# Kinetic theory and transport coefficients of D mesons


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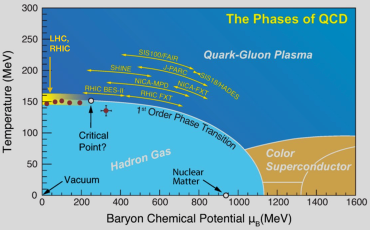
DFG Deutsche Forschungsgemeinschaft

CRC-TR 211


*NA7 STRONG 2020 Workshop*
*Hersonissos, Crete. Oct 5, 2021*

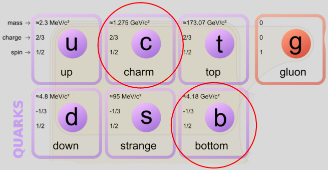
# Introduction: Heavy flavor

(A. Bazavov *et al.*, 1904.09951)




- Infer QCD properties at high temperatures through final state of RHICs
- Find clean and solid observables to connect detections to early stages
- **Hard Probes:** Jets, hard electromagnetic emission, heavy flavor (quarkonia, open-heavy flavor hadrons...)

**Heavy quarks:** formed at the initial stage of the collision (short formation time) and difficult to equilibrate along their evolution (large relaxation time)




**Interactions in a thermal medium?**

**Transport coefficients?**

# Thermal Effective Field Theory for D mesons

Based on:

G. Montaña, À. Ramos, L. Tolos and JMT-R,
Phys.Lett.B 806 (2020) 135464 arxiv

G. Montaña, À. Ramos, L. Tolos and JMT-R,
Phys.Rev.D 102 (2020) 096020 arxiv

# Effective field theory

Effective Lagrangian based on **chiral** and **heavy-quark spin-flavor** symmetries ▸ Effective Lagrangian

- **Chiral expansion** is performed up to NLO
  : also explicitly broken due to light-meson masses $(\pi, K, \bar{K}, \eta)$.
- **Heavy-quark mass expansion** is kept to LO
  : broken by heavy meson masses $(D, D_s, D^*, D_s^*)$.


E.E. Kolomeitsev and M.F.M. Lutz *Phys.Lett. B582 (2004) 39*

J. Hofmann and M.F.M. Lutz *Nucl.Phys. A733 (2004) 142*

F.K.Guo *et al Phys.Lett. B641 (2006) 278*

M.F.M. Lutz and M. Soyeur *Nucl.Phys. A813 (2008) 14*

F.K.Guo, C.Hanhart. S. Krewald, U.G. Meissner *Phys.Lett. B666 (2008) 251*

F-K.Guo, C. Hanhart, U.G. Meissner *Eur.Phys.J. A40 (2009) 171*

L.S. Geng, N. Kaiser, J. Martin-Camalich and W. Weise *Phys.Rev.D82,05422 (2010)*

L.M. Abreu, D. Cabrera, F.J. Llanes-Estrada and JMT-R. *Annals Phys. 326 (2011) 2737*

# Perturbative potential

Tree-level scattering amplitudes at LO in $m_D^{-1}$ expansion:

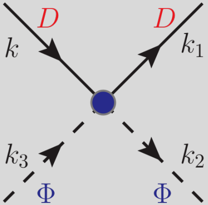
## Perturbative amplitude at lowest order

▸ full tree level

$$V(k, k_3, k_1, k_2) = \frac{C_0}{4 f_\pi^2}\left[(k + k_3)^2 - (k - k_2)^2\right]$$

$f_\pi$: pion decay constant

$C_0$, isospin coefficients: fixed by symmetry




Amplitude accounts for elastic scatterings:

$D\pi$, $DK$, $D\bar{K}$, $D\eta$

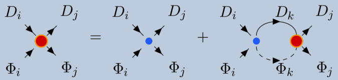
$D_s\pi$, $D_sK$, $D_s\bar{K}$, $D_s\eta$ and their inelastic channels.

# Unitarization

We impose **exact unitarity** to the scattering amplitudes, lost upon truncation of the effective Lagrangian

## Unitarization: Bethe-Salpeter equation

$$T(s) = V(s) + \int VGT \;\; (s)$$




On-shell factorization method

(J.A.Oller and E. Oset *Nucl.Phys.A620 (1997) 438*, L. Roca, E. Oset and J. Singh *Phys.Rev.D72 (2005) 014002*)

## Unitarized scattering amplitude

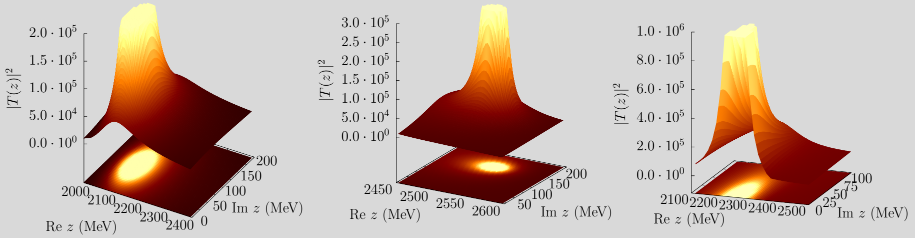
$$T(s) = \frac{V(s)}{1 - G(s)V(s)}$$

# Resonances

## Interpretation of poles

Resonances and Bound states are poles in the complex energy plane

$$m_R = \mathrm{Re}\ z_R\ , \qquad \Gamma_R = 2\mathrm{Im}\ z_R \qquad (z = \sqrt{s} \in \mathbb{C})$$




