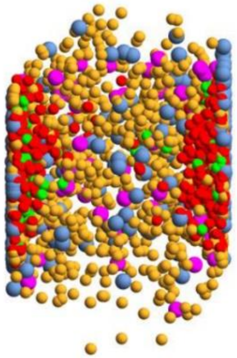
Transport description of strongly interacting systems

Elena Bratkovskaya

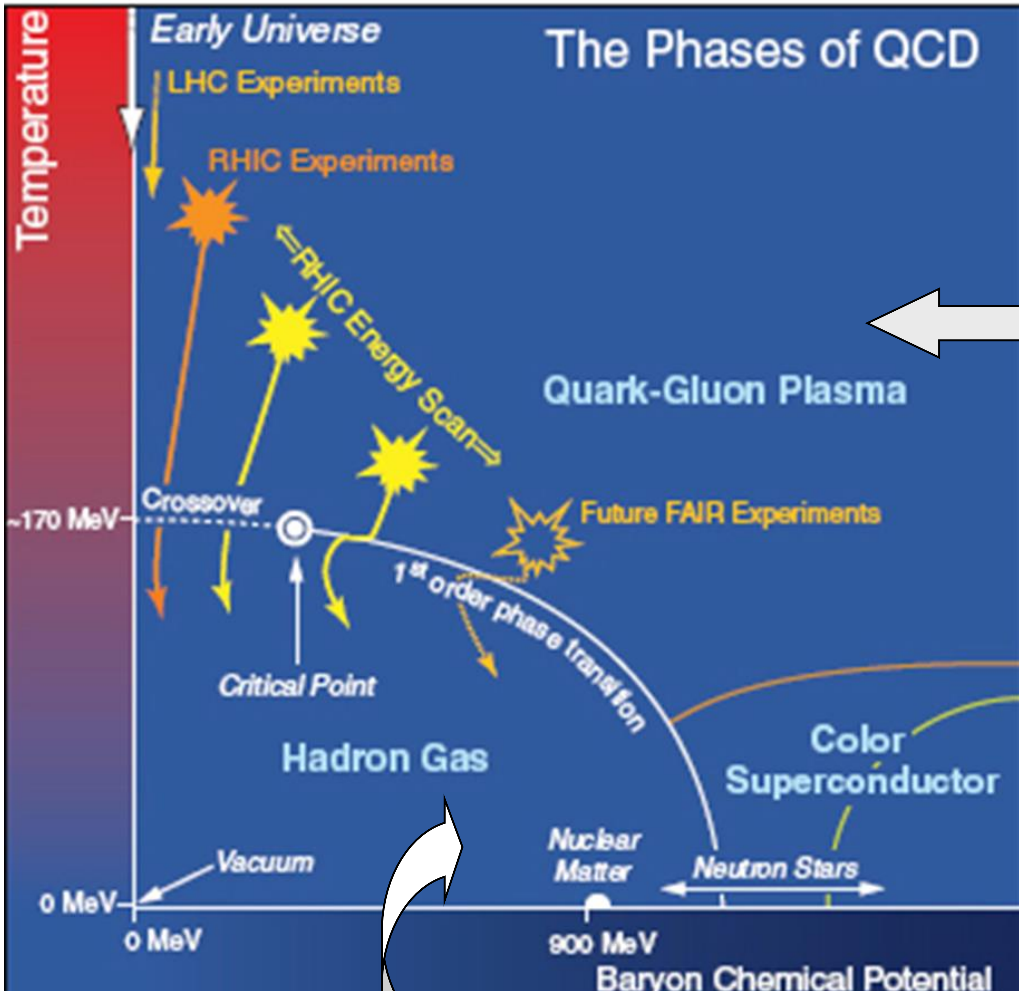
(GSI, Darmstadt & Uni. Frankfurt)

for the PHSD group

Workshop of the Network NA7-Hf-QGP of the European program 'STRONG-2020' and the HFHF, Hersonissos, Crete, Greece, October 4-8, 2021

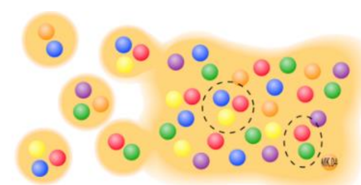


The ‚holy grail‘ of heavy-ion physics:



The phase diagram of QCD

- **Study of the phase transition from hadronic to partonic matter – Quark-Gluon-Plasma**



- **Search for the critical point**
- **Search for signatures of chiral symmetry restoration**
- **Study of the in-medium properties of hadrons at high baryon density and temperature**

Dynamical description of heavy-ion collisions

The goal:
to study the properties of strongly interacting matter under extreme conditions from a microscopic point of view

Realization:
to develop a dynamical microscopic transport approach
1) applicable for strongly interacting systems,
which includes:
2) phase transition from hadronic matter to QGP
3) chiral symmetry restoration

History: Semi-classical BUU equation

Ludwig Boltzmann

Boltzmann-Uehling-Uhlenbeck equation (non-relativistic formulation)
- propagation of particles in the self-generated Hartree-Fock mean-field potential *U(r,t)* with an on-shell collision term:

$$\frac{\partial}{\partial t} f(\vec{r},\vec{p},t) + \frac{\vec{p}}{m}\vec{\nabla}_{\vec{r}} f(\vec{r},\vec{p},t) - \vec{\nabla}_{\vec{r}} U(\vec{r},t)\,\vec{\nabla}_{\vec{p}} f(\vec{r},\vec{p},t) = \left(\frac{\partial f}{\partial t}\right)_{coll}$$

collision term: elastic and inelastic reactions

$f(\vec{r},\vec{p},t)$ **is the single particle phase-space distribution function**
- probability to find the particle at position *r* with momentum *p* at time *t*

- **self-generated Hartree-Fock mean-field potential:**

$$U(\vec{r},t) = \frac{1}{(2\pi\hbar)^3}\sum_{\beta_{occ}}\int d^3r'\, d^3p\;\; V(\vec{r}-\vec{r}',t)\; f(\vec{r}',\vec{p},t) \;+\; (Fock\;\; term)$$

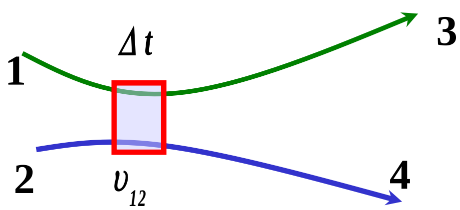
- **Collision term for 1+2→3+4 (let‘s consider fermions) :**

$$I_{coll} = \frac{4}{(2\pi)^3}\int d^3p_2\, d^3p_3 \int d\Omega\;\; |v_{12}|\,\delta^3(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4)\cdot\frac{d\sigma}{d\Omega}(1+2\rightarrow 3+4)\cdot P$$

Probability including Pauli blocking of fermions:

$$P = f_3 f_4 (1-f_1)(1-f_2) - f_1 f_2 (1-f_3)(1-f_4)$$

Gain term: 3+4→1+2 **Loss term: 1+2→3+4**



History: developments of relativistic transport models

Low energy HIC ⇒ **High energy HIC**

Non-relativistic semi-classical BUU

⇒ **Relativistic transport models**

'Relativistic Vlasov-Uehling-Uhlenbeck model for heavy-ion collisions'
Che-Ming Ko, Qi Li, Phys.Rev. C37 (1988) 2270

'Covariant Boltzmann-Uehling-Uhlenbeck approach for heavy-ion collisions'
Bernhard Blaettel, Volker Koch, Wolfgang Cassing, Ulrich Mosel, Phys.Rev. C38 (1988) 1767

'Relativistic BUU approach with momentum dependent mean fields'
T. Maruyama, B. Blaettel, W. Cassing, A. Lang, U. Mosel, K. Weber, Phys.Lett. B297 (1992) 228

'The Relativistic Landau-Vlasov method in heavy ion collisions'
C. Fuchs, H.H. Wolter, Nucl.Phys. A589 (1995) 732

. . .

