NA7-Hf-QGP “STRONG 2020” Workshop

Fluid dynamics of multiple conserved charges

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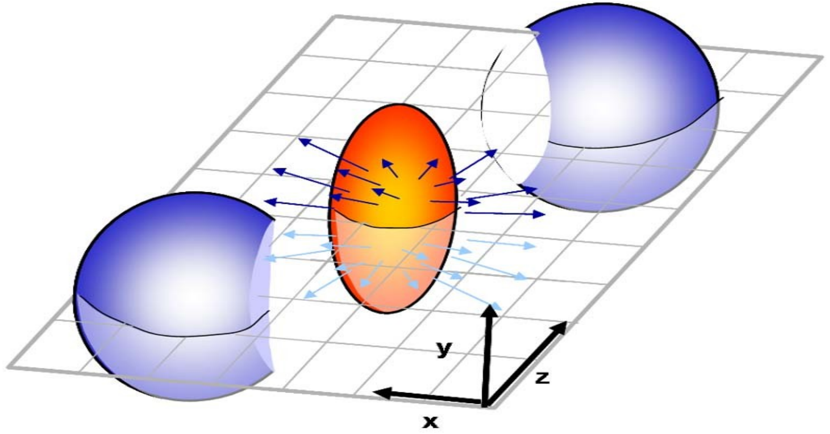
Fluid dynamics of multi-component systems

Traditionally:

Viewed as ‘blob’ of one type of matter (single component) with one velocity field

- usually ‘blob’ of energy without charge



https://www.quantumdiaries.org/wp-content/uploads/2011/02/FlowPr.jpg

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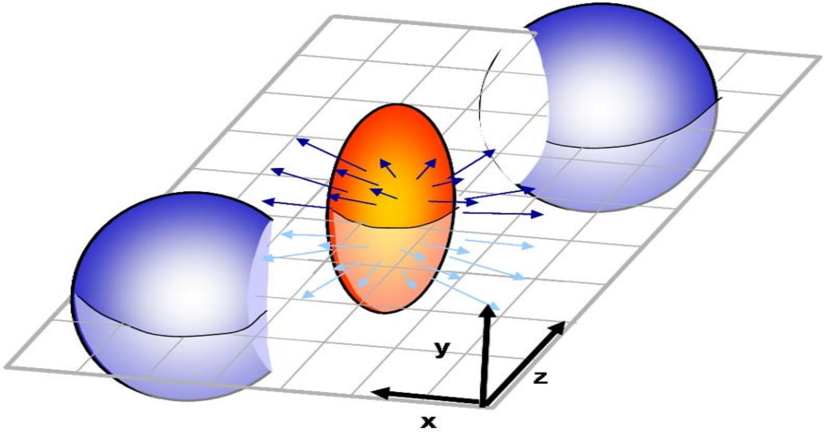

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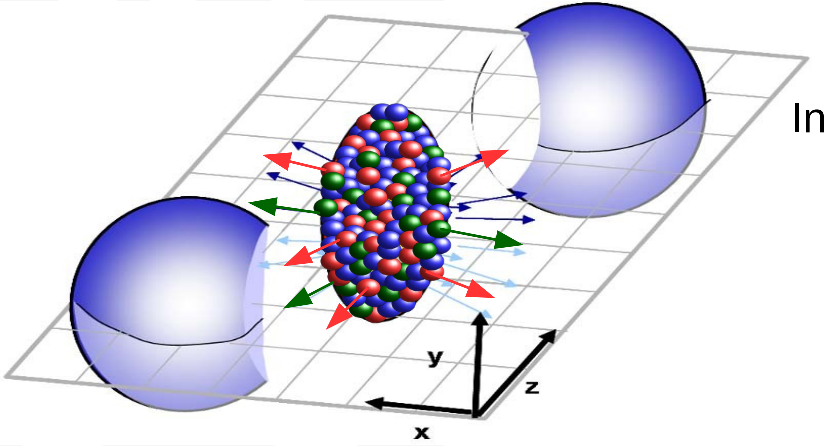

In general:

Consists of multiple components with various properties with multiple velocity fields

- with **multiple conserved quantities**
(e.g. energy, electric charge, baryon number, strangeness, …)
- mixed chemistry → **coupled charge currents!**

Fluid dynamics of multi-component systems

Hydrodynamics: macroscopic effective field theory of thermal matter close to local equilibrium

Here: single fluid velocity field u^μ

$$T^{\mu\nu} = \sum_i T_i^{\mu\nu} = \epsilon u^\mu u^\nu - (P_0 + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

$$N_q^\mu = \sum_i q_i N^\mu = n_q u^\mu + V_q^\mu$$

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q-th conserved charge (eg. B,Q,S)

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$10 + 4N_{\mathrm{ch}}$ degrees of freedom, $4 + N_{\mathrm{ch}}$ equations $\rightarrow$ $6 + 3N_{\mathrm{ch}}$ unknowns

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What needs to be known:

- Equation of state
- Equations of motion for dissipative fields & transport coefficients
- Initial state
- Freeze-out and δf-correction

