PHQMD: Effects of EOS : Flow , Clustering

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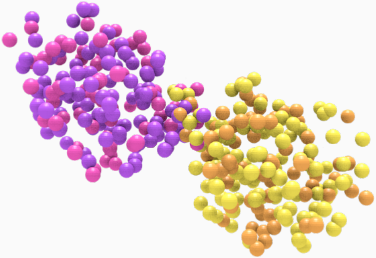

PHQMD

PHQMD(Parton Hadron Quantum Molecular Dynamics)[1]is a simulation code that is based on PHSD(Parton Hadron String Dynamics)[2](quasi particle description of QGP of PHSD)but uses N-body dynamics for propagation of nucleons

Quantum Molecular Dynamics

Using n-body theory to track nucleon by nucleon the collisions, and calculate the two body interactions between all of the nucleons.

- IQMD[3](Limited to lower energies 1.5 GeV/A)
- UrQMD[4](Relativistic, used for coalescence, no potential)

[1] J. Aichelin and, E. Bratkovskaya et al. *PR C101, 044905*
[2] W. Cassing and E. Bratkovskaya *PR C78,034919*
[3] C. Hartnack et al. *Eur.Phys.J.A1:151*
[4] S. A. Bass et al. *NP A41:225*

Why is it needed ?
Current models are not well adapted to address cluster formation, and current QMD models are not usable at relativistic energies

Challenges
Production of clusters with binding energy of $B_E \approx -8[MeV]$ in environment of the fireball at $E \approx 100[MeV]$

Hyper-clusters production[5], [6]is an area of renewed interest, and which is not currently addressed

Transition form low energy, hadron dominated, interactions and high energy, quark and gluon dominated, reactions

Transition regime is characterised by a finite chemical potential thus a finite net baryonic density

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[5] C. Rappold et al. *PL B747,129*
[6] J. Adam et al *PR C93, 024917*

Future experimental data

New energy range from 2 GeV/A - 100 GeV/A, will be used to investigate the first order phase transition from hadronic to QGP matter, and degrees of freedom of hadronic matter (strangeness) .

- FAIR
- NICA

Experimental results so far

- 0.6 [GeV/A] ALADIN [7]
- 1.23 [GeV/A] HADES [8]
- 1.5 [GeV/A] FOPI [9]

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[7] A. Schüttauf et al *NP A607,457*
[8] J. Adamczewski-Musch et al *NP A982*
[9] W. Reisdorf et al. *NP A876*

Brief PHQMD model overview

Generalised Ritz variational principle

$$\delta \int_{t_1}^{t_2} \mathrm{dt} \, \langle \Psi(t) | i \frac{\mathrm{d}}{\mathrm{d}t} - H | \Psi(t) \rangle = 0 \tag{1}$$

Gaussian test wave function which yields the Wigner density:

$$f(\vec{r}_i, \vec{p}_i, \vec{r}_{i0}, \vec{p}_{i0}, t) = \frac{1}{\pi^3 \hbar^3} \exp\left[-\frac{2}{L}(\vec{r}_i - \vec{r}_{i0})^2 \right] \exp\left[-2L(\vec{p}_i - \vec{p}_{i0})^2 \right] \tag{2}$$

Classical type equations of motion for expectation value of the Hamiltonian for Gaussian wavefunctions

$$\dot{r}_{i0} = \frac{\partial \langle H \rangle}{\partial p_{i0}} \qquad \dot{p}_{i0} = -\frac{\partial \langle H \rangle}{\partial r_{i0}} \tag{3}$$

Brief PHQMD model overview

Hamiltonian composed of Kinetic term and Two body potentials

$$\langle H \rangle = \sum_{i}^{N} \langle H_i \rangle = \sum_{i}^{N} \left(\langle T_i \rangle + \sum_{i \neq j}^{N} \langle V_{i,j} \rangle \right) \tag{4}$$