Alternative: QMD (cf. talks by J. Aichelin, M. Bleicher)

Covariant transport equation

- **Covariant relativistic on-shell BUU equation :**

  from many-body theory by connected Green functions in phase-space + mean-field limit for the propagation part (VUU)

$$\left\{ \left(\Pi_\mu - \Pi_\nu (\partial_\mu^p U_V^\nu) - m^* (\partial_\mu^p U_S^\nu) \right) \partial_x^\mu + \left(\Pi_\nu (\partial_\mu^x U_V^\nu) + m^* (\partial_\mu^x U_S^\nu) \right) \partial_p^\mu \right\} f(x,p) = I_{coll}$$

$$I_{coll} \equiv \sum_{2,3,4} \int d2\ d3\ d4\ \ [G^+ G]_{1+2\to 3+4}\ \delta^4(\Pi + \Pi_2 - \Pi_3 - \Pi_4)$$

$$d2 \equiv \frac{d^3 p_2}{E_2}$$

$$\times \{ f(x,p_3)\ f(x,p_4)\ (1 - f(x,p))\ (1 - f(x,p_2))$$

Gain term $3+4 \to 1+2$

$$- f(x,p)\ f(x,p_2)\ (1 - f(x,p_3))\ (1 - f(x,p_4))\ \}$$

Loss term $1+2 \to 3+4$

where $\partial_\mu^x \equiv (\partial_t, \vec{\nabla}_r)$

$m^*(x,p) = m + U_s(x,p)$ - effective mass

$\Pi_\mu(x,p) = p_\mu - U_\mu(x,p)$ - effective momentum

$U_s(x,p)$, $U_\mu(x,p)$ are scalar and vector part of particle self-energies

$\delta(\Pi_\mu \Pi^\mu - m^{*2})$ – mass-shell constraint

W. Ehehalt, W. Cassing, Nucl. Phys. A 602 (1996) 449

Dynamical transport model: collision terms

- BUU eq. for different particles of type $i=1,\dots n$

$$Df_i \equiv \frac{d}{dt} f_i = I_{coll}\left[f_1, f_2, \dots, f_n\right]$$

Drift term=Vlasov eq. collision term

$$i: \quad Baryons: \; p, n, \Delta(1232), N(1440), N(1535), \dots, \Lambda, \Sigma, \Sigma^*, \Xi, \Omega; \; \Lambda_C$$

$$Mesons: \; \pi, \eta, K, \bar{K}, \rho, \omega, K^*, \eta', \phi, a_1, \dots, D, \bar{D}, J/\Psi, \Psi', \dots$$

➔ coupled set of BUU equations for different particles of type $i=1,\dots n$

$$\begin{cases} Df_N = I_{coll}\left[f_N, f_\Delta, f_{N(1440)}, \dots, f_\pi, f_\rho, \dots\right] \\ Df_\Delta = I_{coll}\left[f_N, f_\Delta, f_{N(1440)}, \dots, f_\pi, f_\rho, \dots\right] \\ \dots \\ Df_\pi = I_{coll}\left[f_N, f_\Delta, f_{N(1440)}, \dots, f_\pi, f_\rho, \dots\right] \\ \dots \end{cases}$$

Elementary hadronic interactions

Consider **all possible interactions – eleastic and inelastic collisions -** for the sytem of *(N,R,m)*, where *N*-nucleons, *R*- resonances, *m*-mesons, and **resonance decays**

Low energy collisions:

- binary 2←→2 and 2←→3(4) reactions
- 1←→2 : formation and **decay** of baryonic and mesonic resonances

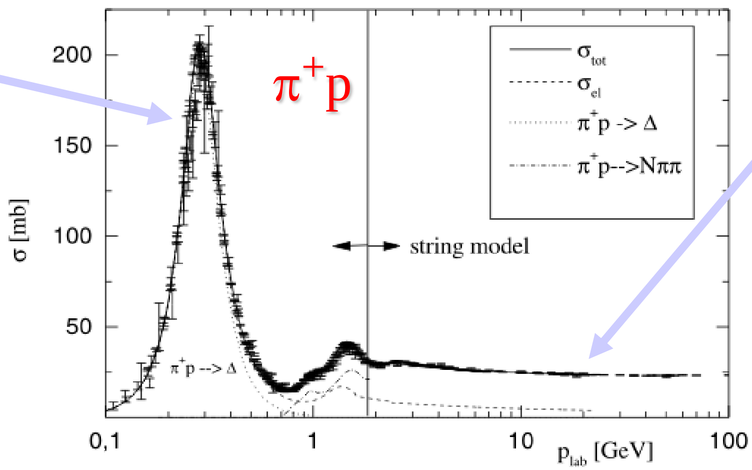
BB ←→ B´B´
BB ←→ B´B´m
mB ←→ m´B´
mB ←→ B´
mm ←→ m´m´
mm ←→ m´ . . .

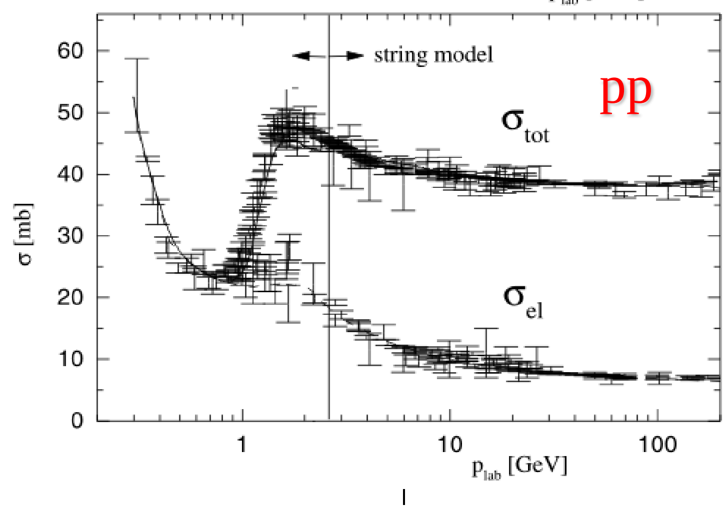
Baryons:

B = p, n, Δ(1232), N(1440), N(1535), ...

Mesons:

$M = \pi, \eta, \rho, \omega, \phi, \ldots$





High energy collisions:

(above $s^{1/2}$~2.5 GeV)

Inclusive particle production:

BB→X , mB→X, mm→X

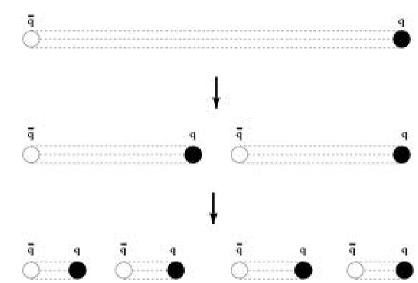
X =many particles

described by

string formation and decay

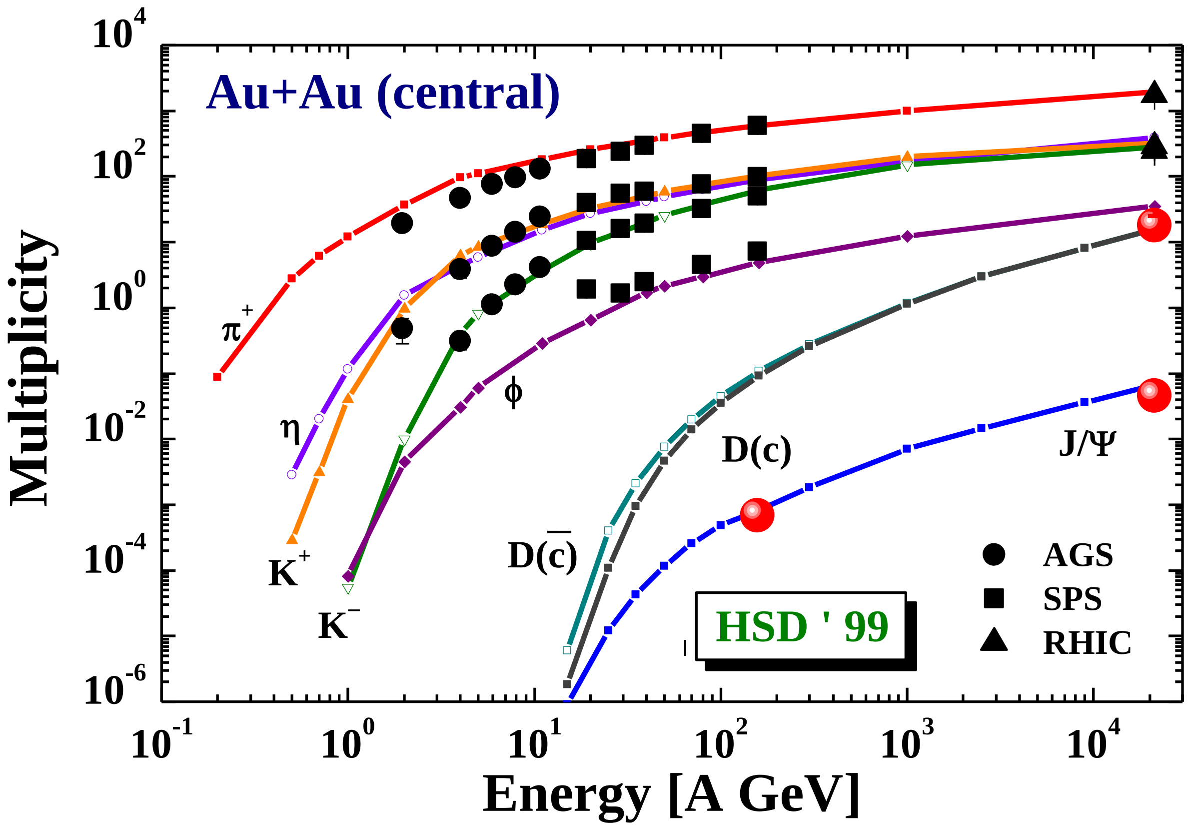
(string = excited color singlet states *q-qq*, *q-qbar*)

using LUND string model



Hadron-String-Dynamics – a microscopic transport model for heavy-ion reactions

- very good description of particle production in pp, pA, pA, AA reactions
- unique description of nuclear dynamics from low (~100 MeV) to ultrarelativistic (>20 TeV) energies



