Recent improvements in extended linear sigma model (axial) vector mesons in-medium masses

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Vector and axial vector meson **E**xtended Polyakov **L**inear **S**igma **M**odel.
Effective model to study the phase diagram of strongly interacting matter at finite T and μ.
Phys. Rev. D 93, no. 11, 114014 (2016)

- **E**xtended: Vector and Axial vector nonets (besides to Scalar and Pseudoscalar)
  Isospin symmetric case: 16 mesonic degrees of freedom.
- Polyakov: Polyakov loop variables give 2 order parameters Φ, $\bar{\Phi}$.
- **L**inear **S**igma **M**odel: "simple" quark-meson model
  The mesonic Lagrangian $\mathcal{L}_m$ build up from the fields

$$L^\mu = \sum_a (V_a^\mu + A_a^\mu)\, T_a, \quad R^\mu = \sum_a (V_a^\mu - A_a^\mu)\, T_a, \quad M = \sum_a (S_a + iP_a)\, T_a,$$

with terms up to fourth order, taking care of the symmetry properties.

- $\mathcal{L}_m$ contains the dynamical, the symmetry breaking, and the meson-meson interaction terms.
  - $U(1)_A$ anomaly and explicit breaking of the chiral symmetry.
  - Each meson-meson terms upto 4th order that are allowed by the chiral symmetry.
- Constituent quarks ($N_f = 2+1$) in Yukawa Lagrangian

$$\mathcal{L}_Y = \bar{\psi}\left(i\gamma^\mu\partial_\mu - g_F(S - i\gamma_5 P) - g_V\gamma^\mu(V_\mu + \gamma_5 A_\mu)\right)\psi \qquad (1)$$

In the 2016 version $g_V = 0$ was used.
$\Rightarrow$ No (axial) vector-fermion interaction was taken into account.
$\Rightarrow$ These masses contains only tree-level contributions.

- SSB with nonzero vev. for scalar-isoscalar sector ϕ_N, ϕ_S.
  $\Rightarrow m_{u,d} = \frac{g_F}{2}\phi_N$, $m_s = \frac{g_F}{\sqrt{2}}\phi_S$ fermion masses in $\mathcal{L}_Y$.
- Mean field level effective potential $\rightarrow$ the meson masses and the thermodynamics are calculated from this.

Thermodynamics: **Mean field level** effective potential:

- Classical potential.
- Fermionic one-loop correction with vanishing fluctuating mesonic fields.

$$\bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - \mathrm{diag}(m_u, m_d, m_s)\right)\psi$$

  Functional integration over the fermionic fields.
  The momentum integrals are renoralized.

- Polyakov loop potential.

$$\Omega(T,\mu_q) = U_{Cl} + \mathrm{tr}\int_K \log\left(iS_0^{-1}\right) + U(\Phi,\bar{\Phi}) \tag{2}$$

Field equations (FE):

$$\frac{\partial\Omega}{\partial\bar{\Phi}} = \frac{\partial\Omega}{\partial\Phi} = \frac{\partial\Omega}{\partial\phi_N} = \frac{\partial\Omega}{\partial\phi_S} = 0 \tag{3}$$

Parametrization of the model at $T = 0$, $\mu = 0$ with ≈ 30 physical quantities.

The curvature meson masses are calculated from the grand potential: $M^2_{ab} = \frac{\partial^2 \Omega}{\partial \varphi_a \partial \varphi_b}$

- Tree level: S-V and P-A mixing in the quadratic (after SSB) part of the Lagrangian

eg. $\propto S_a \; K_\mu g_1 \phi_{N/S} \delta_{ab} \; V_b^\mu$

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- The usual way: shift the (axial) vectors: $V^\mu_a \to V^\mu_a + \alpha K^\mu S_a$
- For the scalars: $S_a (Z^{-2} K^2 \delta_{ab} + m^2_{ab}) S_b$
- A "wavefunction renormalization factor" for the (pseudo)scalar fields eg.: $Z^2_{K^\star_0} = \frac{m^2_{K^\star}}{\left(m^2_{K^\star} - g_1^2 (\phi_N + \sqrt{2}\phi_S)^2\right)}$

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- Fermionic one-loop correction: can be calculated from the fermionic determinant.