Potentials (Coulomb , Asymmetry , Skyrme and Momentum dependent Skyrme)

$$\langle V \rangle = \langle V_c \rangle (\vec{r}) + \langle V_a \rangle (\vec{r}) \begin{cases} \langle V_s \rangle (\rho_{int}) \text{ Static} \\ \langle V_s \rangle (\rho_{int}) + \langle U_{opt} \rangle (\Delta \vec{p}^2) \text{ Momentum dependent} \end{cases} \tag{5}$$

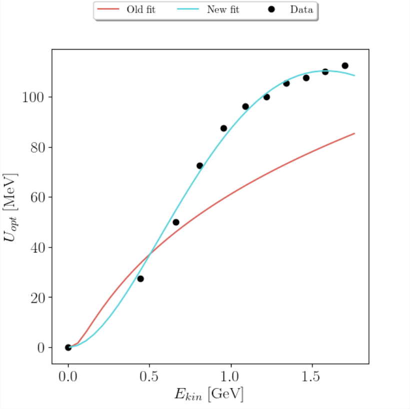
Interaction density

$$\rho_{int}(i,t) = C \sum_{j,j \neq i}^{N} \exp\left[-\frac{1}{L} (\vec{r_i} - \vec{r_j})^2 \right] \tag{6}$$

Momentum dependent Skyrme parametrisation

$$\langle V_{tot}\rangle = \langle V_s\rangle(\rho_{int}) + \langle U_{opt}\rangle(\Delta p^2) \quad (7)$$

$$U_{opt}(\Delta p^2) = \exp\left[-c\sqrt{\Delta p}\right]\left(a\Delta p + b\Delta p^2\right)\rho(\vec{r}) \quad (8)$$



Optical potential fit to expermimental data[10]

[1] S. Hama, et al., Phys. Rev. C 41, 2737

E.o.S parametrisation

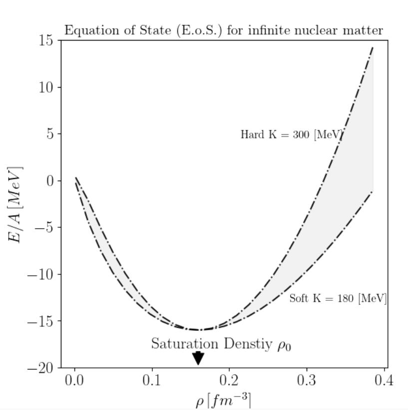
Static Skyrme parametrisation

$$\langle V_s \rangle (\rho_{int}) \propto \alpha\rho + \beta\rho^{\gamma} \qquad (9)$$

Constraints for the potentials

$$\begin{cases} \rho_0 = 0.1695\,[fm^{-3}] \\ E_0 = -16\,[MeV] \\ K_0 = 200 \text{ or } 380\,[MeV] \\ \text{(Soft/Hard)} \end{cases}$$

$$K_0 = 9\frac{\partial^2 E/A}{\partial \rho^2}\bigg|_{\rho=\rho_0}$$



Equations of state parametrised for soft and hard nuclear matter compressibility

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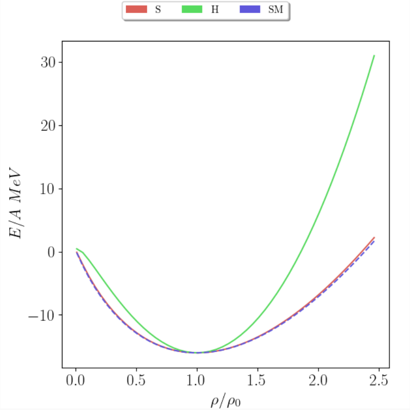


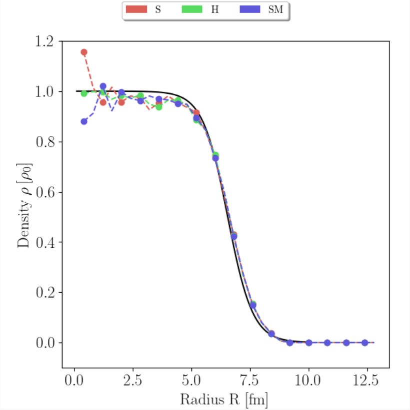
Equations of state parametrised for soft and hard nuclear matter compressibility

Nucleon distribution

Wood-Saxon distribution in position space of nucleons

$$\rho^{WS}(r) = \frac{\rho_0}{1 + \exp\left[\frac{r - R_A}{a}\right]} \tag{11}$$

$$\begin{cases} R_A = r_0 A^{1/3} \\ r_0 = 1.125[fm] \\ a = 0.535[fm] \end{cases}$$



Initial Wood-Saxon position distribution for the three E.o.S.