$D_0^*(2300)$ $\qquad$ $D_{s0}^*(2317)$

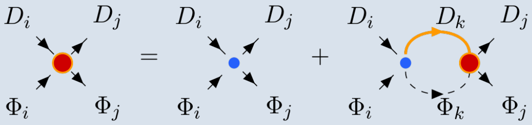
**Double pole structure** of $D_0^*(2300)$

M. Albadalejo *et al.* Phys.Lett.B 767 (2017) 465 , Z.-H. Guo *et al.* Eur.Phys.J.C79 (2019)13, U.G Meißner, Symmetry 12 (2020) 6, 981

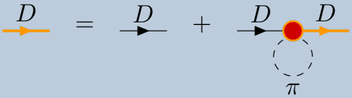
# Finite temperature

At $T \neq 0$ we use **Imaginary Time Formalism**
(energies become Matsubara frequencies, $q^0 \to \omega_n = i2\pi Tn$)

## T-matrix equation




## Propagator equation

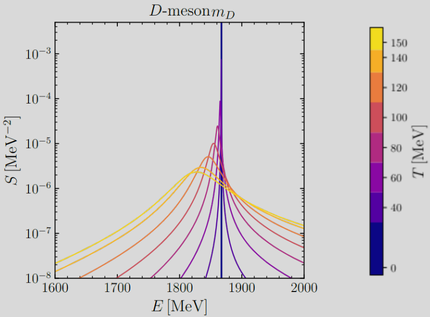


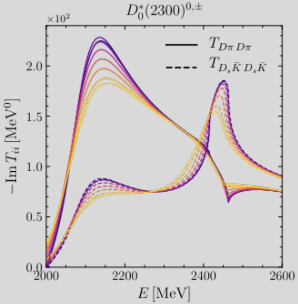

**Self-consistency is required at $T \neq 0$**
M. Cleven, V.K. Magas. À. Ramos,
Phys.Rev.C 96 (2017) 045201

# Spectral functions

$$S_D(E, \vec{k} = 0; T) = -\frac{1}{\pi} \mathrm{Im}\, \mathcal{D}_D(E, \vec{k} = 0; T)$$

(probability distribution of an excitation with energy $E$ and momentum $\vec{k}$)




G. Montaña *et al.*, Phys.Lett.B 806 (2020) 135464, Phys.Rev.D 102 (2020) 9, 096020

other spectral functions

*D* meson gets lighter and broader with increasing temperature mass

# Off-shell Kinetic Theory for D mesons

Based on:

JMT-R, G. Montaña, À. Ramos, L. Tolos,
arXiv:2106.01156 ▸ arxiv

# Kadanoff-Baym approach

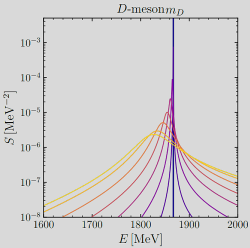
Kinetic theory description with off-shell effects

$\Downarrow$

**Kadanoff-Baym equations**

L. Kadanoff, G.Baym, “Quantum statistical mechanics” 1962, P. Danielewicz, Annals Phys. 152, 239 (1984), W. Botermans and R. Malfliet, Phys. Rept. 198, 115 (1990), J.-P. Blaizot and E. Iancu, Nucl. Phys. B557, 183 (1999), J. Rammer ”Quantum field theory of non-equilibrium states” (2007), W. Cassing, Eur. Phys. J.168, 3 (2009)

- Transport equation from QFT
- Relativistic and quantum effects
- Generic non-equilibrium evolution
- Off-shell effects of quasiparticles $\leftarrow$




(Boltzmann equation can be recovered in the appropriate limit)

Kinetic theory description with off-shell effects

$\Downarrow$

**Kadanoff-Baym equations**

L. Kadanoff, G.Baym, “Quantum statistical mechanics” 1962, P. Danielewicz, Annals Phys. 152, 239 (1984), W. Botermans and R. Malfliet, Phys. Rept. 198, 115 (1990), J.-P. Blaizot and E. Iancu, Nucl. Phys. B557, 183 (1999), J. Rammer ”Quantum field theory of non-equilibrium states” (2007), W. Cassing, Eur. Phys. J.168, 3 (2009)

$$\overbrace{\left(k^\mu - \frac{1}{2}\frac{\partial \mathrm{Re}\ \Pi^R(X,k)}{\partial k_\mu}\right)\frac{\partial i G_D^<(X,k)}{\partial X^\mu}}^{\text{Advective term}} = \overbrace{\underbrace{\frac{1}{2} i\Pi^<(X,k) iG_D^>(X,k)}_{\text{Gain term}} \underbrace{- \frac{1}{2} i\Pi^>(X,k) iG_D^<(X,k)}_{\text{Loss term}}}^{\text{Collision term}}$$

