Relativistic density functional approach to unified description of quark-hadron matter

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Outline

- **Relativistic density functional approach**
- **Mean field**
- **Beyond mean field**

Relativistic density functional

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ - \hat{m})q + \mathcal{L}_V + \mathcal{L}_D - \mathcal{U}$$

- **Vector and diquark interaction (important for astrophysical applications)**

$$\mathcal{L}_V = -G_V(\overline{q}\gamma_\mu q)^2, \quad \mathcal{L}_D = G_D \sum_{A=2,5,7} (\overline{q}i\gamma_5\tau_2\lambda_A q^c)(\overline{q}^c i\gamma_5\tau_2\lambda_A q)$$

- **χ-symmetric density functional**

$$(\overline{q}q)^2 + (\overline{q}i\vec{\tau}\gamma_5 q)^2 \ \textit{is}\ \chi - \textit{invariant} \quad \Rightarrow \quad \mathcal{U} = \mathcal{U}\left[(\overline{q}q)^2 + (\overline{q}i\vec{\tau}\gamma_5 q)^2\right]$$

$$\mathcal{U} = G_0\left[(1+\alpha)\langle\overline{q}q\rangle^2 - (\overline{q}q)^2 - (\overline{q}i\vec{\tau}\gamma_5 q)^2\right]^{\frac{1}{3}}$$

G_0 – coupling constant

$\alpha \geq 0$ – controls quark effective mass in the vacuum

Expansion of RDF around $\langle\overline{q}q\rangle$ and $\langle\overline{q}i\vec{\tau}\gamma_5 q\rangle=0$

$$\mathcal{U} = U + \left(\overline{q}q - \langle\overline{q}q\rangle\right)\Sigma_{MF} - G_S\left(\overline{q}q - \langle\overline{q}q\rangle\right)^2 - G_{PS}\left(\overline{q}i\vec{\tau}\gamma_5 q\right)^2 + \ldots$$

- 0^{th} **order**

$$U = \mathcal{U}_{\langle\overline{q}q\rangle,\langle\overline{q}i\vec{\tau}\gamma_5 q\rangle}$$

- 1^{st} **order – mean field self-energy of quarks**

$$\Sigma_{MF} = \frac{\partial U}{\partial\langle\overline{q}q\rangle}$$

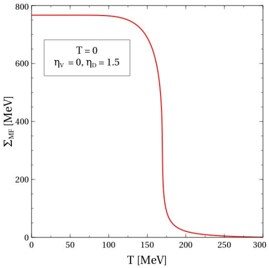
- 2^{nd} **order – effective couplings in (pseudo)scalar channels**

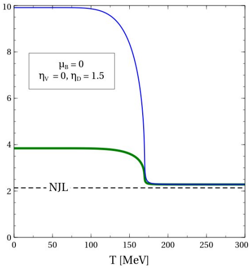
$$G_S = -\frac{1}{2}\frac{\partial^2 U}{\partial\langle\overline{q}q\rangle^2}, \qquad G_{PS} = -\frac{1}{6}\frac{\partial^2 U}{\partial\langle\overline{q}i\vec{\tau}\gamma_5 q\rangle^2}$$

Effective couplings attain medium dependence

Medium dependent couplings and quark self energy

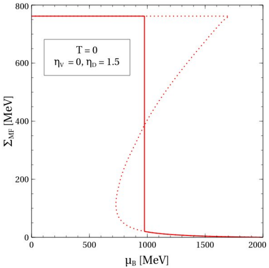
- **Zero chemical potential**

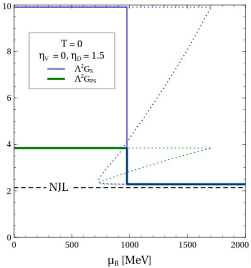




$$\eta_i \equiv \frac{G_i}{G_S}\Big|_{T,\mu_B\to\infty}$$

- **Zero temperature**





Comparison to NJL model

$$\mathcal{L} = \overline{q}(i\partial\!\!\!/ - \underbrace{(m + \Sigma_{MF})}_{\text{effective mass m}^*})q + G_S(\overline{q}q)^2 + G_{PS}\left(\overline{q}i\vec{\tau}\gamma_5 q\right)^2 + \ldots$$