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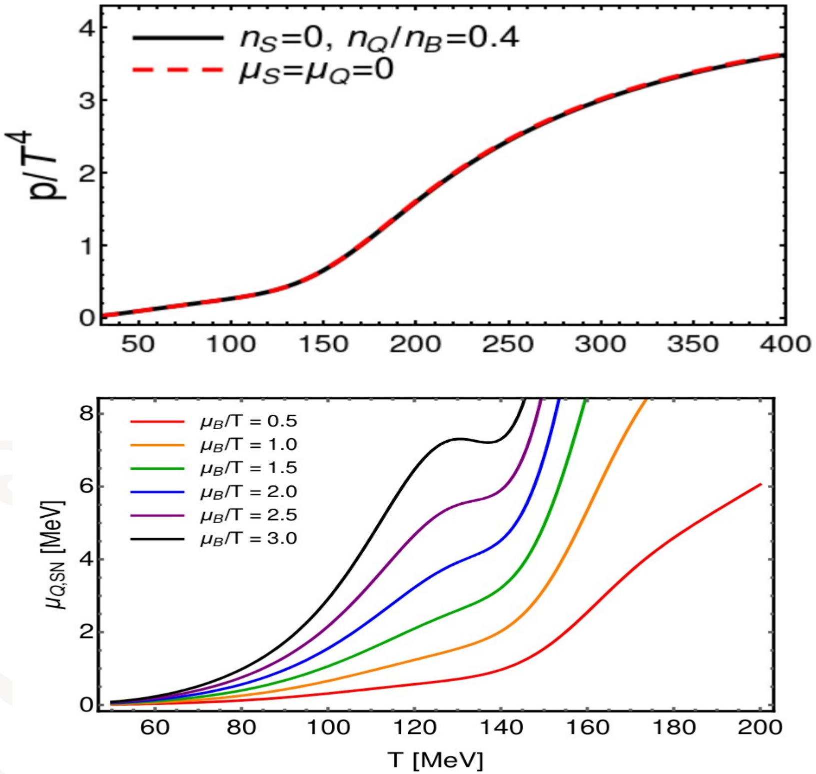
Fluid dynamics with conserved baryon number:
Denicol et al., PRC 98, 034916 (2018)
Du et al., Comp. Phys. Comm. 251, 107090 (2020)
Li et al., PRC 98, 064908 (2018)

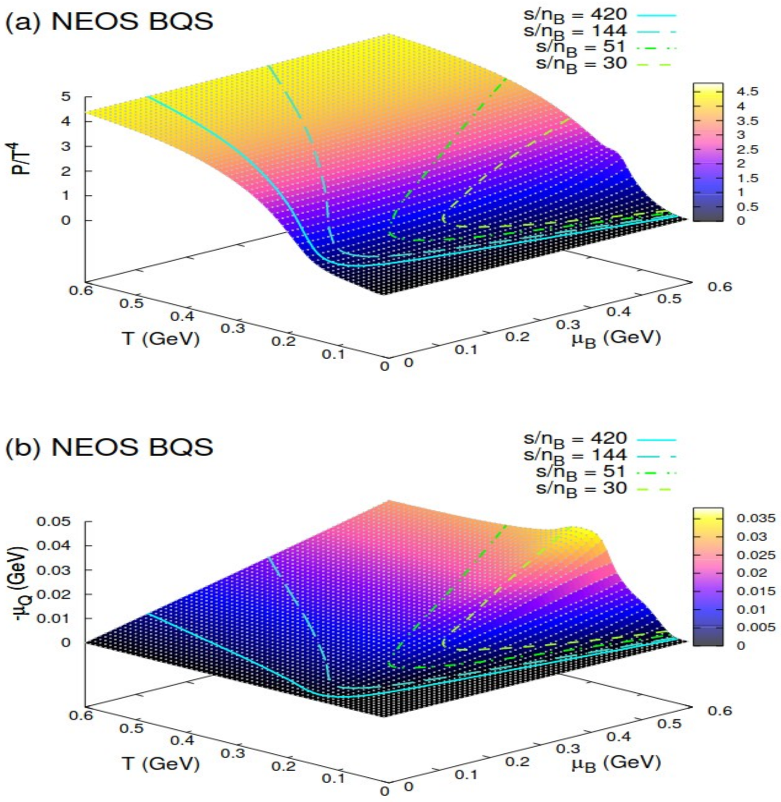
Equation of state with multiple conserved charges

$$P_0(T) \quad \rightarrow \quad P_0(T, \mu_\mathrm{B}, \mu_\mathrm{Q}, \mu_\mathrm{S})$$

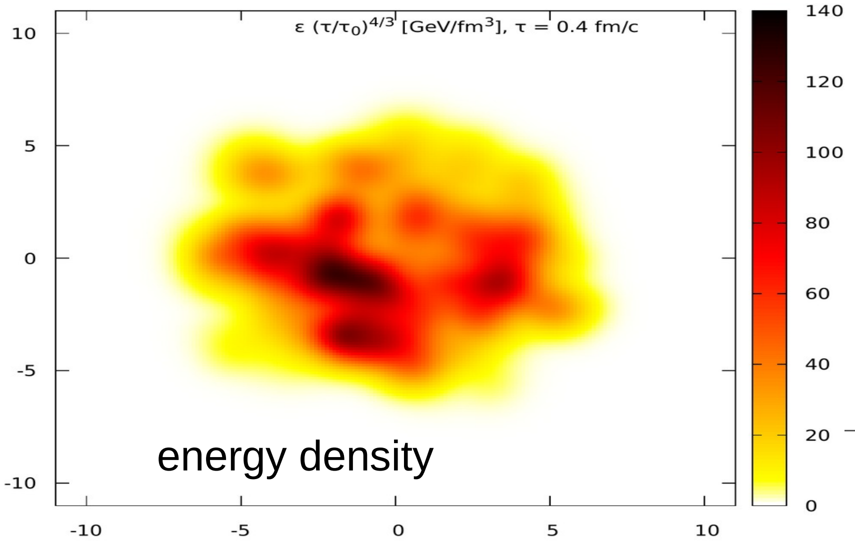
Noronha-Hostler et al., PRC 100, 064910 (2019)