Tree-level + ferm. vacuum + ferm. matter $\qquad M^2_{ab} = m^2_{ab} + \Delta m^2_{ab} + \delta m^2_{ab}$

→ Including (axial) vector-fermion interaction, i.e. setting $g_V \neq 0$

$$\mathcal{L}_Y = \bar{\psi}\left(i\gamma^\mu\partial_\mu - g_F(S - i\gamma_5 P) - g_V\gamma^\mu(V_\mu + \gamma_5 A_\mu)\right)\psi \tag{4}$$

From the fermionic one-loop self-energy corrections come to the (axial) vector masses.

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→ Including one-loop mesonic contribution into the effective potential via ring resummation. (The fermion determinant expanded to 2nd order in the mesonic fields and Gaussian integral performed.)

$$U(\phi) = U_{Cl} + \mathrm{tr}\int_K \log\left(iS_0^{-1}\right) - \frac{i}{2}\mathrm{tr}\int_K \log\left(i\mathcal{D}^{-1}_{(\mu\nu),ab}(K) - \Pi_{(\mu\nu),ab}(K)\right) \tag{5}$$

$i\mathcal{D}^{-1}(K)$ the tree-level inverse propagator and $\Pi(K)$ the fermionic one-loop SE.

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It can be easily seen that the fermionic contribution to the curvature masses can be obtained as the self-energy at vanishing external momentum $\Pi(K=0)$.
⇒ One needs the self-energy.

Generally one has

$$\Pi_{ab}^{(X)}(Q) = iN_c s_X c_X^2 \int_K \mathrm{tr}\Big[\Gamma_X \frac{\lambda_a}{2}\bar{\mathcal{S}}(K)\Gamma'_X \frac{\lambda_b}{2}\bar{\mathcal{S}}(K-Q)\Big] \tag{6}$$

where the trace goes over flavor and Dirac space, too, $\bar{\mathcal{S}} = \mathrm{diag}\,(\mathcal{S}_u, \mathcal{S}_d, \mathcal{S}_s)$, $s_X = \pm 1$ for S, P and V, A while $c_X = -ig_S,\ -g_S,\ -ig_V,\ -ig_V$ and $\Gamma_X = \mathbb{1}, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5$ for $X = S, P, V, A$ respectively. With $m_{a/b} = m_{u/d/s} \propto \phi_{N/S}$

$$\Pi_{ab}^{(V/A)\mu\nu}(Q) = i2N_c g_V^2 \int_K \frac{g^{\mu\nu}(\pm\, m_a m_b - K^2 + K\cdot Q) + 2K^\mu K^\nu - K^\mu Q^\nu - Q^\mu K^\nu}{(K^2 - m_a^2)((K-Q)^2 - m_b^2)} \tag{7}$$

- At $T = 0$ only the vacuum self-energy contributes, that has to be renormalized $\Rightarrow$ Dimensional regularization
- At $T \neq 0$ the matter part (with statistical function) also gives contribution $\Rightarrow$ At finite temperature: Wick rotation, Matsubara frequencies, $\int_K \to iT\sum_n \int \frac{d^3k}{(2\pi)^3}$

PROJECTOR DECOMPOSITION OF $\Pi^{\mu\nu}_{\text{VAC}}(Q)$

Single reference vector at $T=0$: $Q^\mu \Rightarrow$ 4-longitudonal and 4-transversal projectors:

$$P_L^{\mu\nu} = \frac{Q^\nu Q^\mu}{Q^2}, \qquad P_T^{\mu\nu} = g^{\mu\nu} - P_L^{\mu\nu} \tag{8}$$

The **vacuum** contribution can be written up as

$$\Pi^{\mu\nu}_{\text{vac}}(Q) = \Pi_{\text{vac},L}(Q) P_L^{\mu\nu} + \Pi_{\text{vac},T}(Q) P_T^{\mu\nu} \tag{9}$$

We need only the vanishing external momentum case.

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- For the vector self-energy containing two fermion propagators with the same mass in the loop integral one can see that: $Q_\mu \Pi^{\mu\nu}(Q) = 0$ and $\Pi^{\mu\nu}(0) = 0$ (as in QED)

$$\Pi_{\text{vac},L/T}(0) = 0 \tag{10}$$

  Renormalization method that can reproduce this $\Rightarrow$ **Dimensional regularization**
- For the axial vector self-energy and vector self-energy with two different fermion propagators in the loop integral

$$\Pi_{\text{vac},L}(0) = \Pi_{\text{vac},T}(0) = \Pi^{00}_{\text{vac}}(0) = -\Pi^{11}_{\text{vac}}(0) \neq 0 \tag{11}$$

There is another reference vector: 4-velocity of the thermal bath u_μ (with $u^2 = 1$).