HSD predictions from 1999; data from the new millenium

W. Cassing, E.B., Phys.Rept. 308 (1999) 65

From weakly to strongly interacting systems

In-medium effects (on hadronic or partonic levels!) = changes of particle properties in the hot and dense medium

Examples: **hadronic medium** - vector mesons, strange mesons
QGP – dressing of partons

Many-body theory:
Strong interaction ➔ **large width** ➔ broad spectral function ➔ **quantum object**

Semi-classical on-shell BUU: applies for small collisional width, i.e. for a weakly interacting systems of particles

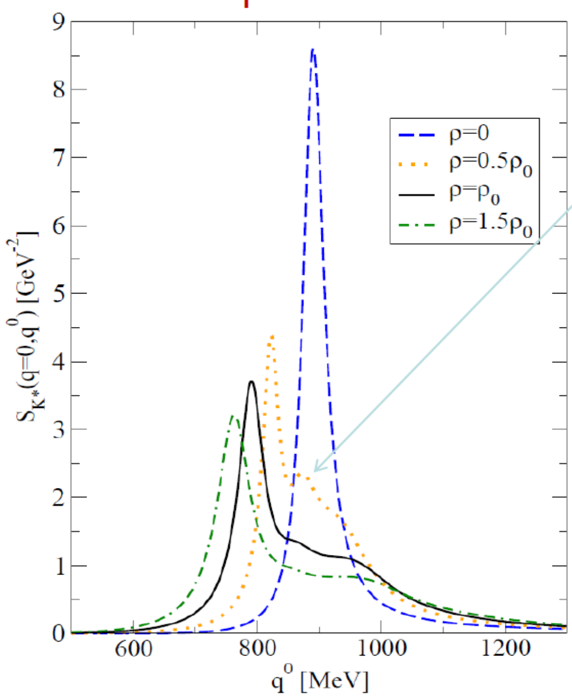
- **How to describe the dynamics of broad strongly interacting quantum states in transport theory?**

❑ semi-classical BUU

first order gradient expansion of quantum Kadanoff-Baym equations

❑ generalized transport equations based on Kadanoff-Baym dynamics

Kbar* spectral function



$\Lambda(1783)N^{-1}$ and $\Sigma(1830)N^{-1}$ exitations

Barcelona / Valencia group

Dynamical description of strongly interacting systems

- **Quantum field theory ➔**

Kadanoff-Baym dynamics for resummed single-particle Green functions $S^<$

$$\hat{S}^{-1}_{0x}\, S^<_{xy} = \Sigma^{ret}_{xz} \odot S^<_{zy} + \Sigma^<_{xz} \odot S^{adv}_{zy}$$

(1962)

Integration over the intermediate spacetime

Green functions $S^<$ / self-energies Σ :

$$iS^<_{xy} = \eta \langle \{ \Phi^+(y) \Phi(x) \} \rangle$$

$$iS^>_{xy} = \langle \{ \Phi(y) \Phi^+(x) \} \rangle$$

$$iS^c_{xy} = \langle T^c \{ \Phi(x) \Phi^+(y) \} \rangle \quad - causal$$

$$iS^a_{xy} = \langle T^a \{ \Phi(x) \Phi^+(y) \} \rangle \quad - anticausal$$

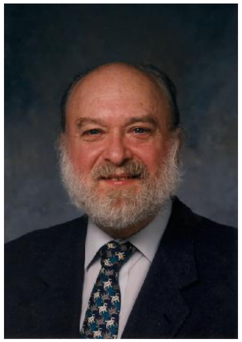
$$S^{ret}_{xy} = S^c_{xy} - S^<_{xy} = S^>_{xy} - S^a_{xy} \quad - retarded$$

$$S^{adv}_{xy} = S^c_{xy} - S^>_{xy} = S^<_{xy} - S^a_{xy} \quad - advanced$$

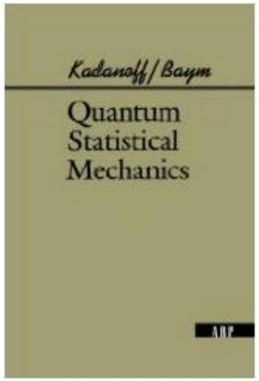
$$\hat{S}^{-1}_{0x} \equiv -(\partial^\mu_x \partial^x_\mu + M^2_0)$$

$$\eta = \pm 1 (bosons\ /\ fermions)$$

$$T^a (T^c) - (anti-) time - ordering\ operator$$



Leo Kadanoff



Gordon Baym

1st application for spacially homodeneous system with deformed Fermi sphere:
P. Danielewicz, Ann. Phys. 152, 305 (1984); … H.S. Köhler, Phys. Rev. 51, 3232 (1995); …

From Kadanoff-Baym equations to generalized transport equations

After the first order gradient expansion of the Wigner transformed Kadanoff-Baym equations and separation into the real and imaginary parts one gets:

Generalized transport equations (GTE):

drift term | Vlasov term | backflow term | collision term = ‚gain‘ - ‚loss‘ term

$$\diamond \{ P^2 - M_0^2 - Re\Sigma^{ret}_{XP} \} \{ S^<_{XP} \} - \diamond \{ \Sigma^<_{XP} \} \{ Re S^{ret}_{XP} \} = \frac{i}{2} [\Sigma^>_{XP} S^<_{XP} - \Sigma^<_{XP} S^>_{XP}]$$

Backflow term incorporates the off-shell behavior in the particle propagation
! vanishes in the quasiparticle limit $A_{XP} \rightarrow \delta(p^2-M^2)$

- GTE: Propagation of the Green‘s function $iS^<_{XP}=A_{XP}N_{XP}$, which carries information not only on the number of particles (N_{XP}), but also on their properties, interactions and correlations (via A_{XP})

Botermans-Malfliet (1990)

- Spectral function:

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - Re\Sigma^{ret}_{XP})^2 + \Gamma^2_{XP}/4}$$

$\Gamma_{XP} = -Im\,\Sigma^{ret}_{XP} = 2\,p_0\Gamma$ – ‚width‘ of spectral function
= reaction rate of particle (at space-time position X)

4-dimentional generalizaton of the Poisson-bracket:

$$\diamond \{ F_1 \} \{ F_2 \} := \frac{1}{2} \left(\frac{\partial F_1}{\partial X_\mu} \frac{\partial F_2}{\partial P^\mu} - \frac{\partial F_1}{\partial P_\mu} \frac{\partial F_2}{\partial X^\mu} \right)$$

- Life time $\tau = \frac{\hbar c}{\Gamma}$

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445

General testparticle off-shell equations of motion

W. Cassing , S. Juchem, NPA 665 (2000) 377; 672 (2000) 417; 677 (2000) 445

- Employ testparticle Ansatz for the real valued quantity $i\, S^{<}_{XP}$

$$F_{XP} = A_{XP} N_{XP} = i\, S^{<}_{XP} \sim \sum_{i=1}^{N} \delta^{(3)}(\vec{X} - \vec{X}_i(t))\ \delta^{(3)}(\vec{P} - \vec{P}_i(t))\ \delta(P_0 - \epsilon_i(t))$$

insert in generalized transport equations and determine equations of motion !

➔ Generalized testparticle Cassing-Juchem off-shell equations of motion for the time-like particles:

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[2\,\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} Re\Sigma^{ret}_{i} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right],$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1-C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial Re\Sigma^{ret}_{(i)}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right],$$

with $F_{(i)} \equiv F(t, \vec{X}_i(t), \vec{P}_i(t), \epsilon_i(t))$

$$C_{(i)} = \frac{1}{2\epsilon_i} \left[\frac{\partial}{\partial \epsilon_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial}{\partial \epsilon_i} \Gamma_{(i)} \right]$$

Note: the common factor $1/(1-C_{(i)})$ can be absorbed in an ‚eigentime‘ of particle (i) !

