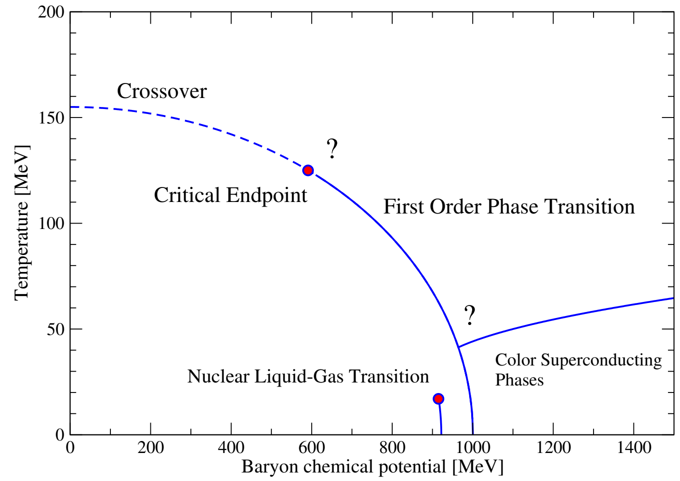
STRONG2020
Crete, October 2021

The QCD phase diagram with functional methods

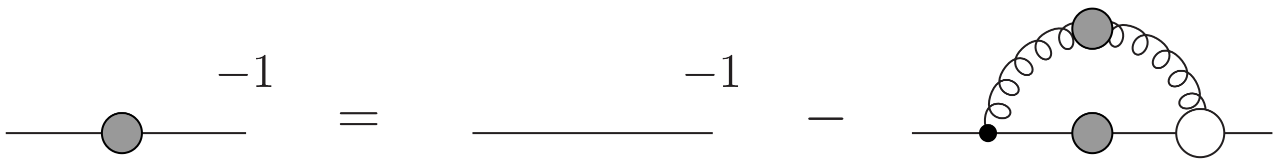
Review: CF, PPNP 105 (2019) [1810.12938]

Overview

1. Introduction



2. Gluons, quarks and DSEs



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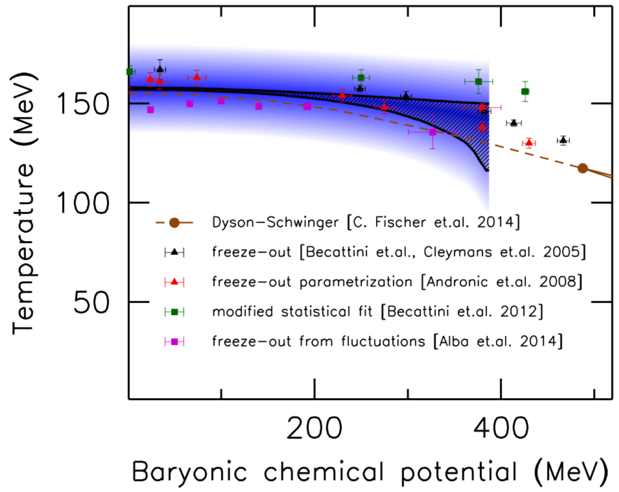
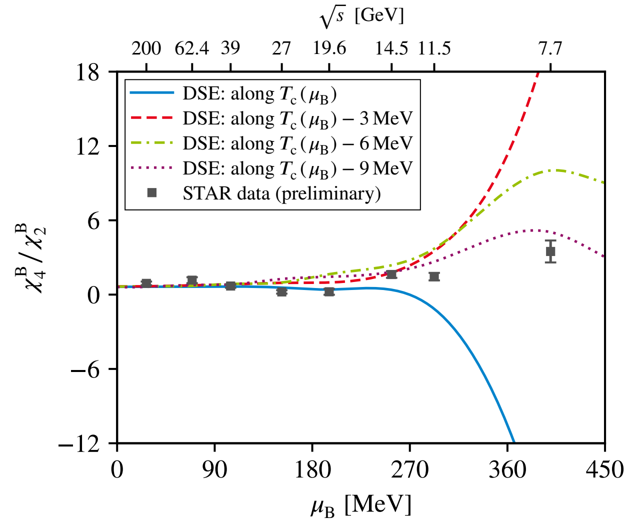


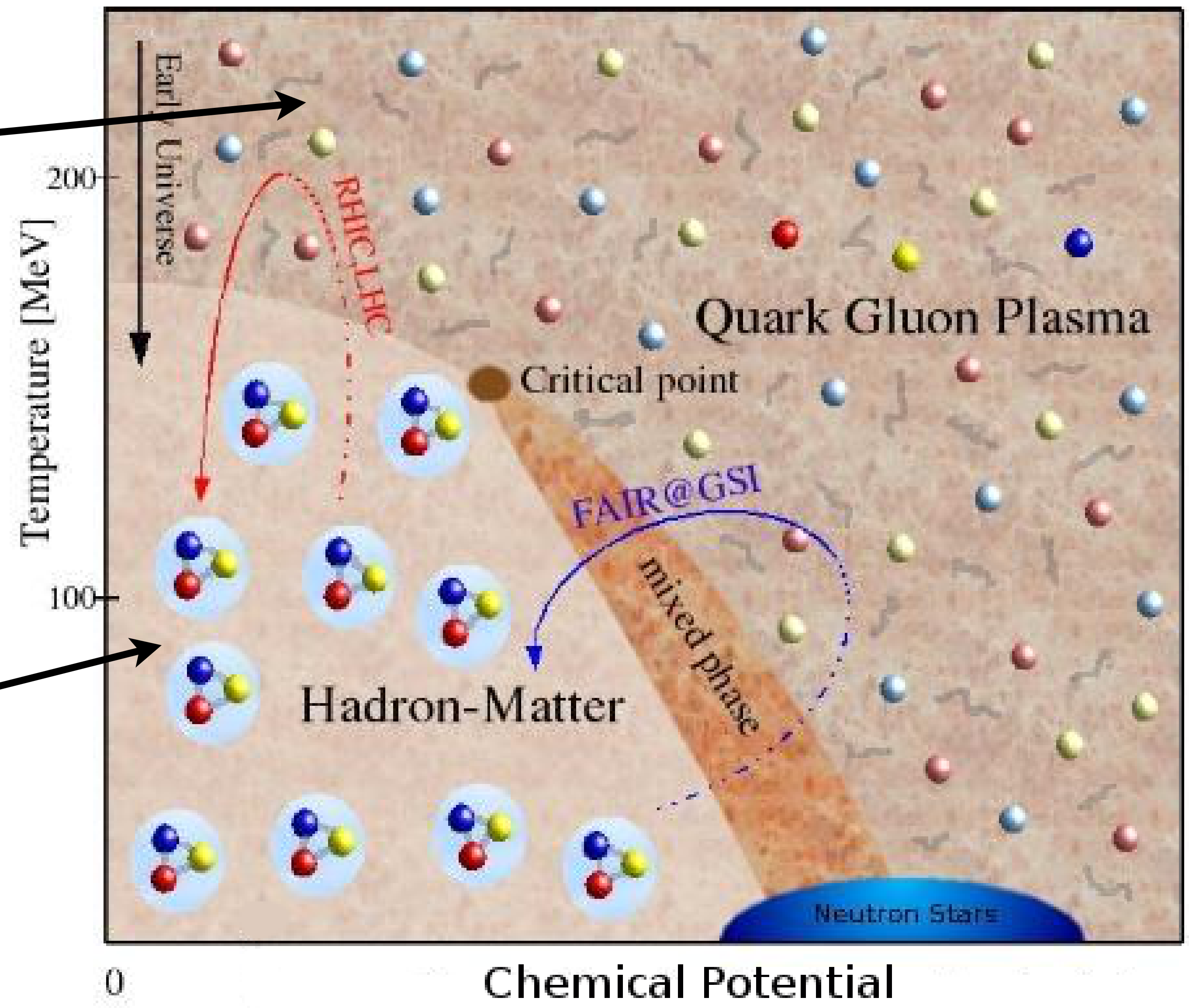
4. Fluctuations and large densities



QCD phase diagram

Quarks de-confined and (almost) massless

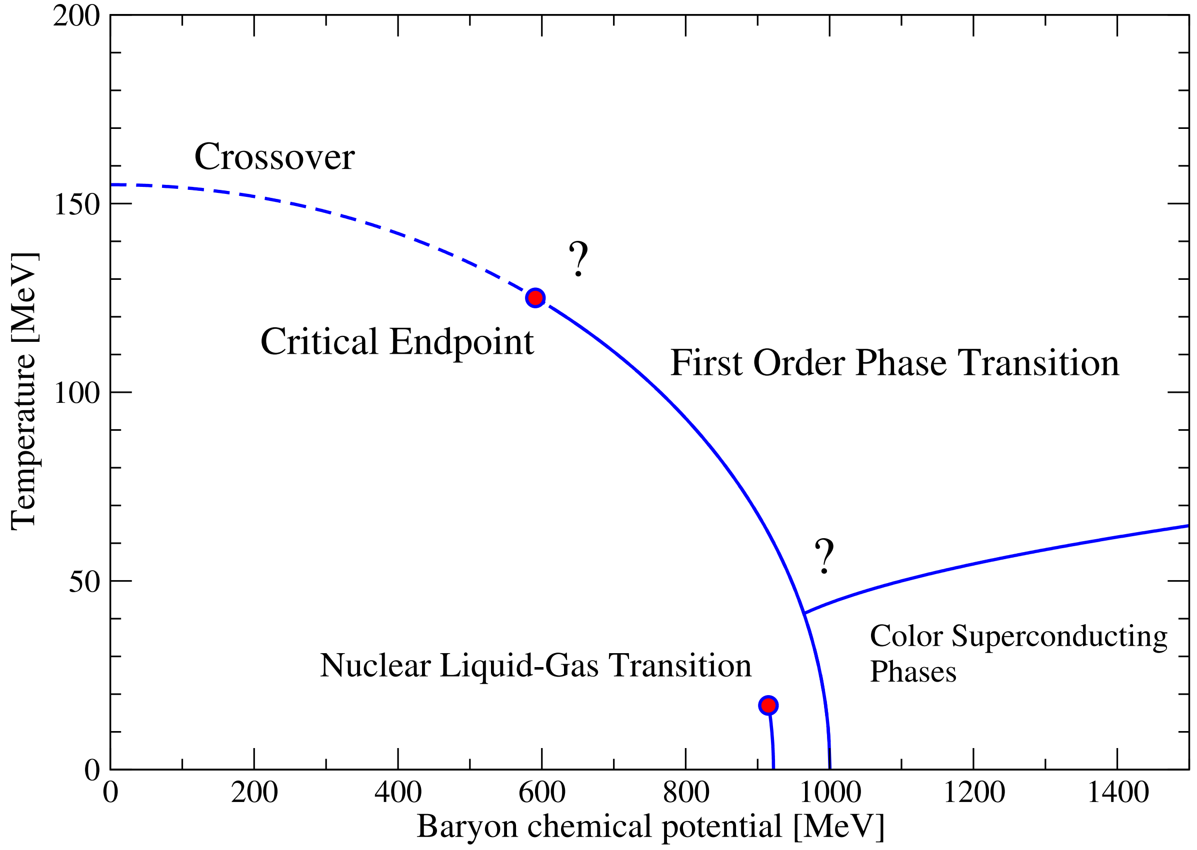
Quarks confined and massive



Many interesting open questions:

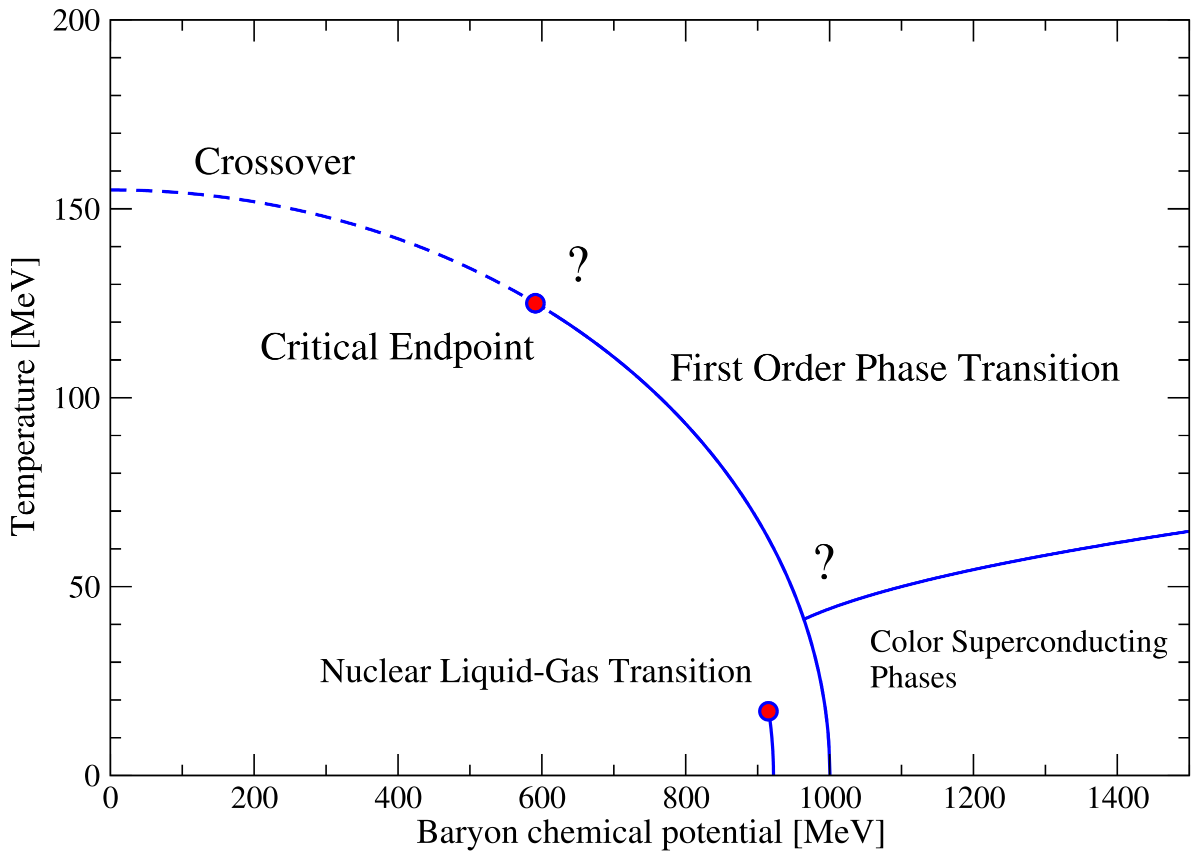
- Existence and location of critical point ?
- Details of phase transitions ??
- Consequences for early universe and physics of neutron stars

QCD phase transitions



CF, PPNP 105 (2019) [1810.12938]

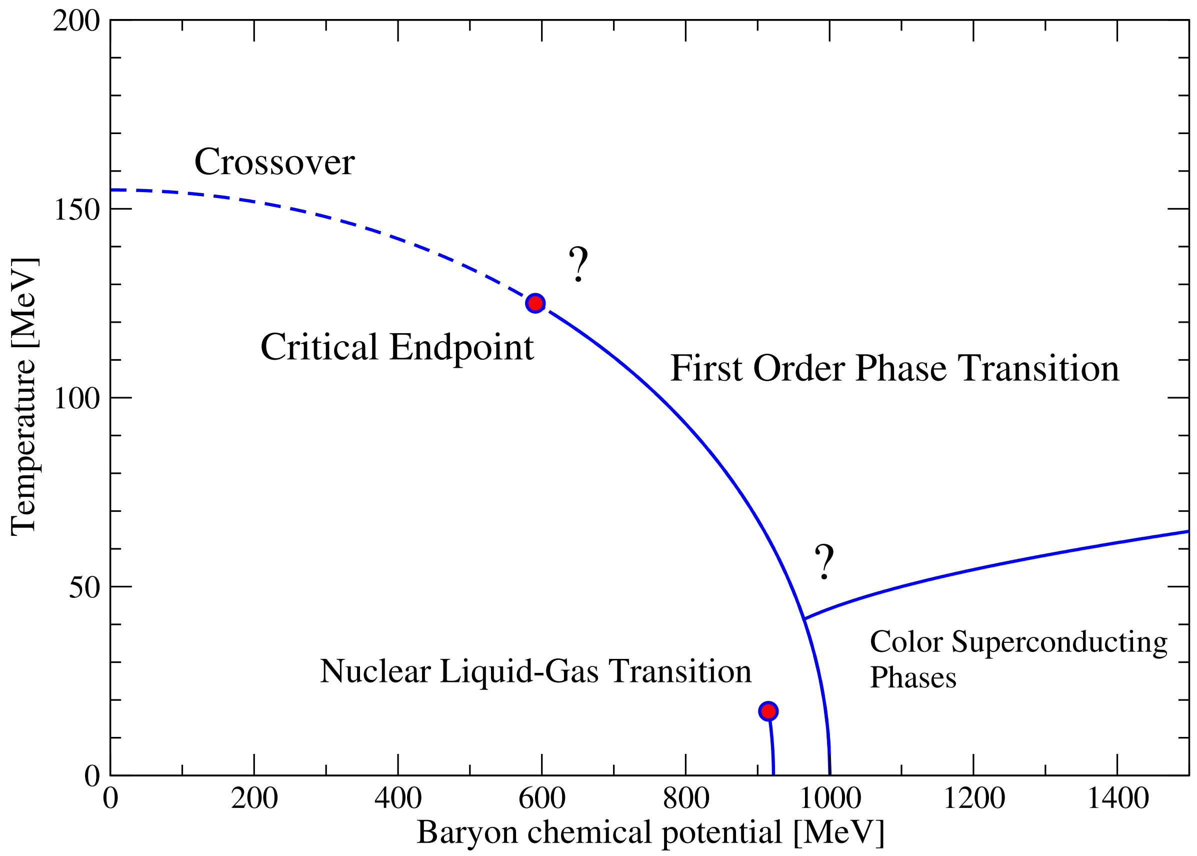
QCD phase transitions



$$Z\left(\frac{\mu_B}{T}\right) = Z\left(-\frac{\mu_B}{T}\right)$$

CF, PPNP 105 (2019) [1810.12938]

QCD phase transitions



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$$\left(\frac{T_c(\mu_B)}{T_c}\right)^2 = 1 - 2\kappa\left(\frac{\mu_B}{T_c}\right)^2$$

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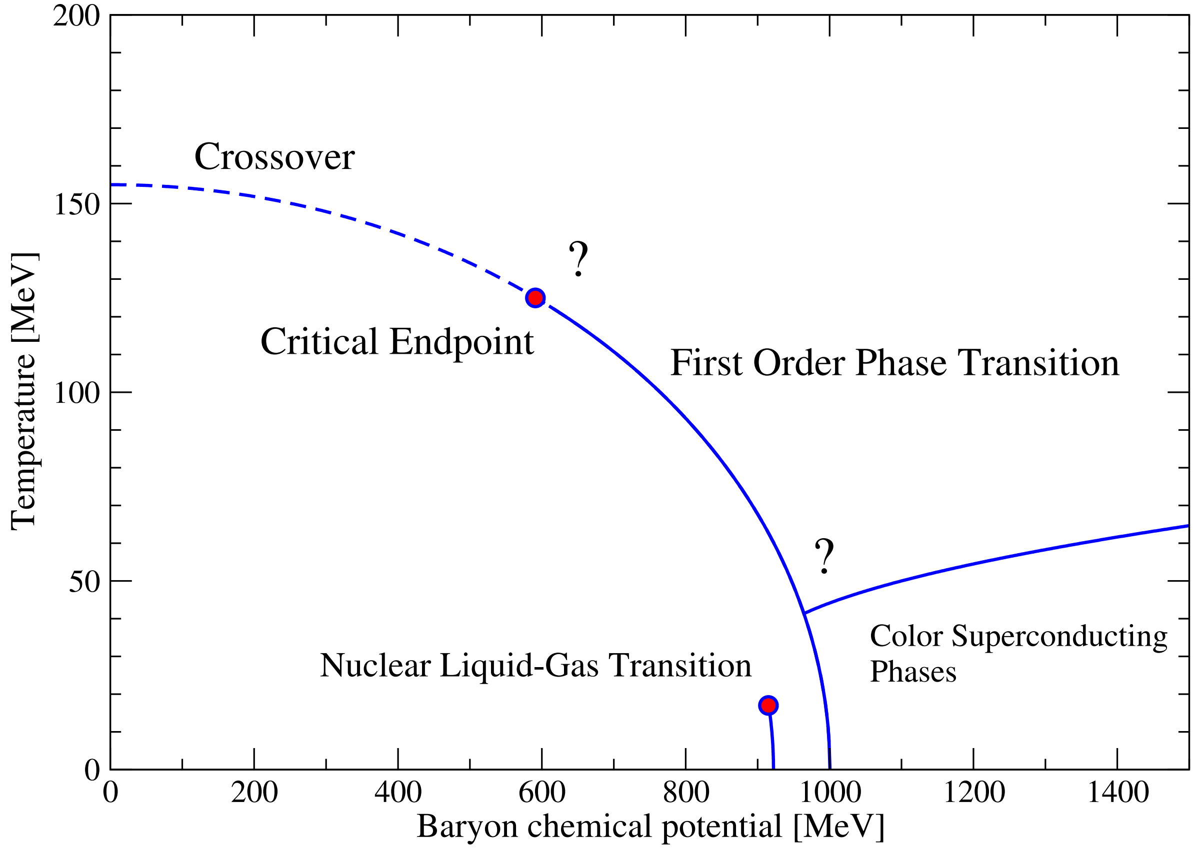

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$$\mu_B^{lg} \approx 922\,\mathrm{MeV} \rightarrow \kappa \leq 0.0141$$

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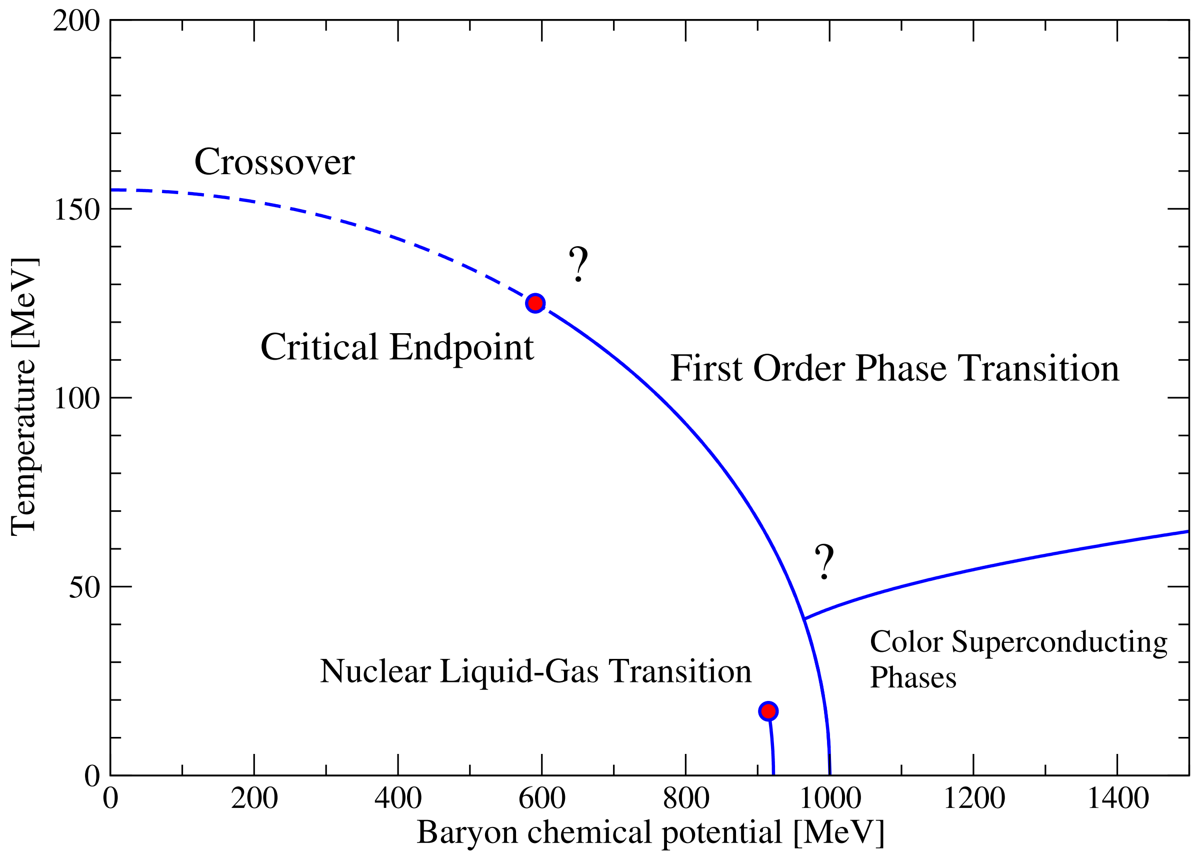

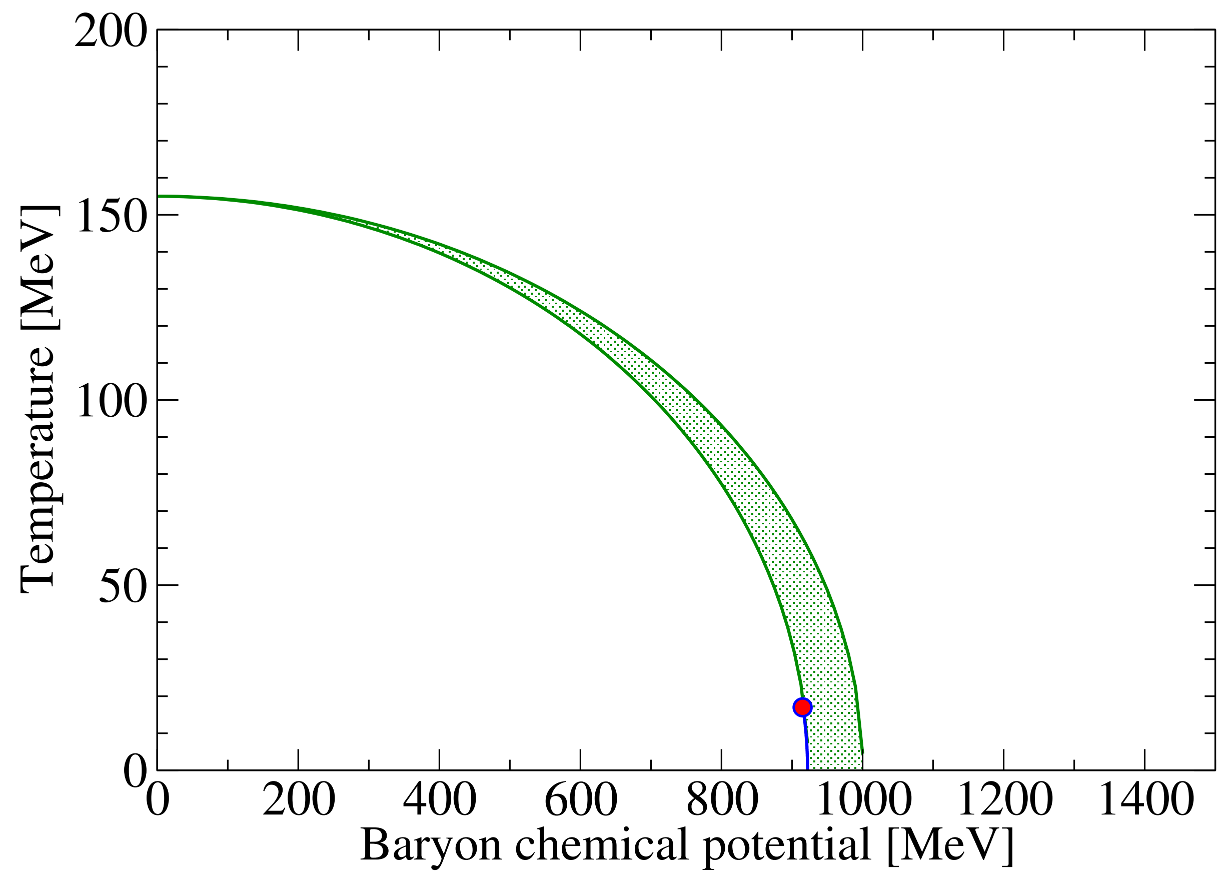

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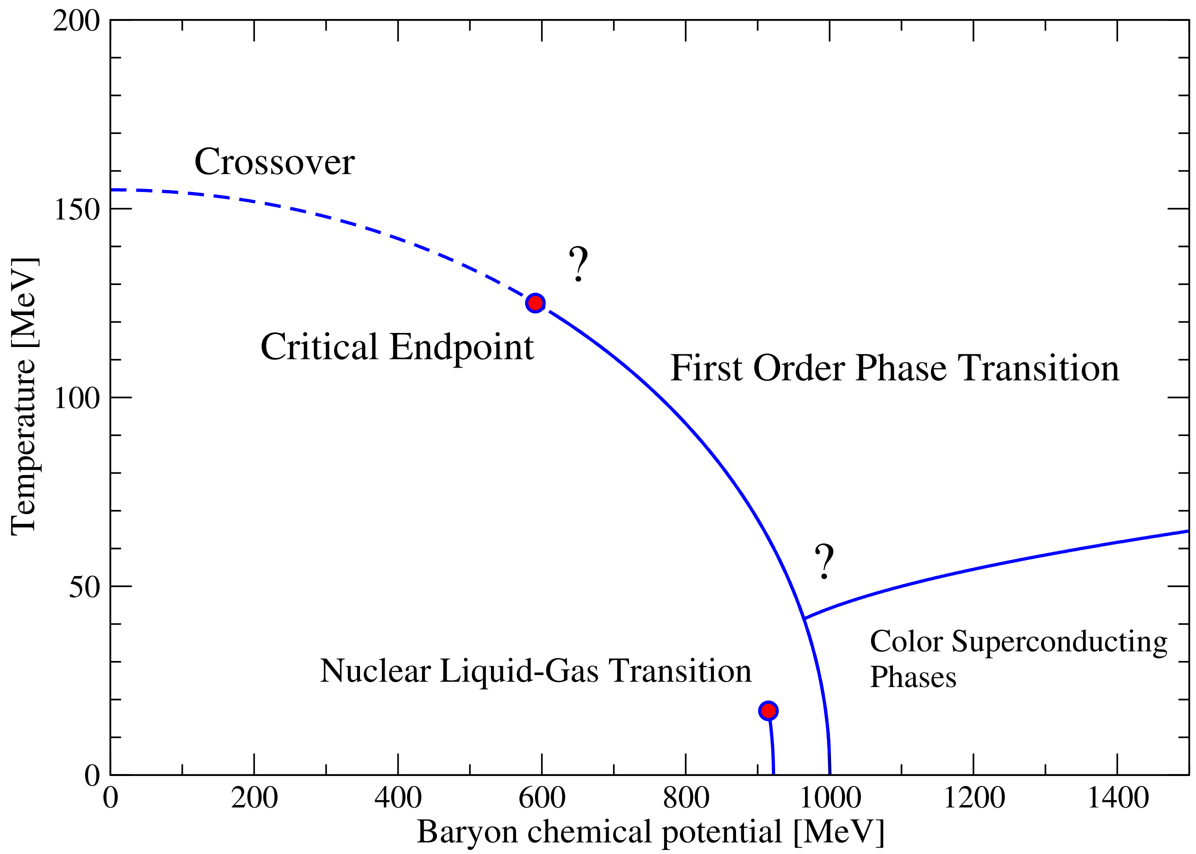

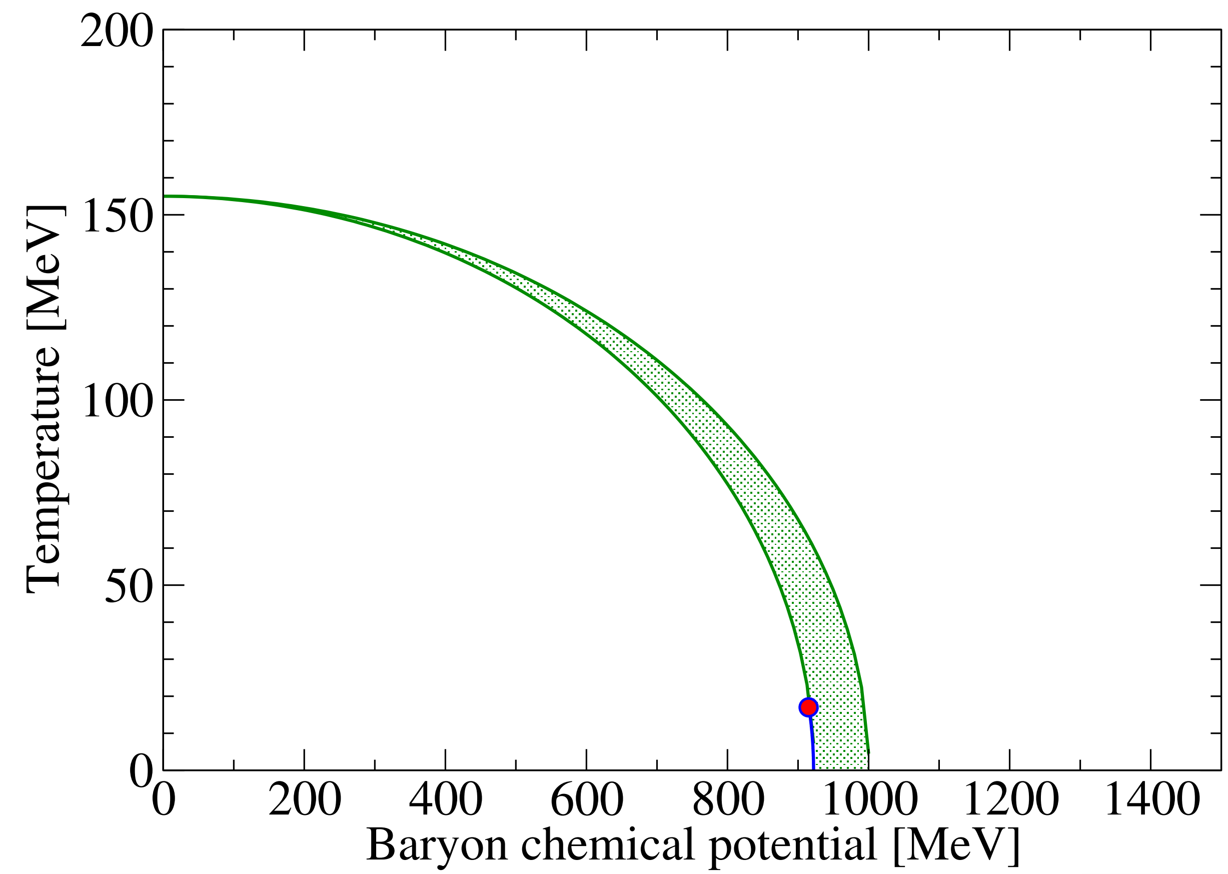

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Lattice QCD:

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa\left(\frac{\mu_B}{T_c}\right)^2 - \lambda\left(\frac{\mu_B}{T_c}\right)^4 \cdots,$$

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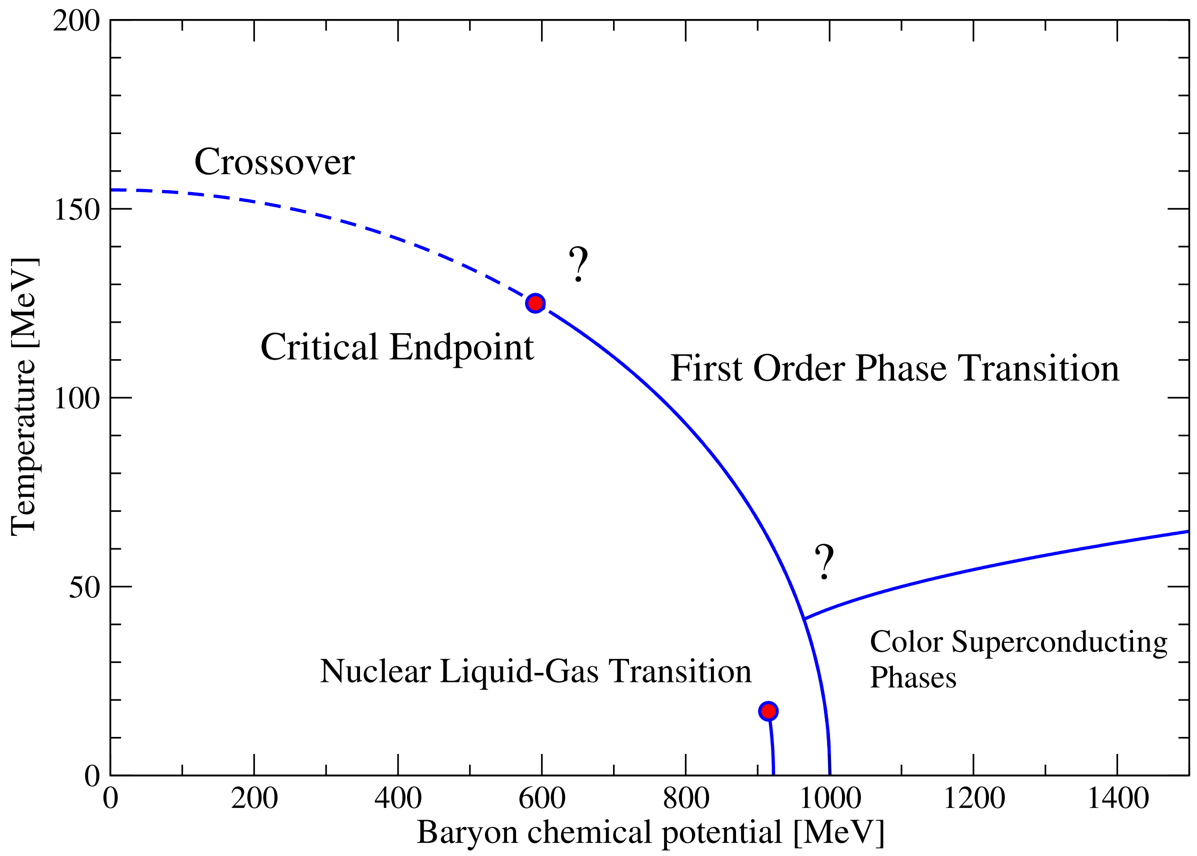
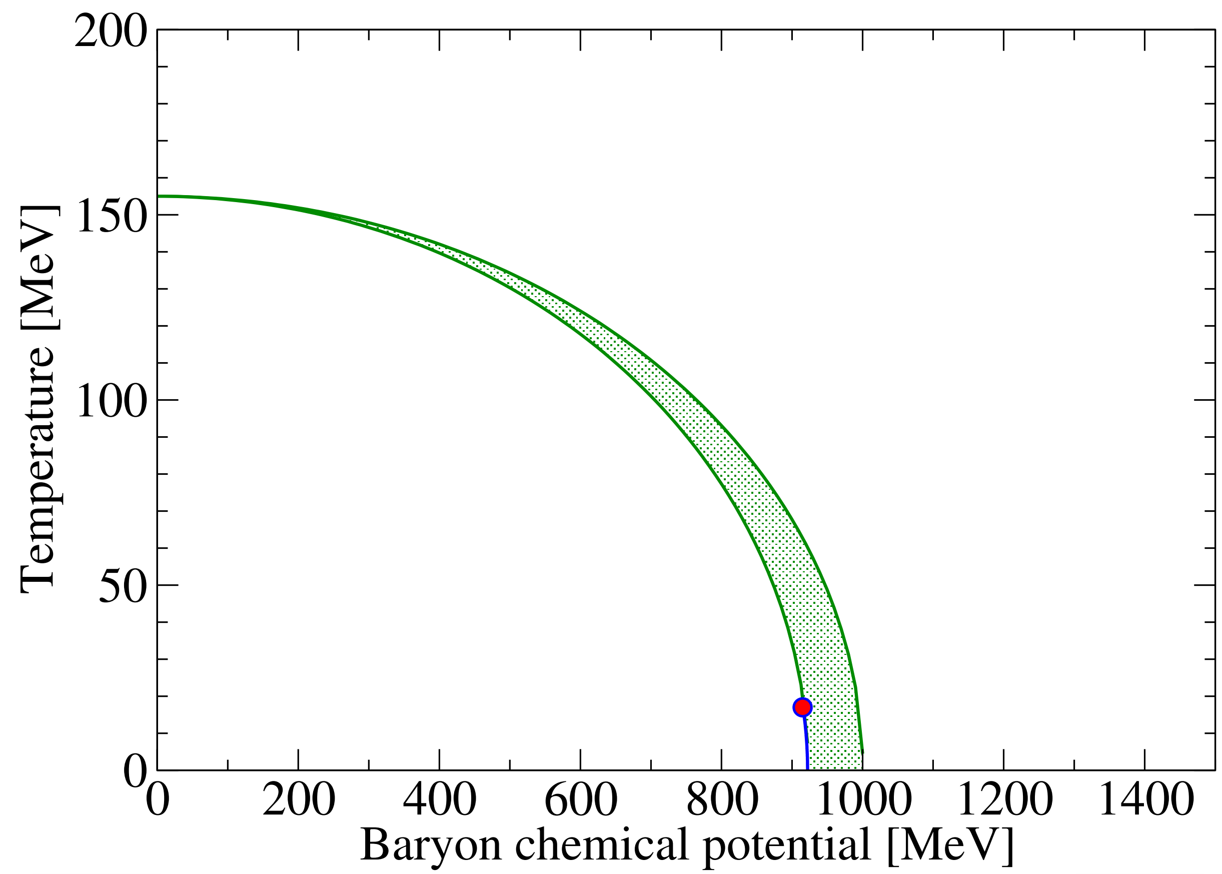
QCD phase transitions





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$$\rightarrow \lambda = \frac{\kappa^2}{2} \leq 0.0001$$

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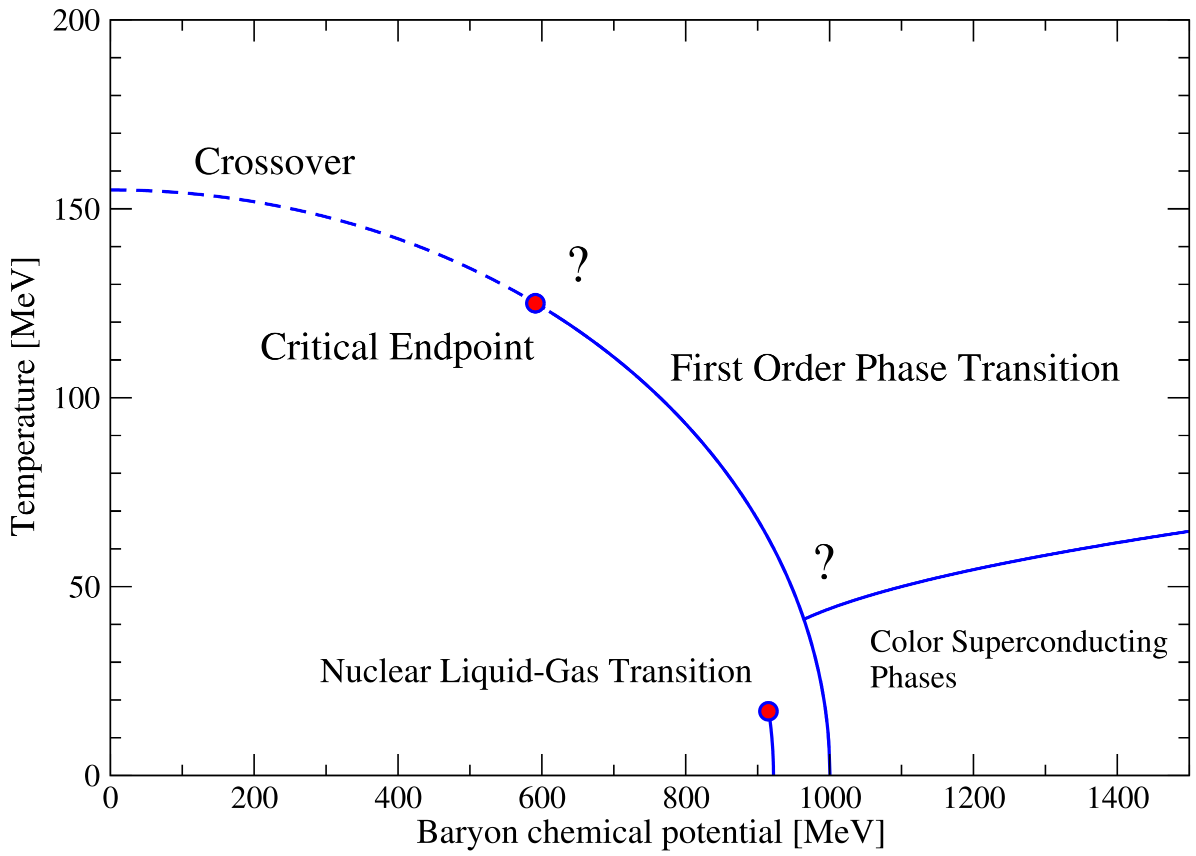
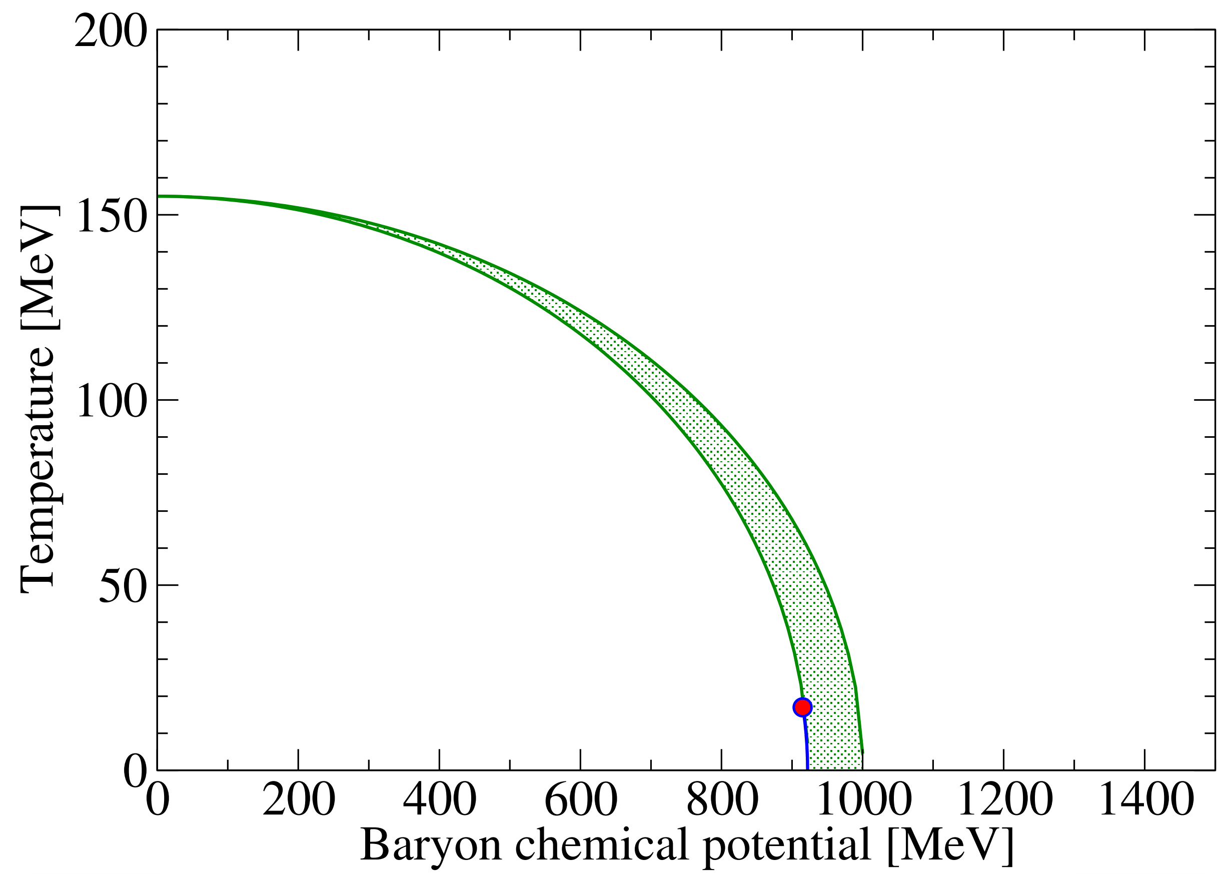
QCD phase transitions





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$$\kappa = \begin{cases} 0.0145(25) \\ 0.0120(40) \\ 0.0153(18) \end{cases}$$

- 0.0145(25) — Bonati et al., PRD 98 (2018)
- 0.0120(40) — Bazavov et al., PLB 795 (2018)
- 0.0153(18) — Borsanyi et al., PRL 125 (2020)

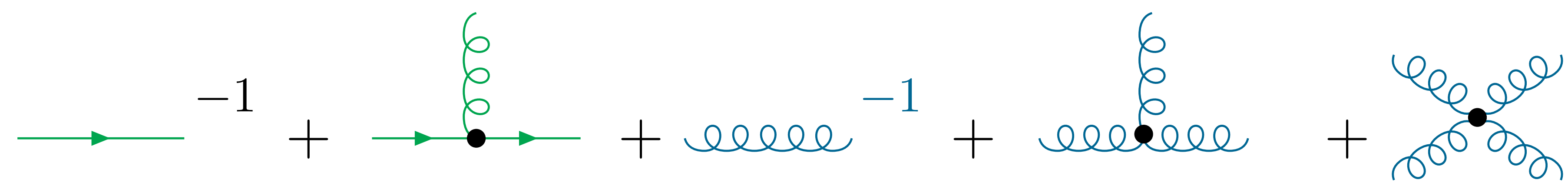
$$\lambda = \begin{cases} 0.000(4) \\ 0.00032(67) \end{cases}$$

- 0.000(4) — Bazavov et al., PLB 795 (2018)
- 0.00032(67) — Borsanyi et al., PRL 125 (2020)

The QCD generating functional

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \; \exp\left\{ - \int d^4x \left(\overline{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \text{gauge fixing} \right) \right\}$$

$S_{QCD} = \int d^4x \; ($



$)$

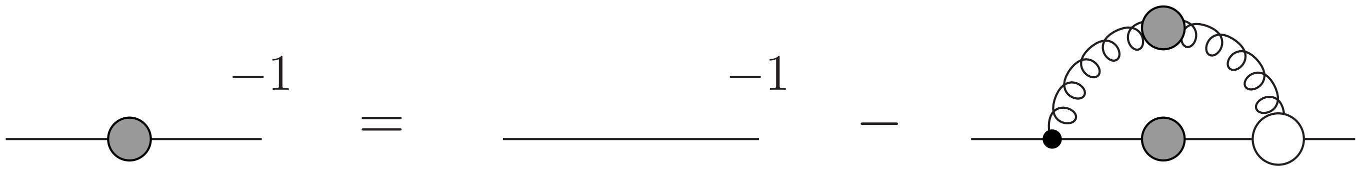
- Euclidean space — cf. talk by Jana
- $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$
- $D_\mu = \partial_\mu + i g t^a A^a_\mu$
- Temperature: — see talk by Jana
- Landau gauge: $\partial_\mu A^a_\mu = 0$

Chiral symmetry breaking: dynamical quark mass

Dynamical quark masses
via weak and strong force

Yoichiro Nambu,
Nobel prize 2008

	u	d	s	c	b	t
M_{weak} $[MeV/c^2]$	3	5	80	1200	4500	176000
M_{strong} $[MeV/c^2]$	350	350	350	350	350	350
M_{total} $[MeV/c^2]$	350	350	450	1500	4800	176000



$$S^{-1}(p) = [i\not{p} + M(p^2)]/Z_f(p^2)$$

Motivation to look at propagators !

Chiral symmetry breaking: dynamical quark mass

Dynamical quark masses
via weak and strong force

Yoichiro Nambu,
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Input parameters in N_f=2+1 QCD

	u	d	s	c	b	t
M_{weak} $[MeV/c^2]$	3	5	80	1200	4500	176000
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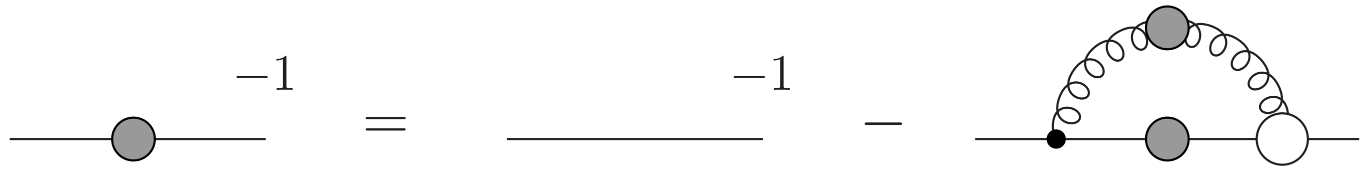

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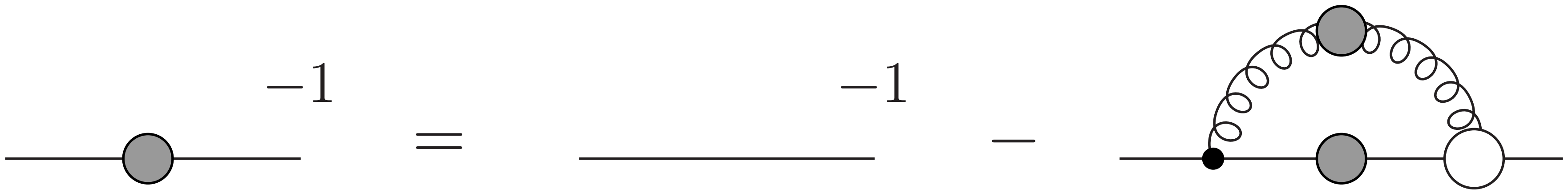
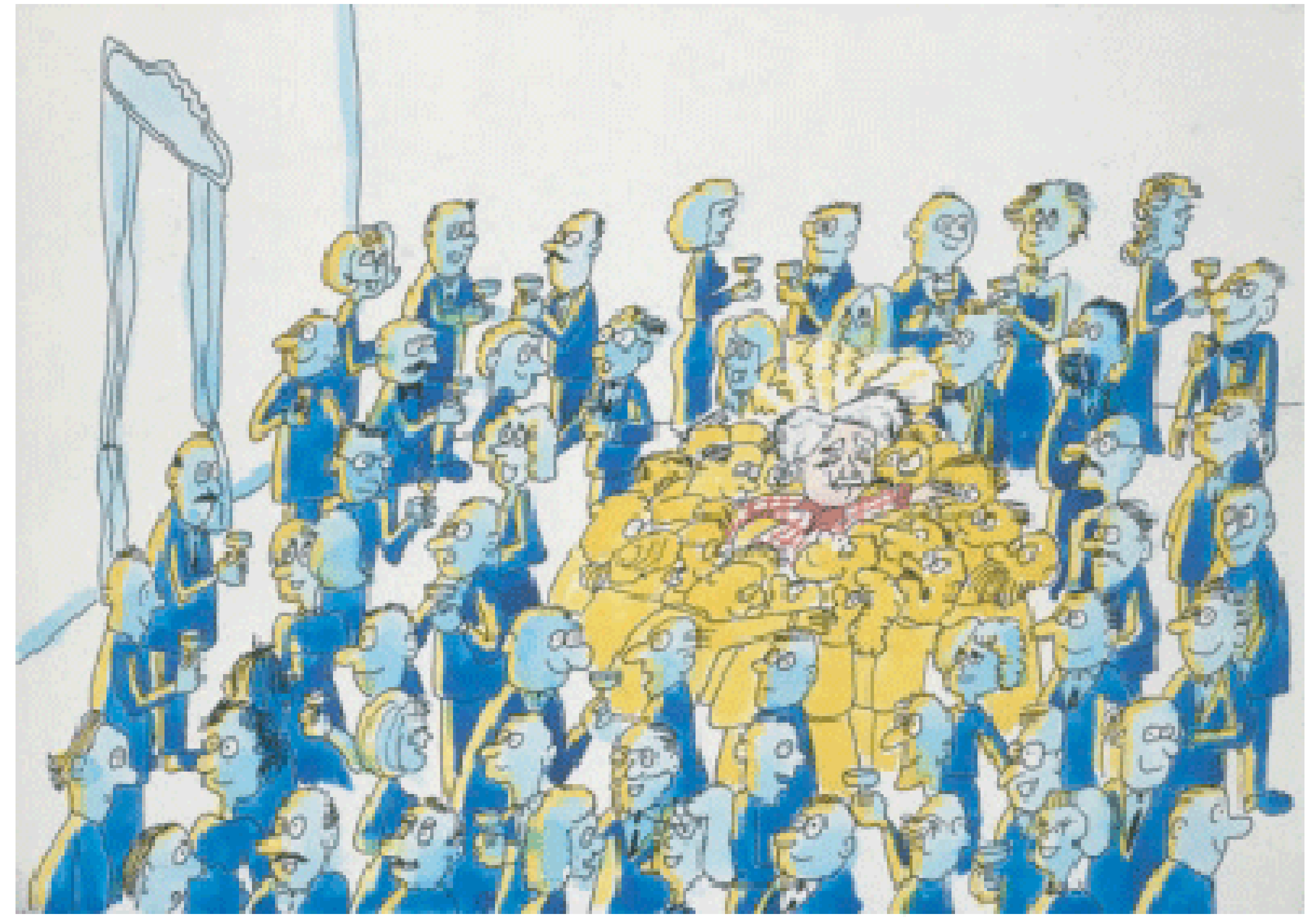
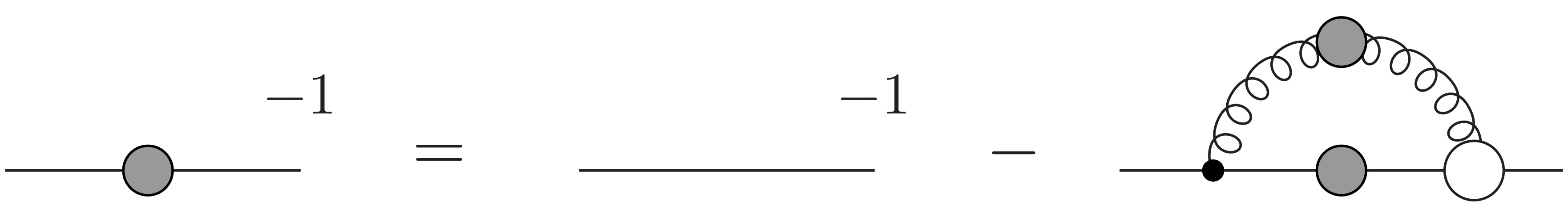
Motivation to look at propagators !