Lorentz covariant quantities: $\omega \equiv Q \cdot u$, $p \equiv \sqrt{\omega^2 - Q^2}$.
We use $u^\mu = (1, \mathbf{0})$, thus, $\omega = q_0$, $p = |\mathbf{q}|$.
New operators ($u_T^\mu = u^\mu - (Q \cdot u)Q^\mu / Q^2$):

$$P_L^{\mu\nu}, \quad P_l^{\mu\nu} = \frac{u_T^\mu u_T^\nu}{u_T^2}, \quad P_t^{\mu\nu} = g^{\mu\nu} - P_L^{\mu\nu} - P_l^{\mu\nu}, \quad C^{\mu\nu} = \frac{Q^\mu u_T^\nu + Q^\nu u_T^\mu}{\sqrt{(Q \cdot u)^2 - Q^2}} \tag{12}$$

Hence, $$\Pi^{\mu\nu}(Q) = \sum_{x=l,t,L} \Pi_x(Q) P_x^{\mu\nu} + \Pi_C(Q) C^{\mu\nu}.$$

$C^{\mu\nu}$ is not a projector, e.g.:

$$C^2 = -P_l - P_L, \qquad C \cdot P_l = P_L \cdot C, C \cdot P_L = P_l \cdot C \tag{13}$$

M. Le Bellac, Thermal Field Theory, (1996)
Buchmuller, Helbig and Walliser, Nucl. Phys. B 407, 387-411 (1993)

We need only the vanishing external momentum case.
To get the curvature mass one need $\lim\limits_{\mathbf{q}\to 0} \lim\limits_{q_0 \to 0}$ in this order.

$$\Pi_l^{\text{mat}}(0,\mathbf{q}) = \Pi_{00}(0,\mathbf{q}), \quad \Pi_L^{\text{mat}}(0,\mathbf{q}) = -\frac{q_i q_j}{\mathbf{q}^2}\Pi_{ij}^{\text{mat}}(0,\mathbf{q}), \quad \Pi_C^{\text{mat}}(0,\mathbf{q}) = -\frac{q_i}{|\mathbf{q}|}\Pi_{0i}^{\text{mat}}(0,\mathbf{q}) = 0$$

Thus, $\quad \Pi_{l/t/L}(0) = \Pi_{\text{vac}}(0) + \Pi_{l/t/L}^{\text{mat}}(0)$

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Thus, $\Pi_{l/t/L}(0) = \Pi_{\text{vac}}(0) + \Pi_{l/t/L}^{\text{mat}}(0)$

- Vector SE with two fermion propagators with equal masses

$$\Pi_L^{\text{mat}}(0) = 0, \qquad \Pi_l^{\text{mat}}(0) = \Pi_{00}^{\text{mat}}(0), \qquad \Pi_t^{\text{mat}}(0) = -\frac{3}{2}\Pi_{11}^{\text{mat}}(0) \; [= 0 \text{ for ELSM}]$$

- Axial vector SE and vector SE with two different fermion propagators

$$\Pi_{t/L}^{\text{mat}}(0) = -\Pi_{11}^{\text{mat}}(0), \qquad \Pi_l^{\text{mat}}(0) = \Pi_{00}^{\text{mat}}(0)$$

Mixing in the Gaussian approximation

Contribution of the self-energy at vanishing external momentum

$$\begin{aligned} i\mathcal{D}^{-1}(K) &\to i\mathcal{G}_{\mathrm{loc}}^{-1}(K) = i\mathcal{D}^{-1}(K) - \Pi(0) \\ i\mathcal{D}_{\mu\nu}^{-1}(K) &\to i\mathcal{G}_{\mathrm{loc},\mu\nu}^{-1}(K) = i\mathcal{D}_{\mu\nu}^{-1}(K) + \Pi_{\mu\nu}(0) \end{aligned} \tag{14}$$

For V/A:
$i\mathcal{G}_{\mathrm{loc},\mu\nu}^{-1}(K) = M_L^2 P_{\mu\nu}^L(K) + \sum_{x=l,t} \left(M_x^2 - K^2\right) P_{\mu\nu}^x(K), \qquad M_{L/l/t}^2 = m^2 + \Pi_{L/l/t}(0)$

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Specially in the scalar-vector $4-5$ sector:

$$\det \mathbf{M}^{SV,45}_{\mu\nu} = -\left(M^2_{L,55} - c^2_{45}\right)\left(K^2 - M^2_{44}\right)\left(K^2 - M^2_{l,55}\right)\left(K^2 - M^2_{t,55}\right)^2, \quad 1+1+1+2 \text{ modes}$$

where $M^2_{44} = Z^2_{S,44}\left(m^{2,(S)}_{44} + \Pi^{(S)}_{44}(0)\right)$ with $Z^2_{S,44} = M^2_{L,55}/\left(M^2_{L,55} - c^2_{45}\right)$.