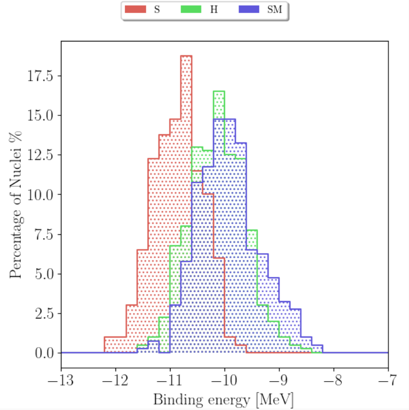
Binding energy

Total binding energy of our nucleus ($< B_E >= -10[MeV]$)

$$B_E = E_{kin}(\vec{p}) + E_c(\vec{r}) + E_a(\vec{r}) \begin{cases} E_s(\vec{r}) \\ E_s(\vec{r}) + E_m(\vec{r}, \Delta p) \end{cases} \quad (12)$$

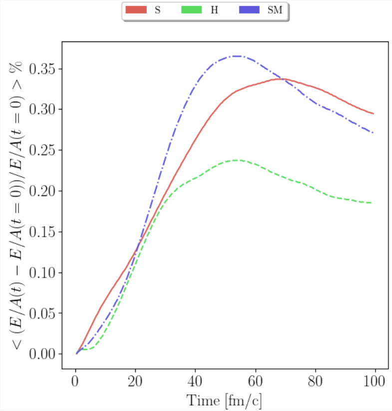
Constraint on momentum distribution

$$0 \leq \sqrt{m^2 + \vec{p}_i^2} - m \leq -V(\rho_{int}) \quad (13)$$

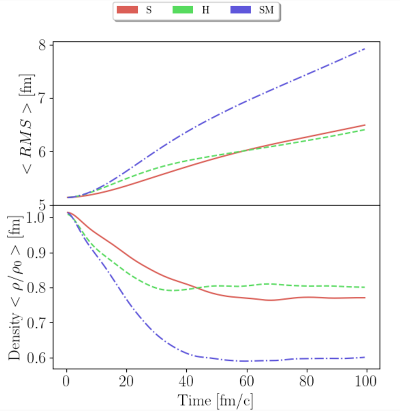


Initial binding energy of the nuclei

Propagation



Conservation of the total system energy



Evolution of the average radial distance of the nucleons, and the average density of the nuclei

Cluster formation

SACA[11] and MST[12]

To form the cluster we use two methods

- MST : simple form of spanning tree to create clusters based on distance from other nucleons
- SACA : complex simulated annealing of all possible cluster patterns of the nucleons to find the lowest sum of binding energies of all clusters

All the following results are for the SACA method

[10] R. K. Puri, C. Hartnack and J. Aichelin *PR C54, R28*

[11] J. Aichelin, Phys. Rept 202, 233

Cluster formation

SACA[13] and MST[14]
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- MST :
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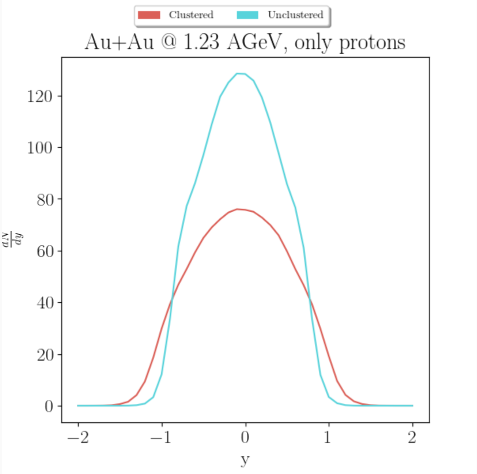