*Kadanoff-Baym Ansatz*:

$$iG_D^<(X,k) = 2\pi\ S_D(X,k)\ f_D(X,k^0)$$

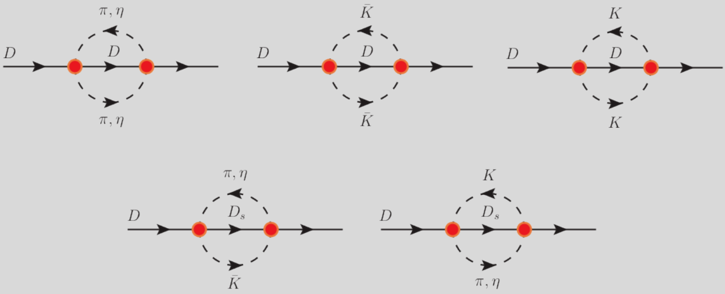
$$iG_D^>(X,k) = 2\pi\ S_D(X,k)\ [1 + f_D(X,k^0)]$$

Specify self-energies $\Pi^<(X,k), \Pi^>(X,k), \Pi^R(X,k)$ to close equation

# $T-$matrix approximation

**$T-$matrix approximation:** L. Kadanoff, G.Baym, "Quantum statistical mechanics" 1962, P. Danielewicz, Annals Phys. 152, 239 (1984), W. Botermans and R. Malfliet, Phys. Rept. 198, 115 (1990)

$$i\Pi^{<}(X,k) = \sum_{\{a,b,c\}} \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \int \frac{d^4k_3}{(2\pi)^4} (2\pi)^4 \delta^{(4)}(k_1+k_2-k_3-k)$$
$$\times \, |T(k_1^0+k_2^0+i\epsilon, \mathbf{k}_1+\mathbf{k}_2)|^2 \; iG^{<}_{D_a}(X,k_1) iG^{<}_{\Phi_b}(X,k_2) iG^{>}_{\Phi_c}(X,k_3)$$

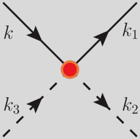


# Kinetic Theory

Employ mass scale hierarchy,

$$m_D \gg m_\Phi, T$$

to exploit $\mathbf{k} - \mathbf{k_1} \ll \mathbf{k}$




small momentum transfer in collision

**On-shell *D* meson**

**Boltzmann Equation**

$\Downarrow$

**Fokker-Planck Equation**

E.M. Lifshitz and L. P. Pitaevskii, ”Physical Kinetics”

B. Svetitsky, Phys. Rev. D37, 2484 (1988)

R. Rapp and H. van Hees, in ”Quark-Gluon Plasma 4”

L.M. Abreu, D. Cabrera, F.J. Llanes-Estrada, and JMTR, Annals Phys. 326 (2011) 2737

**Off-shell *D* meson**

Equivalent reduction in the off-shell case?

JMTR, G. Montaña, À. Ramos, L. Tolos, arXiv:2106.0115

# Fokker-Planck equation reduction

## Off-shell Fokker-Planck equation

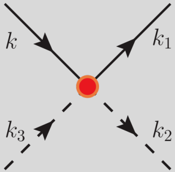
▸ details

$$\frac{\partial}{\partial t} G_D^<(t,k) = \frac{\partial}{\partial k^i}\left\{ \hat{A}(k;T) k^i G_D^<(t,k) + \frac{\partial}{\partial k^j}\left[ \hat{B}_0(k;T)\Delta^{ij} + \hat{B}_1(k;T)\frac{k^i k^j}{\mathbf{k}^2} \right] G_D^<(t,k) \right\}$$

where $\Delta^{ij} = \delta^{ij} - k^i k^j/\mathbf{k}^2$

**Off-shell Transport Coefficients**

$$\begin{cases} \hat{A}(k^0,\mathbf{k};T) \equiv \left\langle 1 - \frac{\mathbf{k}\cdot\mathbf{k}_1}{\mathbf{k}^2} \right\rangle \\ \hat{B}_0(k^0,\mathbf{k};T) \equiv \frac{1}{4}\left\langle \mathbf{k}_1^2 - \frac{(\mathbf{k}\cdot\mathbf{k}_1)^2}{\mathbf{k}^2} \right\rangle \\ \hat{B}_1(k^0,\mathbf{k};T) \equiv \frac{1}{2}\left\langle \frac{[\mathbf{k}\cdot(\mathbf{k}-\mathbf{k}_1)]^2}{\mathbf{k}^2} \right\rangle \end{cases}$$




with

$$\langle \mathcal{F}(\mathbf{k},\mathbf{k}_1)\rangle \equiv \frac{1}{2k^0}\sum_{\lambda,\lambda'=\pm}\lambda\lambda' \int dk_1^0 \int \prod_{i=1}^{3}\frac{d^3k_i}{(2\pi)^3}\frac{1}{2E_2 2E_3} S_D(k_1^0,\mathbf{k}_1)(2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}_3-\mathbf{k}_1-\mathbf{k}_2)$$

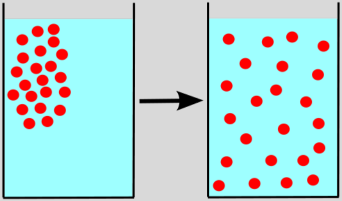
$$\times (2\pi)\delta(k^0+\lambda' E_3 - \lambda E_2 - k_1^0)|T(k^0+\lambda' E_3, \mathbf{k}+\mathbf{k}_3)|^2 f^{(0)}(\lambda' E_3)\tilde{f}^{(0)}(\lambda E_2)\,\mathcal{F}(\mathbf{k},\mathbf{k}_1)$$