- (Pseudo)scalar curvature masses

$$\begin{array}{ccccc} \text{Tree-level} & & T=0 & & T\neq 0 \\ m^2 & \longrightarrow & M^2 = m^2 + \Pi_{\text{vac}}(0) & + & \Pi_{\text{mat}}(0) \end{array}$$

Already calculated by Schaefer and Wagner and part of the latest version ELSM. Momentum has to be kept in the determinant for the (axial) vectors because those couple to the momentum to form a Lorentz scalar.

Phys. Rev. D 79 , 014018 (2009)

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- (Axial) vector curvature masses

$$\begin{array}{ccc} \text{Tree-level} & & \text{Fermionic correction} \\ m^2 = m_L^2 = m_T^2 & \xrightarrow{T=0} & M^2_{\text{vac}} = M^2_{\text{vac},L/T} = m^2_{L/T} + \Pi_{\text{vac},L/T}(0) \\ & \xrightarrow{T\neq 0} & M^2_{L/l/t} = m^2_{L/l/t} + \Pi_{L/l/t}(0) \end{array}$$

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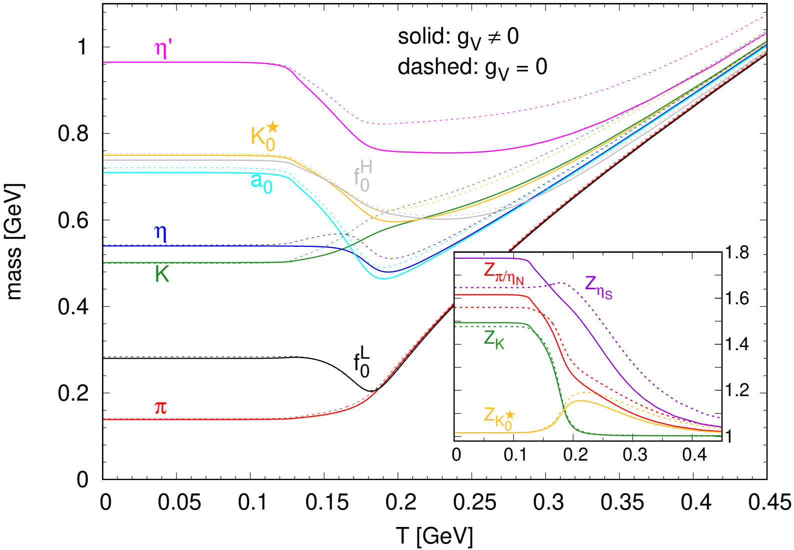
Thus, both T and L get the same vacuum correction and at $T \neq 0$ the 4-transversal splits to 3-transversal + 3-longitudinal, and each modes (L, l, t) gets separate matter correction. In ELSM $\Pi_L(0) = \Pi_t(0) \neq \Pi_l(0)$.



