + dx_3^2\right), \qquad H = \dot R / R$$

Boltzmann-like equations for mesons:

$$\left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right)f^\sigma(t,\vec p) = \mathcal{I}^{b.}_\sigma(t,\vec p_1) + \mathcal{I}^{b.s.}_\sigma(t,\vec p_1) + \mathcal{I}^{f.s.}_\sigma(t,\vec p_1)$$
$$= \mathcal{C}^{b.}_{\sigma\sigma\leftrightarrow\sigma\sigma} + \sum_i \mathcal{C}^{b.}_{\sigma\pi_i\leftrightarrow\sigma\pi_i} + \sum_i \mathcal{C}^{b.}_{\sigma\sigma\leftrightarrow\pi_i\pi_i}$$
$$+ \mathcal{C}^{b.s.}_{\sigma\phi\leftrightarrow\sigma\sigma} + \sum_i \mathcal{C}^{b.s.}_{\sigma\phi\leftrightarrow\pi_i\pi_i} + \mathcal{C}^{f.s.}_{\sigma\leftrightarrow\psi\bar\psi},$$

$$\left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right)f^{\pi_i}(t,\vec p) = \mathcal{I}^{b.}_{\pi_i}(t,\vec p_1) + \mathcal{I}^{b.s.}_{\pi_i}(t,\vec p_1) + \mathcal{I}^{f.s.}_{\pi_i}(t,\vec p_1)$$
$$= \mathcal{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_i\pi_i} + \sum_{j\neq i} \mathcal{C}^{b.}_{\pi_i\pi_j\leftrightarrow\pi_i\pi_j} + \sum_{j\neq i} \mathcal{C}^{b.}_{\pi_i\pi_i\leftrightarrow\pi_j\pi_j} + \mathcal{C}^{b.}_{\pi_i\sigma\leftrightarrow\pi_i\sigma} + \mathcal{C}^{b.}_{\pi_i\pi_i\leftrightarrow\sigma\sigma}$$
$$+ \mathcal{C}^{b.s.}_{\pi_i\phi\leftrightarrow\pi_i\sigma} + \mathcal{C}^{f.s.}_{\pi_i\leftrightarrow\psi\bar\psi}$$

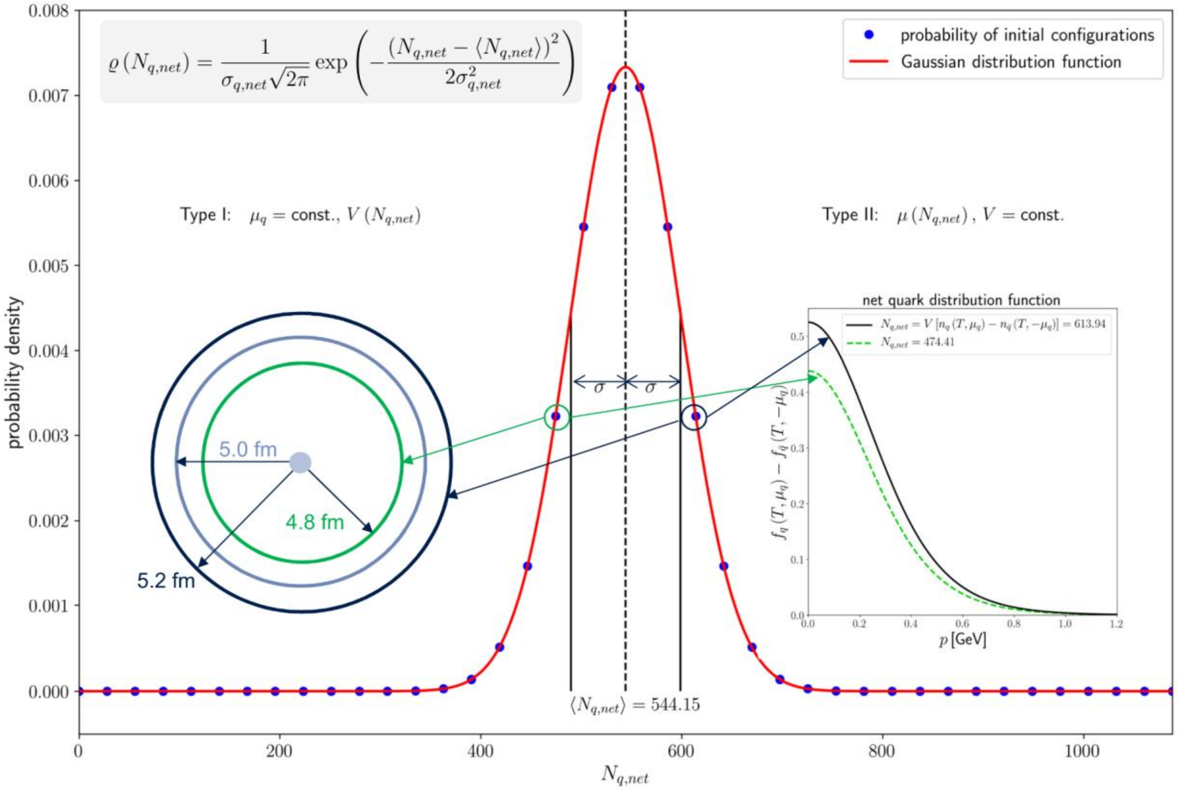
$$\left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right)f^{\psi}(t,\vec p_1) = \mathcal{I}^{f.s.}_{\psi}(t,\vec p_1) = \mathcal{C}^{f.s.}_{\psi\bar\psi\leftrightarrow\sigma} + \sum_i \mathcal{C}^{f.s.}_{\psi\bar\psi\leftrightarrow\pi_i}$$

$$\left(\frac{\partial}{\partial t} - Hp\frac{\partial}{\partial p}\right)f^{\bar\psi}(t,\vec p_1) = \mathcal{I}^{f.s.}_{\bar\psi}(t,\vec p_1) = \mathcal{C}^{f.s.}_{\bar\psi\psi\leftrightarrow\sigma} + \sum_i \mathcal{C}^{f.s.}_{\bar\psi\psi\leftrightarrow\pi_i},$$

initial net quark number:

$$\langle N_{q,net}\rangle := n_{q,net}V = \frac{4}{3}\pi R_0^3 \int \frac{d^3\vec p}{(2\pi)^3}\left(f_q - f_{\bar q}\right)$$

distribution of initial configurations with respect to $N_{q,net}$



1. mean net quark number:

$$\langle N_{q,net}\rangle := \frac{4}{3}\pi R_0^3 \int \frac{d^3\vec{p}}{(2\pi)^3} \left(f_q(T,\mu_q) - f_{\bar{q}}(T,-\mu_q)\right)$$

2. define standard deviation:

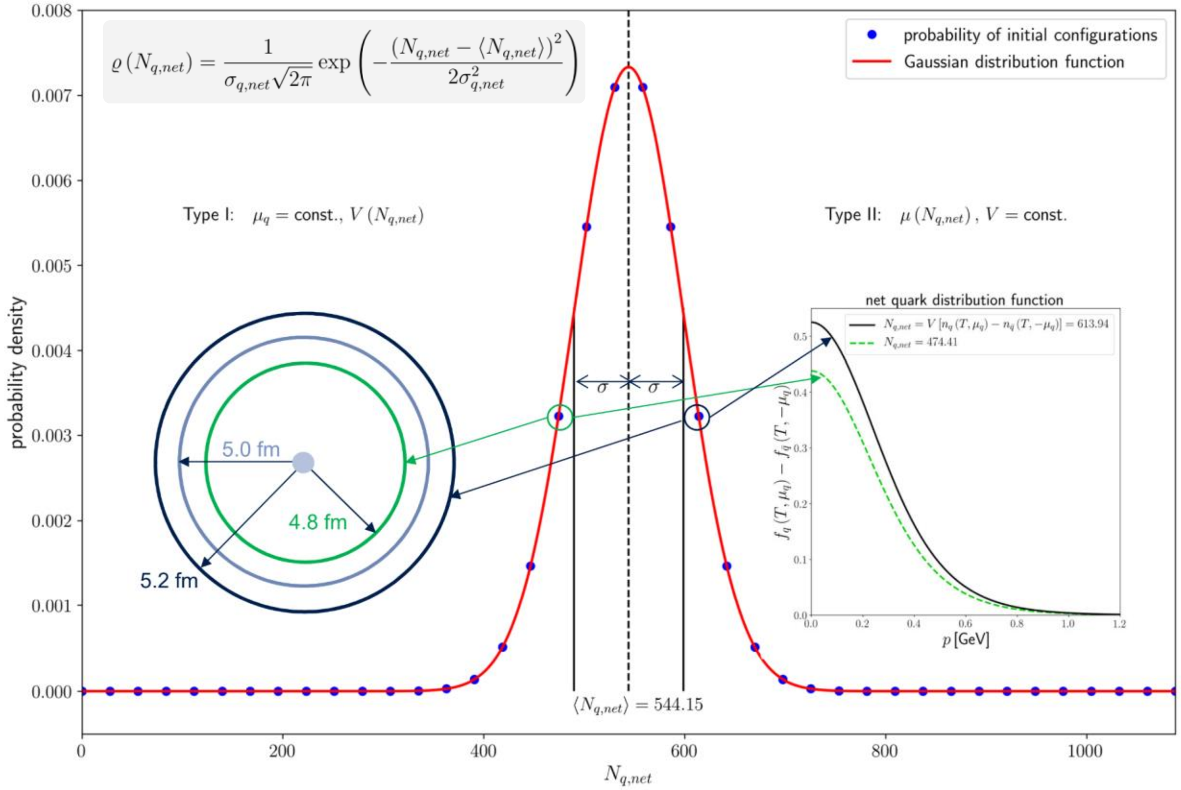
$$\sigma_{q,net} = \langle N_{q,net}\rangle / 10 , \ldots$$

3. choose equidistant grid points:

$200 - 1000$ values for $N_{q,net}$

4. initialize type I or type II for each $N_{q,net}$

distribution of initial configurations with respect to $N_{q,net}$



general observables

$$\langle \mathbf{O} \rangle = \frac{p_0 \mathbf{O}_0 + p_M \mathbf{O}_M}{2} + \sum_{k=1}^{M-1} p_k \mathbf{O}_k$$

$$p_k = \varrho(N_k)/M$$

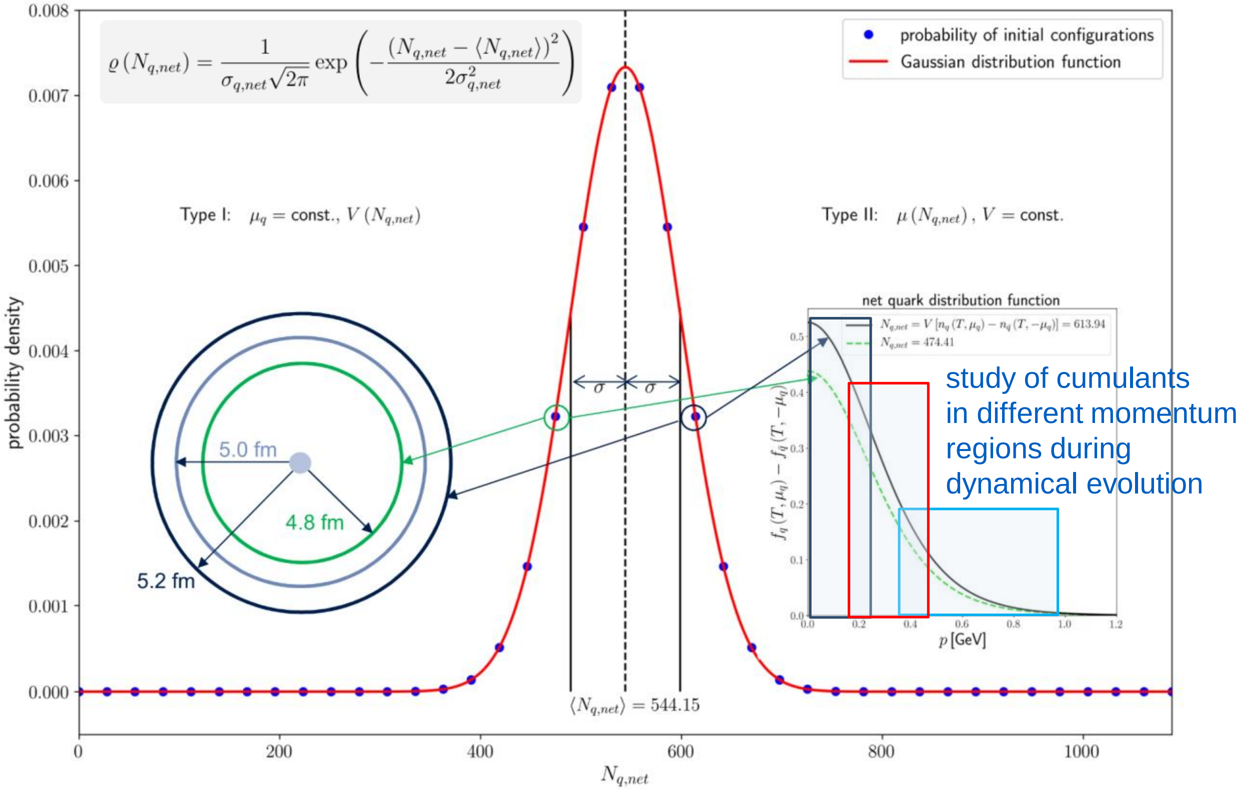
cumulant ratios

$$R_{3,1} = \frac{\kappa_3}{\kappa_1}, \quad R_{4,2} = \frac{\kappa_4}{\kappa_2} \equiv \kappa\sigma^2$$

$$\kappa_1 = \langle m \rangle ,$$
$$\kappa_2 = \tilde{m}_2 \equiv \sigma^2 ,$$
$$\kappa_3 = \tilde{m}_3 ,$$
$$\kappa_4 = \tilde{m}_4 - 3\tilde{m}_2^2 ,$$
$$\kappa_5 = \tilde{m}_5 - 10\tilde{m}_3\tilde{m}_5 ,$$
$$\kappa_6 = \tilde{m}_6 - 15\tilde{m}_4\tilde{m}_2 - 10\tilde{m}_3^2 + 30\tilde{m}_2^3 ,$$