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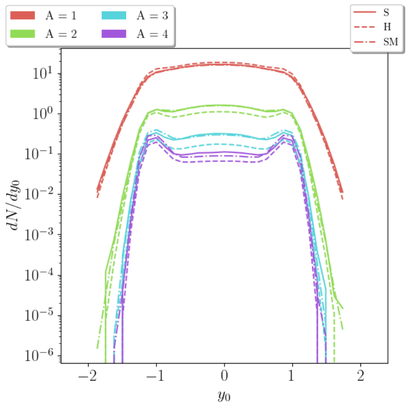
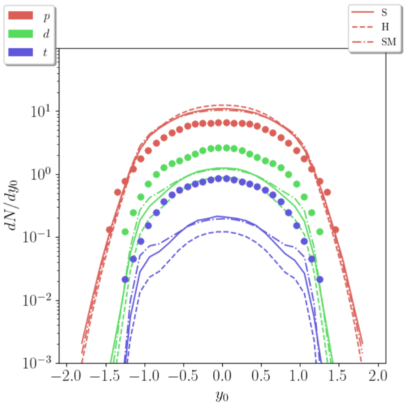
[10] R. K. Puri, C. Hartnack and J. Aichelin *PR C54, R28*
[11] J. Aichelin, Phys. Rept 202, 233

FOPI results for AuAu E = 1.5 [GeV/A] Multiplicity

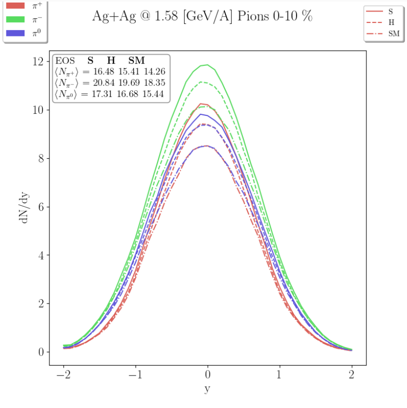


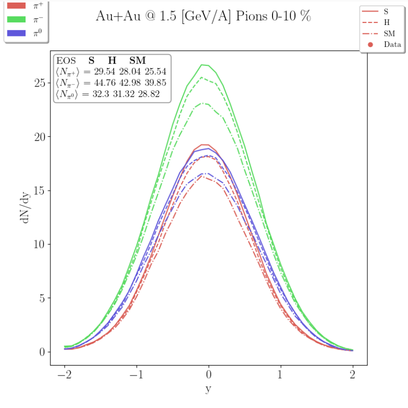
Fragment multiplicity



Z=1 isotopes multiplicities for central collisions (b < 2 [fm])

Effect of EOS on pion Multiplicity



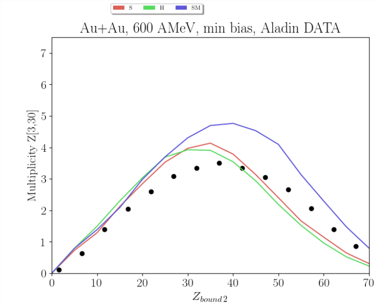


Pion multiplicity for Ag Ag 0-10 % @ 1.58

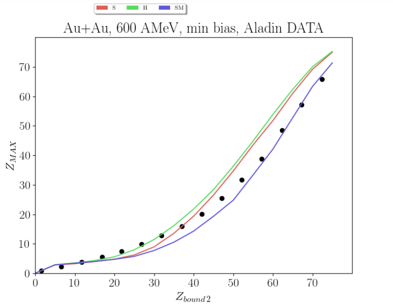
Pion multiplicity for Au Au 0-10 % @ 1.5

Aladin results for AuAu E = 0.6 [GeV/A]

Z_{bound2} the sum of the charges all fragments with $Z \geq 2$, and Z_{MAX} the charge of the largest cluster

