Quantum Mechanical Bound State Formation in Time Dependent Potentials

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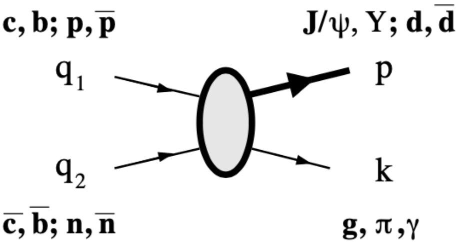
Table of Contents

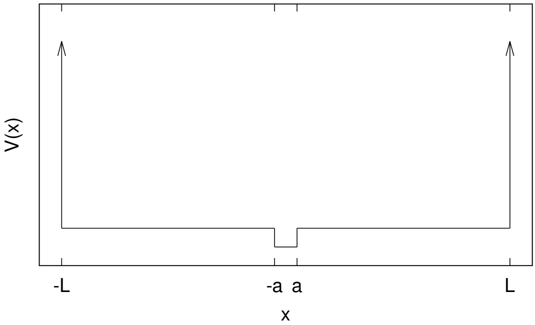
Hadronic bound states in strongly interacting matter in high-energy heavy-ion collisions

- bound states can be formed and destroyed during specific stages of evolution of the medium ('snowball in hell')
- two important kinds of bound states: heavy quarkonia ($J/\Psi, \Upsilon$ and excited states) and (anti-)deuterons ($d, \bar{d}$)
- bound states can form in the evolving medium and are subsequently dissociated. How this happens is regulated by transition probabilities and by the medium constituents' phase-space density.
- possible reactions $c\bar{c} \leftrightarrow J/\Psi g(\pi), b\bar{b} \leftrightarrow \Upsilon g(\pi), pn \leftrightarrow d\pi(\gamma), \bar{p}\bar{n} \leftrightarrow \bar{d}\pi(\gamma)$



Particle with one bound state

Approximate bound state by potential box:



$$V(x) = \begin{cases} V_\infty & \text{for } -\infty < x < -L, \\ 0 & \text{for } -L \leq x \leq -a, \\ -V_0 & \text{for } -a \leq x \leq a, \\ 0 & \text{for } a \leq x \leq L, \\ V_\infty & \text{for } L \leq x \leq \infty \end{cases}$$

with parameters

a, L, V_0

Solving the stationary Schrödinger equation for boundary conditions

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right]\psi(x) = E\psi(x)$$

Consider parity:

$$\psi_1^s(-x) = \psi_3^s(x),$$
$$\psi_2^s(x) = \psi_2^s(-x)$$

and

$$\psi_1^a(-x) = -\psi_3^a(x),$$
$$\psi_2^a(-x) = -\psi_2^a(x)$$

and ansatz

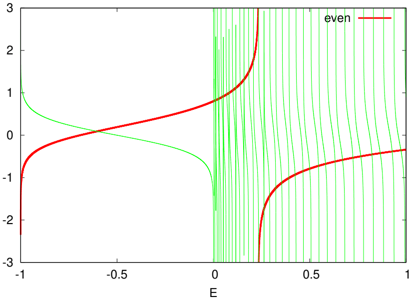
$$\psi_{1,2}(x) = A_{1,2}e^{k_{1,2}x} \pm B_{1,2}e^{k_{1,2}x}$$

with

$$k_1^2 = -\frac{2m}{\hbar^2}E$$
$$k_2^2 = -\frac{2m}{\hbar^2}(E - V)$$

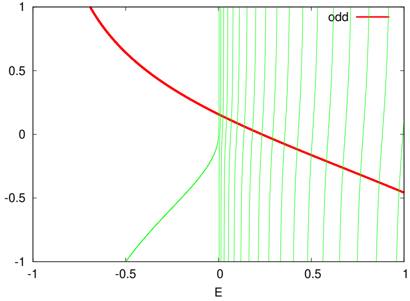
and distinguish $E < 0$ and $E > 0$!!