solid: $g_V \neq 0$
dashed: $g_V = 0$
η'
$K_0^\star$
f_0^H
a_0
η
K
f_0^L
π
Z_{π/η_N}
Z_{η_S}
Z_K
$Z_{K_0^\star}$
1.8
1.6
1.4
1.2
1
0
0.1
0.2
0.3
0.4
1
0.8
0.6
0.4
0.2
0
mass [GeV]
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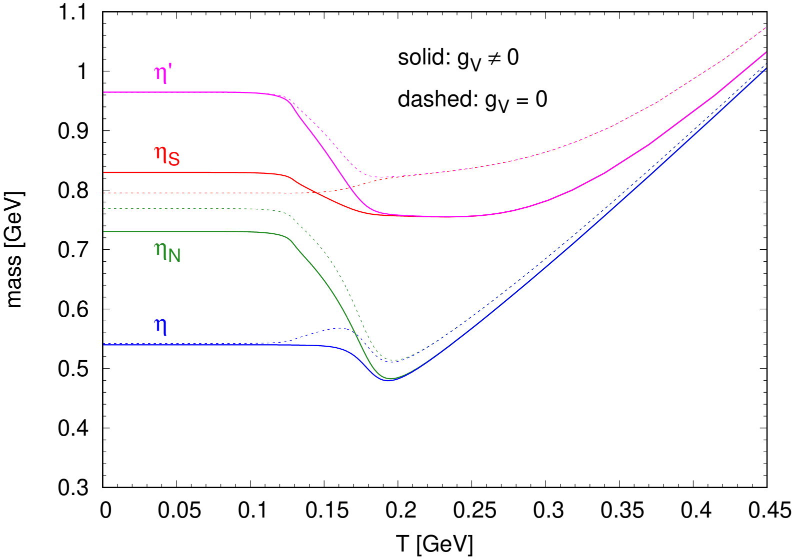
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
T [GeV]


solid: $g_V \neq 0$
dashed: $g_V = 0$
η'
η_S
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mass [GeV]
T [GeV]
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1
0.9
0.8
0.7
0.6
0.5
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0
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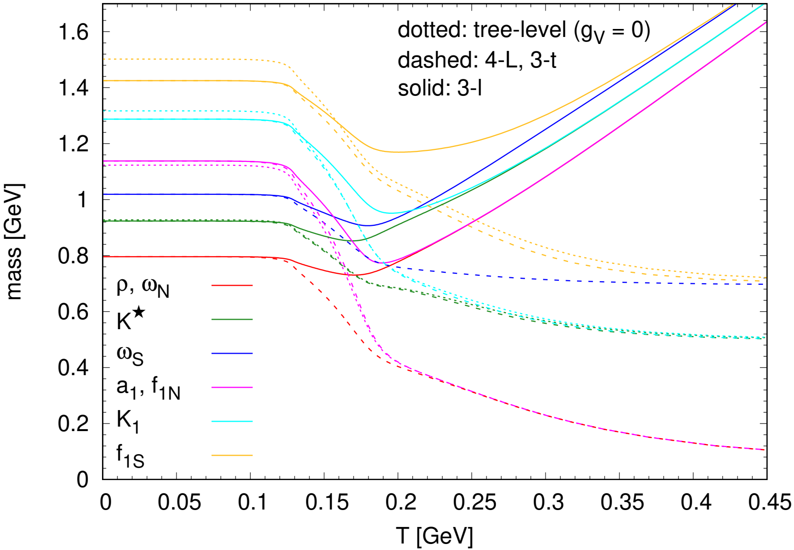
0.35
0.4
0.45


dotted: tree-level (g_V = 0)
dashed: 4-L, 3-t
solid: 3-l
ρ, ω_N
K⋆
ω_S
a_1, f_1N
K_1
f_1S
mass [GeV]
T [GeV]
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45

- There is a mixing between the (pseudo)scalars and (axial) vectors at tree-level.
- The one-loop fermionic self-energy of the (axial) vectors can be calculated with the usual technics.
- One has to decompose the (axial) vector self-energy the find the physical modes for which the masses has to be calculated.
- In the Gaussian approximation the mode decomposition and the reduction of the (pseudo)scalar – (axial) vector naturally appear.
- The T-dependence of the curvature masses of various modes was investigated.
- Using the effective potential in Eq. (5) we can (and plan to) investigate the thermodynamics of the ELSM including fermion and meson one-loop corrections and ring resummation with $g_V \neq 0$.

Thank you!

Classical level mixing

$$\delta\mathcal{L}^{\text{quad}}_{g_1} = -\frac{g_1}{2} i K_\mu \Big[d_{ijk}\big(\tilde{A}^\mu_i \bar{P}_j - \tilde{P}_i \bar{A}_j\big) + f_{ijk}\big(\tilde{V}^\mu_i \bar{S}_j + \tilde{S}_i \bar{V}^\mu_j\big)\Big]\phi_k, \quad i,j,k = 0,\ldots,8$$

Specially for $S-V$ in the $4-5$ sector

$$\frac{1}{2}\tilde{S}_4\big(K^2 - m^{2,(S)}_{44}\big)\bar{S}_4 - \frac{1}{2}\tilde{V}^\mu_5\big(g^{\mu\nu}(K^2 - m^{2,(V)}_{55}) - K^\mu K^\nu\big)\bar{V}^\nu_5 - \frac{i}{2}\tilde{V}^\mu_5 c_{54} K^\mu \bar{S}_4 + \frac{i}{2}\tilde{S}_4 c_{45} K^\nu \bar{V}^\nu_5$$