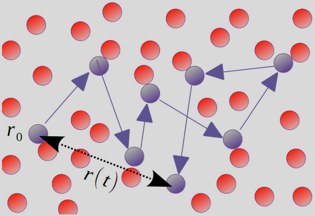
# Diffusion Coefficient

**Fick's diffusion law**

$$\vec{j}_i = -D_s \vec{\nabla} n_i$$

$D_s$ depends on interaction $(\sigma)$
and medium properties $(T, \mu_i)$






**Brownian motion**
**Mean squared displacement**

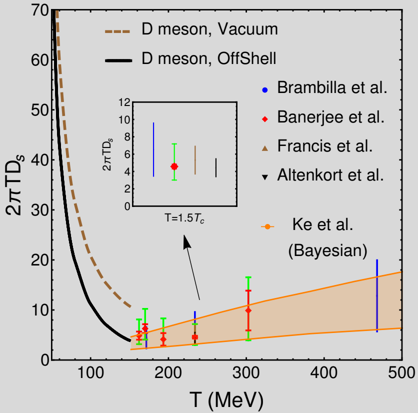
$$\langle [r(t) - r_0]^2 \rangle = 6 D_s t$$

# Spatial diffusion coefficient

## Spatial diffusion coefficient

$$2\pi T D_s(T) = \frac{2\pi T^3}{B_0(k^0 = E_k, \mathbf{k} \to 0; T)}$$

JMTR, G. Montaña, À. Ramos, L. Tolos, arxiv: 2106.01156




**Lattice-QCD calculations**

- N. Brambilla *et al.* Phys. Rev. D102, 074503 (2020)
- D. Banerjee *et al.* Phys. Rev. D85, 014510 (2012)
- A. Francis *et al.* Phys. Rev. D92, 116003 (2015)
- L. Altenkort *et al.* Phys. Rev. D103, 014511 (2021)

**Bayesian study of RHICs**

- W. Ke *et al.* Phys. Rev. C98, 064901 (2018)

# Conclusions

1. We extend the **EFT description of *D* mesons to finite temperature** in a self-consistent manner
2. We describe the **thermal dependence of masses and widths** of ground states, bound states, and resonances
3. We revisit the ***D*-meson kinetic theory** from QFT. Kadanoff-Baym equations to obtain an **off-shell Fokker-Planck equation**
4. We compute of **heavy-flavor transport coefficients** below $T_c$ including thermal amplitudes and off-shell effects. Good agreement with lattice-QCD and Bayesian analyses above $T_c$ .

# Kinetic theory and transport coefficients of D mesons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*Hersonissos, Crete. Oct 5, 2021*

# Effective Lagrangian at NLO

L.S. Geng, N. Kaiser, J. Martin-Camalich and W. Weise *Phys.Rev.D82,05422 (2010)*

$$\mathcal{L}_{\mathbf{LO}} = Tr[\nabla^\mu D \nabla_\mu D^\dagger] - m_D^2 Tr[DD^\dagger] - Tr[\nabla^\mu D^{*\nu} \nabla_\mu D^{*\dagger}_\nu] + m_{D^*}^2 Tr[D^{*\mu} D^{*\dagger}_\mu]$$

$$+igTr\left[\left(D^{*\mu} u_\mu D^\dagger - D u^\mu D^{*\dagger}_\mu\right)\right] + \frac{g}{2m_D} Tr\left[\left(D^*_\mu u_\alpha \nabla_\beta D^{*\dagger}_\nu - \nabla_\beta D^*_\mu u_\alpha D^{*\dagger}_\nu\right)\epsilon^{\mu\nu\alpha\beta}\right]$$

$$\mathcal{L}_{\mathbf{NLO}} = -h_0 Tr[DD^\dagger] Tr[\chi_+] + h_1 Tr[D\chi_+ D^\dagger] + h_2 Tr[DD^\dagger] Tr[u^\mu u_\mu] + h_3 Tr[D u^\mu u_\mu D^\dagger]$$

$$+h_4 Tr[\nabla_\mu D \nabla_\nu D^\dagger] Tr[u^\mu u^\nu] + h_5 Tr[\nabla_\mu D \{u^\mu, u^\nu\} \nabla_\nu D^\dagger] \; + \; \{D \to D^\mu\}$$

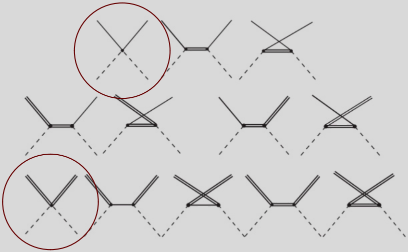
$$D = (D^0, D^+, D_s^+)$$

$$\nabla^\mu = \partial^\mu - \frac{1}{2}(u^\dagger \partial^\mu u + u \partial^\mu u^\dagger)$$

$$u^\mu = i(u^\dagger \partial^\mu u - u \partial^\mu u^\dagger)$$

$$u = \exp\left[\frac{i}{\sqrt{2}F}\Phi\right] = \exp\left[\frac{i}{\sqrt{2}F}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}\right]$$

# Heavy meson—light meson interaction



Tree-level diagrams for $D^{(*)} - \Phi$ scattering (elastic and inelastic).