Odd and even solution



symmetric:

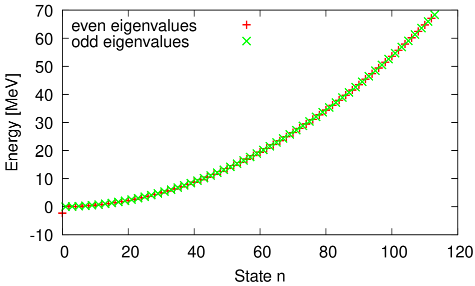
$$\tan(k_2^s a) = \frac{k_1^s}{k_2^s}\frac{1+e^{2k_1^s(a-L)}}{1-e^{2k_1^s(a-L)}}$$



antisymmetric:

$$\cot(k_2^a a) = -\frac{k_1^a}{k_2^a}\frac{1+e^{2k_1^a(a-L)}}{1-e^{2k_1^a(a-L)}}$$

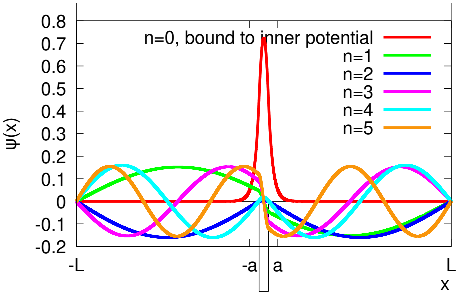
Final stationary solution



here Deuteron:

- $2a = 1.2$ fm
- $V_0 = -15$ MeV
- $m = \frac{m_p + m_n}{2}$
- $L \approx 200$ MeV
- $E_{\text{bind}} = -2.3 MeV$

- $L \gg a$
- numerical evaluation of the first 110 states, $n(L, a, V_0)$



Solving the time-dependent Schrödinger equation

$$i\hbar\partial_t\psi(x,t) = \left[-\frac{\hbar^2}{2m}\partial_x^2 + \hat{V}(x,t)\right]\psi(x,t) = \hat{H}\psi(x,t)$$

where

$$\psi(x,t) = \sum_n c_n(t)\psi_n(x)$$

and therefore

$$\hat{H}\left|\psi\right\rangle = \sum_n c_n(t)\left[E_n + \hat{V}(t)\right]\left|\psi_n\right\rangle, \qquad \left|\psi_n\right\rangle = \psi_n(x)$$

leeds to ODE $\in \mathbb{C}$

$$\boxed{i\dot{\tilde{c}}_j(t) = \sum_n V_{jn}\exp\left(i(E_j - E_n)t\right)\tilde{c}_n(t)}$$

with

$$\tilde{c}_j = c_j \exp\left(iE_j t\right)$$

Solved by a 4th order Runga-Kutta method

- Single pulse, Gaussian

$$V(x,t) = V \exp\left(-a(x-x_0)^2\right) \exp\left(-b(t-t_0)^2\right)$$

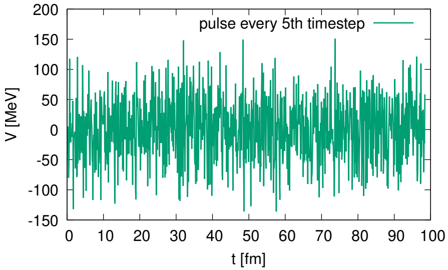
- Several pulses, Gaussian

$$V(x,t) = V \exp\left(-a(x-x_0)^2\right) \left[\exp\left(-b(t-t_0)^2\right) + \exp\left(-b(t-t_1)^2\right) + \ldots\right]$$

with $a = \frac{1}{2\sigma_x^2}$ and $b = \frac{1}{2\sigma_t^2}$

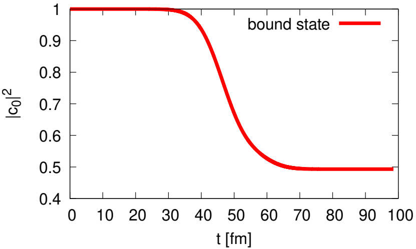
- Stochastic pulses, Gaussian distributed with $\langle V_{\text{stoch}} \rangle = 0$ and σ_V, each pulse $\delta-$like