Dynamical mass generation

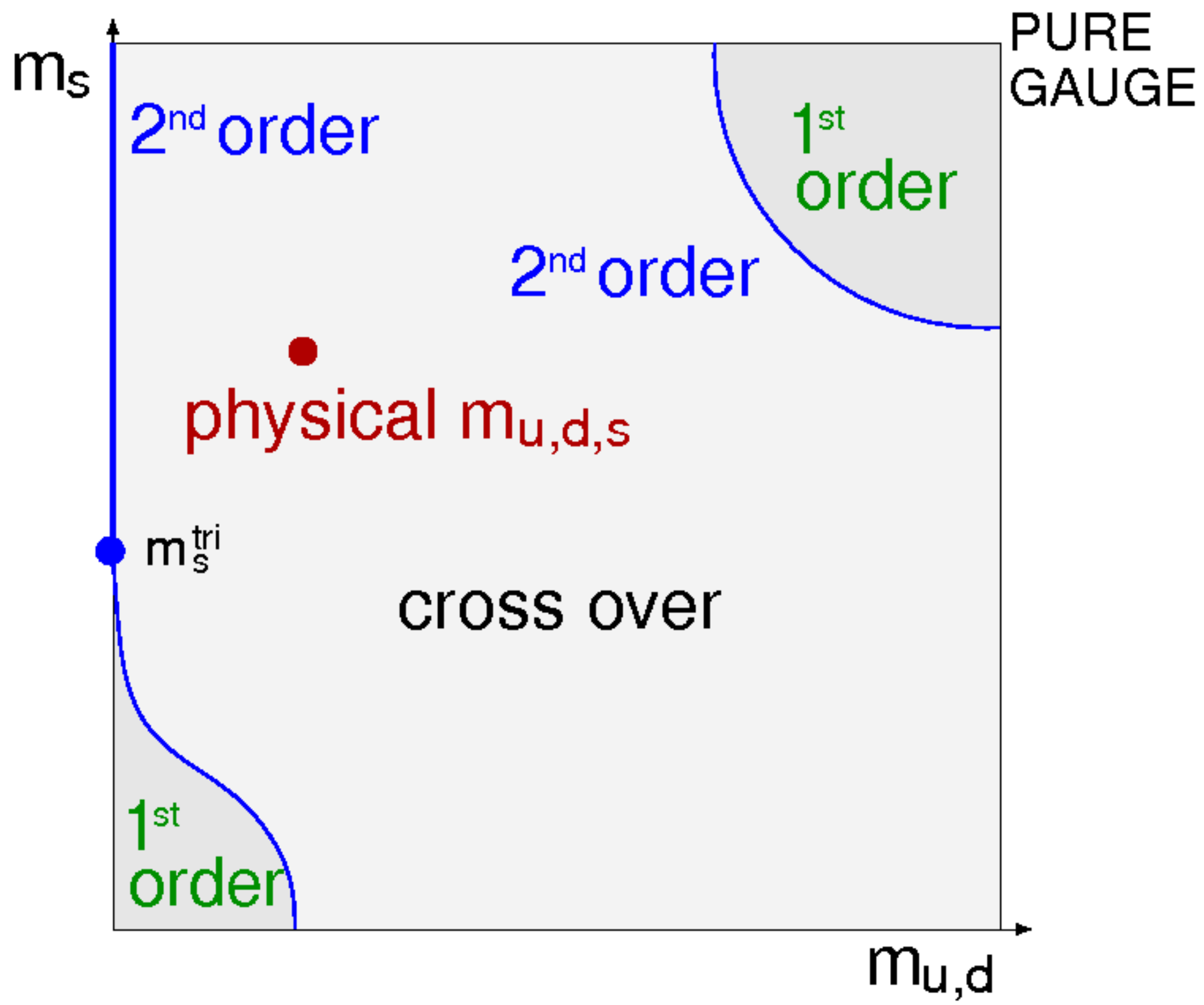


Dynamical mass generation





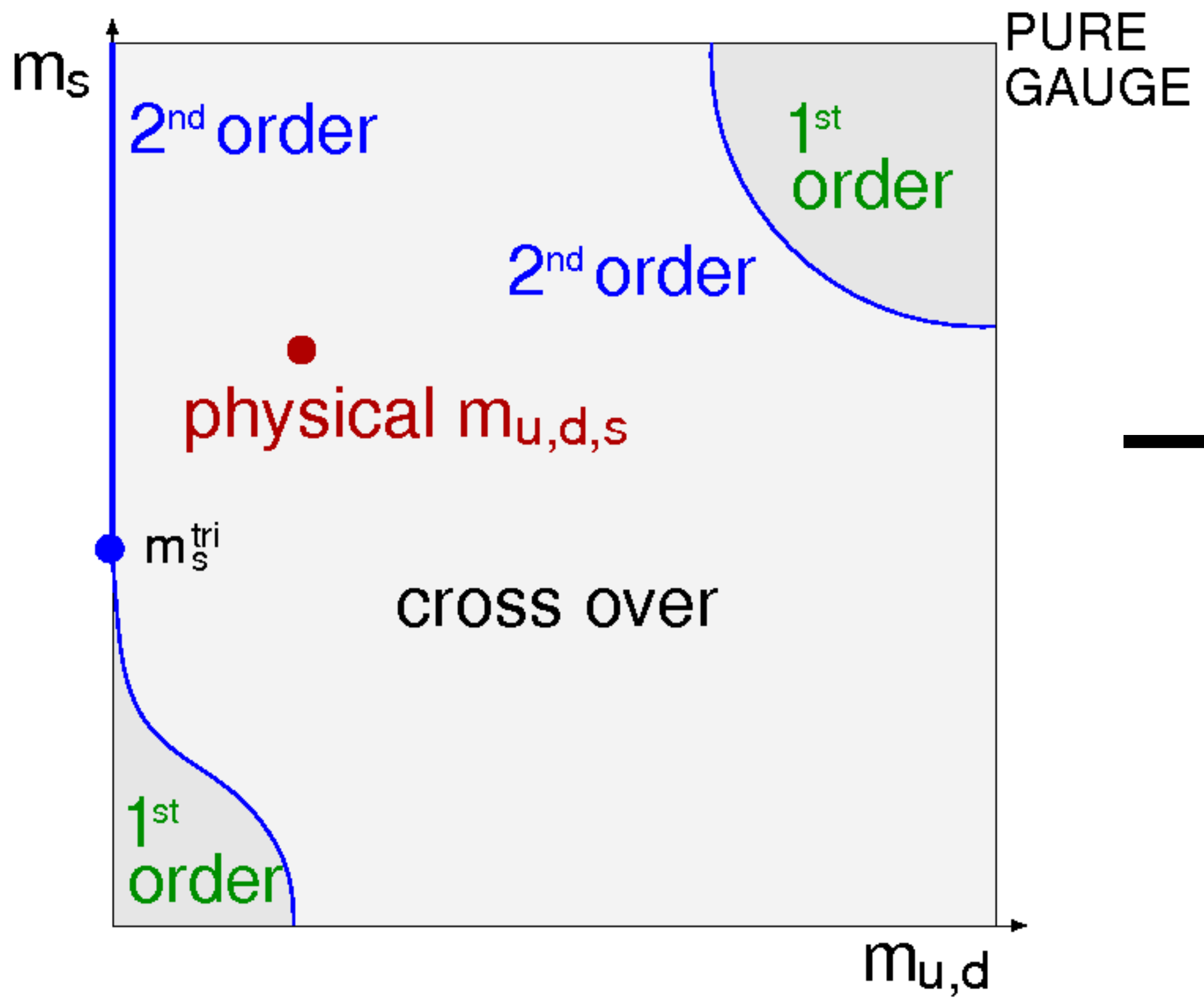
QCD phase transitions

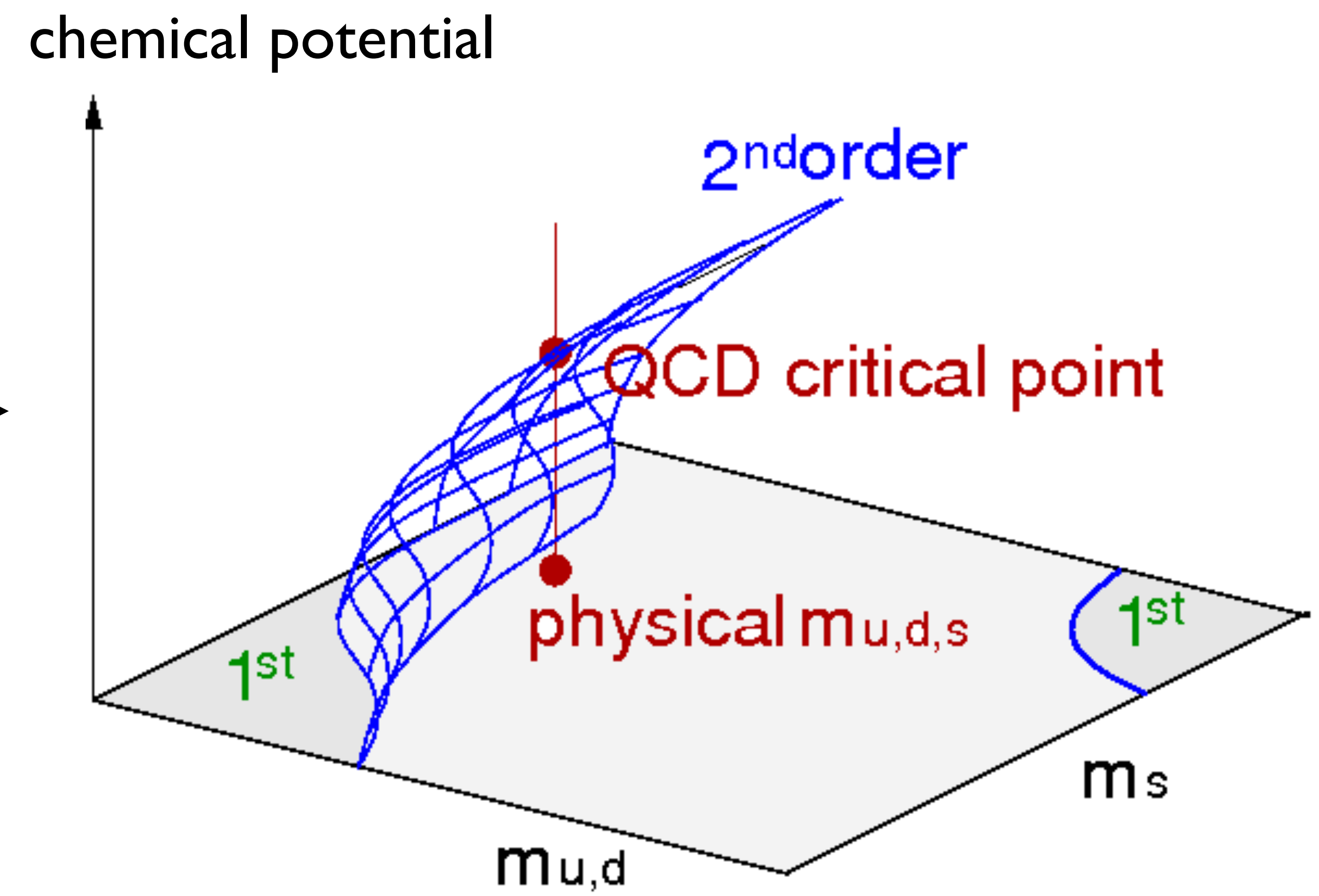


Is this happening ??
Maybe yes, maybe not..

de Forcrand, Philipsen, JHEP 0811 (2008) 012;
NPB 642 (2002) 290

QCD phase transitions





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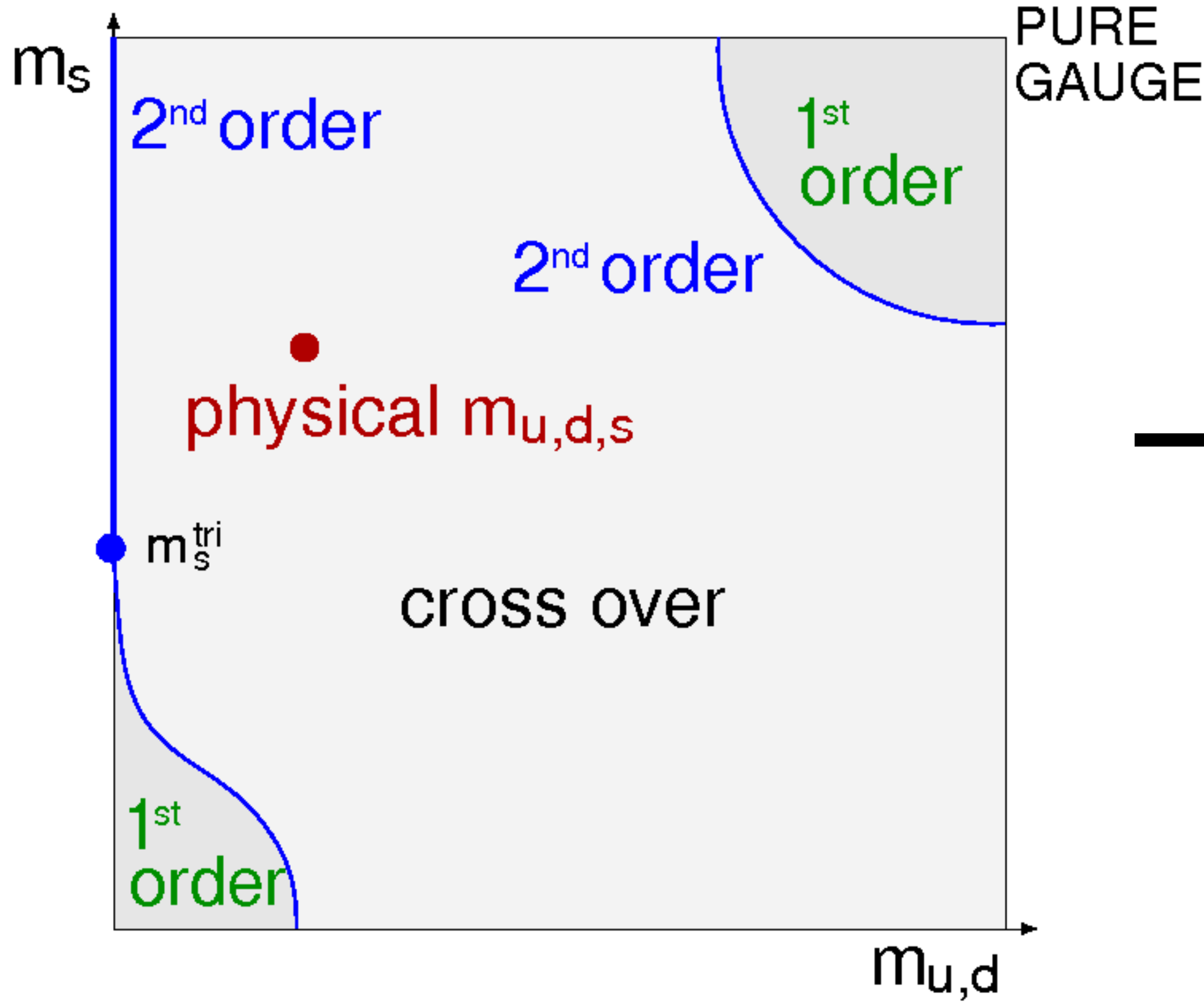

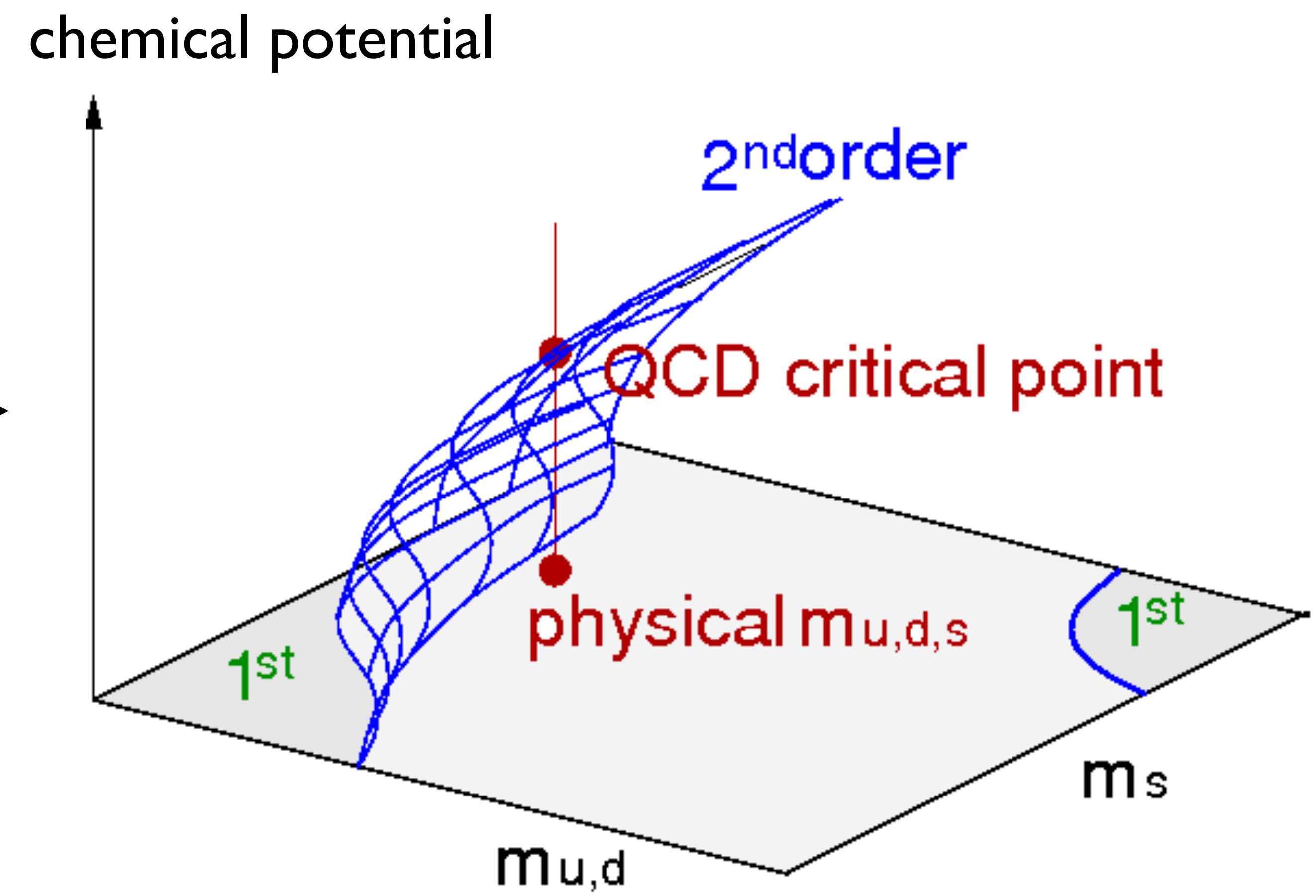

- Lattice-QCD
  - present: extrapolation
  - future: exact methods ?
- DSE/FRG
  - can do ! but typical errors 5-30%

Is this happening ??
Maybe yes, maybe not..

de Forcrand, Philipsen, JHEP 0811 (2008) 012;
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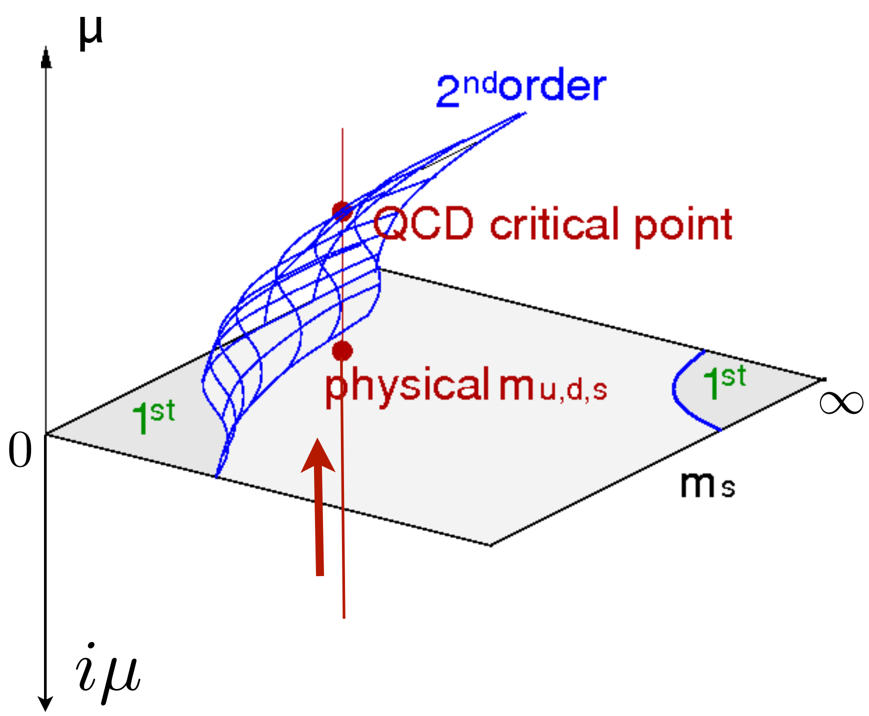

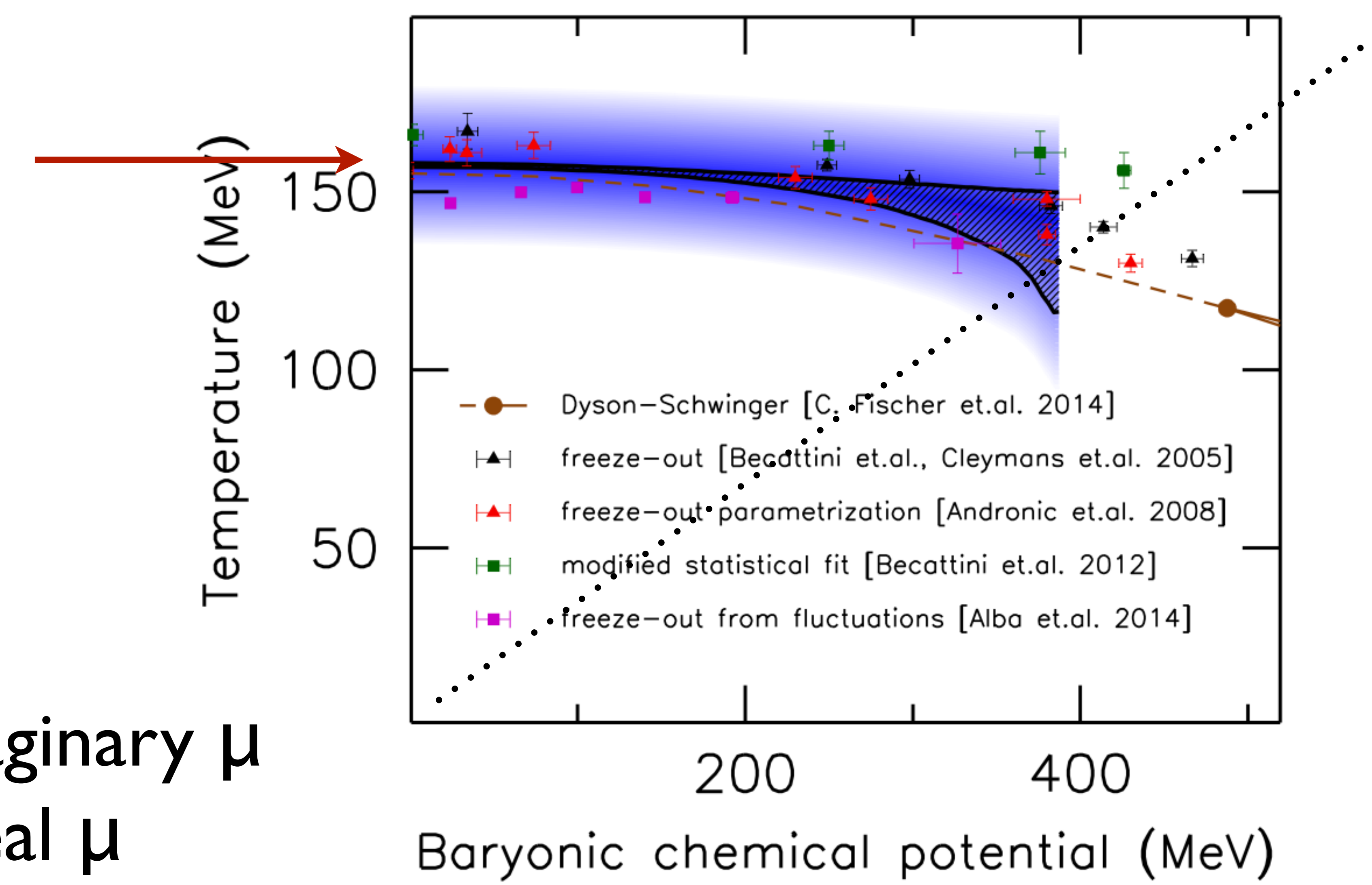

Bellwied, Borsanyi, Fodor, Günther, Katz, Ratti and Szabo, PLB 751 (2015) 559

Lattice method:

- Det. crossover at imaginary μ and extrapolate to real μ
- Control systematics

HOT-QCD: similar results

Main result:

- No transition for $\mu_B/T < 2\text{-}3$

Overview

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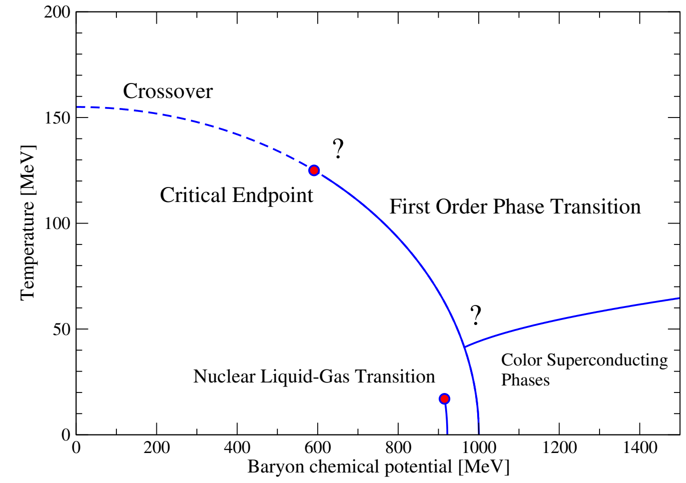

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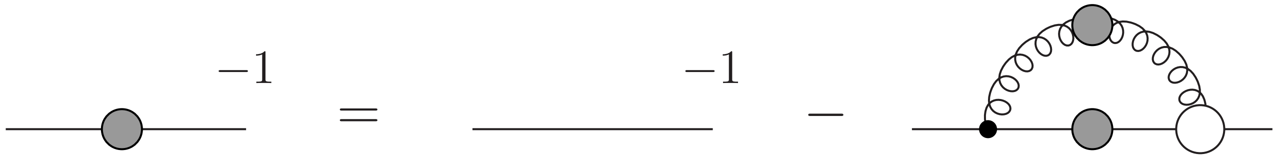

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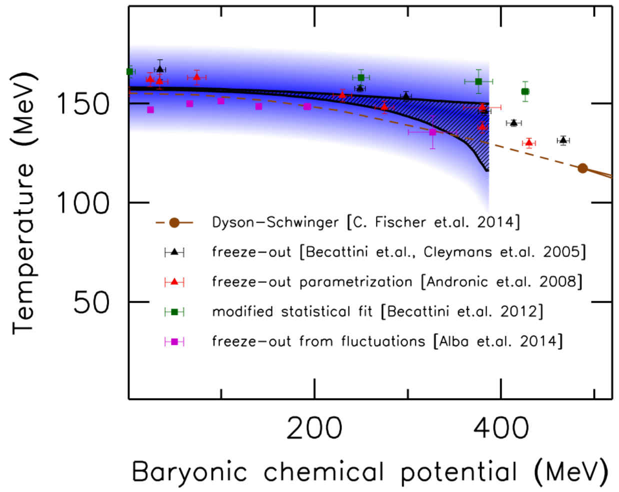

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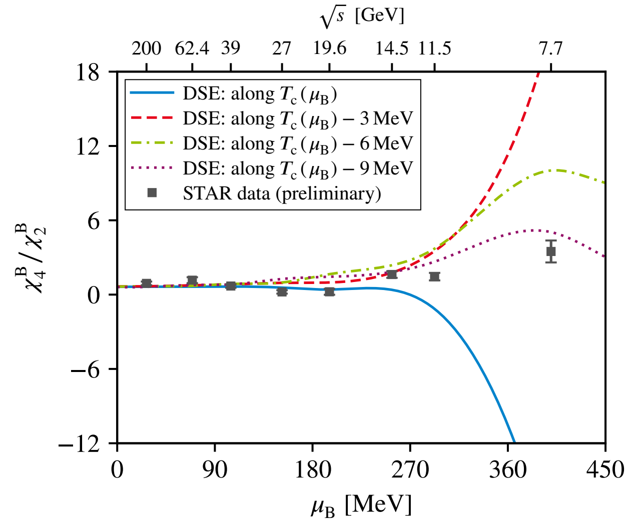
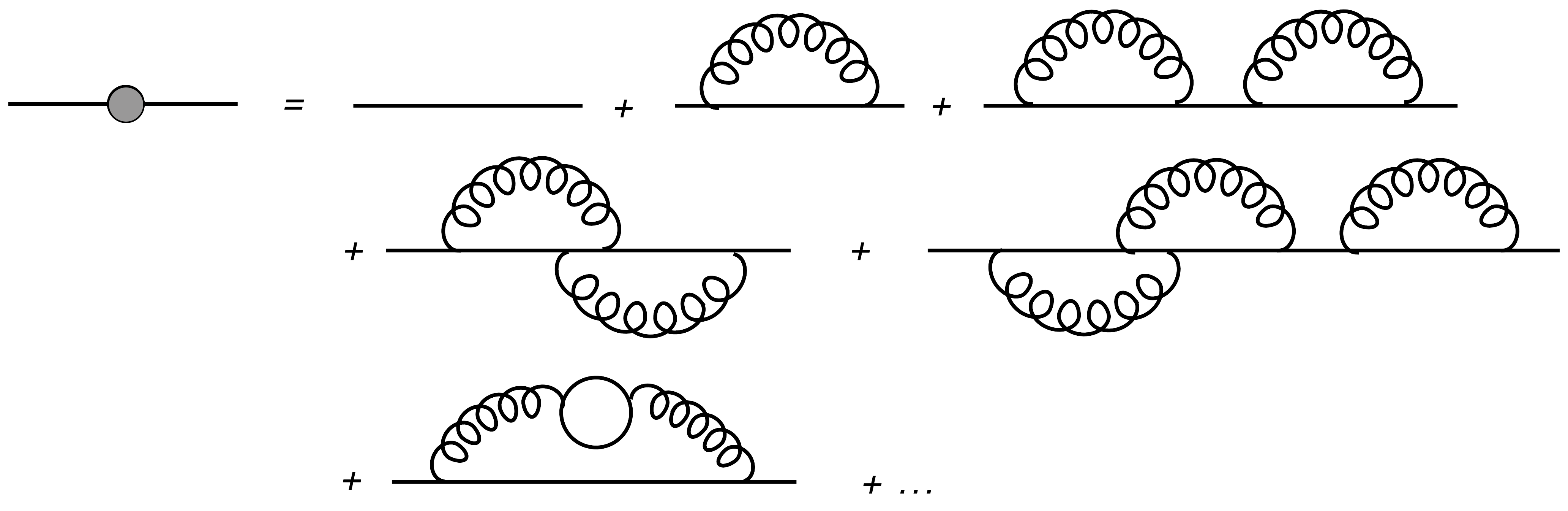


Graphical: start with perturbation theory and resum



$$S_0^{-1} = i\not{p} + m \qquad \longrightarrow \qquad S^{-1}(p) = [i\not{p} + M(p^2)]/Z_f(p^2)$$

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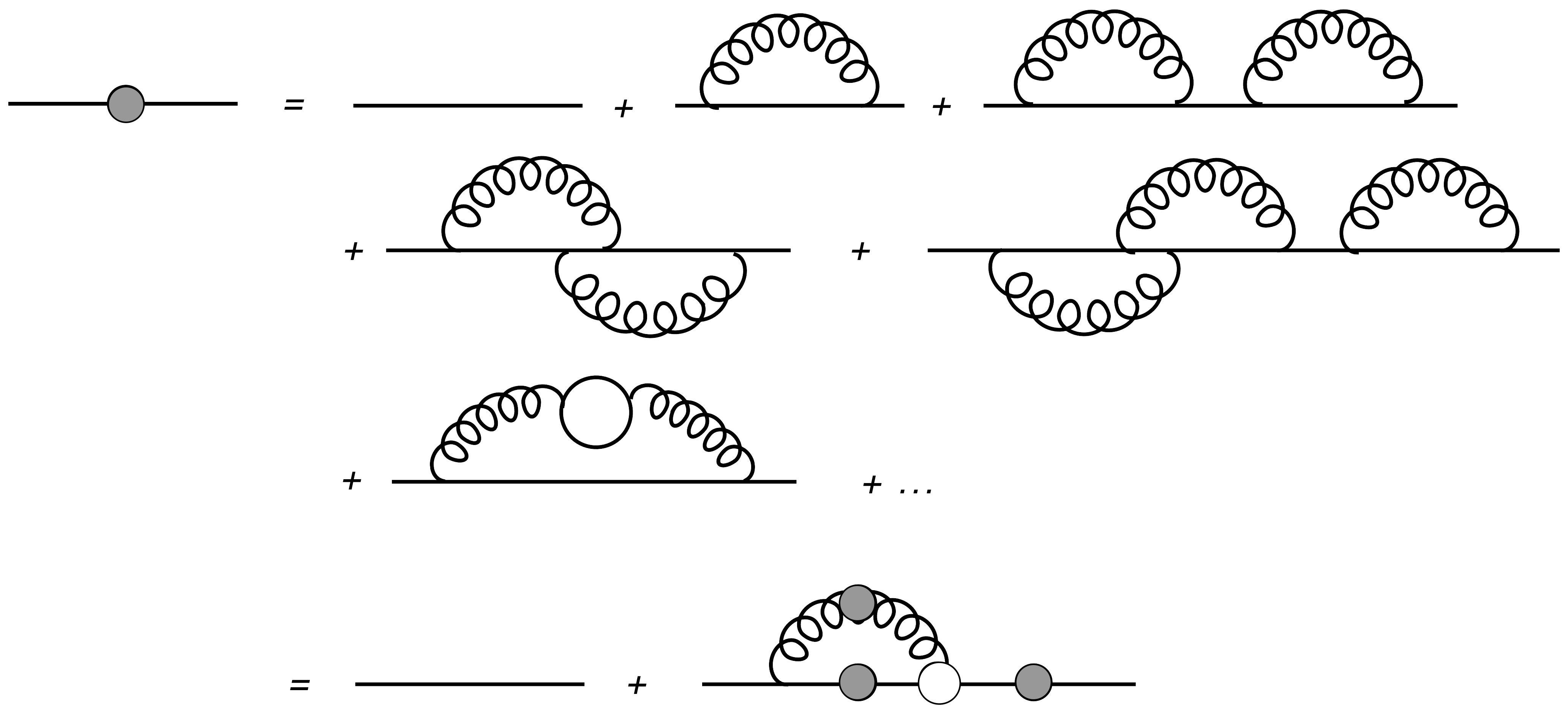

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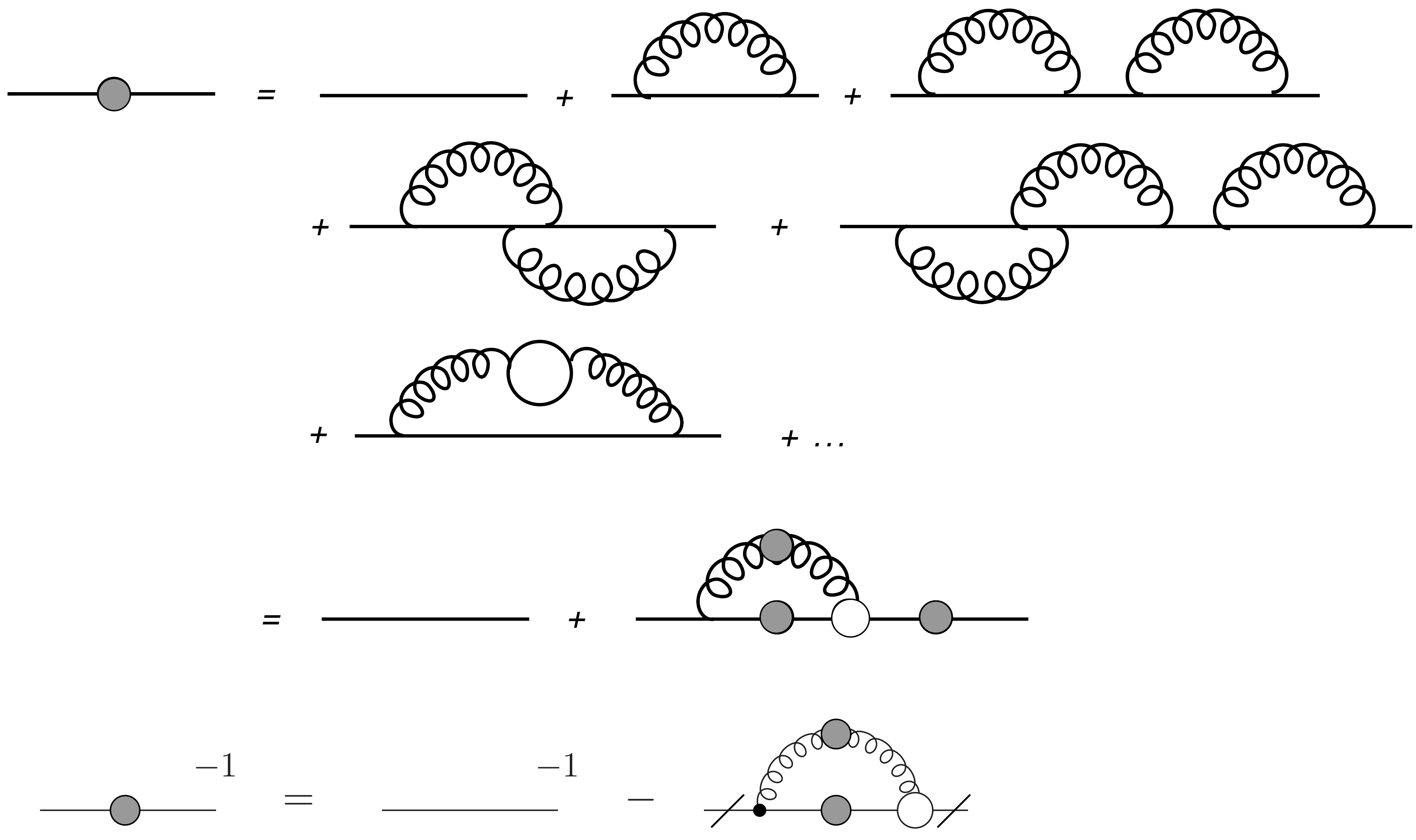
Derivation of DSEs

Graphical: start with perturbation theory and resum



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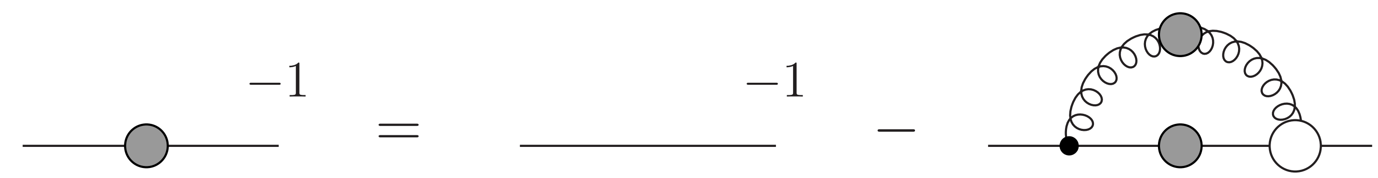
QCD order parameters from propagators

Chiral order parameter:

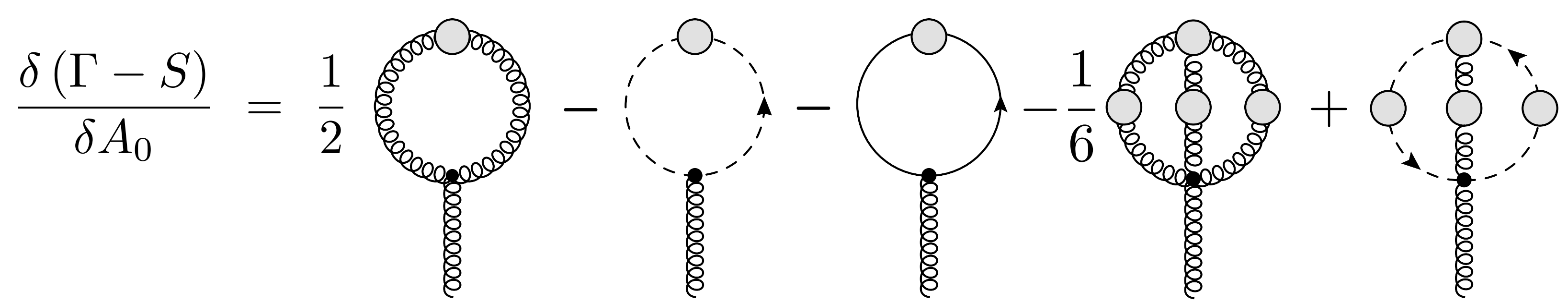
$$\langle \bar{\Psi}\Psi \rangle = Z_2 N_c Tr_D \frac{1}{T} \sum_{\omega} \int \frac{d^3p}{(2\pi)^3} S(\vec{p},\omega)$$



Deconfinement:

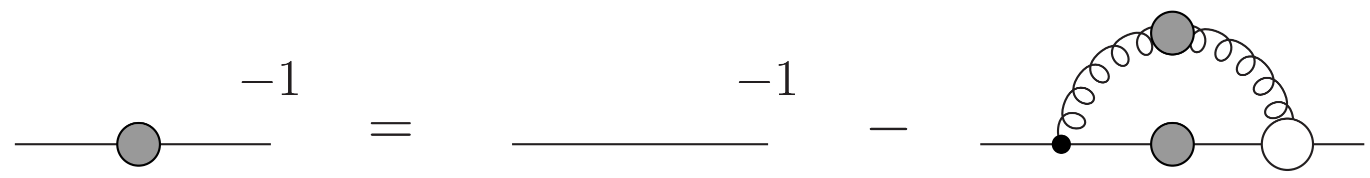
- Polyakov loop potential

$$L = \frac{1}{N_c} Tr\, e^{ig\beta A_0}$$



Braun, Gies, Pawlowski, PLB 684, 262 (2010)
Braun, Haas, Marhauser, Pawlowski, PRL 106 (2011)
Fister, Pawlowski, PRD 88 045010 (2013)
CF, Fister, Luecker, Pawlowski, PLB 732 (2013)

The DSE for the quark propagator

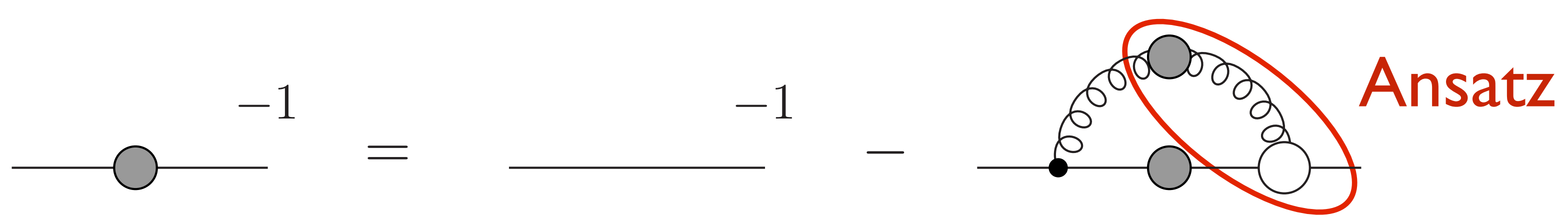


Approximations:

I) NJL/contact model:

Buballa, Phys. Rept., 2005, 407, 205-376

II) Rainbow-ladder:



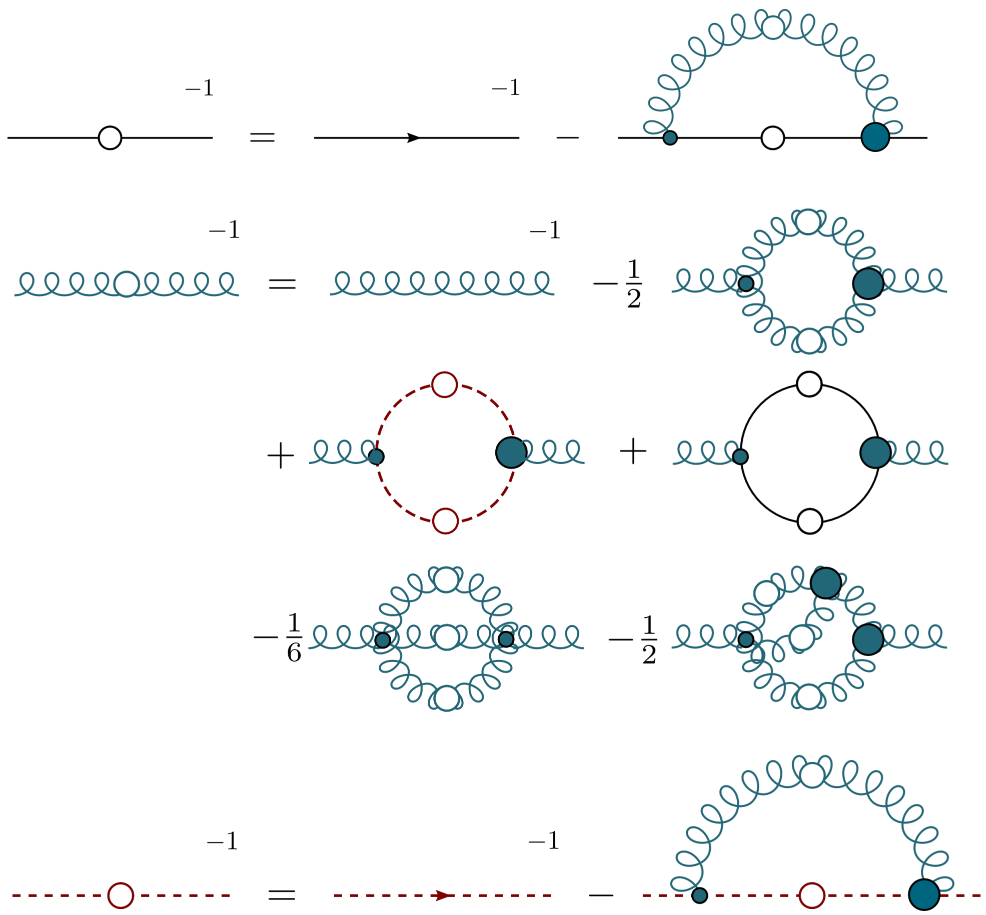
→ valuable for exploratory studies

→ not good enough for quantitative and/or systematic studies at finite T, μ

III) Solve tower of DSEs: (next slide)

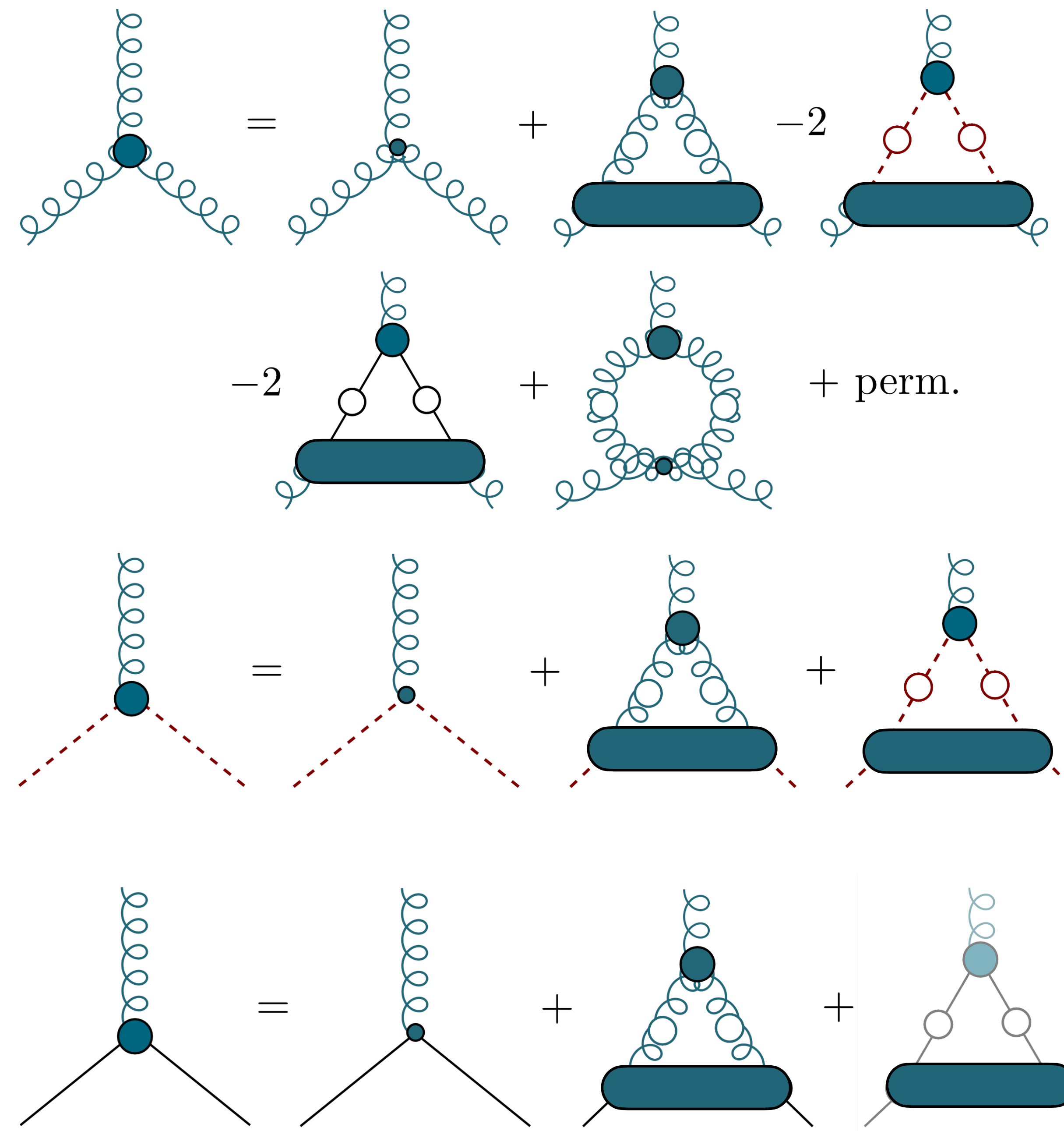
CF, PPNP 105 (2019) [1810.12938]

propagators



for different BRL approaches see work of
Aguilar, Alkofer, Binosi, Blum, Chang, Cyrol, Eichmann, Fister, Huber, Maas, Mitter, Papavassiliou, Pawlowski, Roberts, Smekal, Strodthoff, Vujinovic, Watson, Williams…

vertices

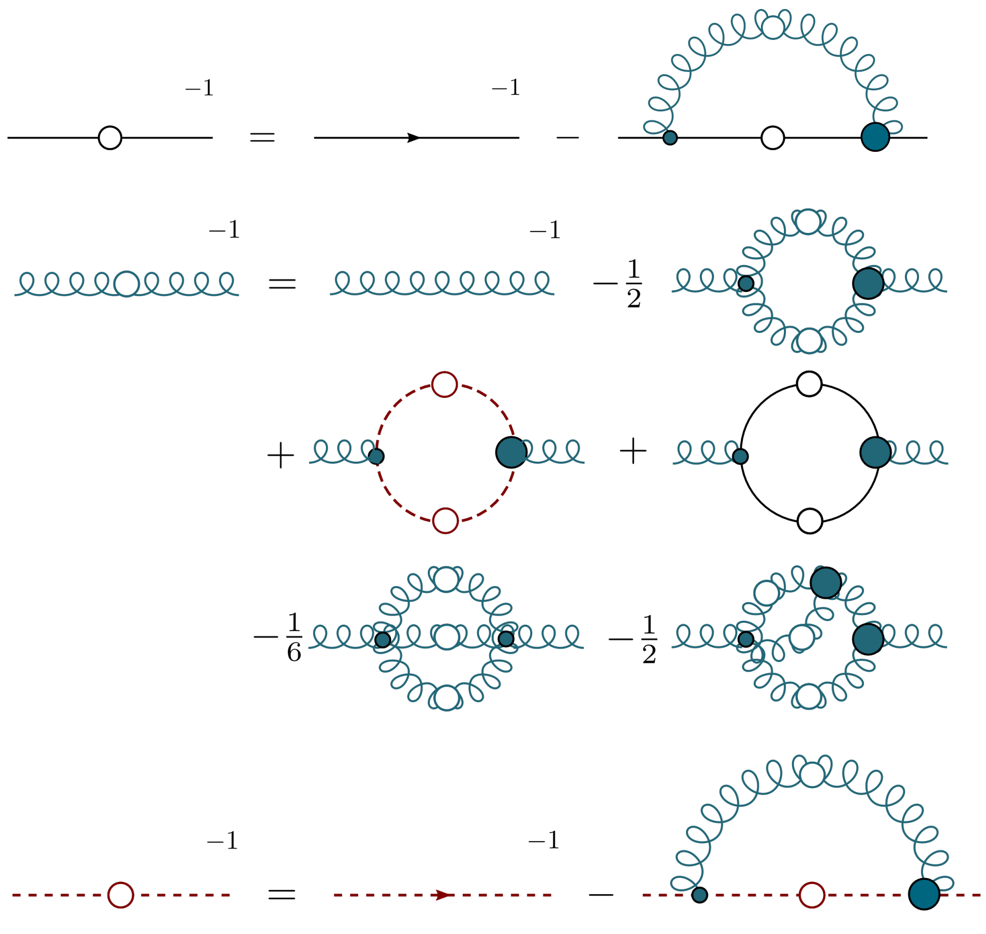


Williams, CF, Heupel, PRD 93 (2016) 034026

Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

3PI-truncation (T=0, μ=0)

propagators



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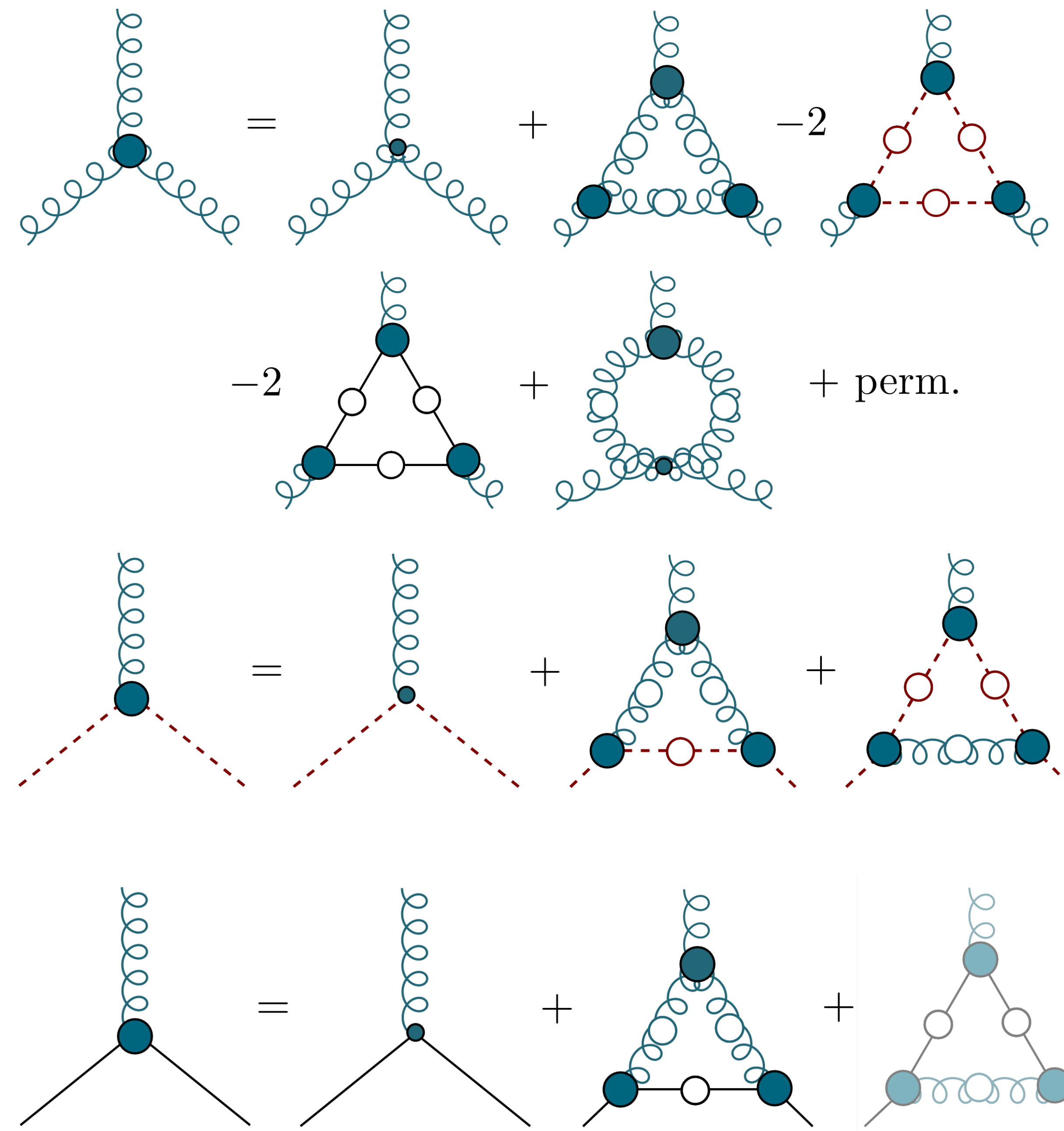