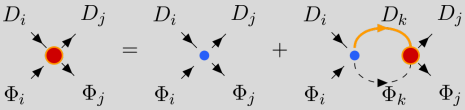
Solid line: $D$ meson, Double solid line: $D^*$ meson, Dashed line: light meson ($\Phi = \{\pi, K, \bar{K}, \eta\}$)

- Born exchanges are suppressed by $1/m_H$.
  In particular, spin-flip processes vanish in the HQ limit.
- Only contact terms survive at lowest order!

back

# Finite temperature

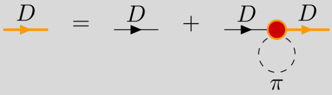
At $T \neq 0$ we use the **Imaginary Time Formalism**
(energies become Matsubara frequencies, $q^0 \to \omega_n = i2\pi Tn$)




$$T_{ij} = [1 - G_{D\Phi} V]^{-1}_{ik} V_{kj}$$

$$G_{D\Phi}(E, \vec{p}; T) = \int \frac{d^3k}{(2\pi)^3} \int d\omega \int d\omega' \frac{S_D(\omega, \vec{k}; T) S_\Phi(\omega', \vec{p} - \vec{k}; T)}{E - \omega - \omega' + i\varepsilon} [1 + f(\omega, T) + f(\omega', T)]$$

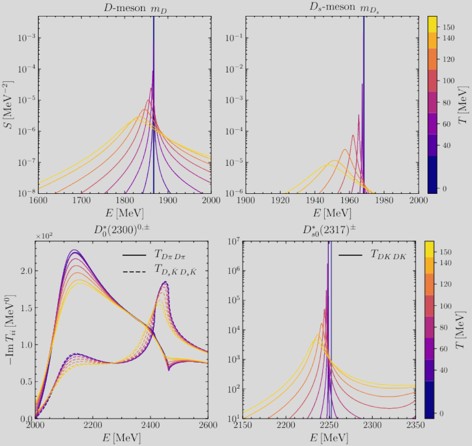
$$S_D(\omega, \vec{k}; T) = -\frac{1}{\pi} \mathrm{Im}\, \mathcal{D}_D(\omega, \vec{k}; T) = -\frac{1}{\pi} \mathrm{Im} \left( \frac{1}{\omega^2 - \vec{k}^{\,2} - m_D^2 - \Pi_D(\omega, \vec{k}; T)} \right)$$




**Self-consistency is required at $T \neq 0$**

$$\Pi_D(\omega_n, \vec{k}; T) = T \int \frac{d^3p}{(2\pi)^3} \sum_m \mathcal{D}_\pi(\omega_m - \omega_n, \vec{p} - \vec{k}) T_{D\pi}(\omega_m, \vec{p})$$

# Spectral functions




$$S_D(E,\mathbf{q}) = -\frac{1}{\pi}\mathrm{Im}\left(\frac{1}{E^2-\mathbf{q}^2-m^2-\Pi^R(E,\mathbf{q})}\right)$$

quasiparticle peak

$$E_q^2 - \mathbf{q}^2 - m^2 - \mathrm{Re}\,\Pi^R(E_q,\mathbf{q}) = 0$$

$$-\mathrm{Im}\,T_{ii}(E,\mathbf{q}) = -\mathrm{Im}\left(\frac{V(E,\mathbf{q})}{1-G(E,\mathbf{q})V(E,\mathbf{q})}\right)$$

G. Montaña *et al.*, Phys.Lett.B 806 (2020) 135464, Phys.Rev.D 102 (2020) 9, 096020

Ground and bound states reduce their mass and acquire a width.
Resonant states remain stable with temperature.

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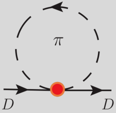
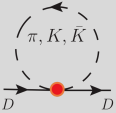
# Thermal masses

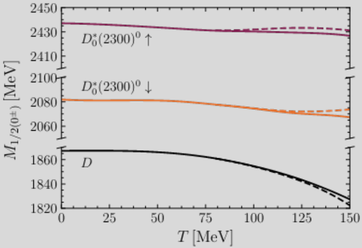
solid line:

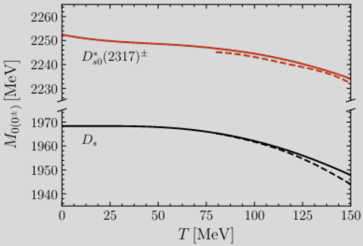



dashed line:




G. Montaña *et al.* (JMT-R),
Phys.Lett.B 806 (2020) 135464,
Phys.Rev.D 102 (2020) 9, 096020




Small reduction with temperature.
Negligible contribution of $K, \bar{K}$ to self-energy

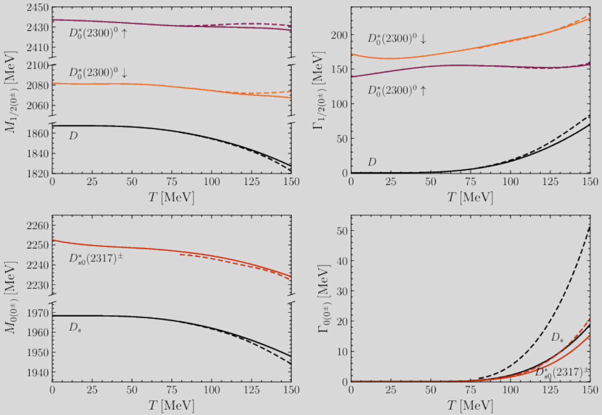
$(T = 100\ \text{MeV}/k_B = 1.1 \times 10^{12}\ \text{K})$