Dynamical Evolution
Initial Conditions

distribution of initial configurations with respect to $N_{q,net}$



general observables

$$\langle \mathbf{O} \rangle = \frac{p_0 \mathbf{O}_0 + p_M \mathbf{O}_M}{2} + \sum_{k=1}^{M-1} p_k \mathbf{O}_k$$

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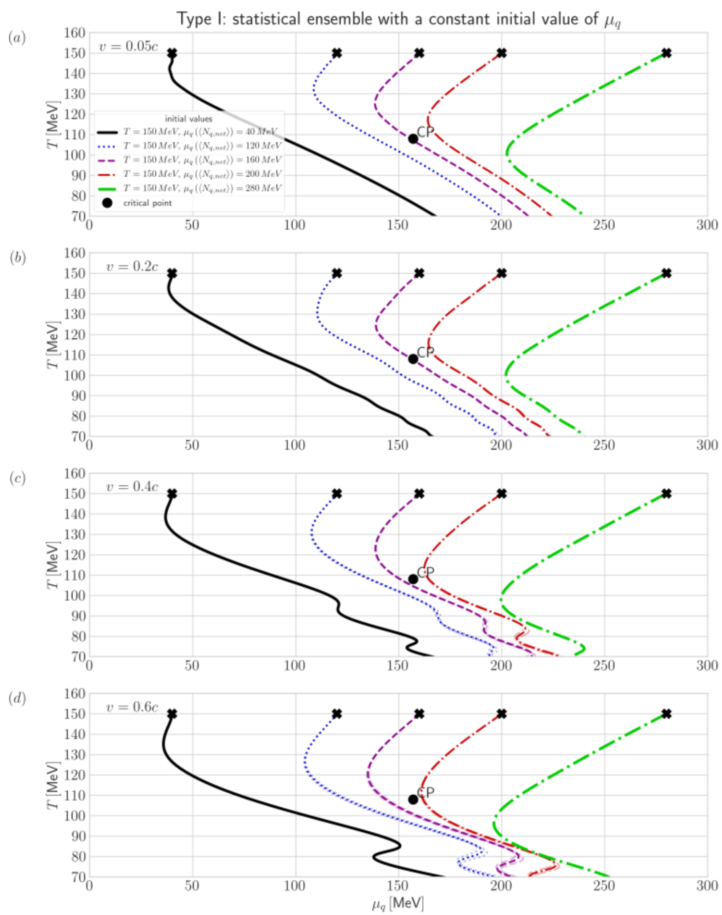
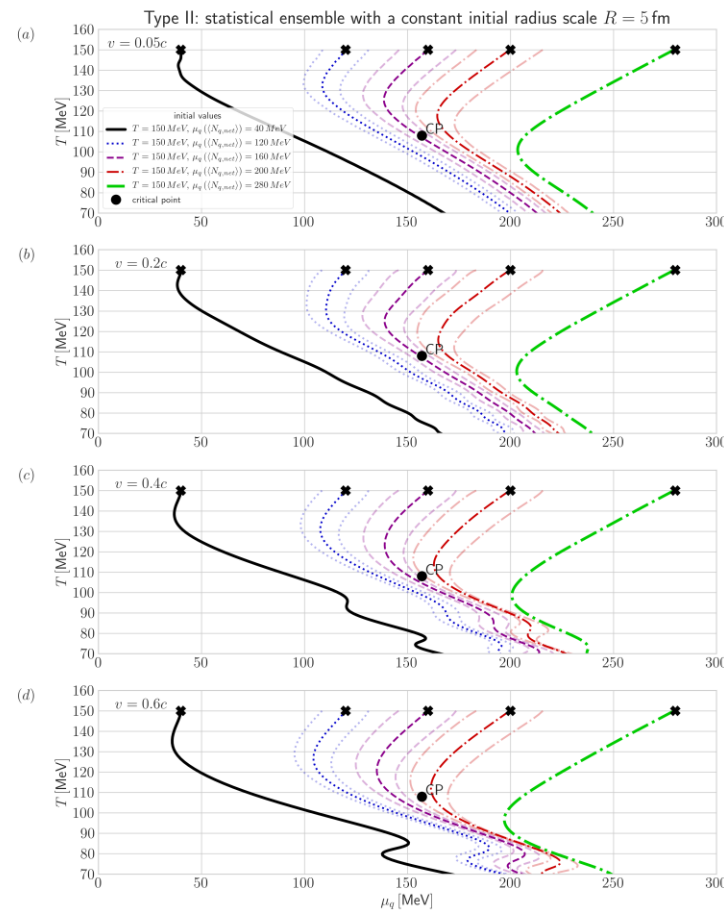
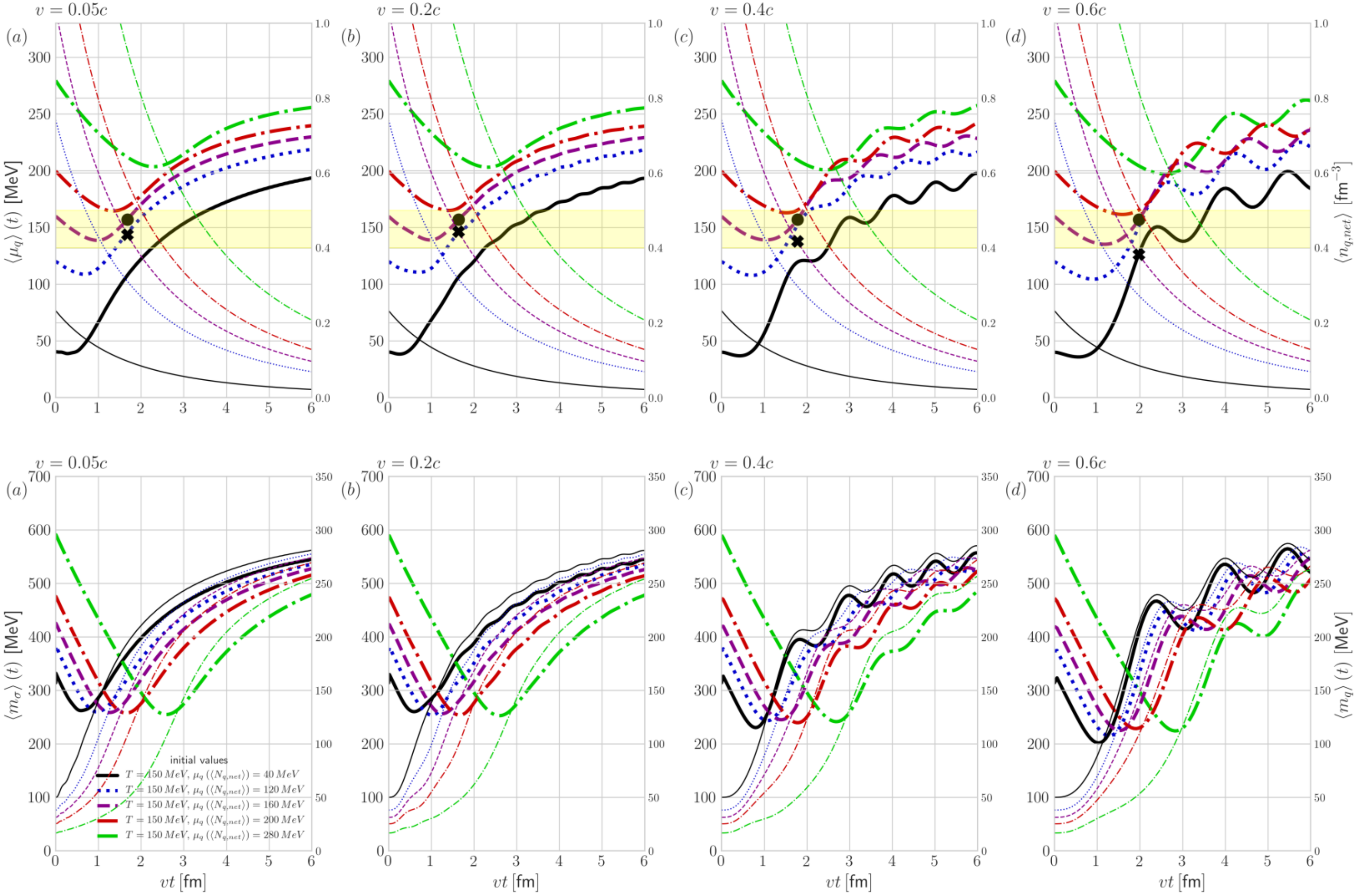
Dynamical Evolution
Phase Diagram