The usual way to handle the mixing: shift the (axial) vectors: $V^\mu_i \to V^\mu_i + \alpha K^\mu S_i$

$$\frac{1}{2}\tilde{S}_4\big(K^2\,(m^{2,(V)}_{55} - c^2_{45})/m^{2,(V)}_{55} - m^{2,(S)}_{44}\big)\bar{S}_4 - \frac{1}{2}\tilde{V}^\mu_5\big((g^{\mu\nu}K^2 - K^\mu K^\nu) - g^{\mu\nu} m^{2,(V)}_{55}\big)\bar{V}^\nu_5$$

To get the canonical $K^2 - m^2$ form for the scalars one defines a "wave function renormalization" for the scalars with $S_4 \to Z_{K_0^{\star\pm}} S_4$ with $Z^2_{K_0^{\star\pm}} = m^2_{K^{\star\pm}}/(m^2_{K^{\star\pm}} - c^2_{45})$

Thus one will get: $\frac{1}{2}\tilde{S}_4\big(K^2 - Z^2_{K_0^{\star\pm}} m^{2,(S)}_{44}\big)\bar{S}_4$

Classical level mixing

$$\delta\mathcal{L}_{g_1}^{\text{quad}} = -\frac{g_1}{2} iK_\mu \Big[d_{ijk}\big(\tilde{A}_i^\mu \bar{P}_j - \tilde{P}_i \bar{A}_j\big) + f_{ijk}\big(\tilde{V}_i^\mu \bar{S}_j + \tilde{S}_i \bar{V}_j^\mu\big)\Big]\phi_k, \quad i,j,k = 0,\ldots,8$$

Specially for $S - V$ in the $4-5$ sector (**with a new way**)

$$\delta\mathcal{L}_{45}^{SV} = \frac{1}{2}\left[(\tilde{S}_4, \tilde{V}_5^\mu)\mathbf{M}_{\mu\nu}^{45}\begin{pmatrix}\bar{S}_4\\ \bar{V}_5^\nu\end{pmatrix} + (\tilde{S}_5, \tilde{V}_4^\mu)\mathbf{M}_{\mu\nu}^{45*}\begin{pmatrix}\bar{S}_5\\ \bar{V}_4^\nu\end{pmatrix}\right], \quad \mathbf{M}_{\mu\nu}^{45} = \begin{pmatrix}\mathcal{D}_{44}^{-1}(K) & -iK_\nu c_{45}\\ iK_\mu c_{45} & -i\mathcal{D}_{\mu\nu,44}^{-1}(K)\end{pmatrix}$$

The propagators: $i\mathcal{D}_{44/55}^{-1} = K^2 - m_{44}^{2,(S)}$ and $i\mathcal{D}_{\mu\nu,44/55}^{-1} = m_{K^\star\pm}^2 P_{\mu\nu}^L + \big(m_{K^\star\pm}^2 - K^2\big)P_{\mu\nu}^T$

In the Gaussian approximation one has the determinant:

$$\begin{aligned}\det \mathbf{M}_{\mu\nu}^{45} &= i\mathcal{D}_{44}^{-1}(K)\det\big(i\mathcal{D}_{\mu\nu,44}^{-1}(K) + ic_{45}^2\mathcal{D}_{44}(K)K^2P_{\mu\nu}^L\big)\\ &= -\big(m_{K^\star\pm}^2 - c_{45}^2\big)\big(K^2 - m_{K_0^\star\pm}^2\big)\big(K^2 - m_{K^\star\pm}^2\big)^3, \qquad 1+1+3 \text{ modes}\end{aligned}$$

where $m_{K_0^\star\pm}^2 = Z_{K_0^\star\pm}^2 m_{44}^{2,(S)}$ with $Z_{K_0^\star\pm}^2 = m_{K^\star\pm}^2/\big(m_{K^\star\pm}^2 - c_{45}^2\big)$.