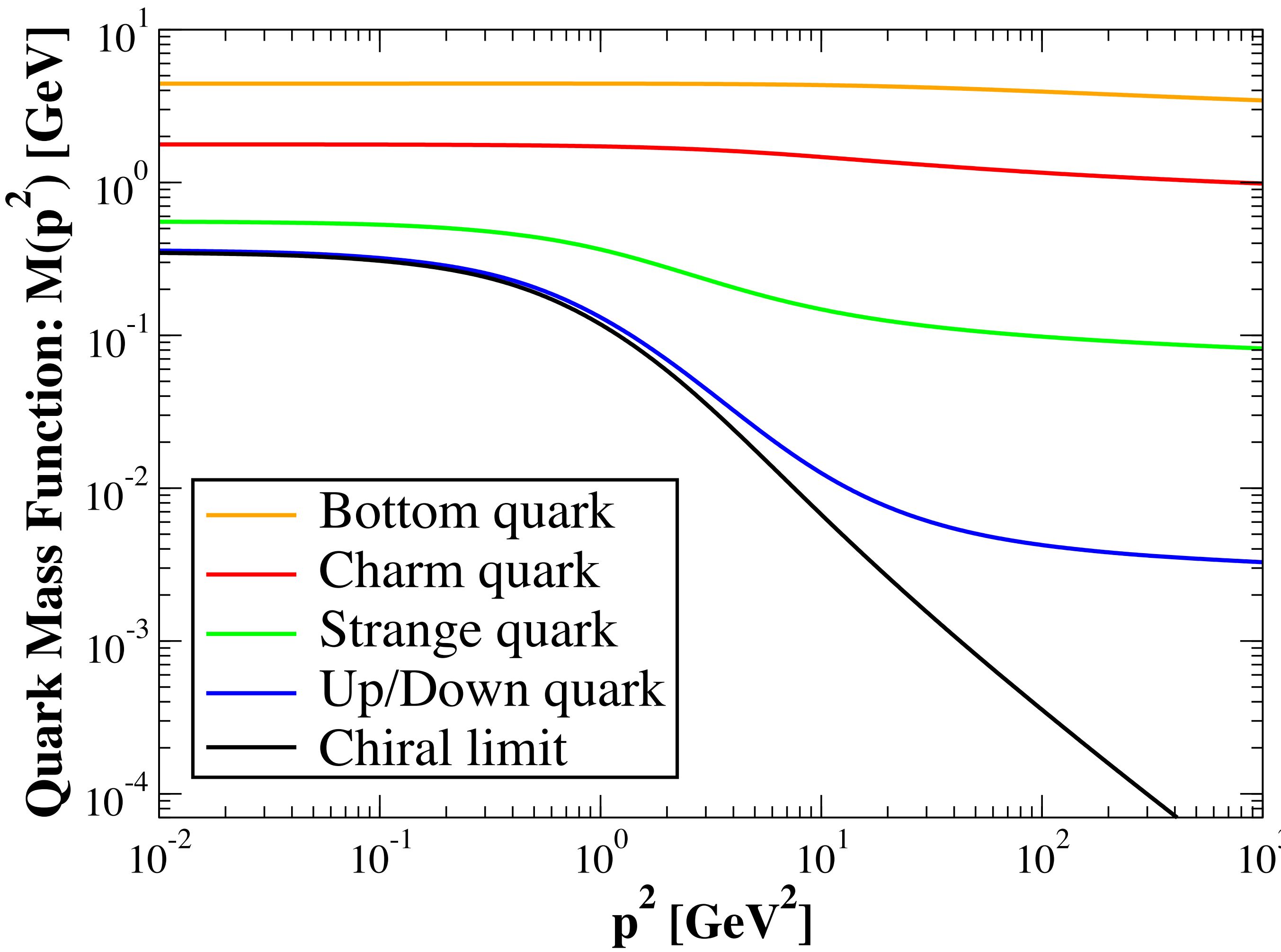
Williams, CF, Heupel, PRD 93 (2016) 034026

Review: Eichmann, Sanchis-Alepuz, Williams, Alkofer, CF, PPNP 91, 1-100 [1606.09602]

Quark mass: flavor dependence

$$S(p) = Z_f(p^2)\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

Typical solution:



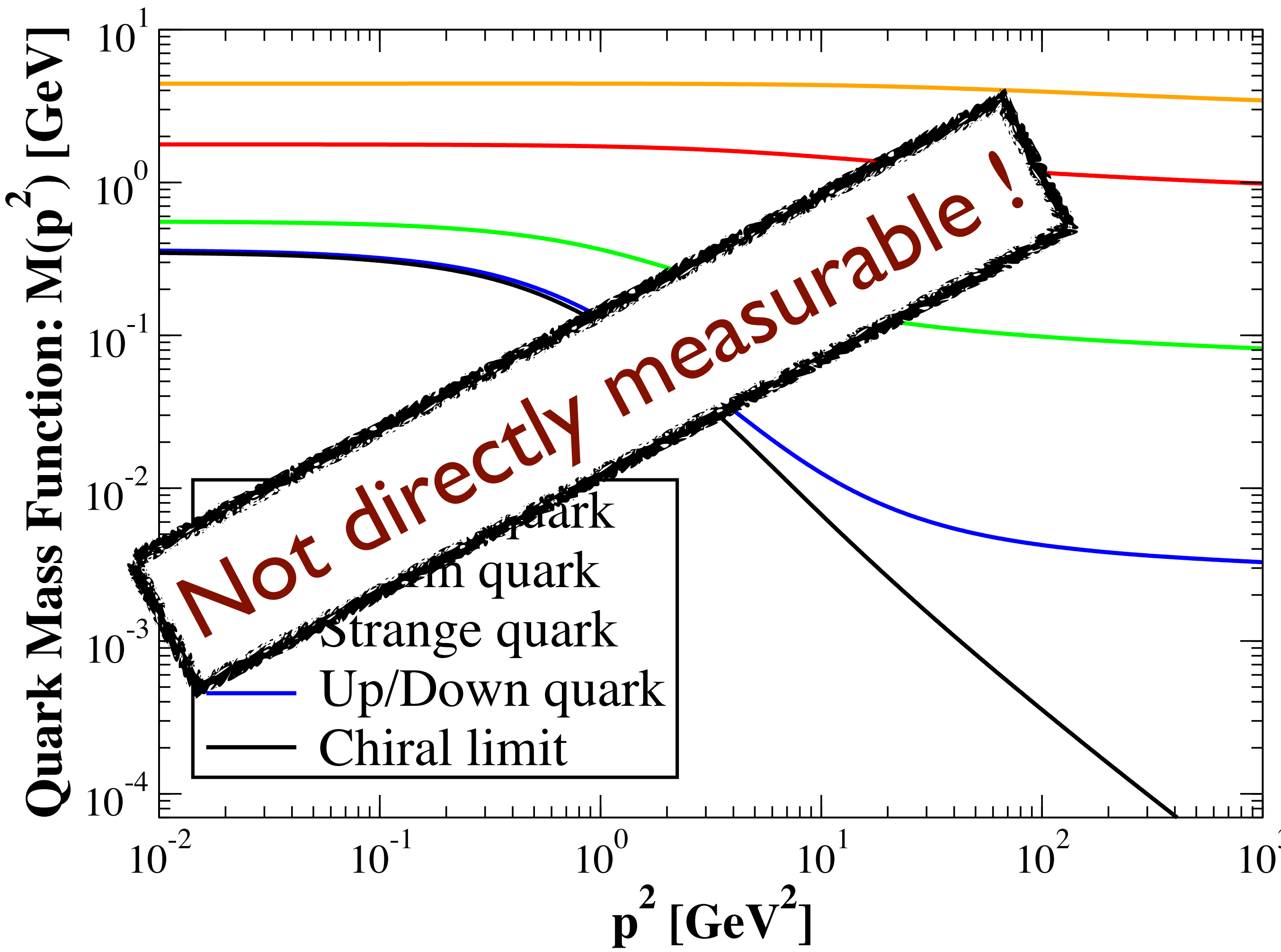
- $M(p^2)$: momentum dependent!
- Dynamical mass: $M_{strong} \approx 350$ MeV
- Flavour dependence because of m_{weak}
- Chiral condensate: $-\langle\bar{\Psi}\Psi\rangle \approx (250\,\mathrm{MeV})^3$

$$-\langle\bar{\Psi}\Psi\rangle = Z_2 Z_m N_c \int_p Tr S(p)$$

Quark mass: flavor dependence

Typical solution:

$$S(p) = Z_f(p^2)\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$



- $M(p^2)$: momentum dependent!
- Dynamical mass: $M_{strong} \approx 350$ MeV
- Flavour dependence because of m_{weak}
- Chiral condensate: $-\langle\bar{\Psi}\Psi\rangle \approx (250\,\mathrm{MeV})^3$

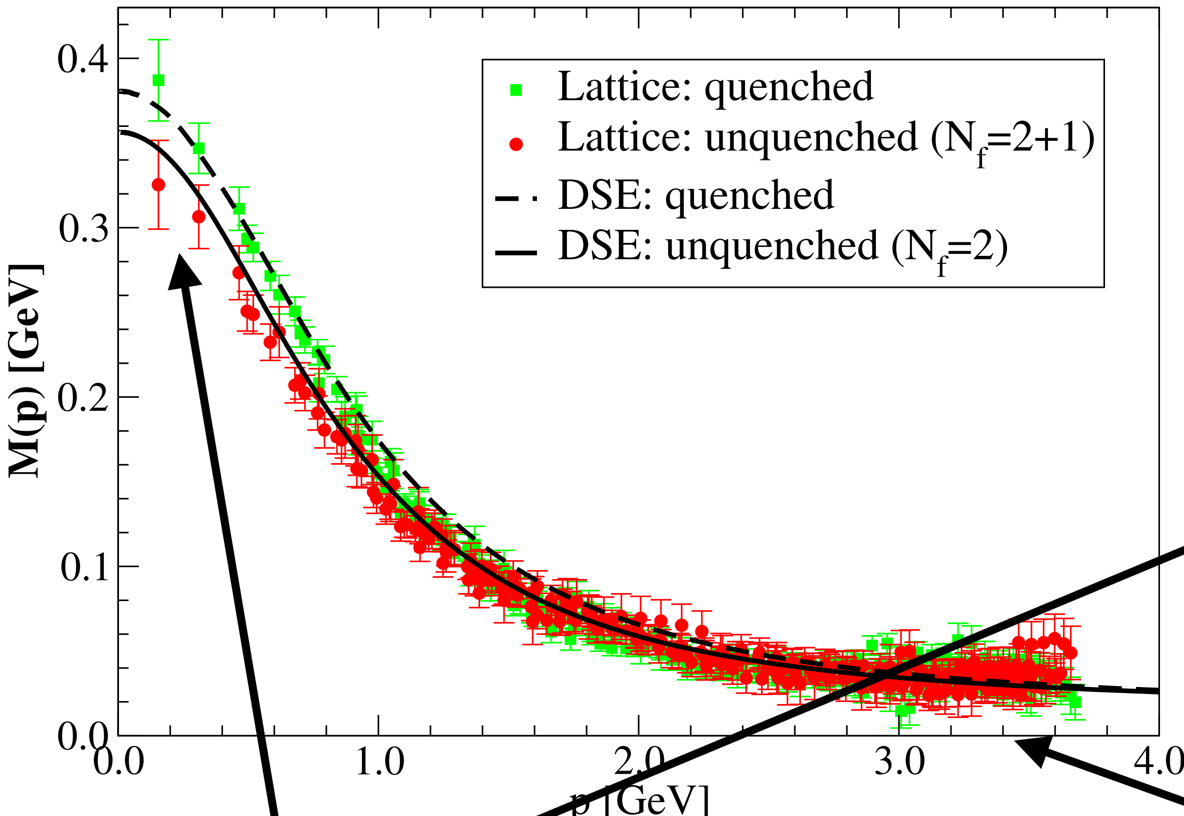
$$-\langle\bar{\Psi}\Psi\rangle = Z_2 Z_m N_c \int_p TrS(p)$$

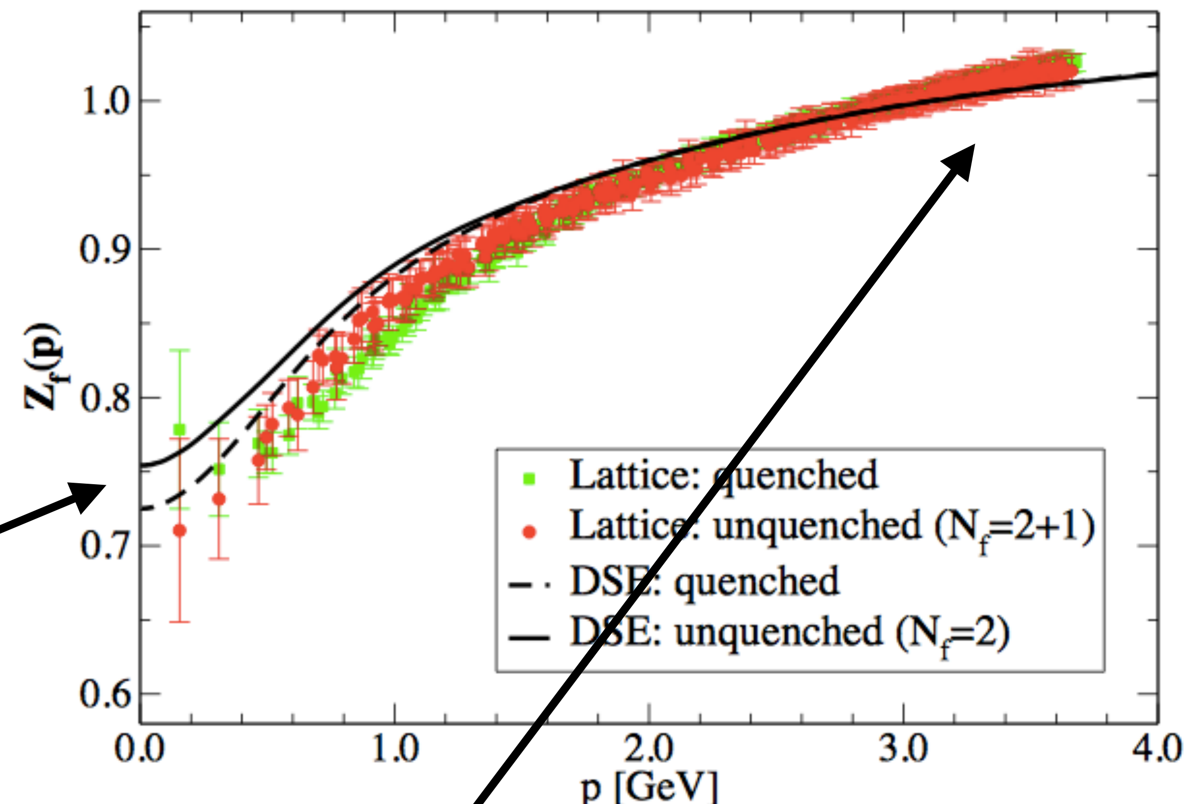
Quark dressing - comparison with lattice

Beyond rainbow-ladder:

$$S(p) = Z_f(p^2)\frac{-i\not p + M(p^2)}{p^2 + M^2(p^2)}$$

DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507



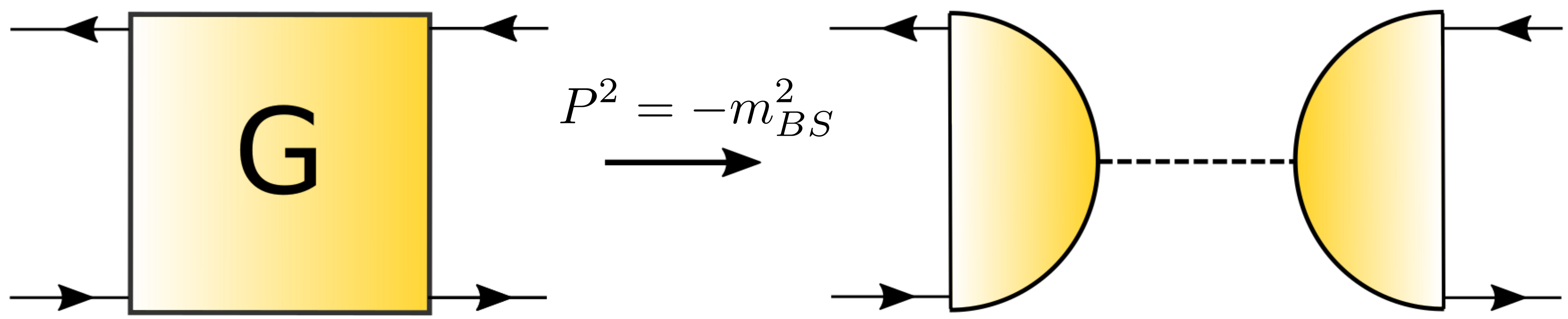


'constituent quark':
large mass; very composite

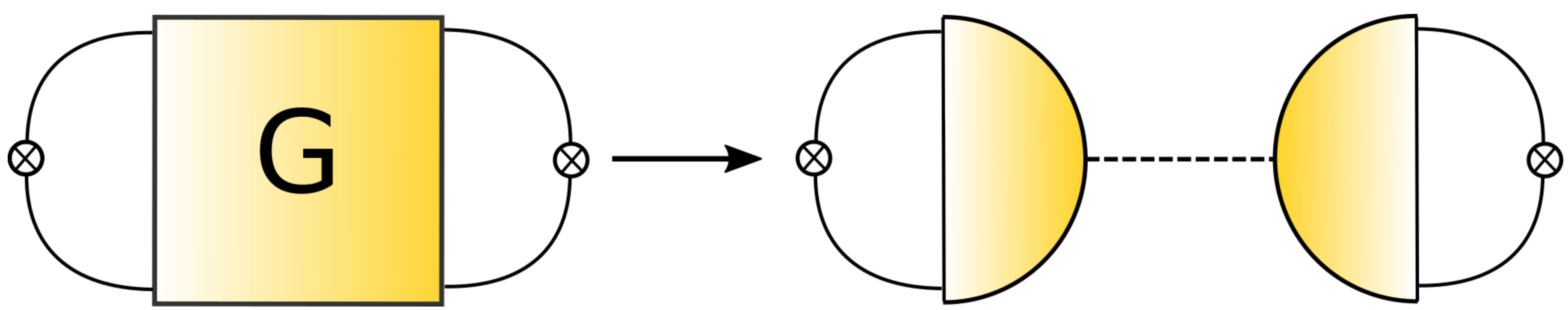
'current quark':
- small mass; non-composite

Extracting spectra from QCD-correlators

functional:



Lattice:



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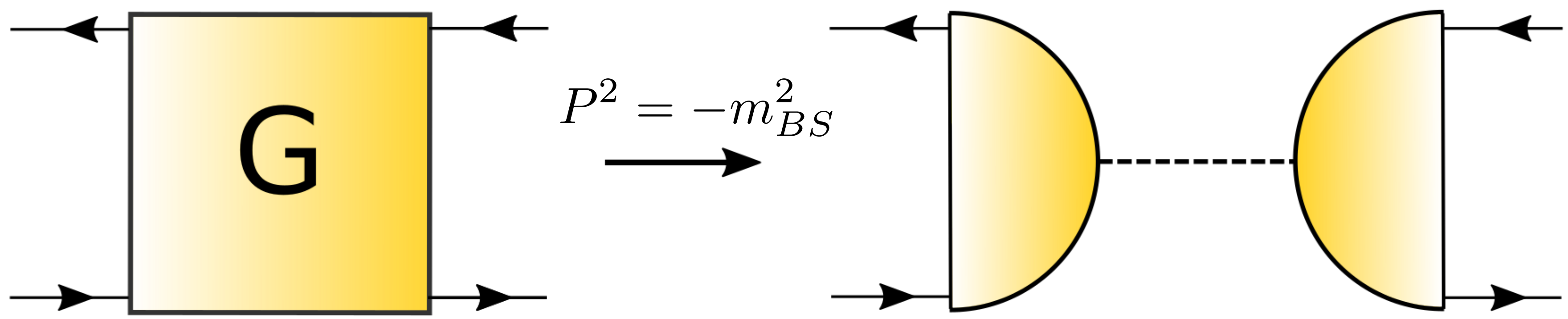

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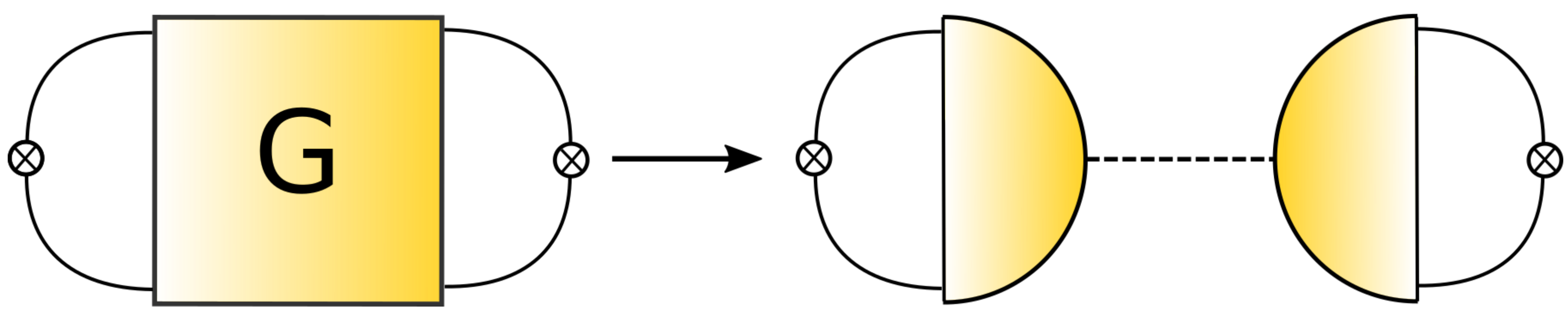
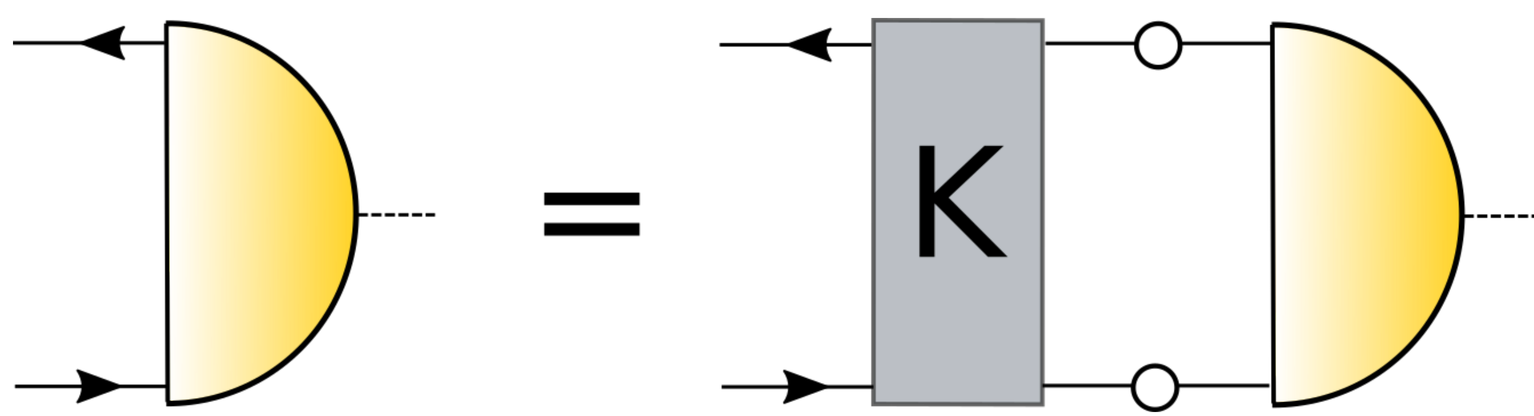
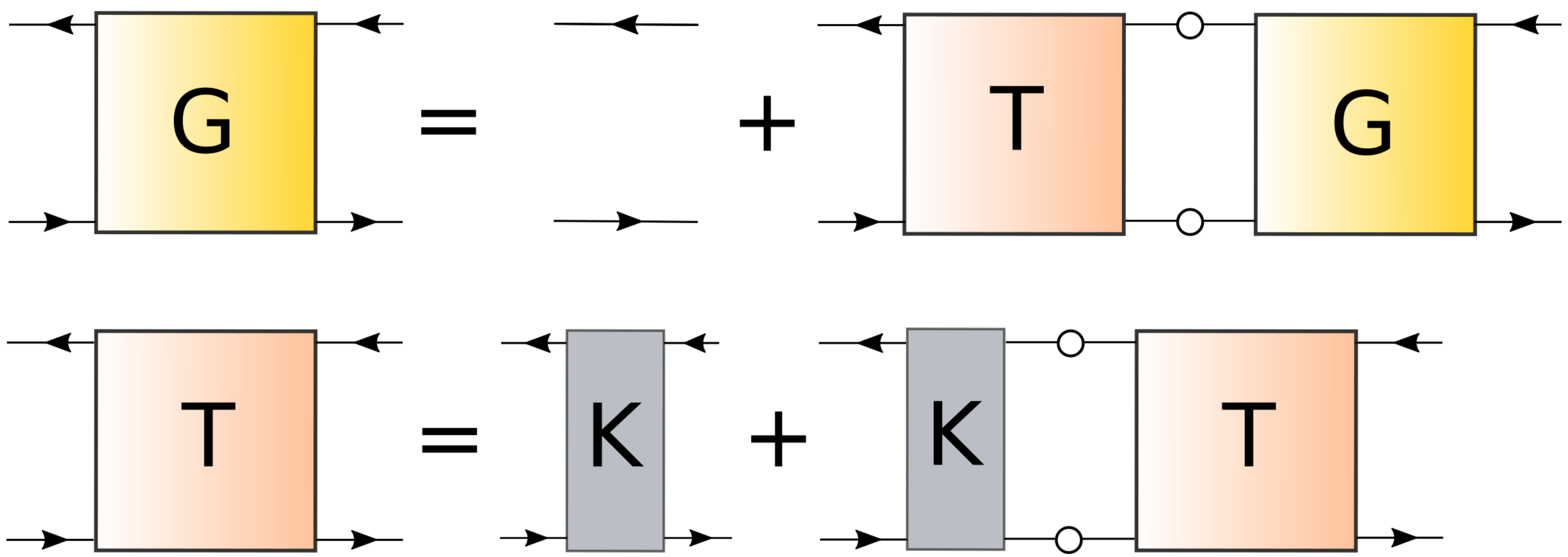
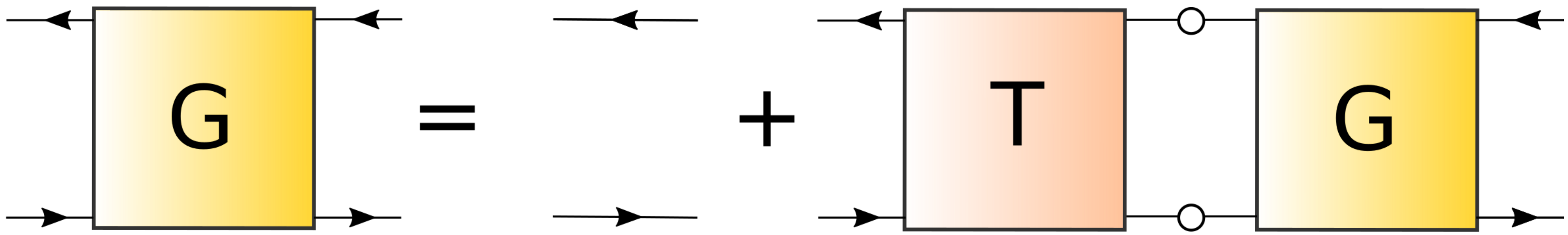


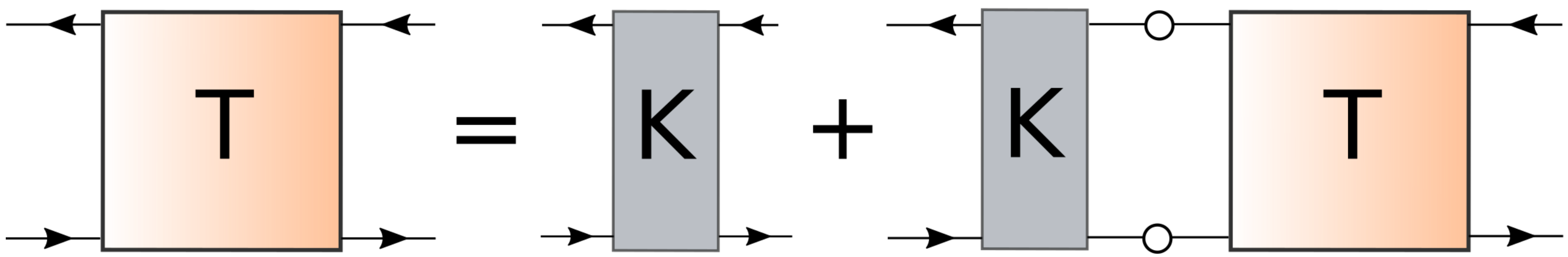
exact BSE:



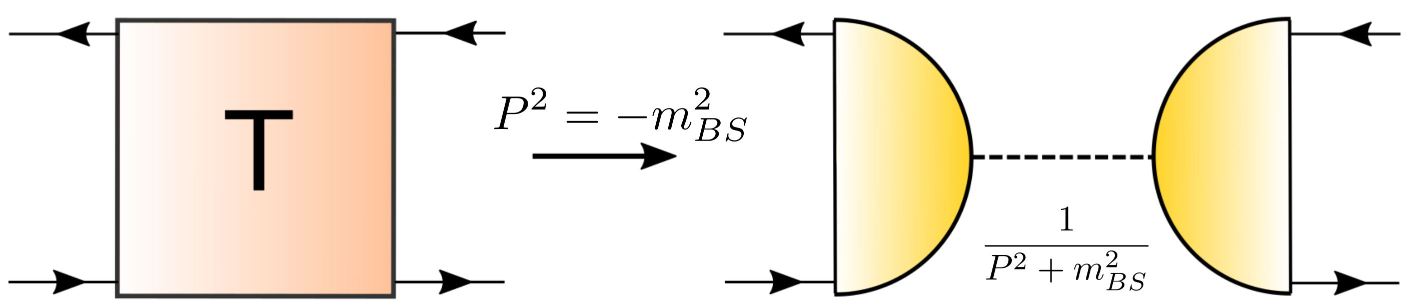
Bound states and Bethe-Salpeter equations



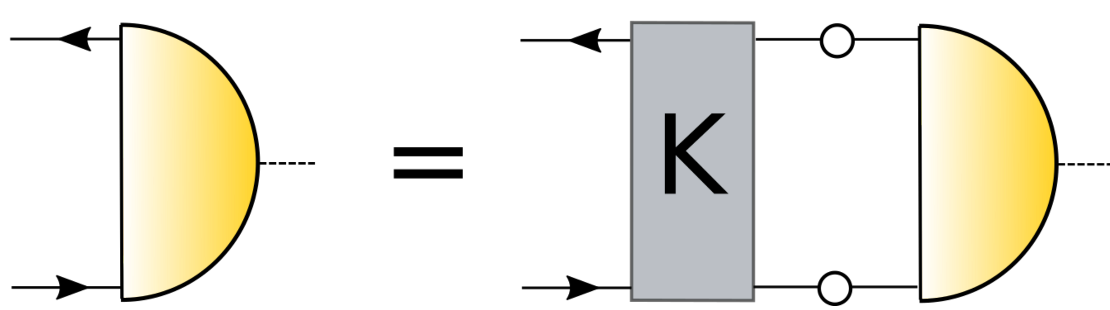




Bound states appear as poles in T:



BSE:



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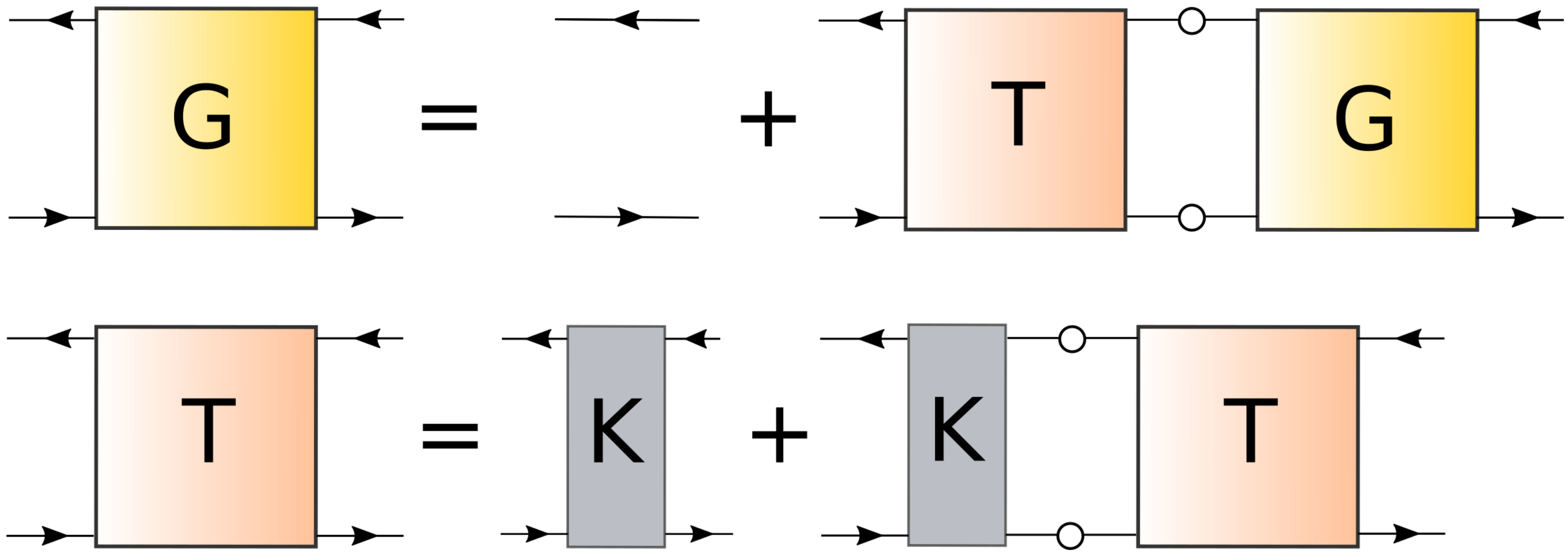

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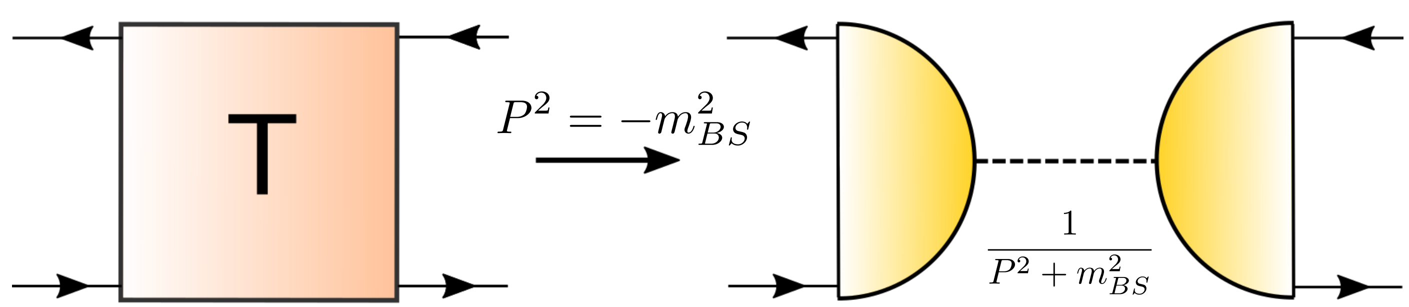
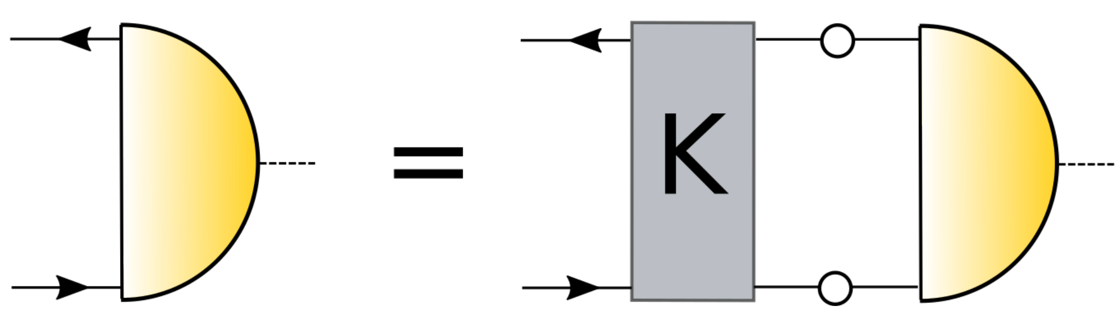
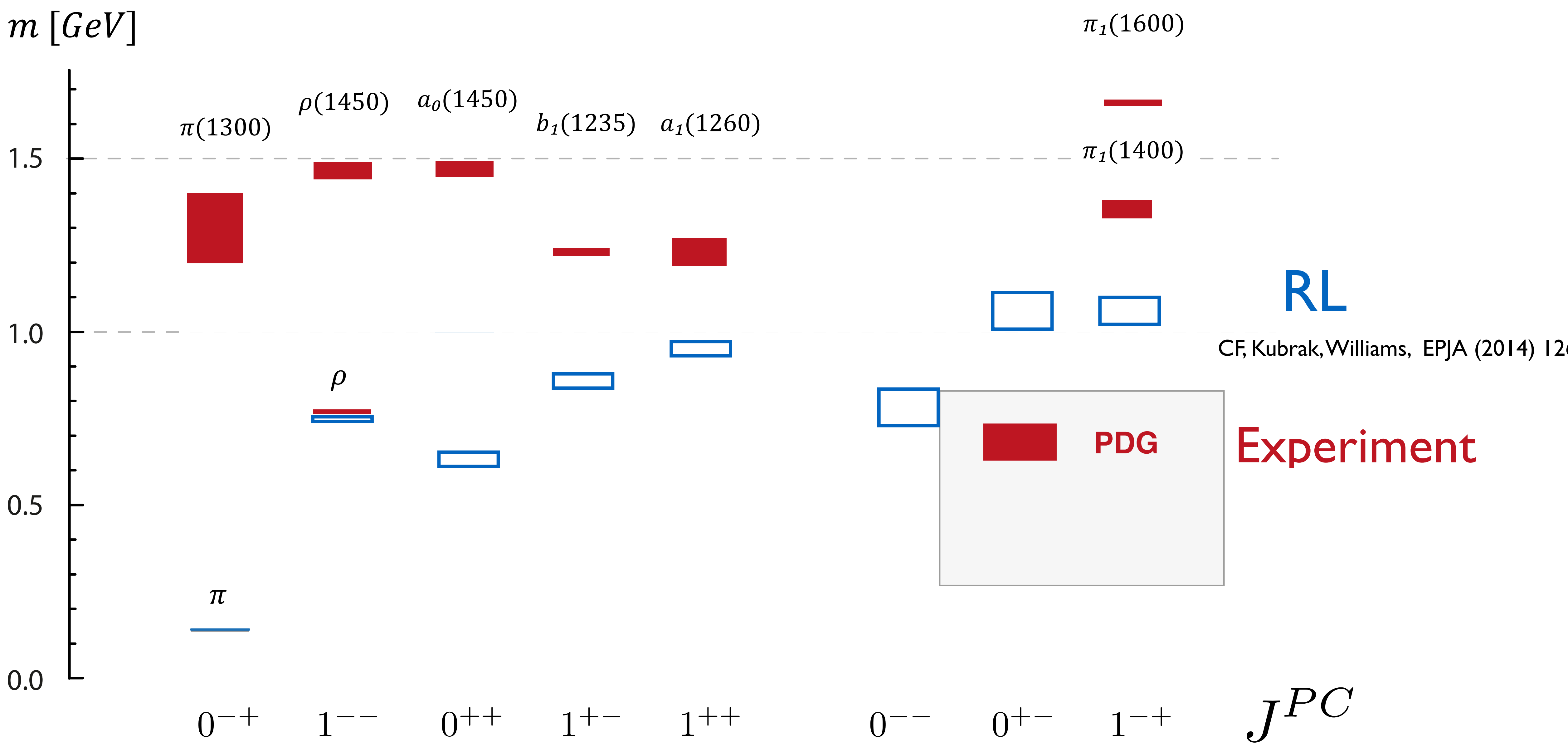


BS-wave functions = residue of bound state pole

BSE:

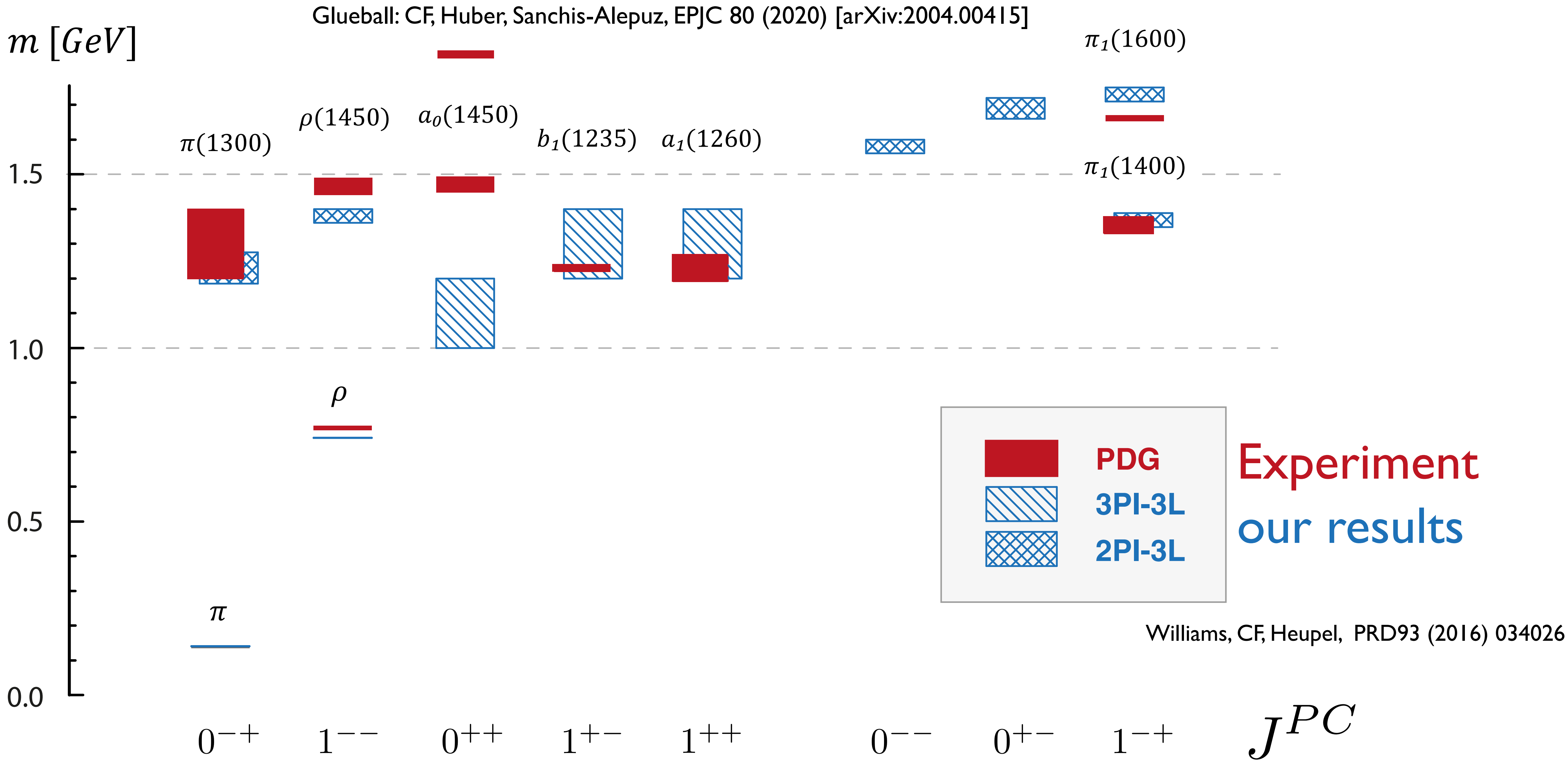


Rainbow-ladder: light meson spectrum



- good channels (ground state): 0^{-+}, 1^{--}
- acceptable channels (ground state) : 2^{++}, 3^{--},...
- clear deficiencies in other channels and excited states

Rainbow-ladder: light meson spectrum

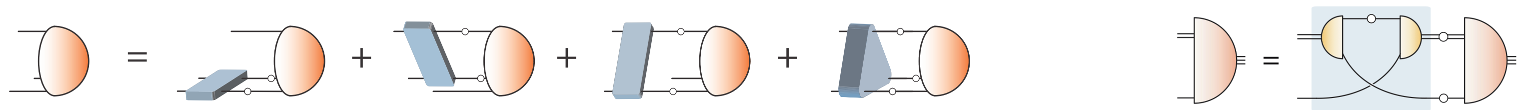


- good agreement with experiment in most channels
- special channels:

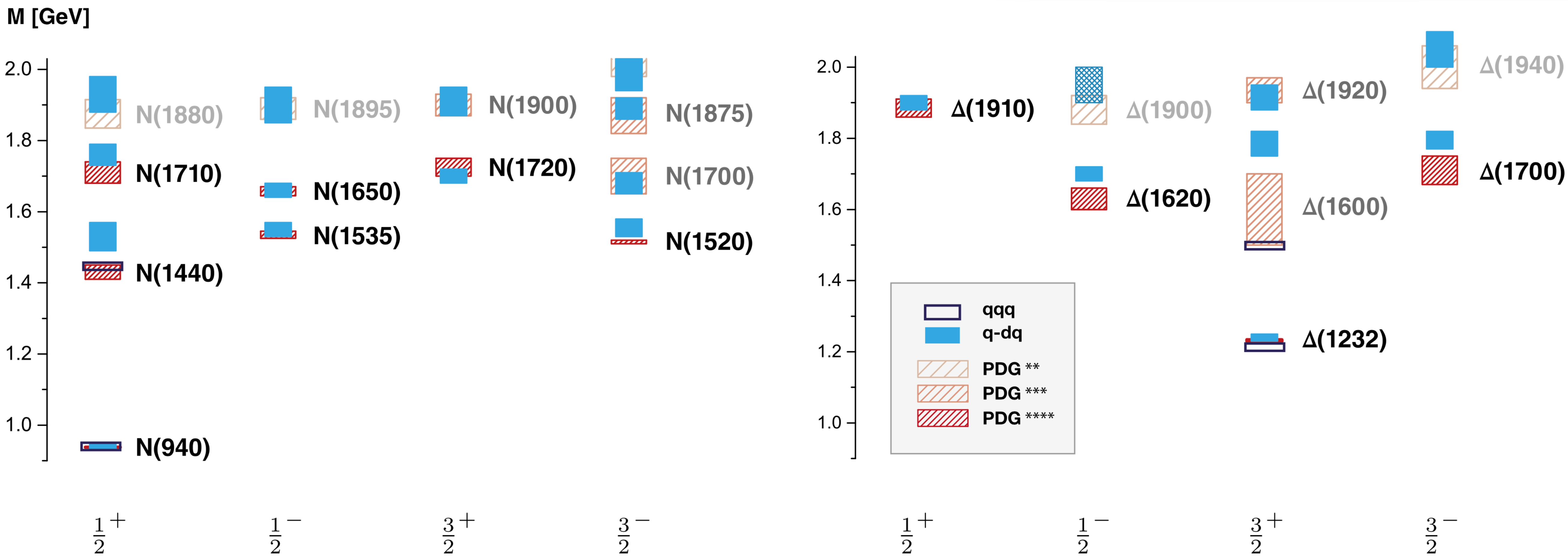
  pseudoscalar 0^{-+} : (pseudo-) Goldstone bosons

  scalar 0^{++} : complicated channel…

Light baryon spectrum:



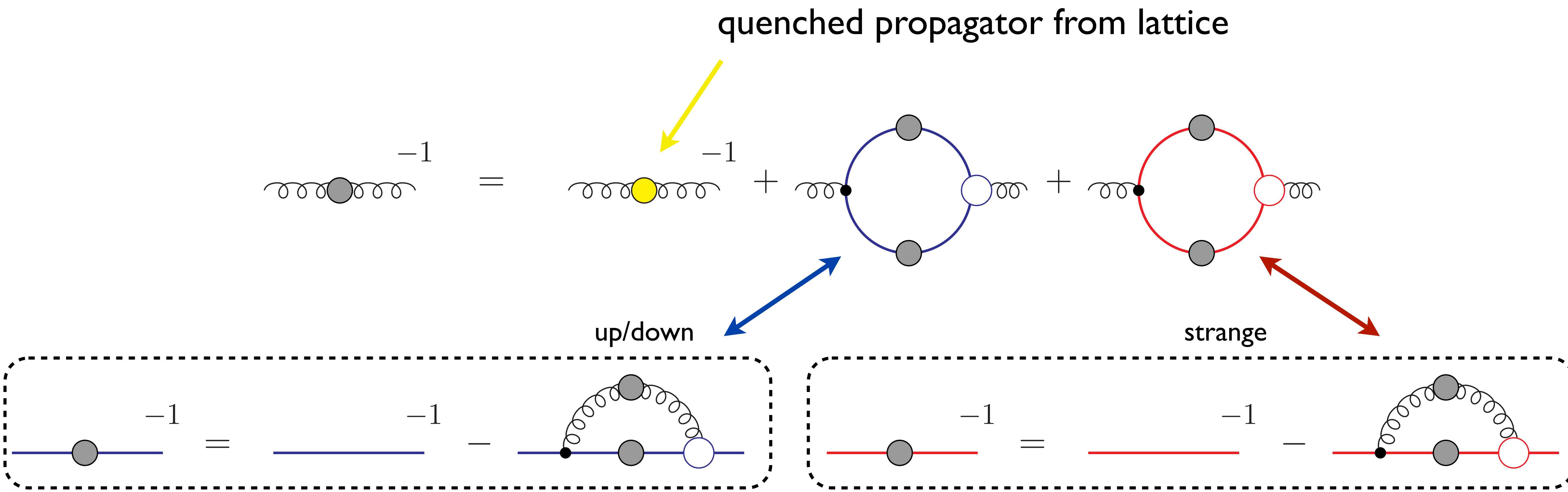
- 3 parameters + $m_{u,d,s}$ (all fixed in meson sector)



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects…)

N_f=2+1-QCD with DSEs



- quenched: without quark-loop
- Nf=2: isospin symmetry
- Nf=2+1: solve coupled system of 2+3+3 equations
- Vertex: ansatz built along STI and known UV/IR behavior

→ T,μ,m-dependent

Approximation for Quark-Gluon interaction

- Lattice input for vertex: not yet available…
- Diagrammatics: vertex-DSE (see later…)

explicit solutions at T=0: Mitter, Pawlowski and Strodthoff, PRD 91 (2015) 054035
Williams, CF, Heupel, PRD PRD 93 (2016) 034026

- Slavnov-Taylor identity: T,μ,m-dependent vertex

$$\Gamma_\nu(q,k,p) = \widetilde{Z}_3 \left(\delta_{4\nu}\gamma_4 \frac{C(k)+C(p)}{2} + \delta_{j\nu}\gamma_j \frac{A(k)+A(p)}{2} \right) \times$$
$$\times \left(\frac{d_1}{d_2+q^2} + \frac{q^2}{\Lambda^2+q^2} \left(\frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2+1]}{4\pi} \right)^{2\delta} \right)$$

STI

PT

- d_1 fixed via T_c
- d_2 fixed to match scale of lattice gluon input

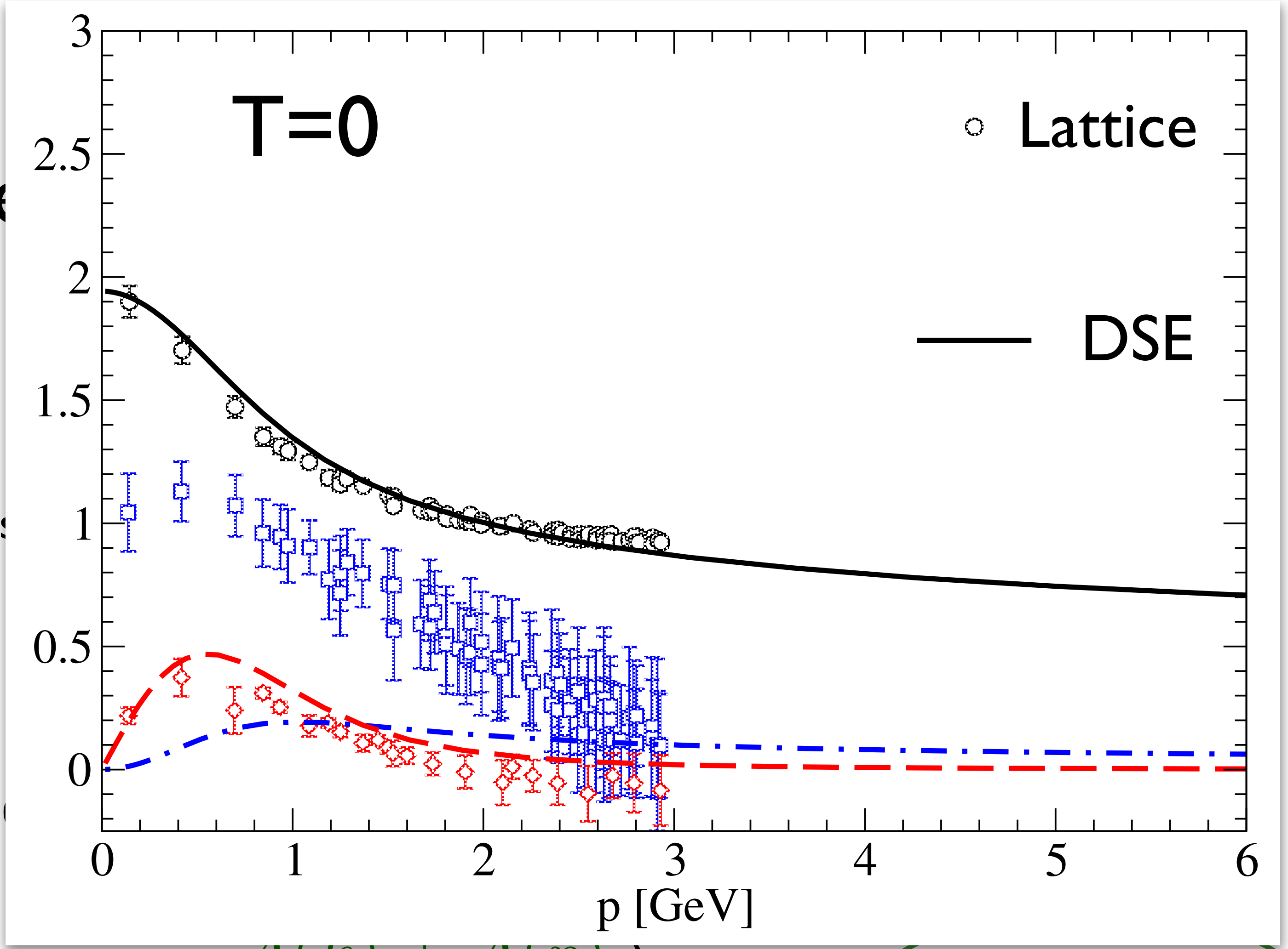
Approximation for Quark-Gluon interaction