- **Similarities:**
  1. current-current interaction
  2. scalar, pseudoscalar, vector, diquark, ... channels
- **Diferences:**
  1. high m^* at low T and/or μ - **phenomenological confinement**

     $$\langle\overline{q}q\rangle = \langle\overline{q}q\rangle_0 \;\Rightarrow\; m^* = m - \frac{2G_0}{3}\langle\overline{q}q\rangle_0^{1/3}\alpha^{-2/3} \; - \text{ diverges at } \alpha \to 0$$

  2. medium dependent couplings

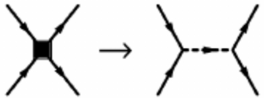
     low T and/or μ $\Rightarrow$ $G_S \neq G_{PS}$ $\Rightarrow$ broken $\chi-$symmetry

     high T and/or μ $\Rightarrow$ $G_S = G_{PS}$ $\Rightarrow$ restored $\chi-$symmetry

Bosonization of (pseudo)scalar interaction

- **Hubbard-Stratonovich transformation**

$$\exp\left[\int dx\ G_\phi(\bar{q}\hat{\Gamma}q)^2\right] = \int[D\phi]\exp\left[-\int dx\left(\frac{\phi^2}{4G_\phi} + \bar{q}\phi\hat{\Gamma}q\right)\right]$$



- **Bosonized Lagrangian ($m^* = m + \Sigma_{MF}$ - effective quark mass)**

$$\begin{aligned}\mathcal{L} &= \bar{q}(i\partial\!\!\!/ - m^*)q + \mathcal{L}_V + \mathcal{L}_D - U + \langle\bar{q}q\rangle\Sigma_{MF} \\ &- \underbrace{\frac{\sigma^2}{4G_S} - \frac{\vec{\pi}^2}{4G_{PS}} - \bar{q}(\sigma + i\vec{\pi}\vec{\tau}\gamma_5)q + \sigma\langle\bar{q}q\rangle}_{\text{fullly beyond mean field terms}}\end{aligned}$$

- **Field equations for σ and $\vec{\pi}$**

$$\begin{cases}\sigma = 2G_S(\langle\bar{q}q\rangle - \bar{q}q) \\ \vec{\pi} = -2G_{PS}\bar{q}i\vec{\tau}\gamma_5 q\end{cases} \Rightarrow \langle\sigma\rangle = \langle\vec{\pi}\rangle = 0 \Rightarrow \sigma,\vec{\pi} - \text{beyond MF}$$

comment: $\langle\sigma\rangle = 0$ does not assume χ-symmetry since $\langle\bar{q}q\rangle \neq 0$

Mean field ($\omega_\mu = g_{\mu 0}\omega,\ \Delta = \text{const},\ \sigma = \vec{\pi} = 0$)

- **Nambu-Gorkov Lagrangian with bosonized vector and diquark channels**

$$\mathcal{L} + q^+ \hat{\mu} q = \overline{Q}\hat{S}_{NG}^{-1} Q + \frac{\omega_\mu \omega^\mu}{4G_V} - \frac{\Delta_A \Delta_A^*}{4G_D} - U + \langle \overline{q} q \rangle \Sigma_{MF}$$

$$Q = \frac{1}{\sqrt{2}} \begin{pmatrix} q \\ q^c \end{pmatrix} \qquad \hat{S}_{NG}^{-1} = \begin{pmatrix} i\partial\!\!\!/ + \omega\!\!\!/ - m^* + \gamma_0 \hat{\mu} & i\Delta_A \gamma_5 \tau_2 \lambda_A \\ i\Delta_A \gamma_5 \tau_2 \lambda_A & i\partial\!\!\!/ + \omega\!\!\!/ - m^* + \gamma_0 \hat{\mu} \end{pmatrix}$$

- **Statistical partition and thermodynamic potential**

$$\mathcal{Z} = \int [D\overline{q}][Dq] \exp\left[\int dx \left(\mathcal{L} + q^+ \hat{\mu} q\right)\right]$$

$$\Omega = -\frac{1}{\beta V} \ln \mathcal{Z} = -\frac{1}{2\beta V} \mathrm{Tr} \ln(\beta S_{NG}^{-1}) - \frac{\omega^2}{4G_V} + \frac{\Delta^2}{4G_D} + U - \langle \overline{q} q \rangle \Sigma_{MF}$$

- **Vector field, diquark gap, χ-condensate**

$$\frac{\partial \Omega}{\partial \omega} = 0, \qquad \frac{\partial \Omega}{\partial \Delta} = 0, \quad \langle \overline{q} q \rangle = \sum_f \frac{\partial \Omega}{\partial m_f}$$