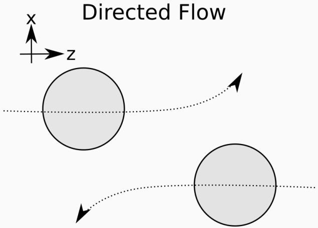


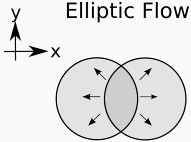
Rise and Fall curve for all three equations of state



Zmax curve for all three equations of state

Directed and Elliptic Flow



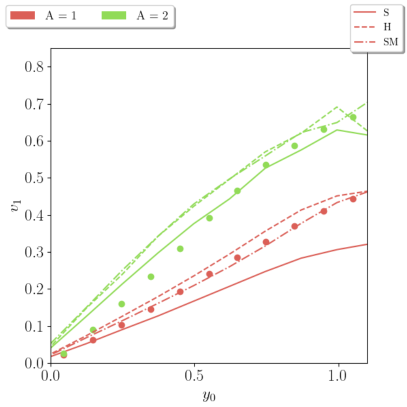


$$E\frac{\mathrm{d}^3N}{\mathrm{d}^3p} = \frac{1}{2\pi}\frac{\mathrm{d}^2N}{p_T\mathrm{d}p_T\mathrm{d}y}\left(1 + 2\sum_{n=1}^{\infty} v_n \cos\left[n(\varphi - \Psi_{RP})\right]\right) \quad (14)$$

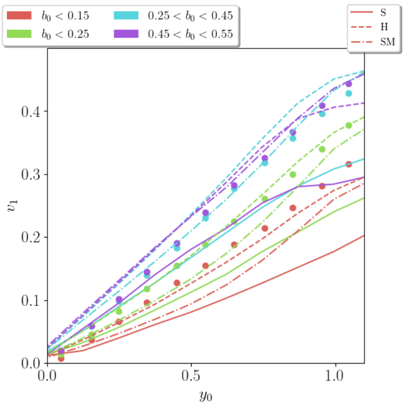
A Fourier expansions allows us to obtain the coefficients:

- Directed Flow $v_1 = \left\langle \frac{p_x}{p_T} \right\rangle$
- Elliptic Flow $v_2 = \left\langle (\frac{p_x}{p_T})^2 - (\frac{p_y}{p_T})^2 \right\rangle$

FOPI results for AuAu E = 1.5 [GeV/A] Flow

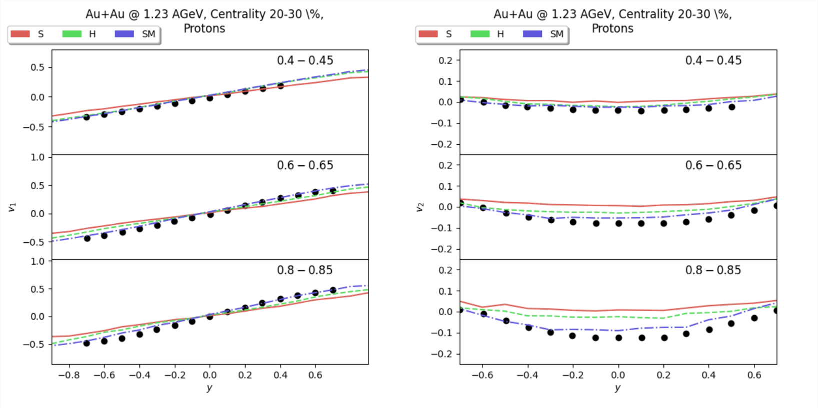


Directed flow for proton (v_1) and deuteron with a threshold on transverse momentum



Directed proton flow (v_1) for 4 different impact parameter classes for all protons with a threshold on transverse momentum