Dynamical Evolution
Effective Chemical Potential, Net Quark Density, Effective Mass

Type II: statistical ensemble with a constant initial radius scale $R = 5\,\text{fm}$



Dynamical Evolution
Effective Chemical Potential, Net Quark Density, Effective Mass

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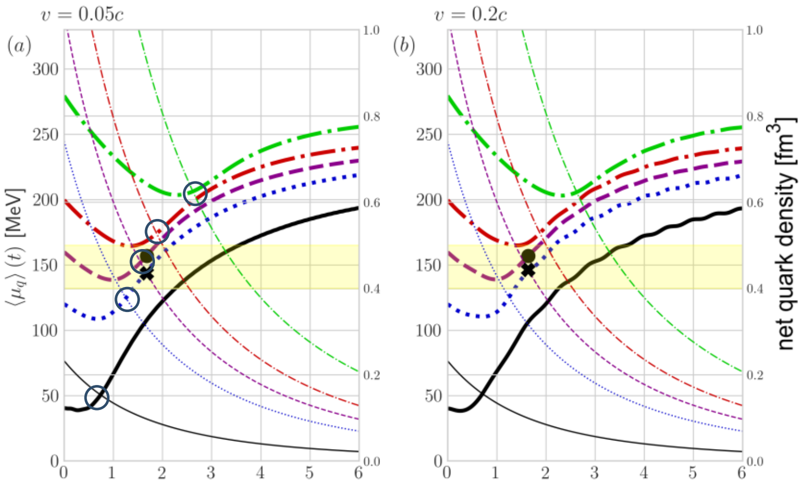
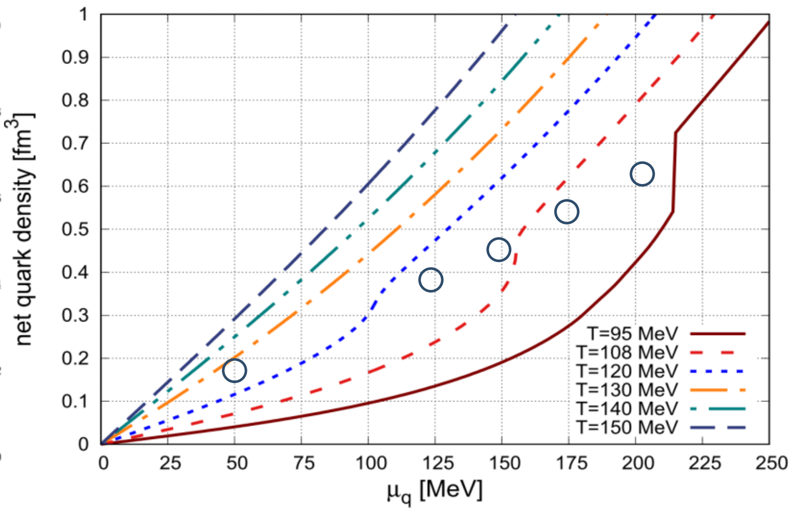
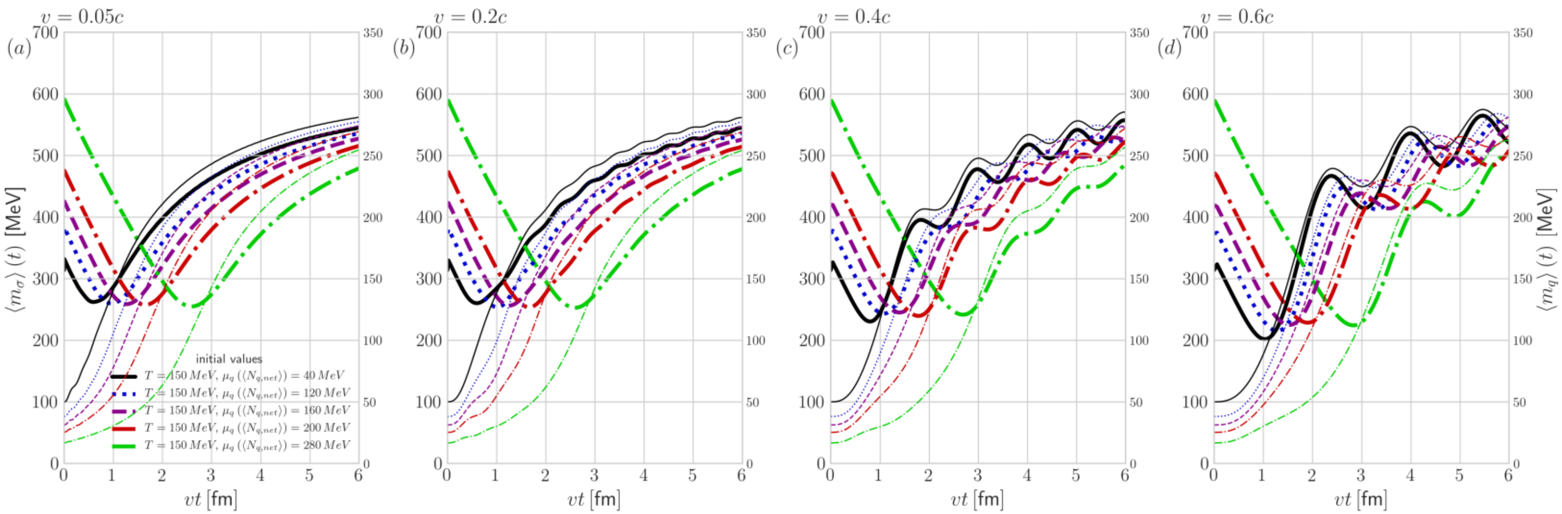