Superconductivity onset

- **Single quark energy and distribution**

$$E_f^{\pm} = \sqrt{(E_f \mp \mu_f)^2 + \Delta^2}$$

$$f_f^{\pm} = [\exp(E_f^{\pm}/T) + 1]^{-1}$$

- **Gap equation**

$$\frac{\partial \Omega}{\partial \Delta} = \frac{\Delta}{2G_D} - 2\Delta \sum_{f,a=\pm} \int \frac{dk}{(2\pi)^3} \frac{1 - 2f_f^a}{E_f^a} = 0$$

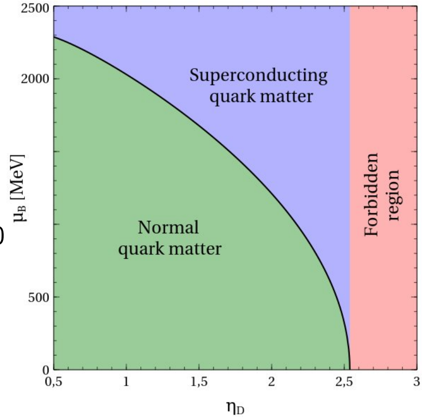
$$\Downarrow$$

two solutions : $\Delta = 0$ or $\Delta \neq 0$

- **Two solutions coincide $\Rightarrow$ SC onset**

$$\left.\frac{\partial^2 \Omega}{\partial \Delta^2}\right|_{\Delta=0} = 0 \quad \Rightarrow \quad \mu_B = \mu_B(G_D)$$

$T = 0$



No vacuum superconductivity

$$\Downarrow$$

$$\eta_D \lesssim 2.5$$

Effective mass and diquark gap

$T = 0$

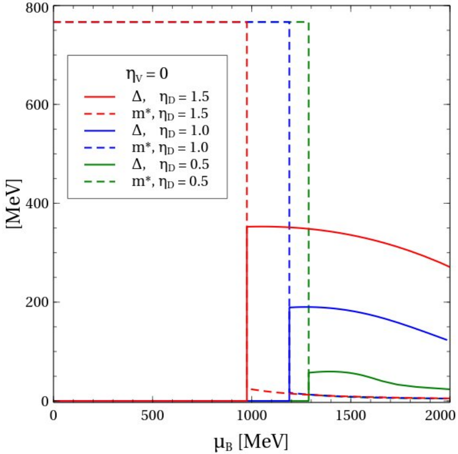
- **Low chemical potential**

  $m^* \gg m$ $\Rightarrow$ $\chi -$ broken
  $\Delta = 0$ normal phase

- **High chemical potential**

  $m^* \to 0$ $\Rightarrow$ $\chi -$ restored
  $\Delta \neq 0$ SC phase

Holds for $T \neq 0$



Hybrid EoS for compact star matter

- **Cold matter:** $T = 0$
- **Charge neutrality:** electrons
- **β-equilibrium:**

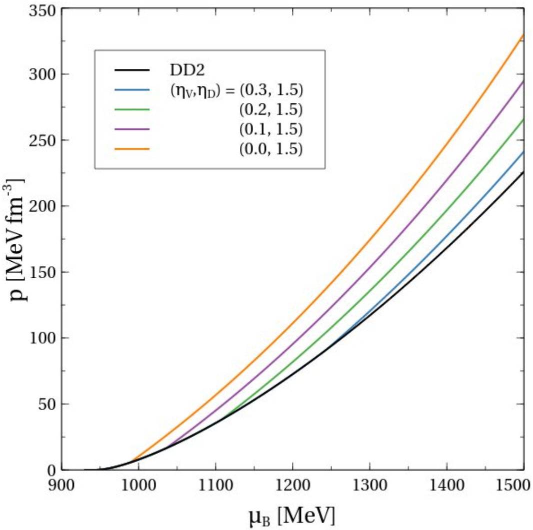
$$\mu_d = \mu_u + \mu_e$$

- **Hadron EoS:** DD2

S. Typel et al., PRC 81, 015803 (2010)

- **Maxwell construction:**

$$p_q(\mu_B^c) = p_h(\mu_B^c)$$



Compact stars with quark core

- **Hydrostatic equilibrium**

$$\frac{dp}{dr} = -\frac{Gm\rho}{r^2}\frac{\left(1+\frac{p}{\rho}\right)\left(1+\frac{4\pi r^3 p}{m}\right)}{\left(1-\frac{2Gm}{r}\right)}$$