$$V(x,t) = \xi_n \delta(t - n\Delta t)\delta(x_0 - 0)$$



Single Pulse

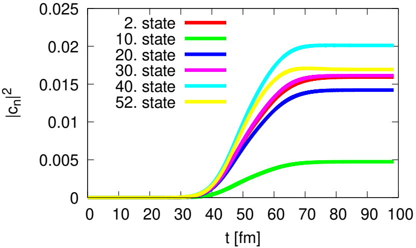
$$V(x,t) = V \exp\left(-\frac{(x-x_0)^2}{2\sigma_x^2}\right) \exp\left(-\frac{(t-t_0)^2}{2\sigma_t^2}\right)$$



$$\Rightarrow V_{mn} = \int \mathrm{d}x \psi_m V(x,t) \psi_n$$

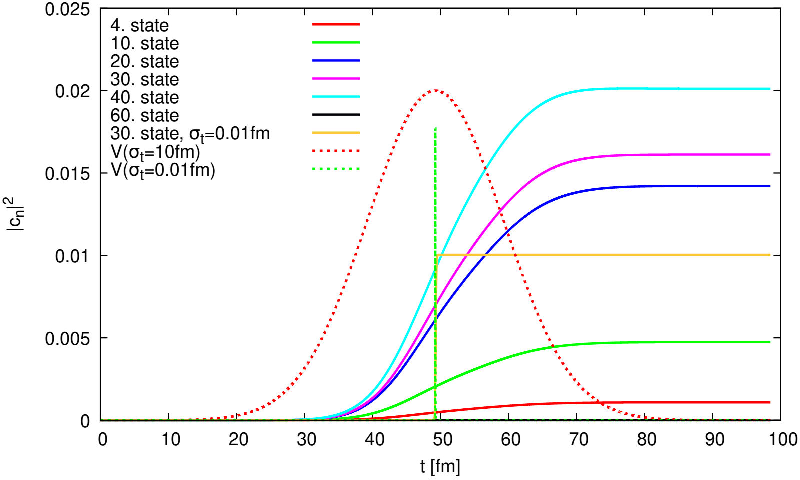
$$\Rightarrow \mathrm{i}\dot{\tilde{c}}_m(t) = \sum_n V_{mn} \mathrm{e}^{\mathrm{i}(E_m - E_n)t} \tilde{c}_n(t)$$

- $|c_n(t)|^2$ depends on σ_t and V
- oscillations appear, non-perturbative
- since $\partial_t V(x,t) = 0$, $\partial_t |c_n(t)|^2 = 0$



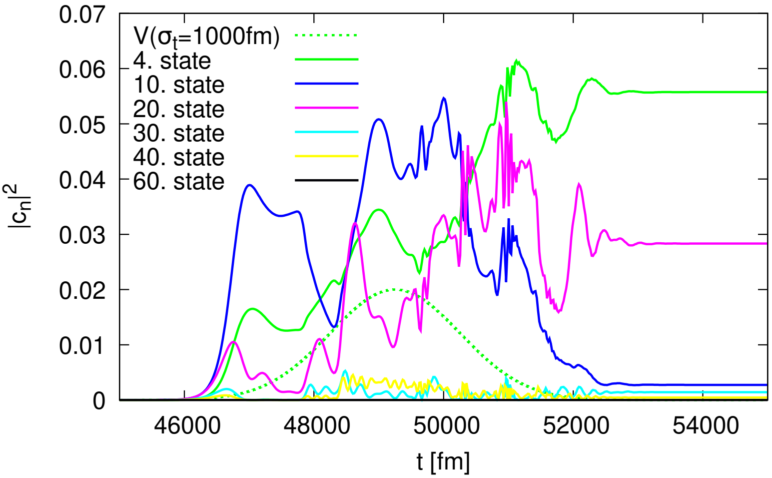
Heisenberg fulfilled???

Impact of the time length of the potential



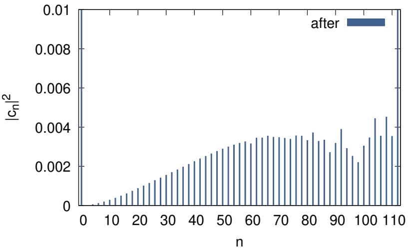
Variation of σ_t has *immediate* impact on $|c_n(t)|^2$.