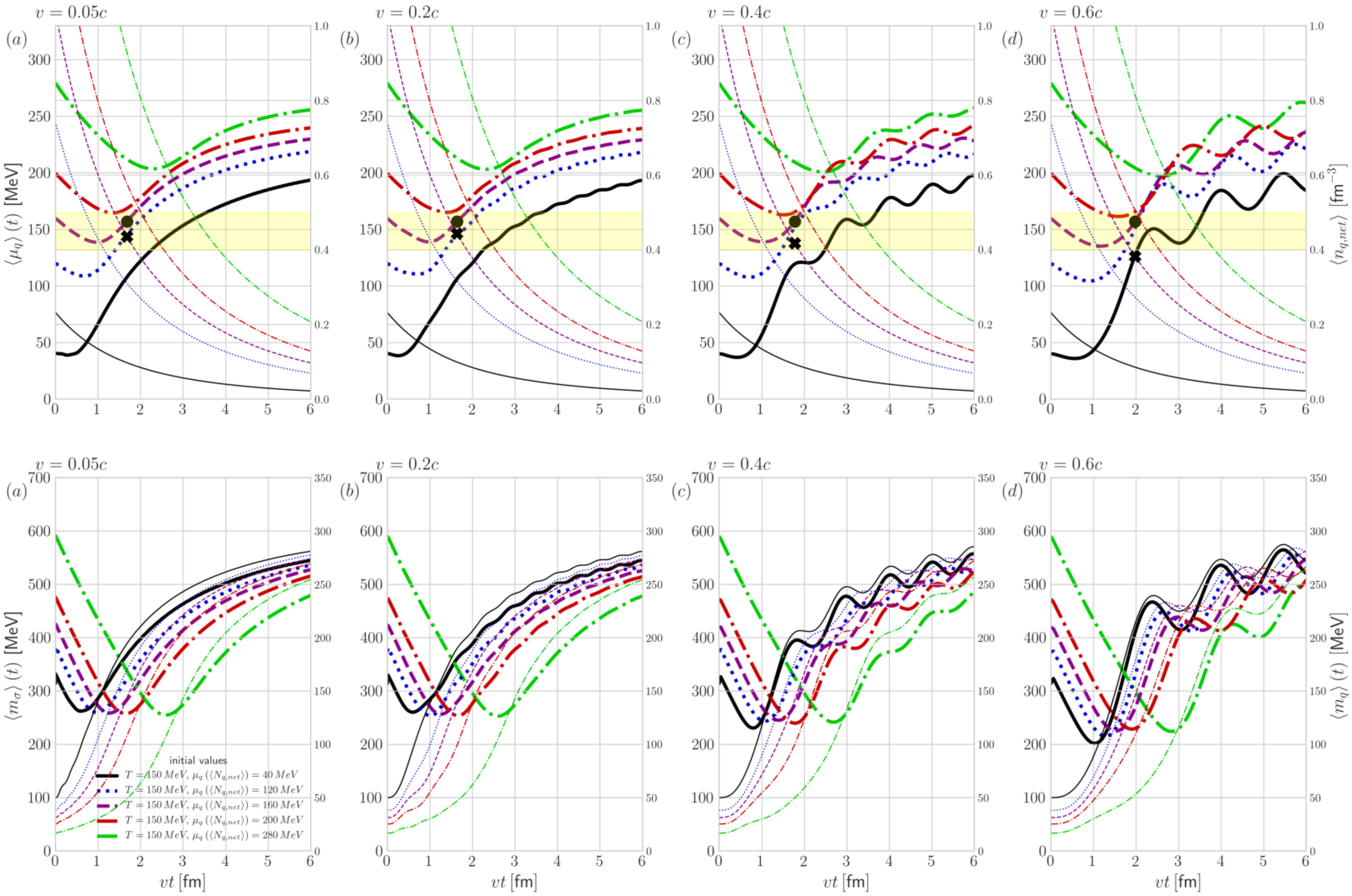




Dynamical Evolution

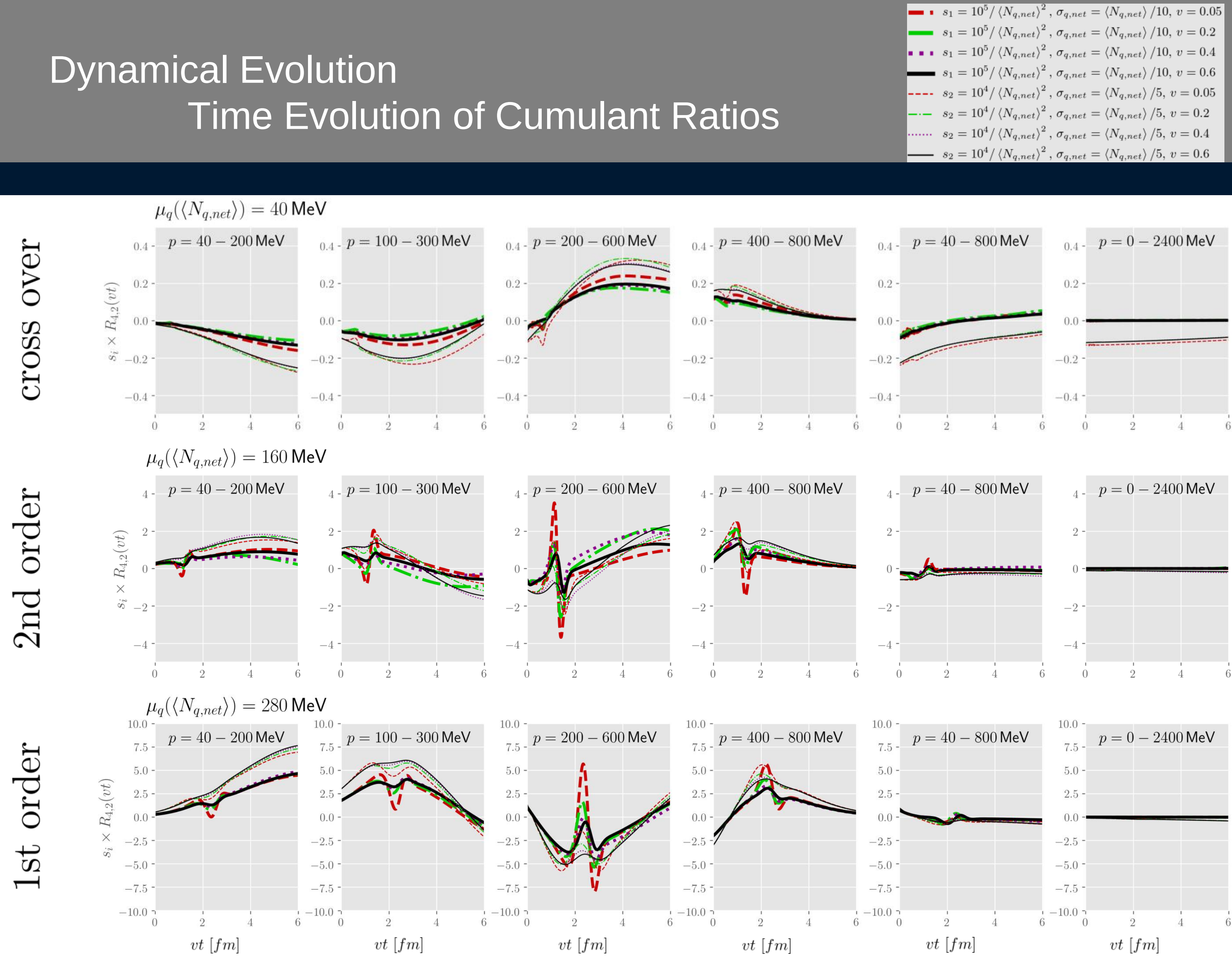
Effective Chemical Potential, Net Quark Density, Effective Mass