Collision term in off-shell transport models

Collision term for reaction 1+2->3+4:

$$I_{coll}(X,\vec{P},M^2) = Tr_2 Tr_3 Tr_4 A(X,\vec{P},M^2) A(X,\vec{P}_2,M_2^2) A(X,\vec{P}_3,M_3^2) A(X,\vec{P}_4,M_4^2)$$

$$|G((\vec{P},M^2)+(\vec{P}_2,M_2^2)\rightarrow(\vec{P}_3,M_3^2)+(\vec{P}_4,M_4^2))|^2_{\mathcal{A},\mathcal{S}}\ \delta^{(4)}(P+P_2-P_3-P_4)$$

$$[\, N_{X\vec{P}_3M_3^2}\, N_{X\vec{P}_4M_4^2}\, \bar{f}_{X\vec{P}M^2}\, \bar{f}_{X\vec{P}_2M_2^2} \;-\; N_{X\vec{P}M^2}\, N_{X\vec{P}_2M_2^2}\, \bar{f}_{X\vec{P}_3M_3^2}\, \bar{f}_{X\vec{P}_4M_4^2}\,]$$

‚gain‘ term ‚loss‘ term

with $\bar{f}_{X\vec{P}M^2} = 1 + \eta\, N_{X\vec{P}M^2}$ and $\eta = \pm 1$ for bosons/fermions, respectively.

The trace over particles 2,3,4 reads explicitly

for fermions

$$Tr_2 = \sum_{\sigma_2,\tau_2} \frac{1}{(2\pi)^4} \int d^3P_2 \frac{dM_2^2}{2\sqrt{\vec{P}_2^2 + M_2^2}}$$

for bosons

$$Tr_2 = \sum_{\sigma_2,\tau_2} \frac{1}{(2\pi)^4} \int d^3P_2 \frac{dP_{0,2}^2}{2}$$

additional integration

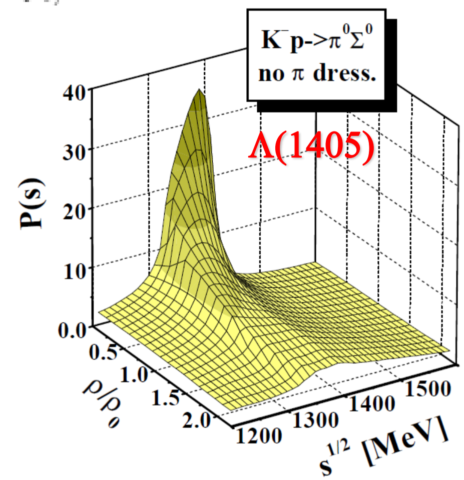
The transport approach and the particle spectral functions are fully determined once the in-medium transition amplitudes G are known in their off-shell dependence!

In-medium transition rates: G-matrix approach

Need to know in-medium transition amplitudes G and their off-shell dependence

$$|G((\vec{P},M^2)+(\vec{P}_2,M_2^2)\rightarrow(\vec{P}_3,M_3^2)+(\vec{P}_4,M_4^2))|^2_{\mathcal{A},\mathcal{S}}$$

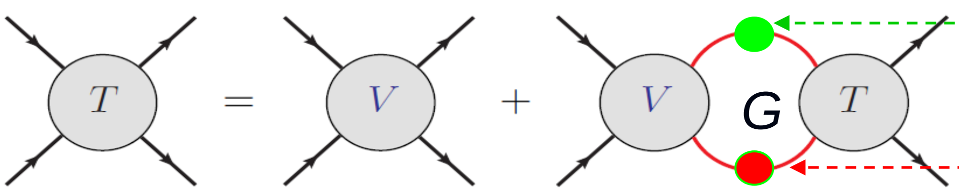
Coupled channel G-matrix approach



Transition probability :

$$P_{1+2\rightarrow 3+4}(s) = \int d\cos(\theta)\ \frac{1}{(2s_1+1)(2s_2+1)} \sum_i \sum_\alpha\ G^\dagger G$$

with G(p,ρ,T) - G-matrix from the solution of coupled-channel equations:



- Meson selfenergy and spectral function
- Baryons: Pauli blocking and potential dressing

$$\mathcal{T}_{ij}(\rho,T) = V_{ij} + V_{il}\ G_l(\rho,T)\ \mathcal{T}_{lj}(\rho,T)$$

For strangeness:
D. Cabrera, L. Tolos, J. Aichelin, E.B., PRC C90 (2014) 055207; W. Cassing, L. Tolos, E.B., A. Ramos, NPA727 (2003) 59; T. Song et al., PRC 103, 044901 (2021)

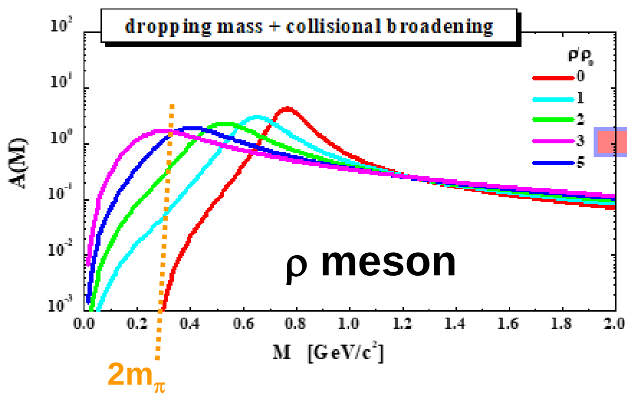
In-medium
$\rho >> \rho_0$

Off-shell vs. on-shell transport dynamics

Time evolution of the mass distribution of ρ and ω mesons for central C+C collisions (b=1 fm) at 2 A GeV for dropping mass + collisional broadening scenario

$$A(M,p,\rho) = \frac{2}{\pi} \frac{M^2 \Gamma_{tot}(M,p,\rho)}{(M^2 - M_0^2 - Re\Sigma^{ret}) + \left(M\Gamma_{tot}(M,p,\rho)\right)^2},$$

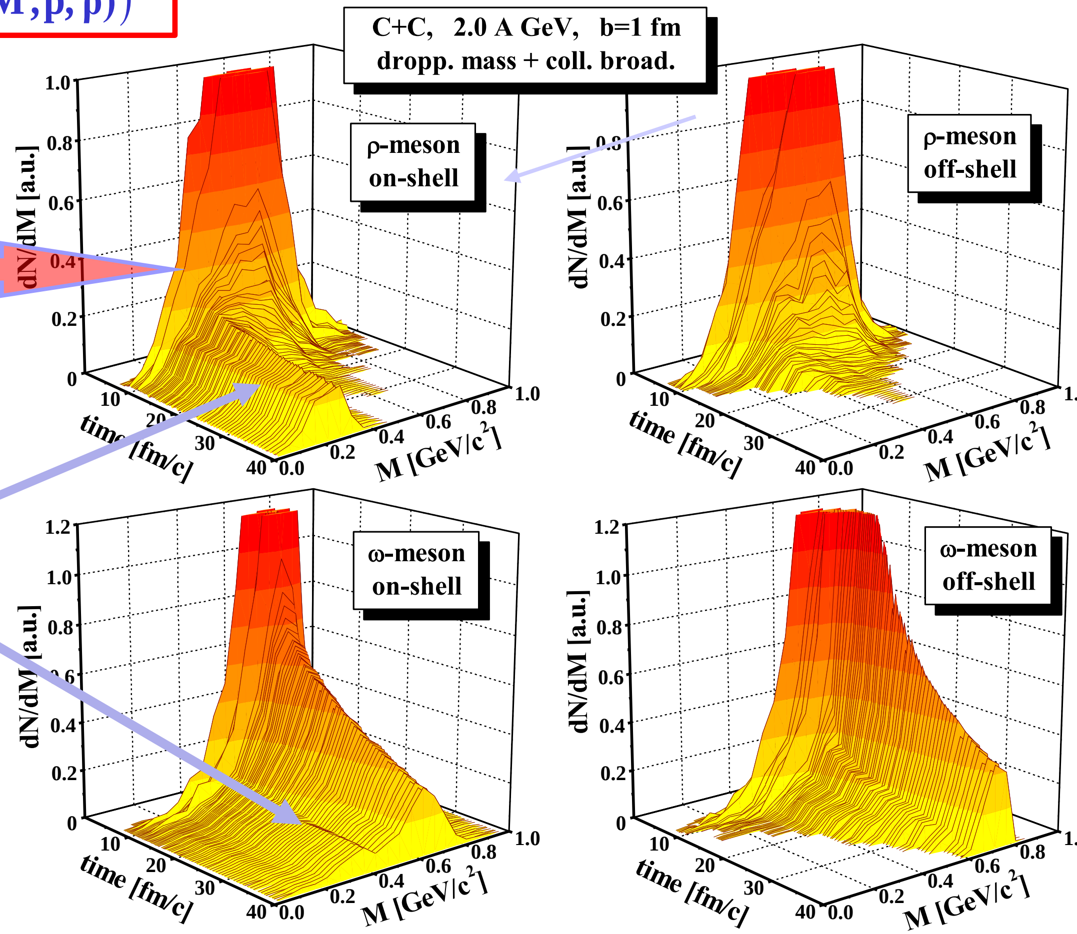
width $\Gamma \sim -Im\,\Sigma^{ret}/M$



On-shell BUU:
low mass ρ and ω mesons live forever (and shine ‚fake' dileptons)!

The off-shell spectral function becomes on-shell in the vacuum dynamically by propagation through the medium!