- Lattice input for vertex: not ye

- Diagrammatics: vertex-DSE (s

explicit solution

- Slavnov-Taylor identity: T,μ,m-



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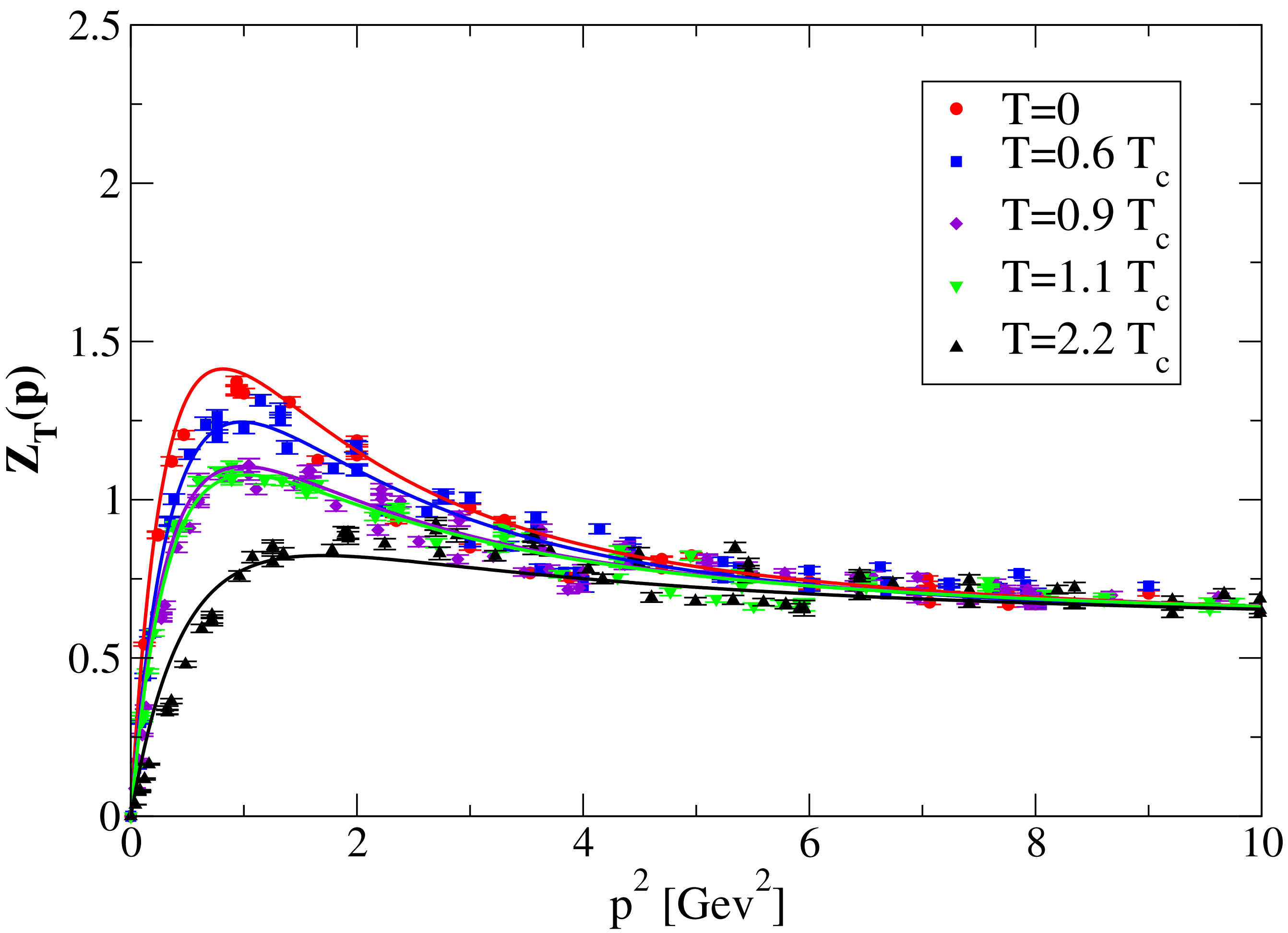
STI

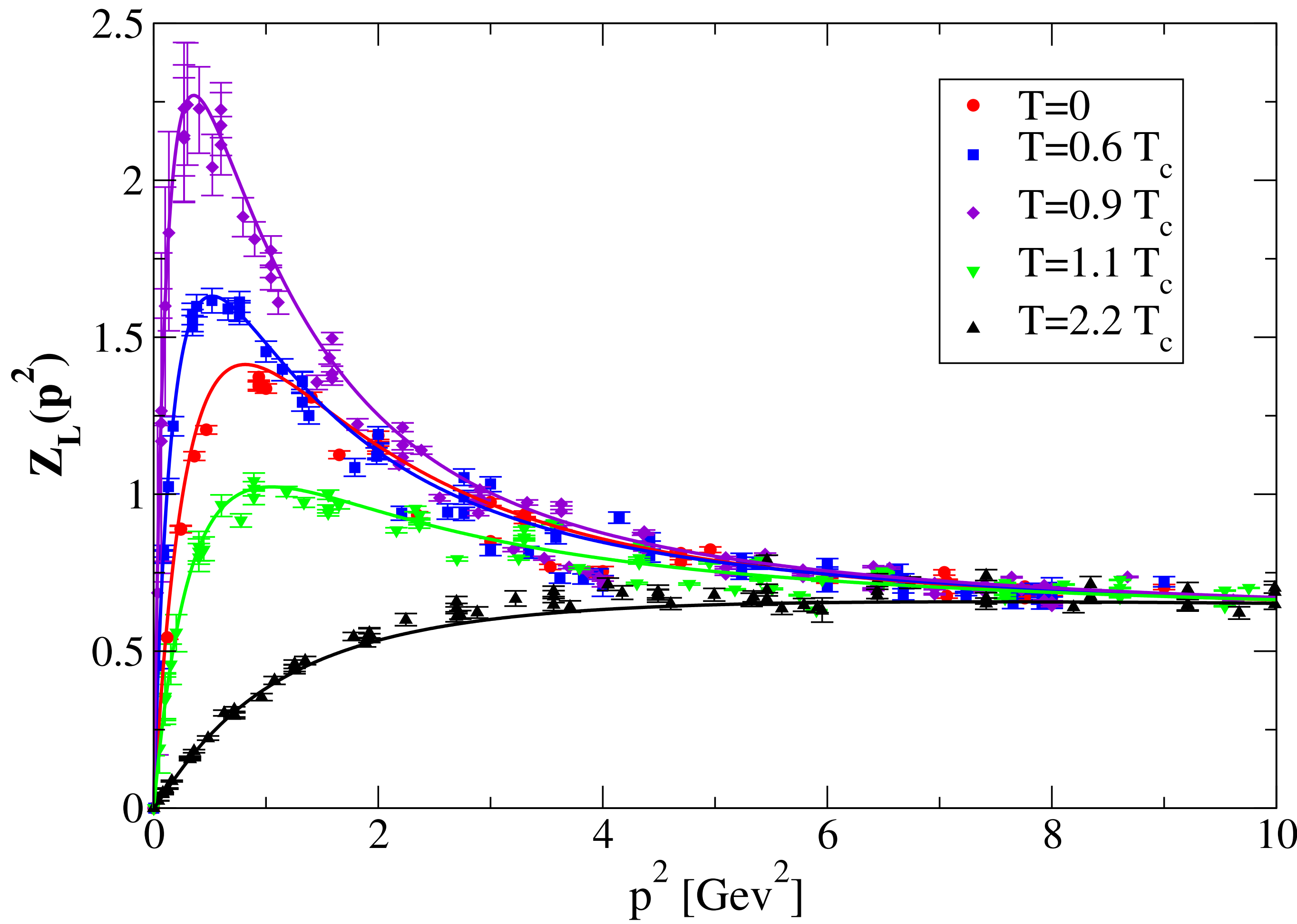
PT

- d_1 fixed via T_c
- d_2 fixed to match scale of lattice gluon input

Glue at finite temperature (T≠0)

T-dependent gluon propagator from quenched lattice simulations:





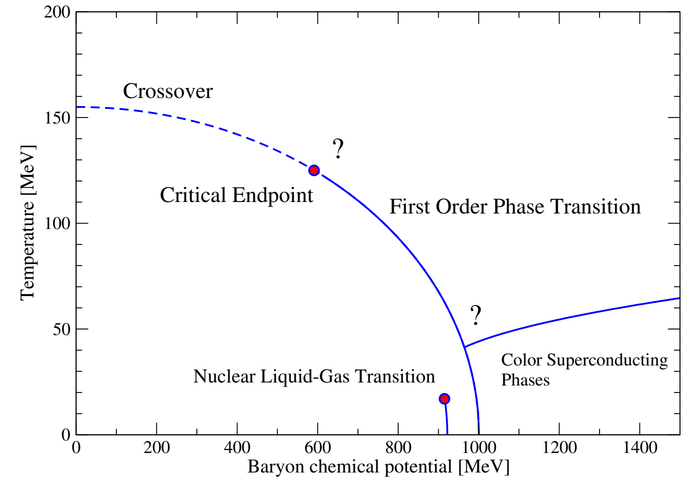
- Crucial difference between magnetic and electric gluon
- Maximum of electric gluon near Tc

Cucchieri, Maas, Mendes, PRD 75 (2007)
CF, Maas, Mueller, EPJC 68 (2010)
Cucchieri, Mendes, PoS FACESQCD 007 (2010)
Aouane, Bornyakov, Ilgenfritz, Mitrjushkin, Muller-Preussker and Sternbeck, PRD 85 (2012) 034501
Silva, Oliveira, Bicudo, Cardoso, PRD 89 (2014) 074503

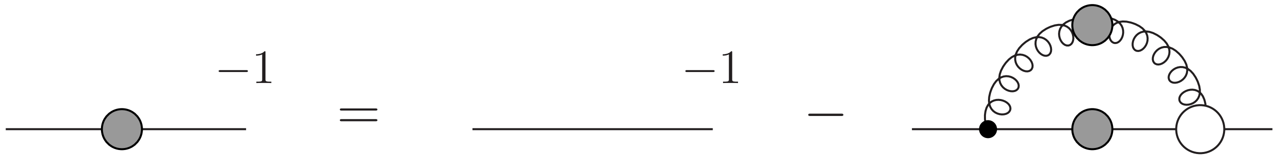
FRG: Fister, Pawlowski, arXiv:1112.5440

Overview

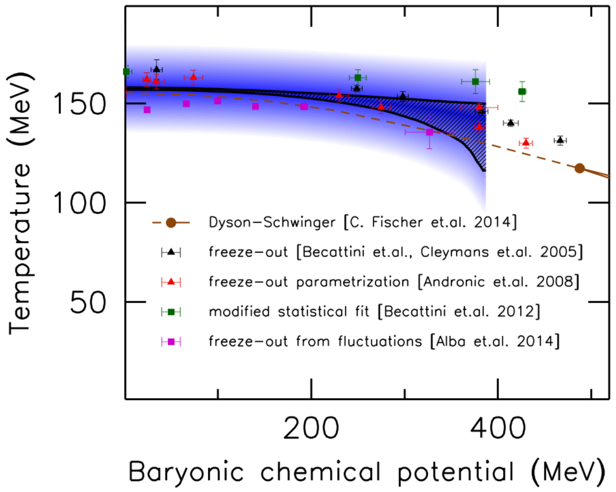
1. Introduction



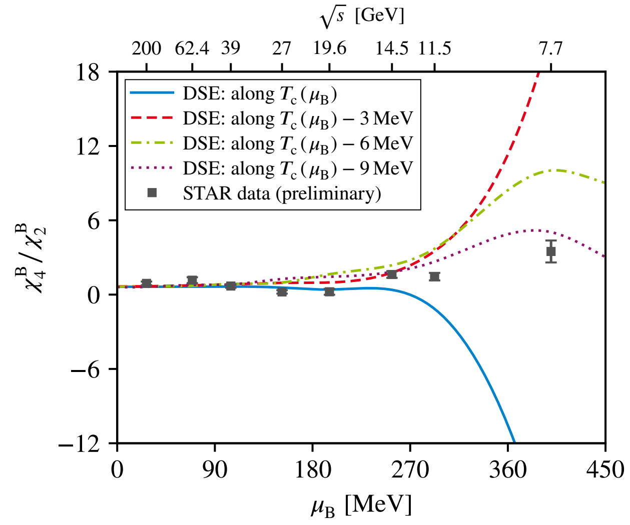
2. Gluons, quarks and DSEs



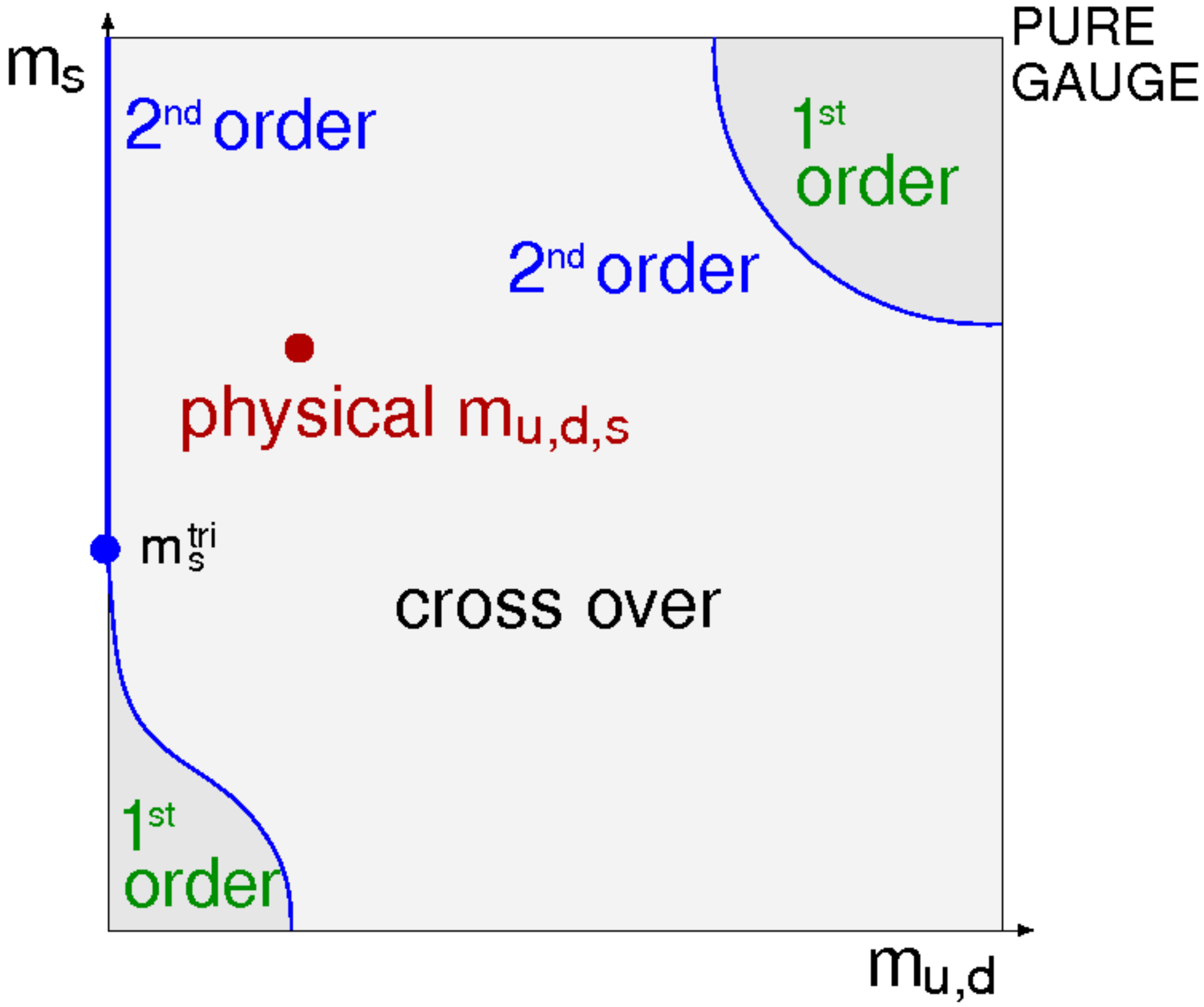
3. The CEP



4. Fluctuations and large densities



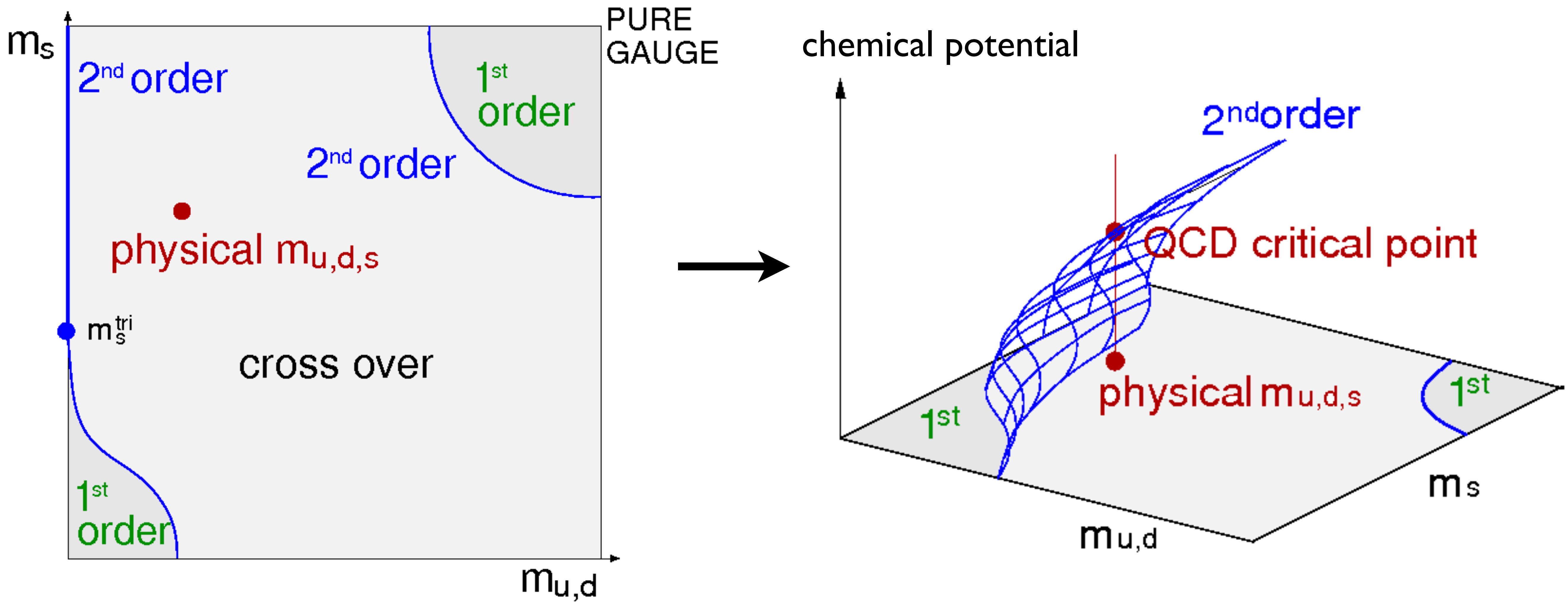
QCD phase transitions



Is this happening ??
Maybe yes, maybe not..

de Forcrand, Philipsen, JHEP 0811 (2008) 012;
NPB 642 (2002) 290

QCD phase transitions



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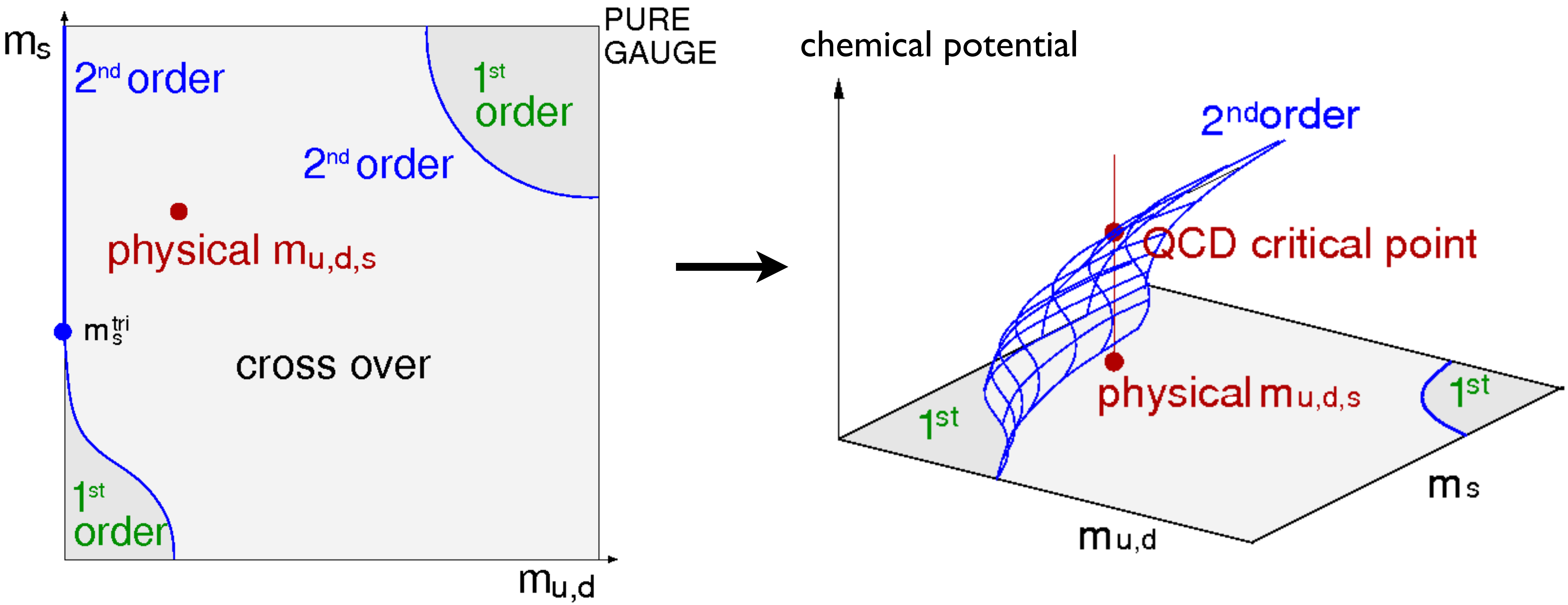


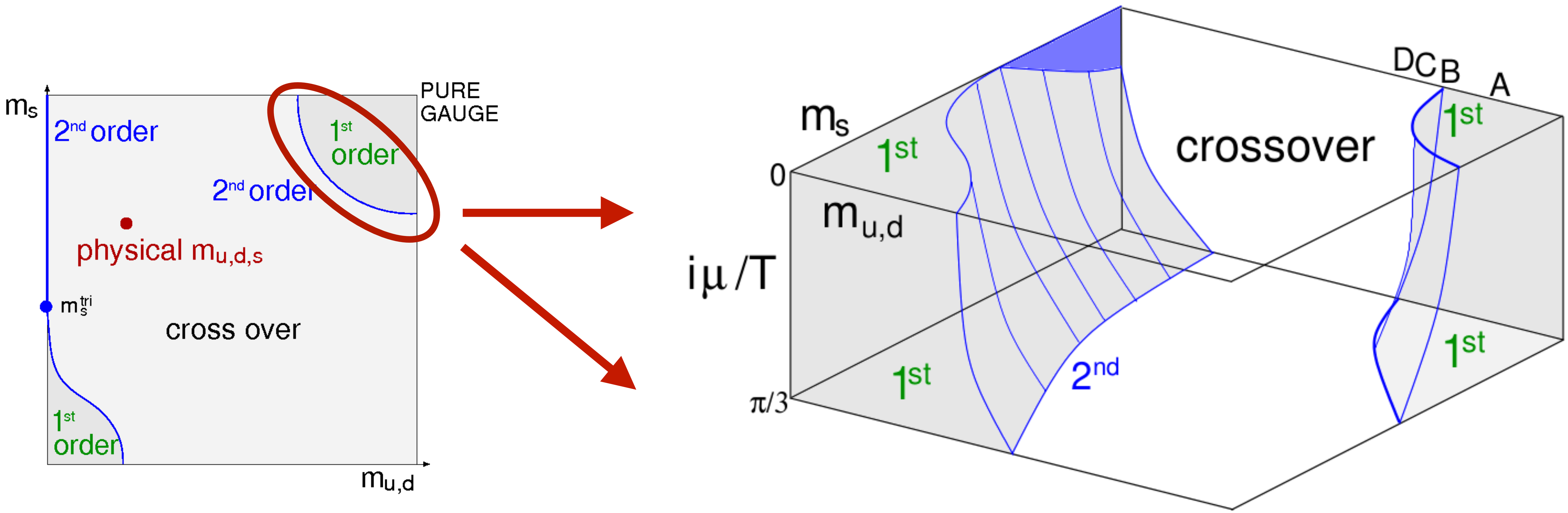
- Lattice-QCD
  - present: extrapolation
  - future: exact methods ?

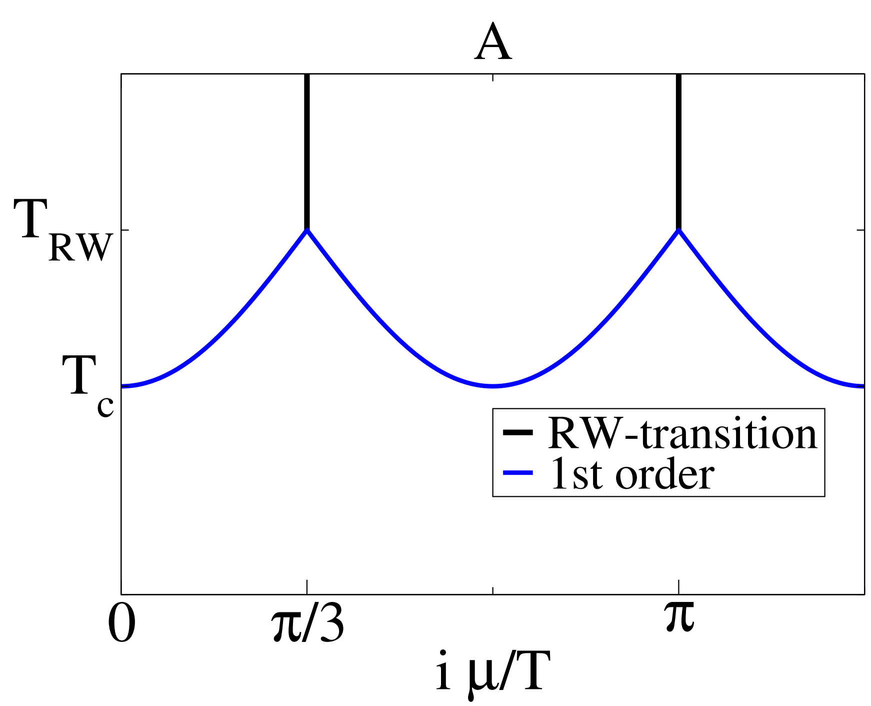
Is this happening ??
Maybe yes, maybe not..

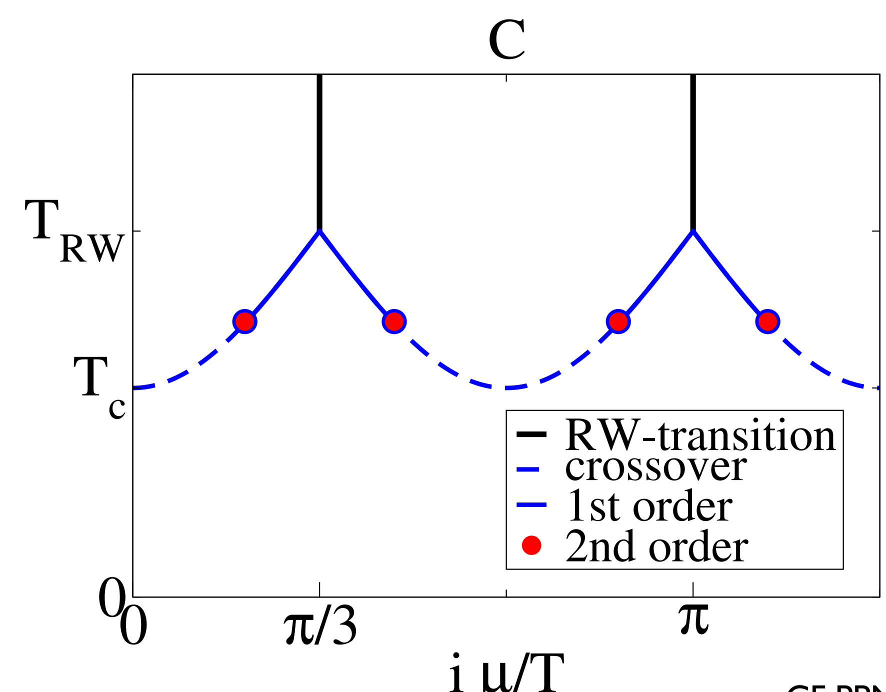
de Forcrand, Philipsen, JHEP 0811 (2008) 012; NPB 642 (2002) 290

- DSE/FRG
  - hardly high precision; typical errors 5-10%

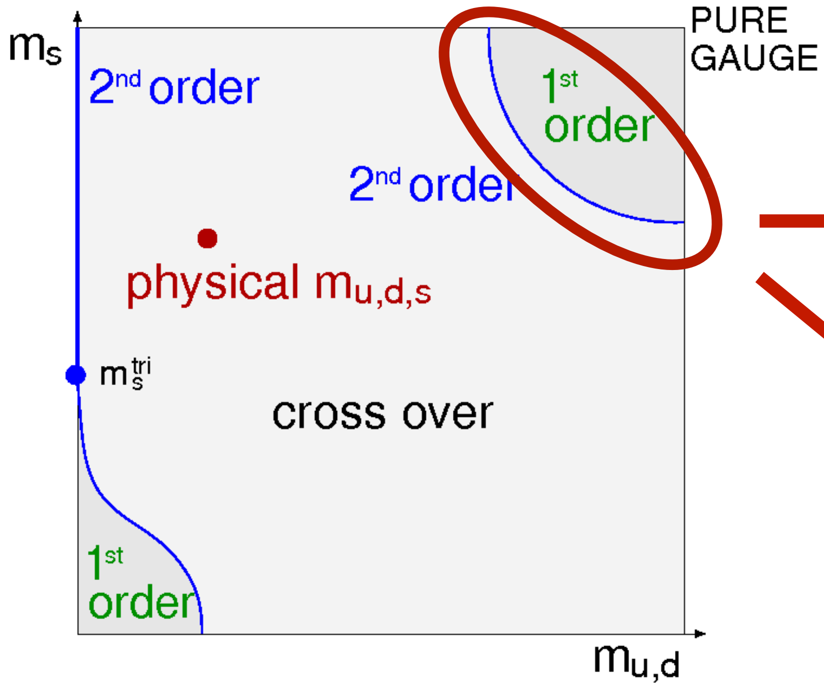
Critical line/surface for heavy quarks



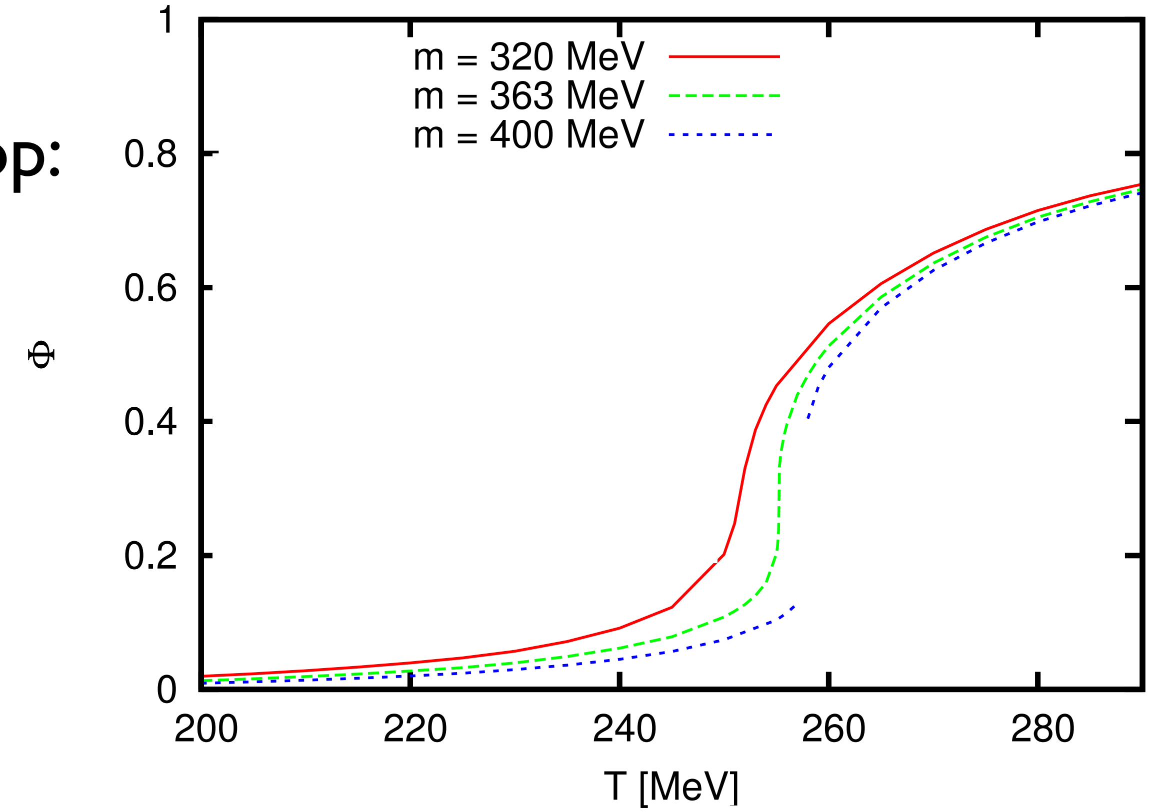


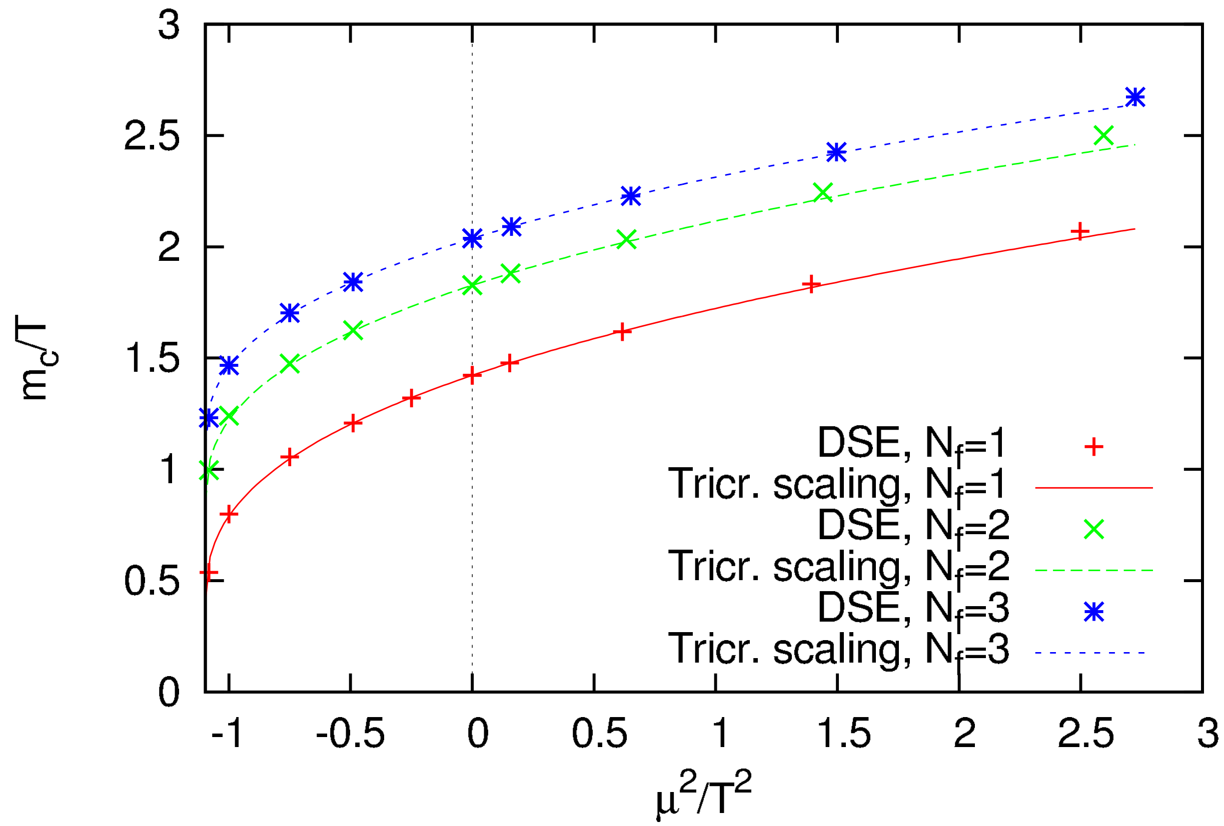


CF, PPNP 105 (2019) [1810.12938]



Polyakov Loop:



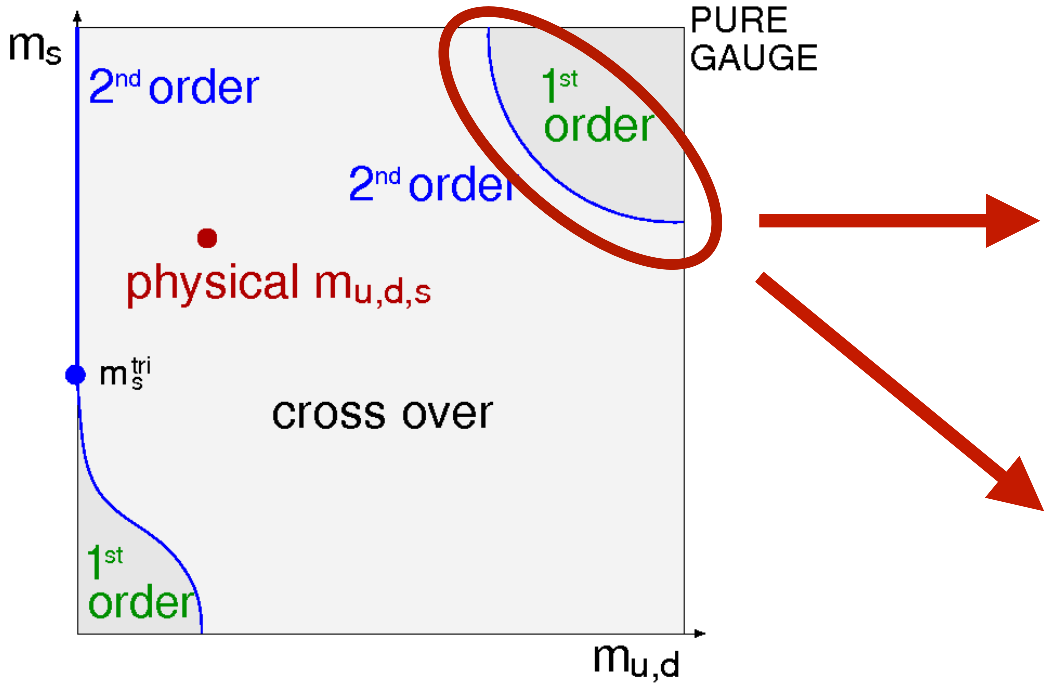


- Deconfinement transition in agreement with lattice QCD
- Correct tricritical scaling
- Roberge-Weiss-transition seen

Lattice:
Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012) 042

CF, Luecker, Pawlowski, PRD 91 (2015) 1

Critical line/surface for heavy quarks



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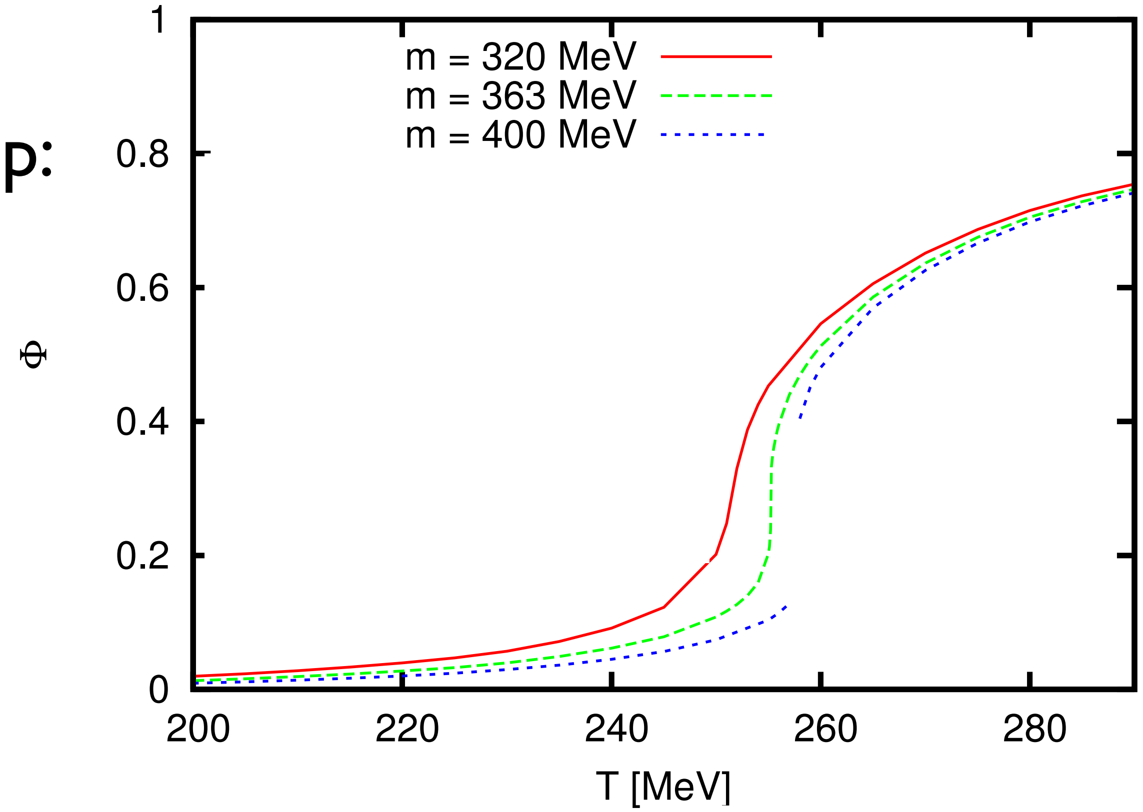

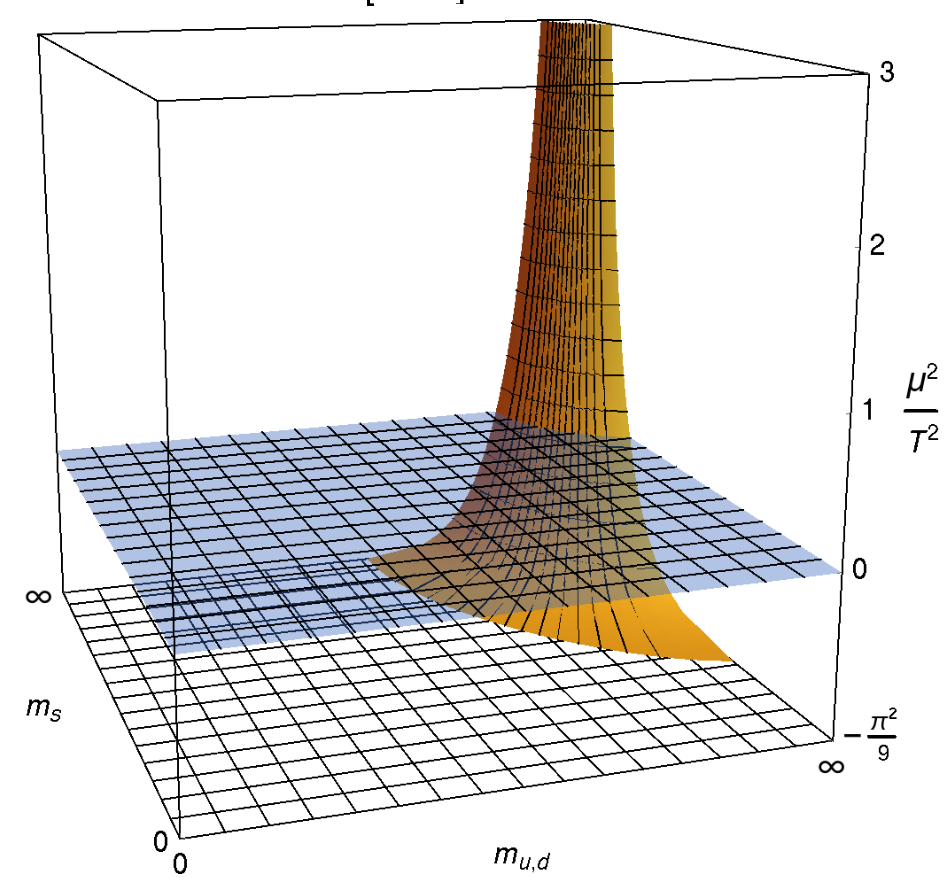

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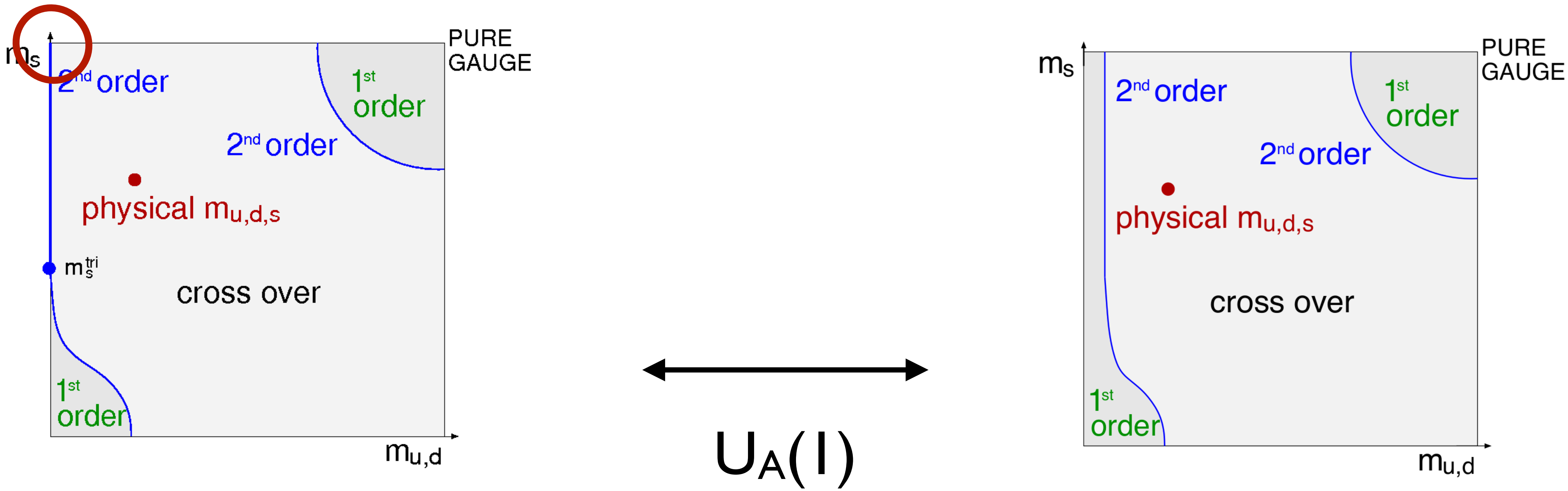
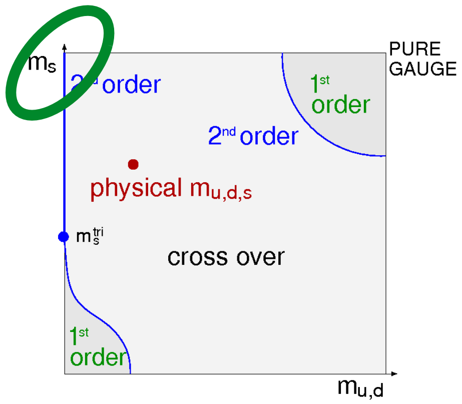
N_f=2 chiral limit

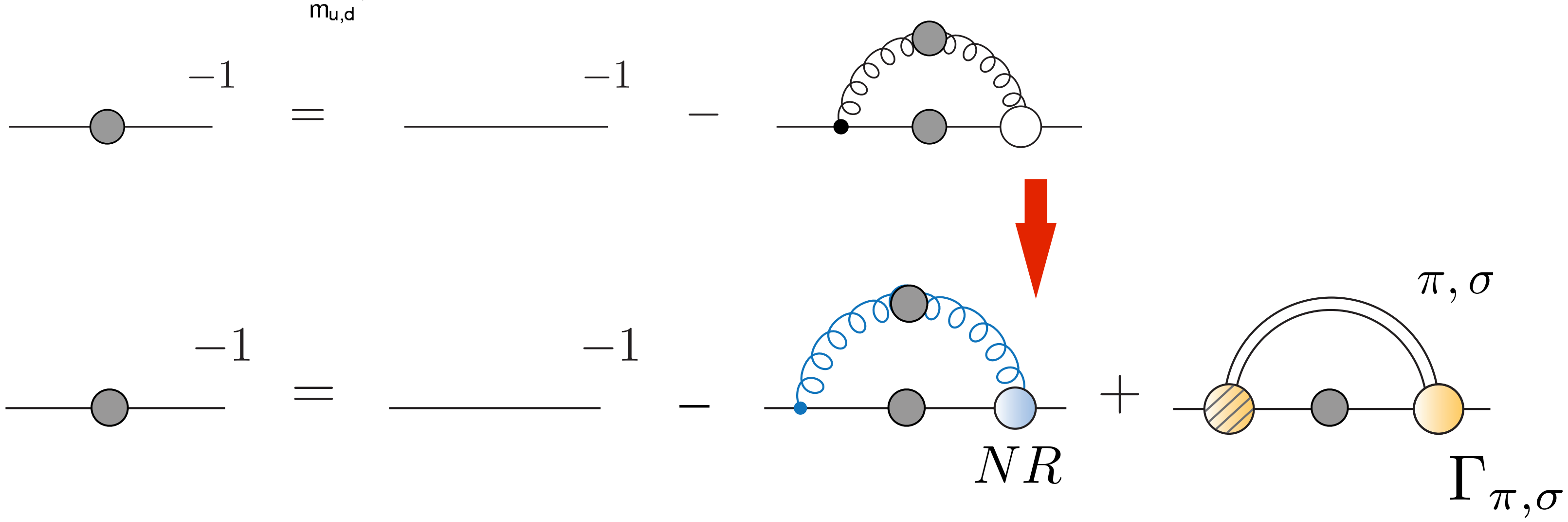


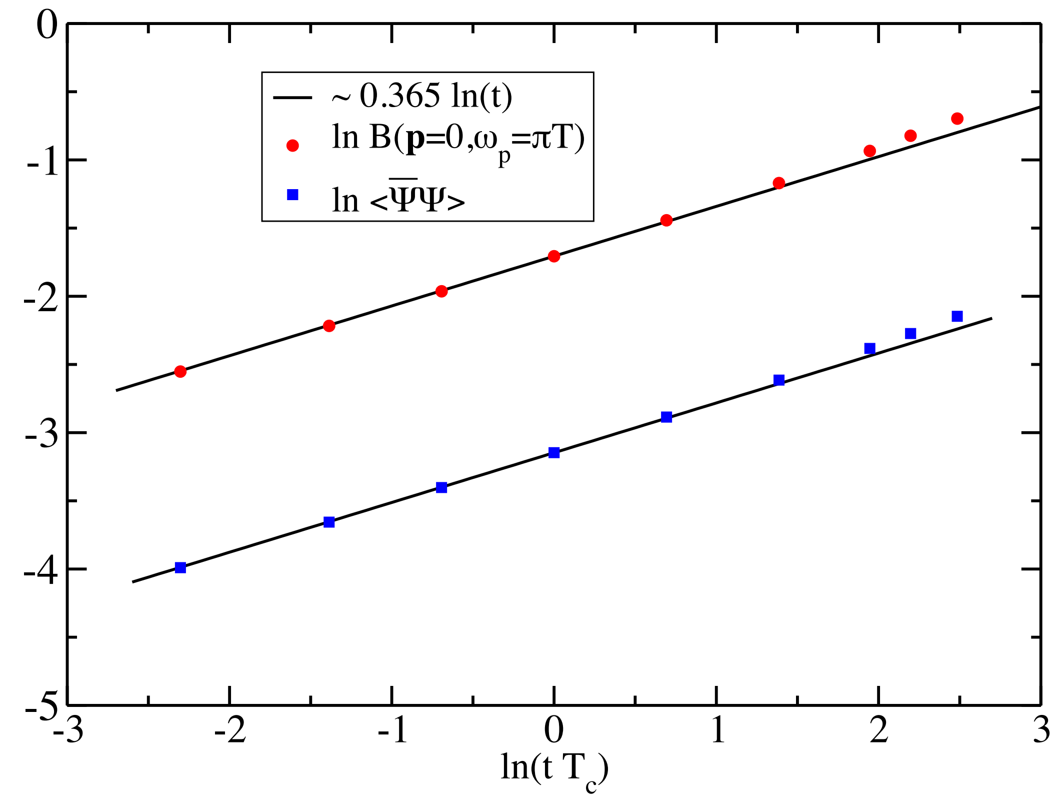
see e.g. Resch, Rennecke, Schaefer, PRD 99 (2019) 7

- N_f=2, chiral limit: phase transition dominated by Goldstone boson physics → (P)-Quark-Meson (QM) model
- N_f=3, chiral limit: don't know !

Critical scaling from DSEs: N_f=2, chiral limit







- T=0: meson contributions of order of 10-20 %

CF, Nickel, Williams EPJC 60 1434 (2008); CF, Williams, PRD 78 (2008) 074006

- T=T_c: meson contributions are dominant - universality !