Type II: statistical ensemble with a constant initial radius scale $R = 5\,\text{fm}$



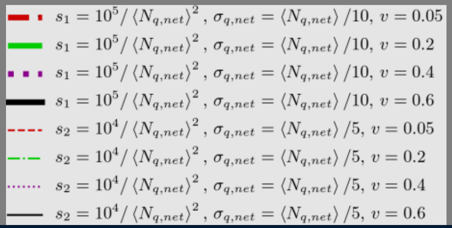
Dynamical Evolution

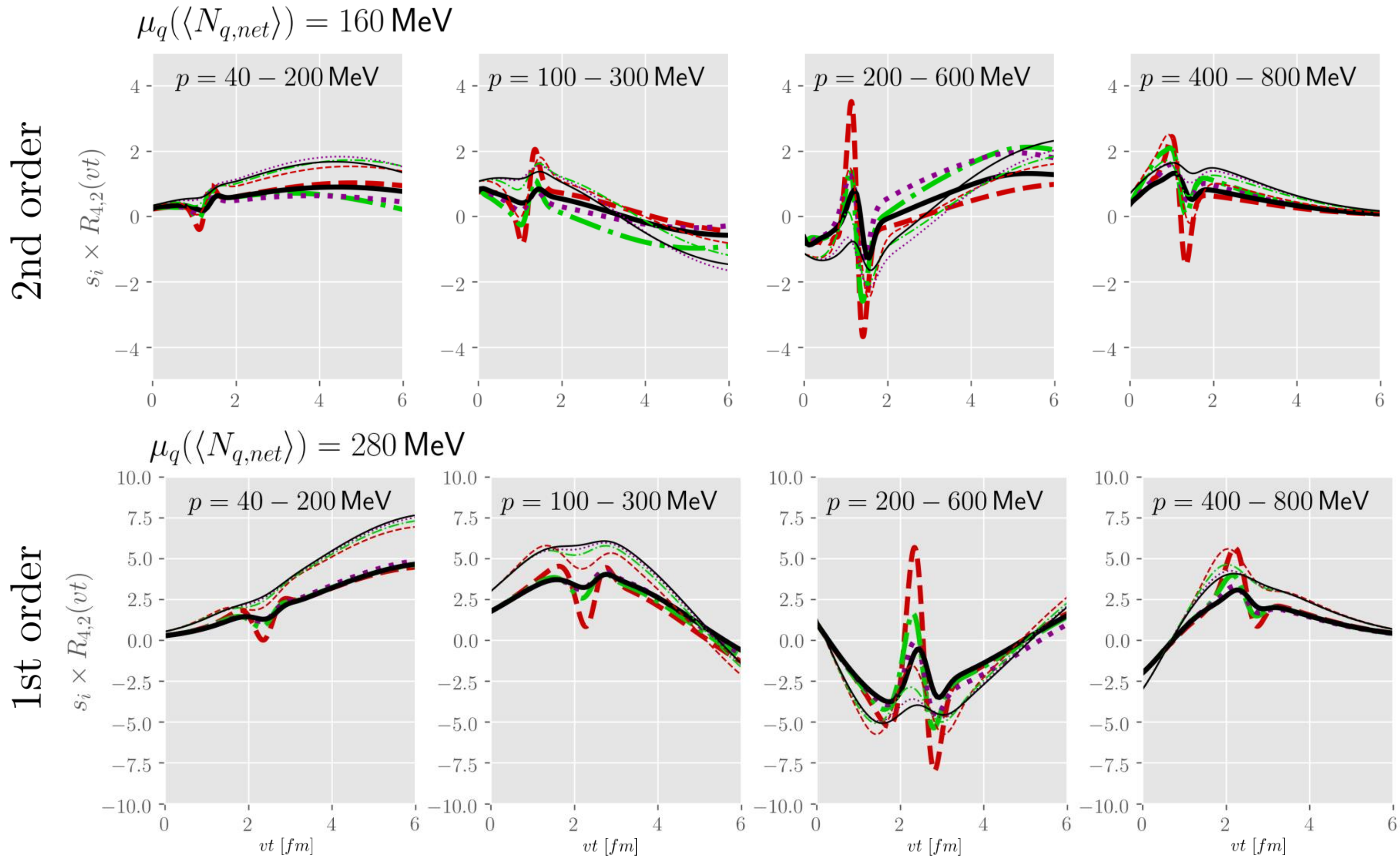
Time Evolution of Cumulant Ratios



Dynamical Evolution

Time Evolution of Cumulant Ratios





Summary

evolution equations with memory effects and expansion

- different ensembles with fluctuative initial conditions for the net quark number
- passing through the chiral phase transition leads to non-trivial fluctuations in momentum bins

- clear differences between crossover and 1st/2nd order during a dynamical expansion
- differences between 1st and 2nd order depend strongly on the expansion velocity and momentum range
- fluctuations can survive the full evolution process