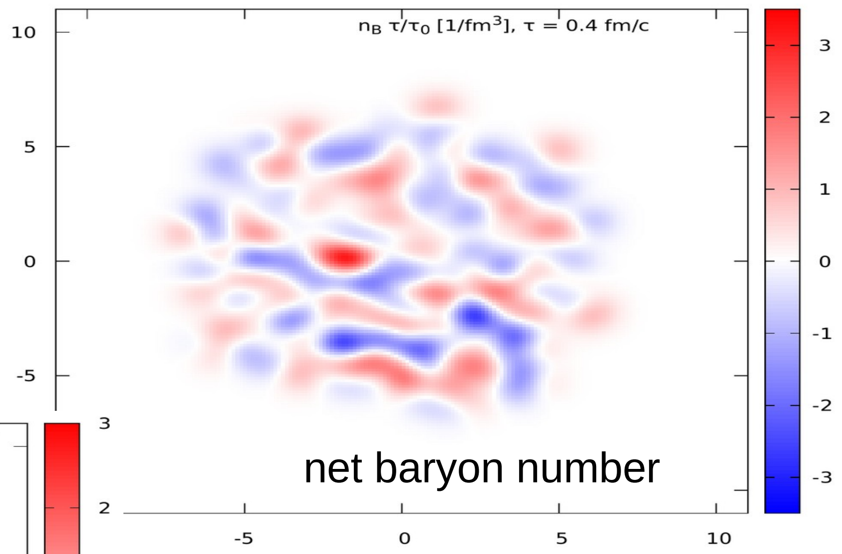


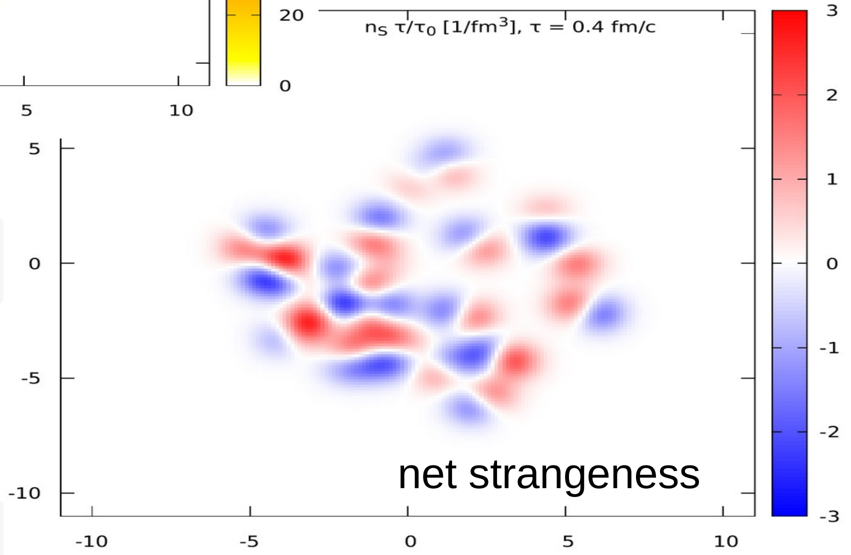
Monnai et al., PRC 100, 024907 (2019)



Initial state with multiple conserved charges







Smoothed initial condition for Pb Pb @ 5 TeV from ICCING

Martinez et al., arXiv:1911.10272, arXiv:1911.12454

Plots not published!

Deriving fluid dynamics from kinetic theory

Denicol et al., PRD 85, 114047 (2012)

On basis of DNMR theory: derivation from the Boltzmann equation

→ **upcoming publication!** (Fotakis, Molnár, Niemi, Denicol, Rischke, Greiner)

Also refer to: Monnai, Hirano, Nucl. Phys. A847:283-314 (2010) or Kikuchi et al. PRC 92, 064909 (2015)

relativistic Boltzmann eq.

$$k_i^\mu \partial_\mu f_{i,\mathbf{k}} = \mathcal{C}_i[f_i]$$

→

2nd-order (multi-component) hydro

$$\dot{\Pi},\ \dot{V}_q^{\langle\mu\rangle},\ \dot{\pi}^{\langle\mu\nu\rangle}$$

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Idea:

Look at off-equilibrium moments:

$$\rho_{i,n}^{\mu\nu} = \sum_{i=1}^{N_{\text{species}}} \int \frac{\mathrm{d}^3\mathbf{k}_i}{(2\pi)^3 E_{i,\mathbf{k}}} E_{i,\mathbf{k}}^n k_i^{\langle\mu} k_i^{\nu\rangle} \left(f_{i,\mathbf{k}} - f_{i,\mathbf{k}}^{(0)}\right)$$

Relate them to the fluid-dynamical fields with constituent's transport coefficients:

$$\rho_{i,n}^{\mu\nu} = \frac{\eta_{i,n}}{\eta}\pi^{\mu\nu} + \mathcal{O}(2)$$

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$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \mathcal{O}(2)[\Pi, V_q^\mu, \pi^{\mu\nu}, \theta, \sigma^{\mu\nu}, \nabla^\mu P_0, \nabla\alpha_q]$$

$$\sum_{q'} \tau_{qq'} \dot{V}_{q'}^{\langle\mu\rangle} + V_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \alpha_{q'} + \mathcal{O}(2)[\Pi, V_q^\mu, \pi^{\mu\nu}, \theta, \sigma^{\mu\nu}, \omega^{\mu\nu}, \nabla^\mu P_0, \nabla\alpha_q]$$

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1st order terms (Navier-Stokes): mixed chemistry already couples diffusion currents!

2nd order terms: couples all currents to each other; depend on all gradients!

→ 3 conserved charges: 70+ transport coefficients (!!) with $(T, \mu_\mathrm{B}, \mu_\mathrm{Q}, \mu_\mathrm{S})$-dependence

Computation of transport coefficients (Example: diffusion coefficients)