On-shell | Off-shell



E.L.B. &W. Cassing, NPA 807 (2008) 214

Advantages of Kadanoff-Baym dynamics vs Boltzmann

Kadanoff-Baym equations:

- propagate two-point Green functions
  $G^<(x,p) \rightarrow A(x,p) * N(x,p)$
  in 8 dimensions $x=(t,\vec{r})$ $p=(p_0,\vec{p})$
- $G^<$ carries information not only on the occupation number N_{XP}, but also on the particle properties, interactions and correlations via spectral function A_{XP}

Boltzmann equations

- propagate phase space distribution function $f(\vec{r},\vec{p},t)$ in 6+1 dimensions
- works well for small coupling
  = weakly interacting system,
  ➔ on-shell approach

- Applicable for strong coupling = strongly interaction system
- Includes memory effects (time integration) and off-shell transitions in collision term
- Dynamically generates a broad spectral function for strong coupling
- KB can be solved exactly for model cases as Φ^4 – theory
- KB can be solved in 1st order gradient expansion in terms of generalized transport equations (in test particle ansatz) for realistic systems of HICs

Detailed balance on the level of 2←→n:
treatment of multi-particle collisions in transport approaches

W. Cassing, NPA 700 (2002) 618

Generalized off-shell collision integral for *n* ←→ *m* reactions:

$$I_{coll} = \sum_n \sum_m I_{coll}[n \leftrightarrow m]$$

$$I^i_{coll}[n \leftrightarrow m] =$$
$$\frac{1}{2} N^m_n \sum_\nu \sum_\lambda \left(\frac{1}{(2\pi)^4}\right)^{n+m-1} \int \left(\prod_{j=2}^{n} d^4p_j \ A_j(x,p_j)\right) \left(\prod_{k=1}^{m} d^4p_k \ A_k(x,p_k)\right)$$
$$\times A_i(x,p) \ W_{n,m}(p,p_j;i,\nu \mid p_k;\lambda) \ (2\pi)^4 \ \delta^4(p^\mu + \sum_{j=2}^{n} p^\mu_j - \sum_{k=1}^{m} p^\mu_k)$$
$$\times [\tilde{f}_i(x,p) \prod_{k=1}^{m} f_k(x,p_k) \prod_{j=2}^{n} \tilde{f}_j(x,p_j) - f_i(x,p) \prod_{j=2}^{n} f_j(x,p_j) \prod_{k=1}^{m} \tilde{f}_k(x,p_k)].$$

$\tilde{f} = 1 + \eta f$ **is Pauli-blocking or Bose-enhancement factors;**
η=1 for bosons and η=-1 for fermions

$W_{n,m}(p,p_j;i,\nu \mid p_k;\lambda)$ **is a transition matrix element squared**

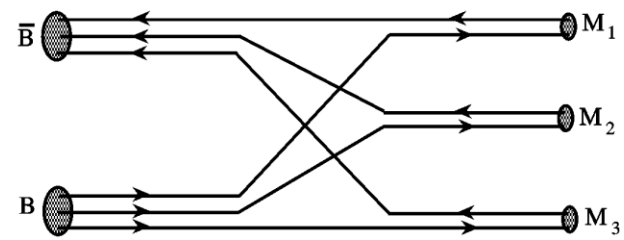
Multi-meson fusion in heavy-ion reactions

W. Cassing, NPA 700 (2002) 618
E. Seifert, W. Cassing, PRC 97 (2018) 024913, (2018) 044907

Multi-meson fusion reactions

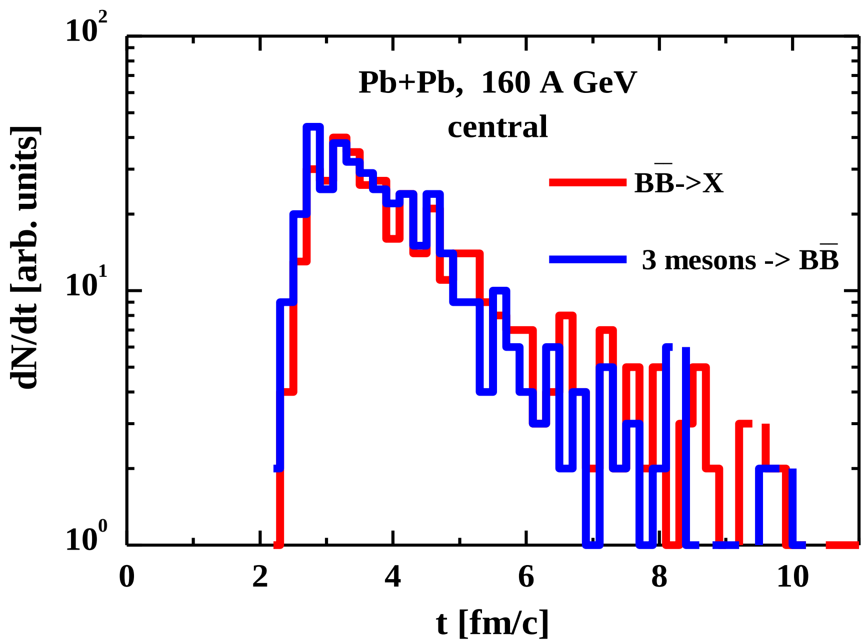
$m_1+m_2+...+m_n$ ←→ B+Bbar

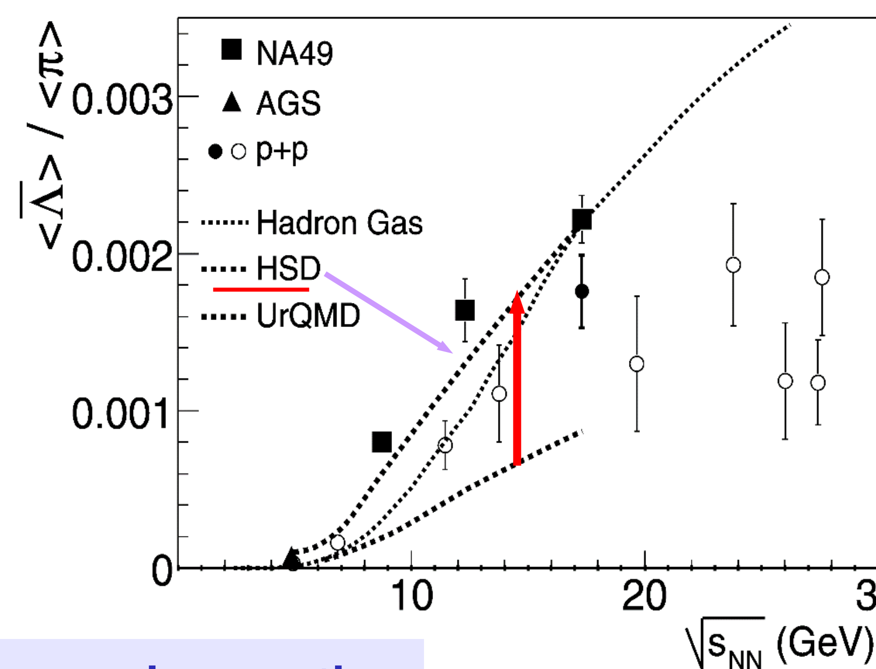
m=π,ρ,ω,.. B=p,Λ,Σ,Ξ,Ω, (>2000 channels)



2←→3

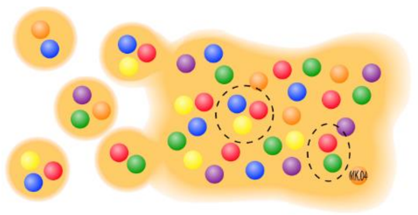
- ❑ important for anti-proton, anti-Λ, anti-Ξ, anti- Ω dynamics !





→ approximate equilibrium of annihilation and recreation

Goal: microscopic transport description of the partonic and hadronic phase



Problems:

- How to model a QGP phase in line with lQCD data?
- How to solve the hadronization problem?