A. Meistrenko, H.v. Hees, CG,
Annals Phys. 431 (2021) 168555
2007.09929 [hep-ph]

Backup

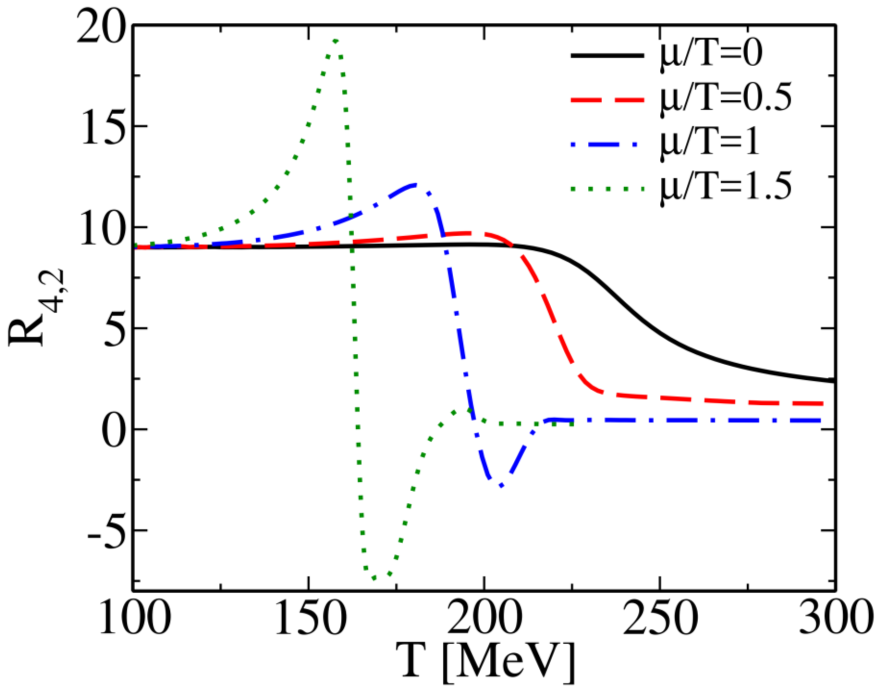
Cumulant Ratios from FRG

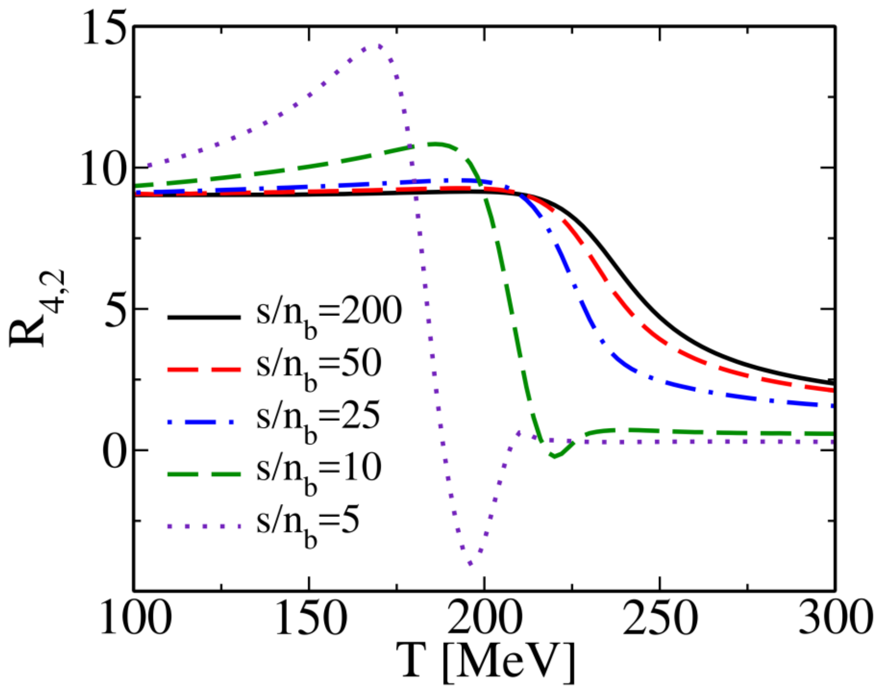
$$c_n(T) = \frac{\partial^n \left[p(T,\mu_q)/T^4\right]}{\partial(\mu_q/T)^n},$$

$$c_1 = \frac{N_{q,net}}{VT^3}, \quad c_2 = \frac{1}{VT^3}\left\langle (N_{q,net} - \langle N_{q,net}\rangle)^2 \right\rangle \equiv \frac{1}{VT^3}\sigma^2_{q,net}$$

$$c_3 = \frac{1}{VT^3}\left\langle (N_{q,net} - \langle N_{q,net}\rangle)^3 \right\rangle, \quad c_4 = \frac{1}{VT^3}\left[\left\langle (N_{q,net} - \langle N_{q,net}\rangle)^4 \right\rangle - 3\sigma^4_{q,net}\right],$$

Figs.: V. Skokov et al.
Phys. Rev. C, 83 : 054904, 2011



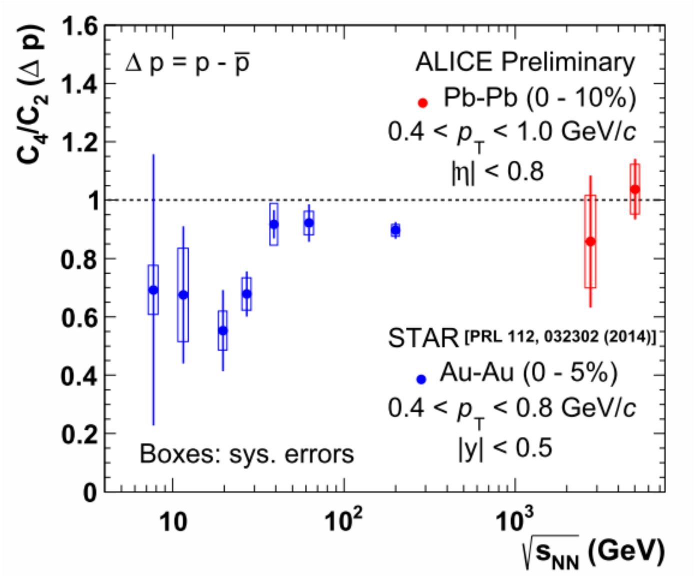


Quantum Chromo Dynamics
Phase Diagram, Observables

$$\langle 0|-m\bar{\psi}\psi|0\rangle = -\frac{m_u+m_d}{2}\langle 0|\bar{\psi}_u\psi_u+\bar{\psi}_d\psi_d|0\rangle = m_\pi^2 f_\pi^2 \neq 0$$

$$\kappa_4(\sigma_V) := \left\langle(\delta\sigma_V)^4\right\rangle - 3\left\langle(\delta\sigma_V)^2\right\rangle^2,$$

$$\kappa_4(N_p) = \kappa_4\left(N_p^{stat.}\right) + \kappa_4(\sigma_V)\left(gd\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{\partial f_p^{eq.}}{\partial m}\right)^4 + \ldots$$
$$= \langle N_p\rangle + \kappa_4(\sigma_V)\left(\frac{gd}{T}\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{\partial E_p}{\partial m}f_p^{eq.}\right)^4 + \ldots,$$



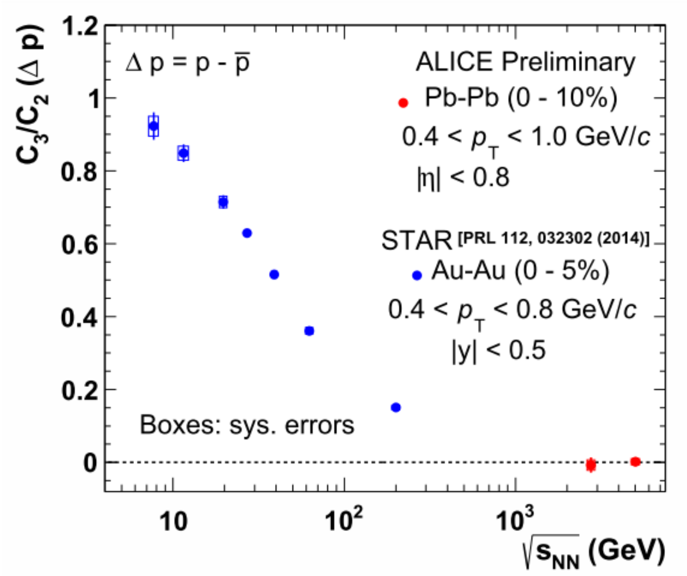


Fig.: Nirbhay Kumar Behera, ALICE collaboration
Nuclear Physics A, 982 : 851–854, 2019