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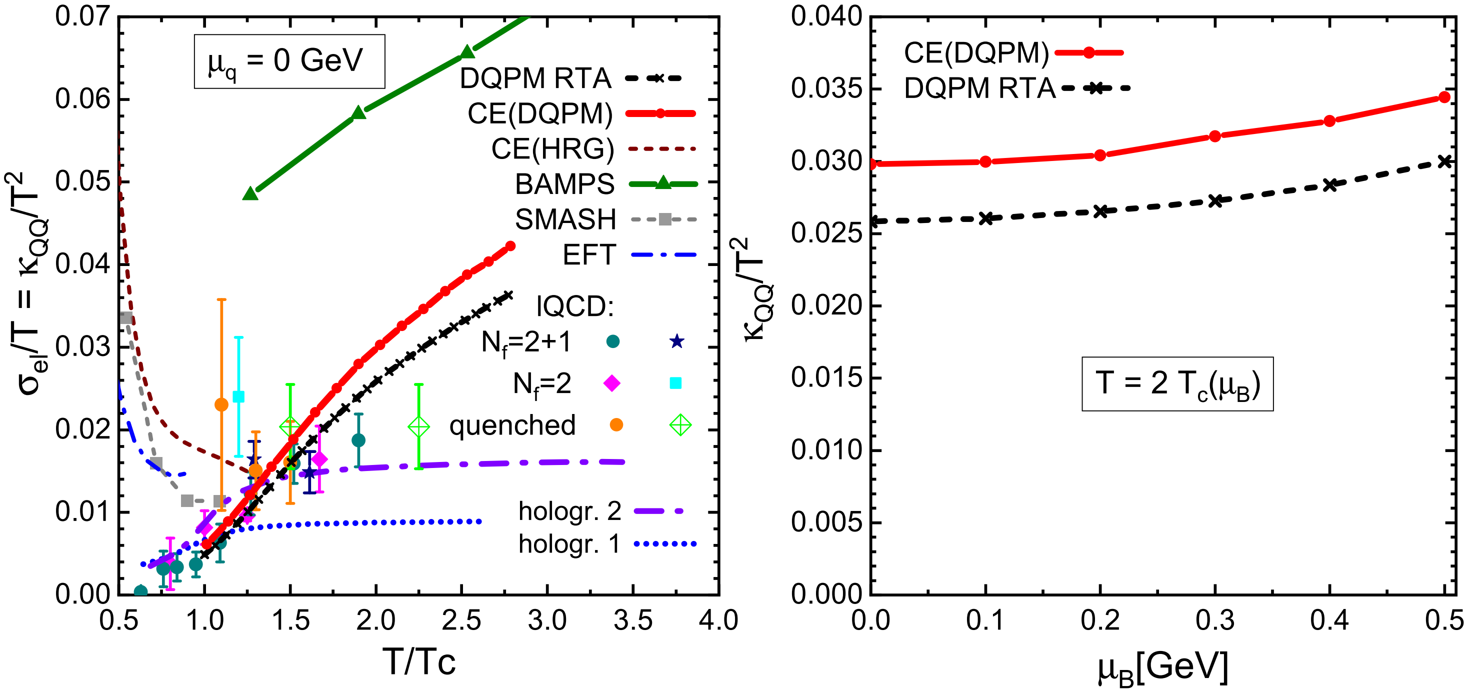
$$\kappa_{qq'} = \sum_{n=0}^{\infty} \sum_{i,j=1}^{N_{\text{species}}} \left(\mathcal{C}^{-1}\right)^{(1)}_{ji,0n} q_j \left(q'_i \mathcal{J}^{(i)}_{n+1,1} - \frac{n_{q'}}{\epsilon + P_0} \mathcal{J}^{(i)}_{n+2,1} \right)$$

Example: introduction of features from LQCD via the usage of DQPM

Fotakis, Soloveva et al, PRD 104, 034014 (2021)



A simplistic case study

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)

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- Investigate longitudinal evolution in Milne coordinates (transversally homogeneous)
- Conserved baryon number and strangeness, ***neglect viscosity***, neglect 2nd order terms

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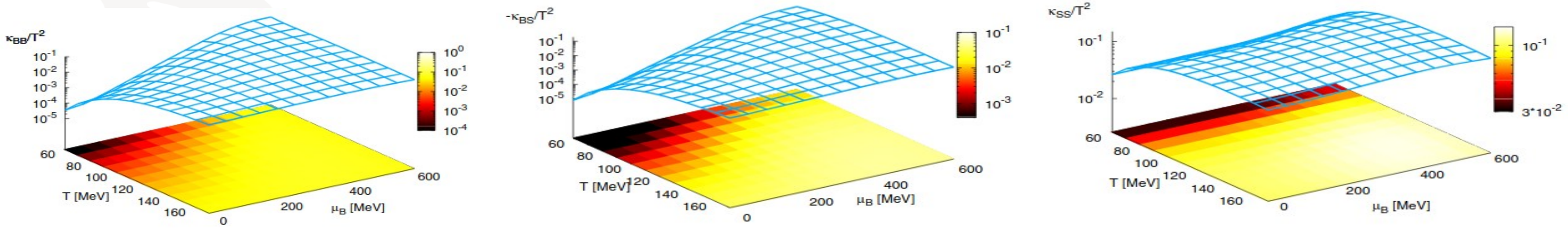
  - Assumed elastic, isotropic, binary cross sections from PDG, SMASH, GiBUU and UrQMD



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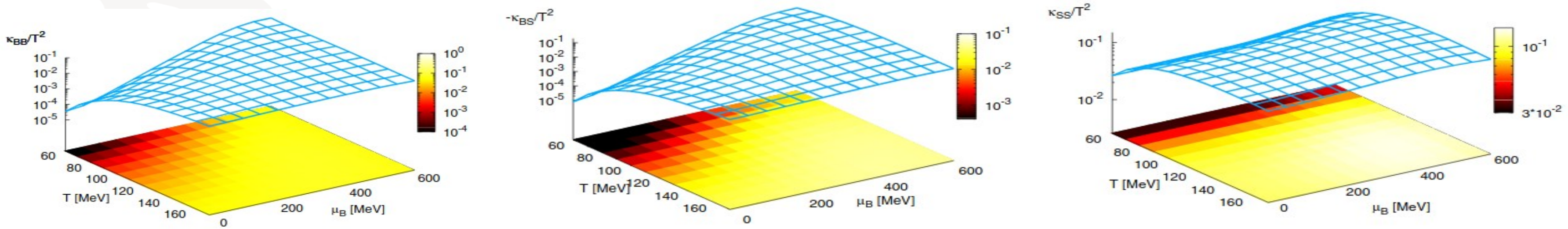

- ***Simple initial state:*** T = 160 MeV, no initial net strangeness, longitudinal double-gaussian profile in net baryon number, no initital dissipative currents