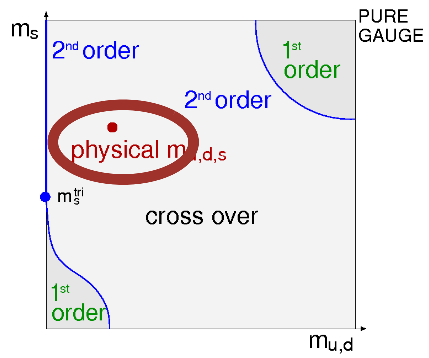
- Critical scaling: $\langle\bar{\Psi}\Psi\rangle(t) \sim B(t) \sim t^{\nu/2}$

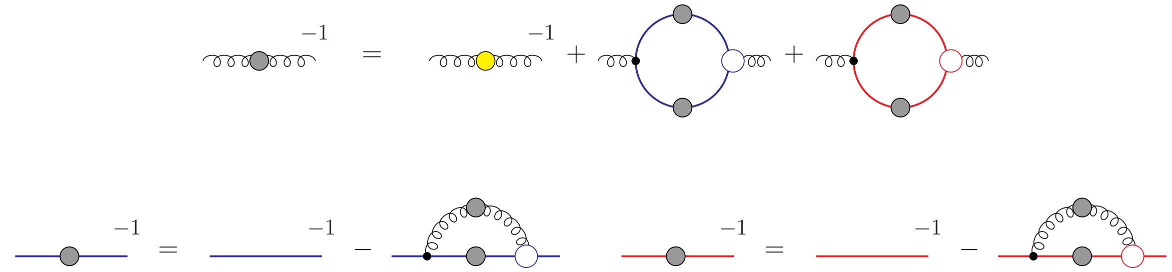
$$f^2_{\pi,s} \sim t^{\nu}$$

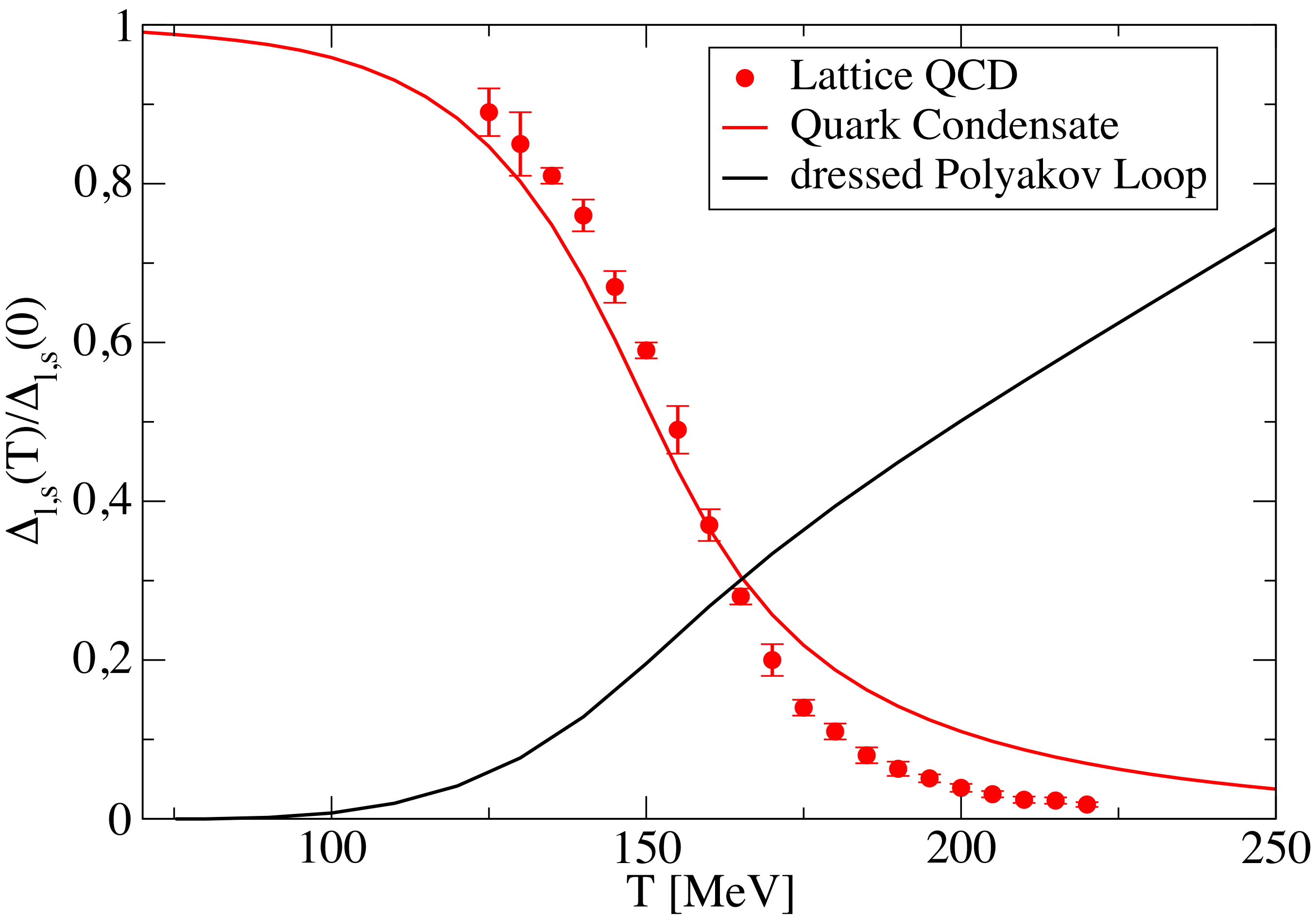
$$t := \frac{T_c - T}{T_c}$$

CF and Mueller, PRD 84 (2011) 054013

N_f=2+1, μ=0, physical point

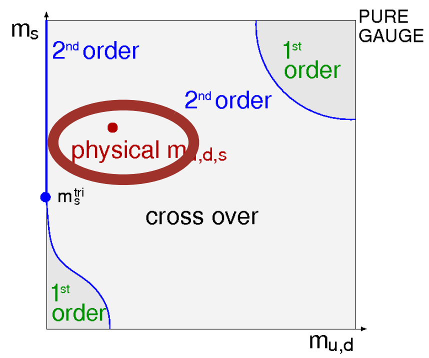


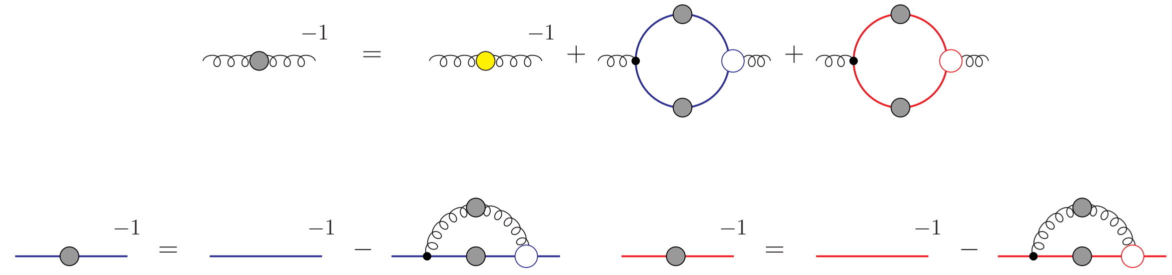


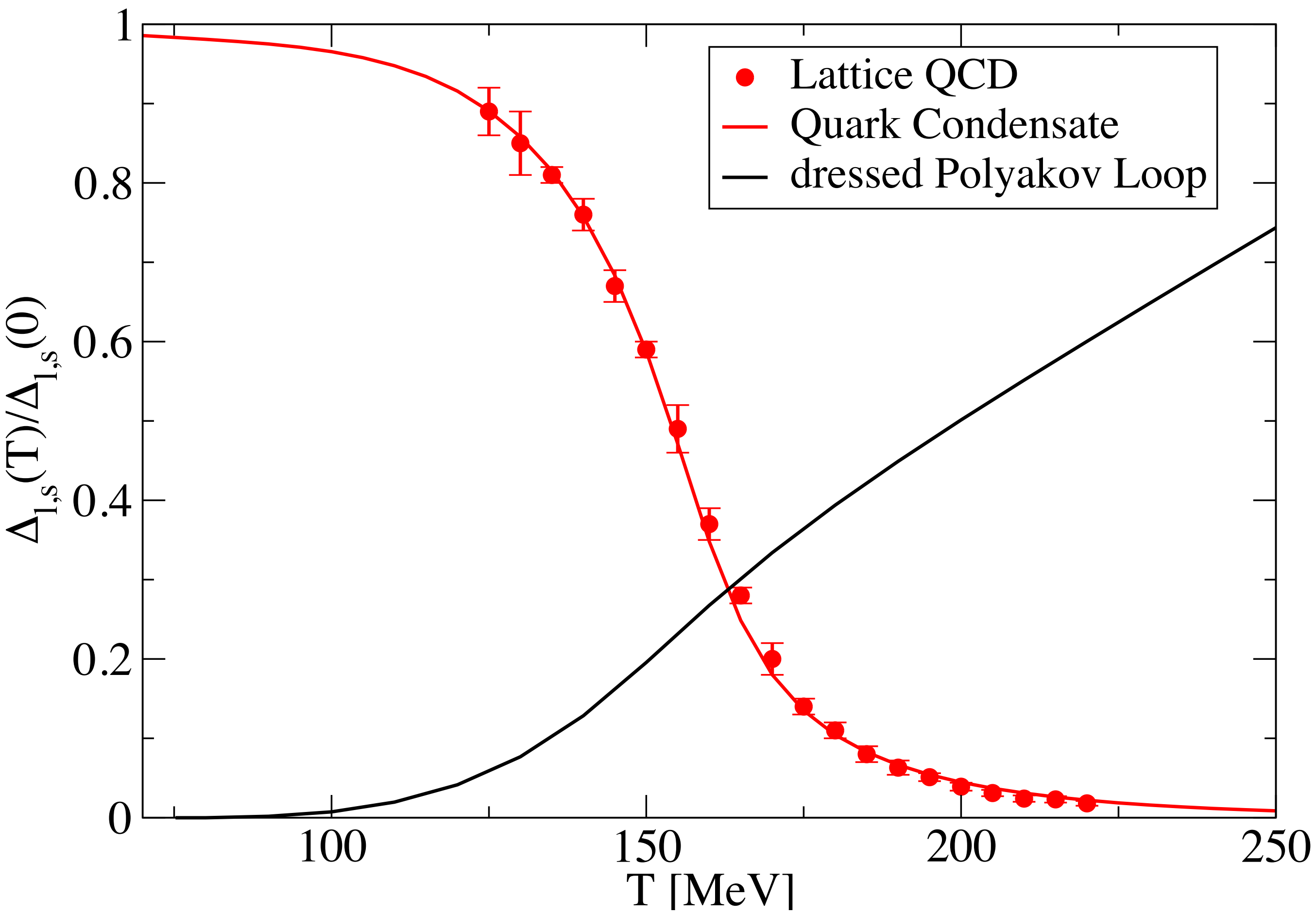


Lattice: Borsanyi *et al.* [Wuppertal-Budapest], JHEP 1009(2010) 073
DSE: CF, Luecker, PLB 718 (2013) 1036,
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

N_f=2+1, μ=0, physical point







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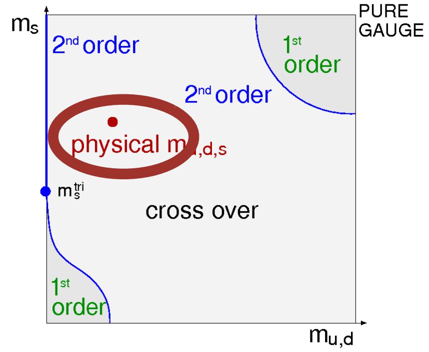
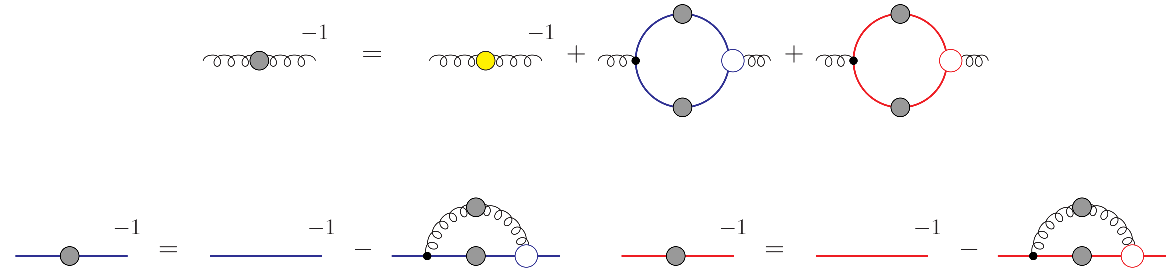
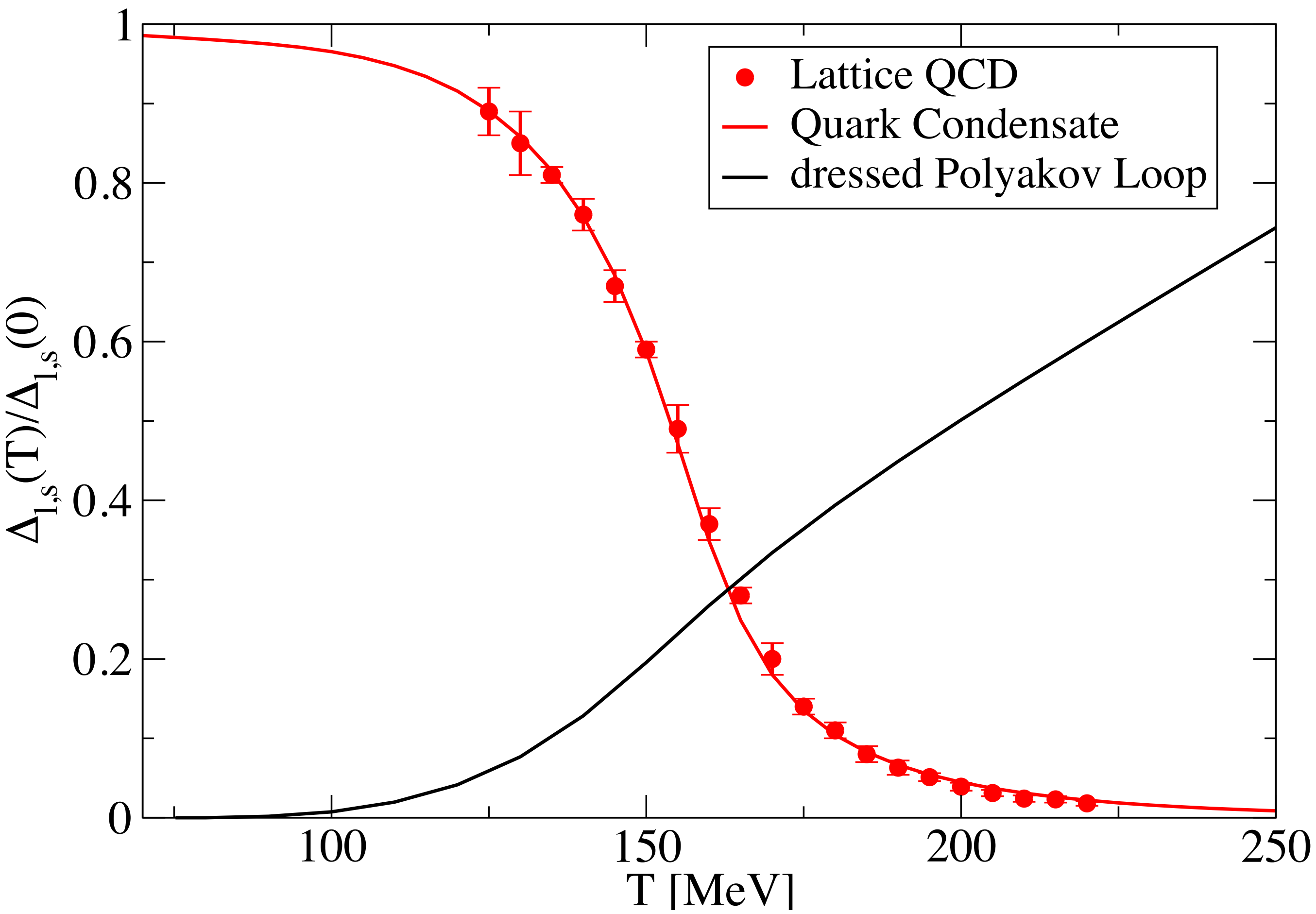
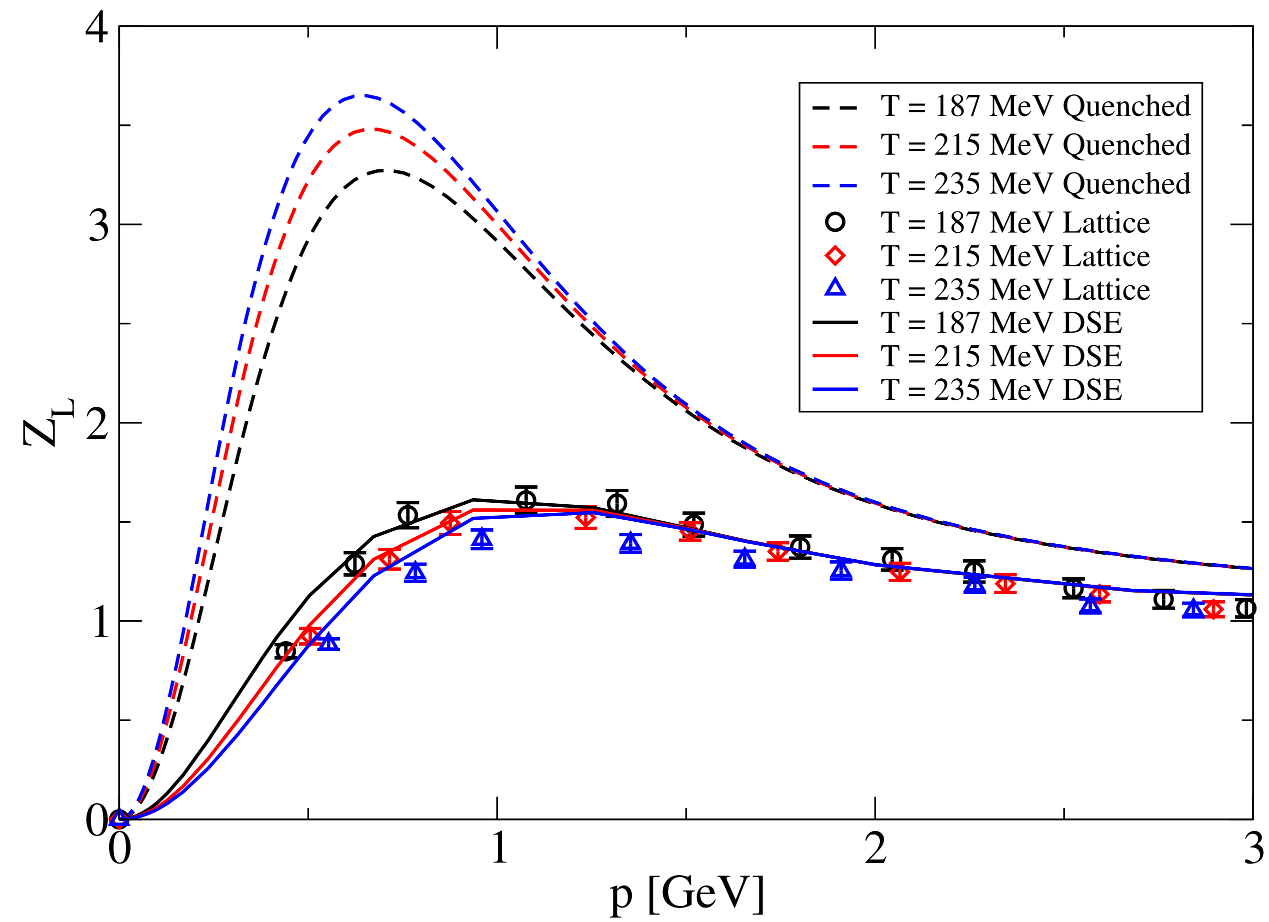






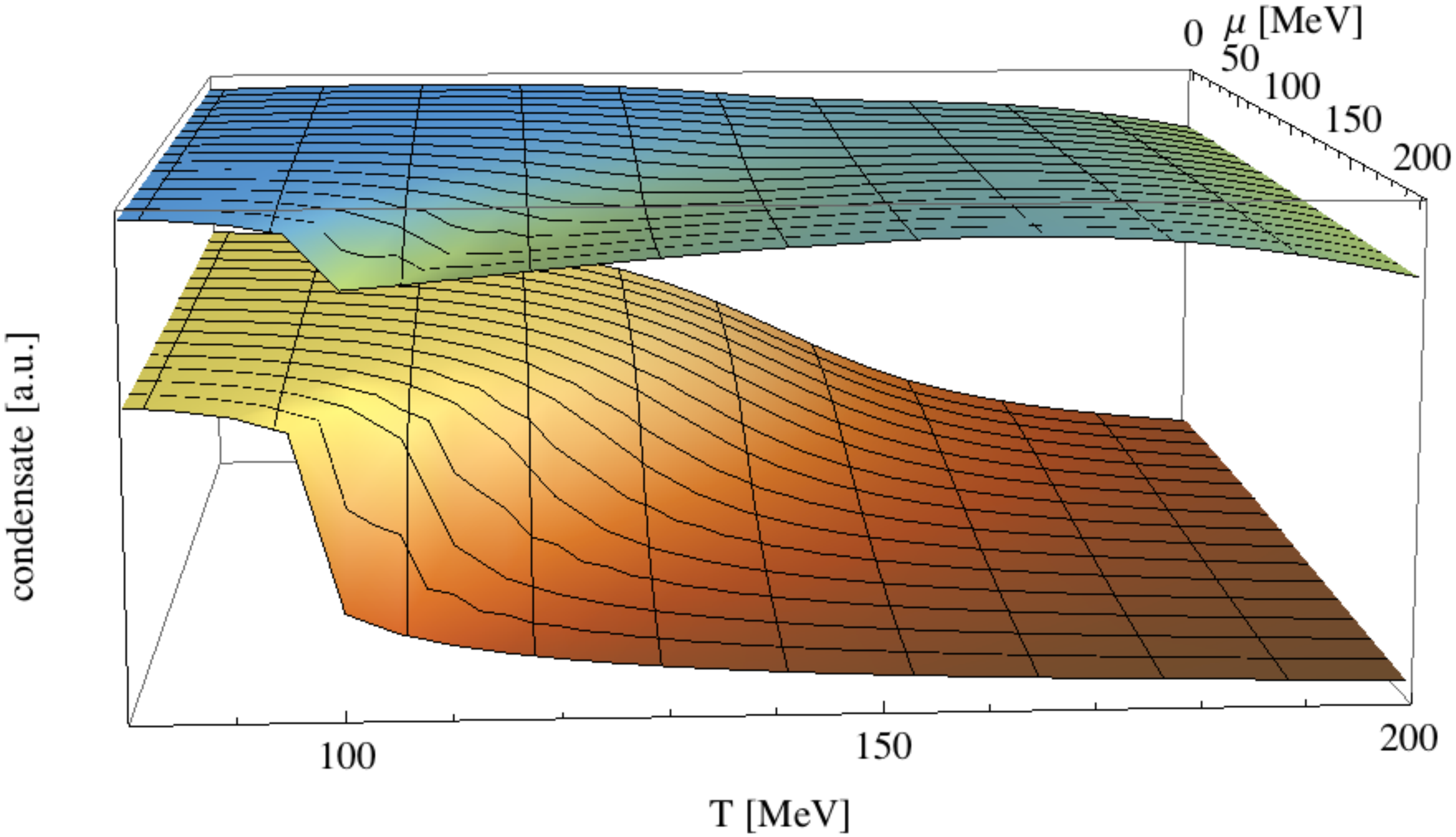


Lattice: Borsanyi *et al.* [Wuppertal-Budapest], JHEP 1009(2010) 073
DSE: CF, Luecker, PLB 718 (2013) 1036,
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

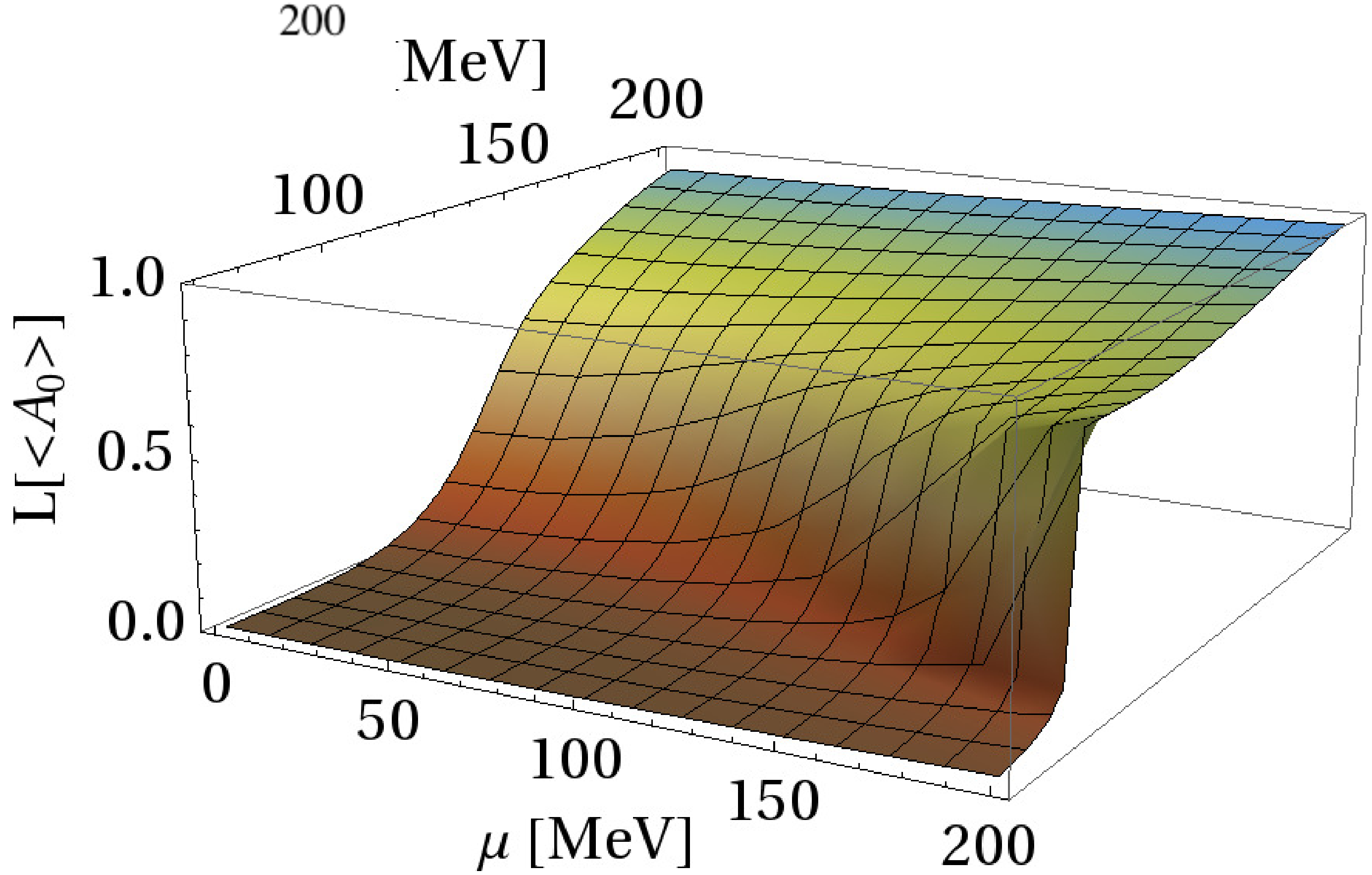
Lattice: Aouane, et al.PRD D87 (2013), [arXiv:1212.1102]
DSE: CF, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]

- quantitative agreement: DSE prediction verified by lattice

Nf=2+1: Condensate and dressed Polyakov Loop



Quark condensate

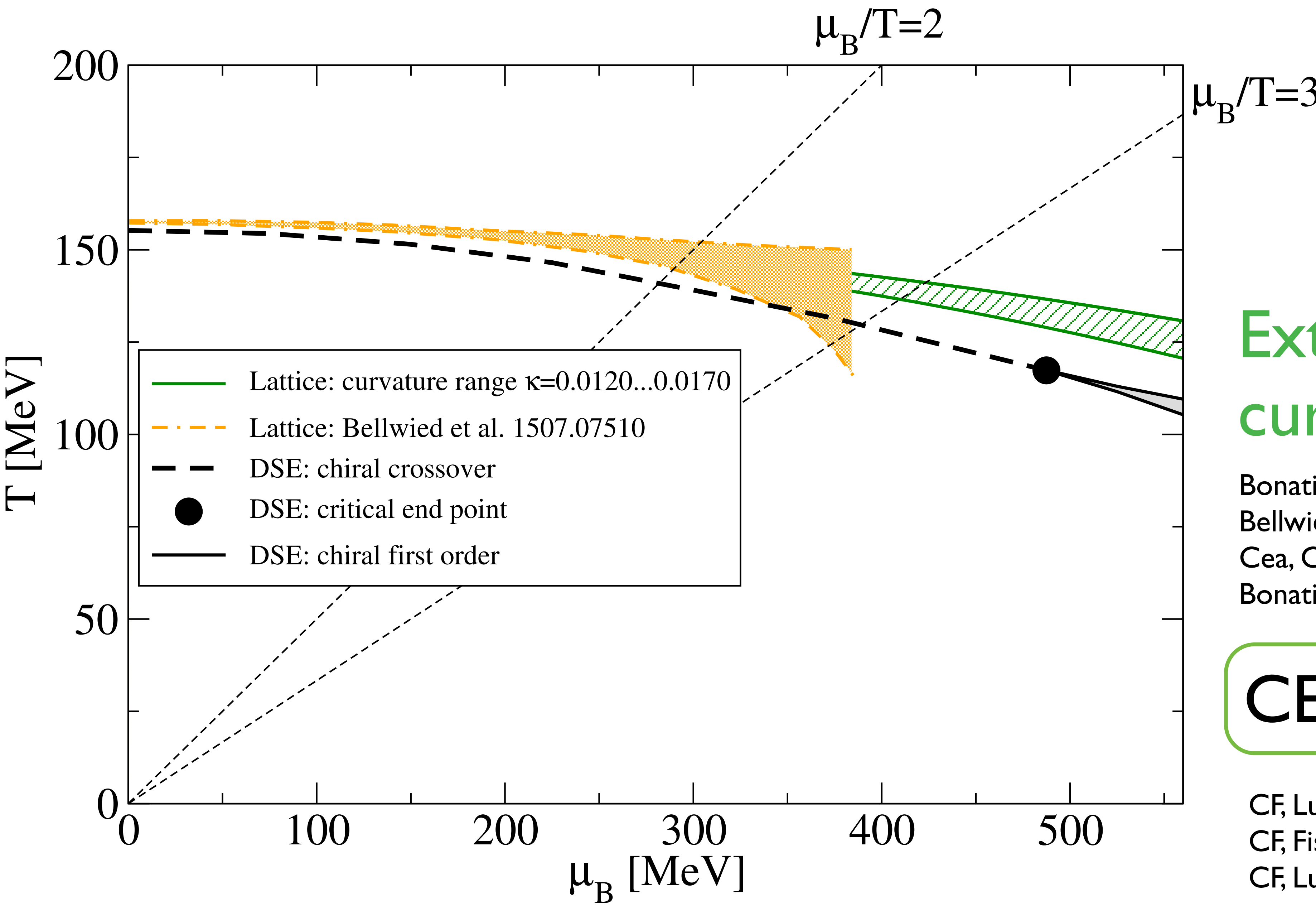


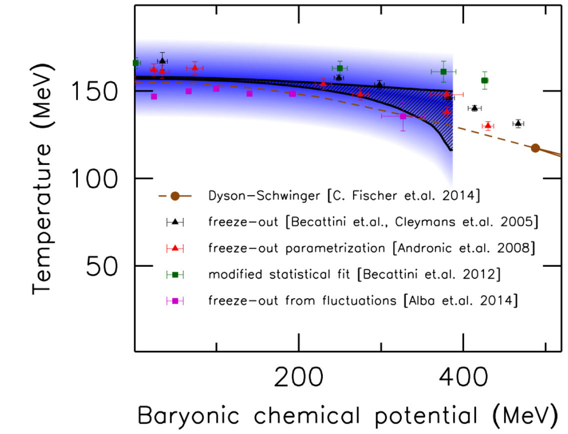
Polyakov-Loop

$$L = \frac{1}{N_c} tr\ e^{ig \int A_0}$$

CF, Fister, Luecker, Pawlowski, PLB 732 (2014) 273

N_f=2+1: phase diagram





Extrapolated curvature from lattice

Bonati et al., PRD 92 (2015) 054503
Bellwied et al. PLB 751 (2015) 559
Cea, Cosmai, Papa, PRD 89 (2014), PRD 93 (2016)
Bonati et al., arXiv:1805.02960

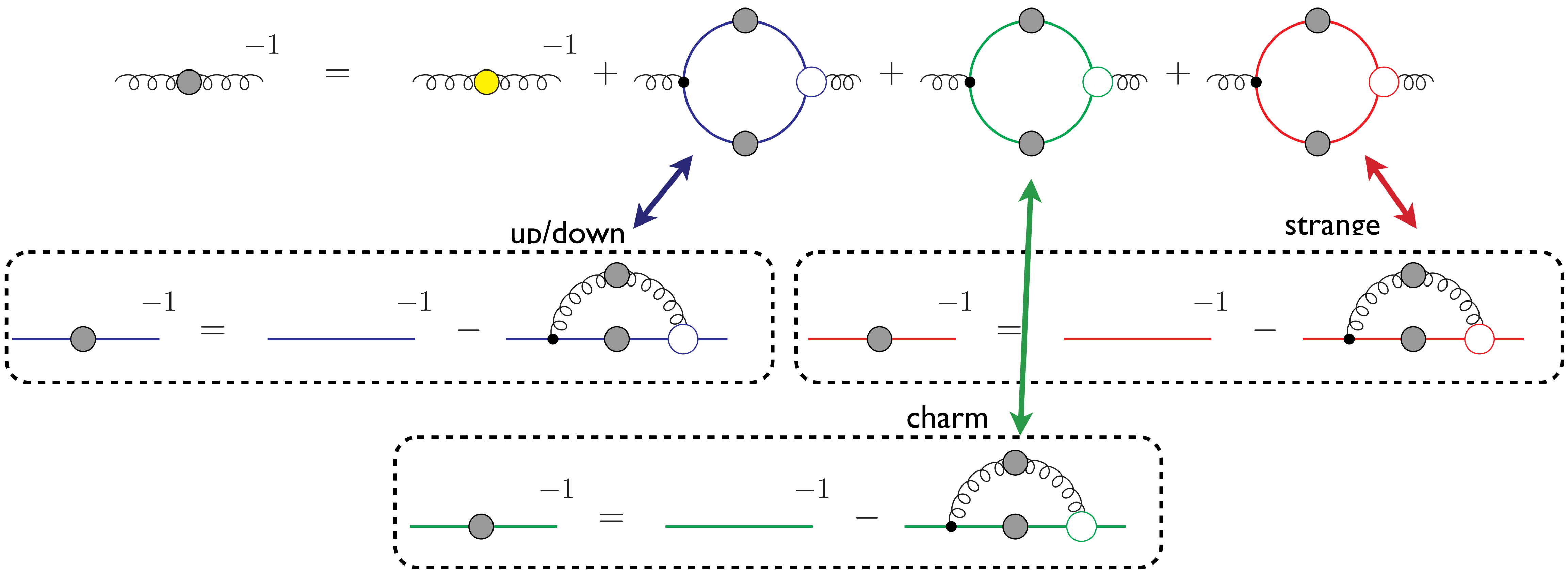
CEP at large μ

CF, Luecker, PLB 718 (2013) 1036,
CF, Fister, Luecker, Pawlowski, PLB 732 (2014) 273
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

- what about truncation error ? how stable is this result ??
  - N_f=2+1+1
  - baryon and meson effects ?
  - crosscheck with FRG

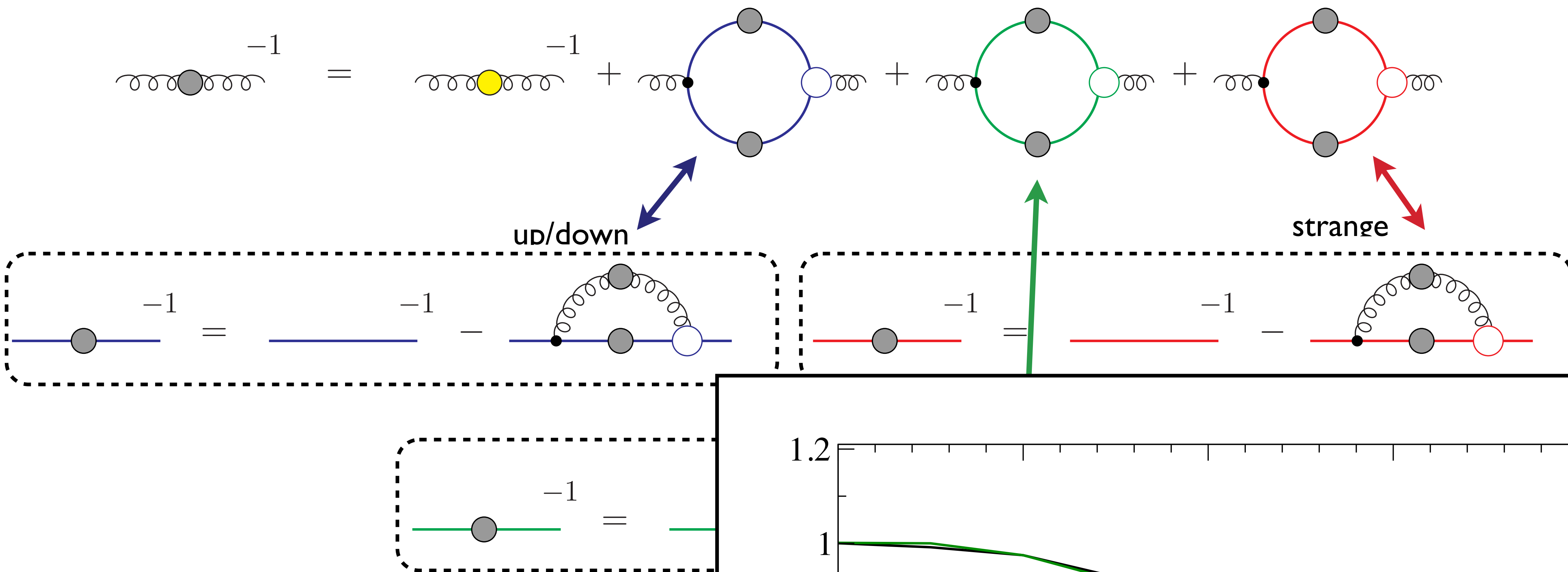
Fu, Pawlowski, Rennecke, PRD 101 (2020) 5
Gao, Pawlowski, PRD 102 (2020) 3, 034027, PLB 820 (2021) 136584
and references therein…

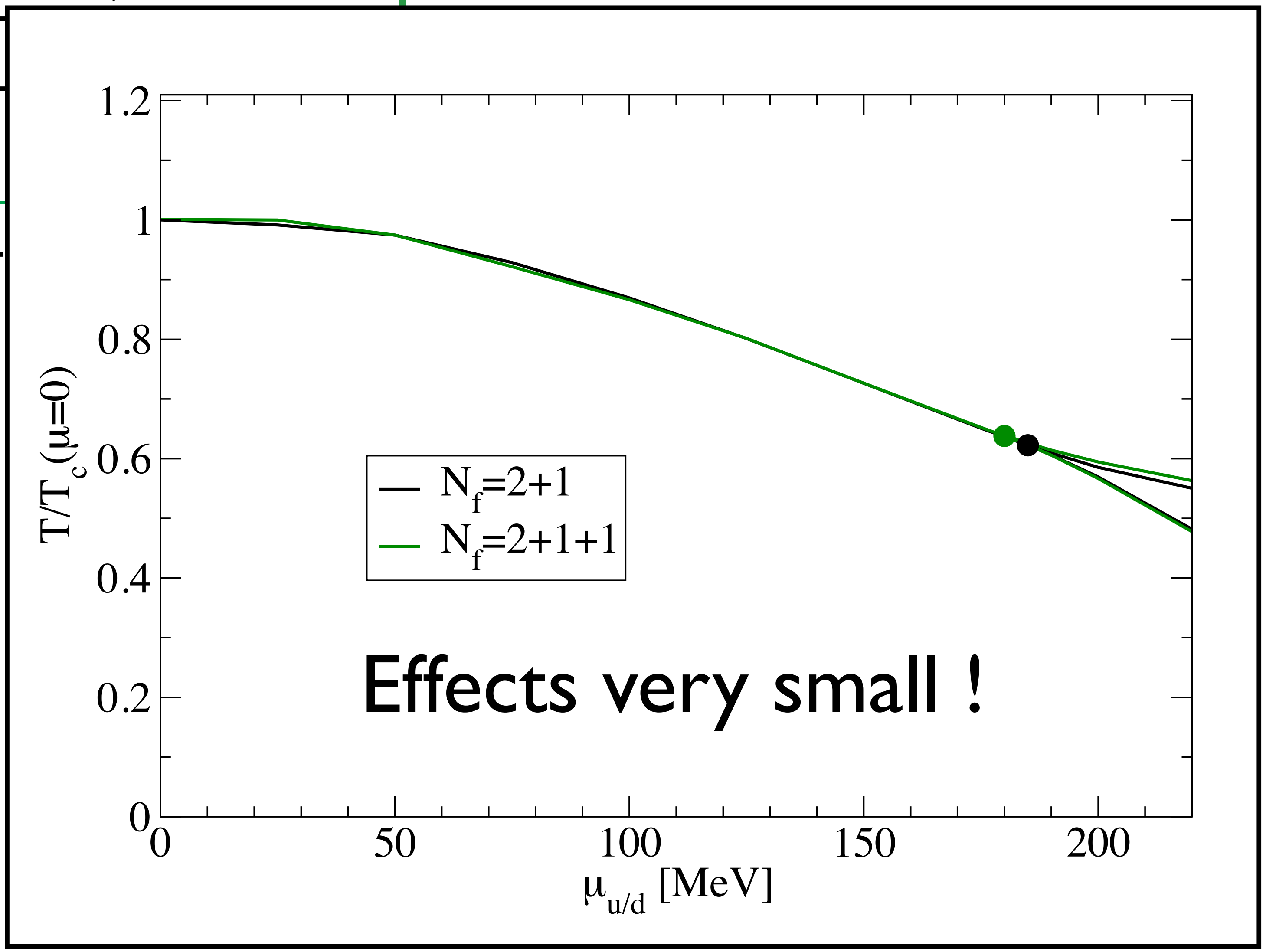
Nf=2+1+1: effects of charm



- Physical up/down, strange and charm quark masses
- Transition controlled by chiral dynamics
- *no lattice or model results available yet*

N_f=2+1+1: effects of charm

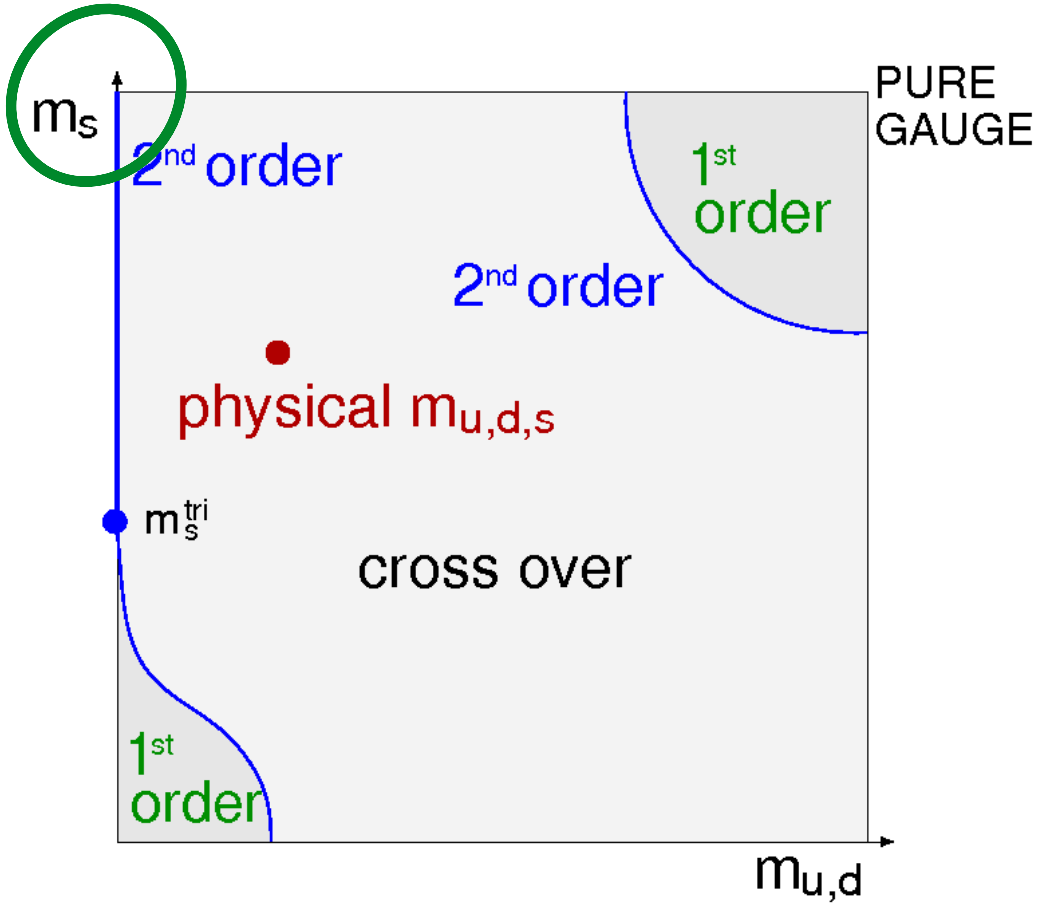


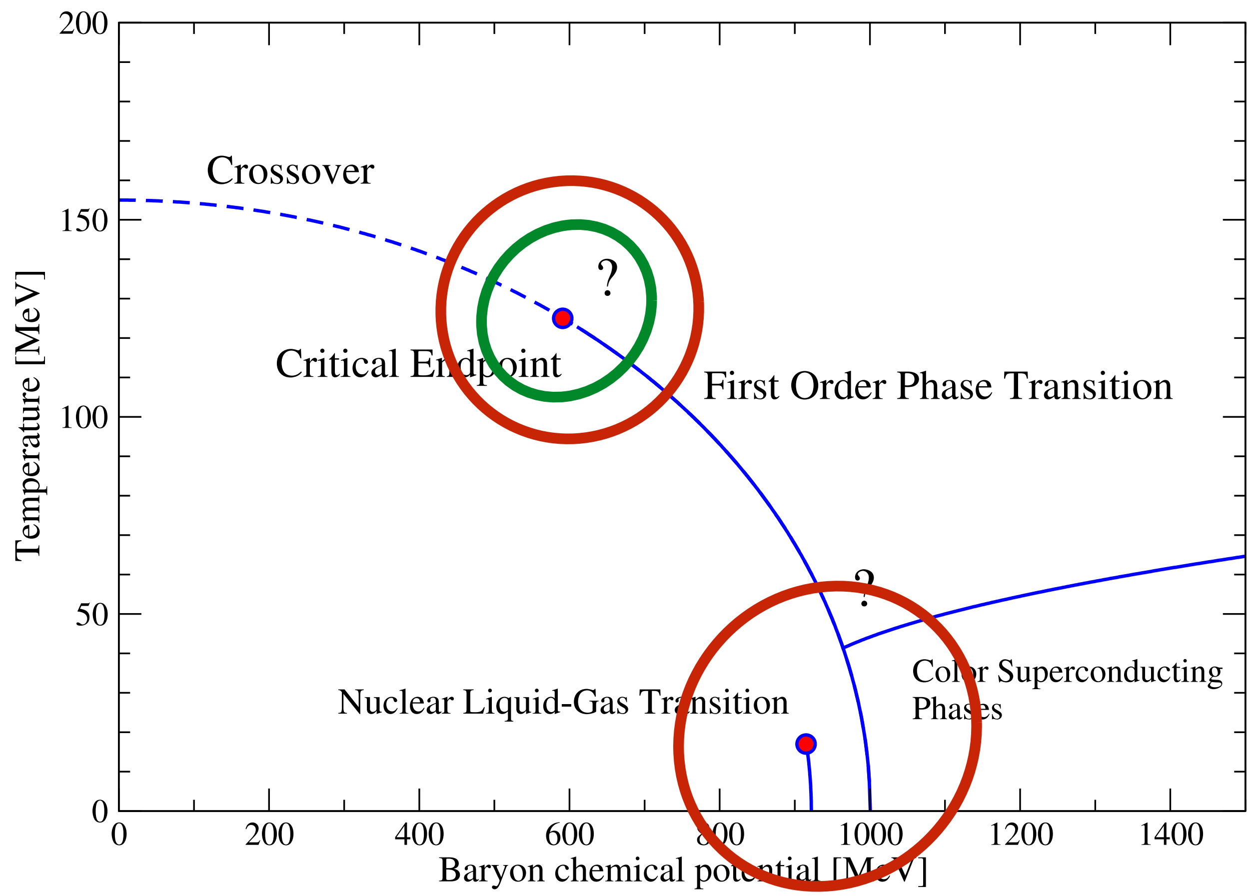


- Physical up/down, strange and charm quark masses
- Transition controlled by chiral dynamics
- *no lattice or model results available yet*

CF, Luecker, Welzbacher, PRD 90 (2014) 034022

Hadron effects in the QCD phase diagram



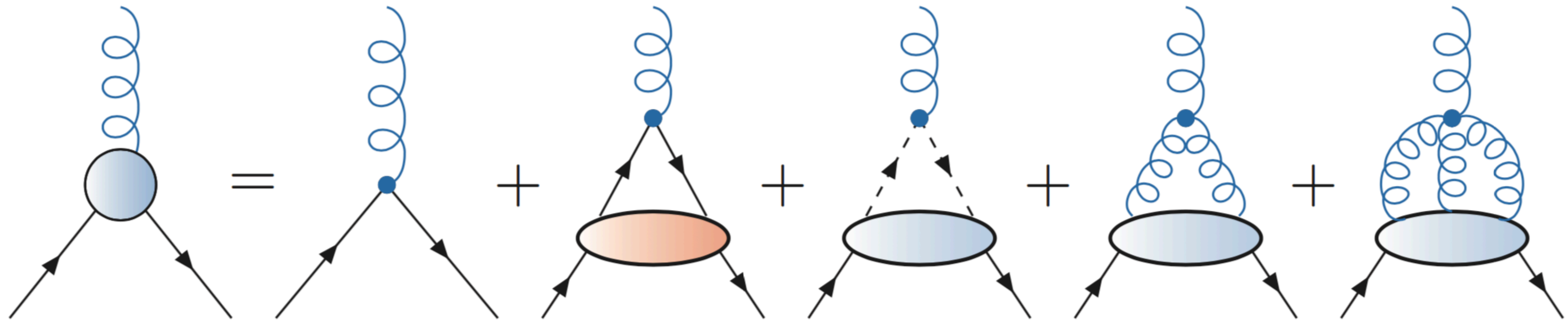


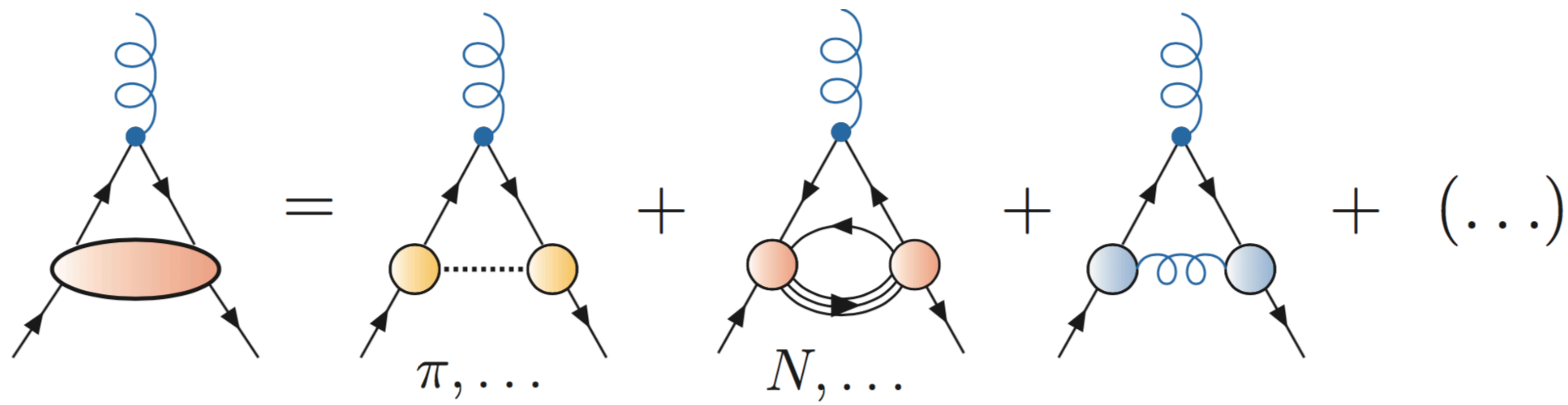
- Meson effects: critical chiral physics, …
- Baryon effects Chiral mirror model: Weyrich, Strodthoff and von Smekal, PRC 92 (2015) no.1, 015214

⟶ truncation of DSEs good enough to include these effects ?

