Evolution of transport coefficients of the QGP along the phase transition

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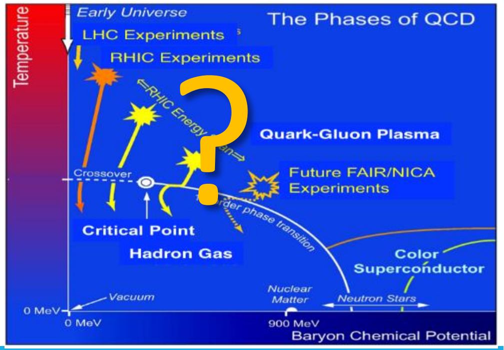
based on arXiv: 2011.03505

2108.08561



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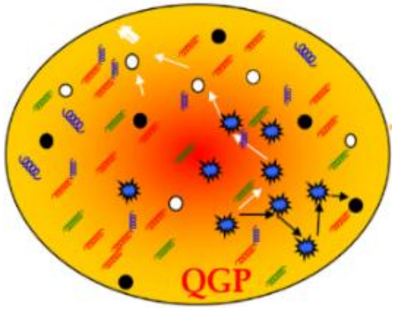


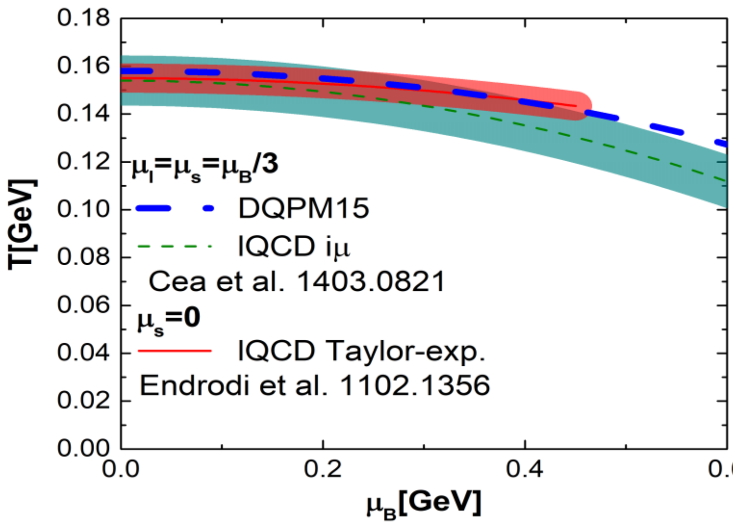
QGP near equilibrium: DQPM and PNJL

Transport coefficients at finite **T** and μ_B

1.) crossover (DQPM model)

2.) CEP and 1st order phase transition (PNJL model)

3.) phenomenological model for partonic phase: DQPM-CP



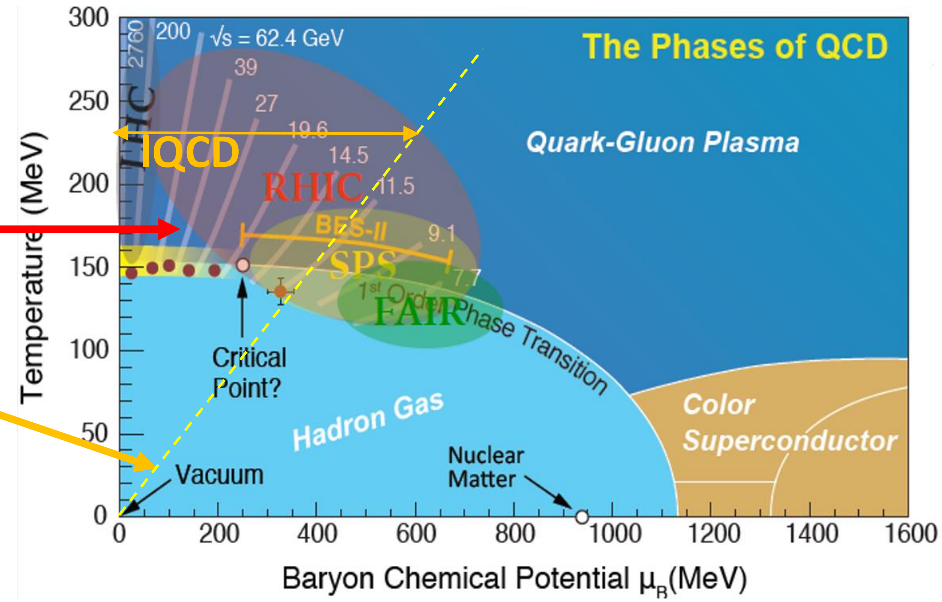
Motivation: Evolution of QGP in HICs

- Explore the **QCD phase diagram** at finite temperature and chemical potential through heavy-ion collisions

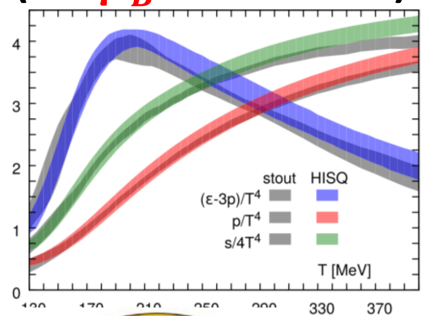
- **Available information:**
  - Experimental data at SPS, BES at RHIC
  - Lattice QCD calculations

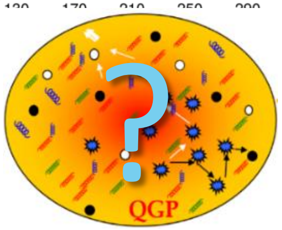
(for μ_B < 450 MeV)

Theoretical scketch of the QCD phase diagram



EoS





$\sigma(\sqrt{s}, m_q, m_q, T, \mu_B)$

$m(T, \mu_B)$

- How to learn about degrees-of-freedom of **QGP ?** ➔ **HIC simulations – transport** PHSD**, hydro description, ..**

! **Problem: Transport models(as well as hydro)** need an input for **the partonic phase**: cross-sections, masses, ...

Solution: effective models

QGP in equilibrium: DQPM and **PNJL**

Transport coefficients of QGP

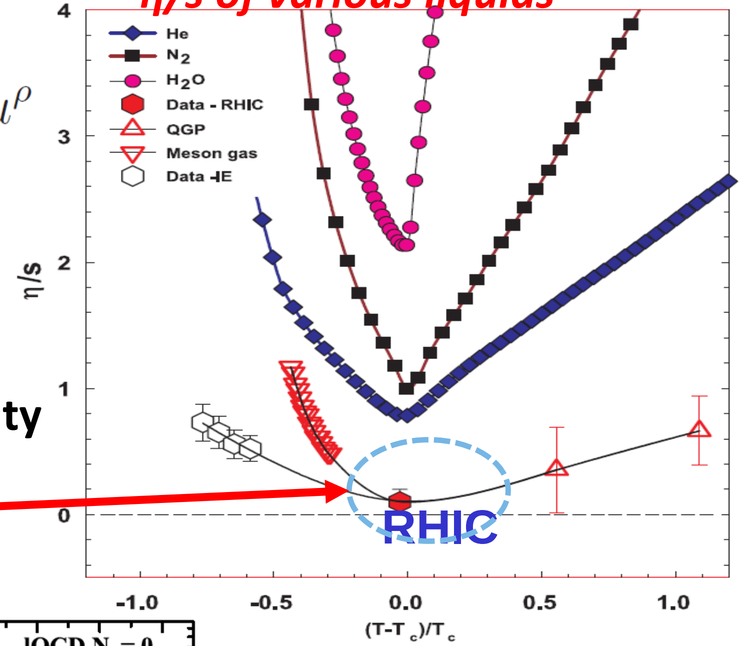
Hydrodynamical model(macroscopic description)

$$\Delta T^{\mu\nu} = \boxed{\eta}\left(D^{\mu}u^{\nu} + D^{\nu}u^{\mu} + \frac{2}{3}\Delta^{\mu\nu}\partial_{\rho}u^{\rho}\right) - \boxed{\zeta}\Delta^{\mu\nu}\partial_{\rho}u^{\rho}$$

input for hydro simulations

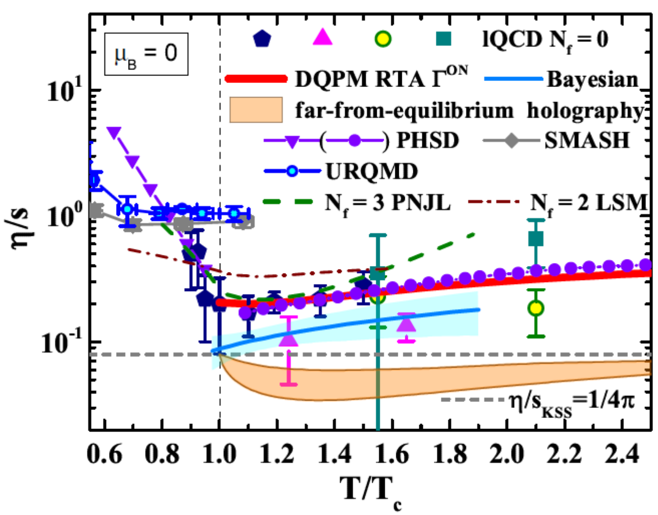
B,**Q**,**S** charge diffusion coefficients : Jan Fotakis

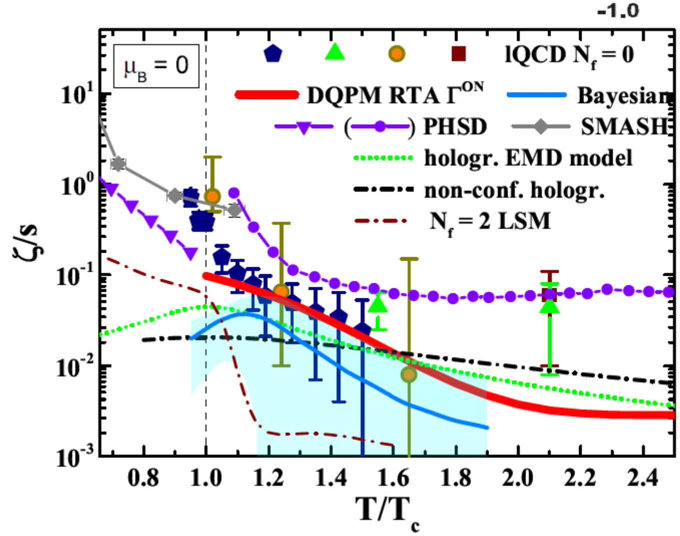
η/s of various liquids



Shear viscosity to entropy density ratio is extremely small

Model predictions:





! Different models using the same EoS can have completely different transport coefficients!

Relaxation time and scattering rate

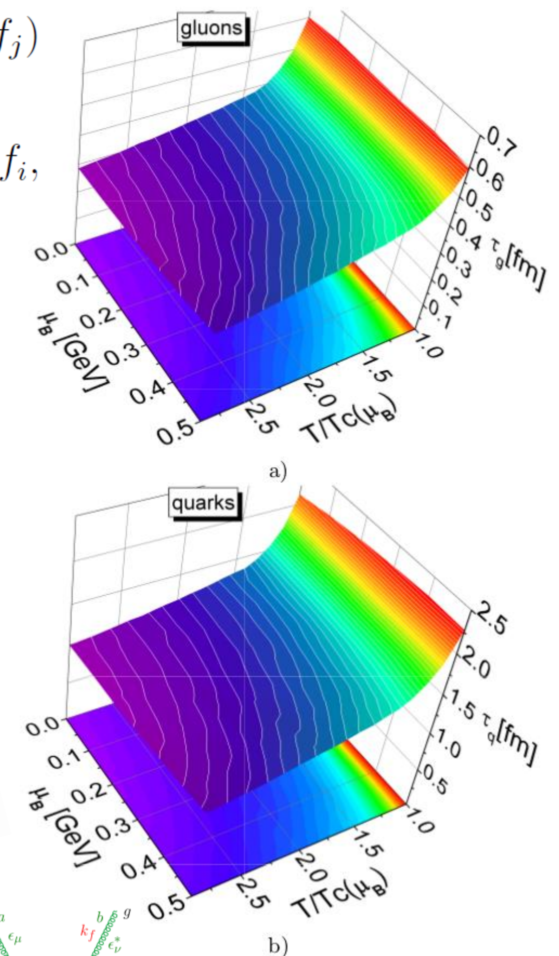
$\delta f_i = f_i^{(0)}\phi_i(\text{while } \phi_j = \phi_c = \phi_d = 0\ ).$

$$\frac{\partial f_i}{\partial t} + v_i \cdot \nabla f_i = \sum_{jcd} \frac{1}{1+\delta_{cd}} \int \frac{d^3p_j d^3p_c d^3p_d}{(2\pi)^9} W(i,j|c,d)(f_c f_d - f_i f_j)$$

$$= \sum_{jcd} \frac{1}{1+\delta_{cd}} \int \frac{d^3p_j d^3p_c d^3p_d}{(2\pi)^9} W(i,j|c,d)(-\delta f_i f_j^{(0)}) = -\Gamma_i(p,T,\mu)\delta f_i,$$

$$\tau_i(\mathbf{p},T,\mu_B) = \frac{1}{\Gamma_i(\mathbf{p},T,\mu_B)}$$

Relaxation times(DQPM)

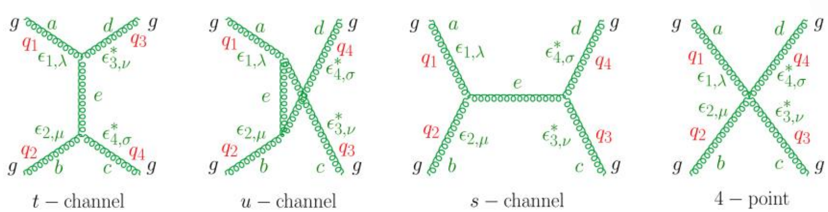


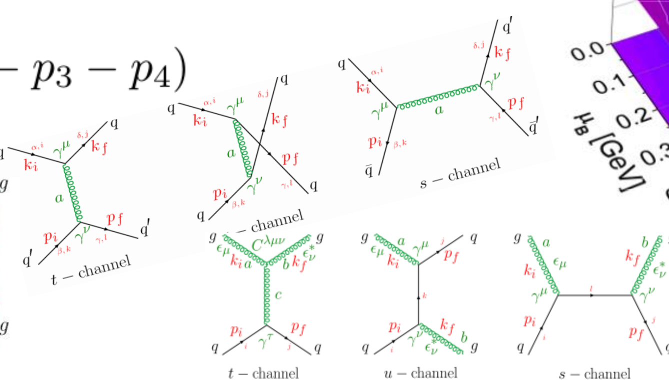
- **on-shell scattering (interaction) rates**

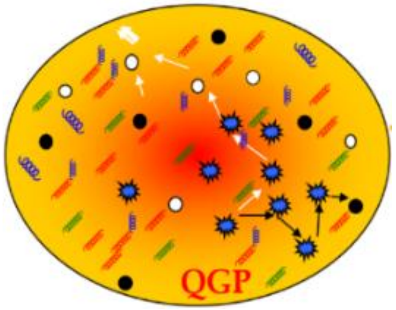
$$\Gamma_i^{\text{on}}(\mathbf{p}_i,T,\mu_q) = \frac{1}{2E_i} \sum_{j=q,\bar{q},g} \int \frac{d^3p_j}{(2\pi)^3 2E_j}\ d_j\ f_j(E_j,T,\mu_q)$$

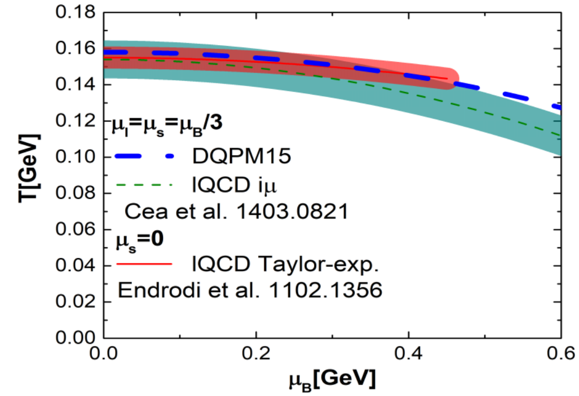
$$\int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} (1 \pm f_3)(1 \pm f_4)$$

$$|\bar{\mathcal{M}}|^2(p_i,p_j,p_3,p_4)\,(2\pi)^4\delta^{(4)}\,(p_i+p_j-p_3-p_4)$$







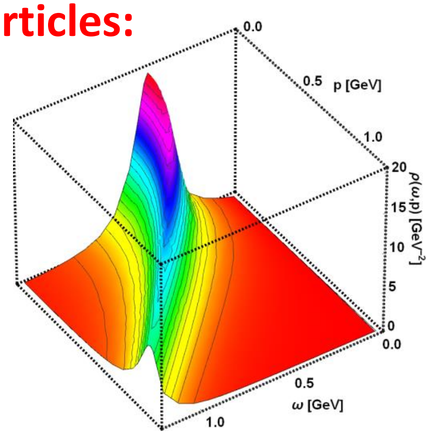


QGP in equilibrium:

Dynamical QuasiParticle Model (DQPM)

DQPM: consider the effects of the nonperturbative nature of the strongly interacting quark-gluon plasma (sQGP) constituents (vs. pQCD models)

- **The QGP phase is described in terms of interacting quasiparticles: quarks and gluons with Lorentzian spectral functions:**



$$\rho_j(\omega,\mathbf{p}) = \frac{\gamma_j}{\tilde{E}_j}\left(\frac{1}{(\omega-\tilde{E}_j)^2+\gamma_j^2} - \frac{1}{(\omega+\tilde{E}_j)^2+\gamma_j^2}\right)$$

$$\equiv \frac{4\omega\gamma_j}{\left(\omega^2-\mathbf{p}^2-M_j^2\right)^2+4\gamma_j^2\omega^2}$$

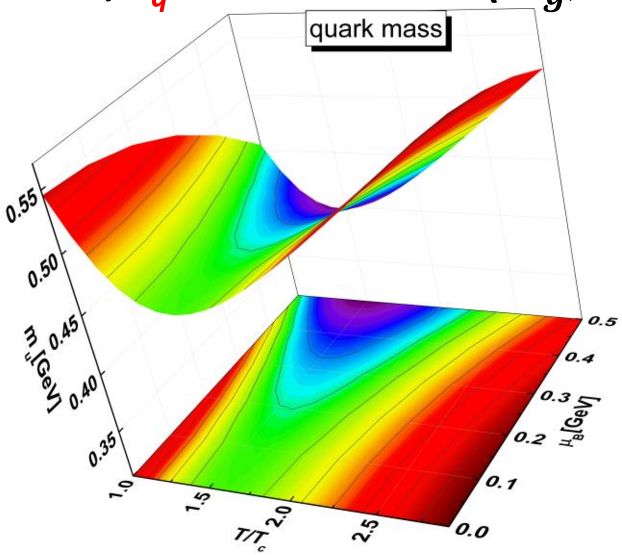
Peshier, Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

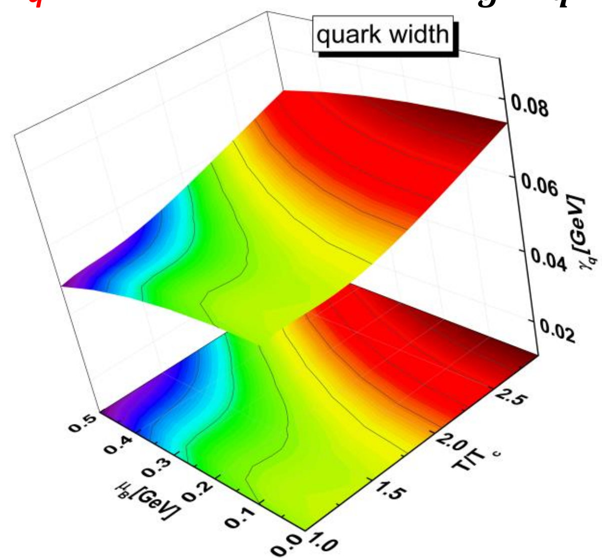
Dynamical QuasiParticle Model

- Resummed properties of the quasiparticles are specified by scalar complex self-energies:

gluon propagator: $\Delta^{-1} = P^2 - \Pi$	&	quark propagator $S_q^{-1} = P^2 - \Sigma_q$
gluon self-energy: $\Pi = M_g^2 - i2\gamma_g\omega$	&	quark self-energy: $\Sigma_q = M_q^2 - i2\,\gamma_q\omega$

- Re Π, Σ_q : thermal mass (M_g, M_q) Im Π, Σ_q : interaction width (γ_g, γ_q)





$$M^2_{q(\bar{q})}(T,\mu_B) = \frac{N_c^2-1}{8N_c} g^2(T,\mu_B)\left(T^2 + \frac{\mu_q^2}{\pi^2}\right)$$

$$\gamma_{q,g}(T,\mu_B) = \frac{c_{A,F}}{3}\frac{g^2(T,\mu_B)T}{8\pi}\ln\left(\frac{2c}{g^2(T,\mu_B)}+1\right)$$

- Modeling of the quark/gluon masses and widths (inspired by HTL calculations)

Peshier, Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

DQPM g^2: fixed within s(lQCD) at μ_B=0

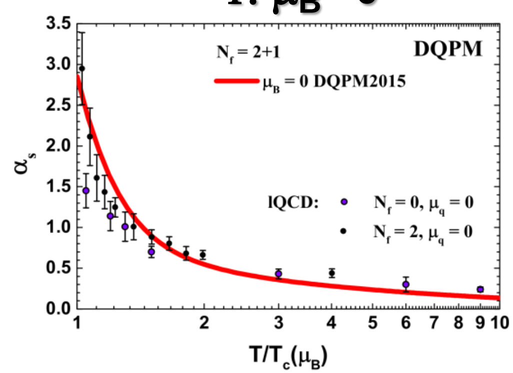
- Input: entropy density as a f(T , $\mu_B = 0$)

$$g^2(s/s_{SB}) = d\left((s/s_{SB})^e - 1\right)^f$$
$$s_{SB}^{QCD} = 19/9\pi^2 T^3$$
$$s^{DQPM}(\Pi, \Delta, S_q, \Sigma) = s^{lattice}$$

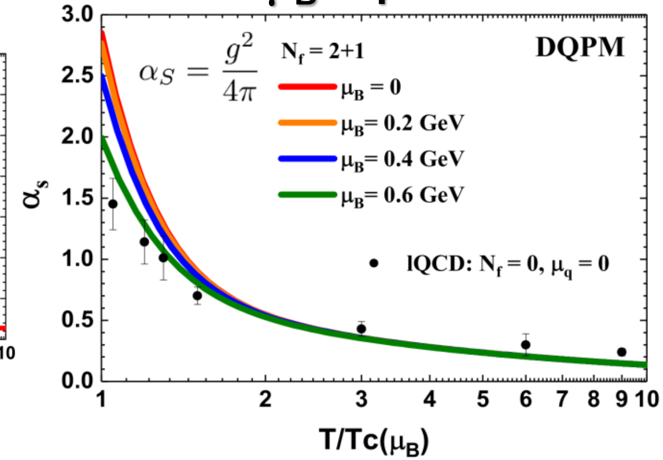
fit S from QP to S from lQCD

fix the model parameters

1. μ_B =0



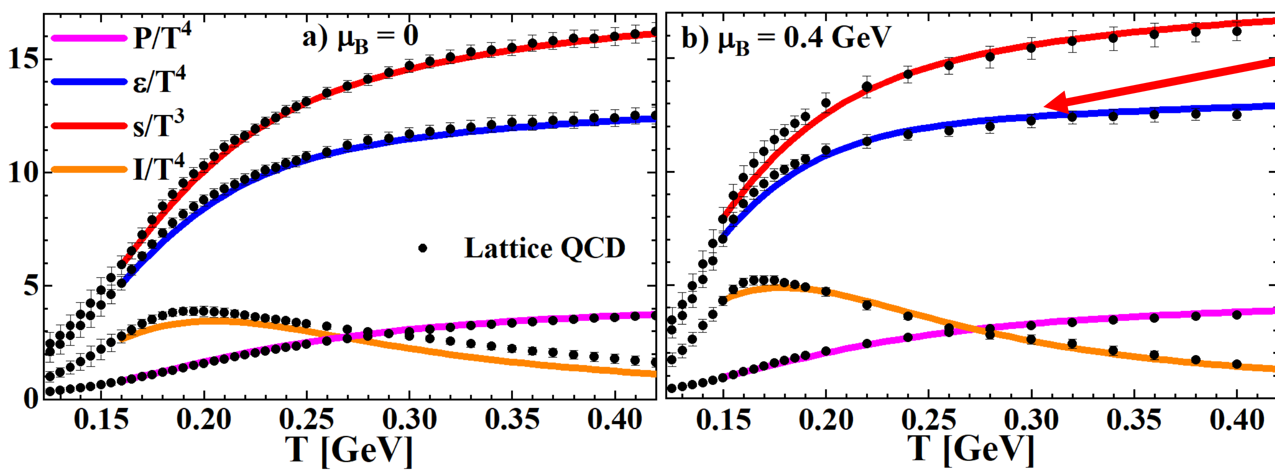
2. μ_B dependence



- Scaling hypothesis at finite $\mu_B \approx 3\mu_q$

$$g^2(T/T_c, \mu_B) = g^2\left(\frac{T^*}{T_c(\mu_B)}, \mu_B = 0\right)$$ with the effective temperature $$T^* = \sqrt{T^2 + \mu_q^2/\pi^2}$$

Input:
lattice EoS
$\mu_B = 0$ (dots)