Ways to go:

pQCD based models:

- QGP phase: pQCD cascade
- hadronization: quark coalescence

➔ AMPT, HIJING, BAMPS

‚Hybrid' models:

- QGP phase: hydro with QGP EoS
- hadronic freeze-out: after burner - hadron-string transport model

➔ Hybrid-UrQMD

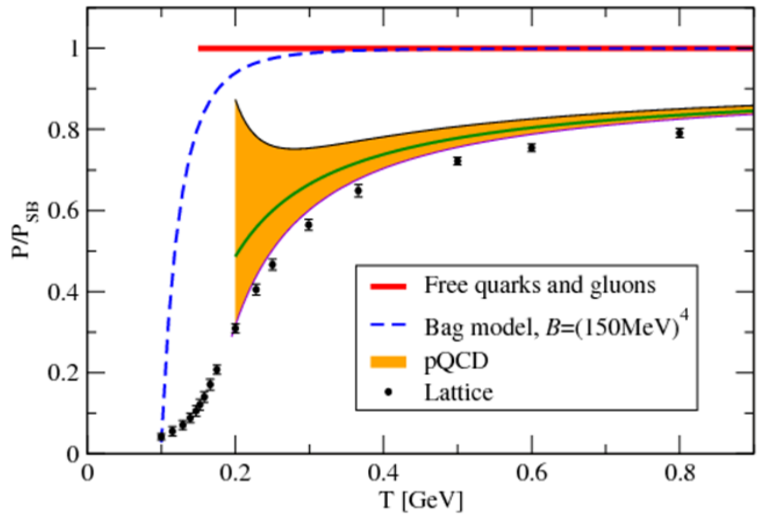
- microscopic transport description of the partonic and hadronic phase in terms of strongly interacting dynamical quasi-particles and off-shell hadrons

➔ PHSD

Degrees-of-freedom of QGP

❖ lQCD gives QGP EoS at finite μ_B

! need to be interpreted in terms of degrees-of-freedom



Non-perturbative QCD ← pQCD

pQCD:

- ❑ weakly interacting system
- ❑ massless quarks and gluons

Thermal QCD
= QCD at high parton densities:

- ❑ strongly interacting system
- ❑ massive quarks and gluons

➔ quasiparticles
= effective degrees-of-freedom

❖ How to learn about degrees-of-freedom of QGP?

Theory ⟺ HIC experiments

Dynamical QuasiParticle Model (DQPM) - Basic ideas:

DQPM describes QCD properties in terms of ‚resummed' single-particle Green's functions (propagators) – in the sense of a two-particle irreducible (2PI) approach:

gluon propagator: $\Delta^{-1} = P^2 - \Pi$ &	quark propagator $S_q^{-1} = P^2 - \Sigma_q$
gluon self-energy: $\Pi = M_g^2 - i2\gamma_g\omega$ &	quark self-energy: $\Sigma_q = M_q^2 - i2\gamma_q\omega$

(scalar approximation)

- the resummed properties are specified by complex (retarded) self-energies:
  - the real part of self-energies (Σ_q, Π) describes a dynamically generated mass (M_q,M_g);
  - the imaginary part describes the interaction width of partons (γ_q, γ_g)
  - Spectral functions : $A_q \sim Im S_q^{ret}, \quad A_g \sim Im \Delta^{ret}$

- Entropy density of interacting bosons and fermions in the quasiparticle limit (2PI) (G. Baym 1998):

QGP

$$s^{dqp} = -d_g \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \frac{\partial n_B}{\partial T} \left(\mathrm{Im} \ln(-\Delta^{-1}) + \mathrm{Im}\, \Pi \, \mathrm{Re}\, \Delta \right) \quad \text{gluons}$$

$$- d_q \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \frac{\partial n_F((\omega - \mu_q)/T)}{\partial T} \left(\mathrm{Im} \ln(-S_q^{-1}) + \mathrm{Im}\, \Sigma_q \, \mathrm{Re}\, S_q \right) \quad \text{quarks}$$

$$- d_{\bar q} \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3} \frac{\partial n_F((\omega + \mu_q)/T)}{\partial T} \left(\mathrm{Im} \ln(-S_{\bar q}^{-1}) + \mathrm{Im}\, \Sigma_{\bar q} \, \mathrm{Re}\, S_{\bar q} \right) \quad \text{antiquarks}$$

A. Peshier, W. Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

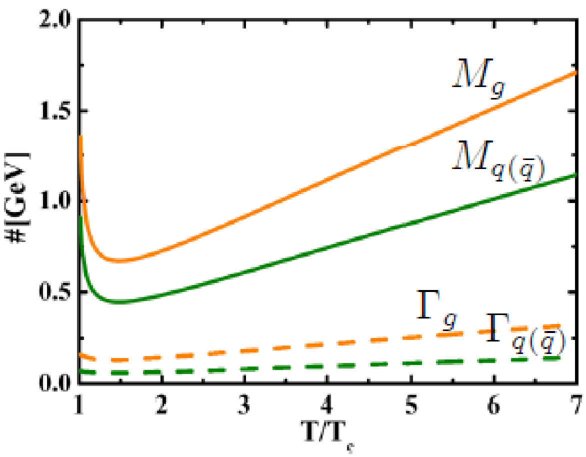
The Dynamical QuasiParticle Model (DQPM)

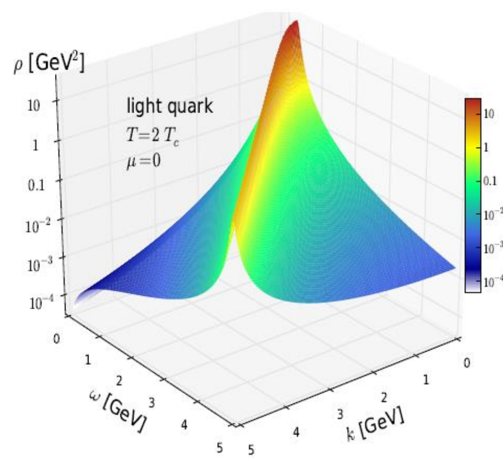
- **Basic idea: interacting quasi-particles: massive quarks and gluons (g, q, q_{bar}) with Lorentzian spectral functions :**

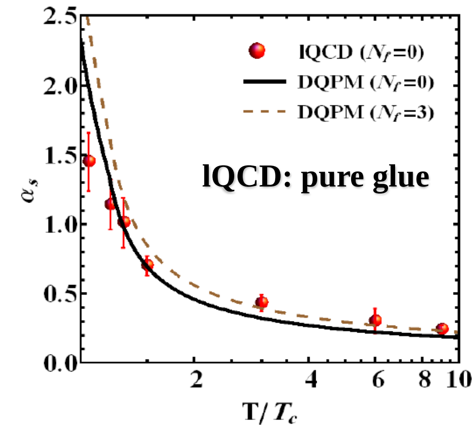
$$\rho_i(\omega,T) = \frac{4\omega\Gamma_i(T)}{\left(\omega^2 - \vec{p}^2 - M_i^2(T)\right)^2 + 4\omega^2\Gamma_i^2(T)} \qquad (i = q, \bar{q}, g)$$

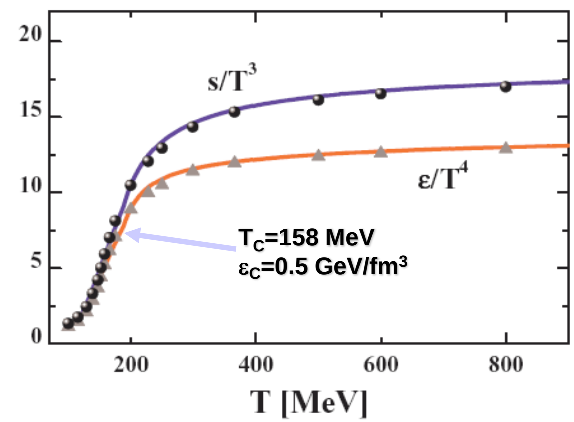
- **Modeling of the quark/gluon masses and widths → HTL limit at high T with 3 model parameters – fited to lattice QCD data**

→ **Quasi-particle properties:**
large width and mass for gluons and quarks









- **DQPM provides mean-fields (1PI) for gluons and quarks as well as effective 2-body interactions (2PI)**
- **DQPM gives transition rates for the formation of hadrons → PHSD**

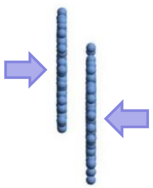
DQPM: Peshier, Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

Parton-Hadron-String-Dynamics (PHSD)

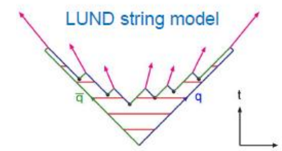
PHSD is a **non-equilibrium microscopic transport approach** for the description of **strongly-interacting hadronic and partonic matter** created in heavy-ion collisions

Dynamics: based on the solution of **generalized off-shell transport equations** derived from Kadanoff-Baym many-body theory

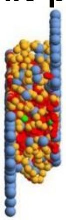
Initial A+A collision



- ❑ **Initial A+A collisions** :
  N+N → string formation → decay to pre-hadrons + leading hadrons

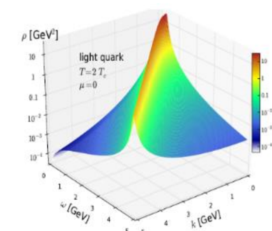


Partonic phase

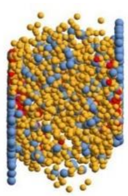


- ❑ **Formation of QGP stage** if local $\varepsilon > \varepsilon_{critical}$:
  dissolution of pre-hadrons → partons

- ❑ **Partonic phase - QGP:**
  QGP is described by the Dynamical QuasiParticle Model (DQPM) matched to reproduce lattice QCD EoS for finite T and μ_B (crossover)