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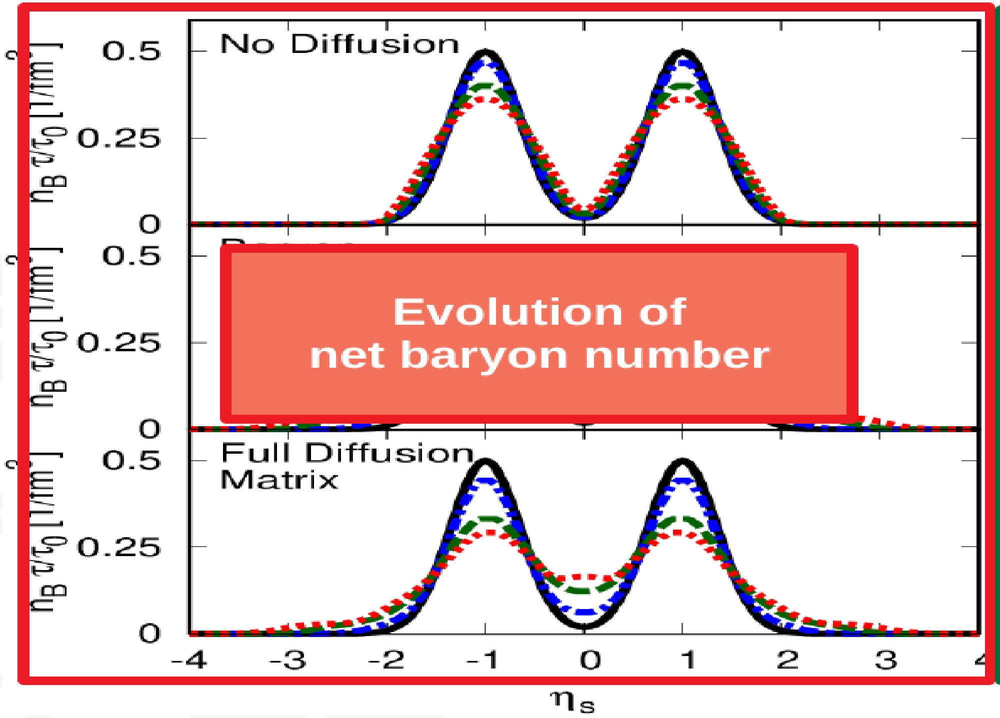

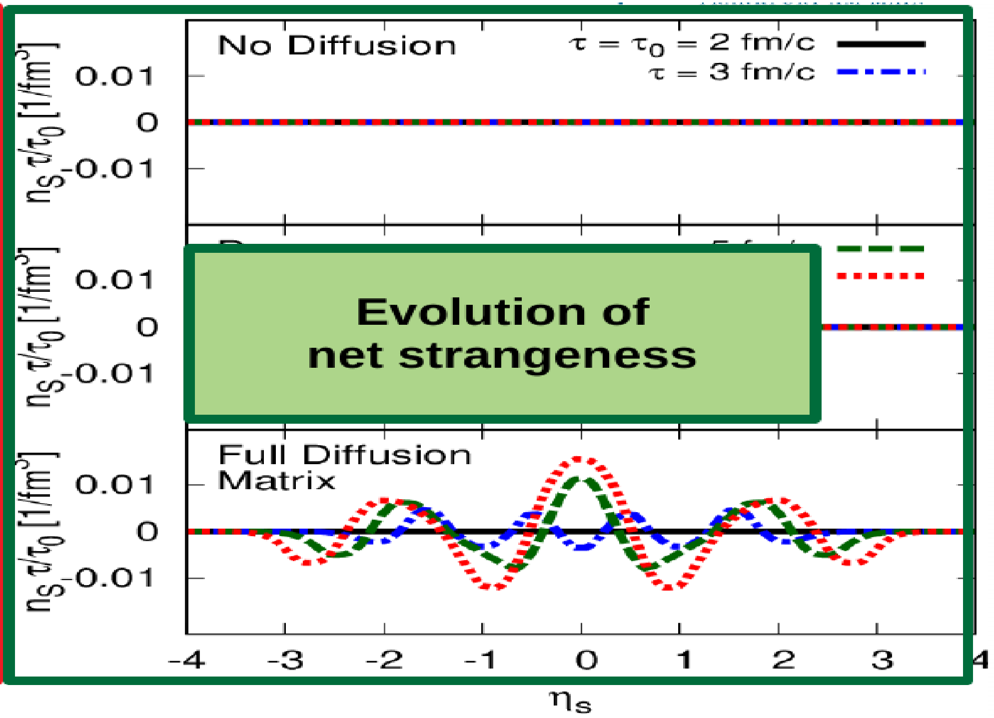

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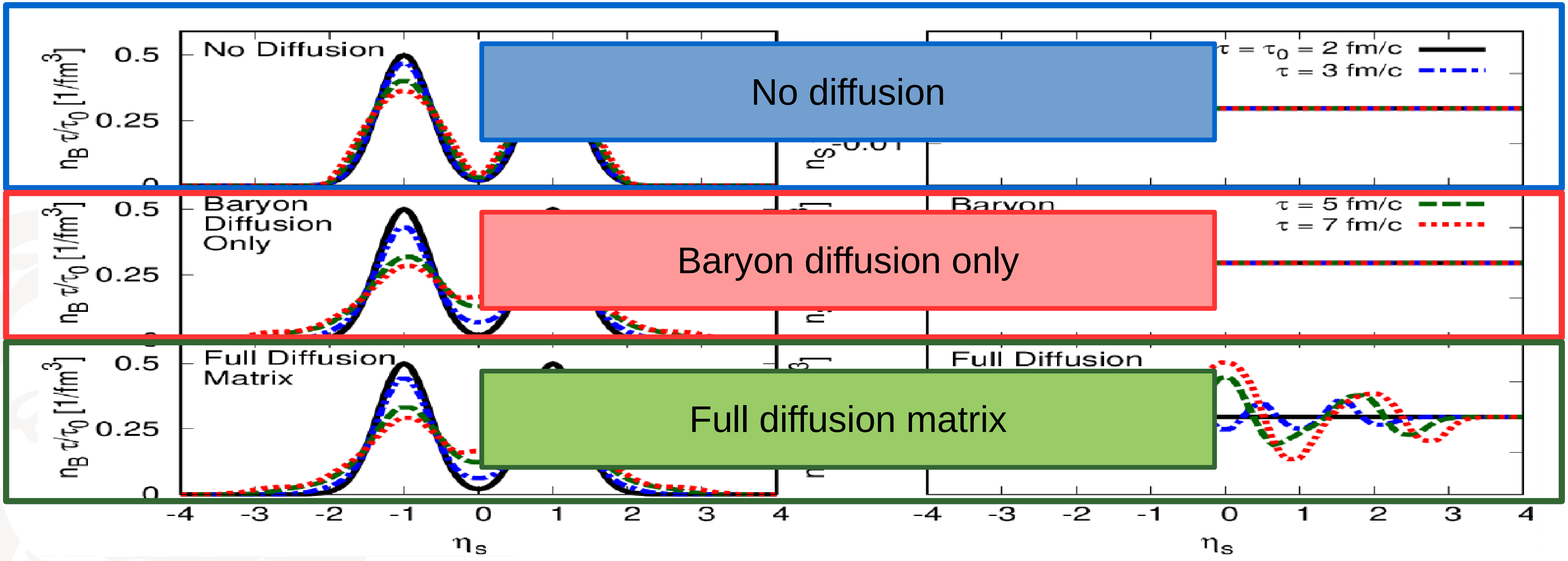