Classical level mixing

$$\delta\mathcal{L}_{g_1}^{\text{quad}} = -\frac{g_1}{2} iK_\mu \Big[d_{ijk}\big(\tilde{A}_i^\mu \bar{P}_j - \tilde{P}_i \bar{A}_j\big) + f_{ijk}\big(\tilde{V}_i^\mu \bar{S}_j + \tilde{S}_i \bar{V}_j^\mu\big)\Big]\phi_k, \quad i,j,k = 0,\dots,8$$

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In the eff. potential: $\int_K \log \mathrm{Det}\big(i\mathcal{D}^{-1}(K) + \Pi(0)\big) = \int_K \log const + \int_K \log S + 3\int_K \log V_T$

Classical level mixing

$$\delta\mathcal{L}_{g_1}^{\rm quad} = -\frac{g_1}{2} iK_\mu \Big[d_{ijk}\big(\tilde{A}_i^\mu \bar{P}_j - \tilde{P}_i \bar{A}_j\big) + f_{ijk}\big(\tilde{V}_i^\mu \bar{S}_j + \tilde{S}_i \bar{V}_j^\mu\big)\Big]\phi_k, \quad i,j,k = 0,\dots,8$$

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In the eff. potential: $\int_K \log \mathrm{Det}\big(i\mathcal{D}^{-1}(K) + \Pi(0)\big) = \int_K \log const + \int_K \log S + 3\int_K \log V_T$
In dimensional regularization one can get rid of the constant.

One-loop fermionic self-energy

Backup frame The expansion of the fermionic functional determinant in powers of some generic mesonic field (in $N_f = 1$)

$$
\begin{aligned}
U_f(\phi,\ \varphi) =& \mathrm{Tr}\log\left(i\mathcal{S}_f^{-1} - g\varphi\right) \\
=& \mathrm{Tr}\log\left(i\mathcal{S}_f^{-1}\right) - \sum_{n=1}^{\infty}\frac{(-ig)^n}{n}\mathrm{tr}_D\Big[\prod_{i=1}^{n}\int \mathrm{d}^4x_i\, \varphi(x_i)\mathcal{S}_f(x_i, x_{i+1})\Big]_{x_{n+1}=x_1},
\end{aligned} \tag{15}
$$

with $i\mathcal{S}_f^{-1} = i\partial\!\!\!/ - m_f$, inverse tree-level fermion propagator, and Tr is the functional trace.

In $N_f = 2+1$:

$$
\begin{aligned}
U_f(\phi, \varphi) =& i\int_K \log\mathrm{Det}\Big[\gamma_0\big(i\gamma^\mu K_\mu + \mathbb{1}\mathrm{diag}(m_u, m_d, m_s) - g_F(\mathbb{1}S^a\lambda^a - i\gamma_5 P^a\lambda^a) \\
& - g_V\gamma^\mu(V_\mu^a\lambda^a + \gamma_5 A_\mu^a\lambda^a)\big)\Big]
\end{aligned} \tag{16}
$$

Second field derivative of $U_f(\phi, \varphi)$ taken at vanishing mesonic fields gives the self-energy.

Backup frame The expansion of the fermionic functional determinant in powers of some generic mesonic field (in $N_f = 1$)

$$\begin{aligned} U_f(\phi,\ \varphi) =& \mathrm{Tr}\log\left(i\mathcal{S}_f^{-1} - g\varphi\right) \\ =& \mathrm{Tr}\log\left(i\mathcal{S}_f^{-1}\right) - \sum_{n=1}^{\infty} \frac{(-ig)^n}{n} \mathrm{tr}_D \left[\prod_{i=1}^{n} \int \mathrm{d}^4 x_i\, \varphi(x_i) \mathcal{S}_f(x_i, x_{i+1})\right]_{x_{n+1}=x_1}, \end{aligned} \tag{15}$$

with $i\mathcal{S}_f^{-1} = i\partial\!\!\!/ - m_f$, inverse tree-level fermion propagator, and Tr is the functional trace.

In $N_f = 2+1$:

$$\begin{aligned} U_f(\phi, \varphi) =& i \int_K \log \mathrm{Det}\Big[\gamma_0 \big(i\gamma^\mu K_\mu + \mathbb{1}\mathrm{diag}(m_u, m_d, m_s) - g_F(\mathbb{1} S^a \lambda^a - i\gamma_5 P^a \lambda^a) \\ & - g_V \gamma^\mu (V_\mu^a \lambda^a + \gamma_5 A_\mu^a \lambda^a)\big)\Big] \end{aligned} \tag{16}$$

(Alternative for masses: brut force derivation of the determinant of a 12×12 matrix.)