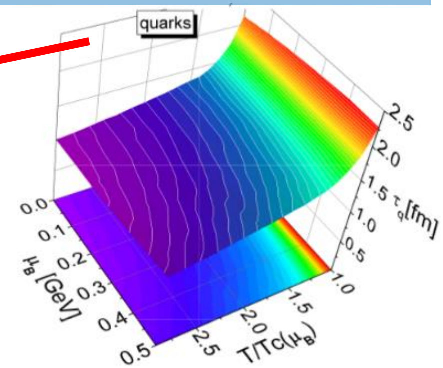


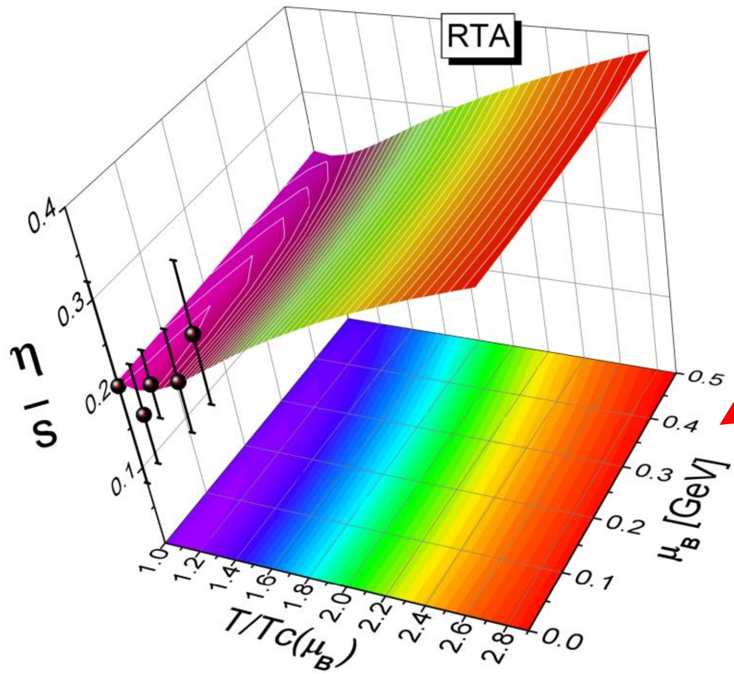
Output:
(lines)
DQPM EoS
$\mu_B > 0$

Specific shear viscosity

Relaxation times

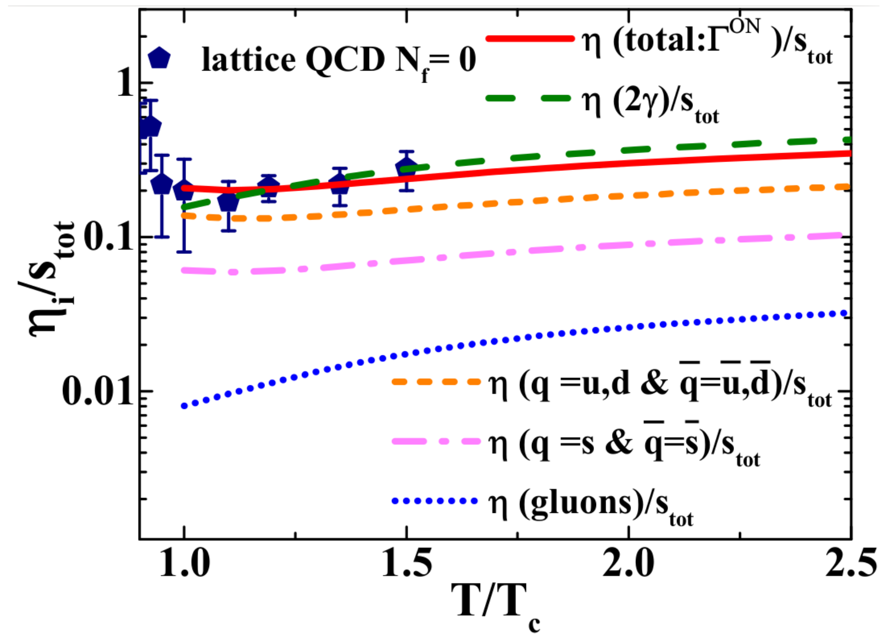
$$\eta^{\mathrm{RTA}}(T,\mu_B) = \frac{1}{15T}\sum_{i=q,\bar{q},g}\int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^4}{E_i^2}\,\tau_i(\mathbf{p},T,\mu_B)\; d_i(1\pm f_i)f_i$$





μ_B

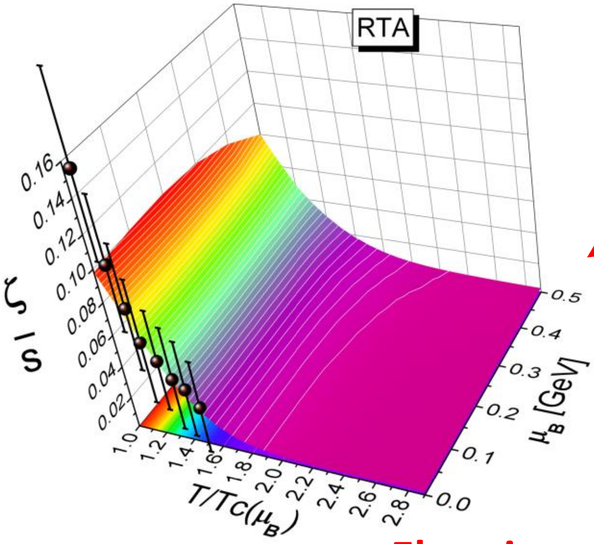
$\mu_B = 0$



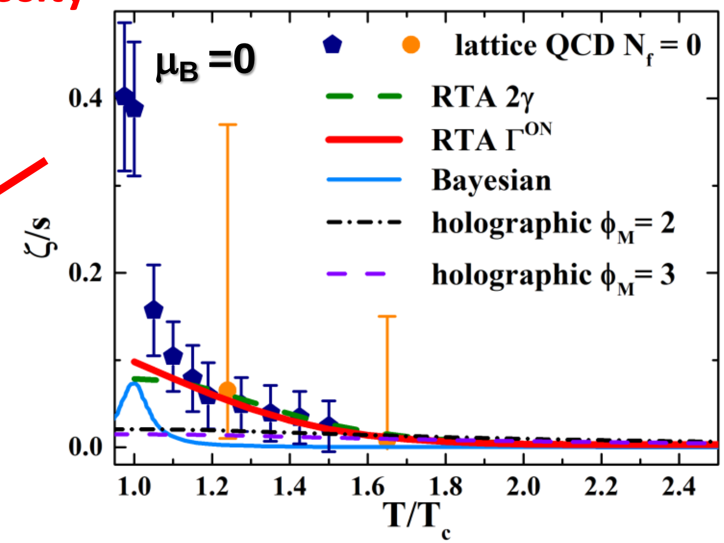
- Light increase with μ_B in the crossover region

Transport coefficients: increasing with μ_B

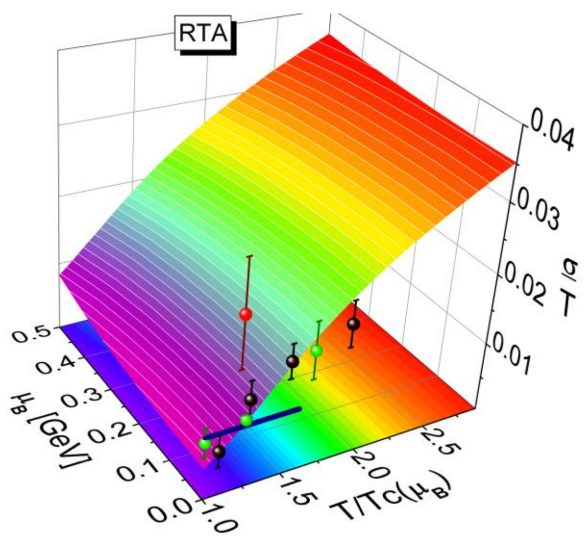
Specific bulk viscosity



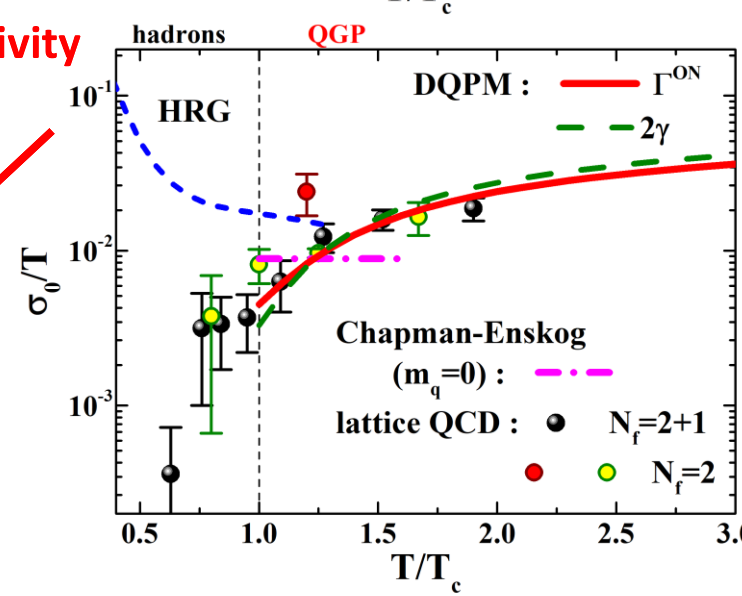
μ_B



Electric conductivity



μ_B



Polyakov Nambu Jona-Lasinio model

- **Effective lagrangian with the same symmetries for the quark dof as QCD**

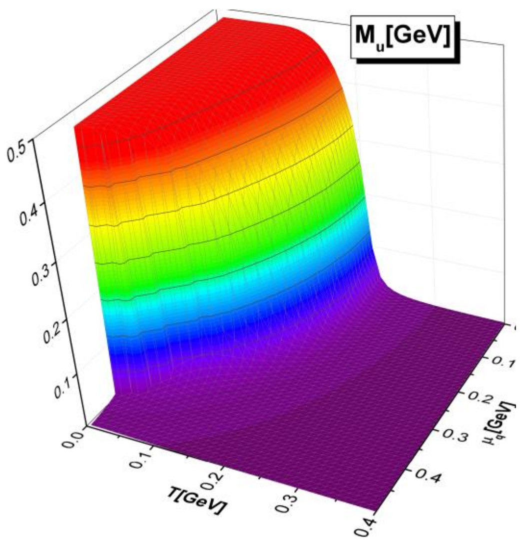
$$\mathscr{L}_{PNJL} = \sum_i \bar{\psi}_i (iD - m_{0i} + \mu_i \gamma_0)\psi_i$$
$$+ G \sum_a \sum_{ijkl} \left[(\bar{\psi}_i \; i\gamma_5 \tau^a_{ij} \psi_j)\; (\bar{\psi}_k \; i\gamma_5 \tau^a_{kl} \psi_l) + (\bar{\psi}_i \tau^a_{ij} \psi_j)\; (\bar{\psi}_k \tau^a_{kl} \psi_l) \right]$$
$$- K \det_{ij} \left[\bar{\psi}_i \; (-\gamma_5) \psi_j \right] - K \det_{ij} \left[\bar{\psi}_i \; (+\gamma_5) \psi_j \right]$$
$$- \mathcal{U}(T; \Phi, \bar{\Phi}) \; .$$

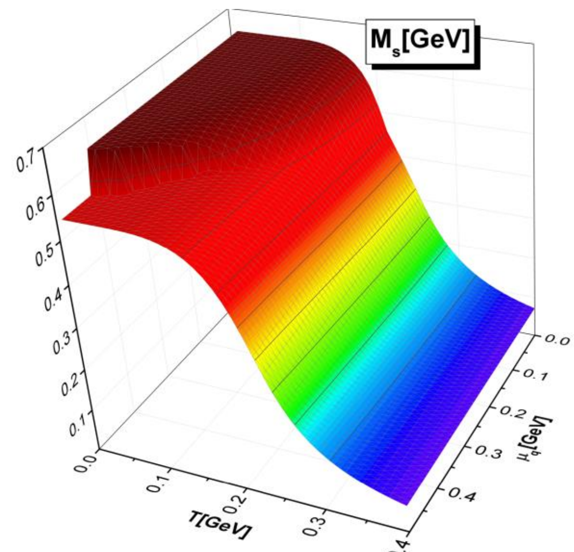
5 parameters fixed by vacuum values K,π masses, η-η'mass splitting , π decay constant, Chiral condensate

Polyakov potential fitted to the YM

J. M. Torres-Rincon, J. Aichelin PRC 96 (2017) 4 045205
D. Fuseau, T. Steinernert, J. Aichelin PRC 101 (2020) 6 065203

- **1st order PT at high μ_B (sudden change of q and meson masses)**





PNJL Relaxation times

$$\tau_i(\mathbf{p}, T, \mu_B) = \frac{1}{\Gamma_i(\mathbf{p}, T, \mu_B)}$$

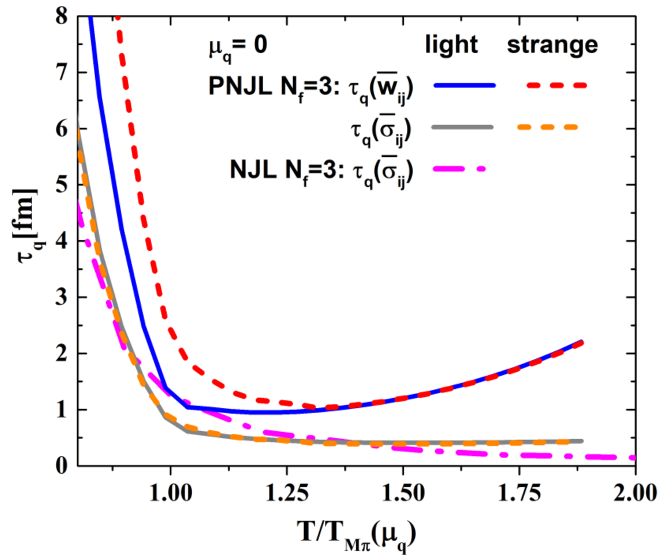
- **on-shell scattering (interaction) rates**

$$\Gamma_i^{\mathrm{on}}(\mathbf{p}_i, T, \mu_q) = \frac{1}{2E_i} \sum_{j=q,\bar{q},g} \int \frac{d^3p_j}{(2\pi)^3 2E_j}\, d_j\, f_j(E_j, T, \mu_q)$$

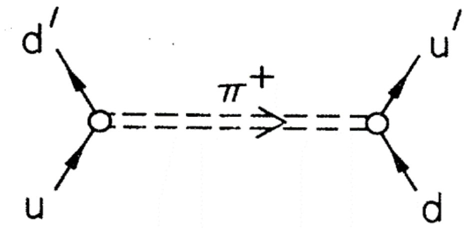
$$\int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} (1 \pm f_3)(1 \pm f_4)$$

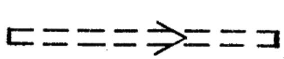
$$|\bar{\mathcal{M}}|^2(p_i, p_j, p_3, p_4)\, (2\pi)^4 \delta^{(4)}(p_i + p_j - p_3 - p_4)$$

Relaxation times(PNJL vs NJL)



4 point interaction -> meson exchange(π,Ϭ,η,η‘,K,.. for s,t,u channels)

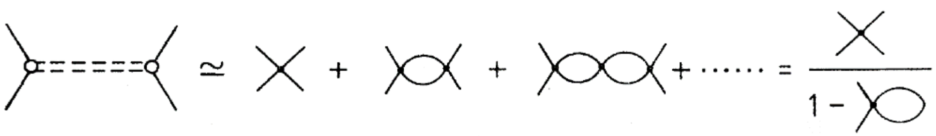




$$= (i\gamma_5)\tau^{(-)} \frac{-ig^2_{\pi qq}}{k^2 - m^2_\pi} (i\gamma_5)\tau^{(+)}$$

meson propagator $\mathscr{D} = \dfrac{2ig_m}{1 - 2g_m \Pi^{\pm}_{ff'}(k_0, \vec{k})}$

Effective interaction in RPA



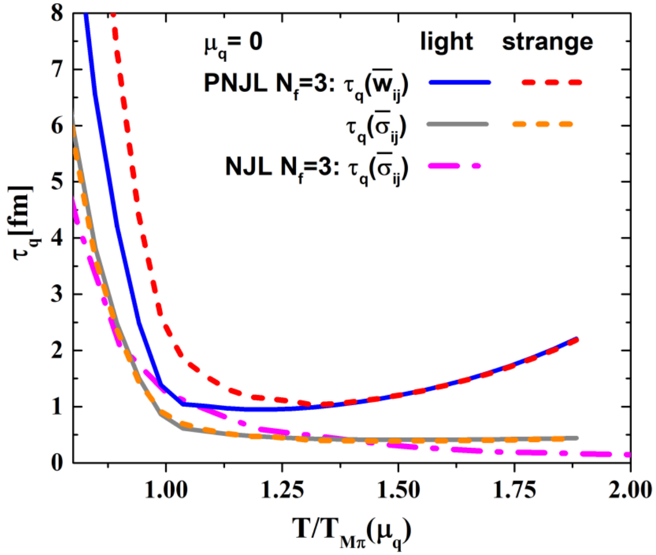
PNJL Relaxation time

$$\tau_i(\mathbf{p}, T, \mu_B) = \frac{1}{\Gamma_i(\mathbf{p}, T, \mu_B)}$$

- **on-shell scattering (interaction) rates**

$$\Gamma_i^{\text{on}}(\mathbf{p}_i, T, \mu_q) = \frac{1}{2E_i} \sum_{j=q,\bar{q},g} \int \frac{d^3p_j}{(2\pi)^3 2E_j}\, d_j\, f_j(E_j, T, \mu_q)$$
$$\int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} (1 \pm f_3)(1 \pm f_4)$$
$$|\bar{\mathcal{M}}|^2(p_i, p_j, p_3, p_4)\ (2\pi)^4 \delta^{(4)}(p_i + p_j - p_3 - p_4)$$

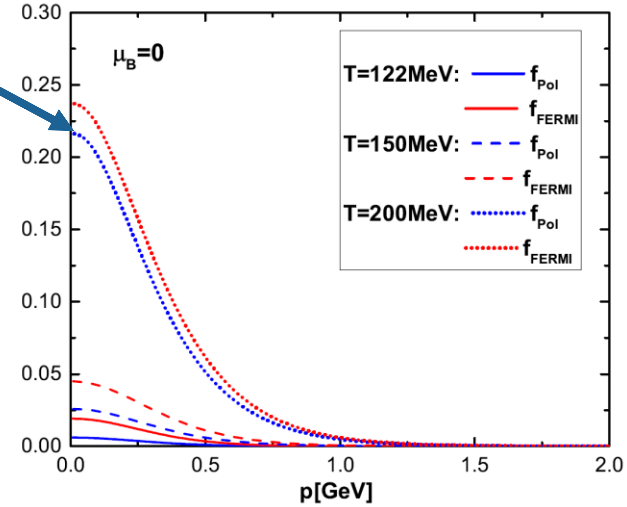
Relaxation times(PNJL)



Modified distribution functions: Polyakov loop contributions

$$f_q \to f_q^{\Phi}(\mathbf{p}, T, \mu) = \frac{(\bar{\Phi} + 2\Phi e^{-(E_\mathbf{p}-\mu)/T}) e^{-(E_\mathbf{p}-\mu)/T} + e^{-3(E_\mathbf{p}-\mu)/T}}{1 + 3(\bar{\Phi} + \Phi e^{-(E_\mathbf{p}-\mu)/T}) e^{-(E_\mathbf{p}-\mu)/T} + e^{-3(E_\mathbf{p}-\mu)/T}},$$

$$f_{\bar{q}} \to f_{\bar{q}}^{\Phi}(\mathbf{p}, T, \mu) = \frac{(\Phi + 2\bar{\Phi} e^{-(E_\mathbf{p}+\mu)/T}) e^{-(E_\mathbf{p}+\mu)/T} + e^{-3(E_\mathbf{p}+\mu)/T}}{1 + 3(\Phi + \bar{\Phi} e^{-(E_\mathbf{p}+\mu)/T}) e^{-(E_\mathbf{p}+\mu)/T} + e^{-3(E_\mathbf{p}+\mu)/T}}.$$

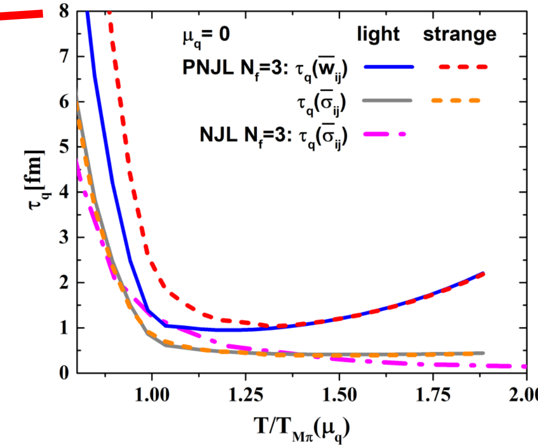


Specific shear viscosity at high μ_B

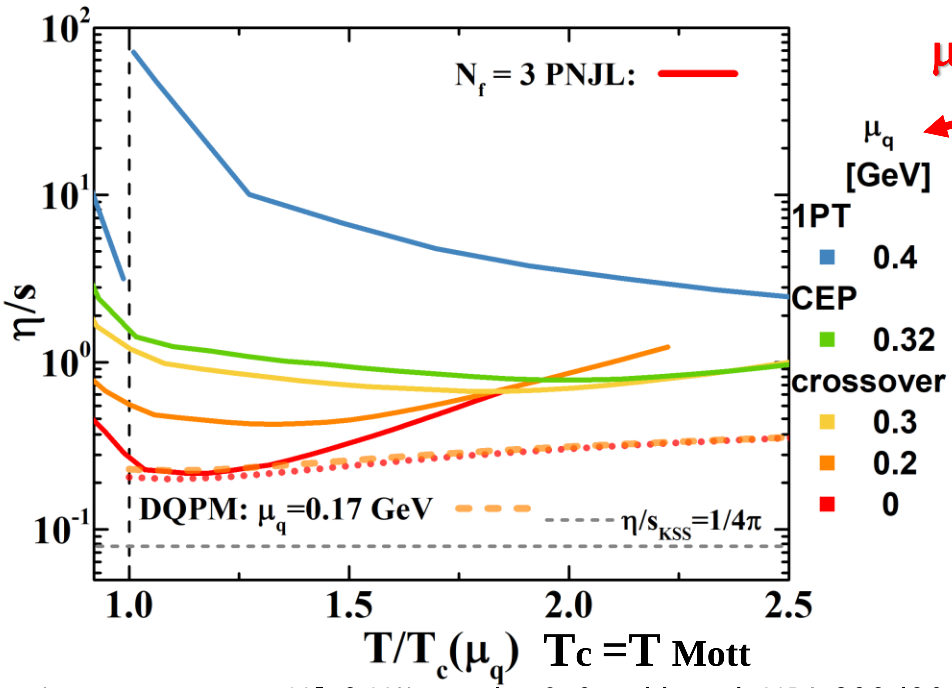
$$\eta^{\rm RTA}(T,\mu_B) = \frac{1}{15T}\sum_{i=q,\bar{q},g}\int\frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^4}{E_i^2}\tau_i(\mathbf{p},T,\mu_B)\, d_q f_i^{\phi}$$

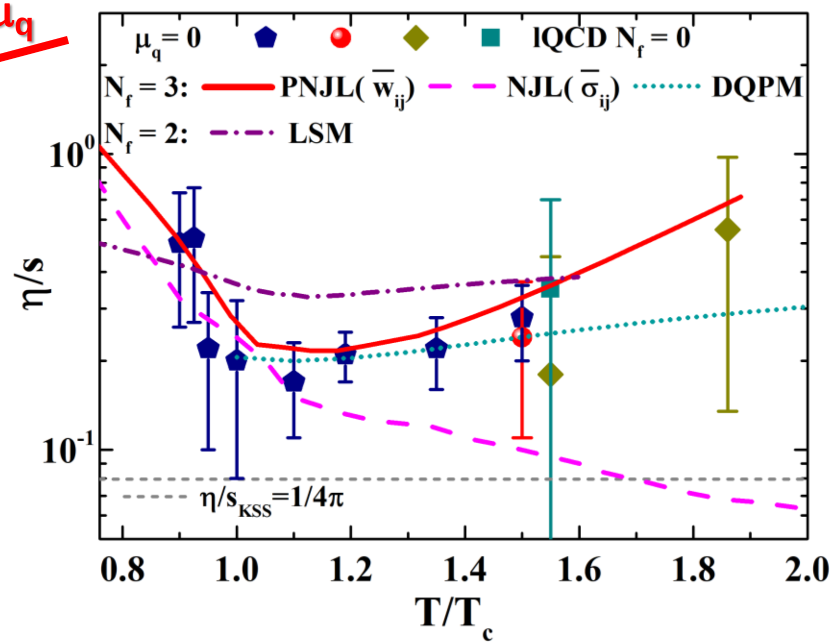
$$f_i^{\phi} = \frac{\phi e^{-(E_i\mp\mu)/T} + 2\bar{\phi}e^{-2(E_i\mp\mu)/T} + e^{-3(E_i\mp\mu)/T}}{1+3\phi e^{-(E_i\mp\mu)/T} + 3\bar{\phi}e^{-2(E_i\mp\mu)/T} + e^{-3(E_i\mp\mu)/T}}$$

with Polyakov loops



$\mu_B = 3\mu_q$





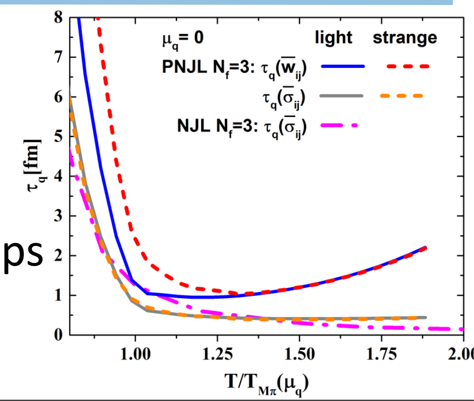
In agreement w Nf=2 NJL results C. Sasaki et al, NPA 832 (2010)