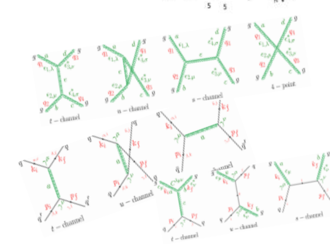


Hadronization

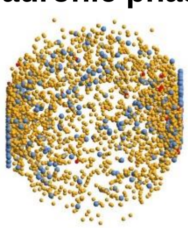


- **Degrees-of-freedom:** strongly interacting quasiparticles: massive quarks and gluons (g,q,q_{bar}) with sizeable collisional widths in a self-generated mean-field potential

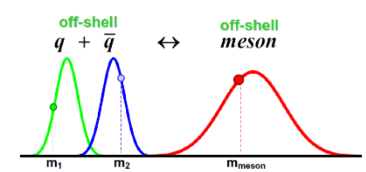
- **Interactions:** (quasi-)elastic and inelastic collisions of partons



Hadronic phase



- ❑ **Hadronization** to colorless off-shell mesons and baryons:
  Strict 4-momentum and quantum number conservation

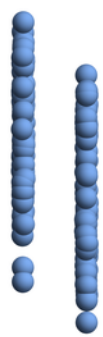


- ❑ **Hadronic phase: hadron-hadron interactions – off-shell HSD**

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215; W. Cassing, EPJ ST 168 (2009) 3

Stages of a collision in PHSD

t = 0.05 fm/c



Au + Au $\sqrt{s_{NN}}$ = 200 GeV

b = 2.2 fm – Section view

Baryons (394)

Antibaryons (0)

Mesons (0)

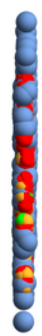
Quarks (0)

Gluons (0)

P.Moreau

Stages of a collision in PHSD

t = 1.6512 fm/c



Au + Au $\sqrt{s_{NN}} = 200$ GeV

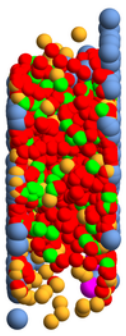
b = 2.2 fm – Section view

- Baryons (394)
- Antibaryons (0)
- Mesons (1523)
- Quarks (4553)
- Gluons (368)

P.Moreau

Stages of a collision in PHSD

t = 3.91921 fm/c



Au + Au $\sqrt{s_{NN}}$ = 200 GeV

b = 2.2 fm – Section view

Baryons (426)

Antibaryons (29)

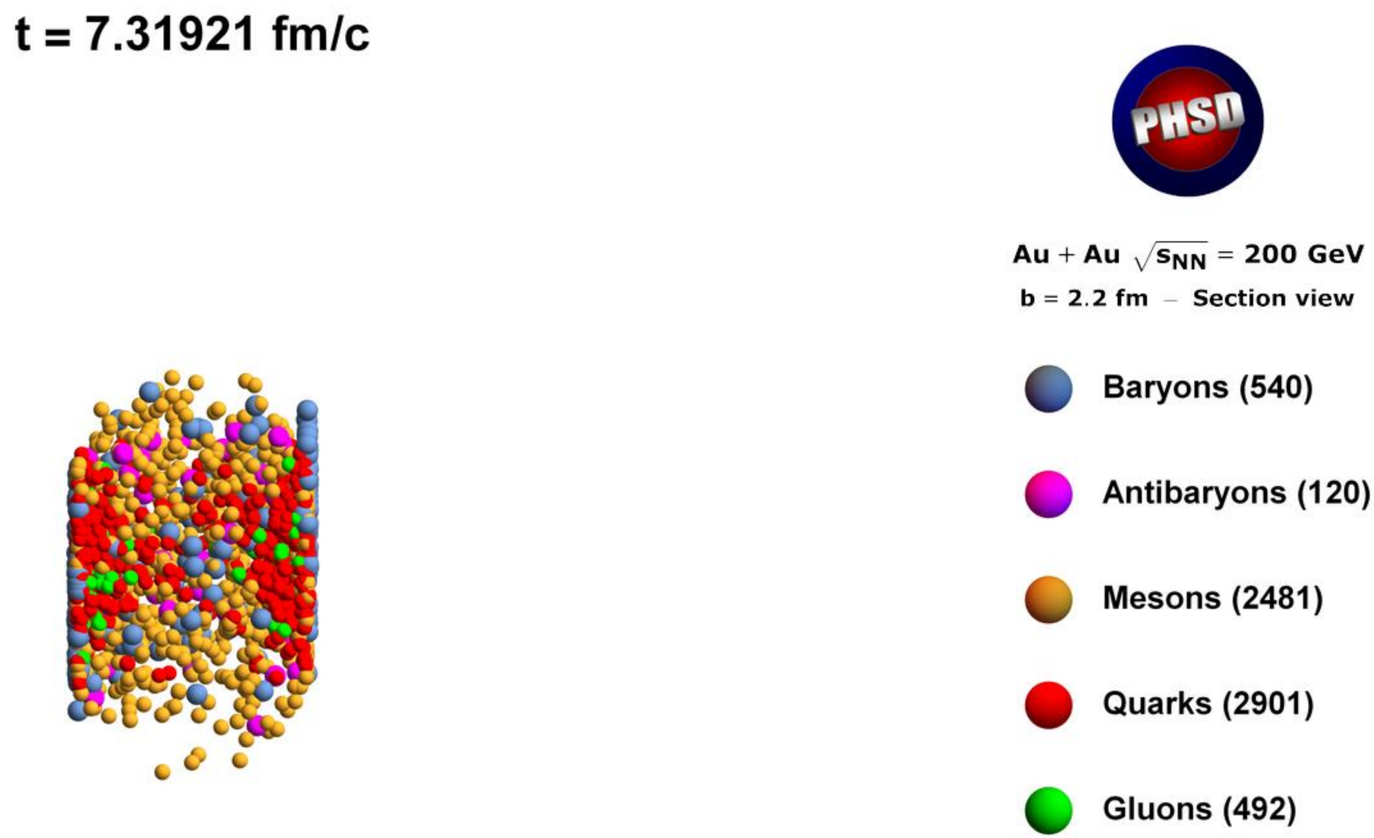
Mesons (1189)

Quarks (4459)

Gluons (783)