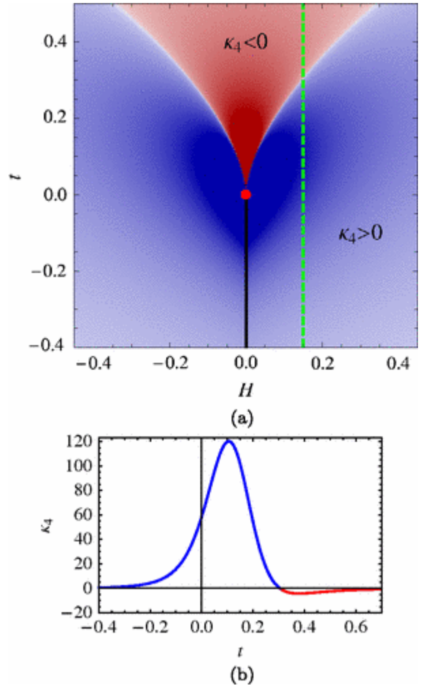
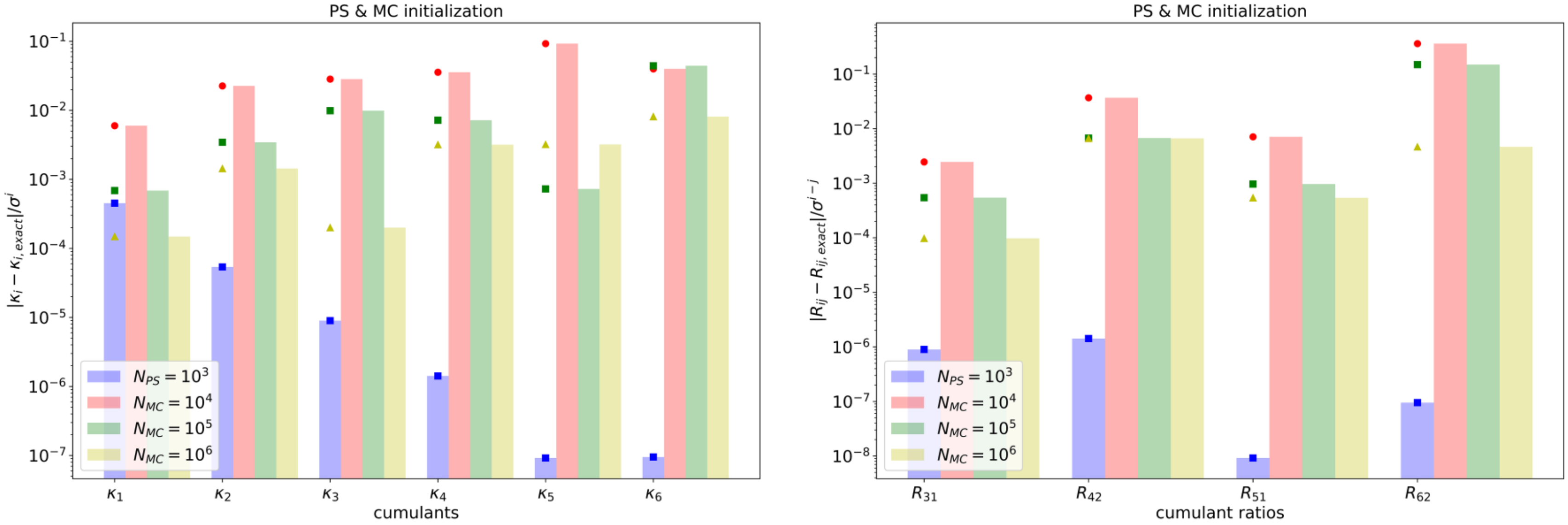


Fig.: M. A. Stephanov
Phys. Rev. Lett.,
107 : 052301, 2011

Backup

Initialization Error for Cumulants

$$err_\kappa := \frac{|\kappa_i - \kappa_{i,exact}|}{\sigma^i_{q,net}}, \quad err_R := \frac{|R_{ij} - R_{ij,exact}|}{\sigma^{i-j}_{q,net}}.$$



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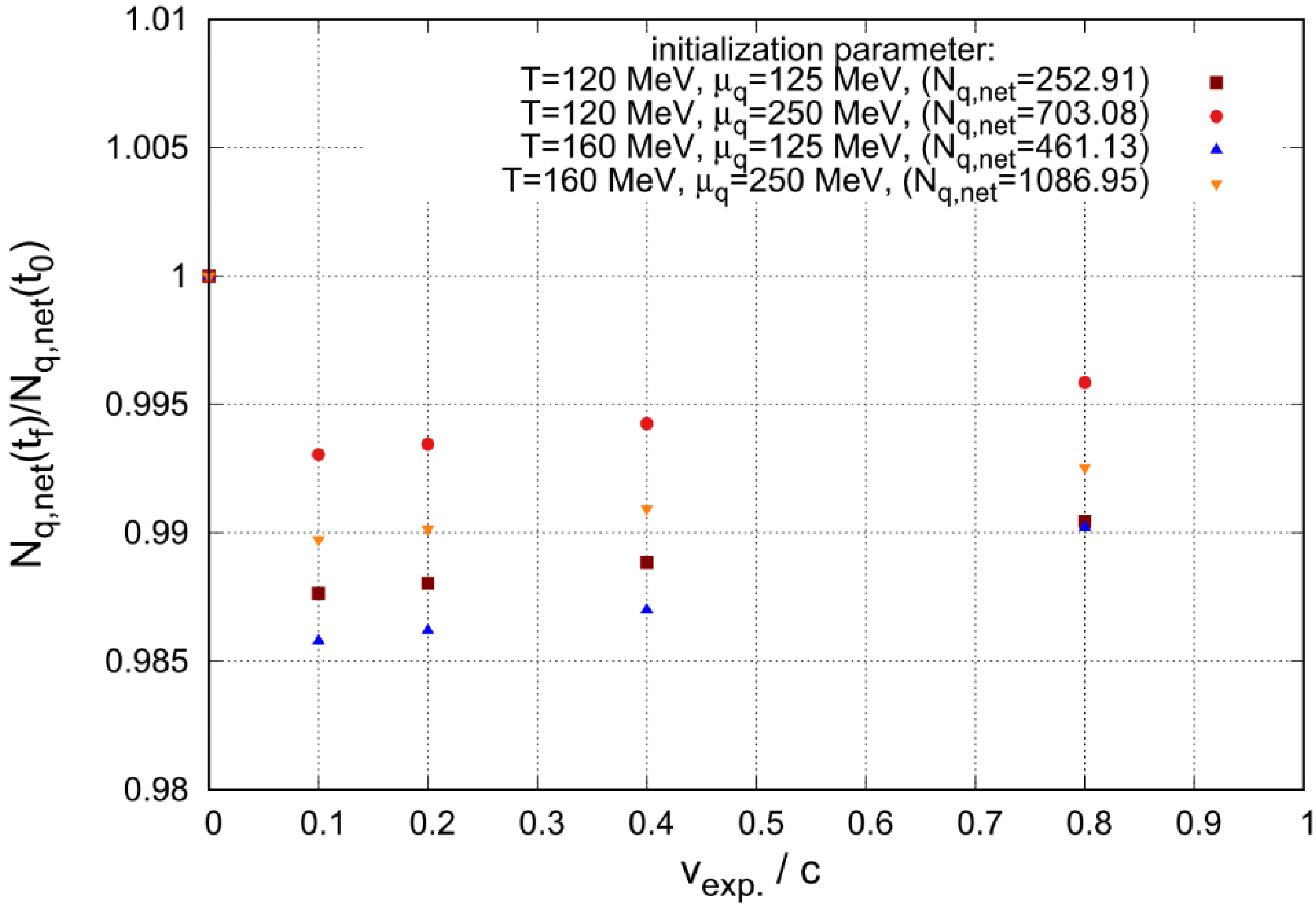
Net Quark Number Conservation



Backup

Evolution von Kumulanten für Typ I

Type I: statistical ensemble with a constant initial value of μ_q