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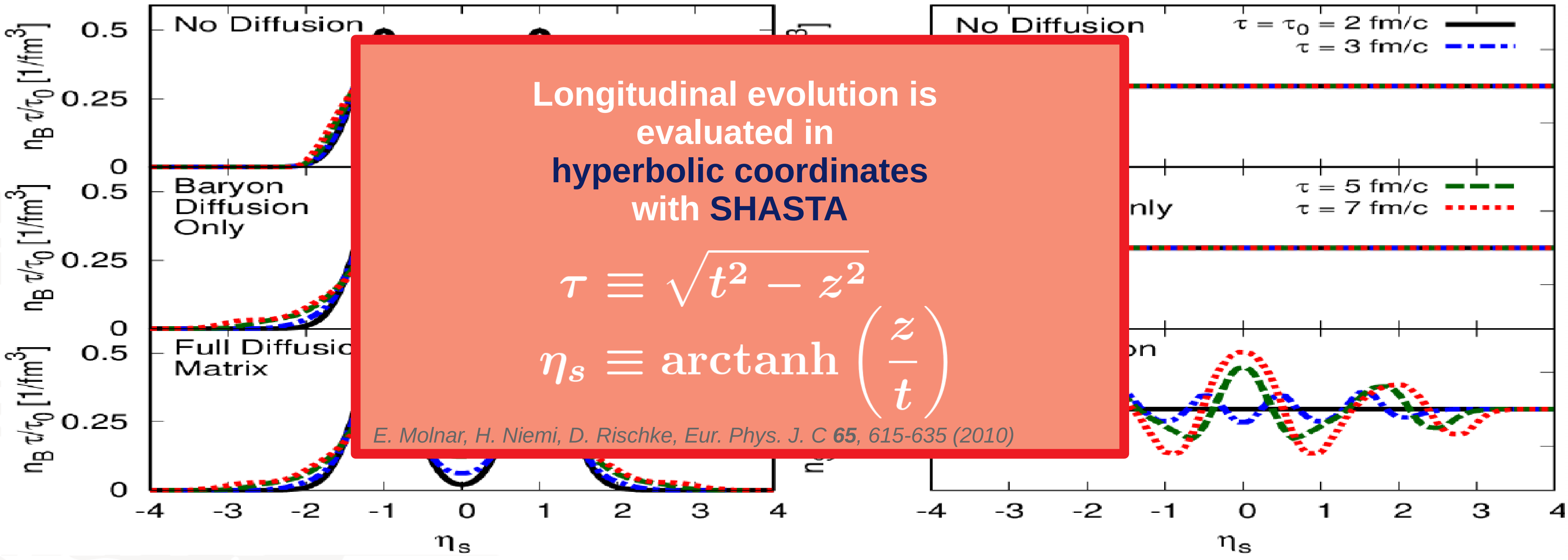

Longitudinal evolution is evaluated in hyperbolic coordinates with SHASTA

$$\tau \equiv \sqrt{t^2 - z^2}$$

$$\eta_s \equiv \operatorname{arctanh}\left(\frac{z}{t}\right)$$

*E. Molnar, H. Niemi, D. Rischke, Eur. Phys. J. C **65**, 615-635 (2010)*

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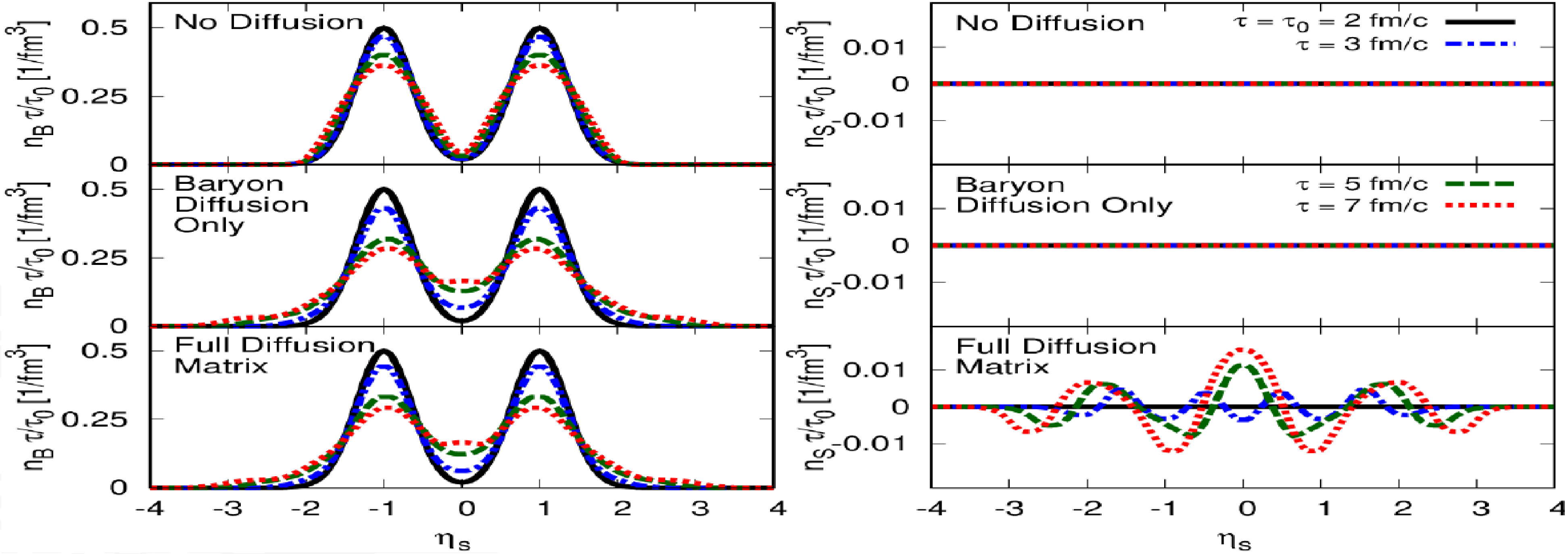