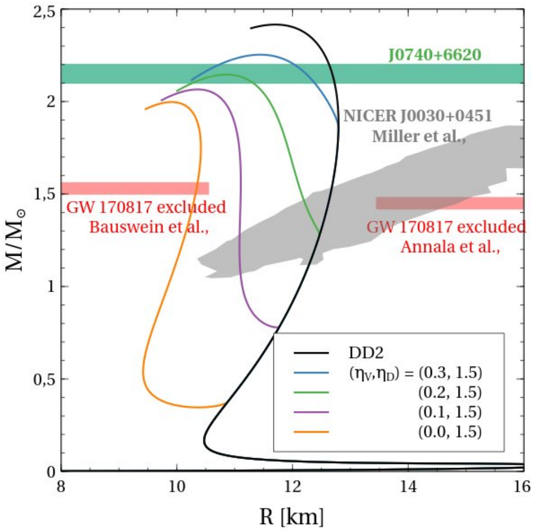
$$\frac{dm}{dr} = 4\pi r^2 \rho$$

- **Cold matter EoS**

$$\begin{cases} p = p(\mu_B) \\ \rho = \mu_B \frac{\partial p}{\partial \mu_B} - p \end{cases} \Rightarrow \; p = p(\rho)$$

- **Total radius R and mass M**

$$r = R \quad \Rightarrow \begin{cases} p = 0 \\ m = M \end{cases}$$



Mean field phase diagram

- **BCS relation**

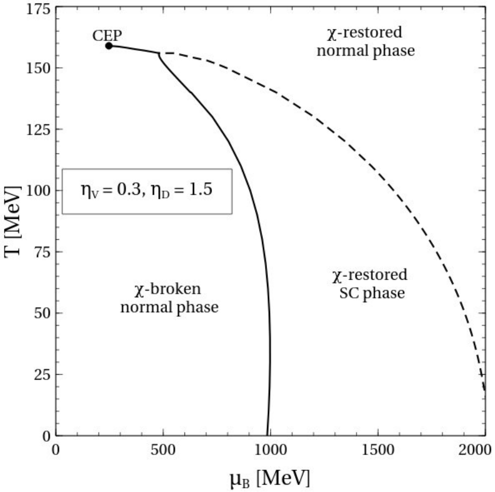
$$\Delta_{T=0} = 1.764\, T_c$$

- $\eta_V = 0.3, \;\; \eta_D = 1.5$

$$\mu_B = 1000 \text{ MeV} \;\Rightarrow\; \frac{\Delta_{T=0}}{T_c} = 2.518$$

$$\mu_B = 1000 \text{ MeV} \;\Rightarrow\; \frac{\Delta_{T=0}}{T_c} = 3.243$$

BCS relation is violated



Beyond mean field ($\hat{\mu} = 0 \;\Rightarrow\; \omega = \Delta = 0$)

$$\mathcal{L} = \overline{q}\hat{S}^{-1}q - \frac{\sigma^2}{4G_S} - \frac{\vec{\pi}^2}{2G_{PS}} + \sigma\langle\overline{q}q\rangle - U + \Sigma_{MF}\langle\overline{q}q\rangle$$

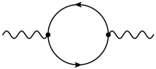
$$\hat{S}^{-1} = \hat{S}^{-1}_{MF} + \Sigma_m, \quad \begin{cases} \hat{S}^{-1}_{MF} = i\partial\!\!\!/ - m^* \; - \; \text{MF propagator} \\ \Sigma_m = -\sigma - i\vec{\tau}\vec{\pi}\gamma_5 \; - \; \text{beyond MF self} - \text{energy} \end{cases}$$

- **Integrating $\overline{\mathrm{q}}$, q and expanding up to $\mathcal{O}(\Sigma_{\mathrm{m}}^3)$**

$$\mathcal{L}^{eff} = \frac{\mathrm{tr}\ln(\beta\hat{S}^{-1})}{\beta V} - \frac{\sigma^2}{4G_S} - \frac{\vec{\pi}^2}{4G_{PS}} + \sigma\langle\overline{q}q\rangle - U + \Sigma_{MF}\langle\overline{q}q\rangle$$

$$\simeq -\Omega_{MF} - \frac{\sigma^2}{4G_S} - \frac{\vec{\pi}^2}{4G_{PS}} + \underbrace{\sigma\langle\overline{q}q\rangle - \mathrm{tr}(\hat{S}_{MF}\Sigma_m)}_{\text{vanishes under } \int dx} + \frac{1}{2}\mathrm{tr}(\hat{S}_{MF}\Sigma_m)^2$$