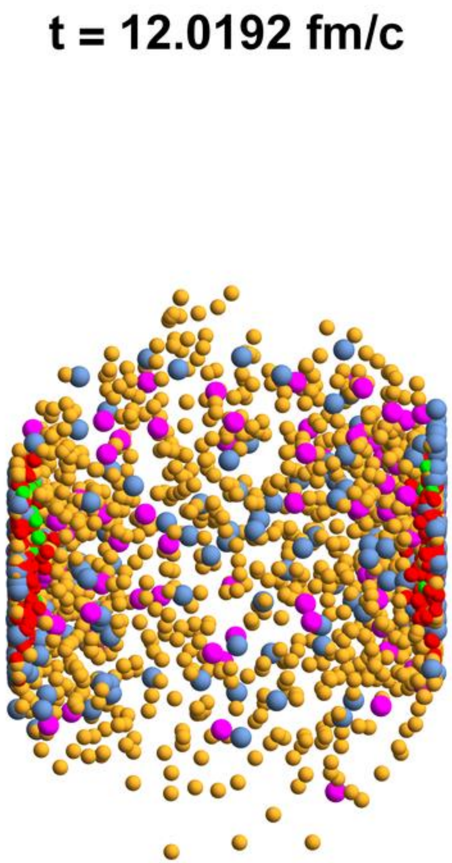
P.Moreau

Stages of a collision in PHSD



P.Moreau

Stages of a collision in PHSD



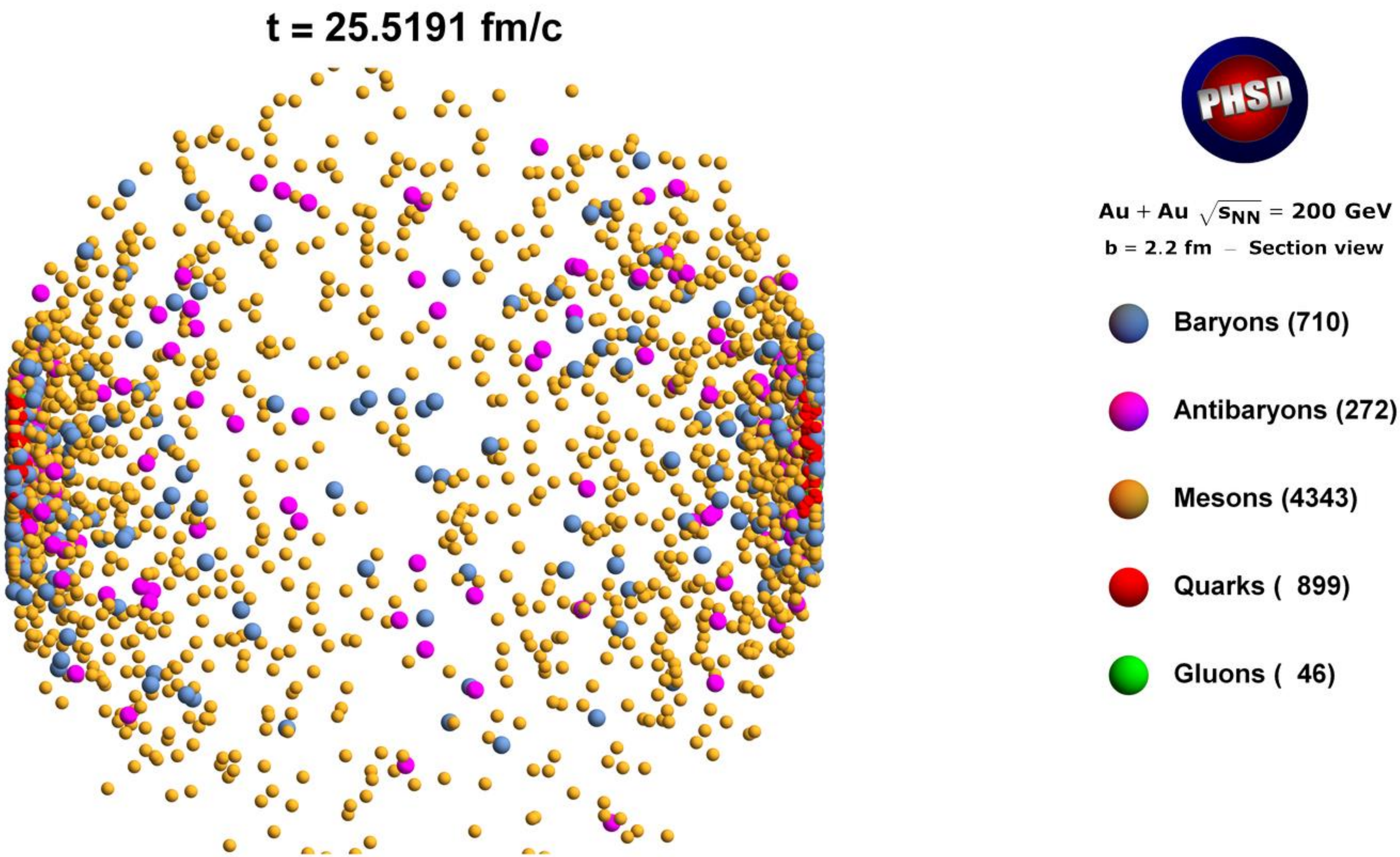
Au + Au $\sqrt{s_{NN}}$ = 200 GeV

b = 2.2 fm – Section view

- Baryons (626)
- Antibaryons (202)
- Mesons (3357)
- Quarks (1835)
- Gluons (269)

P.Moreau

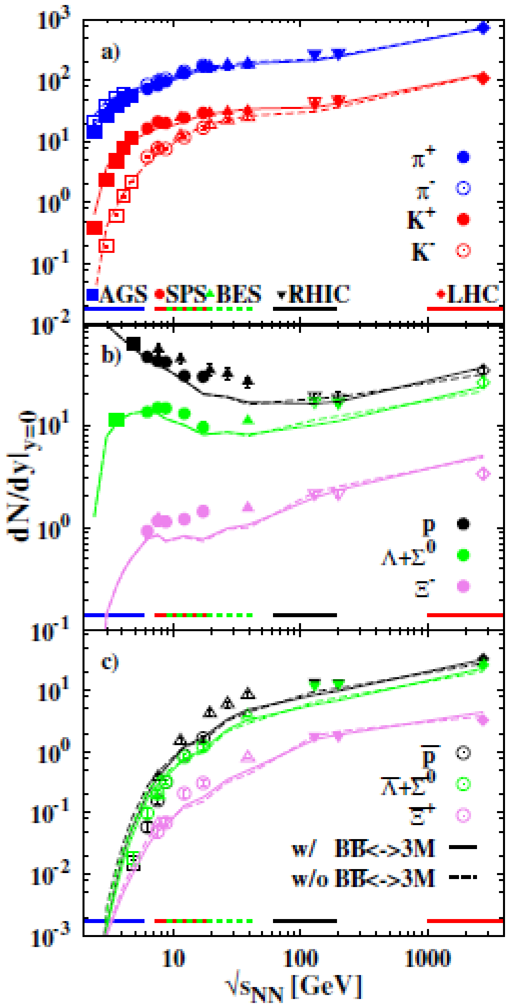
Stages of a collision in PHSD



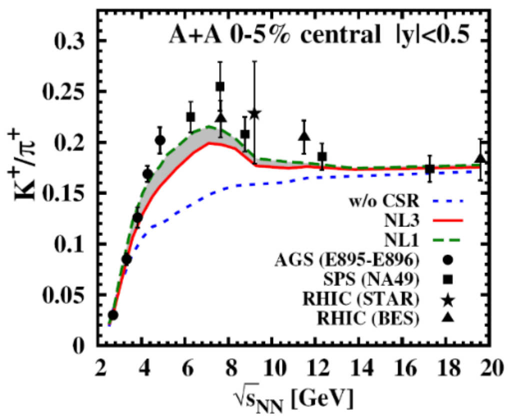
P.Moreau

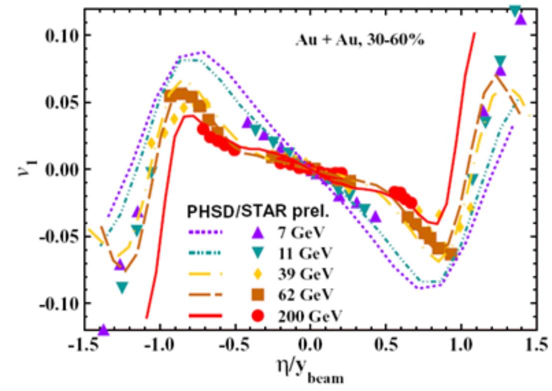
Non-equilibrium dynamics: description of A+A with PHSD

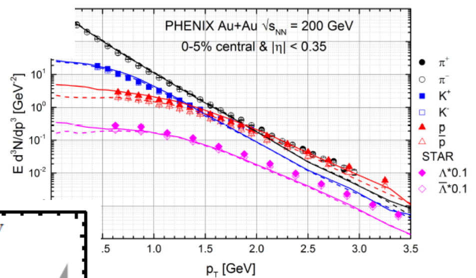
❑ **PHSD: highlights**

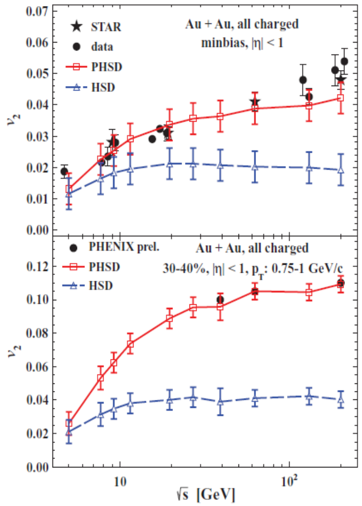


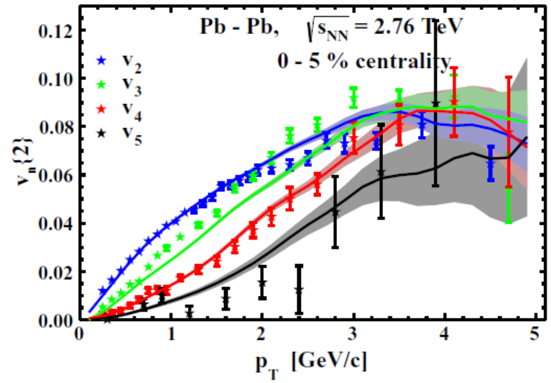
arXiv:1801.07557











PRC 85 (2012) 011902; JPG42 (2015) 055106

❑ **PHSD** provides a **good description of ‚bulk' observables** (y-, p_T-distributions, flow coefficients v_n, …) from SIS to LHC

Related literature

L. P. Kadanoff, G. Baym, ‚*Quantum Statistical Mechanics*‘, Benjamin, 1962

M. Bonitz, ‚*Quantum kinetic theory*‘, B.G. Teubner Stuttgart, 1998

W. Cassing, ‚Transport Theories for Strongly-Interacting Systems’,
Springer Nature: Lecture Notes in Physics 989, 2021

W. Botermans and R. Malfliet, Phys. Rep. 198 (1990) 115

S. Juchem, W. Cassing, and C. Greiner, Phys. Rev. D 69 (2004) 025006;
Nucl. Phys. A 743 (2004) 92

W. Cassing, Eur. Phys. J. ST 168 (2009) 3

J. Berges, Phys.Rev.D7 (2006) 045022; AIP Conf. Proc. 739 (2005) 3

C.S. Fischer, J.Phys.G32 (2006) R253

O. Linnyk, E. Bratkovskaya and W. Cassing,
Progress in Particle and Nuclear Physics 87 (2016) 50-115.