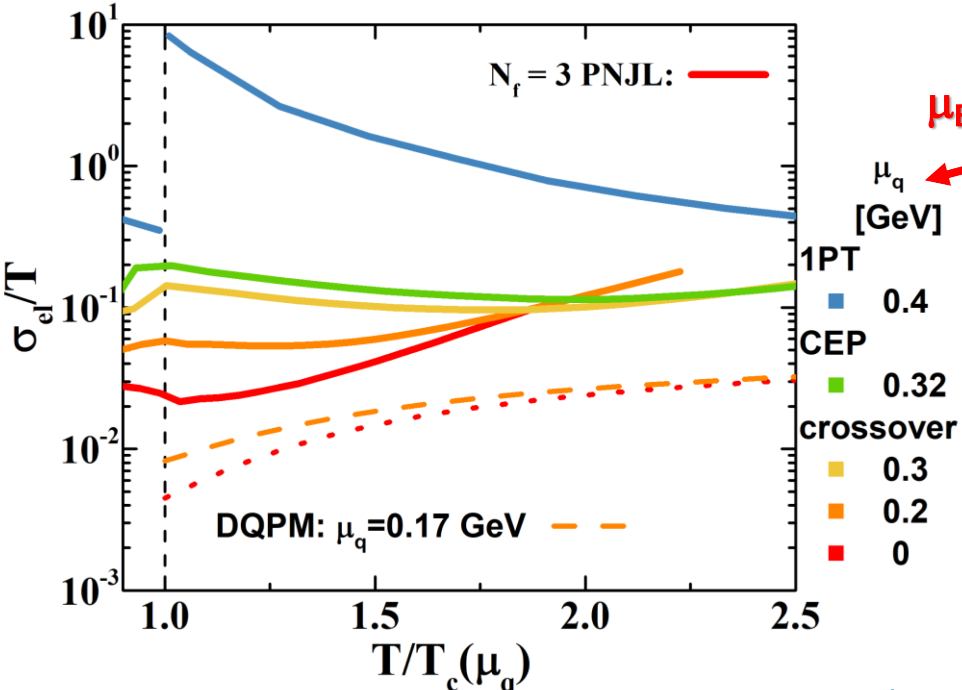
Electric conductivity at high μ_B

$$\sigma_0^{\mathrm{RTA}}(T,\mu_B) = \frac{e^2}{3T}\sum_{i=q,\bar{q}} q_i^2 \int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^2}{E_i^2}\boxed{\tau_i(\mathbf{p},T,\mu_B)}\, d_q f_i^\phi$$

$$f_i^\phi = \frac{\phi e^{-(E_i\mp\mu)/T} + 2\bar{\phi}e^{-2(E_i\mp\mu)/T} + e^{-3(E_i\mp\mu)/T}}{1+3\phi e^{-(E_i\mp\mu)/T} + 3\bar{\phi}e^{-2(E_i\mp\mu)/T} + e^{-3(E_i\mp\mu)/T}}$$

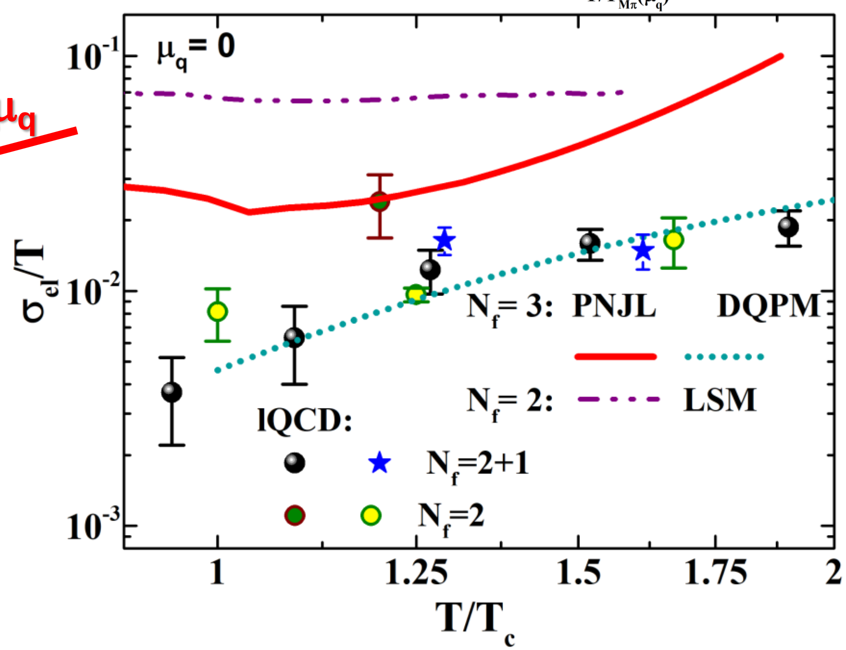
with Polyakov loops



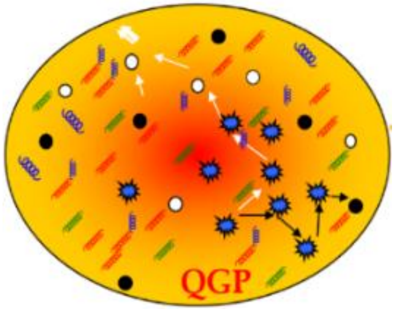


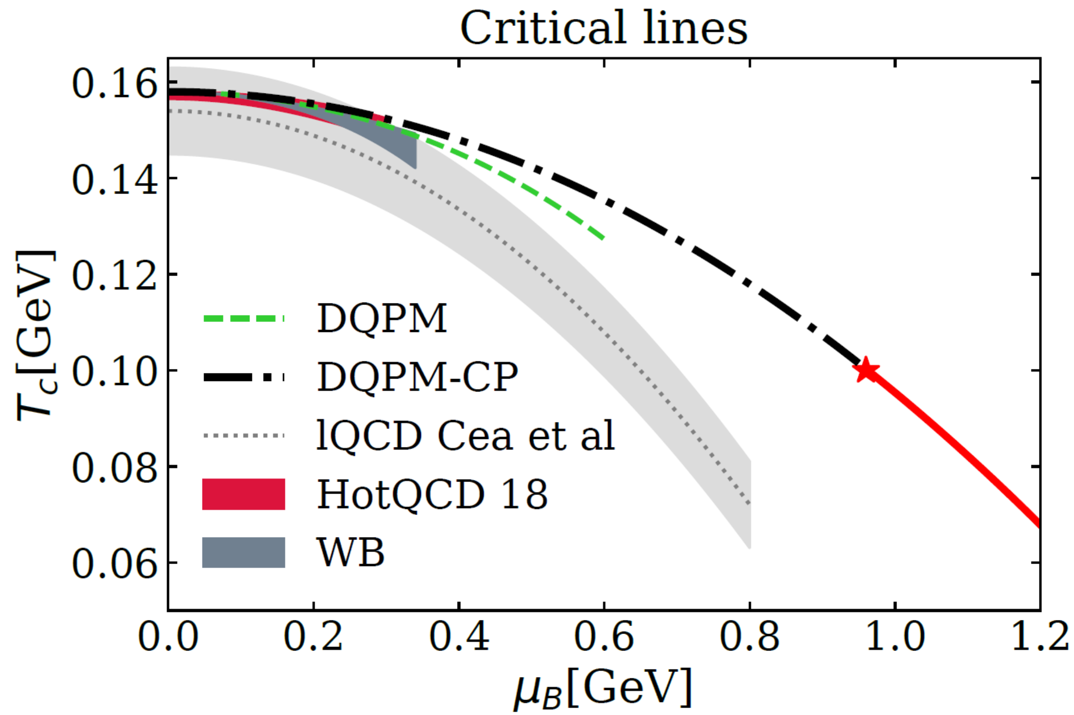
$T_c = T_{\mathrm{Mott}}$

$\mu_B = 3\mu_q$

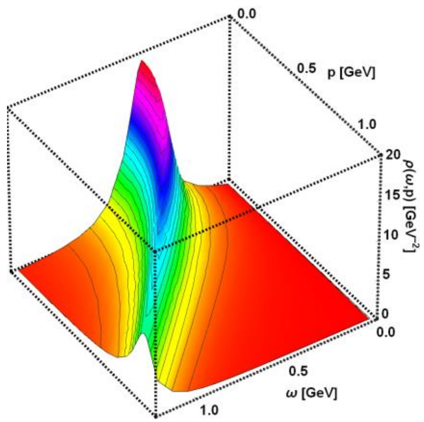


PNJL results: O.S., D. Fuseau, J.Aichelin, E. Bratkovskaya PRC103 5, 054901 (2021)





DQPM-CP: covering wide range of μ_B



O.S, J. Aichelin, E.Bratkovskaya arxiv:2108.08561

Extension of DQPM g^2: finite μ_B

- **Input: entropy density as a f(T , $\mu_B = 0$)**

$$g^2(s/s_{SB}) = d\left((s/s_{SB})^e - 1\right)^f$$
$$s_{SB}^{QCD} = 19/9\pi^2 T^3$$
$$s^{DQPM}(\Pi, \Delta, S_q, \Sigma) = s^{lattice}$$

fit S from QP to S from lQCD

fix the model parameters

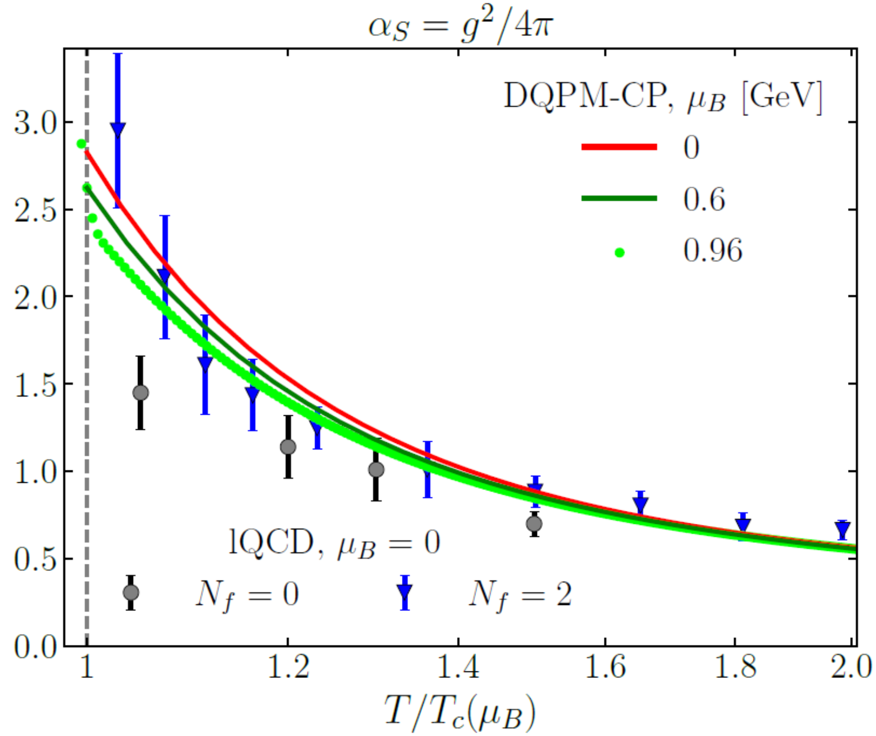
- **Effective coupling constant**

$$\alpha_S^{DQPM-CP} \equiv \begin{cases} \mu_B = \mu_{CEP}: \ \alpha_S^{CEP} = \\ \quad \frac{1-F(T)}{2}\alpha_S^{crit} + \frac{1+F(T)}{2}\alpha_S^{cross} \\ \mu_B \neq \mu_{CEP}: \ \alpha_S^{cross} \end{cases}$$

$$\alpha_S^{crit} = a \cdot (T/T_c)^{-12}$$

fit entropy density from the PNJL

fix the model parameters



Parametrize the DQPM effective coupling

$$\alpha_S^{cross} = a_0 + \frac{a_2}{x_T^2} - \frac{a_3}{x_T^3} + \frac{a_4}{x_T^4} + \frac{a_6 \cdot \sigma(\mu_B)}{x_T^6}$$

$$F(T) = \tanh\left[\frac{T - 0.1004}{\delta T}\right] \quad x_T = T/T_c(\mu_B)$$

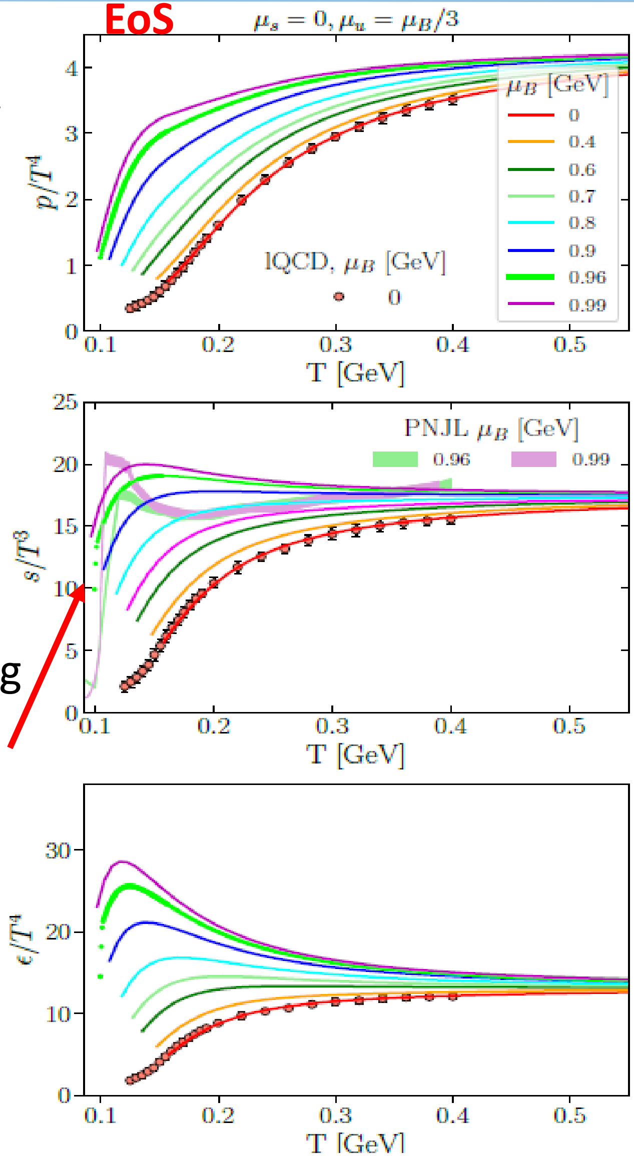
DQPM-CP: Thermodynamic observables

- Input: lQCD entropy density as a $f(T, \mu_B = 0)$
- \+ PNJL s(T/Tc) near the CEP

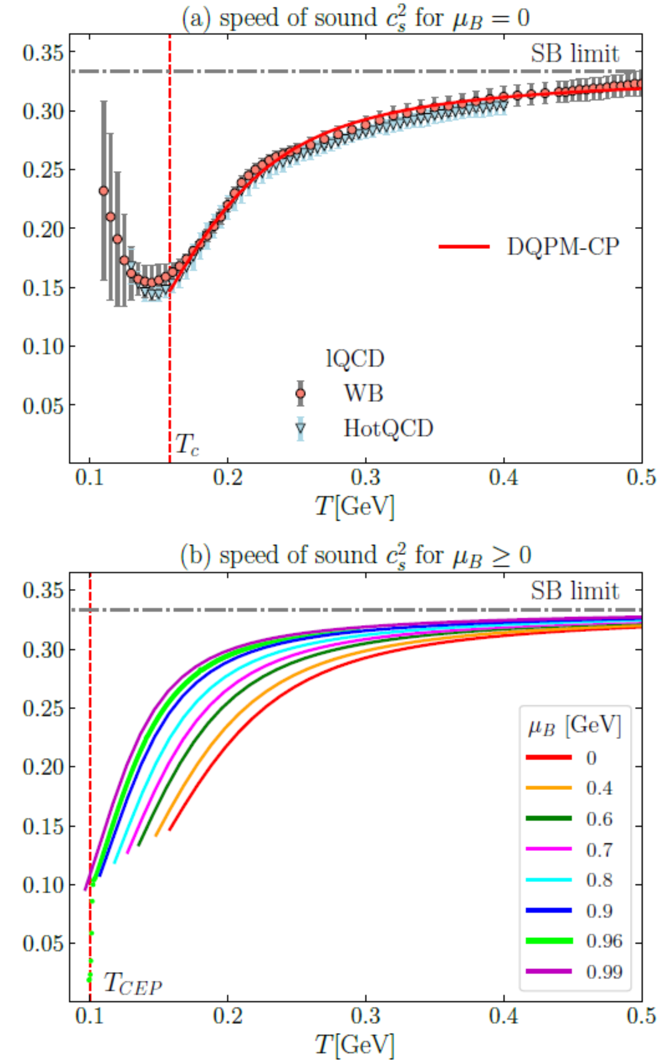
Approaching the CEP

lattice EoS
$\mu_B = 0$ (dots)

EoS



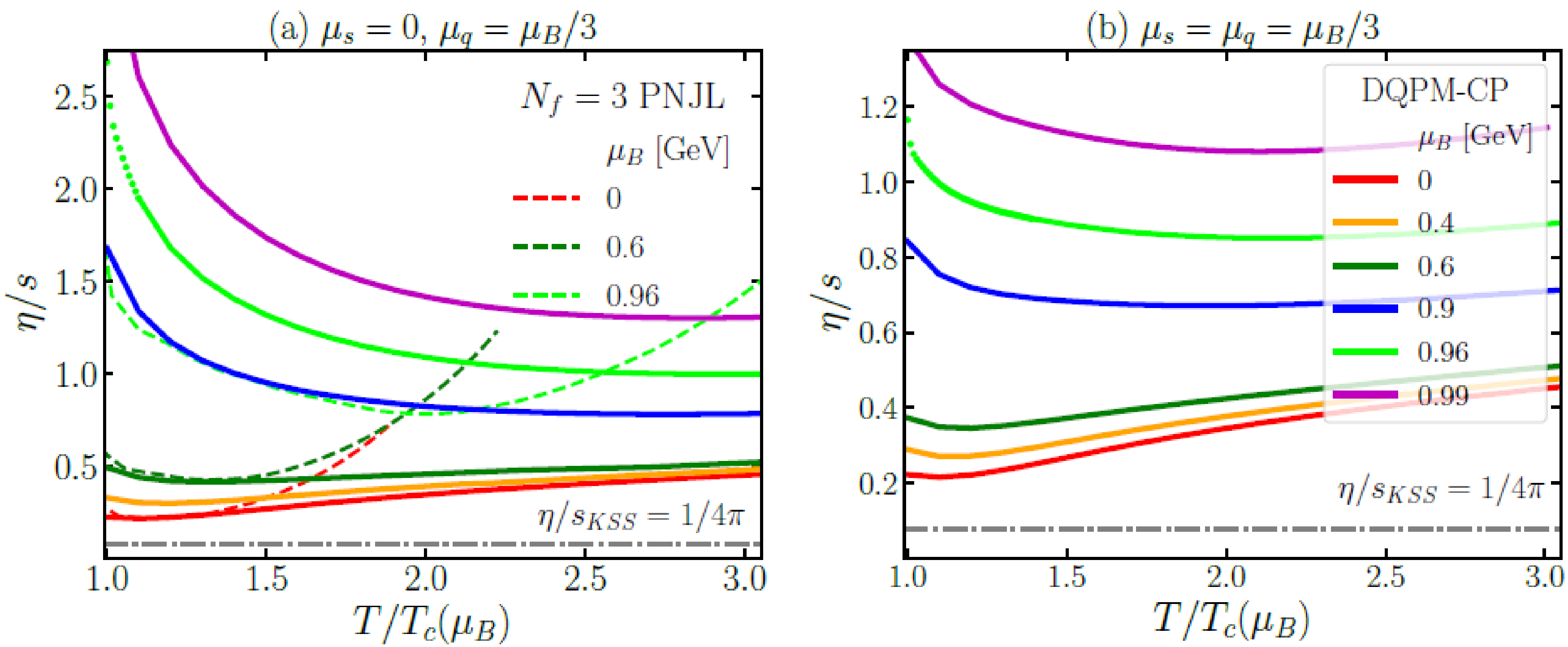
Speed of sound



Specific shear viscosity

$$\eta^{\text{RTA}}(T,\mu_B) = \frac{1}{15T}\sum_{i=q,\bar{q},g}\int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^4}{E_i^2}\boxed{\tau_i(\mathbf{p},T,\mu_B)}\,d_i(1\pm f_i)f_i$$

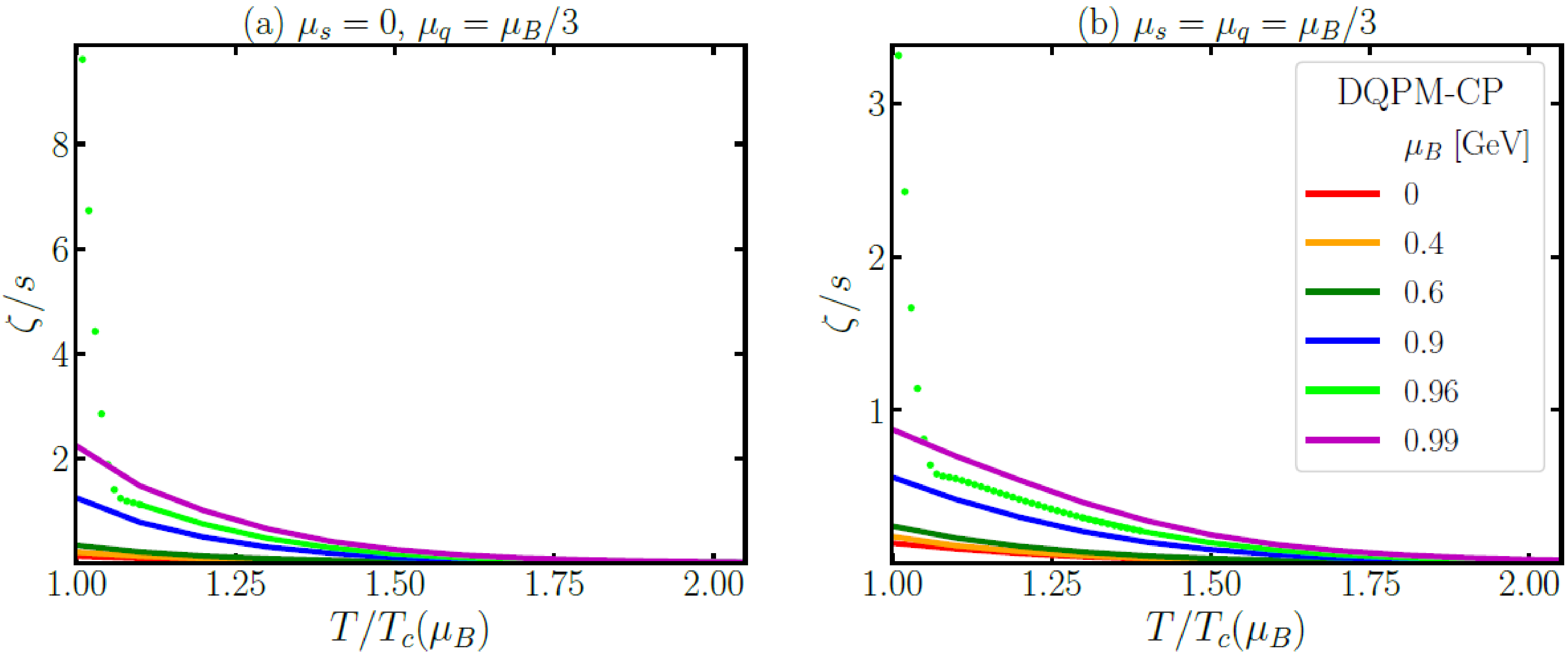
- Two setups for strange quark chemical potential
- Light increase with μ_B in the crossover region



Specific bulk viscosity

$$\zeta^{\rm RTA}(T,\mu_q) = \frac{1}{9T}\int \frac{d^3p}{(2\pi)^3} \sum_{i=q,\bar{q}} \frac{\mathbf{p}^4}{E_i^2} \boxed{\tau_i(\mathbf{p},T,\mu_B)} [\mathbf{p}^2 - 3c_s^2(E_i^2 - T^2\frac{dm_q^2}{dT^2})]^2 d_i(1\pm f_i)f_i$$

- Light increase with μ_B in the crossover region
- Sudden increase approaching the CEP



Summary / Outlook

CRC-TR 211
Strong-interaction matter under extreme conditions

- Transport coefficients at finite T and μ_B have been found using the (T, μ_B)-dependent cross sections in the DQPM and PNJL models
- At $\mu_B = 0$ good agreement with the Bayesian analysis estimations and lQCD estimations of QGP transport coefficients
- DQPM-CP model was parametrized to mimic critical scaling near the CEP
- Increase of η/s, Ϭ/T with μ_B has been found in the both models, near the CEP the transport coefficients has shown critical scaling: bulk viscosity suddenly increase approaching the CEP!
- At large values of μ_B (1.2 GeV in this work) presence of the 1st order phase transition changes T dependence of transport coefficients drastically within the PNJL model

Summary / Outlook

Thank you for your attention!