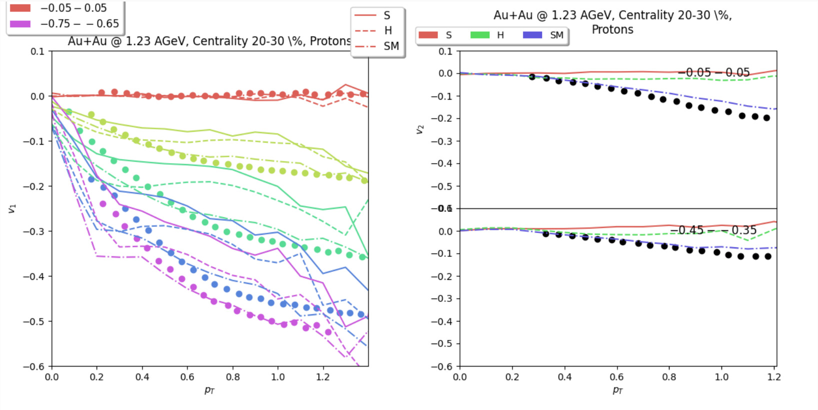
HADES results for AuAu E = 1.23 [GeV/A] Flow



Directed proton flow (v_1) as a function of rapidity for three p_T classes compared with the HADES data

Elliptical proton flow (v_2) as a function of rapidity for three p_T classes compared with the HADES data

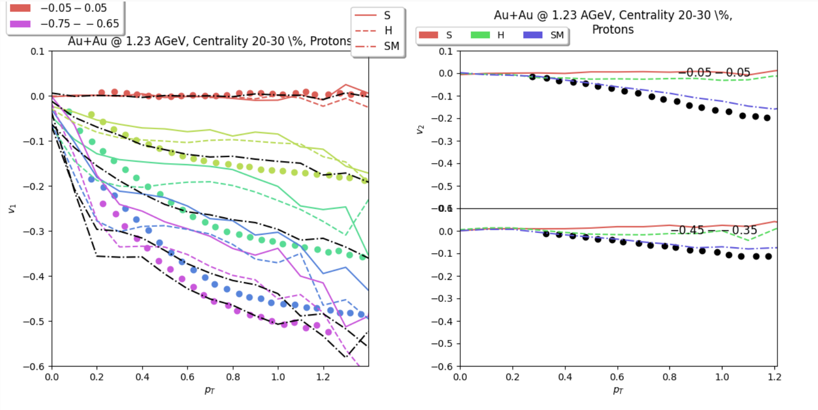
HADES results for AuAu E = 1.23 [GeV/A] Flow



Directed proton flow (v_1) as a function of p_T for various rapidity classes compared with the HADES data

Elliptical deuteron flow (v_2) as a function of rapidity for two p_T classes compared with the HADES data

HADES results for AuAu E = 1.23 [GeV/A] Flow



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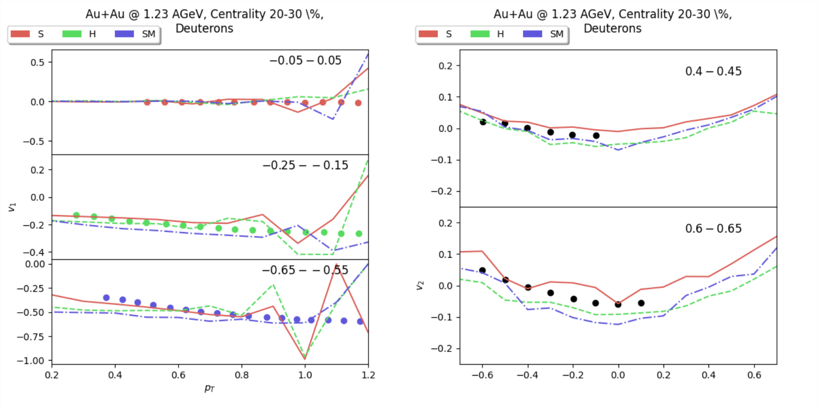


Directed deuteron flow (v_1) as a function of p_T for three various rapidity classes compared with the HADES data