Mixed chemistry couples diffusion currents and introduces charge-correlation through EoS

$$\mu_S \equiv \mu_S(\epsilon, n_B, n_S)$$

e.g.: $\nabla^\mu \alpha_S \sim \nabla^\mu n_B$

→ Generation of domains of non-vanishing local net charge (here net strangeness)!

Outlook

- Implement derived fluid dynamic theory in existing (3+1)D-hydro code
- Evaluate 2nd order transport coefficients from linearized Boltzmann equation
- Use more realistic initial state and equation of state (see above)
- Apply freeze-out routines, take δf-correction
- Find observables sensitive to charge-coupling → investigate impact

Backup

Computation of transport coefficients (Example: diffusion coefficients)

On basis of DNMR theory: derivation from the Boltzmann equation

→ **upcoming publication!** (Fotakis, Molnár, Niemi, Denicol, Rischke, Greiner)

relativistic Boltzmann eq.

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$$\mathcal{C}_{i,n-1}^{\langle\mu\rangle} \equiv \int \frac{\mathrm{d}^3\mathbf{k}_i}{(2\pi)^3 E_{i,\mathbf{k}}} E_{i,\mathbf{k}}^{n-1} k_i^{\langle\mu\rangle} \mathcal{C}_i[f_i]$$

$$= -\sum_{m=0}^{\infty}\sum_j \mathcal{C}_{ij,nm}^{(1)} \rho_{j,m}^\mu + \text{non-linear terms}$$

Entries of „collision matrix“ (for diffusive moments)

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Diffusion coefficient matrix! (equivalent to our PRL and PRD expression)

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)
Fotakis, Soloveva et al, PRD 104, 034014 (2021)

Equation of State - details

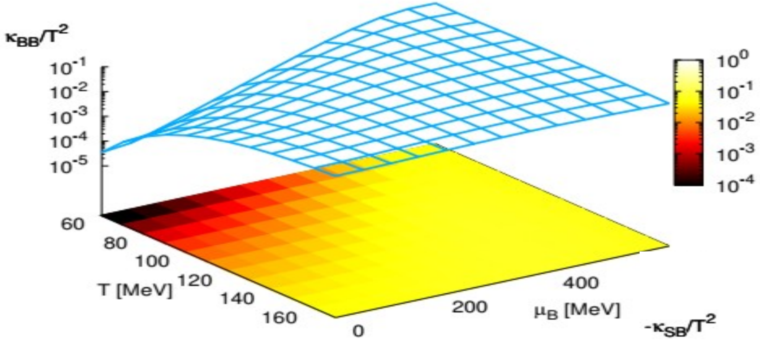
- Hadronic system including lightest 19 species
  $$\pi^\pm, \pi^0, K^\pm, K^0, \bar{K}^0, p, \bar{p}, n, \bar{n}, \Lambda^0, \bar{\Lambda}^0, \Sigma^0, \bar{\Sigma}^0, \Sigma^\pm, \bar{\Sigma}^\pm$$
- Assume classical statistics and non-interacting limit
  $$P_0(T, \{\mu_q\}) \equiv \frac{1}{3} \sum_{i=1}^{N_{\text{species}}} \int \frac{\mathrm{d}p^3}{(2\pi)^3 E_{i,p}} \left(E_{i,p}^2 - m_i^2\right) g_i \exp(-E_{i,p}/T + \sum_q q_i \alpha_q)$$
- Only assume baryon number and strangeness, neglect electric charge
- Tabulate state variables over energy density and net charge densities
  $$T \equiv T(\epsilon, \{n_q\}), \quad \mu_q \equiv \mu_q(\epsilon, \{n_q\}), \quad P_0 \equiv P_0(\epsilon, \{n_q\})$$

Diffusion coefficient matrix - details

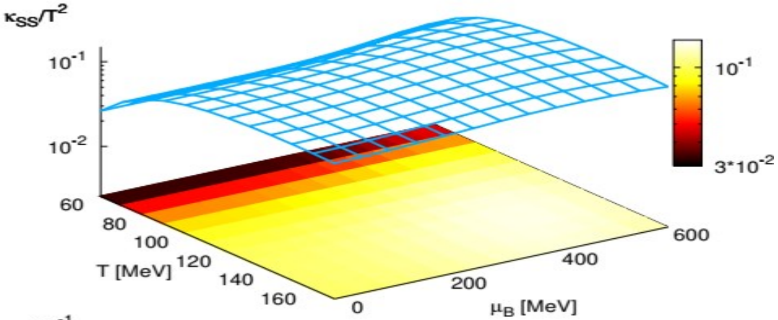
$$\begin{pmatrix} V_B^\mu \\ V_S^\mu \end{pmatrix} \sim \begin{pmatrix} \kappa_{BB} & \kappa_{BS} \\ \kappa_{SB} & \kappa_{SS} \end{pmatrix} \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_S \end{pmatrix}$$