- **Polarization operators of mesons**



$$\begin{cases} \Pi_\sigma = -\mathrm{tr}(\hat{S}_{MF})^2 \\ \Pi_\pi = -\mathrm{tr}(\hat{S}_{MF}i\gamma_5)^2 \end{cases} \Rightarrow \quad -\mathrm{tr}(\hat{S}_{MF}\Sigma_m)^2 = \sigma\Pi_\sigma\sigma + \vec{\pi}\Pi_\pi\vec{\pi}$$

Beyond mean field thermodynamic potential

$$\Omega = \Omega_{MF} + \underbrace{\frac{1}{2\beta V}\mathrm{tr}\ln\left[\beta^{-2}\left(\frac{1}{2G_S} - \Pi_\sigma\right)\right]}_{\Omega_\sigma} + \underbrace{\frac{3}{2\beta V}\mathrm{tr}\ln\left[\beta^{-2}\left(\frac{1}{2G_{PS}} - \Pi_\pi\right)\right]}_{\Omega_\pi}$$

- **Meson propagators**

$$D_\sigma^{-1} = \frac{1}{2G_S} - \Pi_\sigma, \quad D_\pi^{-1} = \frac{1}{2G_{PS}} - \Pi_\pi$$

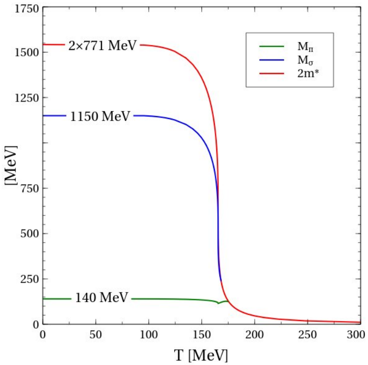
- **Meson masses**

$$D_j^{-1} = 0 \quad \Rightarrow \quad E_j(k), \quad M_j = E_j(0)$$

- **Meson phase shifts**

$$D_j = |D_j| e^{i\delta_j} \quad \Rightarrow \quad \delta_j = \mathrm{Im}\ln(\beta^{-2} D_j)$$

$f_\pi = 90$ MeV



- **Beth-Uhlenbeck EoS**

$$\Omega = \Omega_{MF} + \sum_{j=\sigma,\pi} \Omega_j, \quad \Omega_j = d_j T \int \frac{d^4k}{(2\pi)^4} \ln\left(1 - e^{-\beta k_0}\right) \frac{\partial \delta_j}{\partial k_0}$$

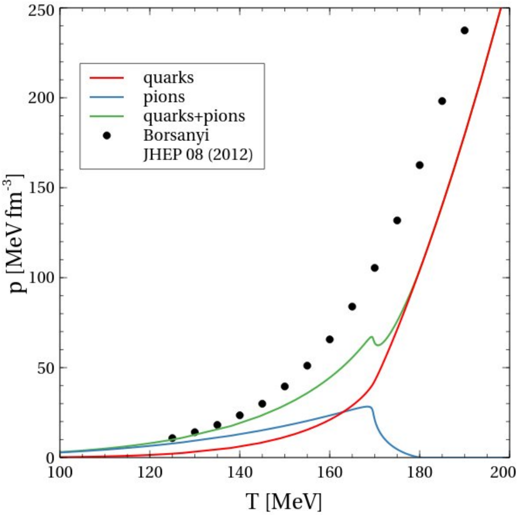
Beyond mean field pressure

- **Total pressure**

$$p = p_{quarks} + p_{hadrons} + \underbrace{\mathcal{U}(\Phi) + p_{pert}}_{\text{missing}}$$

$$\mathcal{U}(\Phi), p_{pert} \text{ - provide } \frac{\partial p}{\partial T} > 0$$

D. Blaschke et al., Symmetry 13 (3), 514 (2021)



Conclusions

- χ-symmetric density functiobal $\Rightarrow$ effective "confining" NJL model
- Medium dependent quark-meson couplings, derived consistently with model and symmetries
- Straightforward addition of vector and diquark channels
- Good agreement with the observational data on compact stars even at weak repulsion
- Lightest mesons
- Next steps:
  1. strangeness
  2. other mesonic states and baryons beyond mean-field
  3. unified quark-hadron EoS (hadron dissociation via Mott effect, hadronic correlations via Beth-Uhlenbeck approach)
  4. ...