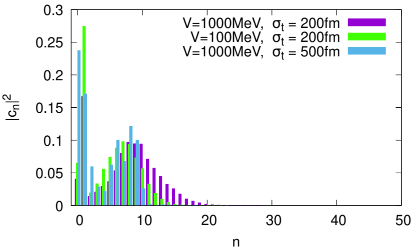
Impact of the time length of the potential



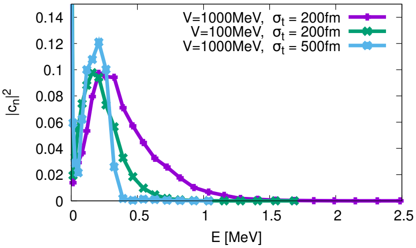
- oscillations from ω_{mn} if σ_t is large
- highly non-perturbative due to V and σ_t

Non-physical: parameters way too large!





- $\sigma_t = 1$ fm
- $\sigma_t = 200, 500$ fm



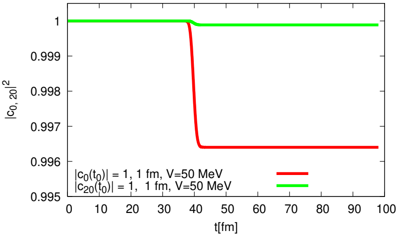
$$\Delta t \Delta E \geq \frac{1}{2} \quad \Leftrightarrow \quad \sigma_t \Delta E \geq \frac{1}{2}$$

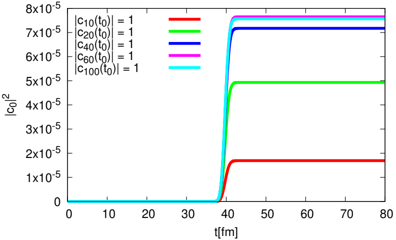
Deuteron - $|c_{n\geq 0}(t=0)| = 1$, $|c_{m\neq n}(t=0)| = 0$

- $r = 1.2$ fm
- $V_0 = -15$ MeV
- $V_{\text{bind}} = -2.23$ MeV
- $m = \frac{m_p + m_n}{2}$

Some elementary thoughts

- collision length $\sim 0.5 - 2$ fm
- collision energy $\sim 50 - 100$ MeV

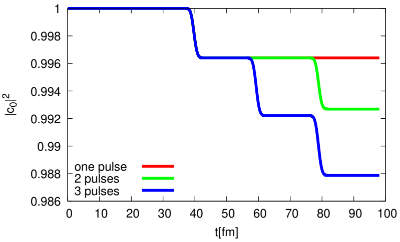


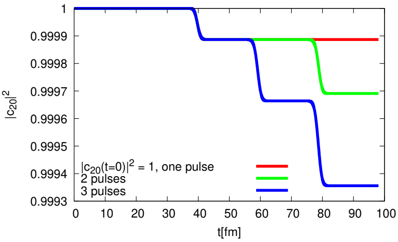


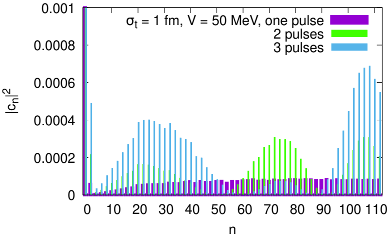
Associated energies: $n = 0, 10, 20, 40, 60, 100,$
$E = -2.3, 0.55, 2.21, 8.72, 19.45, 51.45$ MeV

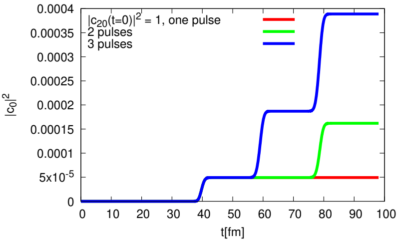
0th state is originally populated:

20th state is originally populated:



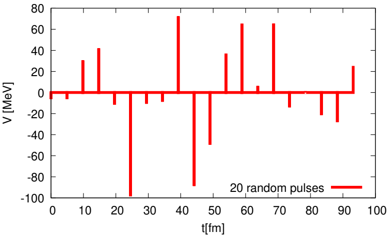


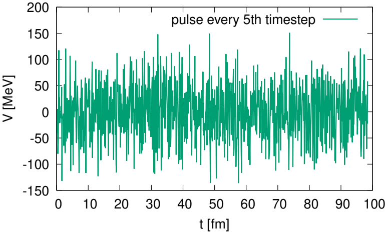


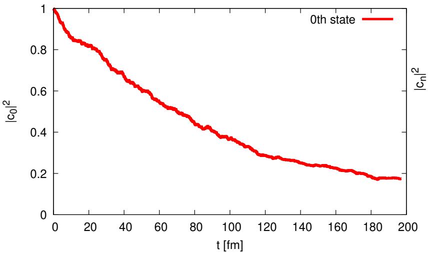


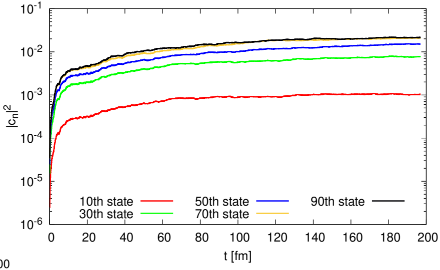
- allow as well positive as negative potential
- make the potential random with equidistant distributed in time...
- ... with Gaussian distributed potentials V with $\sigma_V = 50$ MeV, $\langle V \rangle = 0$ MeV.

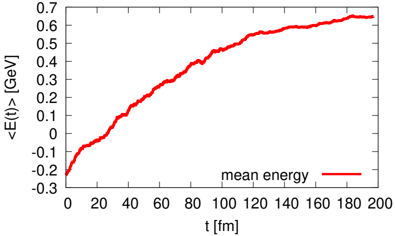
number of pulses from 20.... ... to 2000 pulses



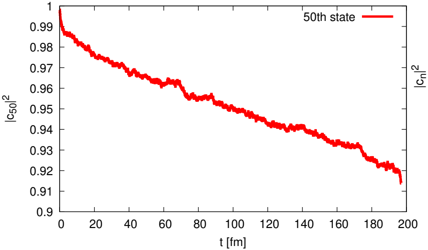


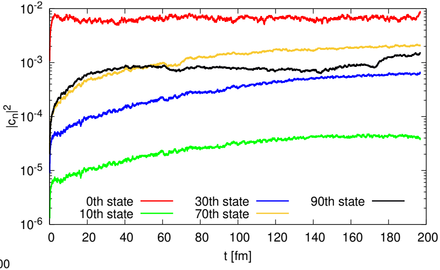


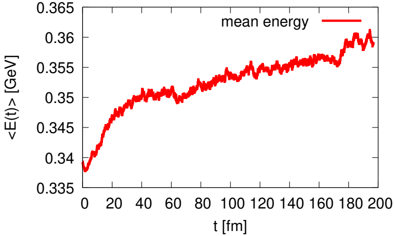




- taking the average over 200 runs
- bound state remains at a certain level
- $|c_0(t=0)|^2 = 1,\ |c_{n\neq 0}(t=0)|^2 = 0$
- 2000 pulses
- $\sigma_V = 50$ MeV







- taking the average over 200 runs
- $|c_{50}(t=0)|^2 = 1$,
  $|c_{n\neq 50}(t=0)|^2 = 0$
- $\sigma_V = 50$ MeV
- 2000 pulses
- how does it go on?

- white noise stochastic potential works as a thermal bath
- damping not described yet