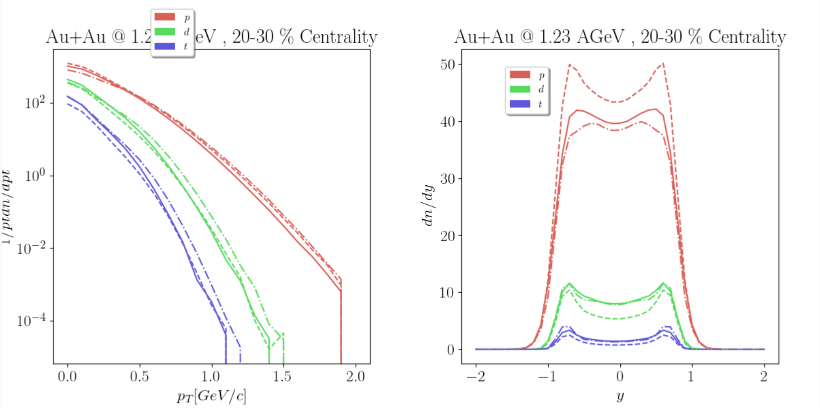
Elliptical deuteron flow (v_2) as a function of rapidity for two p_T classes compared with the HADES data

Conclusions

- PHQMD allows use to study cluster formation in a unique and novel way
- It permits us to study the effect of nuclear EOS on a host of observables
- In most cases for the FOPI data the SM EOS give better agreement with the data In some cases the HADES data is better replicated for the SM EOS

Thank you

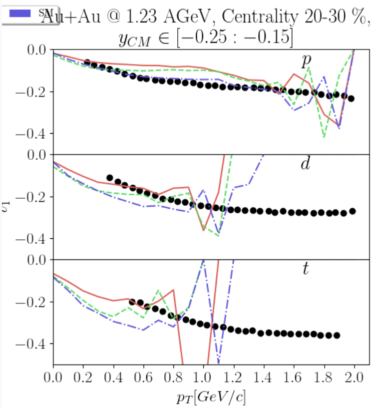
AuAu E = 1.23 [GeV/A] Multiplicity



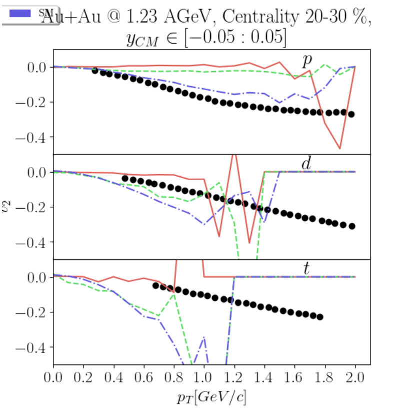
Multiplicity of Z=1 isotopes function of rapidity for Au+AU @ 1.23 AGeV

Multiplicity of Z=1 isotopes function of p_T for Au+AU @ 1.23 AGeV

AuAu E = 1.23 [GeV/A] Flow

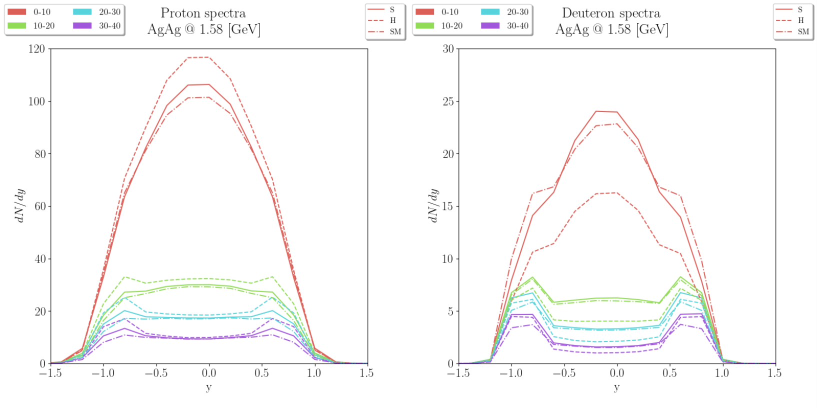


Directed isotope flow of Z=1 isotopes function of p_T for Au+AU @ 1.23 AGeV



Elliptical Z=1 isotopes flow function of p_T for Au+AU @ 1.23 AGeV

HADES results for AgAg E = 1.58 [GeV/A] Flow



Proton multiplicity Ag Ag @ 1.58 AGeV function rapidity

Deuteron multiplicity Ag Ag @ 1.58 AGeV function rapidity