- Matrix is symmetric

L. Onsager, Phys. Rev. ***37****, 405 (1931) & Phys. Rev.* ***38****, 2265 (1931)*

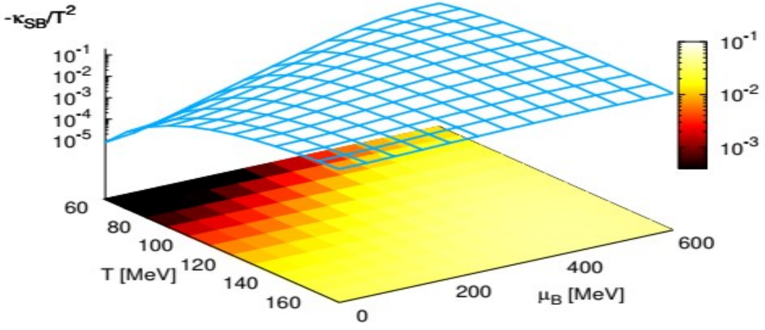


- Elastic isotropic cross sections from PDG, SMASH, GiBUU, UrQMD



κ_{SB} is negative and has similar magnitude as κ_{BB}

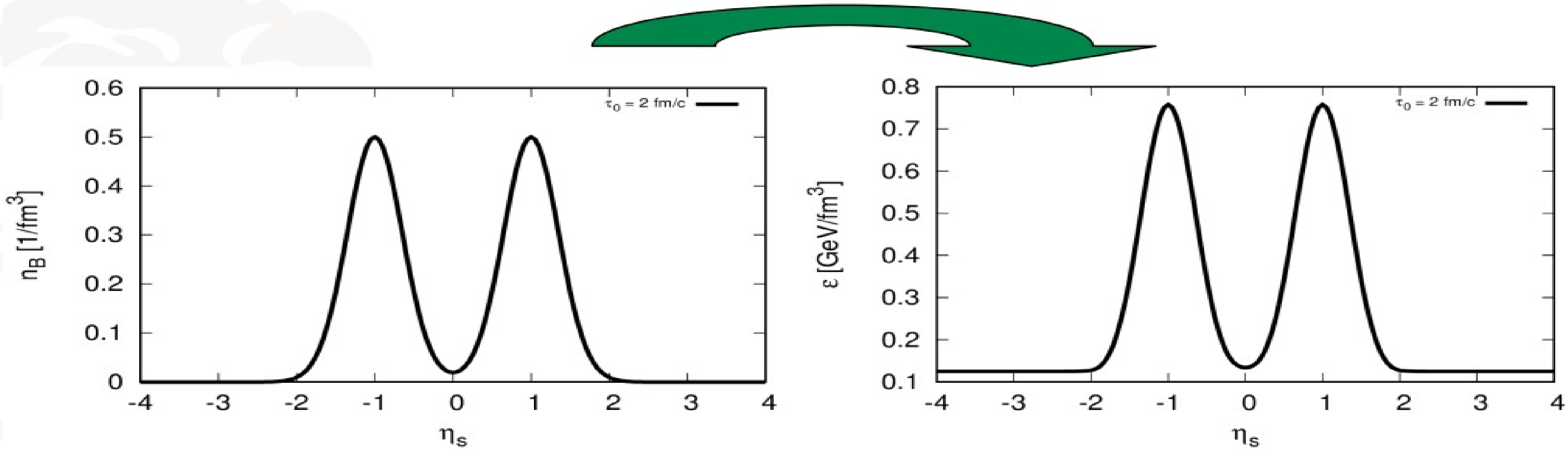
$\Longrightarrow$ significant coupling?



- Tabulate coefficient matrix over $T,\ \mu_B,\ \mu_S$
- $\mu_Q = 0$

Initial conditions - details

- $\tau_0 = 2\ \mathrm{fm}/c$
- Initially: no dissipation and only **Bjorken scaling flow**
- Temperature = 160 MeV
- Double-gaussian profile in net baryon number
- From **EoS**: get energy density



A simplistic case study

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)

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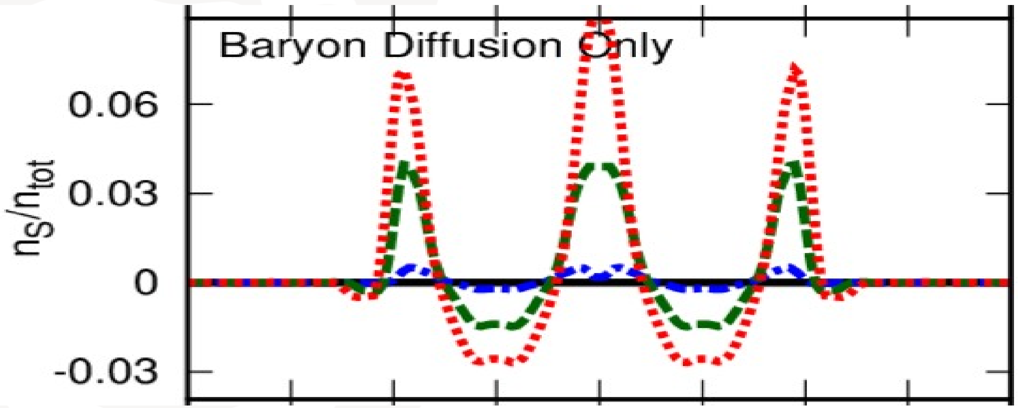
Let's add shear and also account for second-order terms in the diffusion!

Second-order terms in ultrarelativistic limit:

$$\tau_\pi \, \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \frac{4}{3}\pi^{\mu\nu}\theta - 2\pi_\lambda^{\langle\mu}\omega^{\nu\rangle\lambda} - \frac{10}{7}\pi^{\lambda\langle\mu}\sigma_\lambda^{\nu\rangle}$$

With: $\frac{\eta}{s} = \frac{1}{4\pi}$



Second-order transport coefficients not consistent with assumed (massive) system

→ generation of unphysical charge currents

Consistency is important in charge transport!