- **Outlook:**
  - **More precise EoS large μ_B**
  - **Approaching high densities via transport simulations (PHSD)**

Backup slides

Relaxation Time Approximation

- **Boltzmann equation** $f_a = f_a^{\text{eq}}(1+\phi_a)$

$$\frac{d f_a^{\text{eq}}}{dt} = \mathcal{C}_a = -\frac{f_a^{\text{eq}}\phi_a}{\tau_a}$$

RTA: system equilibrates within the relax time τ, Express collisional Integral via τ and f_a

- **Relaxation times:**

$$\frac{1+d_a f_a^{\text{eq}}}{\tau_a(E_a^*)} = \sum_{bcd} \frac{1}{1+\delta_{ab}} \int d\Gamma_b^* \, d\Gamma_c^* \, d\Gamma_d^* \, W(a,b|c,d) f_b^{\text{eq}} \big(1+d_c f_c^{\text{eq}}\big)\big(1+d_d f_d^{\text{eq}}\big) + (cd), (bc)$$

$$T^{\mu\nu} = -Pg^{\mu\nu} + wu^\mu u^\nu + \underline{\Delta T^{\mu\nu}} \qquad J_B^\mu = n_B u^\mu + \underline{\Delta J_B^\mu}$$

$$\Delta T^{\mu\nu} = \eta\left(D^\mu u^\nu + D^\nu u^\mu + \tfrac{2}{3}\Delta^{\mu\nu}\partial_\rho u^\rho\right) - \zeta \Delta^{\mu\nu}\partial_\rho u^\rho$$

$$\Delta J_B^\mu = \lambda \left(\frac{n_B T}{w}\right)^2 D^\mu \left(\frac{\mu_B}{T}\right)$$

hydrodynamics

Energy-momentum tensor and baryon diffusion current can be expressed using f_a :

$T^{\mu\nu}(f_a, m_{q,g}), J_B^\mu(f_a, m_{q,g})$

Obtain the transport coefficients using conservation laws, and f_a:

$$\begin{cases} \partial_\mu J_B^\mu = 0 \\ \partial_\mu T^{\mu\nu} = 0 \end{cases} \Rightarrow \quad \eta^{\text{RTA}}(T,\mu_B) = \frac{1}{15T} \sum_{i=q,\bar{q},g} \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}^4}{E_i^2} \tau_i(\mathbf{p}, T, \mu_B) \, d_i(1 \pm f_i) f_i$$

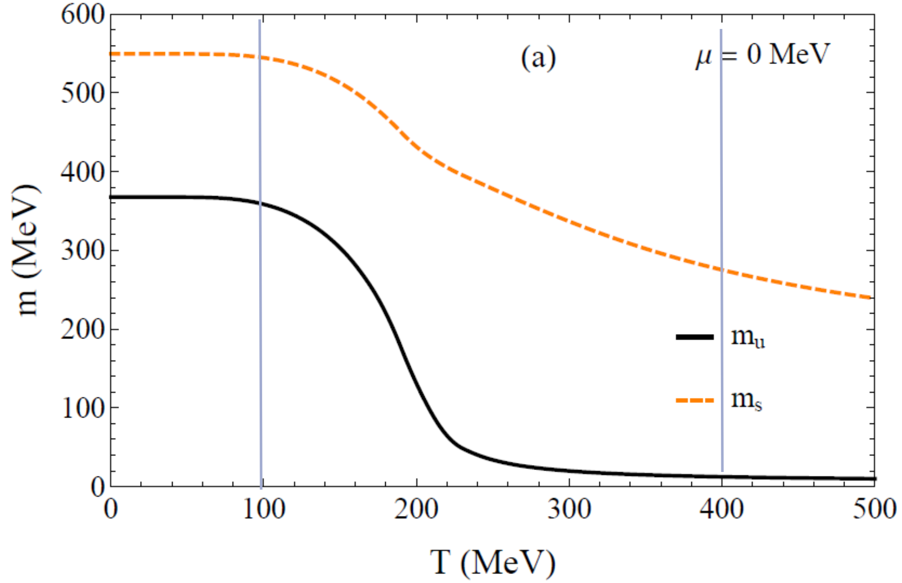
P. Chakraborty and J. I. Kapusta, PRC 83,014906 (2011).

Quark masses for NJL and PNJL

- **Gap equation + minimization of the grand potential** → **Chiral masses** (M_l, M_s)

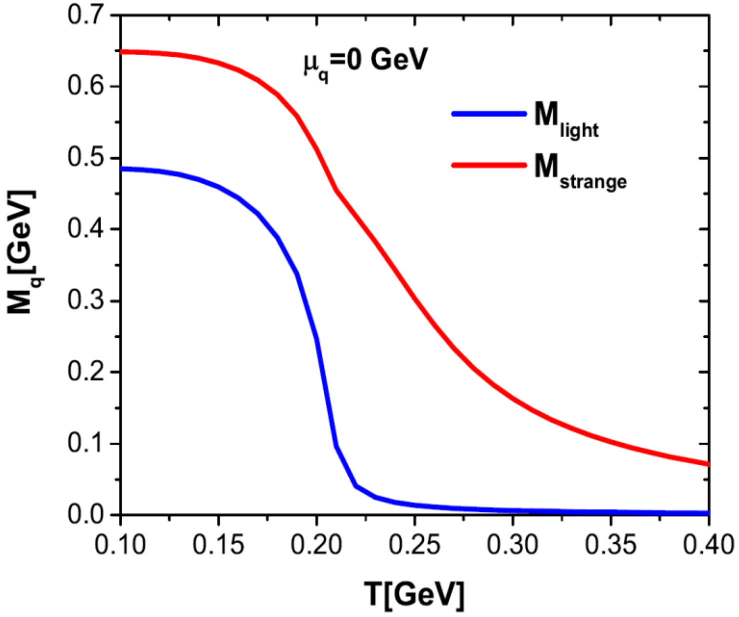
$$m_i = m_{0i} - 4G\langle\langle\bar{\psi}_i\psi_i\rangle\rangle + 2K\langle\langle\bar{\psi}_j\psi_j\rangle\rangle\langle\langle\bar{\psi}_k\psi_k\rangle\rangle$$

Chiral masses(NJL)



R. Marty et al. PRC 88 (2013) 4 045204

Chiral masses(PNJL)



- **in PNJL transition is steeper than in NJL**

Properties of QGP: transport coefficients

Hydrodynamics

$$T^{\mu\nu} = -Pg^{\mu\nu} + wu^\mu u^\nu + \Delta T^{\mu\nu}$$

$$J^\mu_B = n_B u^\mu + \Delta J^\mu_B$$

$$\begin{cases} \partial_\mu J^\mu_B = 0 \\ \partial_\mu T^{\mu\nu} = 0 \end{cases}$$

input for hydro simulations

$$\Delta T^{\mu\nu} = \boxed{\eta}\left(D^\mu u^\nu + D^\nu u^\mu + \tfrac{2}{3}\Delta^{\mu\nu}\partial_\rho u^\rho\right) - \boxed{\zeta}\Delta^{\mu\nu}\partial_\rho u^\rho$$

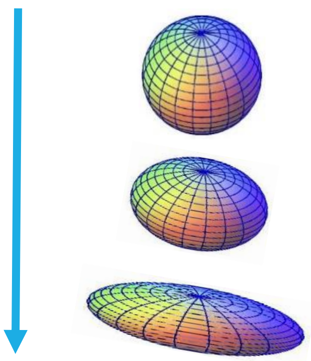
$$\Delta J^\mu_B = \boxed{\kappa_B} D^\mu\left(\frac{\mu_B}{T}\right)$$

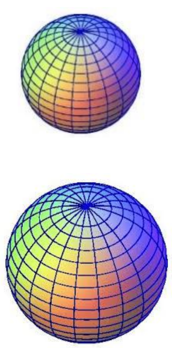
$$D^\mu = \Delta^{\alpha\nu}\partial_\nu \qquad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

Shear viscosity
Resistance to deformation
$\eta \nabla^{<\mu} u^{\nu>}$

Bulk viscosity
Resistance to expansion
$-\zeta \nabla u$

B,Q,S charge **diffusion coefficients**
$\kappa_B \nabla^\mu \frac{\mu_B}{T}$





Relaxation time: increases with μ_B

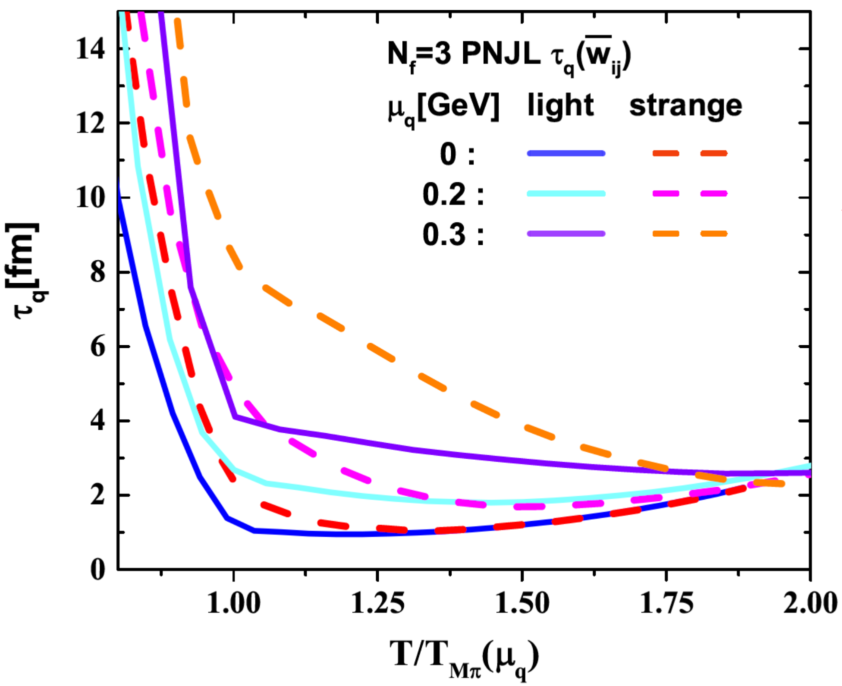
$$\tau_i(\mathbf{p},T,\mu_B) = \frac{1}{\Gamma_i(\mathbf{p},T,\mu_B)}$$

$$\tau_i^{-1}(T,\mu_q) = \frac{1}{n_i(T,\mu_q)} \int \frac{\mathrm{d}^3 p_i}{(2\pi)^3} d_q f_i^{(0)} \tau_i^{-1}(p_i,T,\mu_q)$$

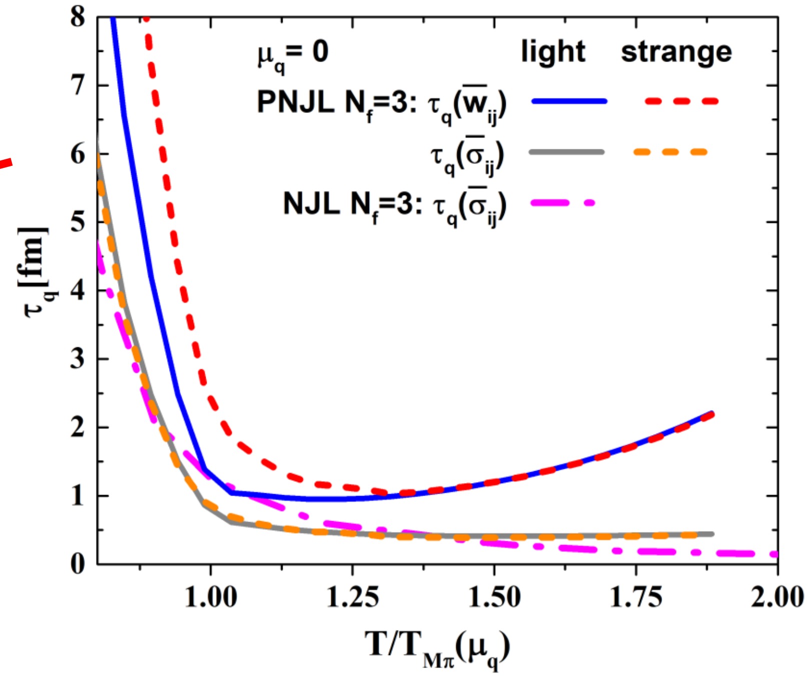
- **on-shell scattering (interaction) rates**

$$\Gamma_i^{\mathrm{on}}(\mathbf{p}_i,T,\mu_q) = \frac{1}{2E_i} \sum_{j=q,\bar{q},g} \int \frac{d^3p_j}{(2\pi)^3 2E_j}\, d_j\, f_j(E_j,T,\mu_q)$$

$$\int \frac{d^3p_3}{(2\pi)^3 2E_3} \int \frac{d^3p_4}{(2\pi)^3 2E_4} (1\pm f_3)(1\pm f_4)\,|\bar{\mathcal{M}}|^2(p_i,p_j,p_3,p_4)\ (2\pi)^4 \delta^{(4)}(p_i+p_j-p_3-p_4)$$

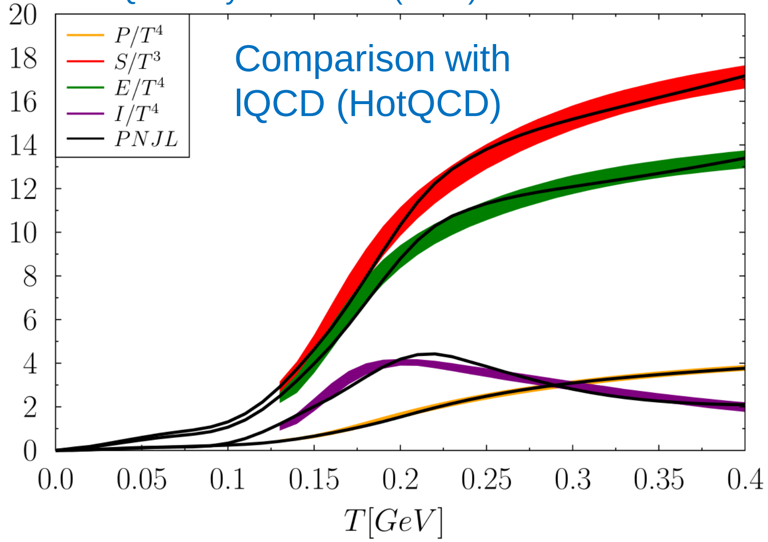


$\mu_B = 3\mu_q$

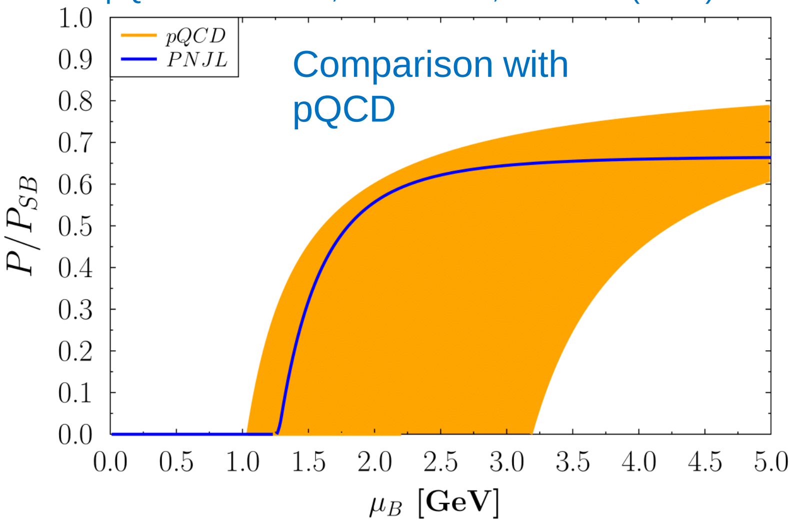


Polyakov Nambu Jona-Lasinio model: EOS

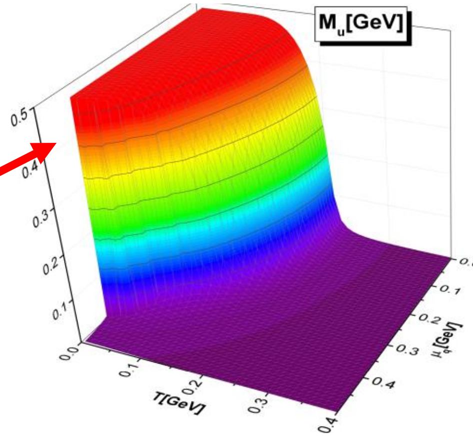
- **PNJL allow for predictions for finite T and μ_B:** D. Fuseau, T. Steinernert, J.Aichelin PRC 101 (2020) 6 065203
- **Parameters fixed, EoS at μ_B =0:**
  HotQCD Phys.Rev. D90 (2014) 094503



- **EoS at high μ_B :**
  pQCD: A.Kurkela, A.Vuorinen, PRL 117 (2016)4 042501



- **CEP**: (T , μ_B) = (110,960) MeV , μ_B/T = 8.73
- **1st order PT** at high μ_B (sudden change of q and meson masses)

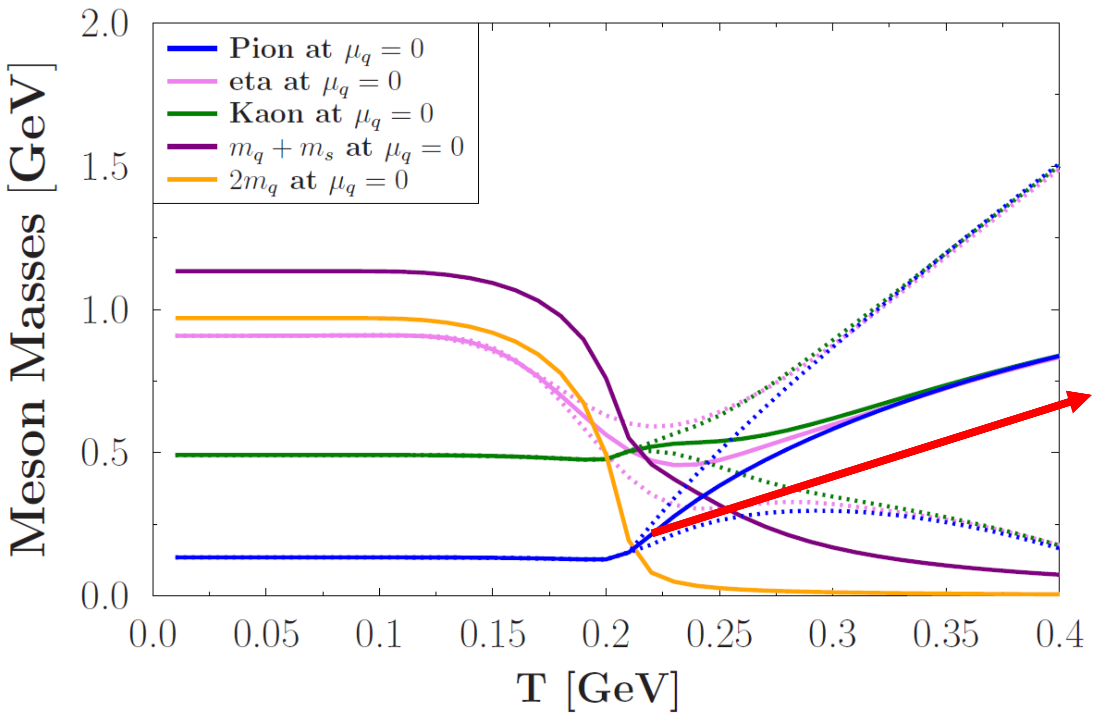


Mesons in PNJL

- **The meson pole mass and the width can be obtained by**

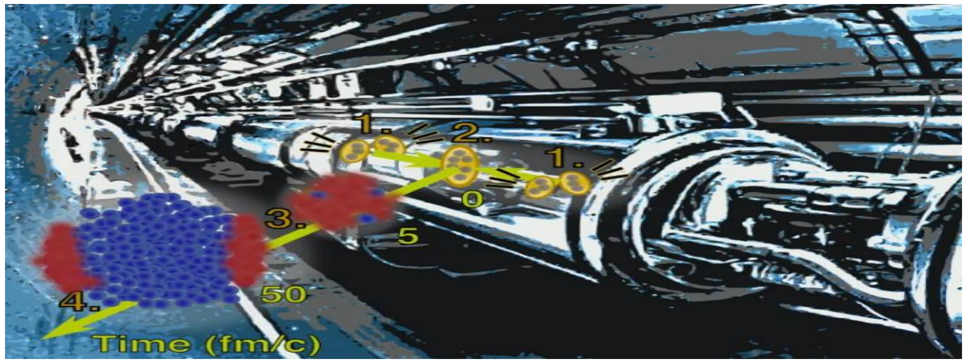
$$1 - 2G_{eff}\ \Pi(p_0 = M_{meson} - i\Gamma_{meson}/2, p = 0) = 0$$

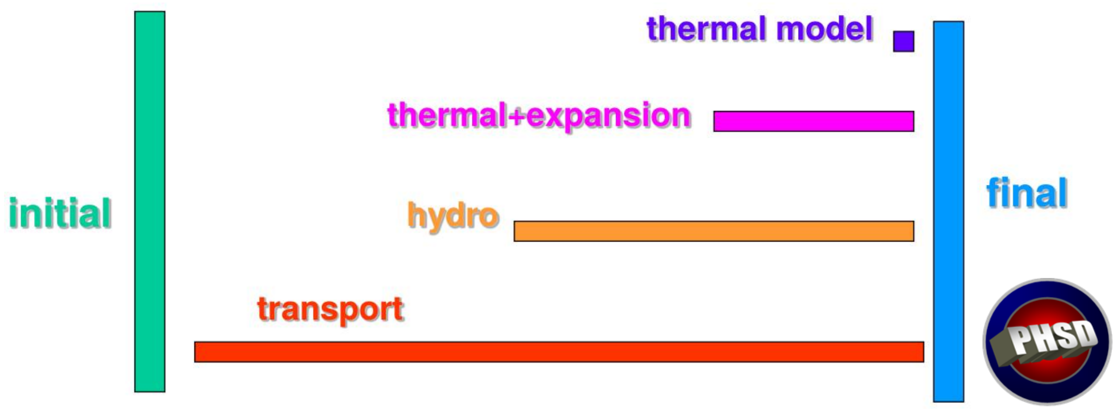
Nf=3 PNJL

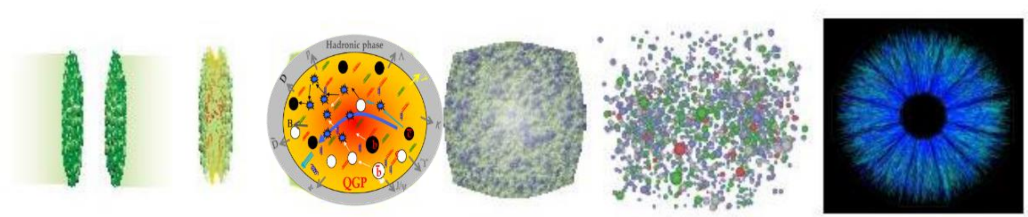


At T=0 good agreement with the physical masses
After T>T$_{Mott}$ mesons become unstable

QGP out-of equilibrium ←→ HIC



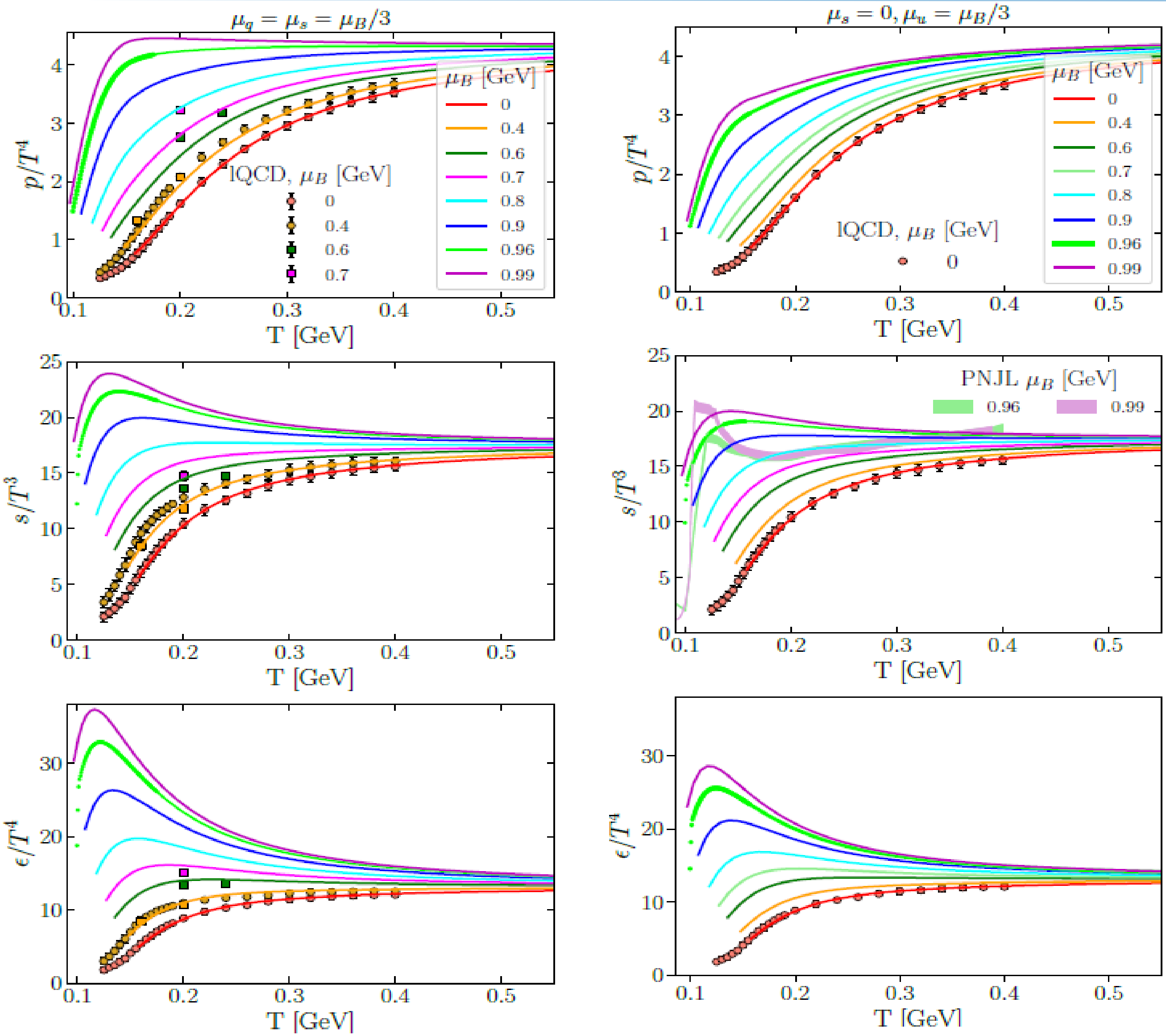


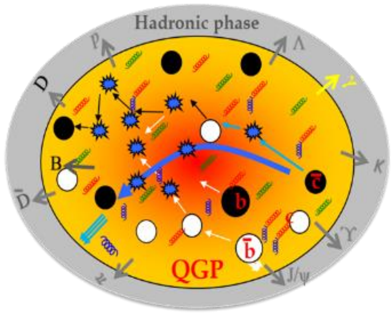


DQPM-CP: Thermodynamic observables

- Input: lQCD entropy density as a
  f(T , $\mu_B = 0$)
- + PNJL s(T/Tc) near the CEP

Input:
lattice EoS
$\mu_B = 0$ (dots)





QGP out-of equilibrium ←→ HIC

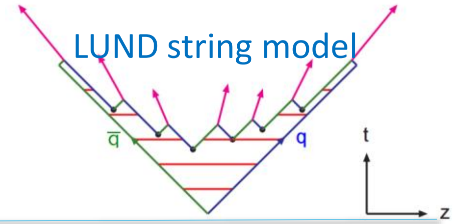
Parton-Hadron-String-Dynamics (PHSD)

- **Transport theory: off-shell transport equations in phase-space representation based on Kadanoff-Baym equations for the partonic and hadronic phase**