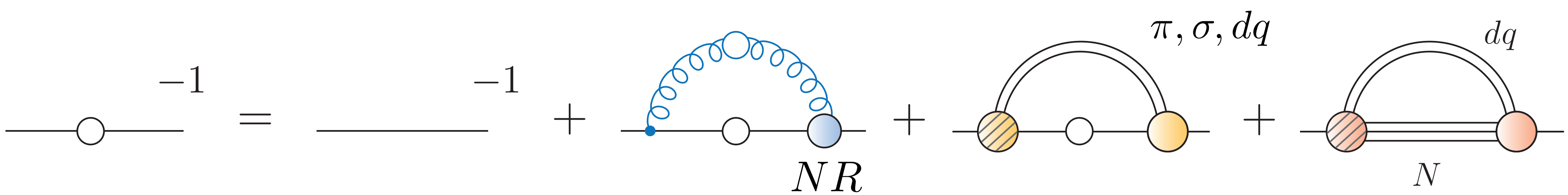
Hadron effects in quark-gluon interaction

quark-gluon vertex:





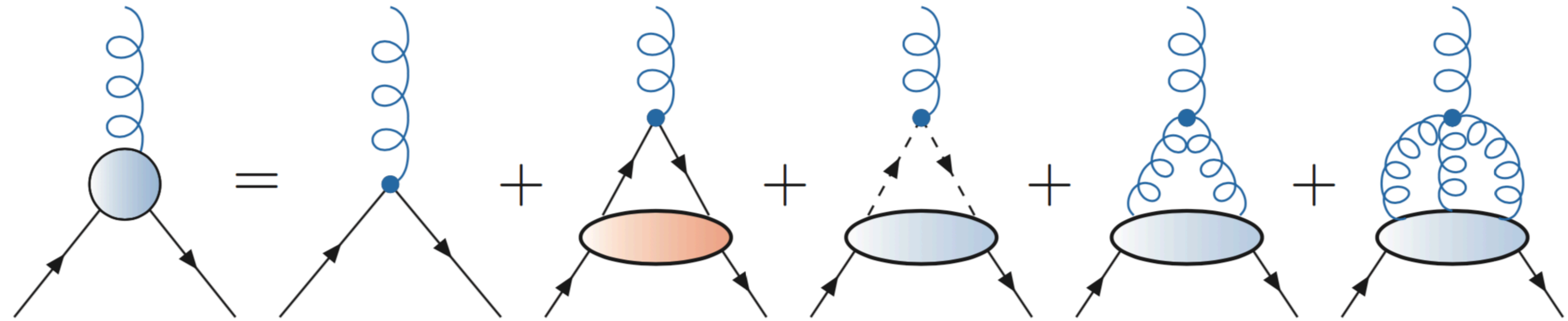
quark:

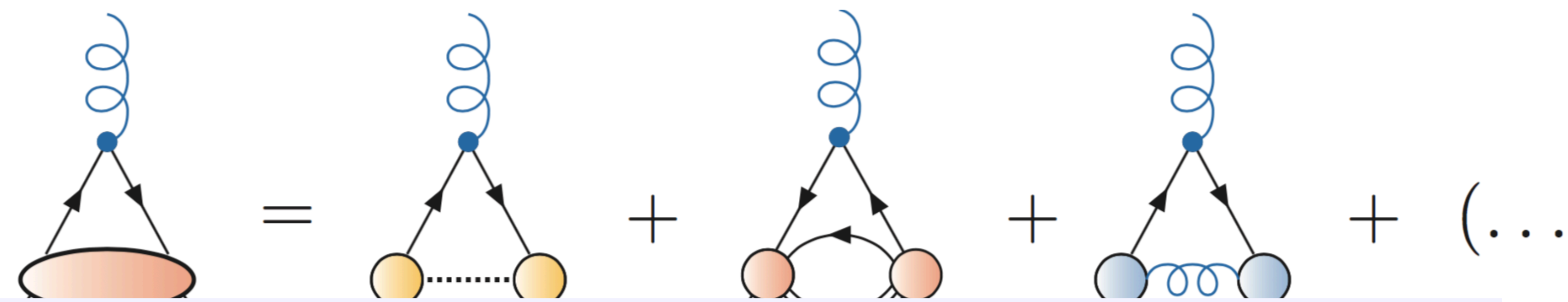


Eichmann, CF, Welzbacher, PRD93 (2016) [1509.02082]

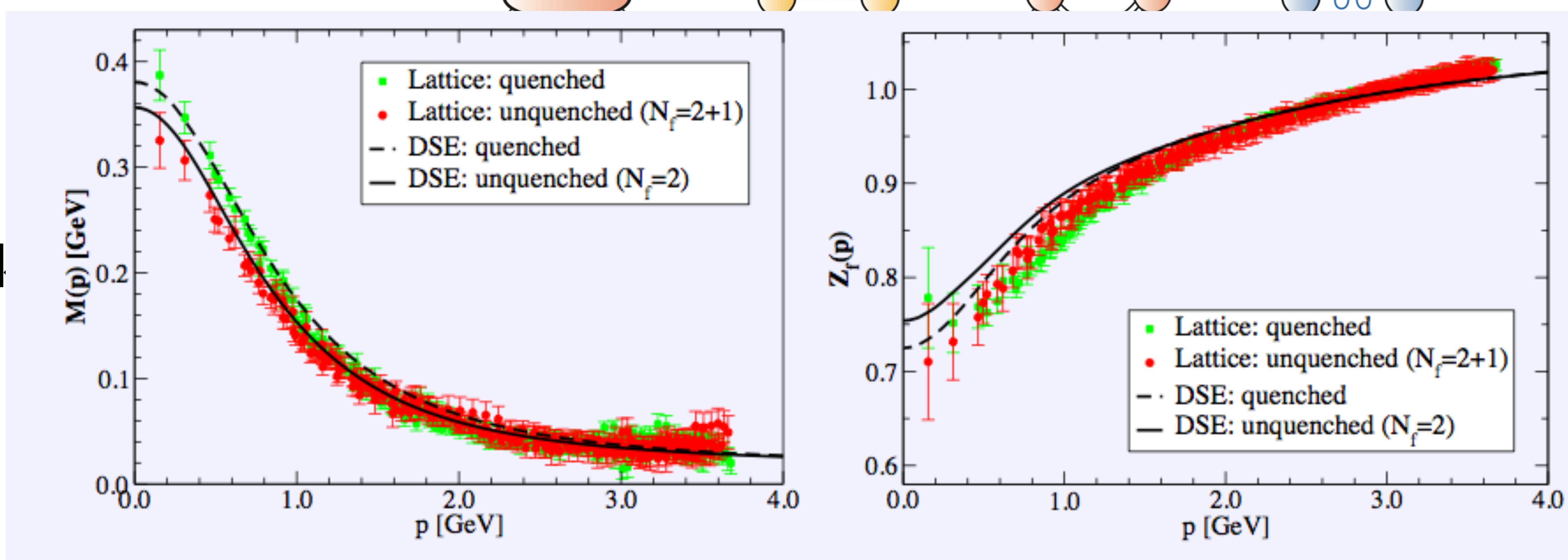
Hadron effects in quark-gluon interaction

quark-gluon vertex:



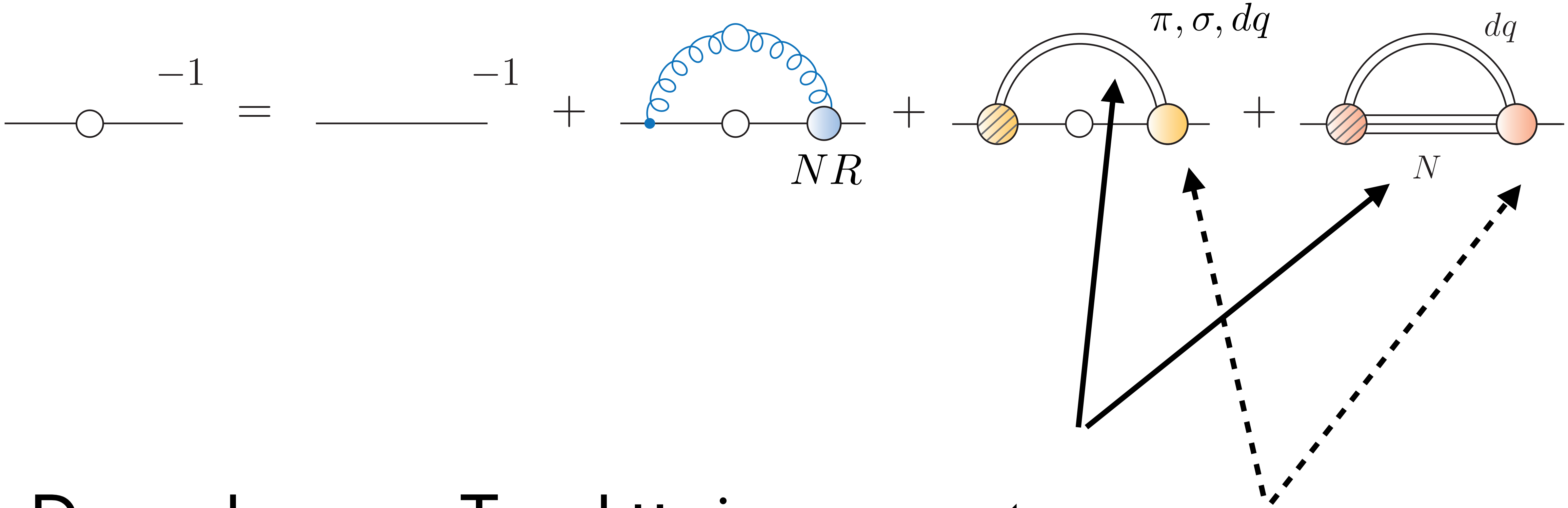


quarl

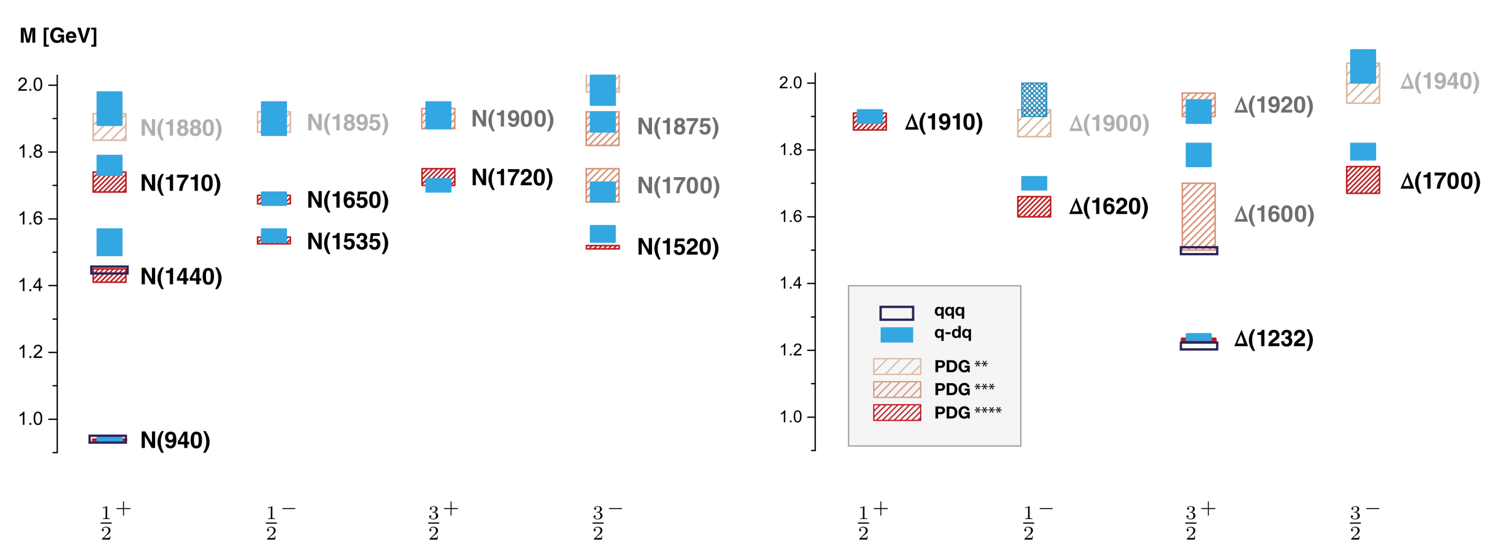


CF, D. Nickel and R. Williams, EPJC **60**, 1434 (2008)

2]

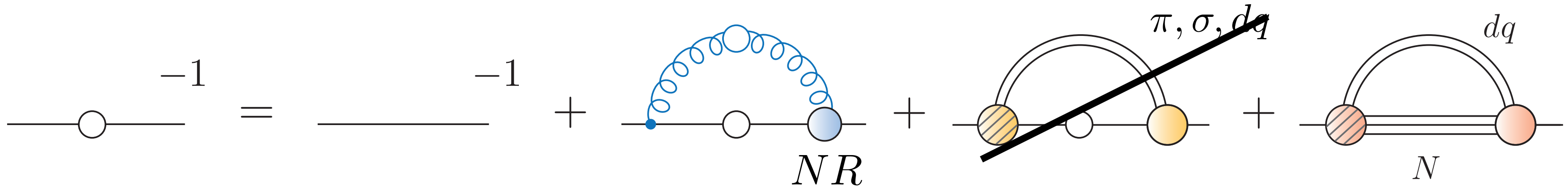


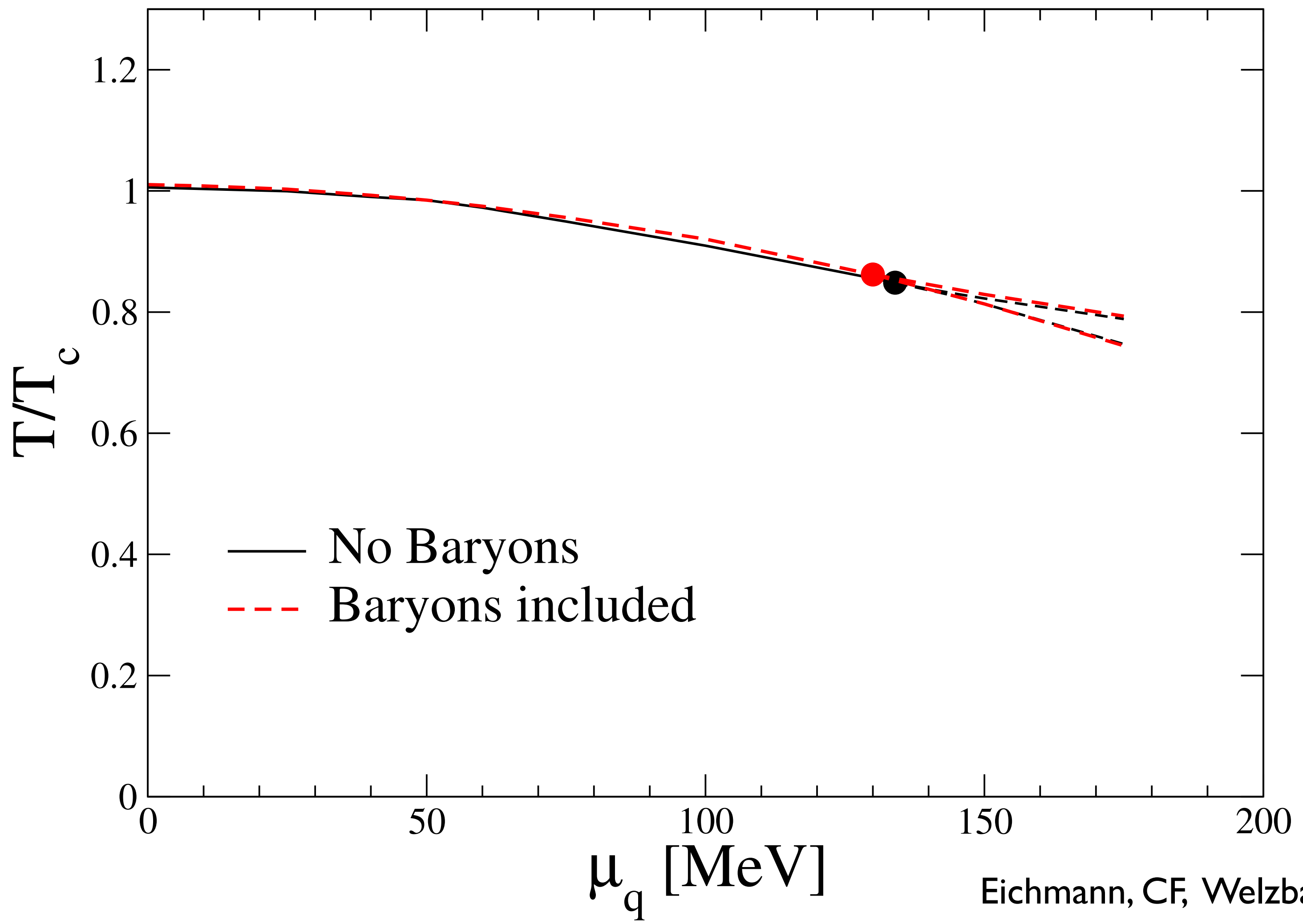
- Dependence on T and μ via -propagators
  -wave functions
- Baryons: exploratory calculation: use wave functions from T=μ=0



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

Baryon effects on the CEP - results (N_f=2)





Eichmann, CF, Welzbacher, PRD93 (2016) [1509.02082]

- Small chemical potential: no effect
- almost no effect on location of CEP

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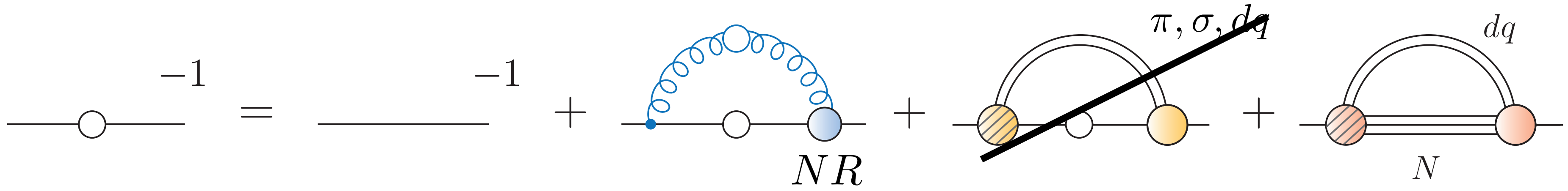
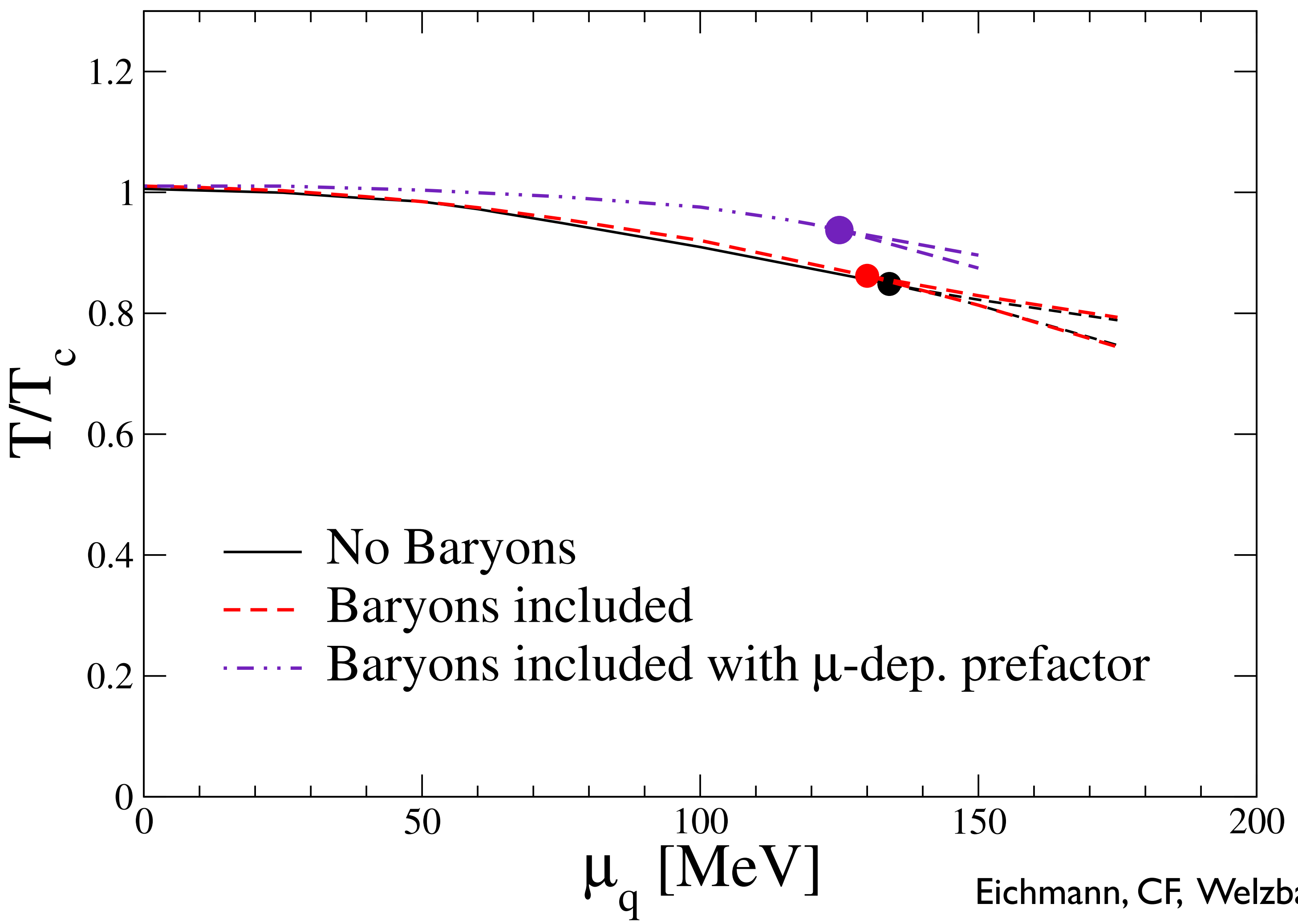
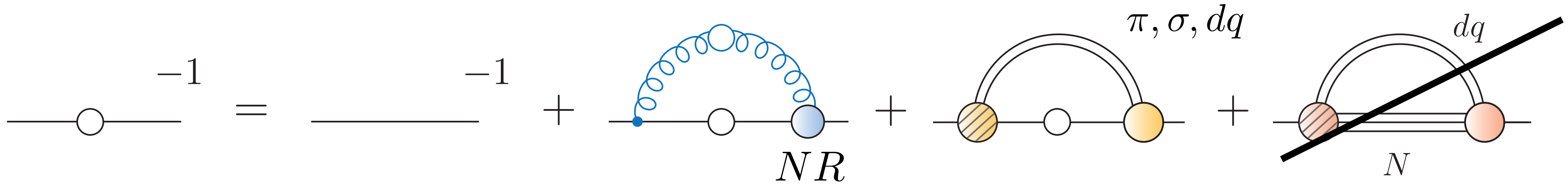


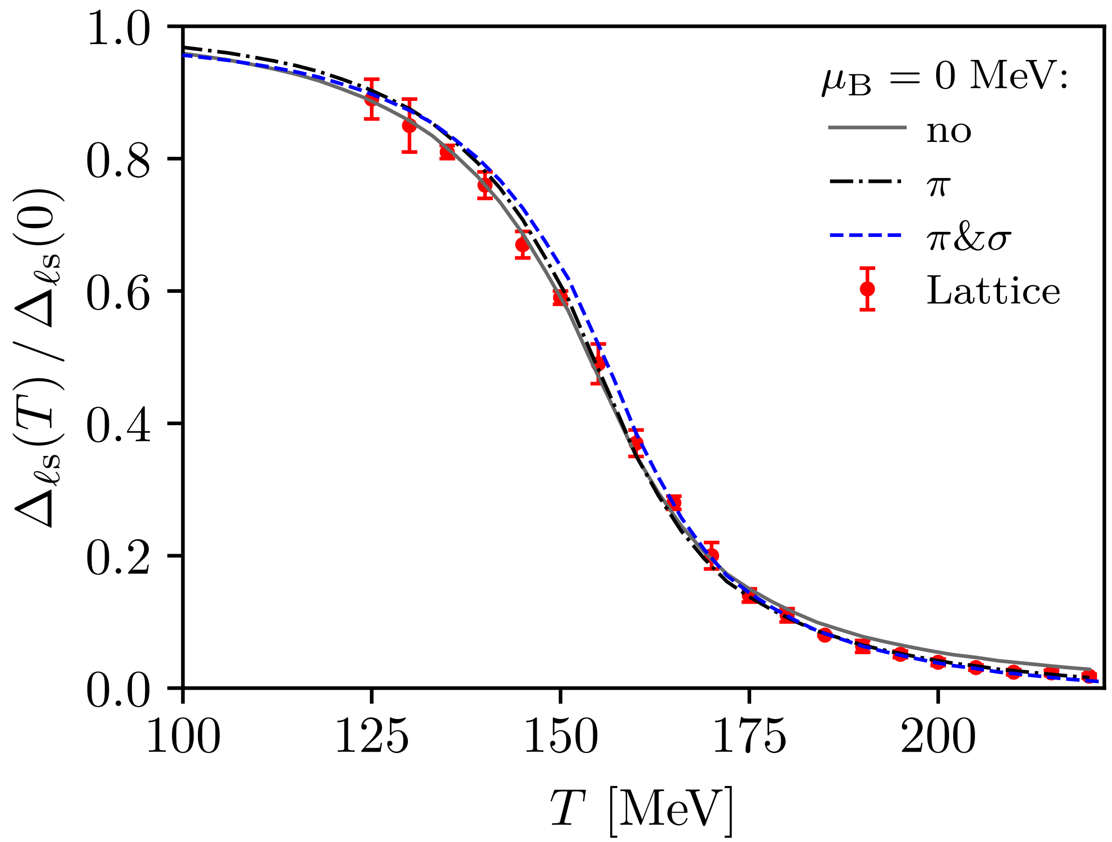


Eichmann, CF, Welzbacher, PRD93 (2016) [1509.02082]

- Small chemical potential: no effect
- almost no effect on location of CEP
- But: strong μ-dependence of baryon wave function may change situation…

Meson effects on the CEP - results (N_f=2+1)

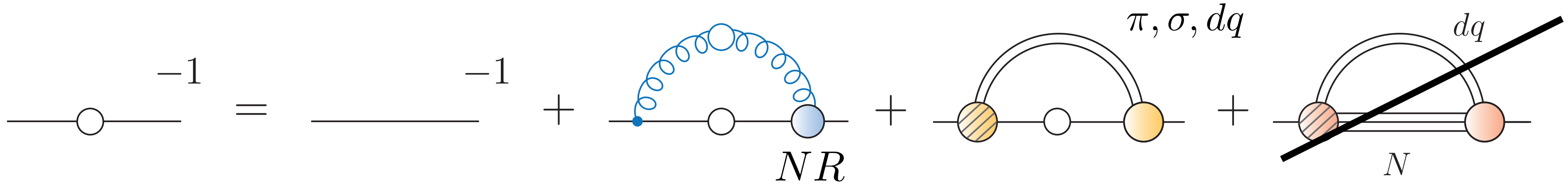


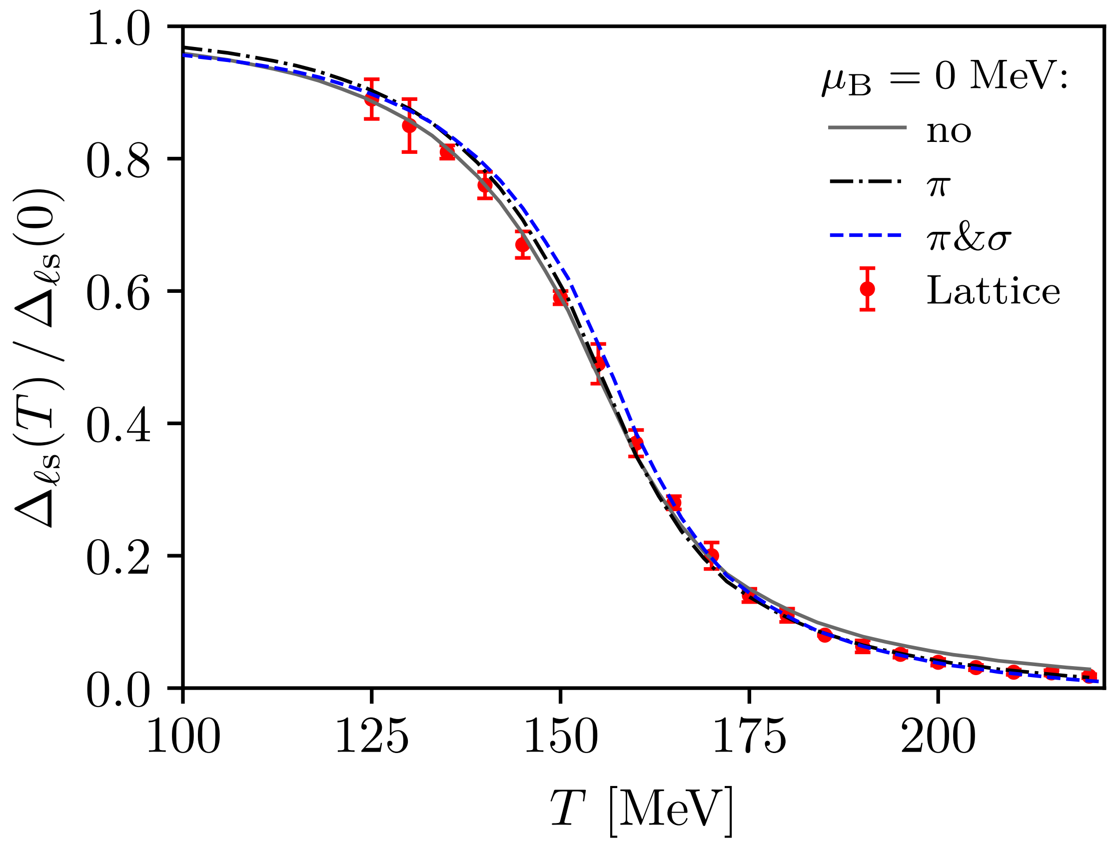


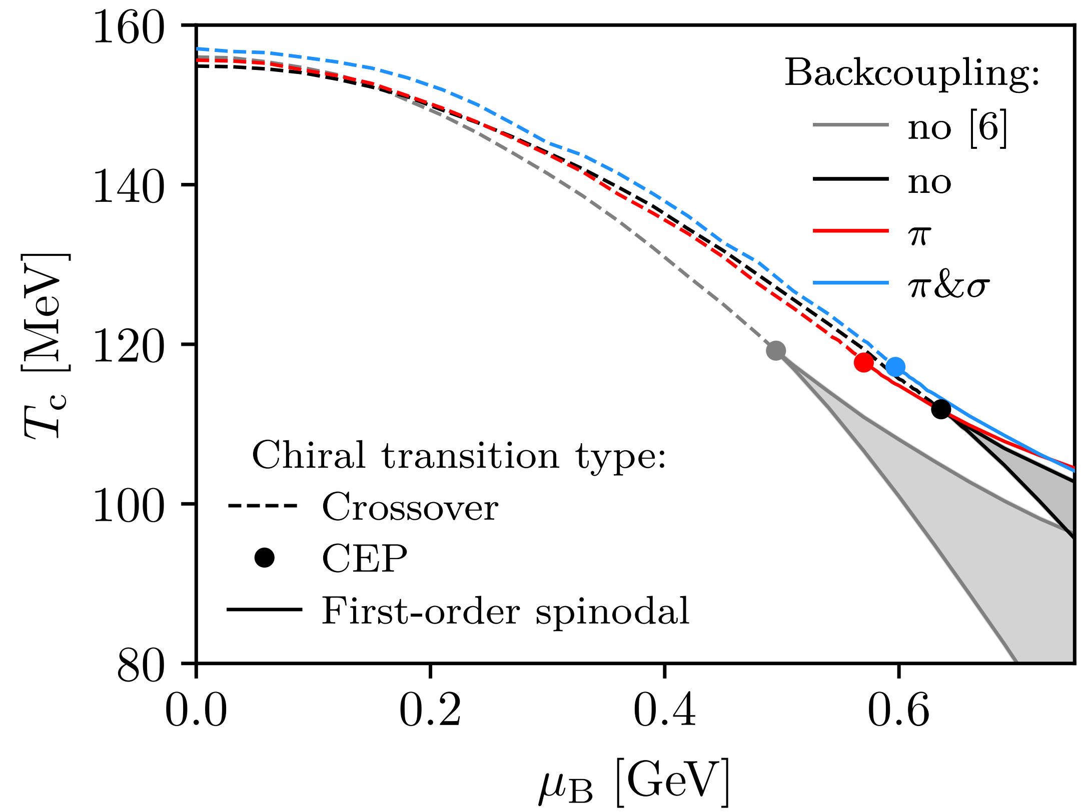
Gunkel, CF, PRD 104 (2021) [2106.08356]

- Vanishing chemical potential: no effect

Meson effects on the CEP - results (N_f=2+1)



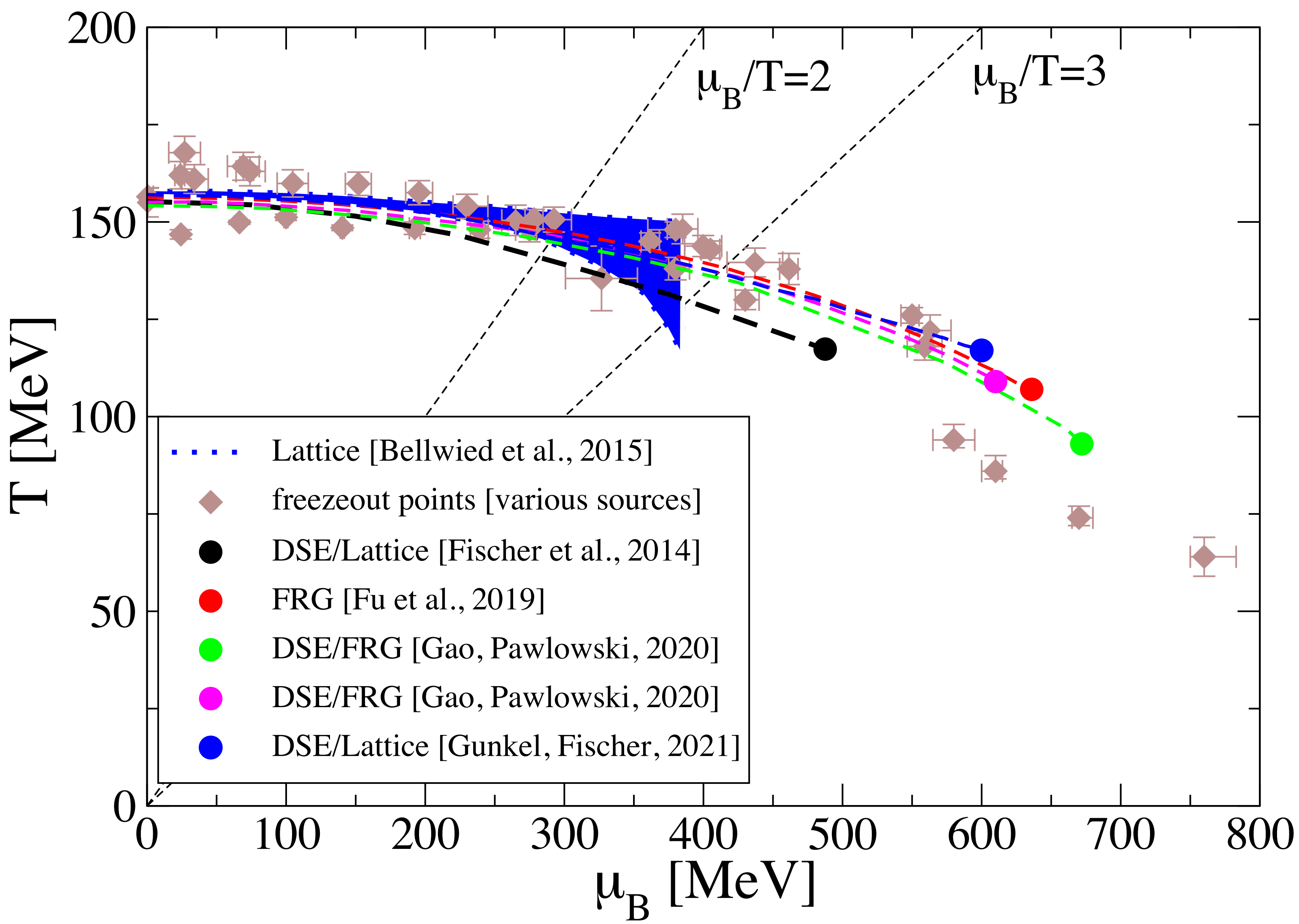




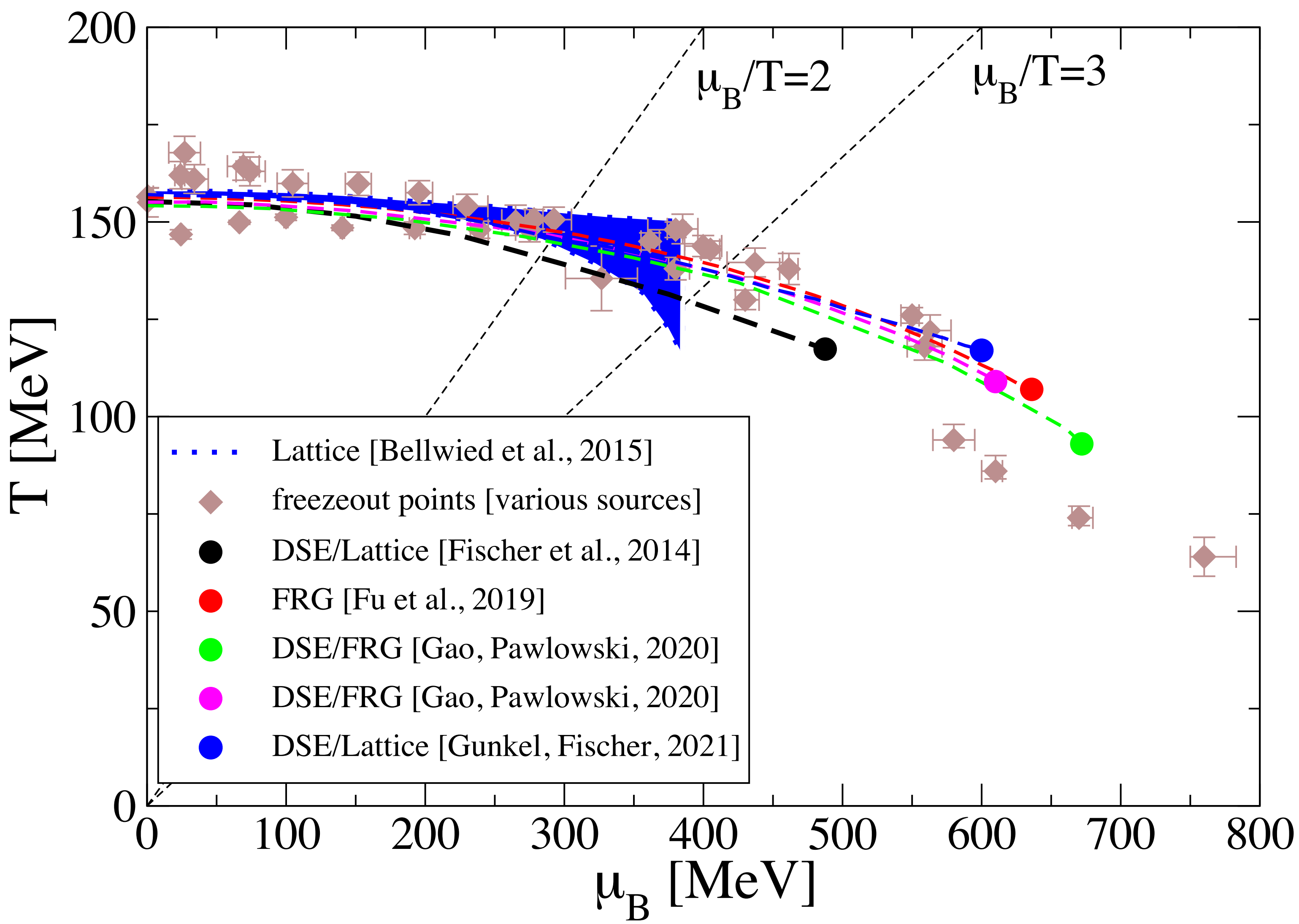
Gunkel, CF, PRD 104 (2021) [2106.08356]

- Vanishing chemical potential: no effect
- small effects on location of CEP
- μ-dependence of meson wave function taken into account

Location of CEP in freece-out landscape



Location of CEP in freece-out landscape



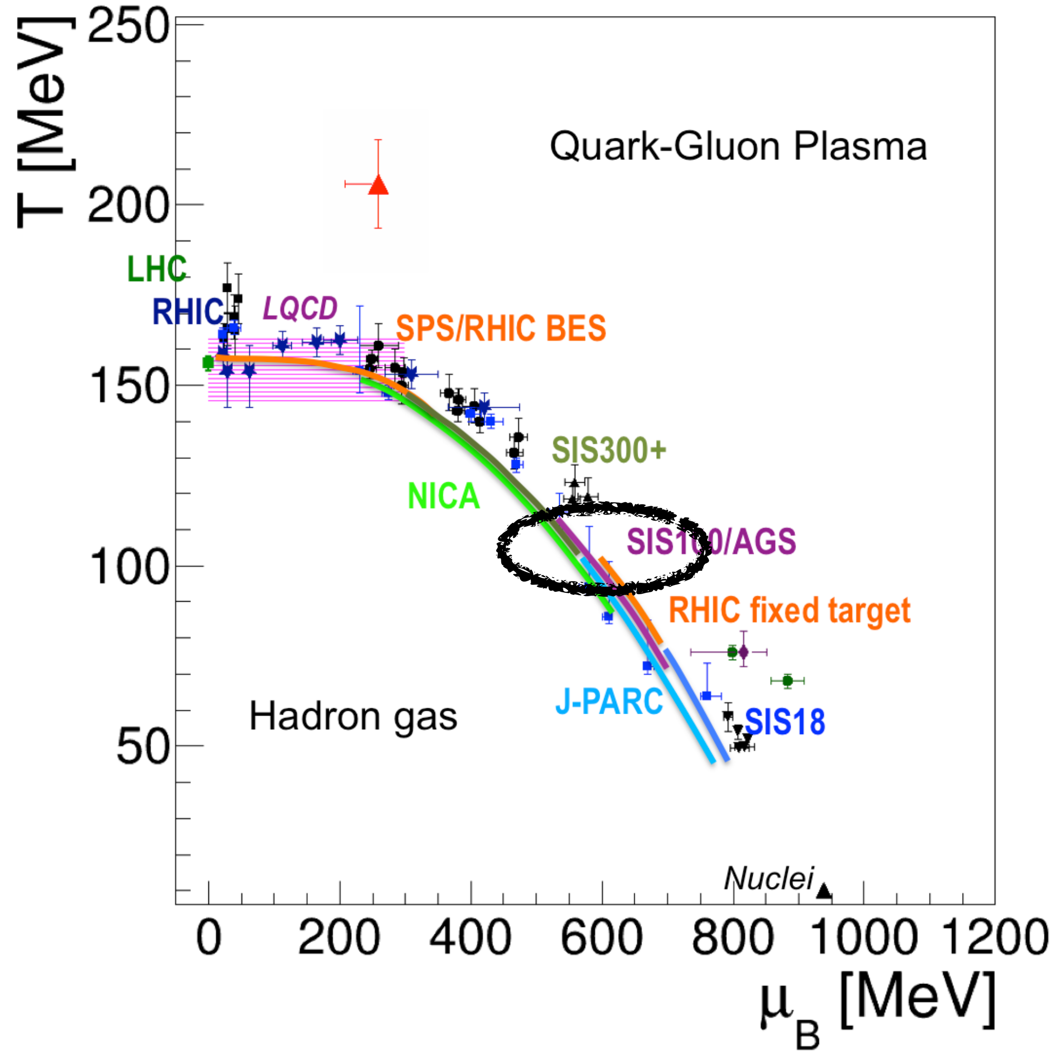
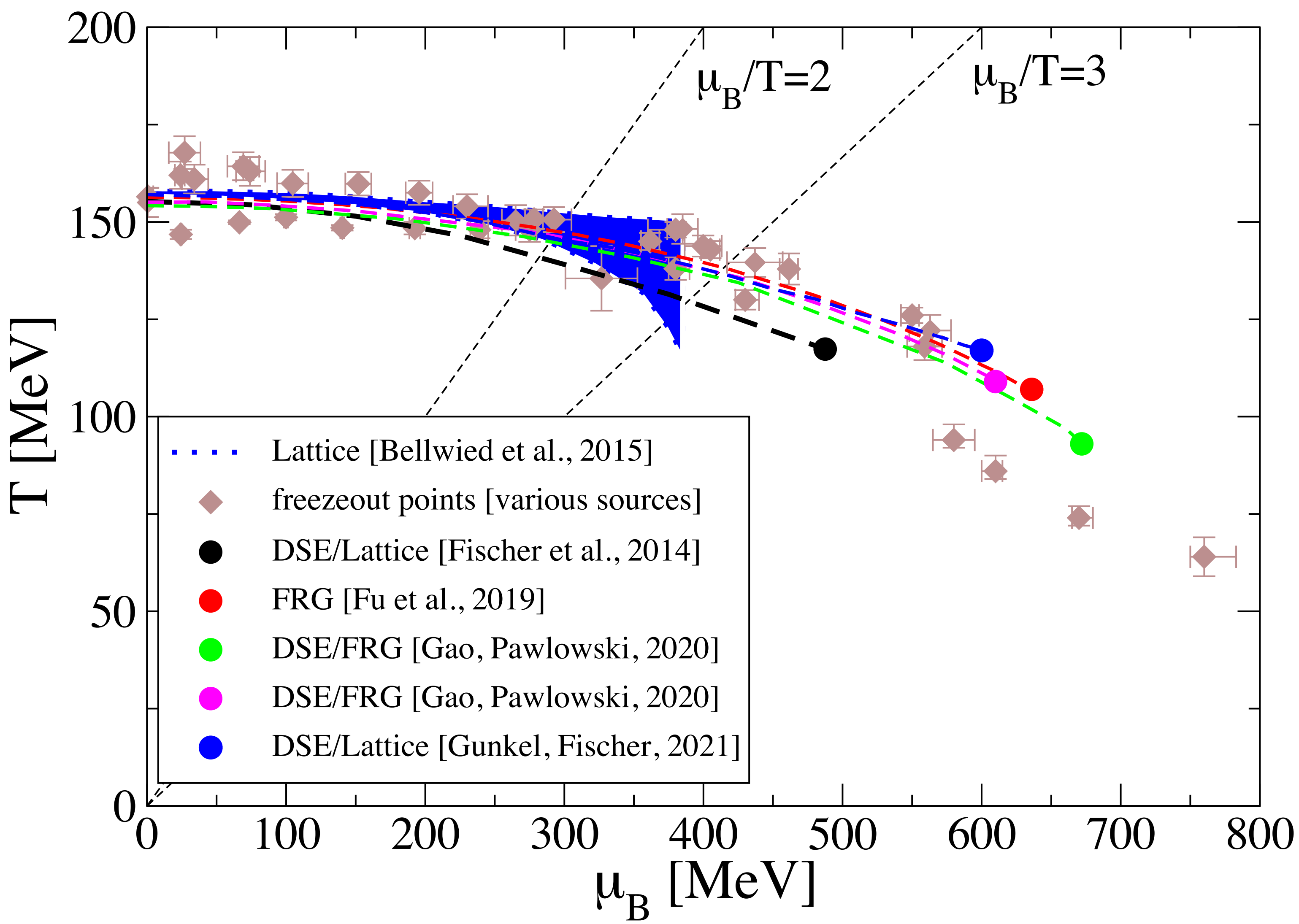


Figure adapted from talk of T. Galatyuk, Erice 2016

Location of CEP in freece-out landscape



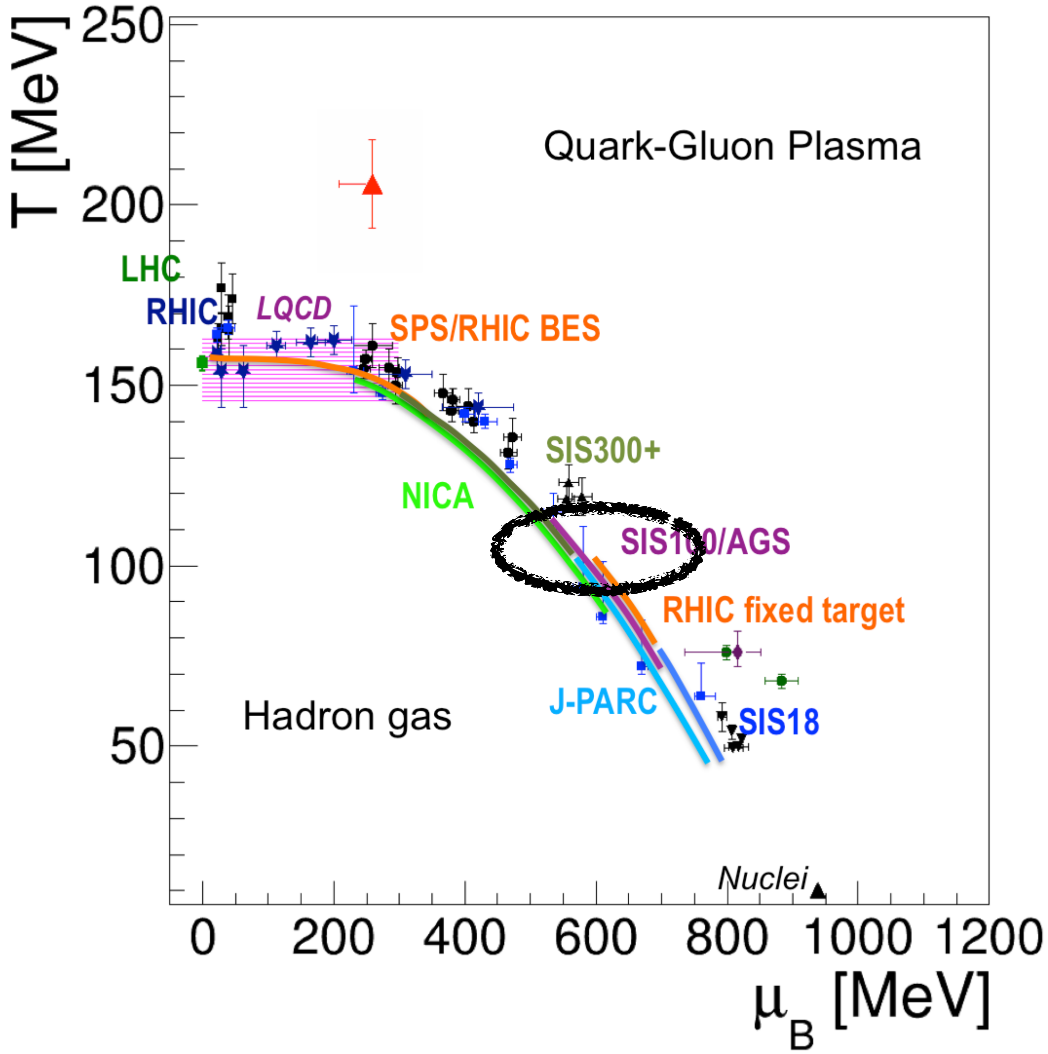


Figure adapted from talk of T. Galatyuk, Erice 2016

Caveats:

- inhomogeneous phases
- …

Buballa and Carignano, PPNP 81 (2015) 39

Contact with experiment: fluctuations

X.~Luo and N.~Xu, Nucl. Sci. Tech. 28 (2017) no.8, 112 [arXiv:1701.02105 [nucl-ex]].

Quark chemical potentials related to those of conserved charges:

$$\mu_u = \mu_B/3 + 2\mu_Q/3$$
$$\mu_d = \mu_B/3 - \mu_Q/3$$
$$\mu_s = \mu_B/3 - \mu_Q/3 - \mu_S$$

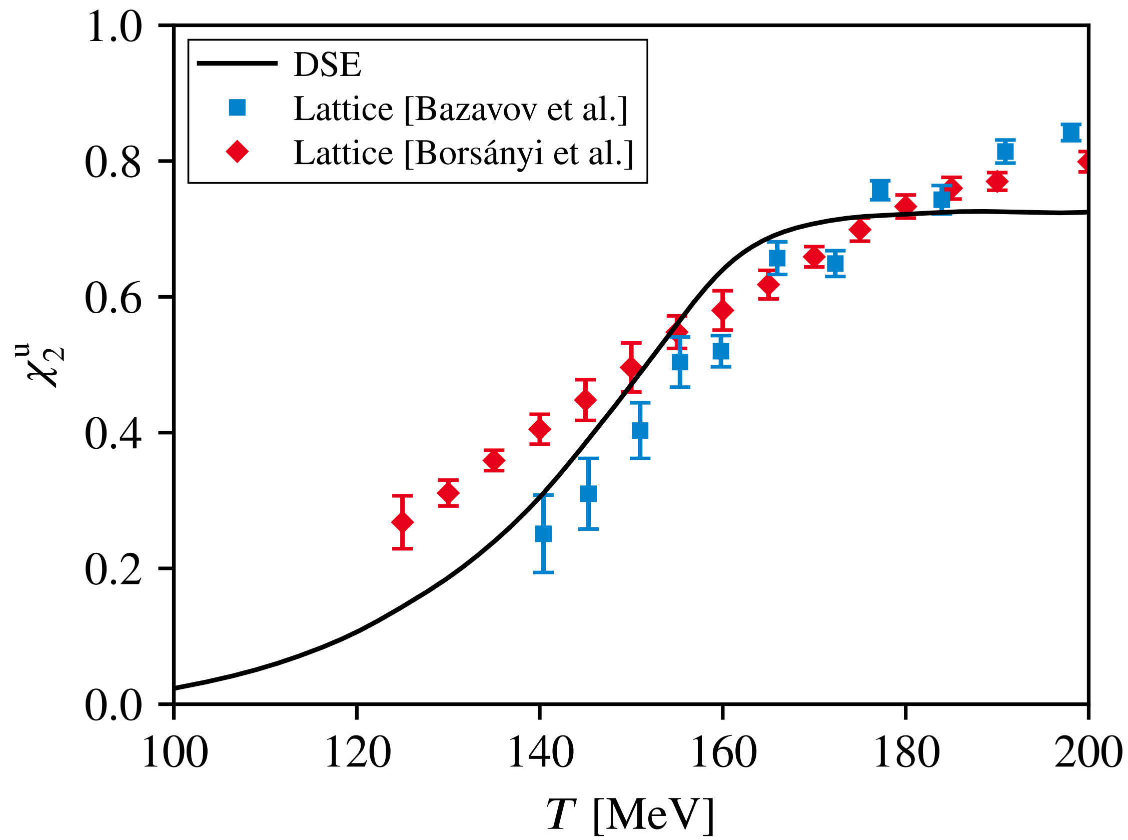
Serve to calculate susceptibilities:

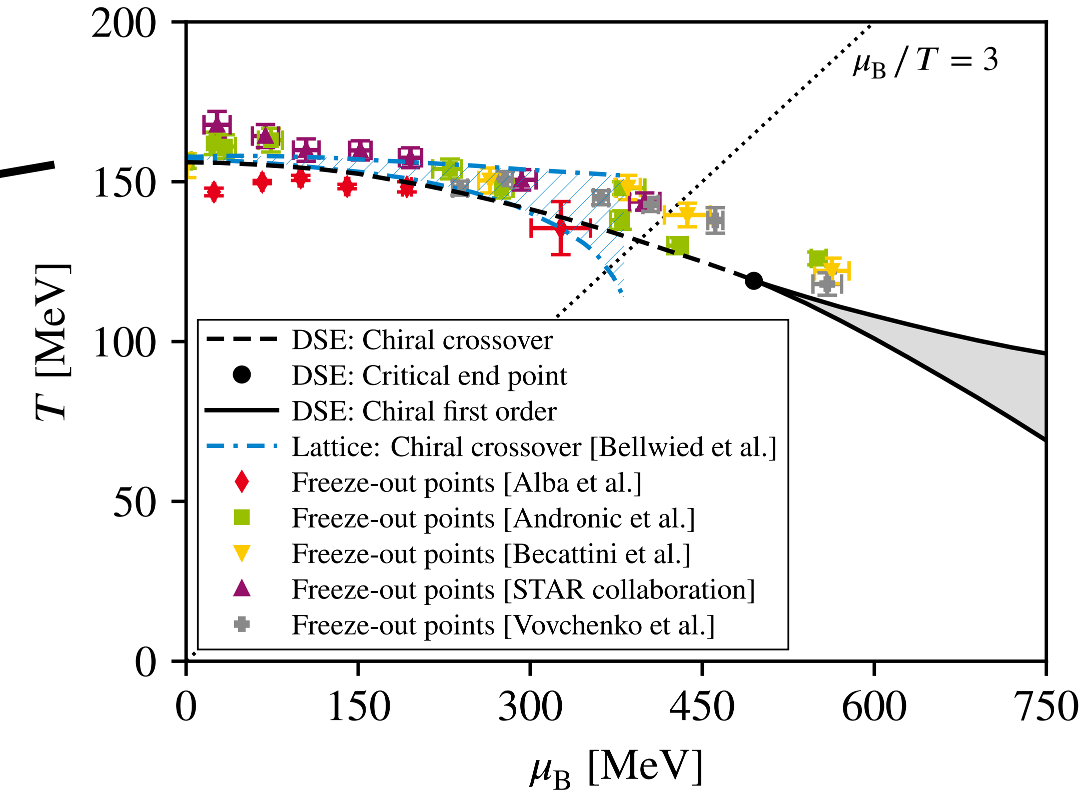
$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n}(p/T^4)}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}$$

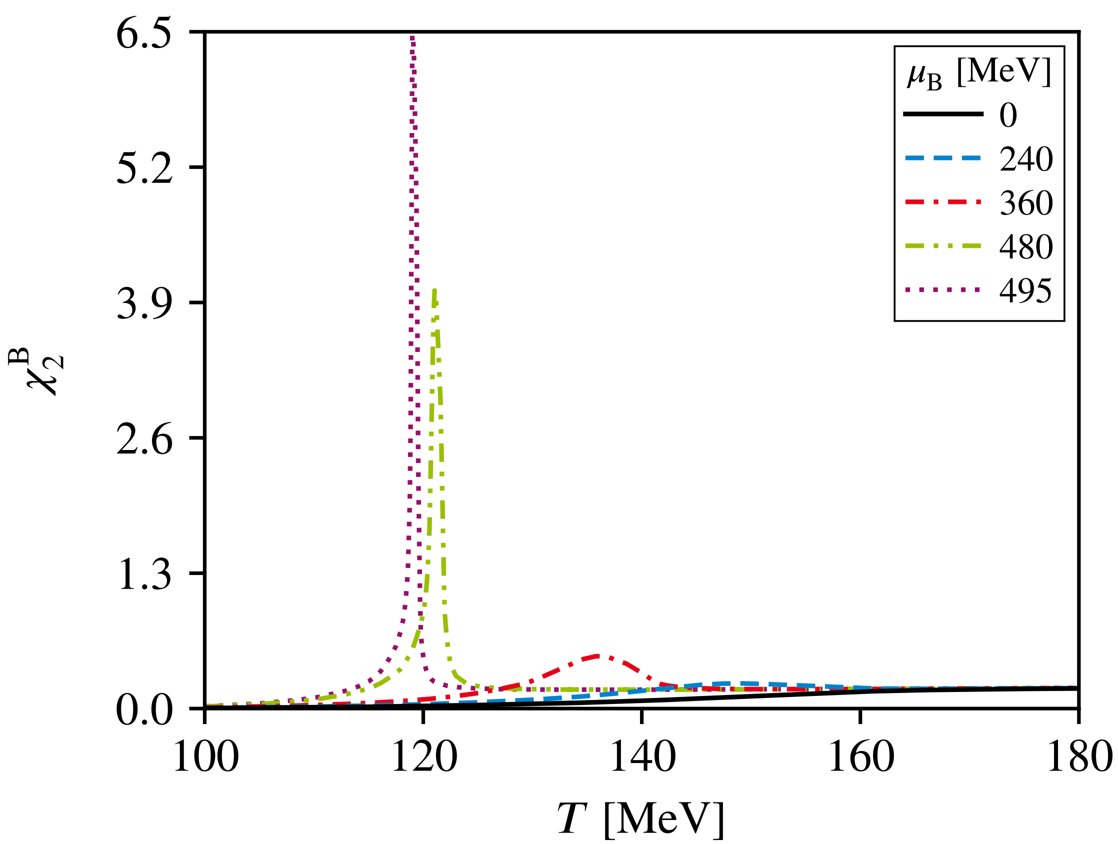
Related to cumulants, which can be extracted from experiment:

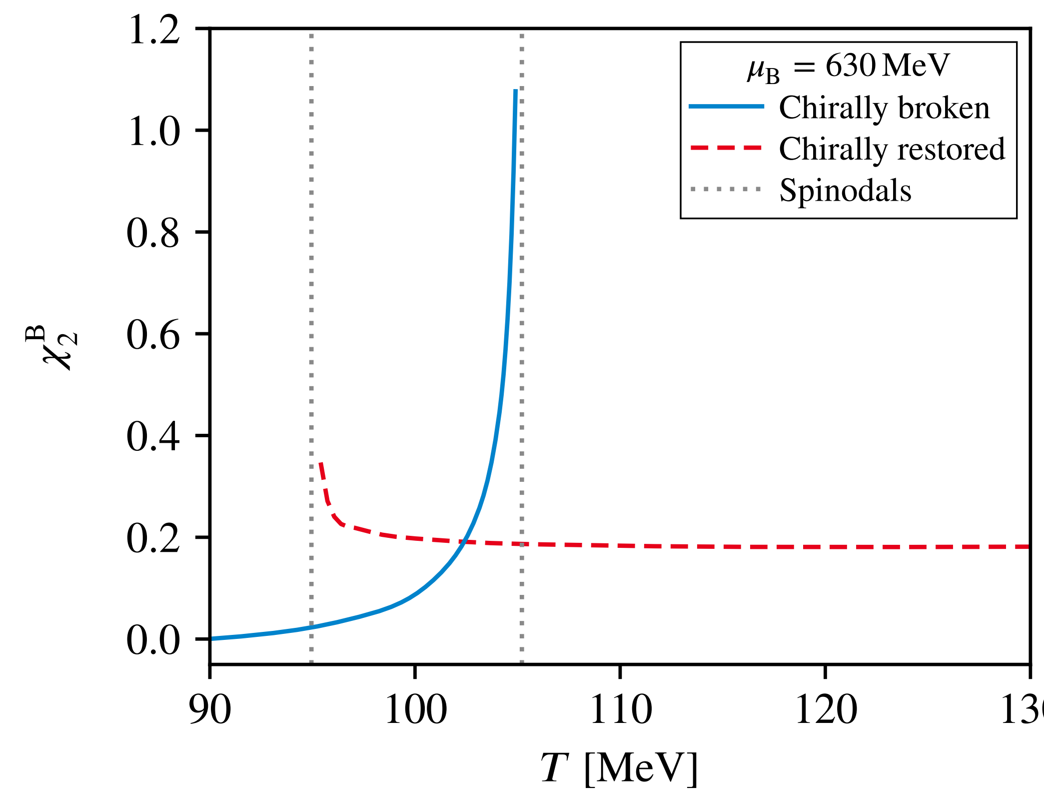
$$C_{lmn}^{BSQ} = VT^3 \chi_{lmn}^{BSQ}$$

Results for fluctuations



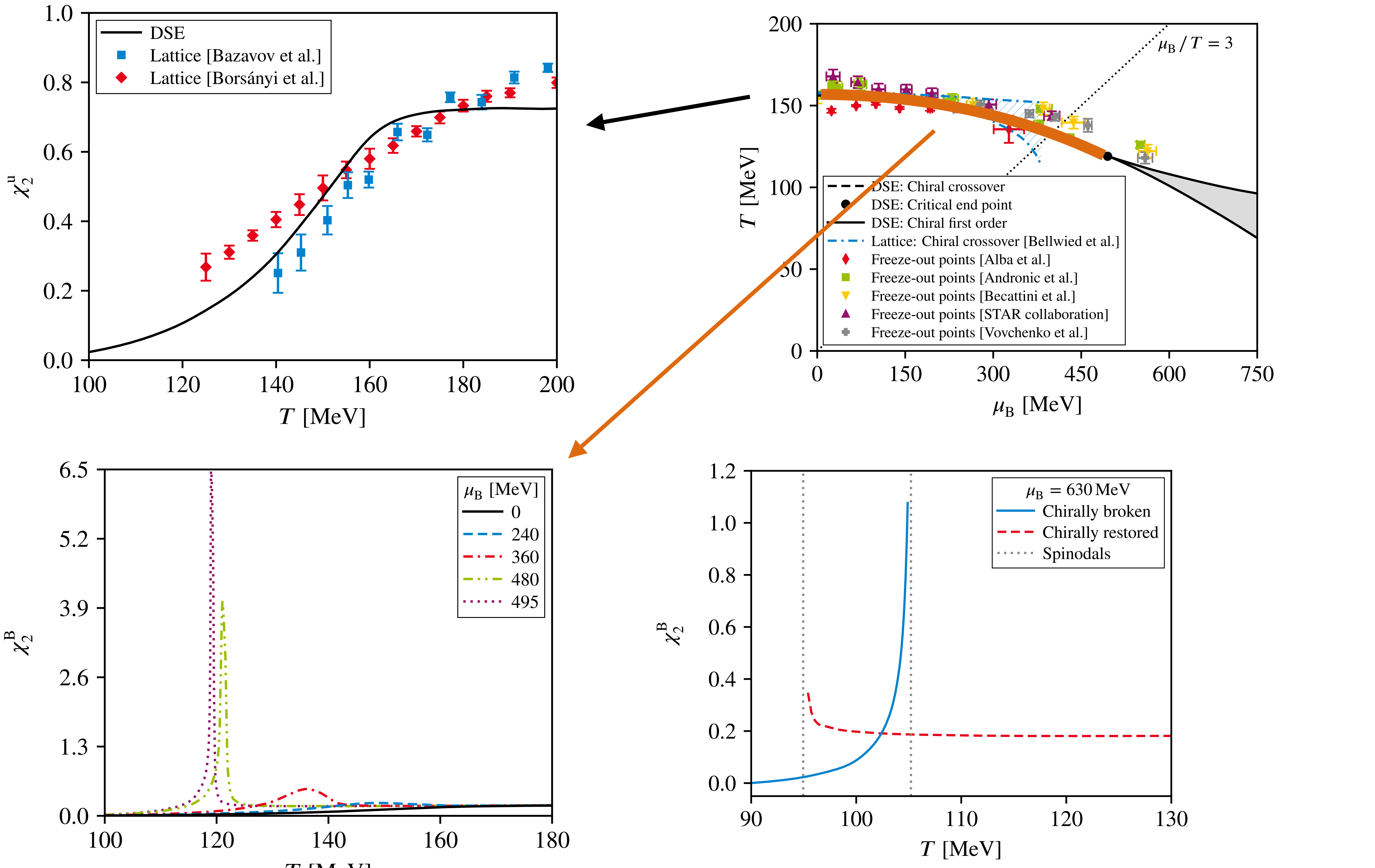






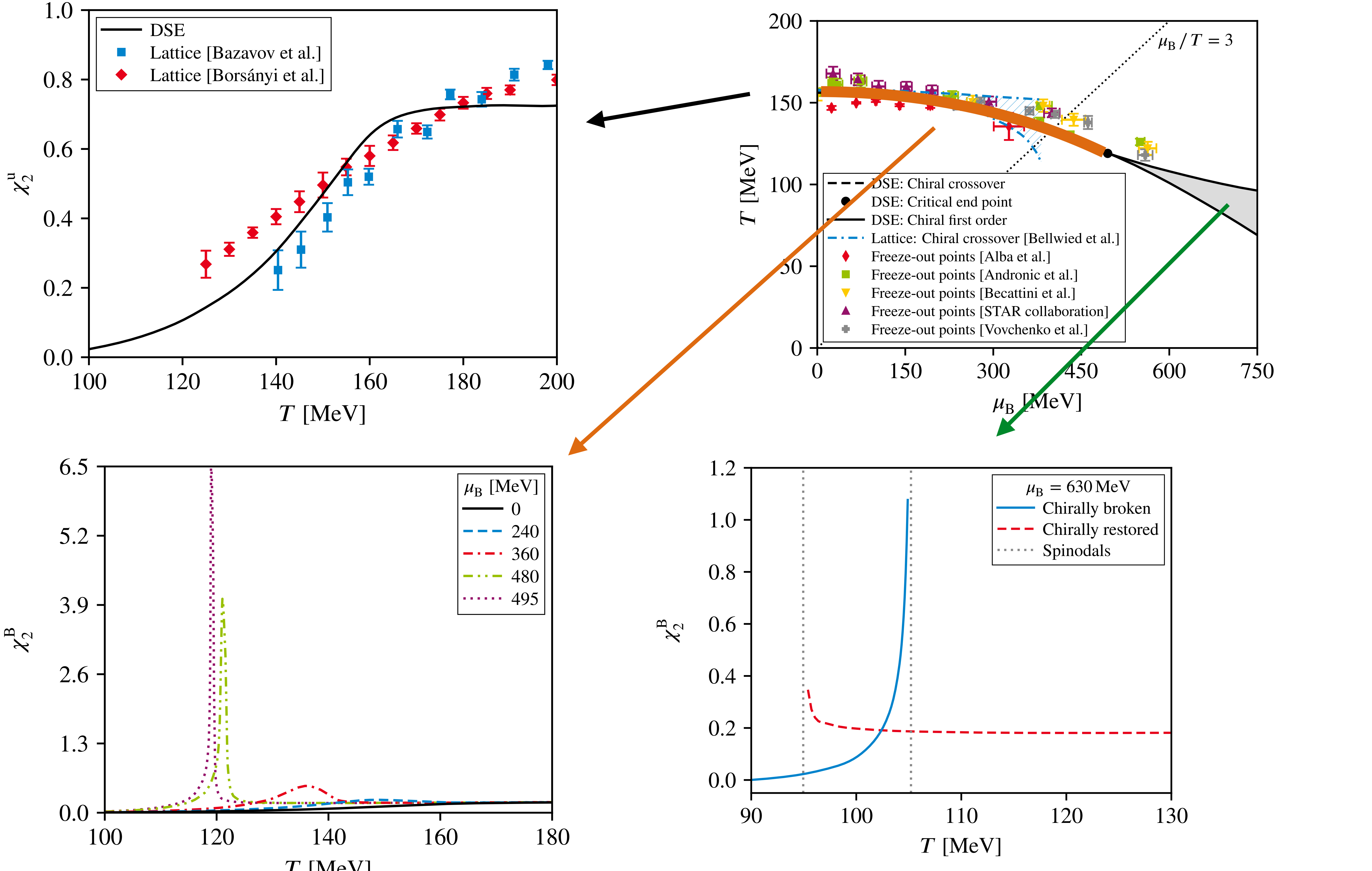
Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011

Results for fluctuations



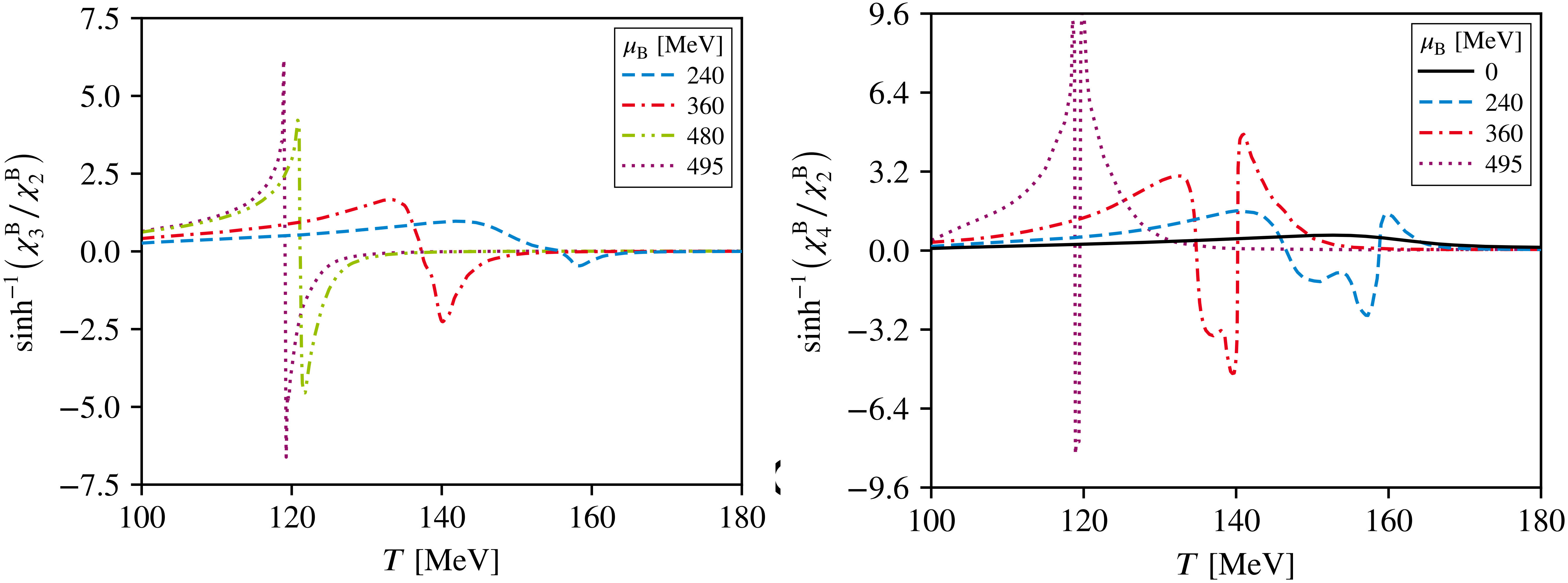
Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011

Results for fluctuations



Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011

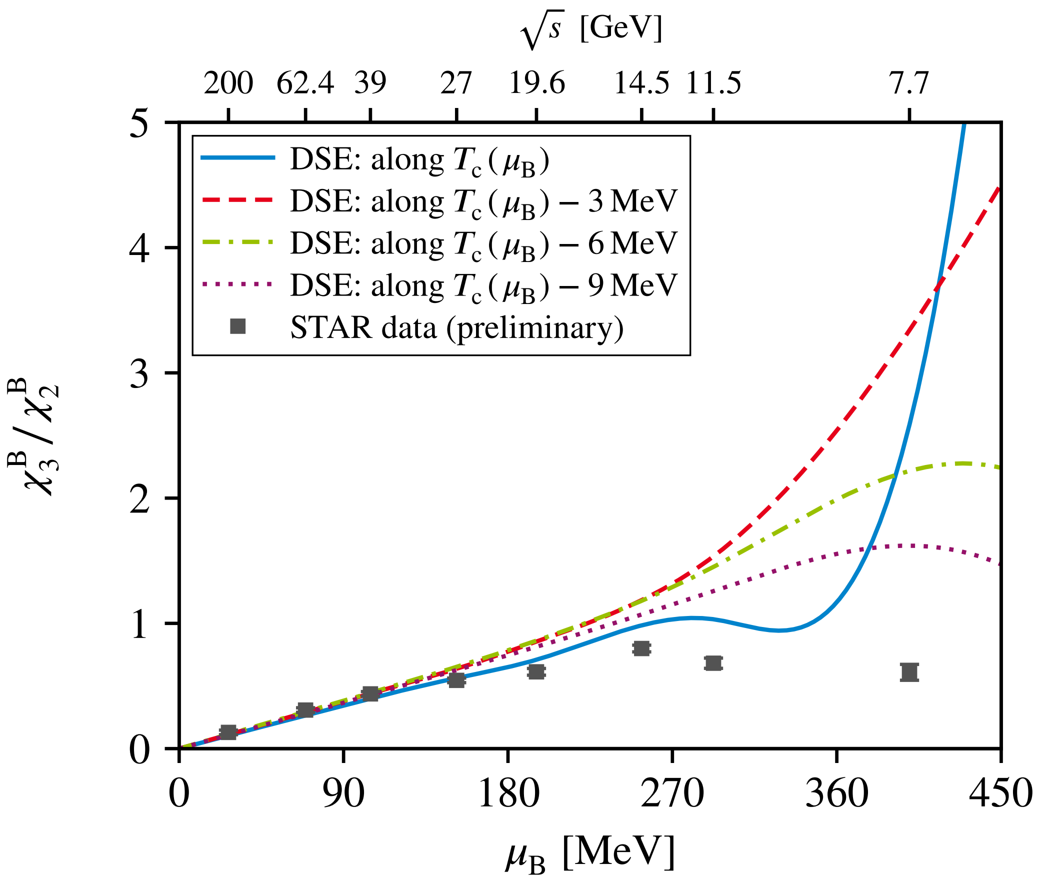
Ratios: skewness and kurtosis

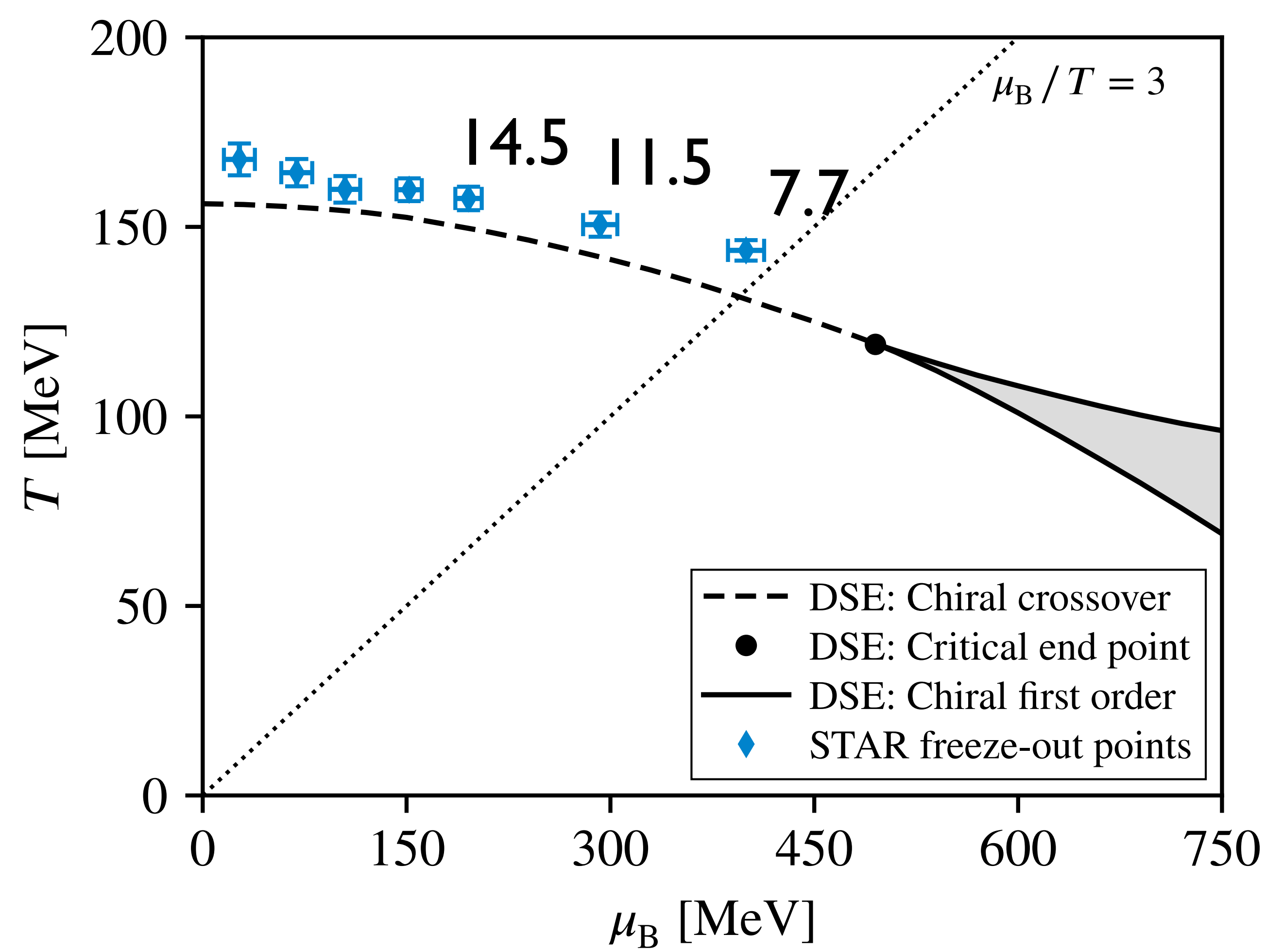


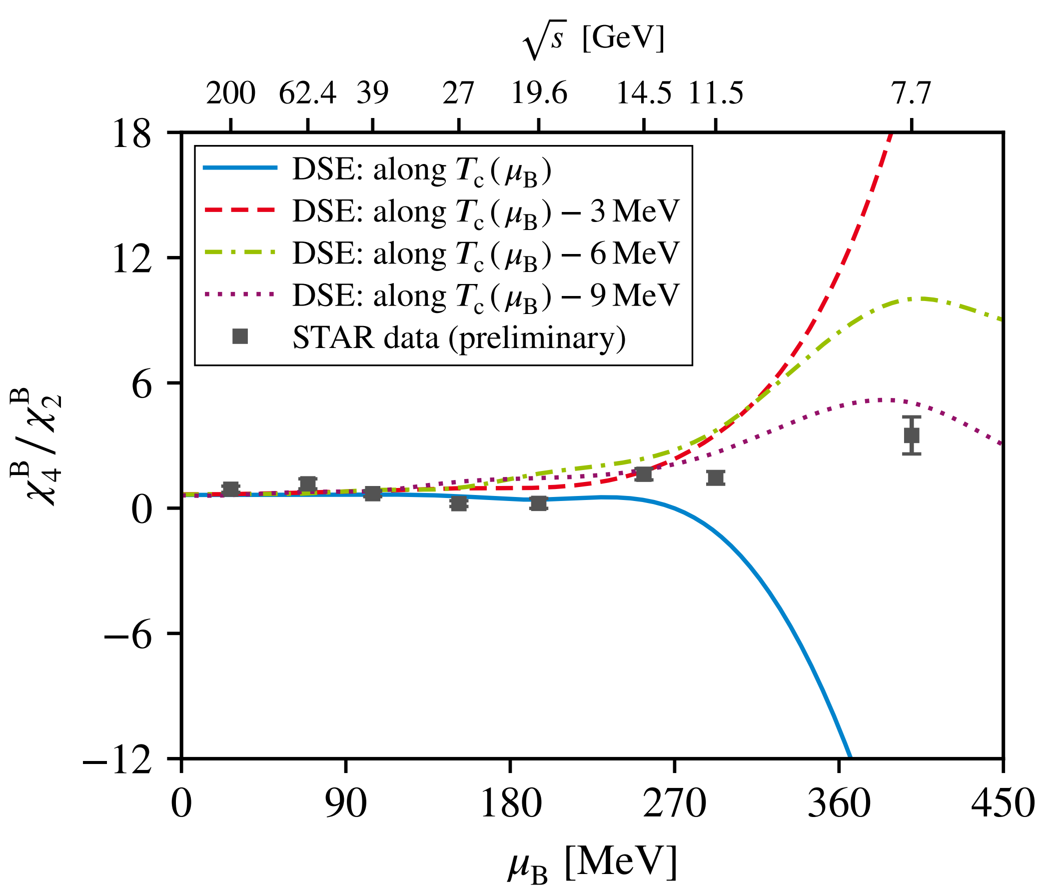
Caveats when comparing with experiment:

- so far only flavor-diagonal elements taken into account
- critical region may be too large… Schaefer and Wambach, PRD 75 (2007) 085015
- experimental extraction not without problems

Ratios: skewness and curtosis







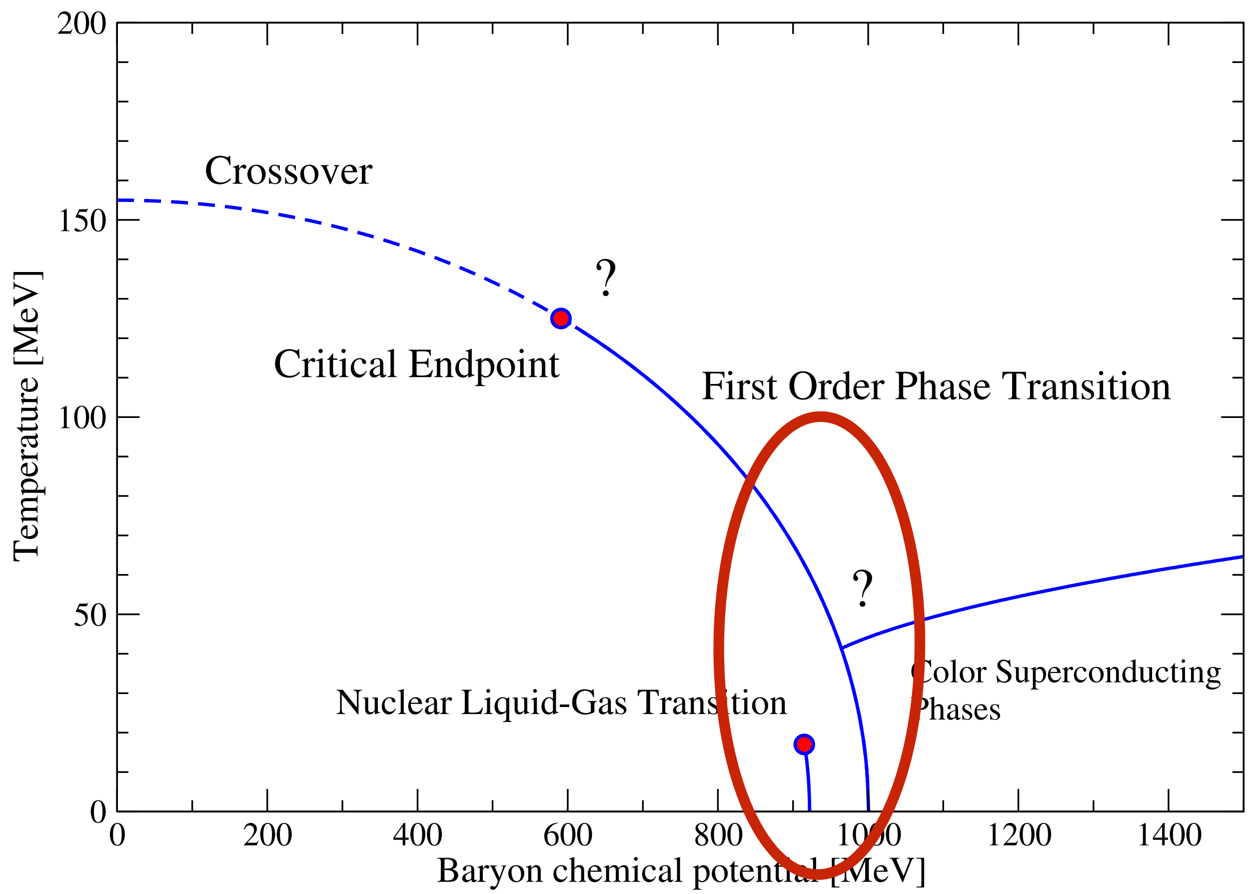
$\sqrt{s} \geq 14.5:$ good agreement

$\sqrt{s} = 11.5:$ trend ok !

$\sqrt{s} \leq 7.7:$ freezeout line $\neq$ transition line ?!

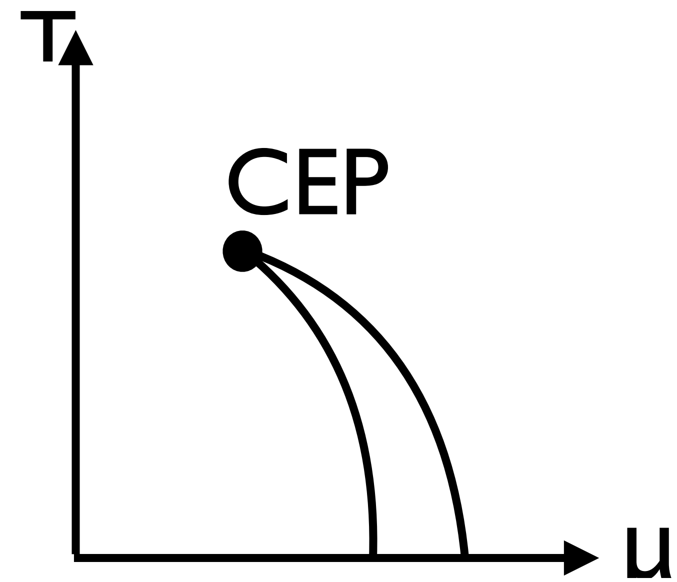
Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011

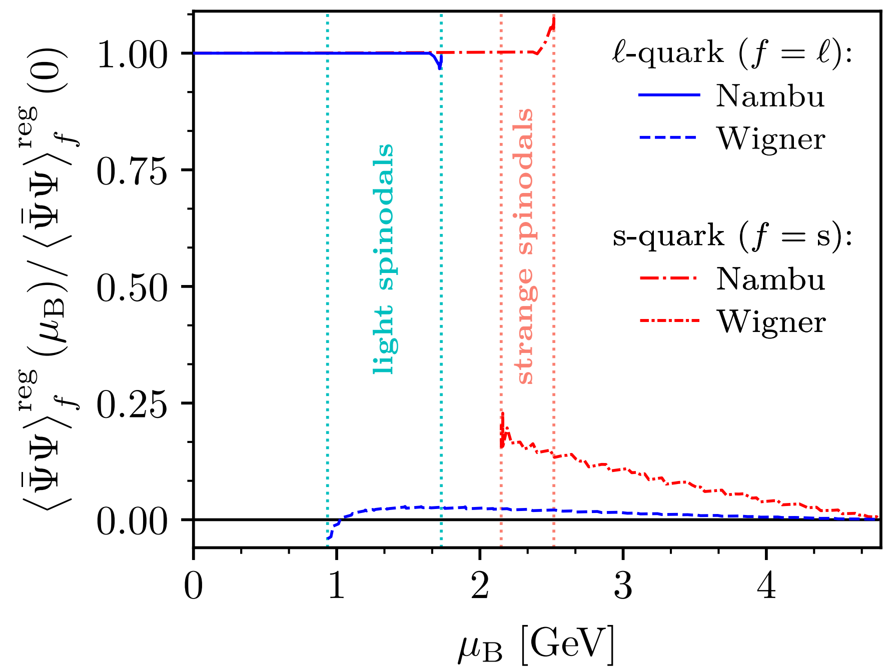
Dense QCD

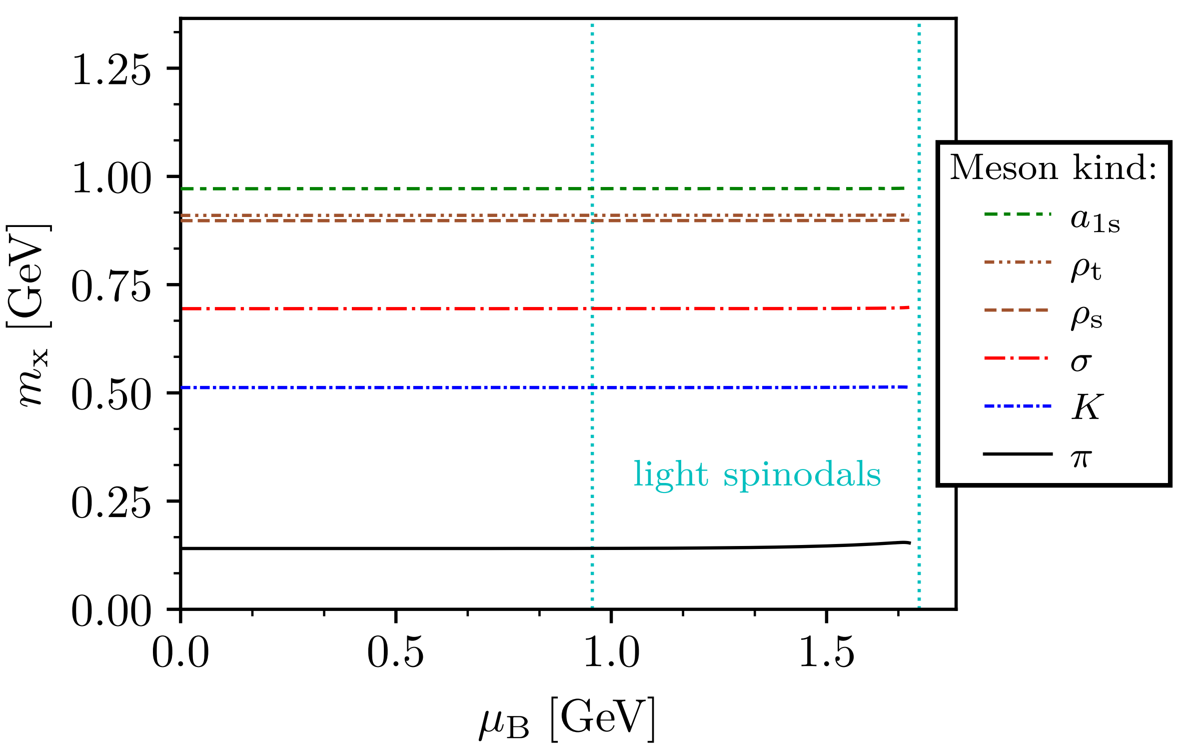


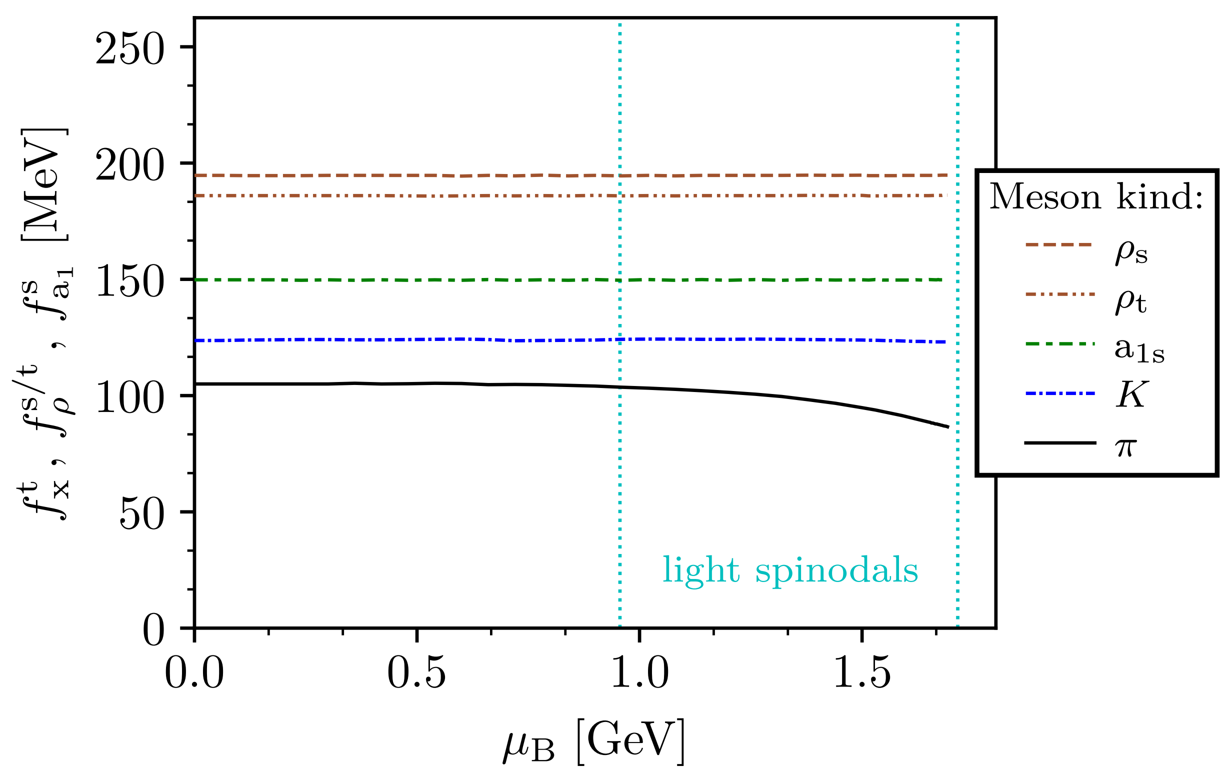
- Relevant for physics of neutron stars/mergers
- Second CEP ?

Meson properties at finite chemical potential









- Silver blaze satisfied — T. D. Cohen, PRL 91 , 222001 (2003)
- But quarks/meson wave functions do change !

Gunkel, CF, Isserstedt, EPJ A 55 (2019) no.9, 169
Gunkel, CF, EPJ A 57 (2021) no. 4, 147

Summary: QCD with functional methods

Main goals:

- one framework for all areas of hadron physics:
  mesons, baryons, 'exotic states', form factors,
  hadronic contributions to precision observables (g-2)
- same framework for QCD phase diagram

Main challenge:

- systematic control over error budget:
  intrinsic, cp with FRG, cp with lattice QCD

Main results:

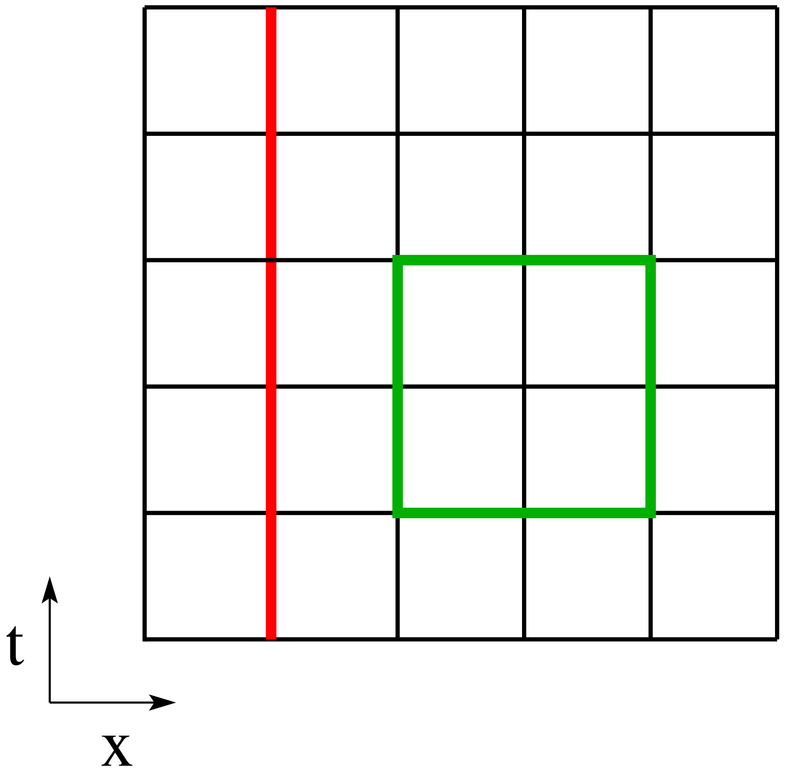
- not high precision physics
  but competitive contributions in many areas
- ONLY QCD-based tool to explore phase diagram at large μ
  at physical quark masses

Backup

Polyakov-Loop and center symmetry

Wilson-Loop: $$U(C) = \hat{P} \exp\left[ig \oint_C dx^\mu A_\mu(x)\right]$$

Polyakov-Loop: $$\Phi = \hat{P} \exp\left[ig \int_0^{1/T} d\tau A_4(\tau, \vec{x})\right]$$



Center of gauge group SU(N_c):

$$z_n = \exp[2\pi i\, n/N_c]\mathbb{1}, \quad n = 0..N_c - 1$$

Polyakov-Loop and center symmetry

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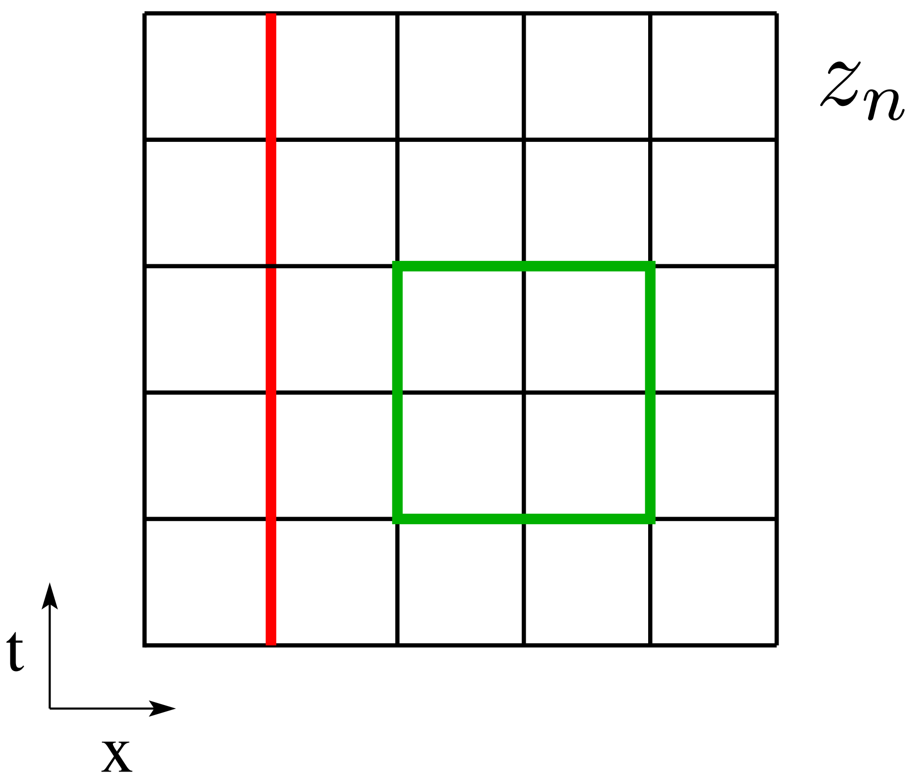

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Center transformation:

$$S_{QCD} \to S_{QCD}$$
$$\Phi \to z_n \Phi$$

Polyakov-Loop and center symmetry

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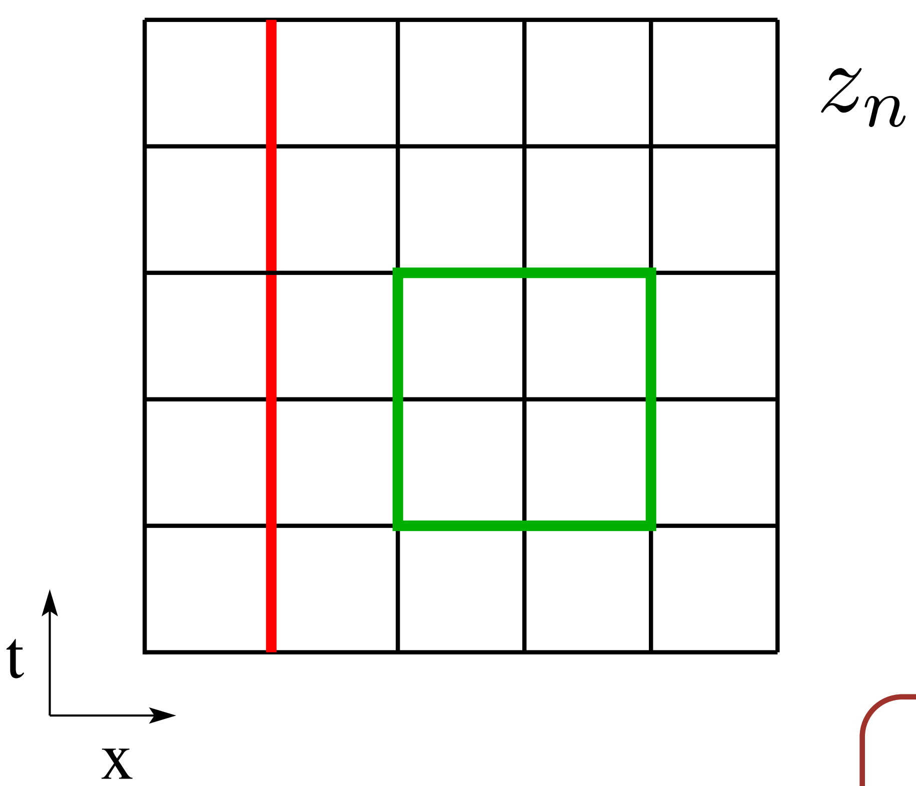

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Center transformation:

$$S_{QCD} \to S_{QCD}$$
$$\Phi \to z_n \Phi$$

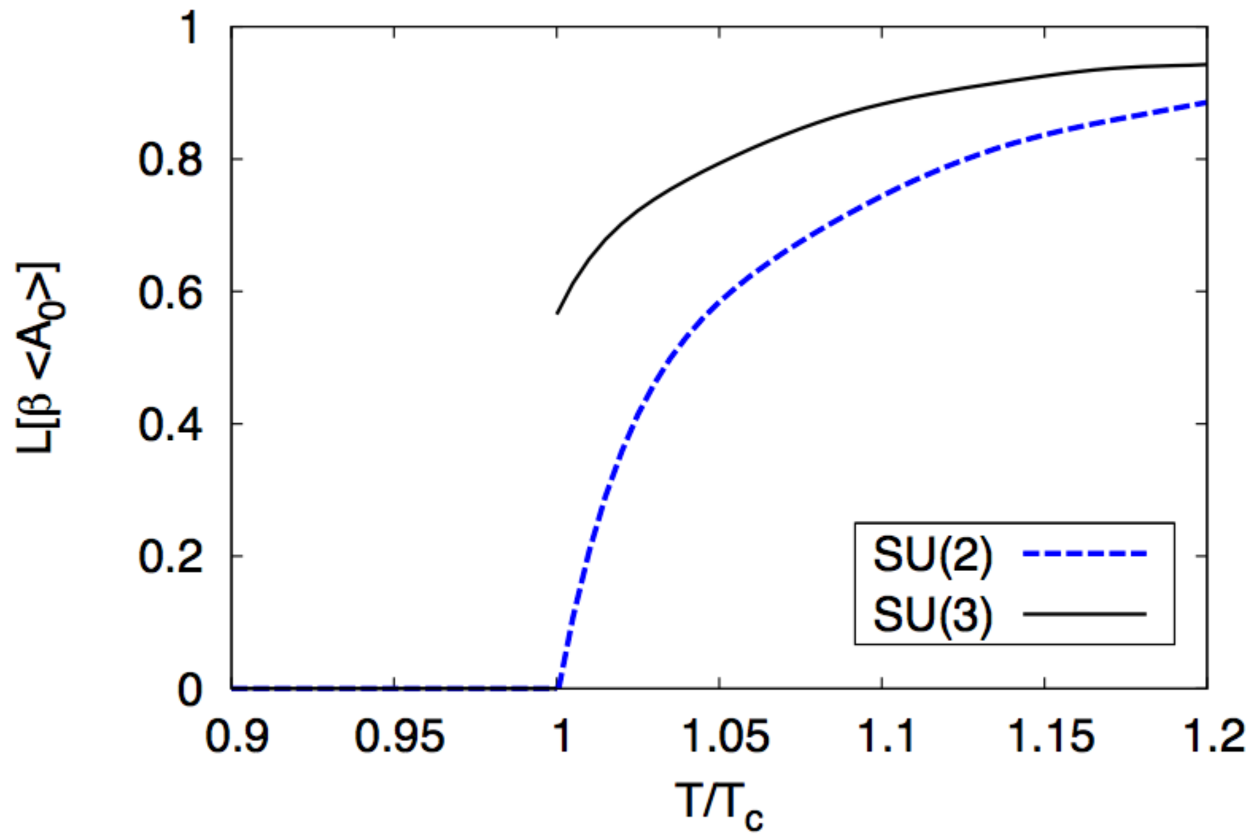
$$\langle Tr\,\Phi\rangle = \begin{cases} 0 & \text{unbroken } z_n \text{ symmetry} \\ \text{non-zero} & \text{broken } z_n \text{ symmetry} \end{cases}$$

Energy of an isolated quark

$$\langle Tr\,\Phi\rangle = \begin{cases} 0 & \text{unbroken } z_n \text{ symmetry} \\ \text{non-zero} & \text{broken } z_n \text{ symmetry} \end{cases}$$

$$\langle Tr\,\Phi\rangle \sim e^{-F_q/T} \qquad F_q = \begin{cases} \infty & \text{unbroken } z_n \text{ symmetry} \\ \text{finite} & \text{broken } z_n \text{ symmetry} \end{cases}$$

F_q: free energy of heavy quark



Braun, Gies, Pawlowski, PLB684 (2010)

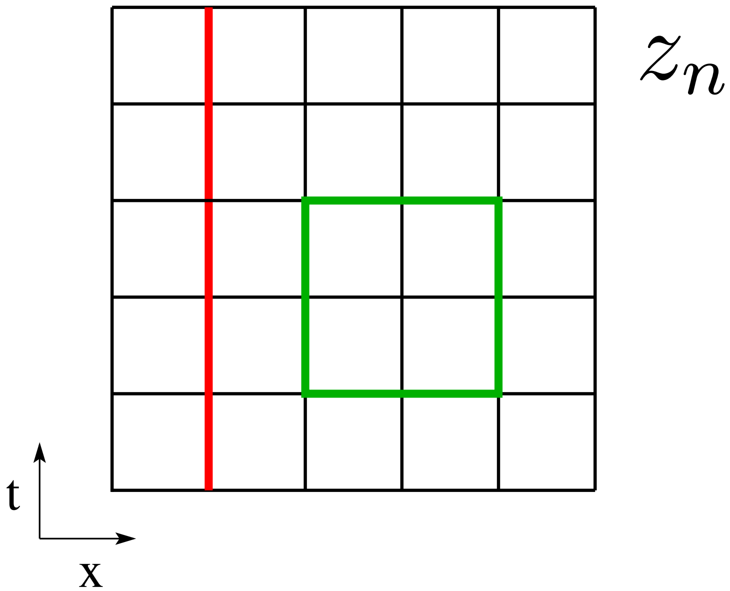
Order parameter!

- SU(2): second order
- SU(3): first order

Order parameter: the dressed Polyakov-loop

ordinary Polyakov-loop:

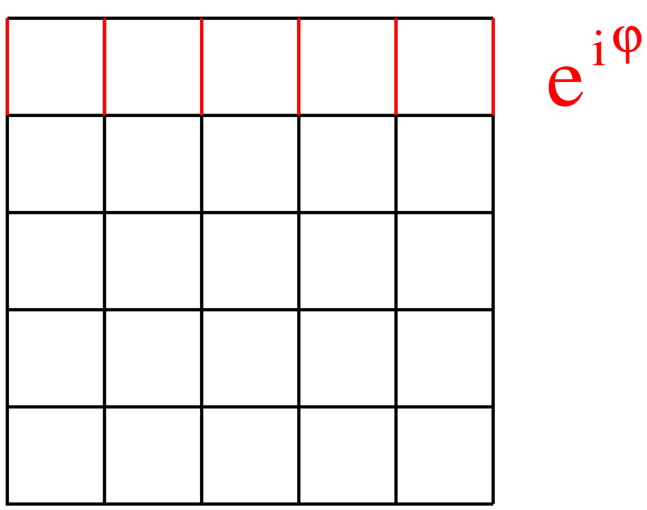
$$\Phi = \hat{P} \exp\left[ig \int_0^{1/T} d\tau\, A_4(\tau, \vec{x}) \right]$$



sensitive to center transformation

$$z_n = \exp[2\pi i\, n/N_c]\mathbb{1}, \quad n = 0..N_c - 1$$

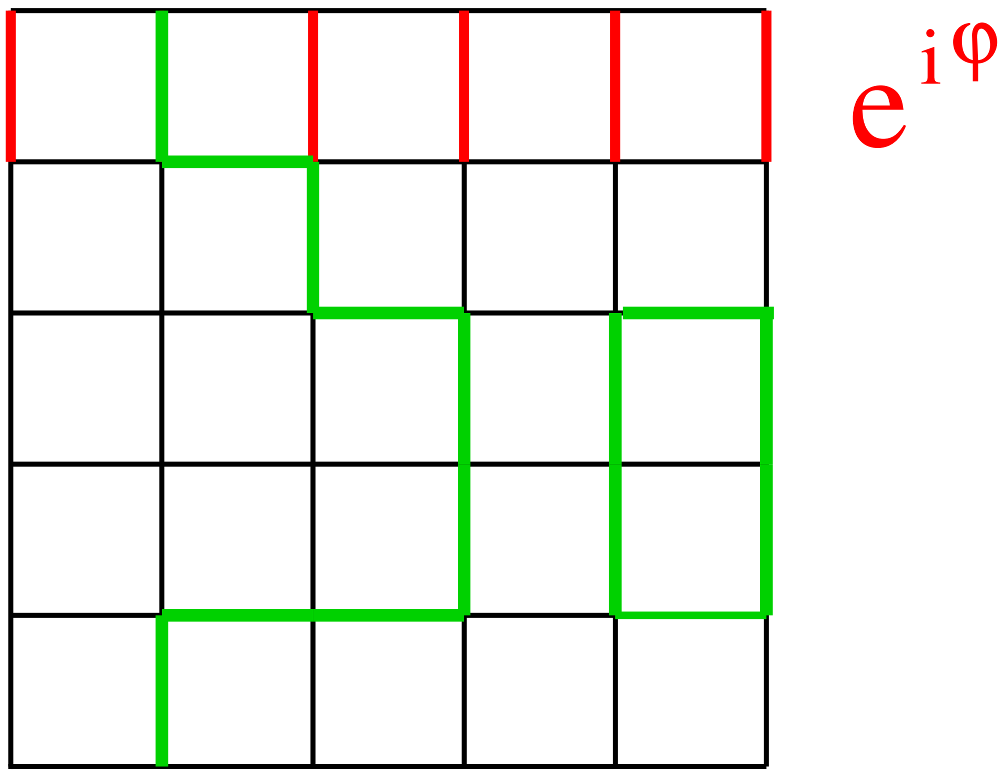
Now consider general U(1)-valued boundary conditions in temporal direction for quark fields:



$$\psi(\vec{x}, 1/T) = e^{i\varphi}\, \psi(\vec{x}, 0)$$

$$\omega(n_t) = (2\pi T)(n_t + \varphi/2\pi)$$

Order parameter: the dressed Polyakov-loop II



$$\langle \overline{\psi}\psi \rangle_{\varphi} = \frac{1}{Vm} \sum_{l} \frac{e^{i\varphi n(l)}}{(2am)^{|l|}} Tr_c U(l)$$

m :explicit quark mass

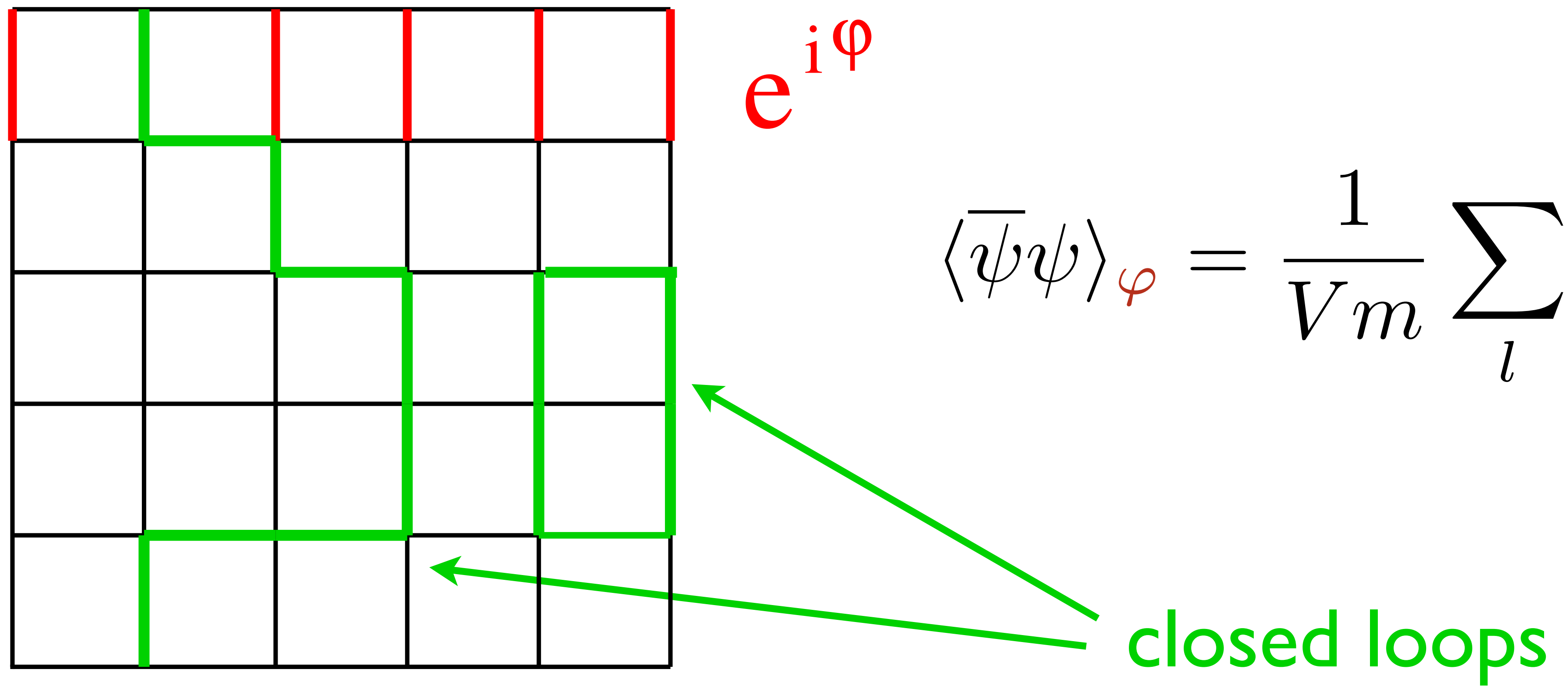
a :lattice spacing

V :volume

$|l|$:Loop length

F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).
E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.