# Thermal masses and widths

## Chiral parity partners

$D(1867)$
$\leftrightarrow$
$D_0^*(2300)$

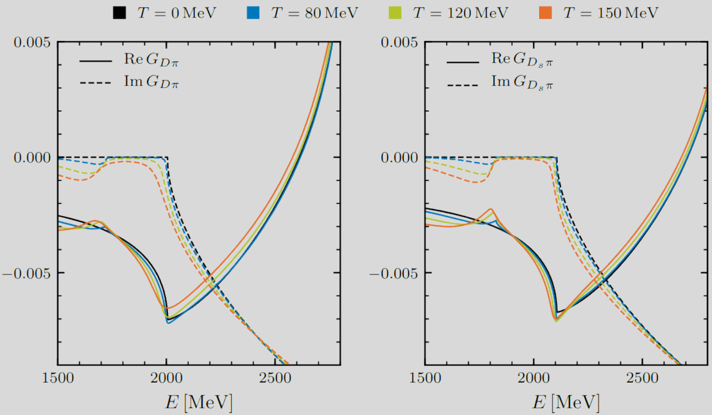
$D_s(1968)$
$\leftrightarrow$
$D_{s0}^*(2317)$




G. Montaña *et al.*, Phys.Lett.B 806 (2020) 135464, Phys.Rev.D 102 (2020) 9, 096020

No evidence of chiral partner degeneracy due to chiral symmetry restoration

# Loop function: Unitary and Landau cuts




$$G_{D\Phi}(i\omega_m, \mathbf{p}; T) = -T \sum_n \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_n^2 + \mathbf{k}^2 + m_D^2} \frac{1}{(\omega_m - \omega_n)^2 + (\mathbf{p} - \mathbf{k})^2 + m_\Phi^2}$$

# Thermal width (damping rate) of $D$ mesons

On-shell $D$ meson with momentum $k$ at equilibrium

$$\Gamma_k = -\frac{1}{E_k}\mathrm{Im}\ \Pi^R(E_k, k) \qquad \Gamma_k = -\frac{1}{2E_k}[\Pi^>(E_k, k) - \Pi^<(E_k, k)]$$

$$\Gamma_k^{(U)} = \frac{1}{2E_k}\frac{1}{\tilde{f}_k^{(0)}}\sum_{\lambda=\pm}\lambda\int dk_1^0\int\prod_{i=1}^{3}\frac{d^3k_i}{(2\pi)^3}\frac{1}{2E_2}\frac{1}{2E_3}|T(E_k+E_3,\mathbf{k}+\mathbf{k}_3)|^2 S_D(k_1^0,\mathbf{k}_1)$$
$$\times (2\pi)^4\delta^{(3)}(\mathbf{k}+\mathbf{k}_3-\mathbf{k}_1-\mathbf{k}_2)\delta(E_k+E_3-k_1^0-\lambda E_2)\tilde{f}^{(0)}(k_1^0)f^{(0)}(E_3)\tilde{f}^{(0)}(\lambda E_2)$$

$$\Gamma_k^{(L)} = \frac{1}{2E_k}\frac{1}{\tilde{f}_k^{(0)}}\sum_{\lambda=\pm}\lambda\int dk_1^0\int\prod_{i=1}^{3}\frac{d^3k_i}{(2\pi)^3}\frac{1}{2E_2}\frac{1}{2E_3}|T(E_k-E_3,\mathbf{k}+\mathbf{k}_3)|^2 S_D(k_1^0,\mathbf{k}_1)$$
$$\times (2\pi)^4\delta^{(3)}(\mathbf{k}+\mathbf{k}_3-\mathbf{k}_1-\mathbf{k}_2)\delta(E_k-E_3-k_1^0-\lambda E_2)\tilde{f}^{(0)}(k_1^0)\tilde{f}^{(0)}(E_3)\tilde{f}^{(0)}(\lambda E_2)$$

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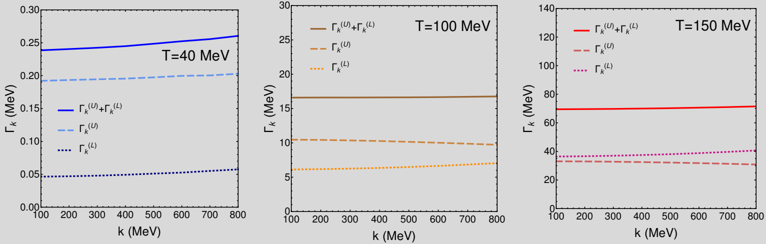


# Langevin equation

It is an alternative (but equivalent) description to the Fokker-Planck equation.

$$\begin{cases} dx^i & = & k^i dt/E_k \,, \\ dk^i & = & -A(k)k^i dt + C^{ij}(k)\rho^j\sqrt{dt} \,, \end{cases}$$

where ($\Delta^{ij} = \delta^{ij} - k^i k^j/k^2$)

$$C^{ij} = \sqrt{2B_0(k)}\Delta^{ij} + \sqrt{2B_1(k)}\,\frac{k^i k^j}{k^2}$$

and $\rho^i$ a stochastic Gaussian noise

$$\langle \rho^i(t) \rangle = 0$$

$$\langle \rho^i(t)\rho^j(t') \rangle = \delta(t - t')$$

# On-shell Fokker-Planck equation

**Narrow quasiparticle limit**

$$G_D^<(t, k^0, \mathbf{k}) = 2\pi S_D(k^0, \mathbf{k}) f_D(t, k^0)$$

$$S_D(k^0, \mathbf{k}) = \frac{1}{2E_k}\left[\delta(k^0 - E_k) + \delta(k^0 + E_k)\right]$$