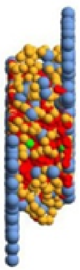
Stages of a collision in the PHSD

Initial A+A collision

- **String formation in primary NN collisions**
  → **decays to pre-hadrons (baryons and mesons)**



Partonic phase



- **Formation of a QGP state if $\varepsilon > \varepsilon_{critical}$:**
  Dissolution of pre-hadrons → DQPM
  → **massive quarks/gluons and mean-field energy**

(quasi-)elastic collisions :

$q+q \to q+q \quad g+q \to g+q$

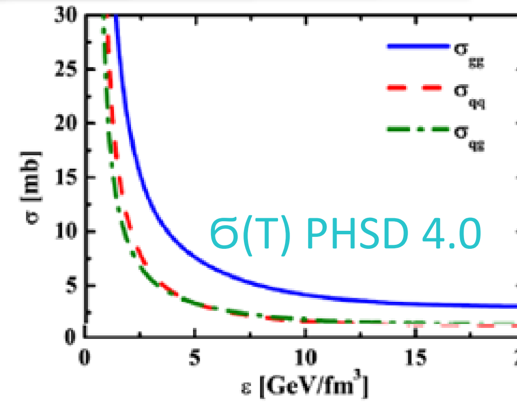
$q+\bar{q} \to q+\bar{q} \quad g+\bar{q} \to g+\bar{q}$

$\bar{q}+\bar{q} \to \bar{q}+\bar{q} \quad g+g \to g+g$

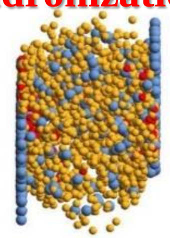
inelastic collisions:

$q+\bar{q} \to g$

$g \to q+\bar{q}$



Hadronization

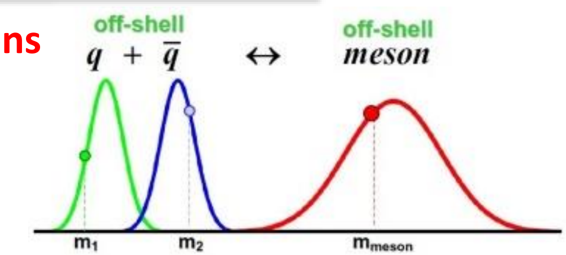


- **Hadronization to colorless off-shell mesons and baryons**

$g \to q+\bar{q}, \quad q+\bar{q} \leftrightarrow meson \ ('string')$

$q+q+q \leftrightarrow baryon \ ('string')$

Strict 4-momentum and quantum number conservation



- **Hadron-string interactions – off-shell HSD**

W. Cassing, E. Bratkovskaya, PRC 78 (2008) 034919; NPA831 (2009) 215;
W. Cassing, EPJ ST 168 (2009) 3

Extraction of (T, μ_B) in PHSD

For **each space-time cell** of the PHSD: $T^{\mu\nu} = \sum_i \frac{p_i^\mu p_i^\nu}{E_i}$ → Diagonalize in LRF → ε^{PHSD}

- Calculate the local energy density ε^{PHSD} and baryon density n_B^{PHSD}

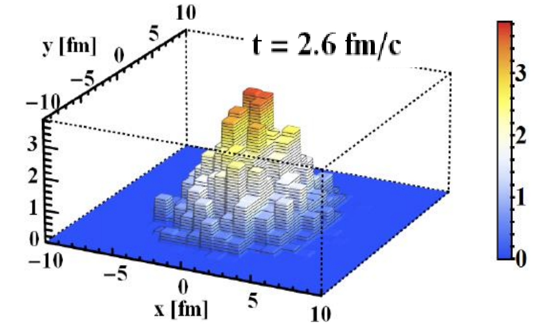
- use lQCD relations (up to 6th order):

$$\begin{cases} \frac{n_B}{T^3} \approx \chi_2^B(T)\left(\frac{\mu_B}{T}\right) + \ldots \\ \Delta\epsilon/T^4 \approx \frac{1}{2}\left(T\frac{\partial\chi_2^B(T)}{\partial T} + 3\chi_2^B(T)\right)\left(\frac{\mu_B}{T}\right)^2 + \ldots \end{cases}$$

Use baryon number susceptibilities χ_n from lQCD

➔ obtain (T, μ_B) by solving the system of coupled equations using ε^{PHSD} and n_B^{PHSD}

$e(x, y, z = 0)$



Input:

ε^{PHSD} and n_B^{PHSD}

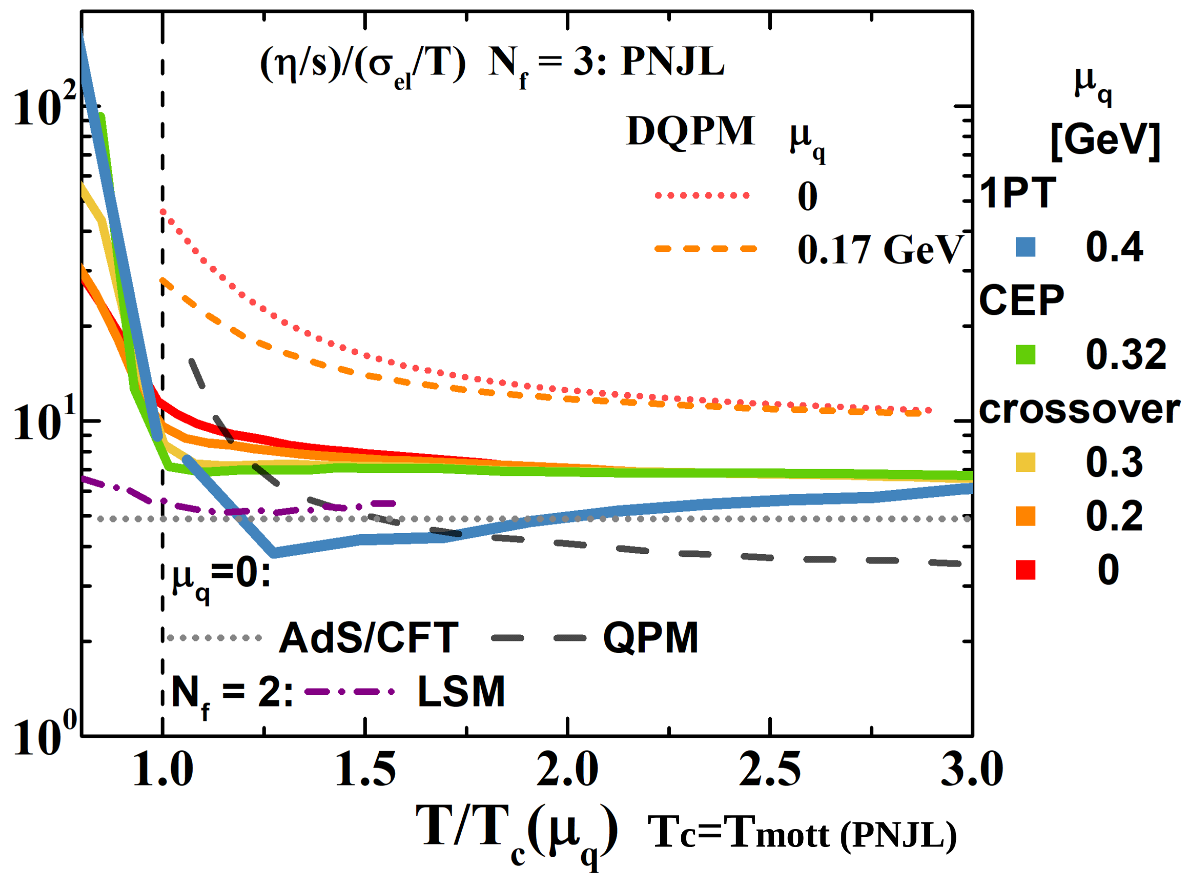
→

Output:

T, μ_B

for details see P. Moreau, O. Soloveva, L. Oliva, T. Song, W. Cassing, E. Bratkovskaya arXiv:1903.10157, PRC 100 (2019) no. 1, 014911

Ratio (η/s)/(σ/T) at finite μ_B



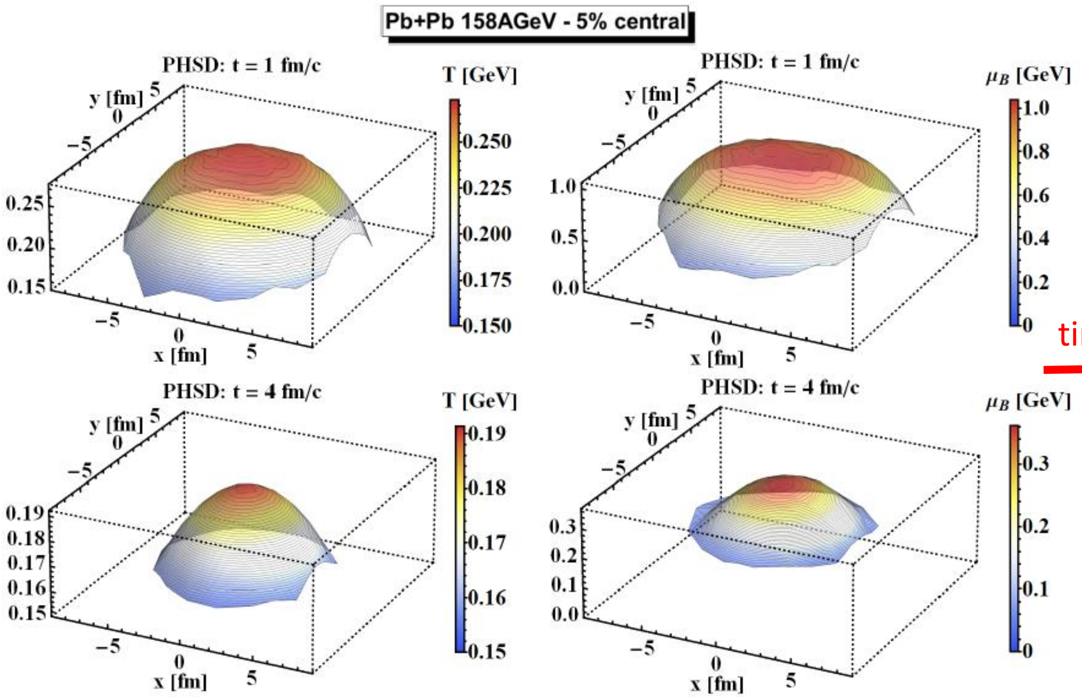
- **ratio (η/s)/(σ/T)** decreases with T , has μ_B depence in the vicinity of the chiral phase transition

PNJL results:arXiv:2011.03505

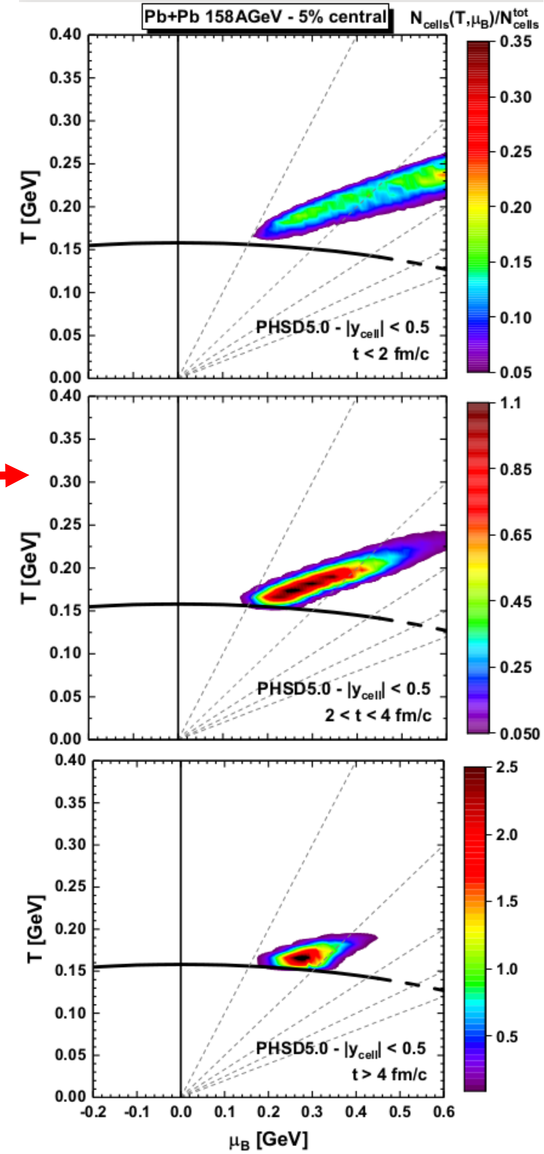
QGP evolution for HIC ($\sqrt{s_{NN}} = 17$ GeV)

The T profile in (x;y) **μ_B profile in (x;y)**

at midrapidity ($|y_{cell}|$< 1) at fixed times(1 and 4 fm/c)



time evolution



for details see P. Moreau, O. Soloveva, L. Oliva, T. Song, W. Cassing, E. Bratkovskaya
arXiv:1903.10157, PRC 100 (2019) no. 1, 014911

Results for HIC: compare 3 versions

- **Comparison between three different results:**

  - **PHSD 4.0 : only isotropic $\sigma(T)$ and ρ(T)**

    partonic cross sections

    parton spectral function (masses and widths)

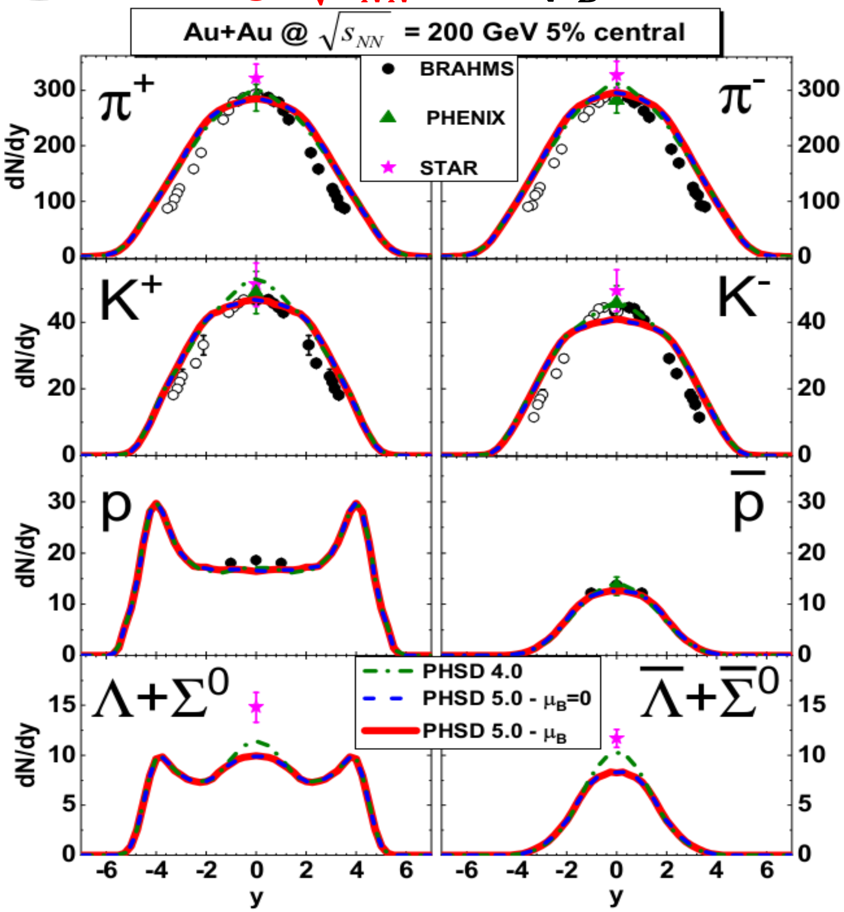
    new PHSD 5 : angular dependence of dσ/d cosθ

  - **PHSD 5.0 : with $\sigma\left(\sqrt{s}, m_1, m_2, T, \mu_B = 0\right)$ and ρ$(T, \mu_B = 0)$**
  - **PHSD 5.0 : with $\sigma\left(\sqrt{s}, m_1, m_2, T, \mu_B\right)$ and ρ(T, μ_B)**

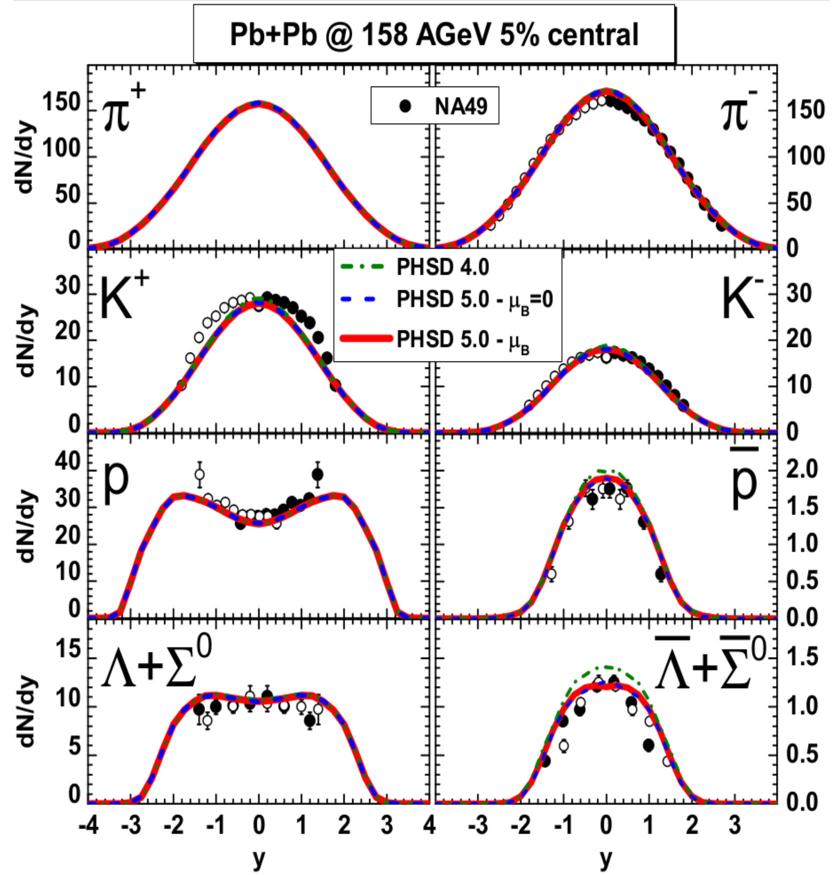
Results for ($\sqrt{s_{NN}} = 200$ GeV vs $\sqrt{s_{NN}} = 17$ GeV)

- No visible effects of μ_B dependence
- Small effect of the angular dependence of **dσ/dcosθ**

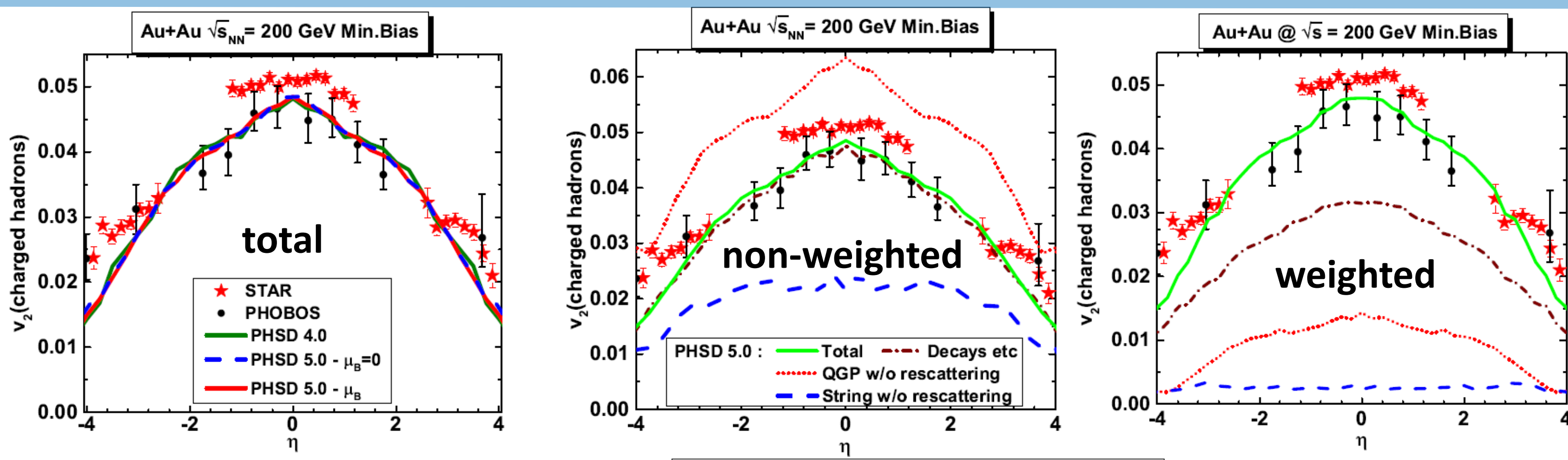
at high $\sqrt{s_{NN}}$ - low μ_B



! QGP fraction is small at low $\sqrt{s_{NN}}$



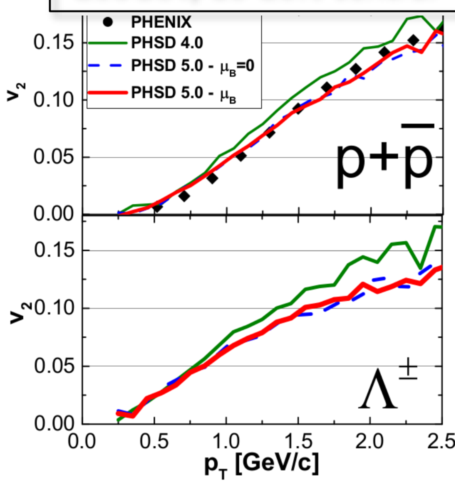
Elliptic flow ($\sqrt{s_{NN}} = 200\ GeV\ vs\ 27 GeV$)

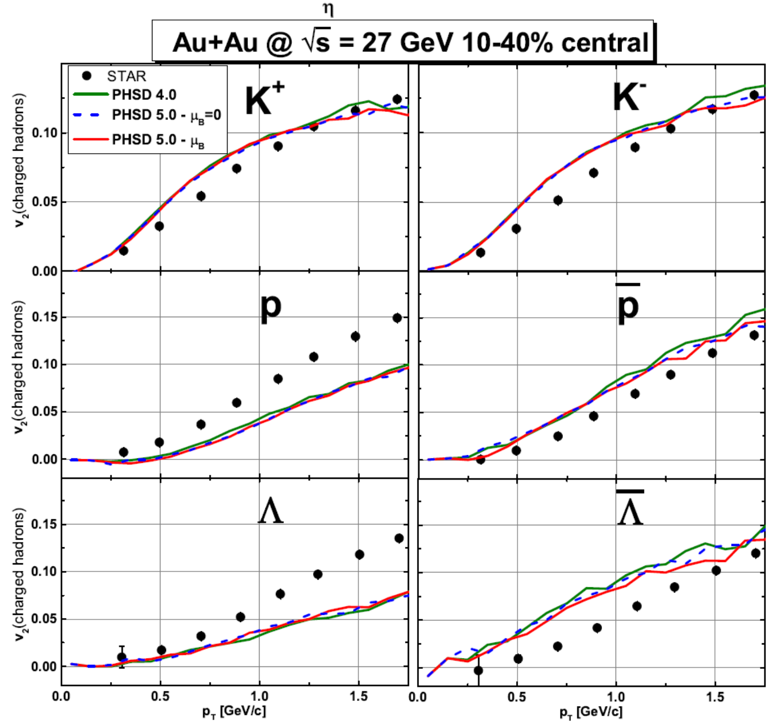


$$\frac{dN}{d\varphi} \propto \left(1+2\sum_{n=1}^{+\infty} v_n \cos\left[n(\varphi-\psi_n)\right]\right)$$

$$v_n = \left\langle \cos n\left(\varphi-\psi_n\right)\right\rangle, \quad n=1,2,3..,$$

200GeV, 10-20% central





- No visible effects of μ_B dependence
- Small effect of the angular dependence of **dσ/dcosθ for v1**

arXiv:2001.05395

Dynamical QuasiParticle model

❖ **How to construct a quasi-particle model:**

1) assume the properties of quasi-particles ➔ some model parameters involved

2) determine the thermal properties of the system from

Grand canonical potential Ω in propagator (D,S) representation (2PI):

$$\beta\Omega[D,S] = \underbrace{\frac{1}{2}\mathrm{Tr}[\ln D^{-1} - \Pi D]}_{\text{bosons}} - \underbrace{\mathrm{Tr}[\ln S^{-1} + \Sigma S]}_{\text{fermions}} + \Phi[D,S]$$

Self-energies:

$$\frac{\delta\Phi}{\delta D} = \frac{1}{2}\Pi$$

$$\frac{\delta\Phi}{\delta S} = -\Sigma$$

Cf. J.P. Blaizot et al, PRD 63 (2001) 065003

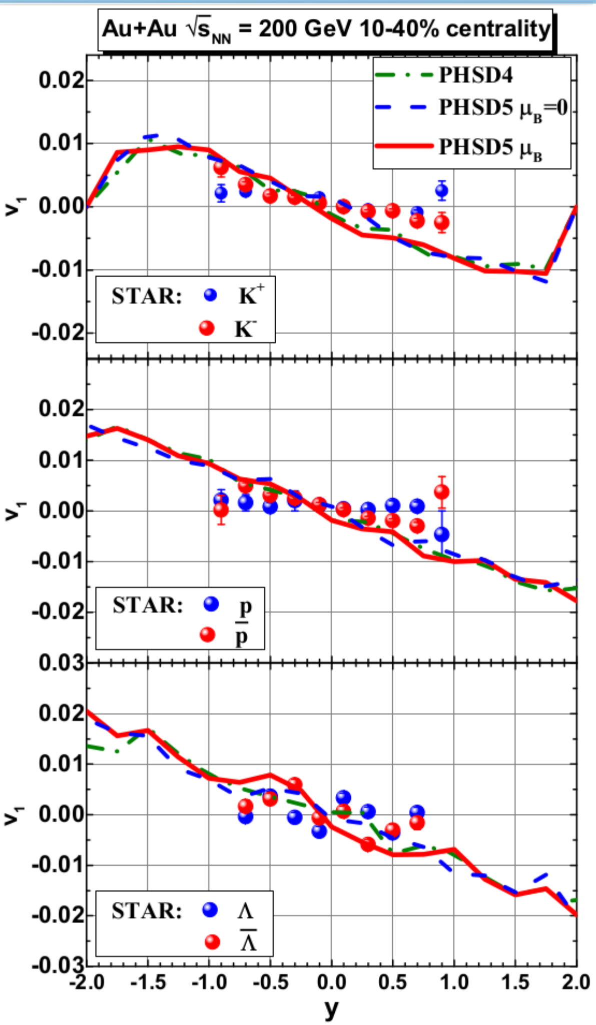
i.e. determine entropy *S*, pressure P etc. for QP:

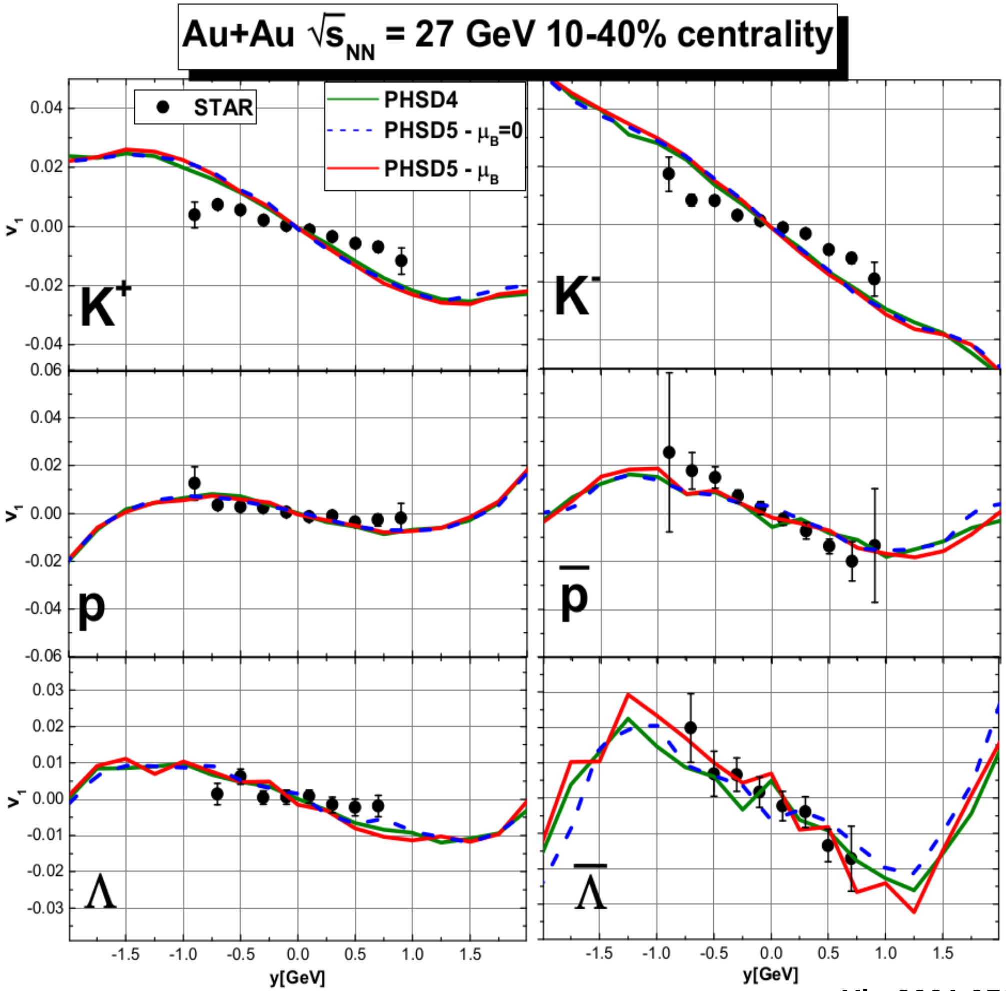
$$\Omega/V = -P \qquad d\Omega = -SdT - PdV - Nd\mu \qquad S = -\frac{\partial\Omega}{\partial T} \qquad N = -\frac{\partial\Omega}{\partial\mu} \qquad P = -\frac{\partial\Omega}{\partial V}$$

3) fit S, P from QP to S, P from lQCD ➔ fix the model parameters

➔ Properties of quasi-particles

Directed flow ($\sqrt{s_{NN}} = 200\ GeV\ vs\ 27\ GeV$)



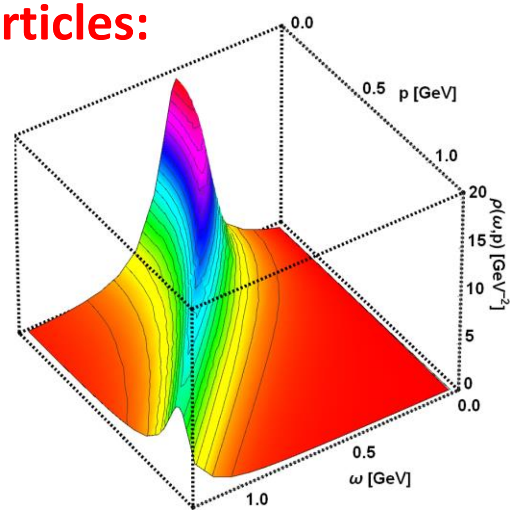


arXiv:2001.05395

No visible effects of μ_B dependence or angular dependence

Dynamical QuasiParticle Model (DQPM)

- **The QGP phase is described in terms of interacting quasiparticles: quarks and gluons with Lorentzian spectral functions:**



$$\rho_j(\omega, \mathbf{p}) = \frac{\gamma_j}{\tilde{E}_j}\left(\frac{1}{(\omega - \tilde{E}_j)^2 + \gamma_j^2} - \frac{1}{(\omega + \tilde{E}_j)^2 + \gamma_j^2}\right)$$

$$\equiv \frac{4\omega\gamma_j}{\left(\omega^2 - \mathbf{p}^2 - M_j^2\right)^2 + 4\gamma_j^2\omega^2}$$

- **Resummed properties of the quasiparticles are specified by scalar complex self-energies:**

gluon propagator: $\Delta^{-1} = P^2 - \Pi$	&	quark propagator $S_q^{-1} = P^2 - \Sigma_q$
gluon self-energy: $\Pi = M_g^2 - i2\gamma_g\omega$	&	quark self-energy: $\Sigma_q = M_q^2 - i2\,\gamma_q\omega$

  - **Real part of the self-energy: thermal mass (M_g, M_q)**
  - **Imaginary part of the self-energy: interaction width of partons (γ_g, γ_q)**

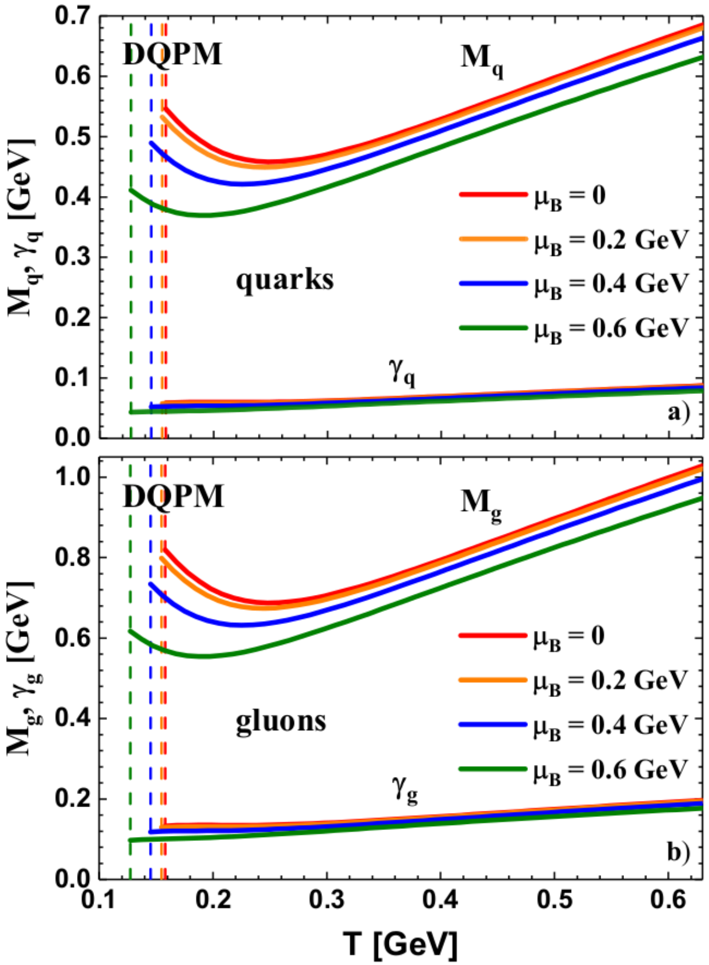
Peshier, Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

Parton properties

- **Modeling of the quark/gluon masses and widths (inspired by HTL calculations)**

$$M^2_{q(\bar{q})}(T,\mu_B) = \frac{N_c^2-1}{8N_c} g^2(T,\mu_B) \left(T^2 + \frac{\mu_q^2}{\pi^2}\right)$$

$$\gamma_{q,g}(T,\mu_B) = \frac{c_{A,F}}{3} \frac{g^2(T,\mu_B) T}{8\pi} \ln\left(\frac{2c}{g^2(T,\mu_B)} + 1\right)$$



- **Only one parameter (c = 14.4) + (T, μ_B)-dependent coupling constant to determine from lattice results**

- **Entropy and baryon density in the quasiparticle limit (G. Baym 1998, Blaizot et al. 2001)**

$$s^{DQPM}(\Pi, \Delta, S_q, \Sigma), n_B^{DQPM}(\Pi, \Delta, S_q, \Sigma)$$

Peshier, Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

DQPM coupling constant

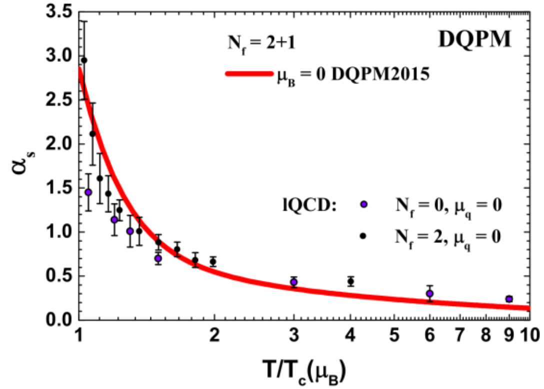
- **Input: entropy density as a f(T , $\mu_B = 0$)**

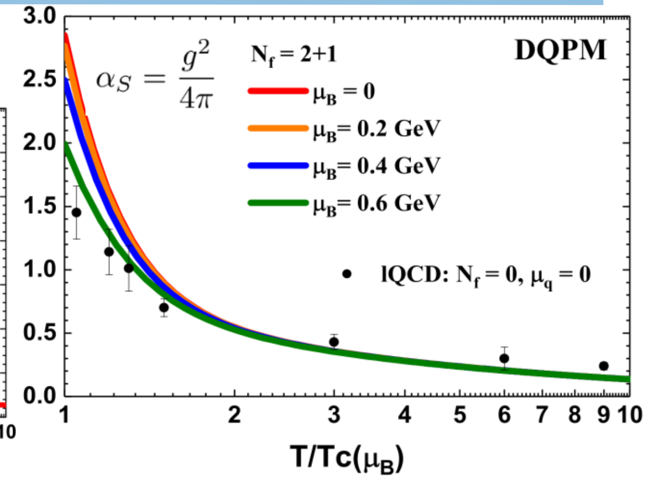
$$g^2(s/s_{SB}) = d\left((s/s_{SB})^e - 1\right)^f$$

$$s_{SB}^{QCD} = 19/9\pi^2 T^3$$

$$s^{DQPM}(\Pi, \Delta, S_q, \Sigma) = s^{lattice}$$

fit S from QP to S from lQCD ⇒ fix the model parameters

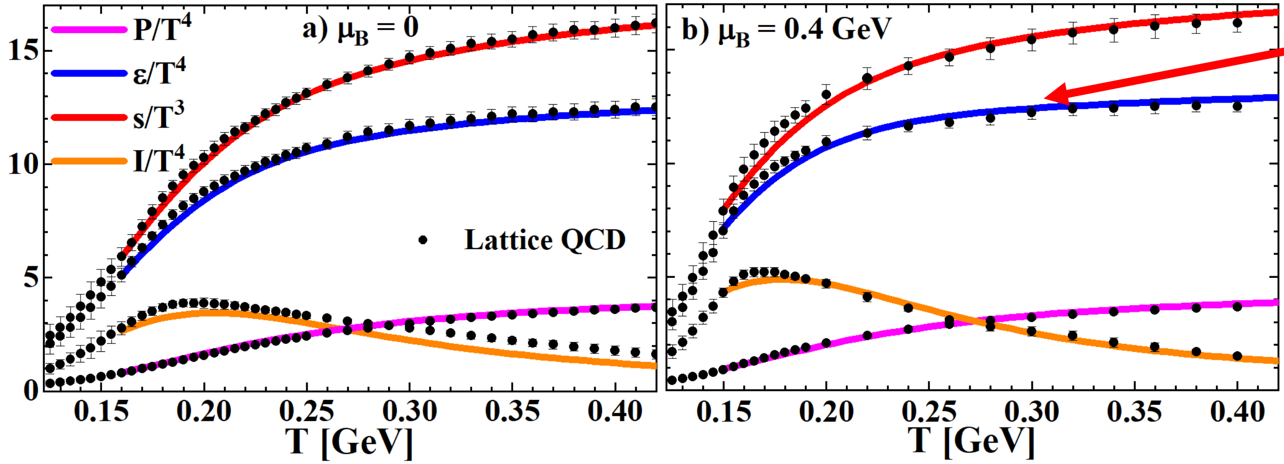




- **Scaling hypothesis at finite $\mu_B \approx 3\mu_q$**

$$g^2(T/T_c, \mu_B) = g^2\left(\frac{T^*}{T_c(\mu_B)}, \mu_B = 0\right)$$ **with the effective temperature** $$T^* = \sqrt{T^2 + \mu_q^2/\pi^2}$$

Input: lattice EoS $\mu_B = 0$ (dots)



Output: (lines) DQPM EoS $\mu_B > 0$

DQPM : Thermodynamics

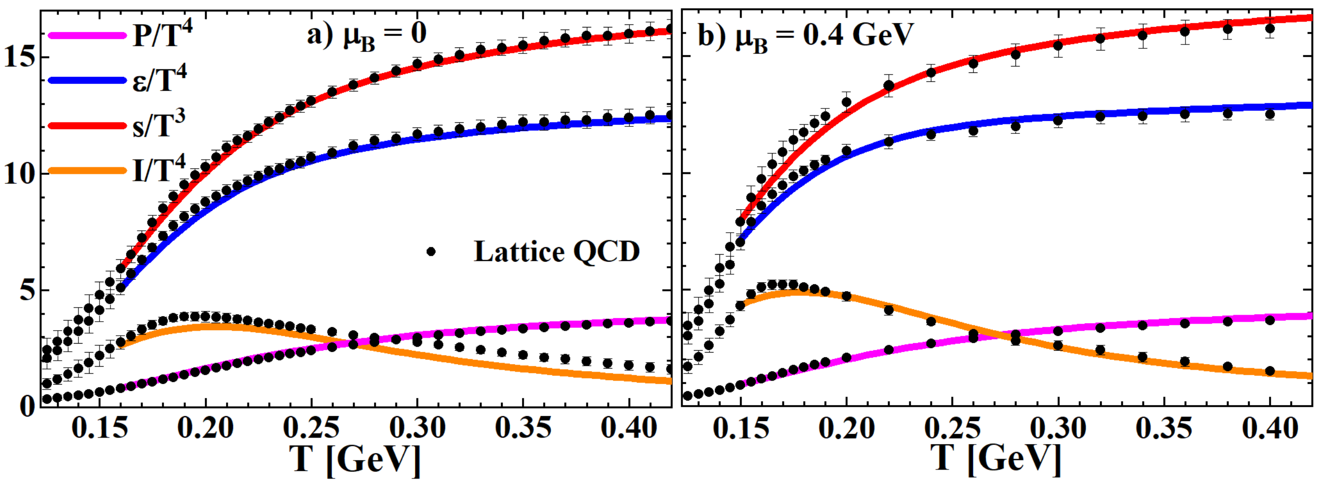
➤ **Entropy and baryon density**

in the quasiparticle limit (G. Baym 1998, Blaizot et al. 2001):

$$s^{dqp} = -\int \frac{d\omega}{2\pi}\frac{d^3p}{(2\pi)^3}\Bigg[d_g\,\frac{\partial n_B}{\partial T}\left(\mathrm{Im}(\ln -\underline{\Delta}^{-1}) + \mathrm{Im}\,\underline{\Pi}\,\mathrm{Re}\,\underline{\Delta}\right) + \sum_{q=u,d,s} d_q\,\frac{\partial n_F(\omega-\mu_q)}{\partial T}\left(\mathrm{Im}(\ln -\underline{S_q^{-1}}) + \mathrm{Im}\,\underline{\Sigma_q}\,\mathrm{Re}\,\underline{S_q}\right) + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}}\,\frac{\partial n_F(\omega+\mu_q)}{\partial T}\left(\mathrm{Im}(\ln -\underline{S_{\bar{q}}^{-1}}) + \mathrm{Im}\,\underline{\Sigma_{\bar{q}}}\,\mathrm{Re}\,\underline{S_{\bar{q}}}\right)\Bigg]$$

$$n^{dqp} = -\int \frac{d\omega}{2\pi}\frac{d^3p}{(2\pi)^3}\Bigg[\sum_{q=u,d,s} d_q\,\frac{\partial n_F(\omega-\mu_q)}{\partial \mu_q}\left(\mathrm{Im}(\ln -\underline{S_q^{-1}}) + \mathrm{Im}\,\underline{\Sigma_q}\,\mathrm{Re}\,\underline{S_q}\right) + \sum_{\bar{q}=\bar{u},\bar{d},\bar{s}} d_{\bar{q}}\,\frac{\partial n_F(\omega+\mu_q)}{\partial \mu_q}\left(\mathrm{Im}(\ln -\underline{S_{\bar{q}}^{-1}}) + \mathrm{Im}\,\underline{\Sigma_{\bar{q}}}\,\mathrm{Re}\,\underline{S_{\bar{q}}}\right)\Bigg]$$

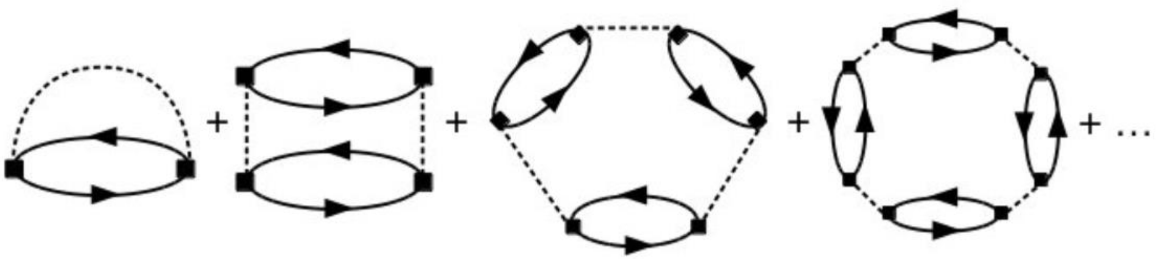
Input:
lattice EoS
$\mu_B = 0$



Output:
DQPM EoS
$\mu_B > 0$

PNJL improvements

- **Next to leading order in Nc(O$(1/Nc)^0$) of the grand-canonical potential : presence of the mesons below Tc**



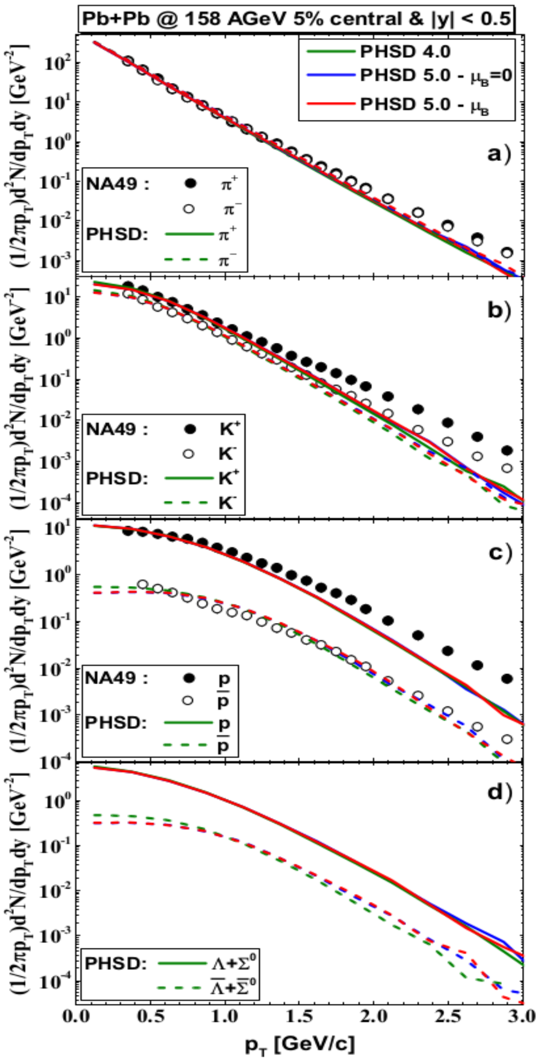
J. M. Torres-Rincon, J. Aichelin PRC 96 (2017) 4 045205

- **Modification of the gluon potential due to the presence of the quark**

$$T_{phen}(T) = a + bT + cT^2 + dT^3 + e\frac{1}{T}$$

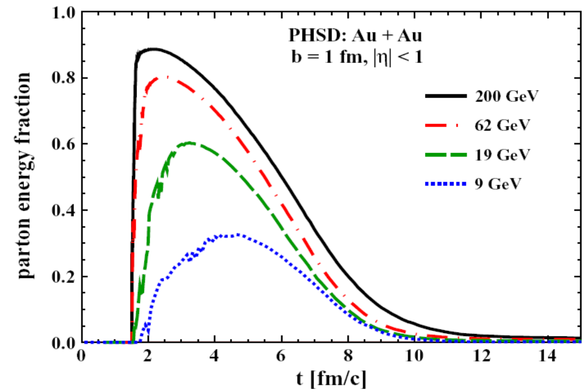
D. Fuseau, T. Steinernert, J. Aichelin PRC 101 (2020) 6 065203

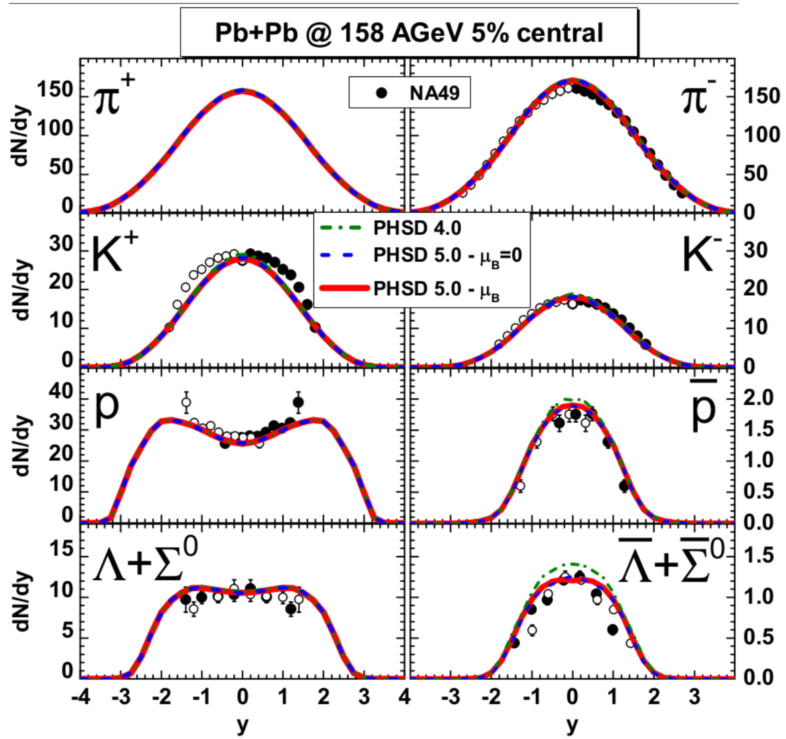
Results for HIC ($\sqrt{s_{NN}} = 17$ GeV)



High-μ_B regions are probed at low $\sqrt{s_{NN}}$ or high rapidity regions

But, QGP fraction is small at low $\sqrt{s_{NN}}$





Transport coefficients: approaches

- **Kubo formalism: transport coefficients are expressed through correlation functions of stress-energy tensor**

used in lattice QCD, transport approaches(hadrons), effective models

$$\eta = \frac{1}{20} \lim_{\omega \to 0} \frac{1}{\omega} \int d^4x \, e^{i\omega t} \left\langle \left[\mathcal{S}^{ij}(t, \mathbf{x}), \, \mathcal{S}^{ij}(0, \mathbf{0}) \right] \right\rangle \theta(t) \qquad \mathcal{S}^{ij} = T^{ij} - \delta^{ij}\mathcal{P}$$

$$\zeta = \frac{1}{2} \lim_{\omega \to 0} \frac{1}{\omega} \int d^4x \, e^{i\omega t} \langle [\mathcal{P}(t, \mathbf{x}), \, \mathcal{P}(0, \mathbf{0})] \rangle \theta(t) \qquad \mathcal{P} = -\tfrac{1}{3} T^i{}_i$$

R. Lang and W. Weise, EPJ. A 50, 63 (2014) (NJL model)
A. Harutyunyan et al,PRD 95, 114021, (2017)

Kinetic theory:

- **Relaxation time approximation(RTA) : consider relaxation time** $\frac{d f_a^{\text{eq}}}{dt} = \mathcal{C}_a = -\frac{f_a^{\text{eq}} \phi_a}{\tau_a}$

P. Chakraborty and J. I. Kapusta, PRC 83,014906 (2011)

- **Chapman-Enskog : expand the distribution in terms of the Knudsen number**

J. A. Fotakis et al, PRD 101 (2020) 7, 076007 (HRG)

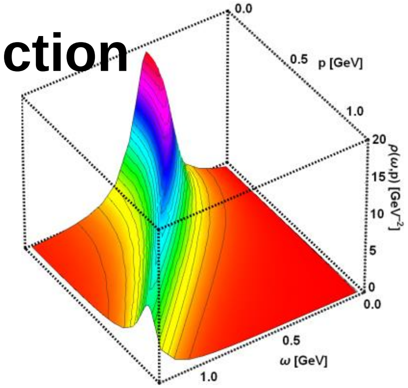
And more!