Order parameter: the dressed Polyakov-loop II



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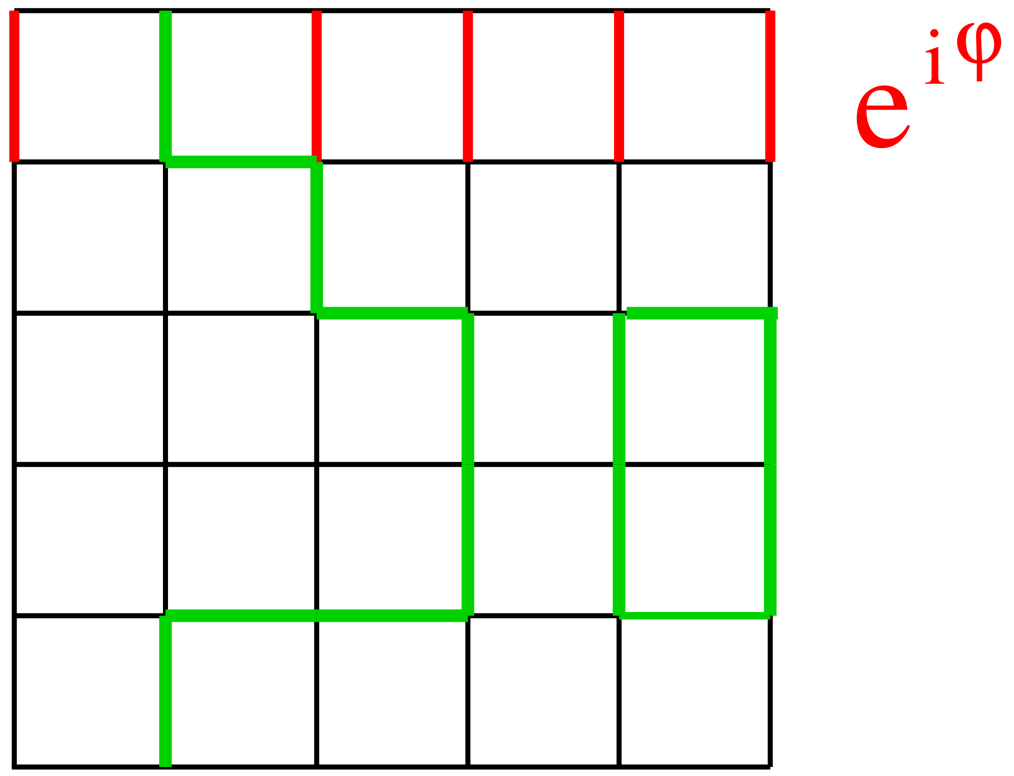

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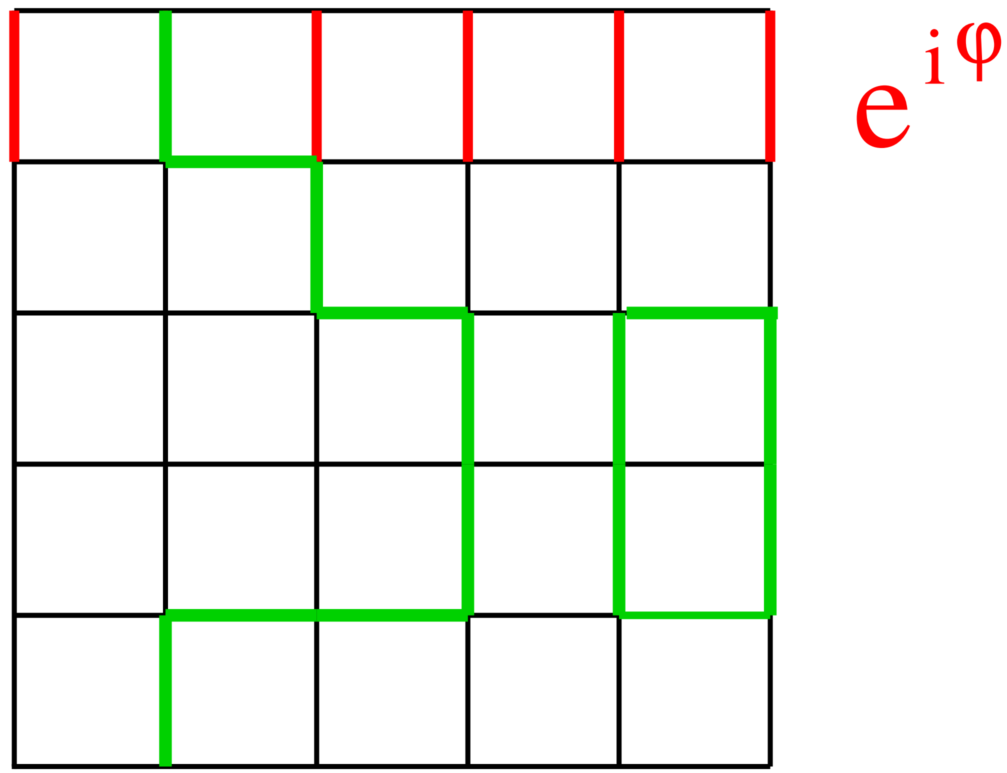
Order parameter: the dressed Polyakov-loop II



winding number

$$\langle \overline{\psi}\psi \rangle_{\varphi} = \frac{1}{Vm} \sum_{l} \frac{e^{i\varphi n(l)}}{(2am)^{|l|}} Tr_c U(l)$$

m :explicit quark mass

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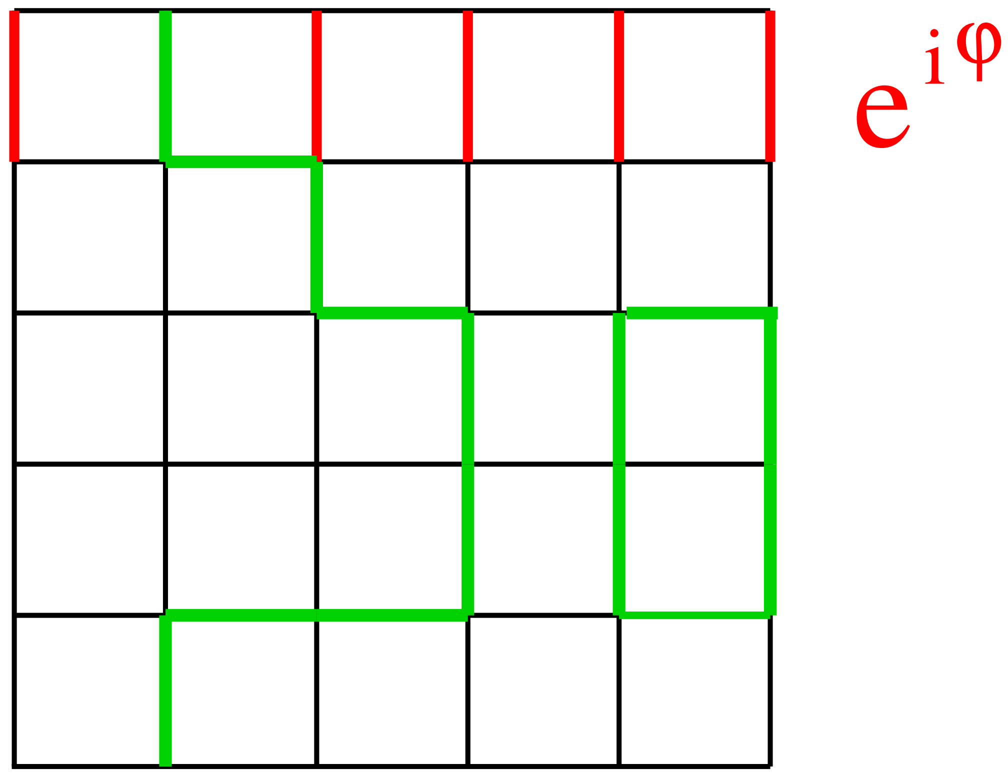
F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).
E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.

Order parameter: the dressed Polyakov-loop II



winding number

$$\langle \overline{\psi}\psi \rangle_\varphi = \frac{1}{Vm} \sum_l \frac{e^{i\varphi n(l)}}{(2am)^{|l|}} Tr_c U(l)$$

$m \to \infty : \quad n(l) = 1$

are ordinary Polyakov-loops

m :explicit quark mass

a :lattice spacing

V :volume

$|l|$:Loop length

F. Synatschke, A. Wipf and C. Wozar, PRD **75**, 114003 (2007).
E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.

Order parameter: the dressed Polyakov-loop III

Define dual condensate:

$$\Sigma_n = -\int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi n}\, \langle \overline{\psi}\psi \rangle_\varphi$$

- n=1 projects out all loops winding once around the torus: dressed Polyakov-loop
- Σ_1 transforms under center transformations exactly like ordinary Polyakov-loop:

$$\begin{aligned} {}^z\Sigma_n &= -\int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi n}\, \langle \overline{\psi}\psi \rangle_{\varphi+2\pi k/N_c} \\ &= -\int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i(\varphi+2\pi k/N_c)n}\, \langle \overline{\psi}\psi \rangle_\varphi \\ &= -z^n \int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi n}\, \langle \overline{\psi}\psi \rangle_\varphi \end{aligned}$$

Order parameter: the dressed Polyakov-loop III

Define dual condensate:

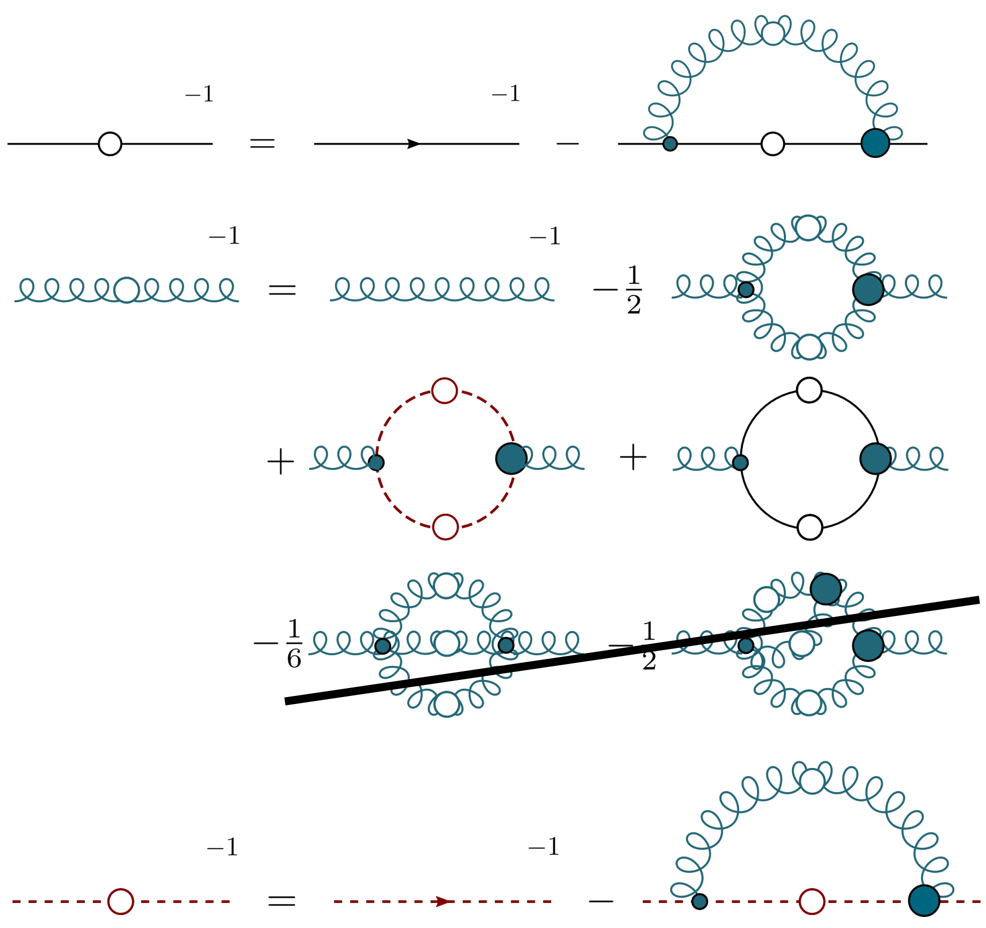
$$\Sigma_n = -\int_0^{2\pi} \frac{d\varphi}{2\pi}\, e^{-i\varphi n} \langle \overline{\psi}\psi \rangle_\varphi$$

- n=1 projects out all loops winding once around the torus: dressed Polyakov-loop
- Σ_1 is order parameter for center symmetry breaking
- Σ_1 is accessible with Dyson-Schwinger equations or the functional renormalization group

C. Gattringer, PRL 97, 032002 (2006)
F. Synatschke, A. Wipf and C. Wozar, PRD 75, 114003 (2007)
E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD 77 094007 (2008)
F. Synatschke, A. Wipf and K. Langfeld, PRD 77, 114018 (2008)
CF, PRL 103 052003 (2009)
CF, J.A. Mueller, PRD 80 (2009) 074029
J. Braun, L. Haas, F. Marhauser, J.M. Pawlowski, PRL 106 022002 (2011)

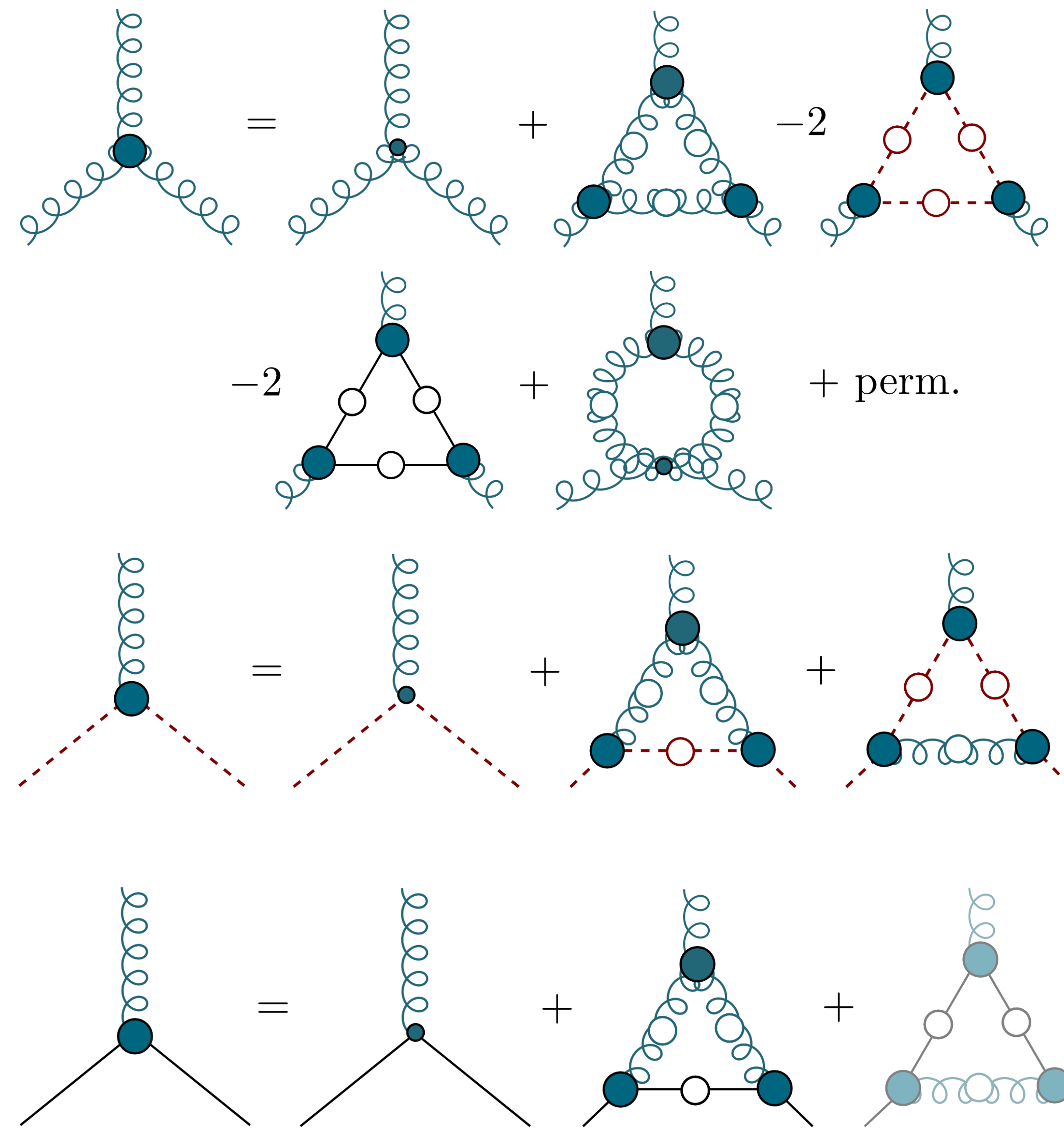
3PI-truncation

propagators



for different BRL approaches see work of
Aguilar, Alkofer, Binosi, Blum, Chang, Cyrol, Eichmann, Fister, Huber, Maas, Mitter, Papavassiliou, Pawlowski, Roberts, Smekal, Strodthoff, Vujinovic, Watson, Williams…

vertices



Williams, CF, Heupel, PRD 93 (2016) 034026
CF, Williams, PRL 103 (2009) 122001

Light meson spectrum

Experiment

our results

J^{PC}

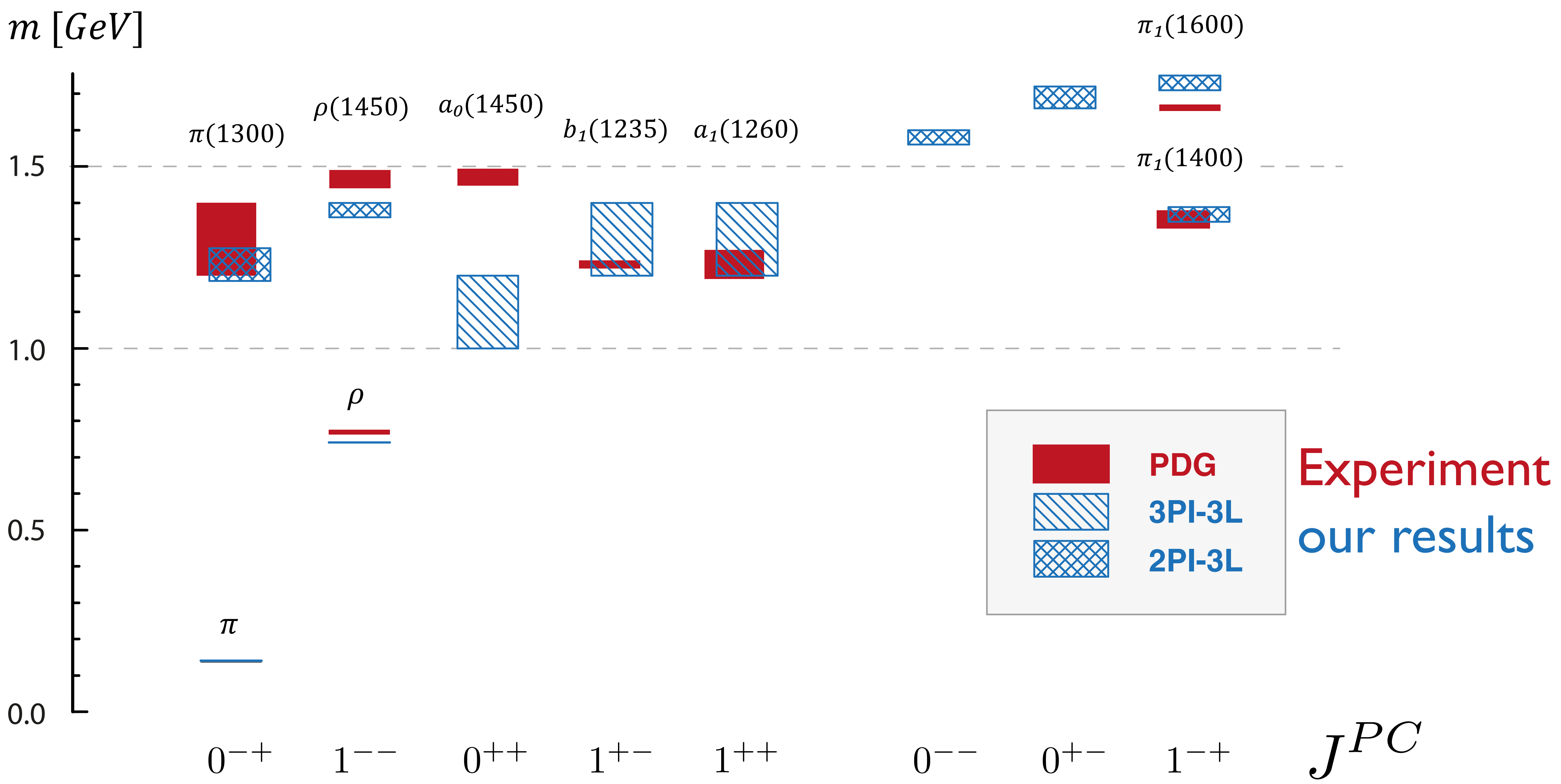
Williams, CF, Heupel, PRD93 (2016) 034026

- good agreement with experiment in most channels
- special channels:

  pseudoscalar 0^{++} : (pseudo-) Goldstone bosons

  scalar 0^{-+} : complicated channel… tetraquarks !

Light meson spectrum



Williams, CF, Heupel, PRD93 (2016) 034026

- good agreement with experiment in most channels
- special channels:

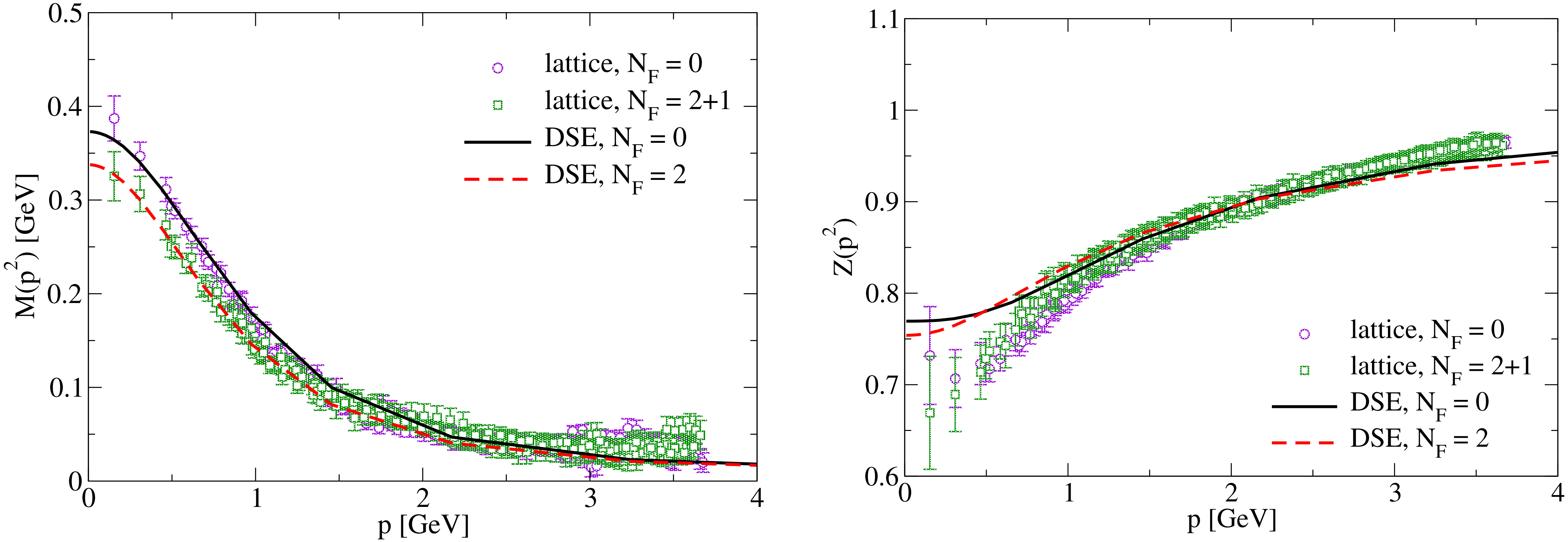
pseudoscalar 0^{++} : (pseudo-) Goldstone bosons

scalar 0^{-+} : complicated channel… tetraquarks !

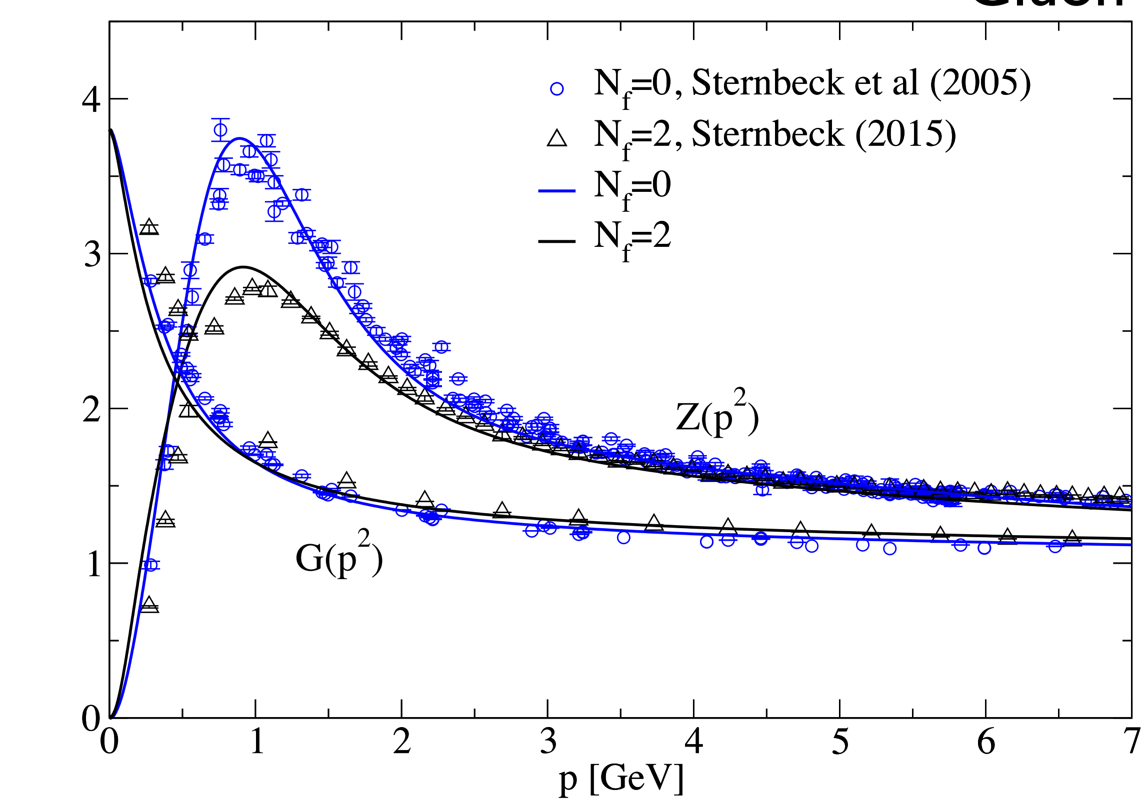
Selected results for Green's functions

Quark

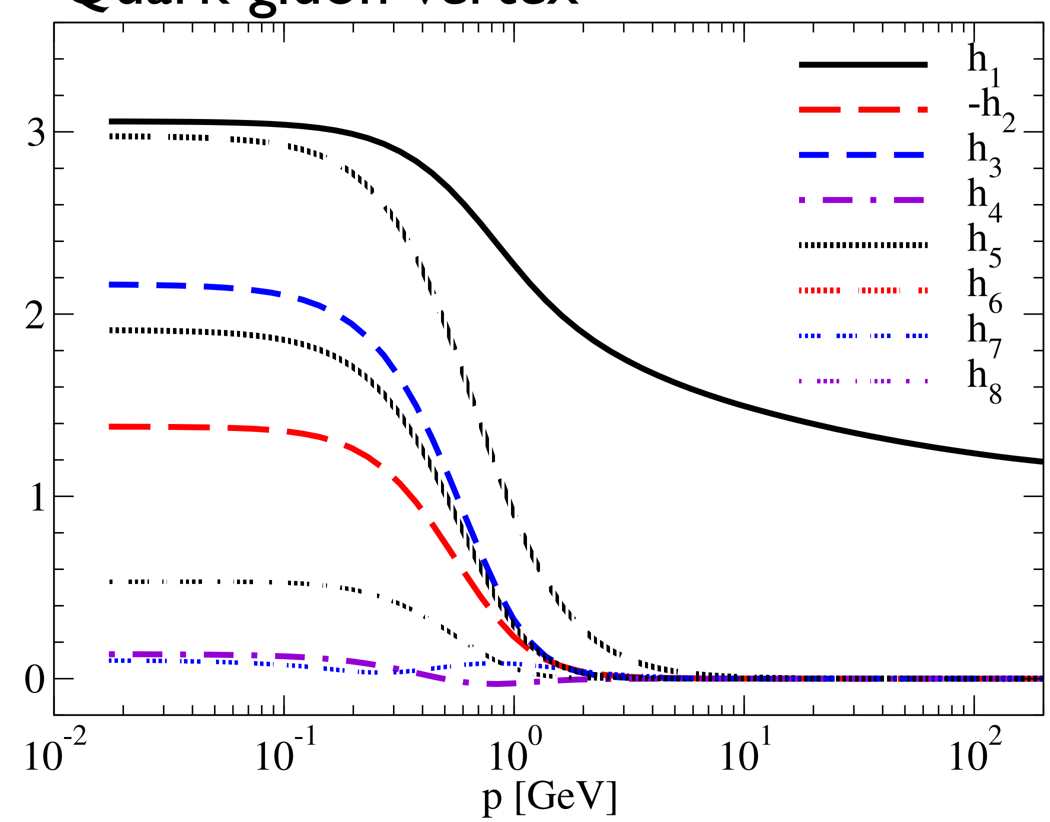
Williams, CF, Heupel, PRD 93 (2016) 034026



Gluon

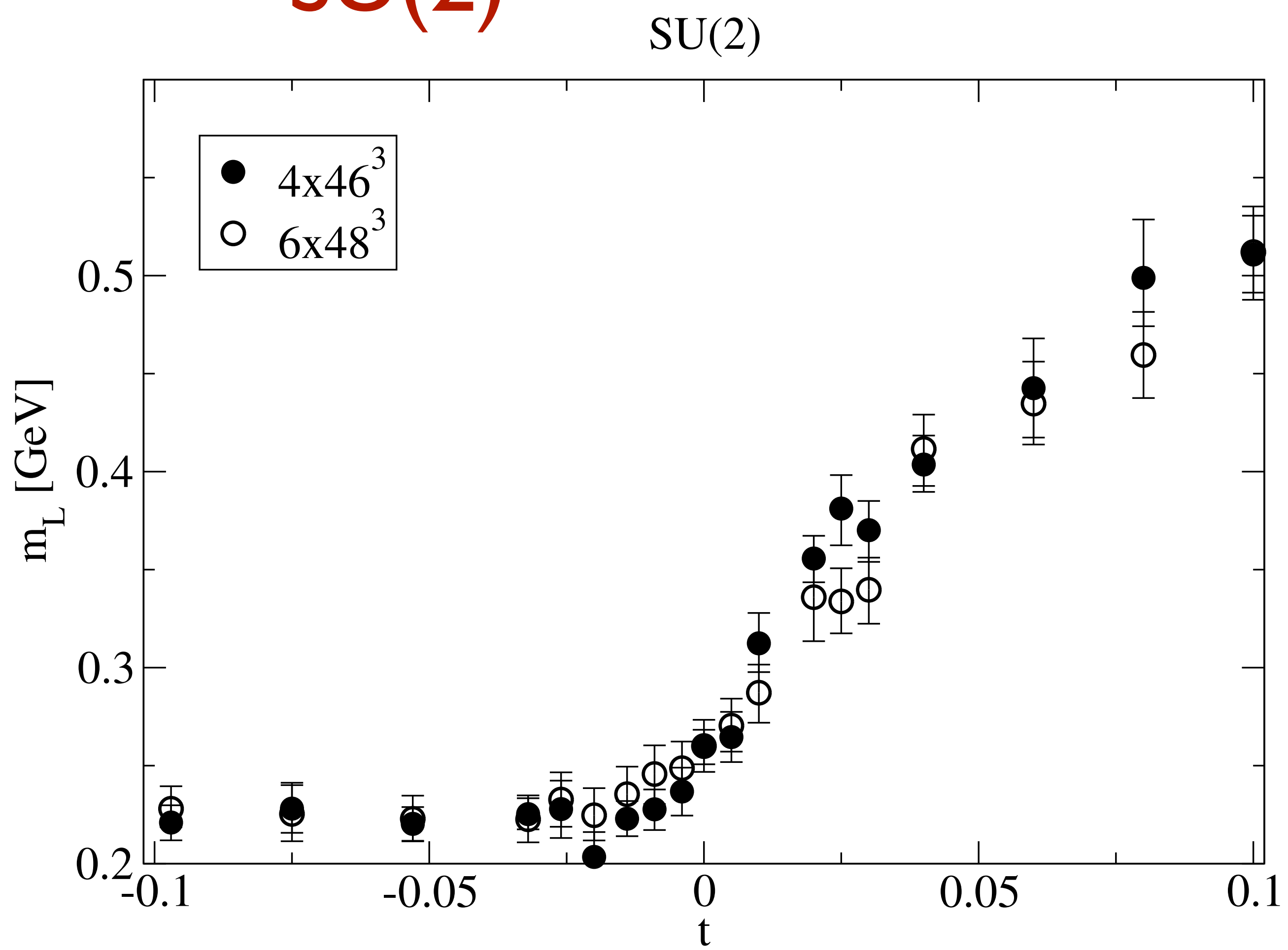


Quark-gluon-vertex

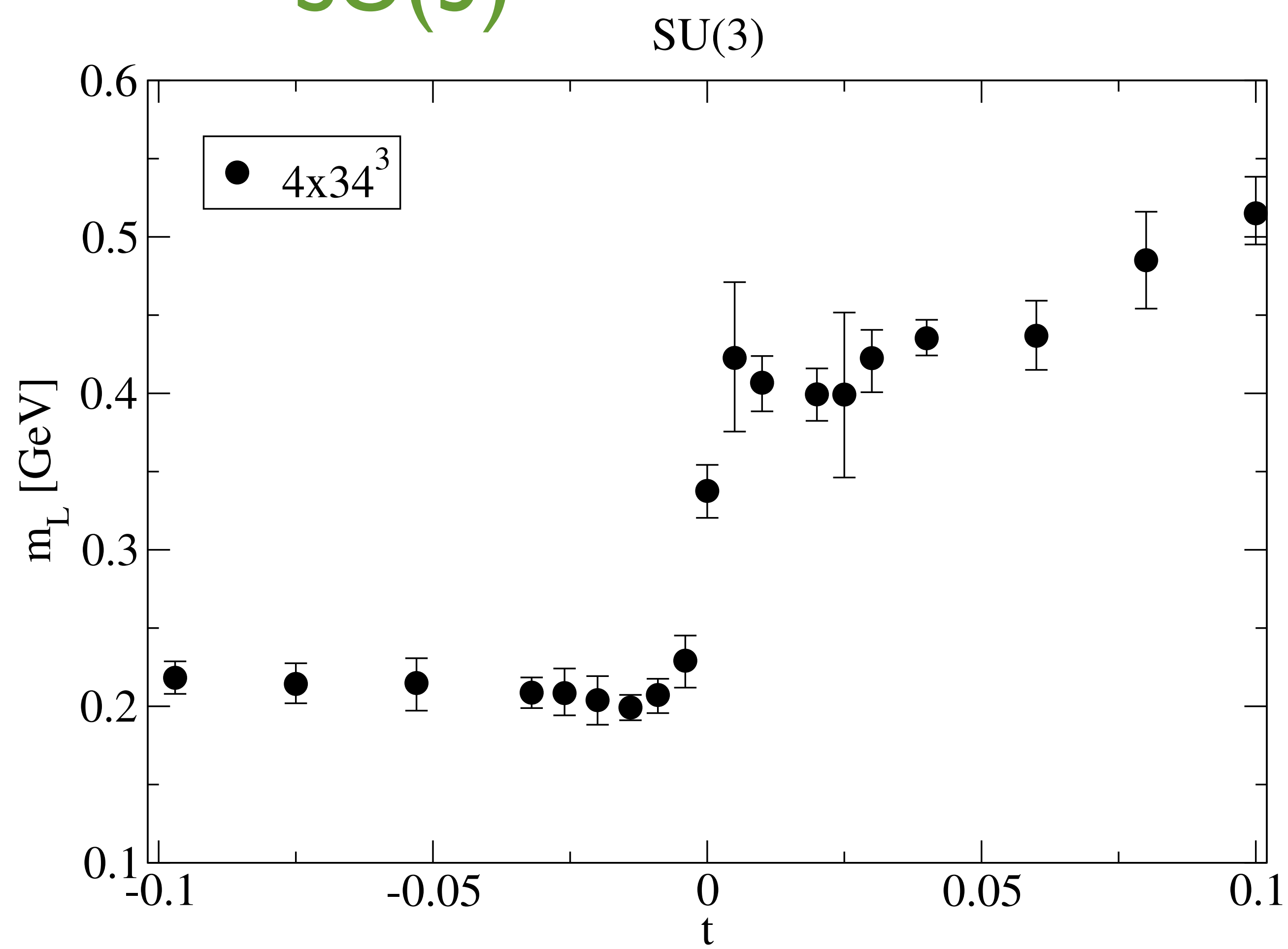


Gluon electric screening mass: SU(2) vs. SU(3)

SU(2)



SU(3)

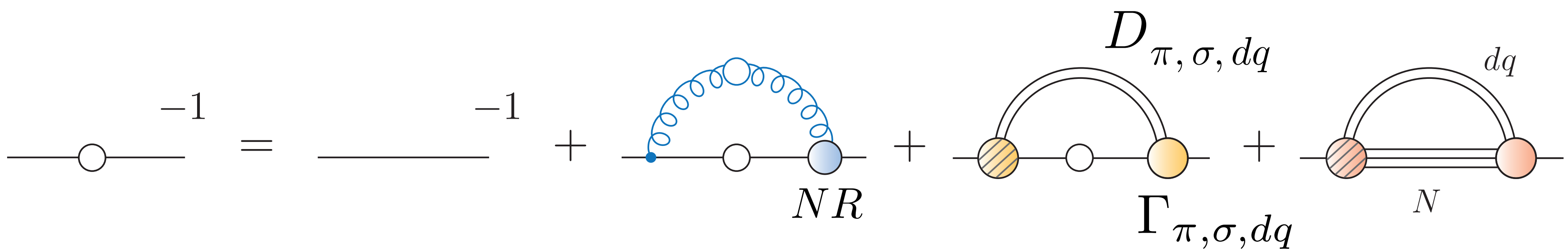


Maas, Pawlowski, Smekal, Spielmann, PRD 85 (2012) 034037
CF, Maas, Mueller, EPJC 68 (2010)

$t = (T-T_c)/T_c$

- phase transition of second and first order visible in electric screening mass

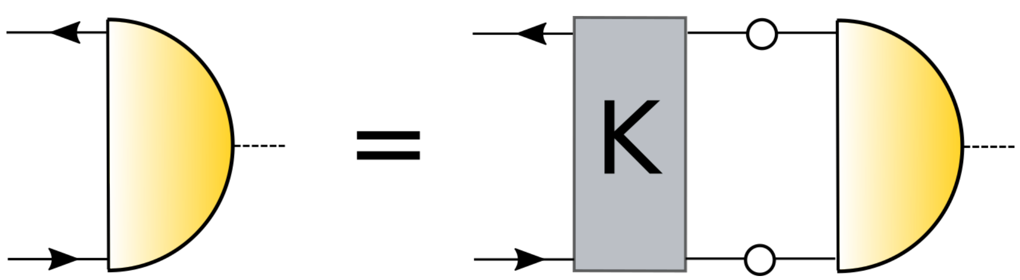
Meson effects at finite T and μ



$$D_\pi(p) = \frac{1}{p_4^2 + u^2(\vec{p}^{\,2} + m_\pi(T,\mu)^2)}$$

$$u = \frac{f_s}{f_t}$$

Son, Stephanov, PRD 66 (2002) 7

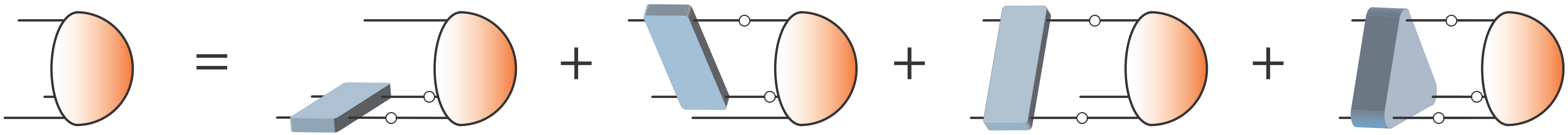


$$\Gamma_\pi(P,q) = \gamma_5\, E(P,q,T,\mu) + \ldots$$

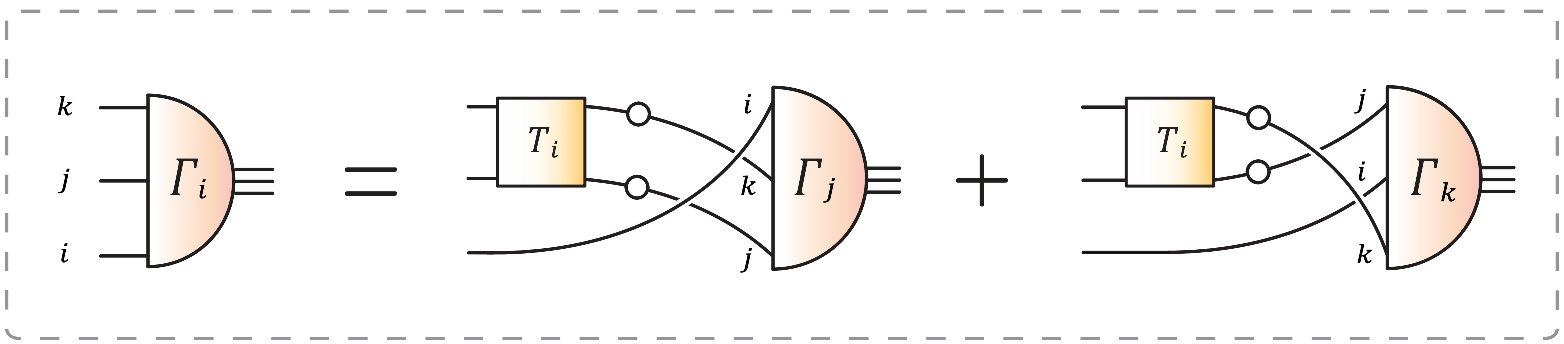
chiral limit: $\Gamma_\pi = \gamma_5 \dfrac{B}{f_t}$

Vacuum: Baryons from BSEs

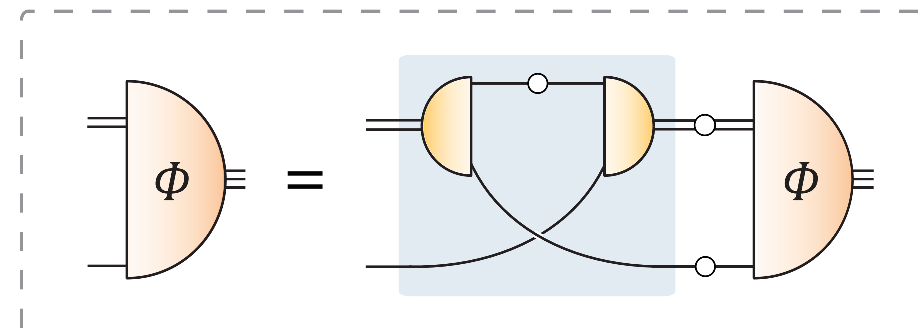
BSE for baryons (derived from equation of motion for G)



Faddeev equation (no three-body forces)

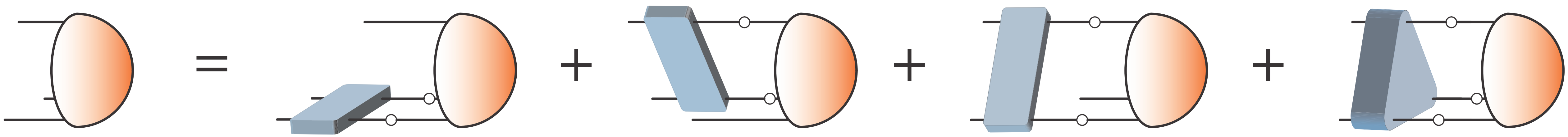


Diquark-quark



Vacuum: Baryons from BSEs

BSE for baryons (derived from equation of motion for G)



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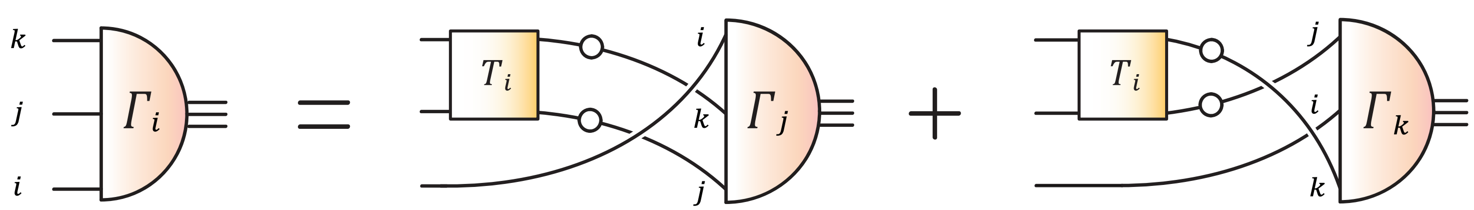
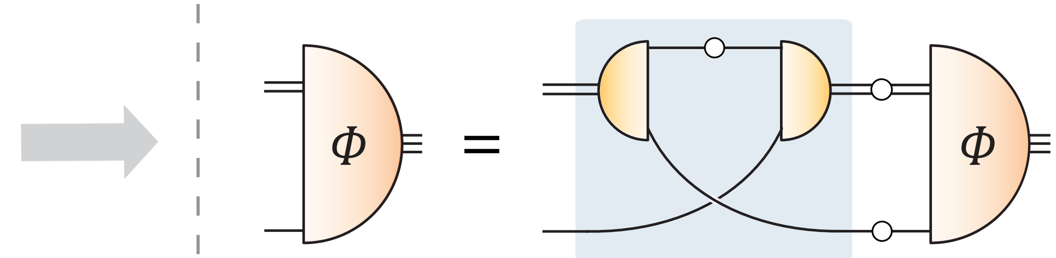
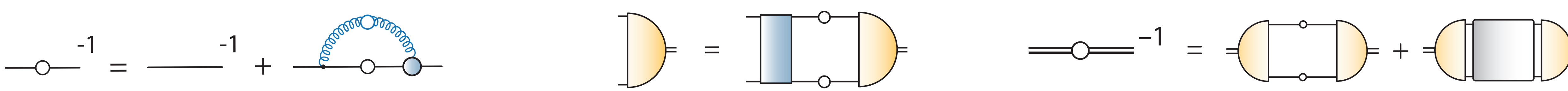


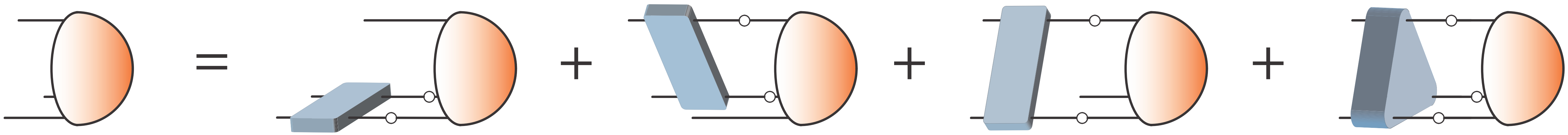
Diquark-quark



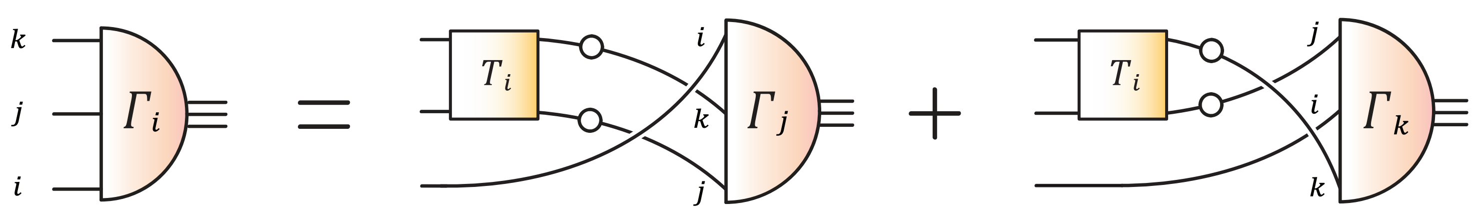


Vacuum: Baryons from BSEs

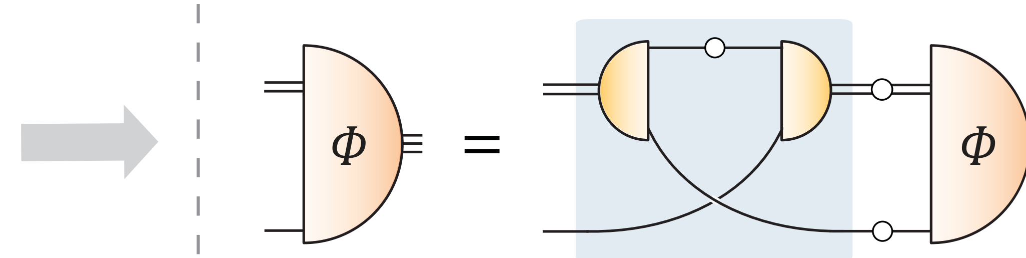
BSE for baryons (derived from equation of motion for G)

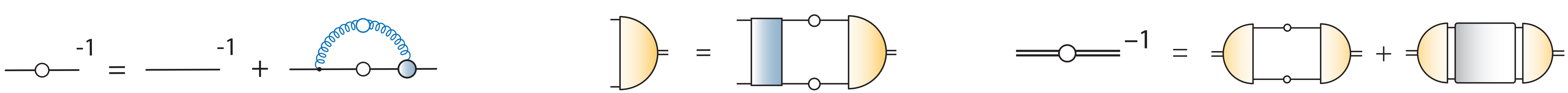


Faddeev equation (no three-body forces)

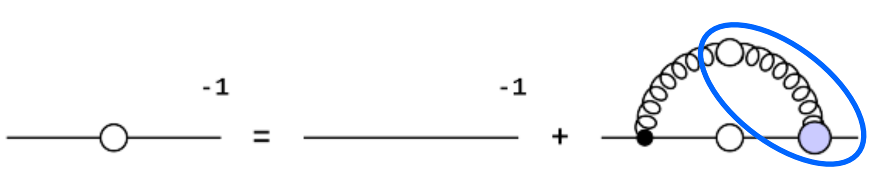


Diquark-quark

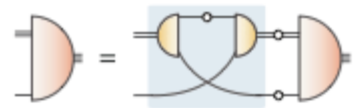
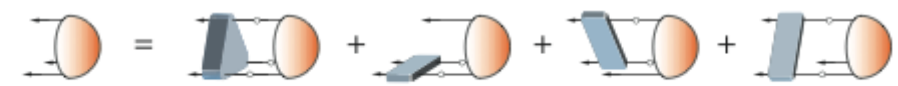




- Input: Non-perturbative quark, quark-gluon interaction (RL)

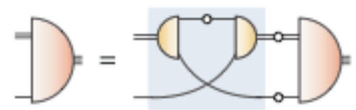
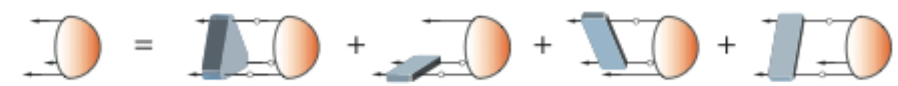


$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

	Quark-diquark			Three-quark		
						
	Contact interaction	**QCD-based model**	**DSE (RL)**	**RL**	**bRL**	**bRL + 3q**
N, Δ masses	√	√	√	√	√	. . .
N, Δ em. FFs	√	√	√	√		
$N \to \Delta\gamma$	√	√	√	. . .		
Roper	√	√		. . .		
$N \to N^*\gamma$	√	√		. . .		
$N^*(1535), \ldots$	. . .	. . .		. . .	. . .	
$N \to N^*\gamma$	. . .	. . .				
	Roberts et al	Oettel, Alkofer Roberts, Bloch Segovia et al.	Eichmann, Alkofer Nicmorus, Krassnigg	Eichmann, Alkofer Sanchis-Alepuz, CF	Sanchis-Alepuz, CF Williams	

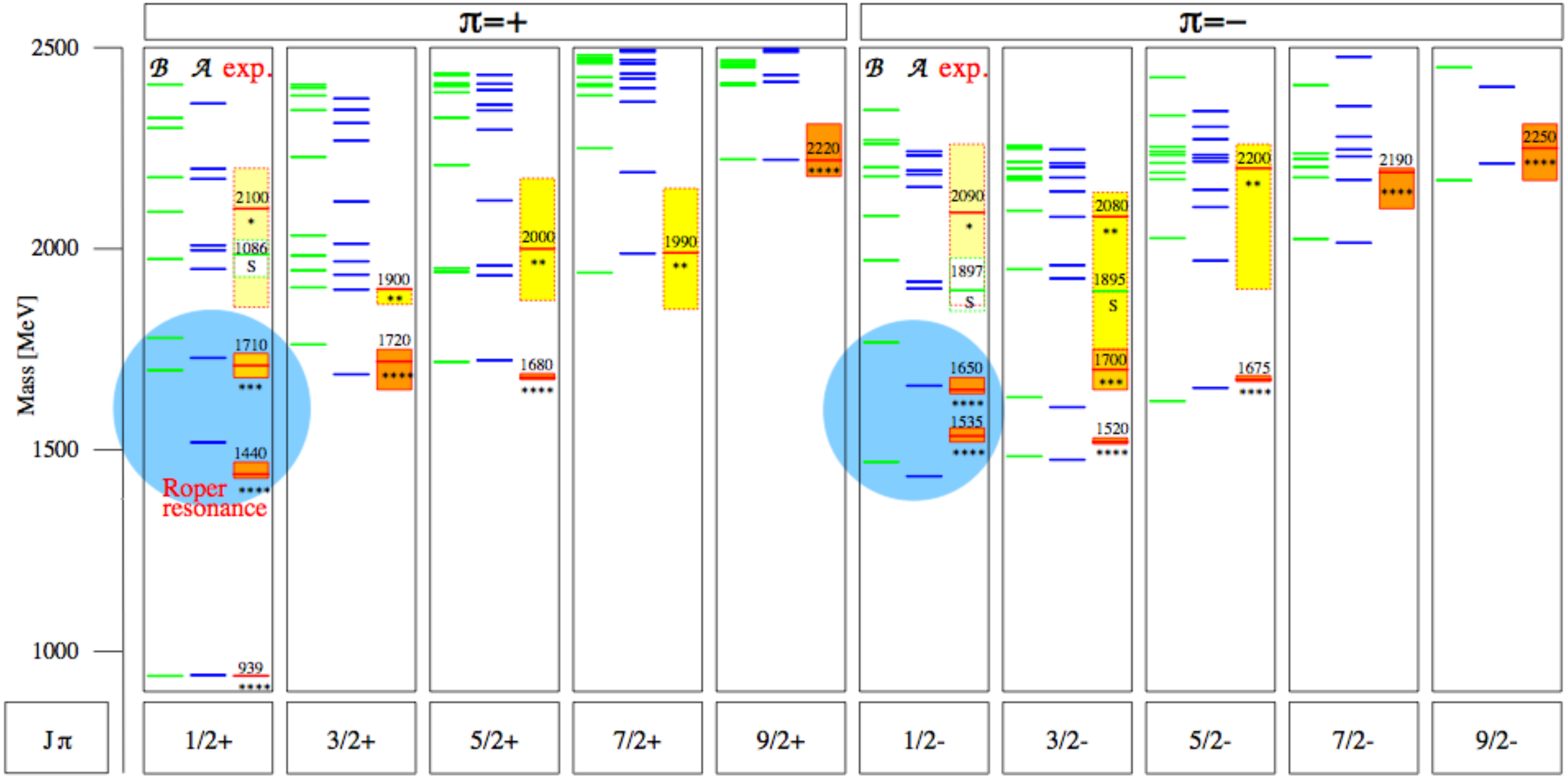
Eichmann, N*-Workshop, Trento 2015

Vacuum: DSE/Faddeev landscape

	Quark-diquark 			Three-quark 		
	Contact interaction	QCD-based model	DSE (RL)	RL	bRL	bRL + 3q
N, Δ masses	√	√	√	√	√	...
N, Δ em. FFs	√	√	√	√		
$N \to \Delta\gamma$	√	√	√	...		
Roper	√	√		...		
$N \to N^*\gamma$	√	√		...		
$N^*(1535), \ldots$	...	...		...	...	
$N \to N^*\gamma$	...	...				
	Roberts et al	Oettel, Alkofer Roberts, Bloch Segovia et al.	Eichmann, Alkofer Nicmorus, Krassnigg	Eichmann, Alkofer Sanchis-Alepuz, CF	Sanchis-Alepuz, CF Williams	

Eichmann, N*-Workshop, Trento 2015

Baryons: Quark model



Loring, Metsch, Petry, EPJA 10 (2001) 395

- 'missing resonances' - three-body vs. quark-diquark
- level ordering: $N\frac{1}{2}^{\pm}$ vs. $\Lambda\frac{1}{2}^{\pm}$