On-shell Fokker-Planck equation

$$\frac{\partial}{\partial t} f_D(t, E_k) = \frac{\partial}{\partial k^i}\left\{A(\mathbf{k})k^i f_D(t, E_k) + \frac{\partial}{\partial k^j}\left[B_0(\mathbf{k})\Delta^{ij} + B_1(\mathbf{k})\frac{k^i k^j}{k^2}\right] f_D(t, E_k)\right\}$$

where $\Delta^{ij} = \delta^{ij} - k^i k^j / \mathbf{k}^2$

with

$$\langle \cdot \rangle = \frac{1}{2E_k}\int \frac{d^3k_1}{(2\pi)^4 2E_1}\frac{d^3k_2}{(2\pi)^3 2E_2}\frac{d^3k_3}{(2\pi)^3 2E_3}(2\pi)^4\delta^{(4)}(k_1 + k_2 - k_3 - k)$$
$$\times\left(|T(E_k + E_3, \mathbf{k} + \mathbf{k}_3)|^2 + |T(E_k - E_2, \mathbf{k} - \mathbf{k}_2)|^2\right) f^{(0)}(E_3)\tilde{f}^{(0)}(E_2)$$

We recover standard formula, **but** with Landau contribution

# Relaxation time

Average momentum loss

$$\left\langle \frac{dk^i}{dt} \right\rangle = -A(k)\, k^i$$

Assuming constant $A$ one can solve the equation for $k(t)$

$$\langle k(t) \rangle = k(0)\, e^{-At}$$

The inverse of $A$ plays the role of a relaxation time $\tau_R$ for the average heavy-hadron momentum

$$\tau_R = \frac{1}{A}$$

# Fluctuation-Dissipation Theorem

$A(k)$ is a deterministic drag force that causes energy loss (dissipation) whereas the diffusion coefficients are related to the strength of a stochastic (or fluctuating) force.
The fluctuation-dissipation theorem relates the 3 coefficients:

## Fluctuation-Dissipation Theorem

$$A(k) + \frac{1}{k}\frac{\partial B_1(k)}{\partial k} + \frac{2}{k^2}\left[B_1(k) - B_0(k)\right] = \frac{B_1(k)}{m_D T}$$

In the static limit, i.e. when $k \to 0$ the two diffusion coefficients become degenerate and the Einstein relation is recovered

## Einstein Relation

$$A = \frac{B}{m_D T}$$

# Off-shell Fokker-Planck equation

$$\left(k^\mu - \frac{1}{2}\frac{\partial \mathrm{Re}\,\Pi^R(X,k)}{\partial k_\mu}\right)\frac{\partial i G_D^<(X,k)}{\partial X^\mu} = \frac{1}{2} i\Pi^<(X,k) i G_D^>(X,k) + -\frac{1}{2} i\Pi^>(X,k) i G_D^<(X,k)$$

Off-shell transport equation can be rewritten as a master equation:

$$2\left(k^\mu - \frac{1}{2}\frac{\partial \mathrm{Re}\Pi^R}{\partial k_\mu}\right)\frac{\partial}{\partial X^\mu} G_D^<(X,k)$$

$$= \int \frac{dk_1^0}{2\pi}\frac{d^3q}{(2\pi)^3}[W(k^0,\mathbf{k}+\mathbf{q},k_1^0,\mathbf{q})G_D^<(X,k^0,\mathbf{k}+\mathbf{q}) - W(k^0,\mathbf{k},k_1^0,\mathbf{q})G_D^<(X,k^0,\mathbf{k})]$$

with transition probability rate

$$W(k^0,\mathbf{k},k_1^0,\mathbf{q}) \equiv \int \frac{d^4k_3}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}(2\pi)^4\delta(k_1^0+k_2^0-k_3^0-k^0)\delta^{(3)}(\mathbf{k}_2-\mathbf{k}_3-\mathbf{q})$$
$$\times|T(k_1^0+k_2^0+i\epsilon,\mathbf{k}-\mathbf{q}+\mathbf{k}_2)|^2 G_\Phi^>(X,k_2)G_\Phi^<(X,k_3)G_D^>(X,k_1^0,\mathbf{k}-\mathbf{q})$$

# Off-shell Fokker-Planck equation

Using $\mathbf{k} \gg \mathbf{q}$ one can Taylor expand

$$f(\mathbf{k}+\mathbf{q}) \simeq f(\mathbf{k}) + q^i \frac{\partial f(\mathbf{k})}{\partial k^i} + \frac{1}{2} q^i q^j \frac{\partial^2 f(\mathbf{k})}{\partial k^i \partial k^j}$$

for the combination

$$f(\mathbf{k}+\mathbf{q}) \equiv W(k^0, \mathbf{k}+\mathbf{q}, k_1^0, \mathbf{q}) G_D^<(X, k^0, \mathbf{k}+\mathbf{q})$$