Holographic models: AdS/CFT correspondence

D. T. Son and A. O. Starinets, JHEP 0603, 052 (2006)
M. Attems et al , JHEP 10 (2016), 155.

DQPM : Thermodynamics

- Masses, widths, coupling are fixed ⟶ spectral function



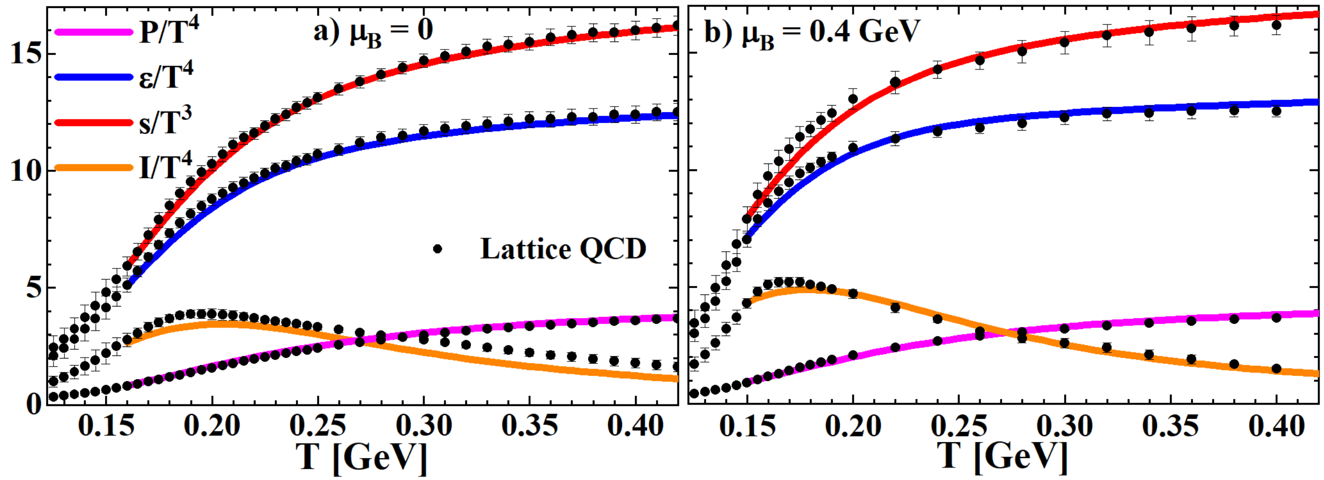
- Entropy and baryon density in the QP limit

$$s^{DQPM}(\Pi, \Delta, S_q, \Sigma), n_B^{DQPM}(\Pi, \Delta, S_q, \Sigma)$$

$$P(T, \mu_B) = P(T_0, 0)$$

$$+ \int_{T_0}^{T} s(T', 0)\ dT' + \int_0^{\mu_B} n_B(T, \mu'_B)\ d\mu'_B \longrightarrow \quad \epsilon = Ts - P + \mu_B n_B$$

Input:
lattice EoS
$\mu_B = 0$

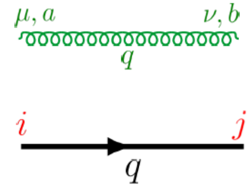


Output:
DQPM EoS
$\mu_B > 0$

Partonic interactions: matrix elements

DQPM partonic cross sections → leading order diagrams

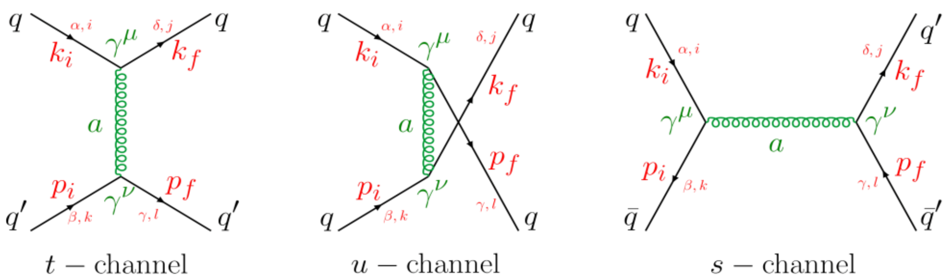
- **Propagators for massive bosons and fermions:**



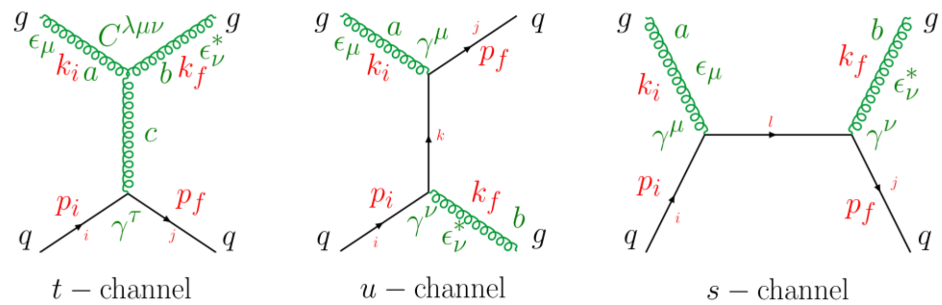
$$= -i\delta_{ab}\frac{g^{\mu\nu} - q^\mu q^\nu / M_g^2}{q^2 - M_g^2 + 2i\gamma_g q_0}$$

$$= i\delta_{ij}\frac{\not{q} + M_q}{q^2 - M_q^2 + 2i\gamma_q q_0}$$

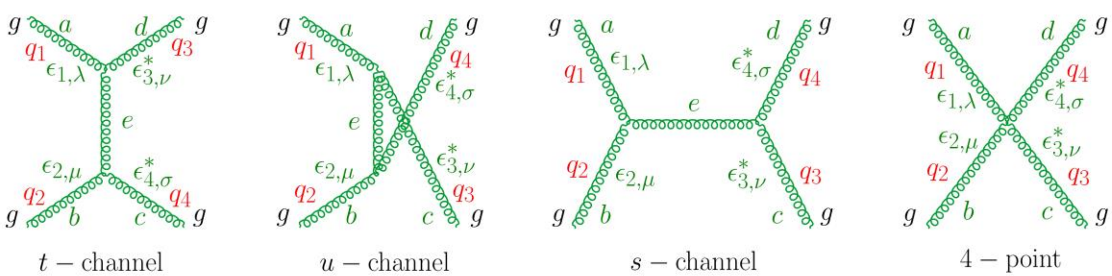
qq' → qq' scattering



gq → gq scattering

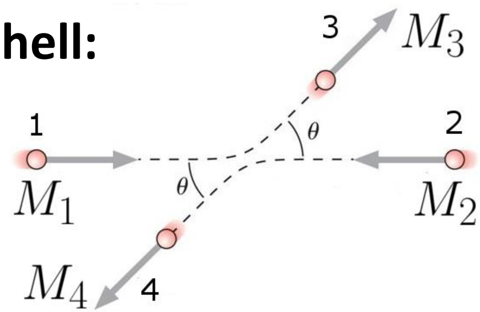


gg→ gg scattering



Total cross sections

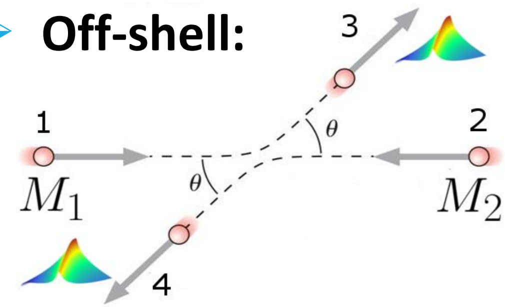
- **On-shell:**



Initial masses: pole masses

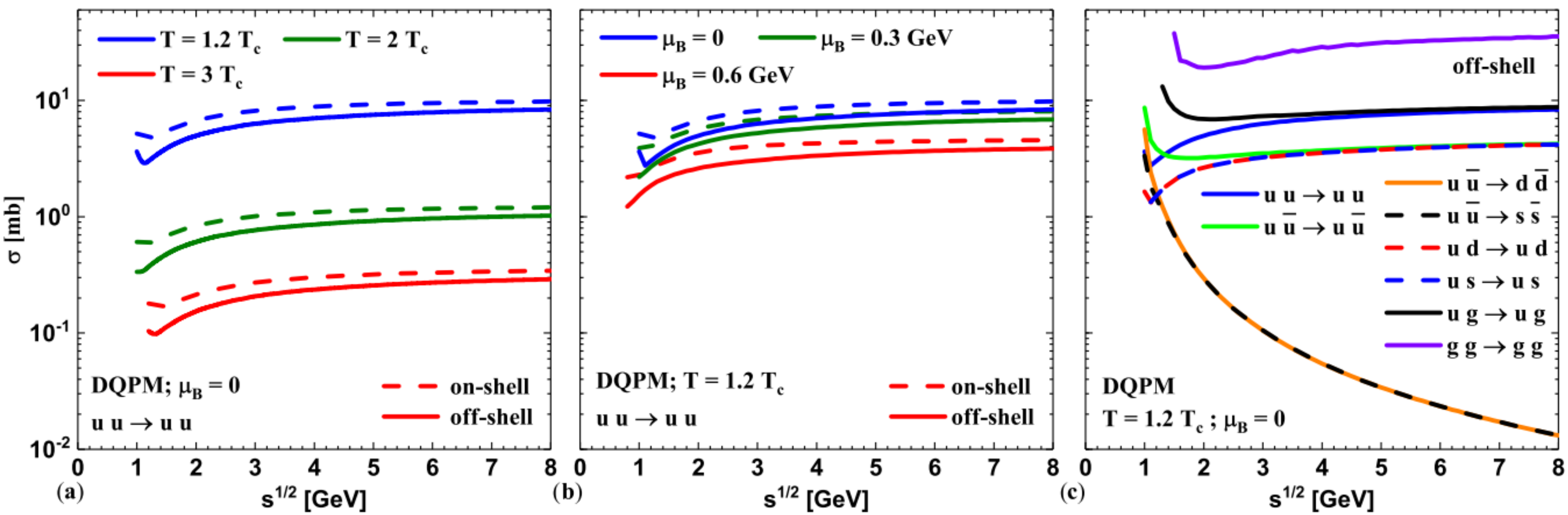
Final masses: pole masses

- **Off-shell:**



Initial masses: pole masses

Final masses: integrated over spectral functions

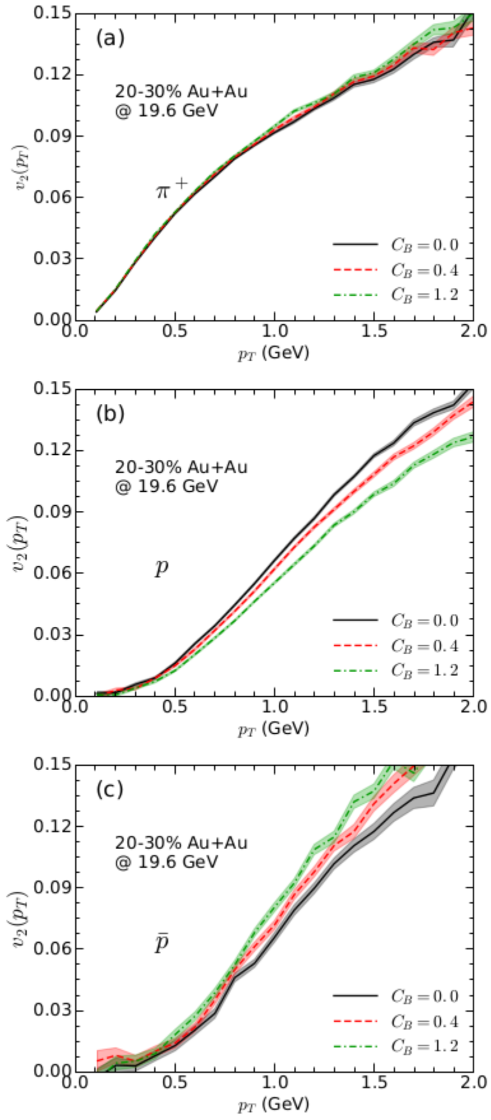


Transport coefficients: baryon diffusion coefficient

- **Relaxation Time Approximation**

$$\kappa_B^{\mathrm{RTA}}(T,\mu_B) = \frac{1}{3}\sum_{i=q,\bar{q}}\int \frac{d^3p}{(2\pi)^3}\mathbf{p}^4\tau_i(\mathbf{p},T,\mu_B)$$
$$\frac{d_i(1\pm f_i)f_i}{E_i^2}\left(b_a - \frac{n_B E_i}{\epsilon + p}\right)^2$$

Baryon diffusion depends on the baryon charge->
Reduces proton v2 and increases antiproton v2



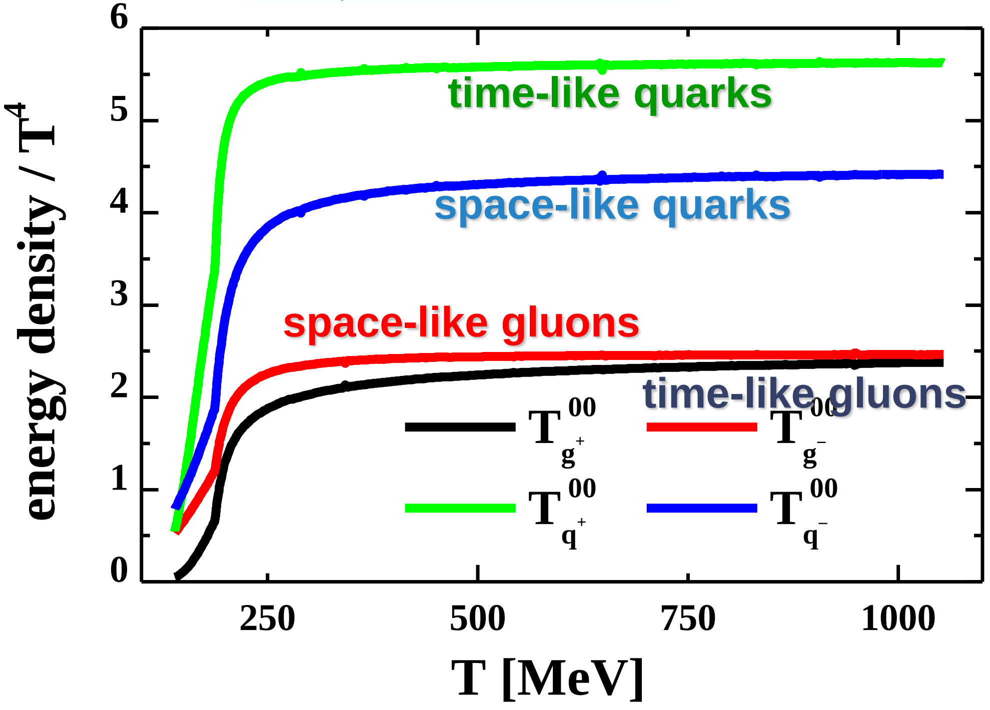
G. S. Denicol et al, PRC 98. 034916 (2018)

DQPM: Time-like and ‚space-like' energy densities

Time/space-like part of energy-momentum tensor $T_{\mu\nu}$ for quarks and gluons:

$$T^{\pm}_{00,x}(T) = \tilde{T}r^{\pm}_{x}\ \omega$$

x: gluons, quarks, antiquarks



- space-like energy density of quarks and gluons = ~1/3 of total energy density
- space-like energy density dominates for gluons
- space-like parts are identified with potential energy densities

Cassing, NPA 791 (2007) 365: NPA 793 (2007)

DQPM EoS at finite (T, μ_B)

- **Taylor series of thermodynamic quantities in terms of (μ_B/T)**
- **With the 6[nd] order susceptibility. Example 2[nd] order:**

$$\Delta P/T^4 = \frac{P(T,\mu_B) - P(T,0)}{T^4} \approx \frac{1}{2}\chi_2^B(T)\left(\frac{\mu_B}{T}\right)^2$$

$$\frac{n_B}{T^3} = \left.\frac{\partial(P/T^4)}{\partial(\mu_B/T)}\right|_T \approx \chi_2^B(T)\left(\frac{\mu_B}{T}\right)$$

$$\Delta s/T^3 = \frac{s(T,\mu_B) - s(T,0)}{T^3} = \frac{1}{T^3}\left.\frac{\partial \Delta P}{\partial T}\right|_{\mu_B}$$

$$= T\left.\frac{\partial(\Delta P/T^4)}{\partial T}\right|_{\mu_B} + 4(\Delta P/T^4) \approx \frac{1}{2}\left(T\frac{\partial \chi_2^B(T)}{\partial T} + 2\chi_2^B(T)\right)\left(\frac{\mu_B}{T}\right)^2$$

$$\Delta\epsilon/T^4 = \frac{\epsilon(T,\mu_B) - \epsilon(T,0)}{T^4}$$

$$= \Delta s/T^3 - \Delta P/T^4 + \left(\frac{\mu_B}{T}\right)\frac{n_B}{T^3} \approx \frac{1}{2}\left(T\frac{\partial \chi_2^B(T)}{\partial T} + 3\chi_2^B(T)\right)\left(\frac{\mu_B}{T}\right)^2$$

A. Bazavov, Phys. Rev. D 96, 054504(2017)

Extraction of (T, μ_B) in PHSD

For each space-time cell of the PHSD:

- **Calculate the local energy density ε^{PHSD} and baryon density n_B^{PHSD}**

1) Energy density ε^{PHSD}

In each space-time cell of the PHSD, the energy-momentum tensor is calculated by the formula:

$$T^{\mu\nu} = \sum_i \frac{p_i^\mu p_i^\nu}{E_i}$$

Diagonalization of the energy-momentum tensor to get the energy density and pressure components expressed in the local rest frame (LRF)

$$T^{\mu\nu} = \begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix} \Rightarrow \begin{pmatrix} \epsilon^{LRF} & 0 & 0 & 0 \\ 0 & P_x^{LRF} & 0 & 0 \\ 0 & 0 & P_y^{LRF} & 0 \\ 0 & 0 & 0 & P_z^{LRF} \end{pmatrix} \Rightarrow \varepsilon^{PHSD}$$

Xu et al., Phys.Rev. C96 (2017), 024902

2) Net-baryon density n_B^{PHSD} ⇒ $n_B = \gamma_E \left(J_B^0 - \vec{\beta}_E \cdot \vec{J}_B\right) = \frac{J_B^0}{\gamma_E}$

Net-baryon current: $J_B^\mu = \sum_i \frac{p_i^\mu}{E_i} \frac{(q_i - \bar{q}_i)}{3}$

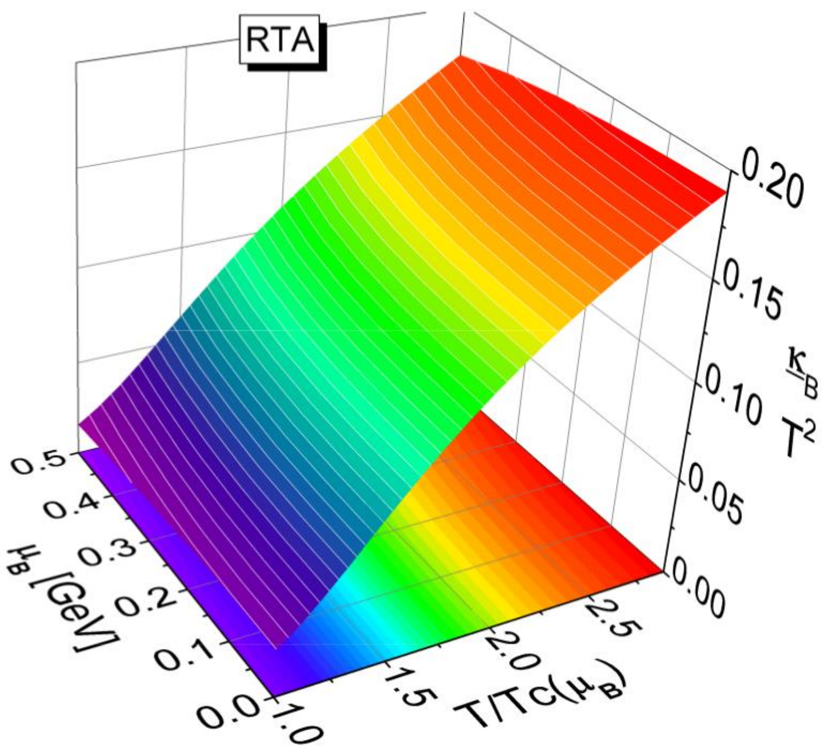
Eckart velocity $\vec{\beta}_E = \vec{J}_B / J_B^0$

Transport coefficients: baryon diffusion coefficient

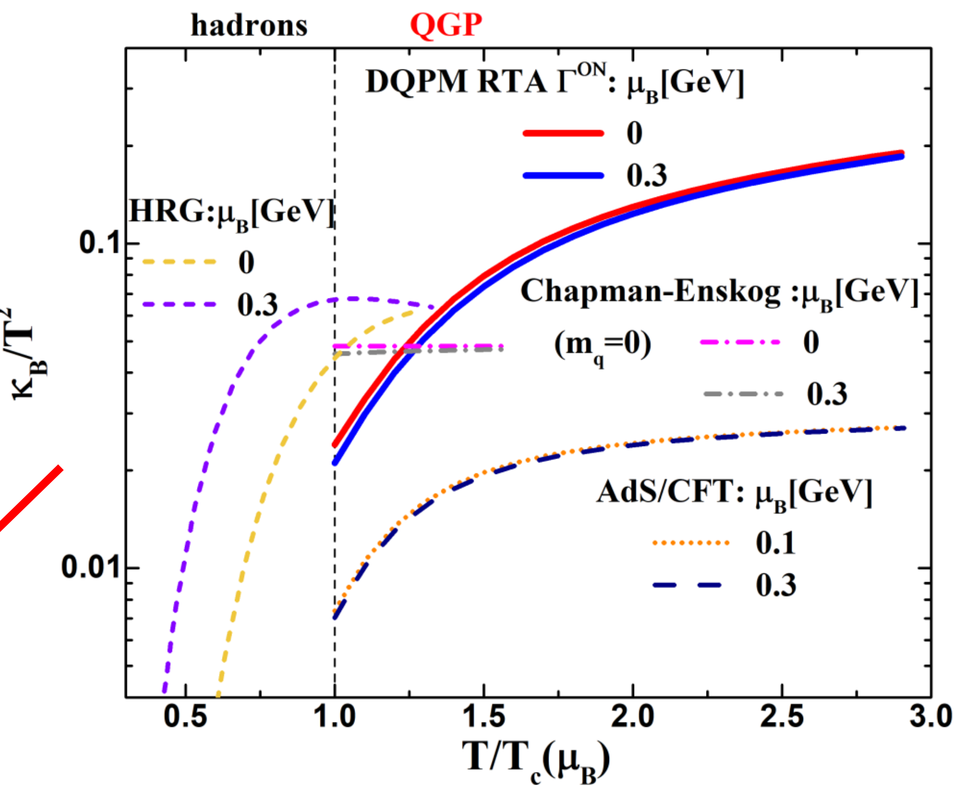
$$\kappa_B^{\rm RTA}(T,\mu_B) = \frac{1}{3}\sum_{i=q,\bar q}\int \frac{d^3p}{(2\pi)^3}\mathbf{p}^4 \tau_i(\mathbf{p},T,\mu_B) \frac{d_i(1\pm f_i)f_i}{E_i^2}\left(b_a - \frac{n_B E_i}{\epsilon+p}\right)^2$$

Relaxation times

DQPM EoS



μ_B



HRG: J. A. Fotakis et al, PRD 101 (2020) 7, 076007
AdS/CFT: T. Son and A. O. Starinets, JHEP 0603, 052 (2006)

Extraction of (T, μ_B) in PHSD

- **In each space-time cell of the PHSD, the energy-momentum tensor is calculated by the formula:**

$$T^{\mu\nu} = \sum_i \frac{p_i^\mu p_i^\nu}{E_i}$$

- **Diagonalization of the energy-momentum tensor to get the energy density and pressure components expressed in the local rest frame (LRF)**

$$T^{\mu\nu} = \begin{pmatrix} T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \end{pmatrix} \Rightarrow \begin{pmatrix} \epsilon^{LRF} & 0 & 0 & 0 \\ 0 & P_x^{LRF} & 0 & 0 \\ 0 & 0 & P_y^{LRF} & 0 \\ 0 & 0 & 0 & P_z^{LRF} \end{pmatrix}$$

Xu et al., Phys.Rev. C96 (2017), 024902

For each space-time cell of the PHSD:

- **Calculate the local energy density ε^{PHSD} and baryon density n_B^{PHSD}**
- **use lQCD relations (up to 6th order):**

$$\begin{cases} \frac{n_B}{T^3} \approx \chi_2^B(T) \left(\frac{\mu_B}{T}\right) + \ldots \\ \Delta\epsilon/T^4 \approx \frac{1}{2}\left(T\frac{\partial \chi_2^B(T)}{\partial T} + 3\chi_2^B(T)\right)\left(\frac{\mu_B}{T}\right)^2 + \ldots \end{cases}$$

➔ **obtain (T, μ_B) by solving the system of coupled equations using ε^{PHSD} and n_B^{PHSD}**

DQPM: Time-like and space-like quantities

Separate time-like and space-like single-particle quantities by Θ(+P²), Θ(-P²):

$$\tilde{\mathrm{Tr}}_g^{\pm} \cdots = d_g \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3}\, 2\omega\, \rho_g(\omega)\, \Theta(\omega)\, n_B(\omega/T)\ \underline{\Theta(\pm P^2)} \cdots \quad \text{gluons}$$

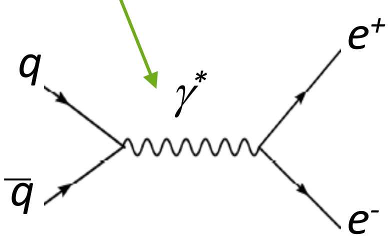
$$\tilde{\mathrm{Tr}}_q^{\pm} \cdots = d_q \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3}\, 2\omega\, \rho_q(\omega)\, \Theta(\omega)\, n_F((\omega-\mu_q)/T)\ \underline{\Theta(\pm P^2)} \cdots \quad \text{quarks}$$

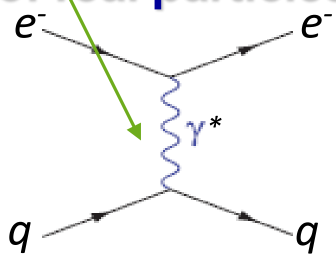
$$\tilde{\mathrm{Tr}}_{\bar q}^{\pm} \cdots = d_{\bar q} \int \frac{d\omega}{2\pi} \frac{d^3p}{(2\pi)^3}\, 2\omega\, \rho_{\bar q}(\omega)\, \Theta(\omega)\, n_F((\omega+\mu_q)/T)\ \underline{\Theta(\pm P^2)} \cdots \quad \text{antiquarks}$$

Time-like: Θ(+P²): particles may decay to real particles or interact

Space-like: Θ(-P²): particles are virtuell and appear as exchange quanta in interaction processes of real particles

Examples:





Cassing, NPA 791 (2007) 365: NPA 793 (2007)

DQPM: Mean-field potential for quasiparticles

Space-like part of energy-momentum tensor $T_{\mu\nu}$ defines the potential energy density:

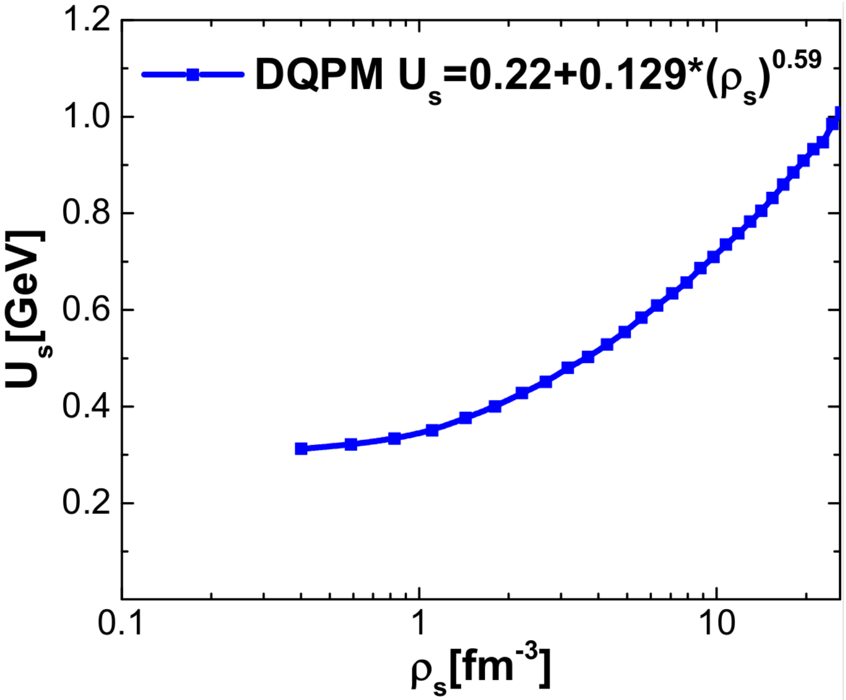
$$V_p(T,\mu_q) = T^{00}_{g-}(T,\mu_q) + T^{00}_{q-}(T,\mu_q) + T^{00}_{\bar{q}-}(T,\mu_q)$$

space-like gluons — space-like quarks+antiquarks

➔ **mean-field scalar potential (1PI) for quarks and gluons (U_q, U_g) vs scalar density ρ_S:**

$$U_s(\rho_s) = \frac{dV_p(\rho_s)}{d\rho_s}$$

$Uq=Us, \; Ug \sim 2Us$



Quasiparticle potentials (U_q, U_g) are repulsive !