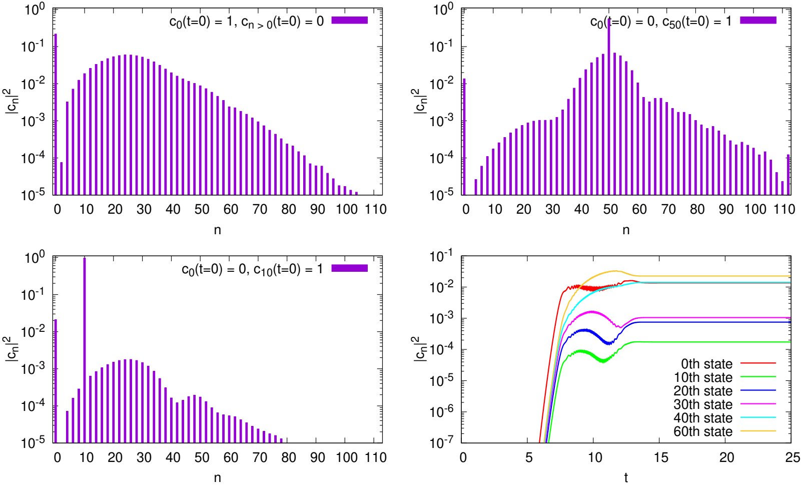
$$\dot{p} = -\gamma p + f(t)$$

- equilibration of system?
- help of influence functional, the Caldeira-Leggett master equation or Kadanoff-Baym with $H = H_S + H_B + H_{SB}$

$$\dot{\rho}_S(t) = -\frac{\mathrm{i}}{\hbar}\left[H'_S, \rho_S(t)\right] - \frac{\mathrm{i}\gamma_0}{\hbar}\left[x, \{p, p_S\}\right] - \frac{2M\gamma_0 k_B T}{\hbar^2}\left[x, \left[x, \rho_S(t)\right]\right]$$

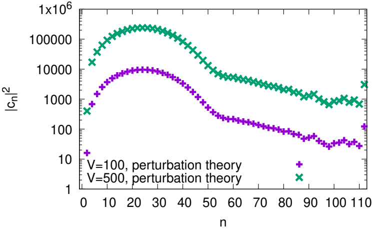
Quantum (dissipation) Langevin-Equation??

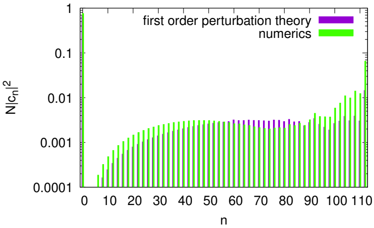
Backup slides

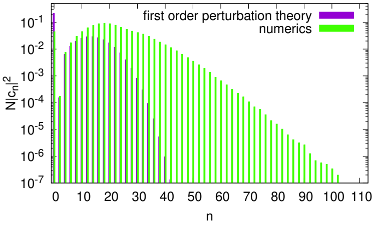


if $|c_0(t=0)|^2 = 1$, $|c_{10}(t=0)|^2 = 1$ or ...if $|c_{50}(t=0)|^2 = 1$

Fermi's golden rule: $c_f^{(1)}(t) = -\frac{\mathrm{i}}{\hbar}\int_{t_0}^{t} \mathrm{d}t'\, V_{fi}(t') \exp\left(\mathrm{i}\omega_{fi} t\right)$

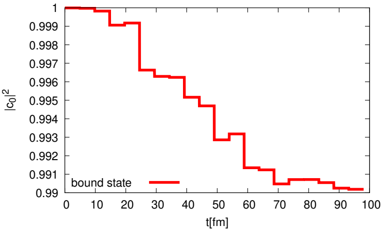


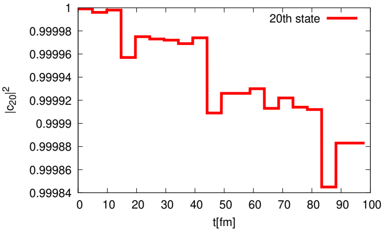


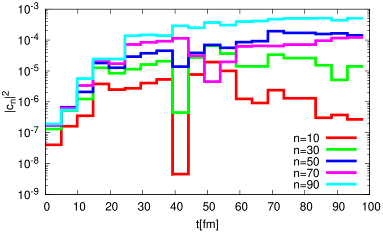


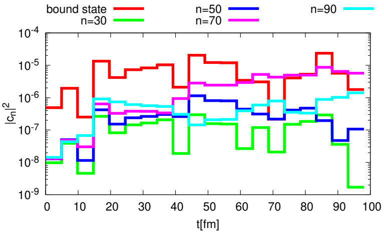
- results way over exact
- normalized for σ_t
- $|c_n(t)|^2 \propto V^2$
- perturbation theory good for small and weak potentials

Stochstic potential









$V \in \mathbb{R}$, originally populated state gets depopulated.