One gets:

$$\frac{\partial}{\partial t} G_D^<(t,k) = \frac{\partial}{\partial k^i} \left\{ \hat{A}^i(k;T) G_D^<(t,k) + \frac{\partial}{\partial k^j} \hat{B}_0^{ij}(k;T) G_D^<(t,k) \right\}$$

with

$$A^i(k;T) \equiv \frac{1}{2k^0} \int \frac{dk_1^0}{2\pi} \frac{d^3q}{(2\pi)^3} W(k^0, \mathbf{k}, k_1^0, \mathbf{q})\, q^i$$

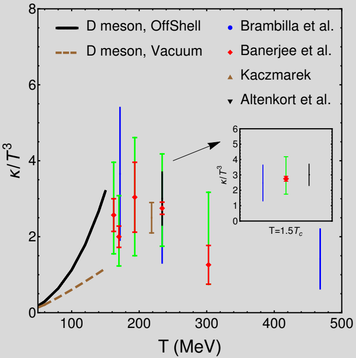
$$B^{ij}(k;T) \equiv \frac{1}{2} \frac{1}{2k^0} \int \frac{dk_1^0}{2\pi} \frac{d^3q}{(2\pi)^3} W(k^0, \mathbf{k}, k_1^0, \mathbf{q})\, q^i q^j$$

back

# Momentum diffusion coefficient

## Diffusion coefficient in momentum space

$$\kappa = 2B_0(k \to 0) = 2B_1(k \to 0)$$




**Lattice-QCD calculations**

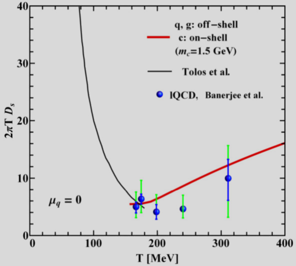
- D. Banerjee *et al.*
  Phys. Rev. D85, 014510 (2012)
- O. Kaczmarek
  Nucl. Phys. A931, 633 (2014)
- N. Brambilla *et al.*
  Phys. Rev. D102, 074503 (2020)
- L. Altenkort *et al.*
  Phys. Rev. D103, 014511 (2021)

Our result with thermal and offshell effects is compatible with lattice-QCD calculations

# Diffusion coefficient

## Spatial diffusion coefficient

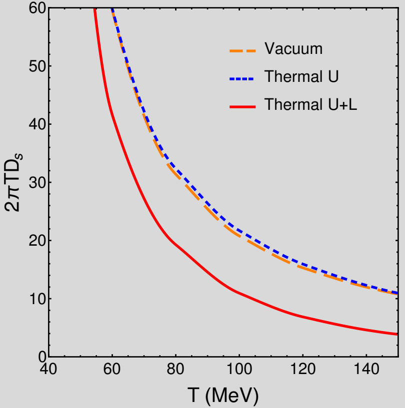
$$2\pi T D_s(T) = \frac{2\pi T^3}{B_0(k \to 0, T)}$$




$T > T_c$: DQPM for $c$ quarks.

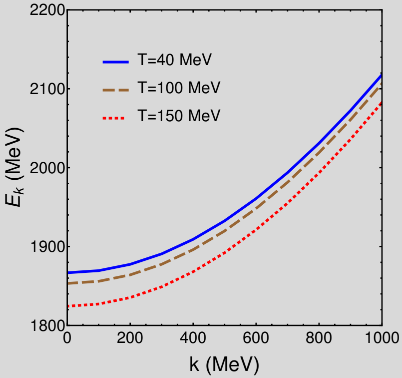
$T < T_c$ : Our result for $D$ meson.

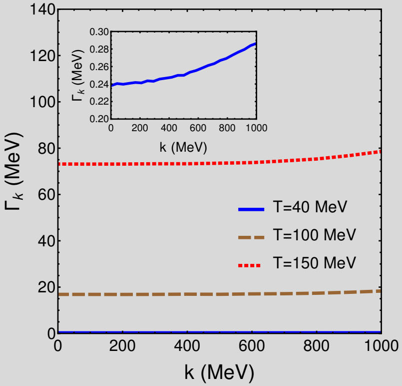
T. Song *et al.*, Phys. Rev. C 96, 014905 (2017)



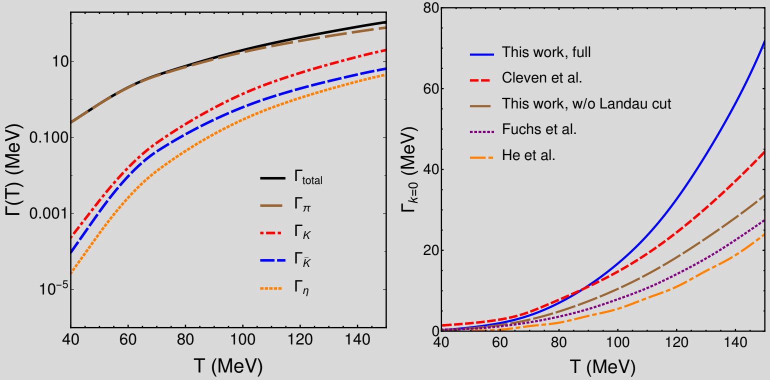

New results

# Quasiparticle properties



# Total Width


Γ(T) (MeV)
10
0.100
0.001
10^−5
40
60
80
100
120
140
T (MeV)
Γ_total
Γ_π
Γ_K
Γ_K̄
Γ_η
Γ_k=0 (MeV)
80
60
40
20
0
This work, full
Cleven et al.
This work, w/o Landau cut
Fuchs et al.
He et al.