➔ **the force acting on a quasiparticle j:**

$$F \sim \; M_j/E_j \nabla U_s(x) = M_j/E_j \; dU_s/d\rho_s \; \nabla\rho_s(x)$$
$$j = g, q, \bar{q}$$

➔ **accelerates particles**

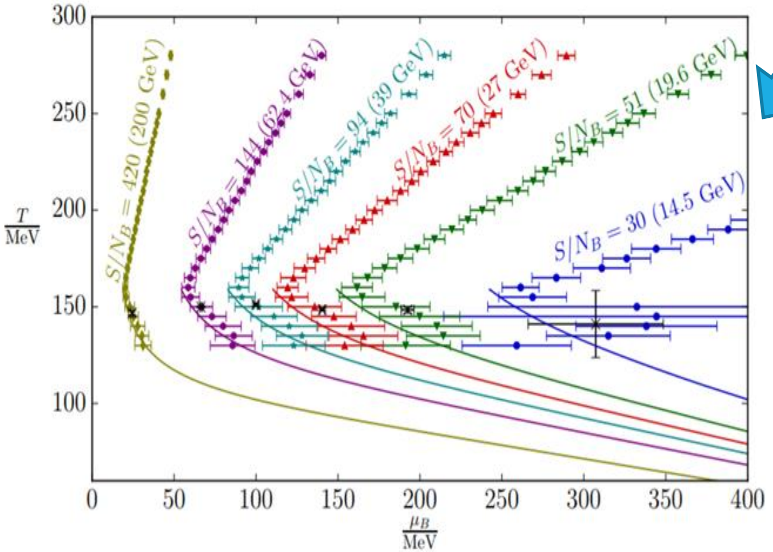
Cassing, NPA 791 (2007) 365: NPA 793 (2007)

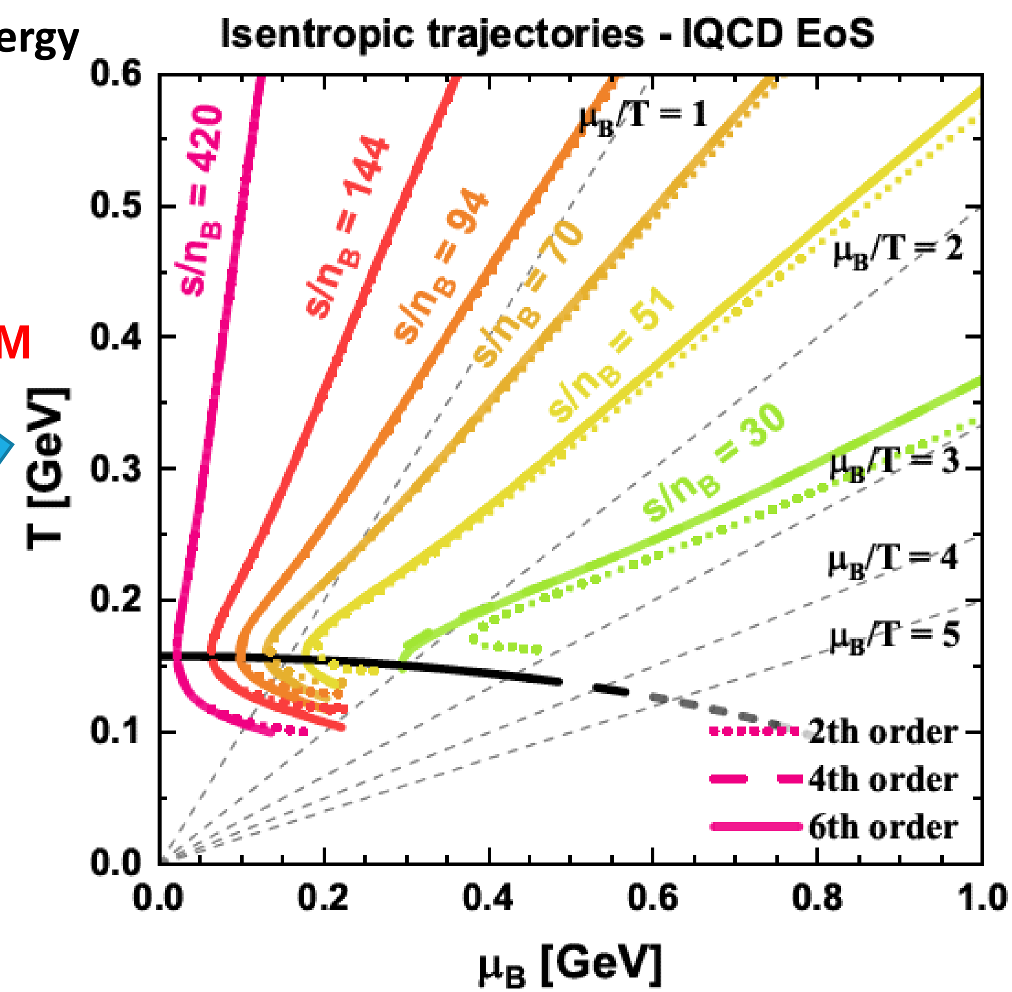
Isentropic trajectories for (T, μ_B)

- Correspondance $s/n_B \leftrightarrow$ collisional energy

s/n_B = 420 ↔ 200 GeV
= 144 ↔ 62.4 GeV
= 94 ↔ 39 GeV
= 70 ↔ 15 GeV
= 51 ↔ 19.6 GeV
= 30 ↔ 14.5 GeV

DQPM





- Safe for $(\mu_B/T) < 3$

lQCD: WB, PoS CPOD2017 (2018) 032

P. Moreau et al., arXiv:1903.10157, PRC (2019)

Energy-momentum tensor in PHSD

- **Diagonalization of the energy-momentum tensor to get the energy density and pressure components expressed in the local rest frame (LRF)**

$$T^{\mu\nu}\ (x_\nu)_i = \lambda_i\ (x^\mu)_i = \lambda_i\ g^{\mu\nu}\ (x_\nu)_i$$

- **Landau-matching condition:** Xu et al., Phys.Rev. C96 (2017), 024902

$$T^{\mu\nu}u_\nu = \epsilon u^\mu = (\epsilon g^{\mu\nu})u_\nu$$

- **Evaluation of the characteristic polynomial:**

$$P(\lambda) = \begin{vmatrix} T^{00}-\lambda & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11}+\lambda & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22}+\lambda & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33}+\lambda \end{vmatrix}$$

- **The four solutions λ_i are identified to $(e, -P_1, -P_2, -P_3)$**

The pressure components P_i do not necessarily correspond to (P_x, P_y, P_z)

Transport coefficients: shear viscosity

- **Kubo formalism**

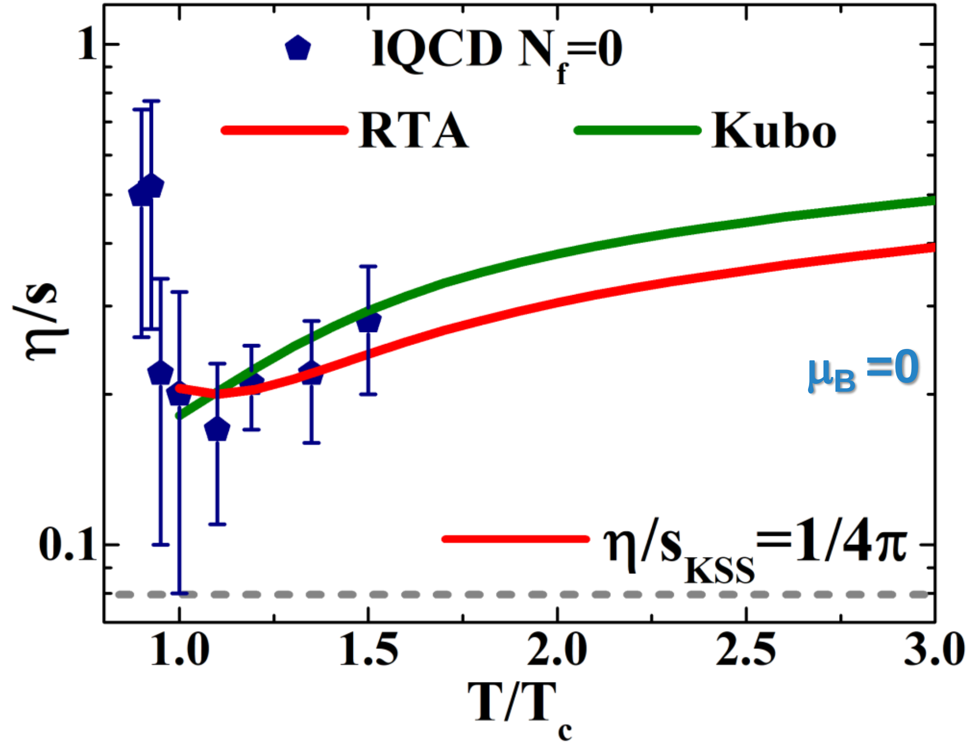
$$\eta^{\text{Kubo}}(T,\mu_q) = -\int \frac{d^4p}{(2\pi)^4}\, p_x^2 p_y^2 \sum_{i=q,\bar{q},g} d_i\, \frac{\partial f_i(\omega)}{\partial \omega}\, \rho_i(\omega,\mathbf{p})^2$$

$$= \frac{1}{15T}\int \frac{d^4p}{(2\pi)^4}\, \mathbf{p}^4 \sum_{i=q,\bar{q},g} d_i\, \big((1\pm f_i(\omega)) f_i(\omega)\big)\, \rho_i(\omega,\mathbf{p})^2$$

- **Relaxation Time Approximation**

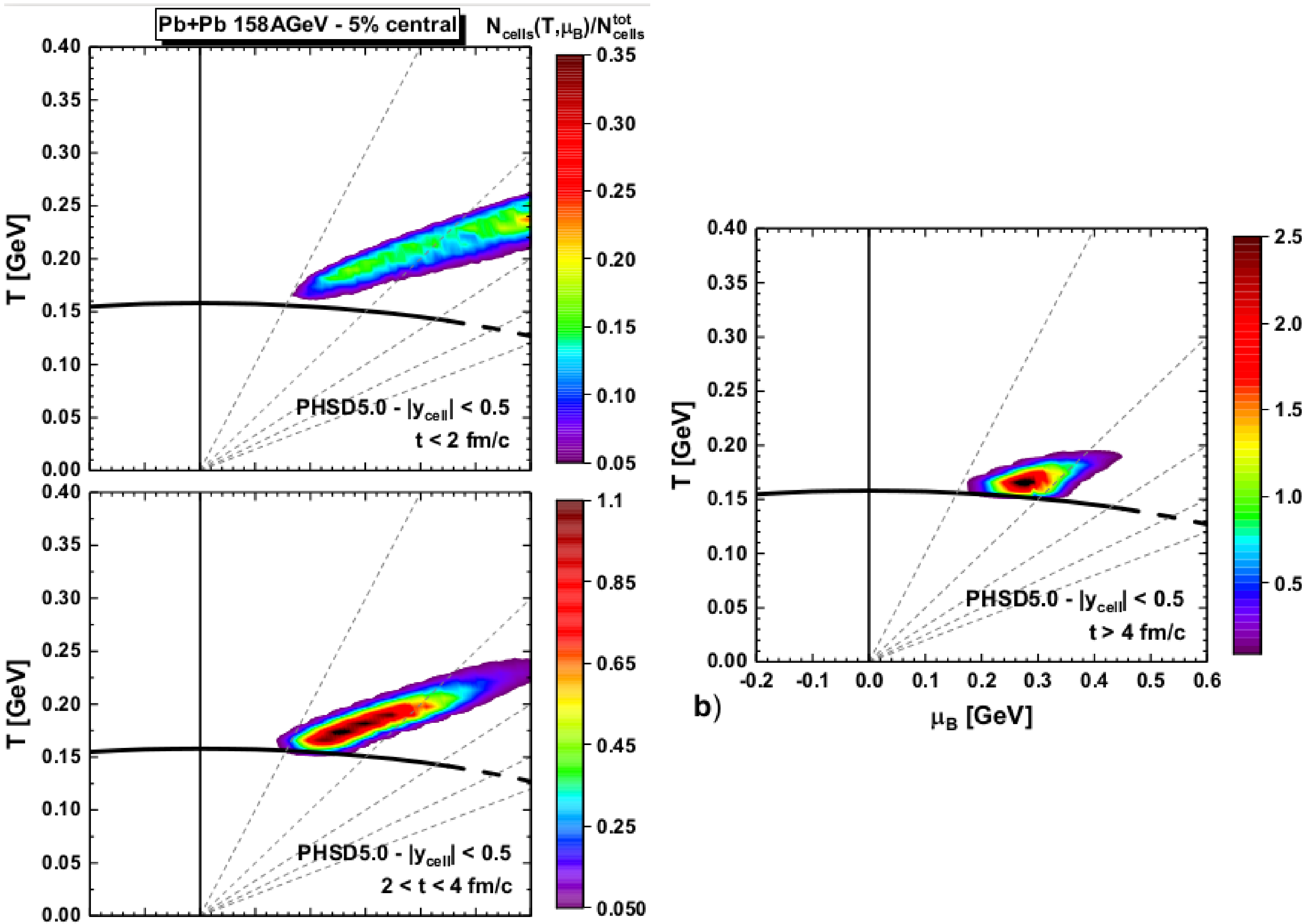
$$\eta^{\text{RTA}}(T,\mu_q) = \frac{1}{15T}\int \frac{d^3p}{(2\pi)^3} \sum_{i=q,\bar{q},g} \left(\frac{\mathbf{p}^4}{E_i^2\, \Gamma_i(\mathbf{p}_i,T,\mu_q)}\, d_i\, \big((1\pm f_i(E_i)) f_i(E_i)\big) \right)$$

Collisional rate

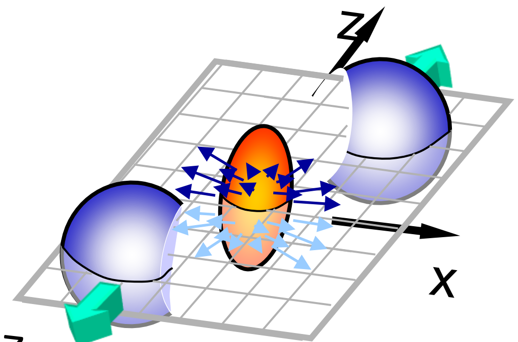


P. Moreau, O. Soloveva et al., arXiv:1903.10157, PRC 100 (2019) no. 1, 014911

QGP evolution for HIC ($\sqrt{s_{NN}} = 17$ GeV)



Anisotropic flow coefficients

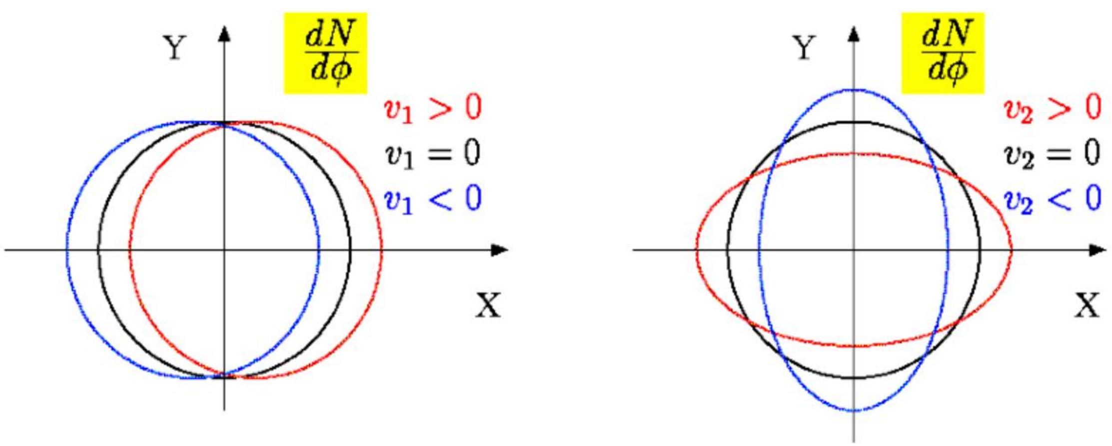


$$\frac{dN}{d\varphi} \propto \left(1 + 2\sum_{n=1}^{+\infty} v_n \cos\left[n(\varphi - \psi_n)\right]\right)$$

$$v_n = \left\langle \cos n\left(\varphi - \psi_n\right)\right\rangle, \quad n = 1,2,3..,$$

Anisotropic flow = correlations
with respect to the reaction plane

$$v_n = \langle\cos(n(\phi - \Psi_r))\rangle$$



Directed flow Elliptic flow

Extraction of (T, μ_B) in PHSD

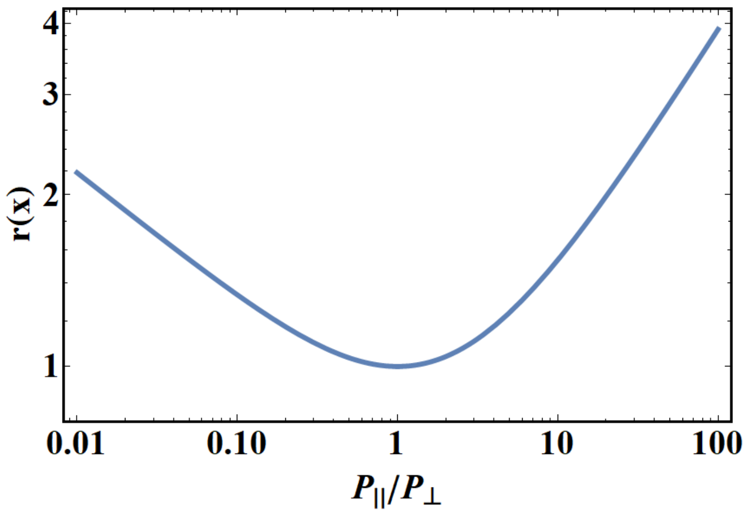
- **Correction for the medium anisotropy to extract values for (T, μ_B)**

$$\epsilon^{\text{anis}} = \epsilon^{\text{EoS}} \; r(x)$$

$$P_\perp = P^{\text{EoS}} \; [r(x) + 3xr'(x)]$$

$$P_{||} = P^{\text{EoS}} \; [r(x) - 6xr'(x)]$$

$$r(x) = \begin{cases} \dfrac{x^{-1/3}}{2}\left[1 + \dfrac{x \,\text{arctanh}\sqrt{1-x}}{\sqrt{1-x}}\right] & \text{for } x \leq 1 \\ \dfrac{x^{-1/3}}{2}\left[1 + \dfrac{x \arctan\sqrt{x-1}}{\sqrt{x-1}}\right] & \text{for } x \geq 1 \end{cases}$$



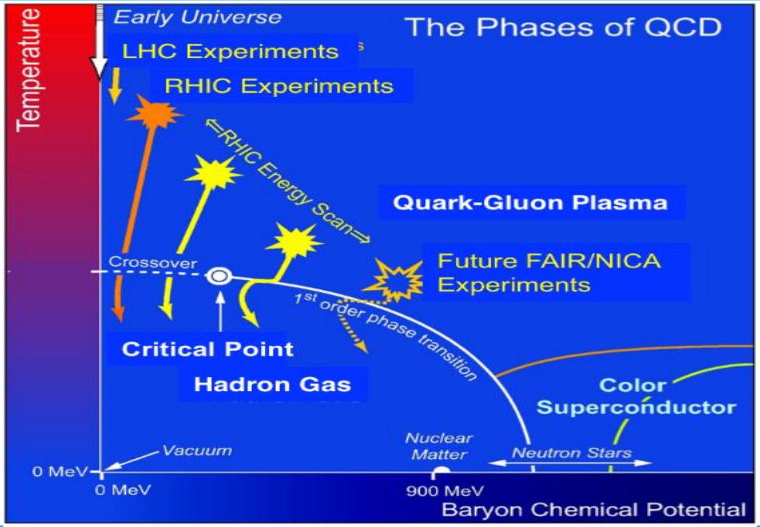
Ryblewski, Florkowski, Phys.Rev. C85 (2012) 064901

- **We have to solve the following system in PHSD:**

$$\begin{cases} \epsilon^{\text{EoS}}(T, \mu_B) = \epsilon^{\text{PHSD}} / r(x) \\ n_B^{\text{EoS}}(T, \mu_B) = n_B^{\text{PHSD}} \end{cases}$$

Done by Newton-Raphson method

Traces of the QGP at finite μ_B in observables in high energy heavy-ion collisions



Transport coefficients: approaches

- **Kubo formalism: transport coefficients are expressed through correlation functions of stress-energy tensor**

used in lattice QCD, transport approaches(hadrons), effective models

$$\eta = \frac{1}{20}\lim_{\omega\to 0}\frac{1}{\omega}\int d^4x\, e^{i\omega t}\left\langle\left[\mathcal{S}^{ij}(t,\mathbf{x}),\,\mathcal{S}^{ij}(0,\mathbf{0})\right]\right\rangle\theta(t) \qquad \mathcal{S}^{ij} = T^{ij} - \delta^{ij}\mathcal{P}$$

$$\zeta = \frac{1}{2}\lim_{\omega\to 0}\frac{1}{\omega}\int d^4x\, e^{i\omega t}\langle[\mathcal{P}(t,\mathbf{x}),\,\mathcal{P}(0,\mathbf{0})]\rangle\theta(t) \qquad \mathcal{P} = -\tfrac{1}{3}T^{i}{}_{i}$$

R. Lang and W. Weise, EPJ. A 50, 63 (2014) (NJL model)
A. Harutyunyan et al, PRD 95, 114021, (2017)

Kinetic theory:

- **Relaxation time approximation(RTA) : consider relaxation time** $\frac{df_a^{eq}}{dt} = \mathcal{C}_a = -\frac{f_a^{eq}\phi_a}{\tau_a}$

P. Chakraborty and J. I. Kapusta, PRC 83,014906 (2011)

$$\tau_i(\mathbf{p},T,\mu_B) = \frac{1}{\Gamma_i(\mathbf{p},T,\mu_B)}$$

- **Chapman-Enskog : expand the distribution in terms of the Knudsen number**

J. A. Fotakis et al, PRD 101 (2020) 7, 076007 (HRG)

And more!

Holographic models: AdS/CFT correspondence

D. T. Son and A. O. Starinets, JHEP 0603, 052 (2006)
M. Attems et al , JHEP 10 (2016), 155.

DQPM: q, qbar, g elastic/inelastic scattering (leading order)