Net-proton number fluctuations in partial chemical equilibrium

Based on: B. Tomasik, P. Hillmann, and M. Bleicher, (2021), arXiv:2107.03830 [nucl-th]

Paula Hillmann, Strong Workshop, Herssonisos, Crete, 05/10/21

Overview

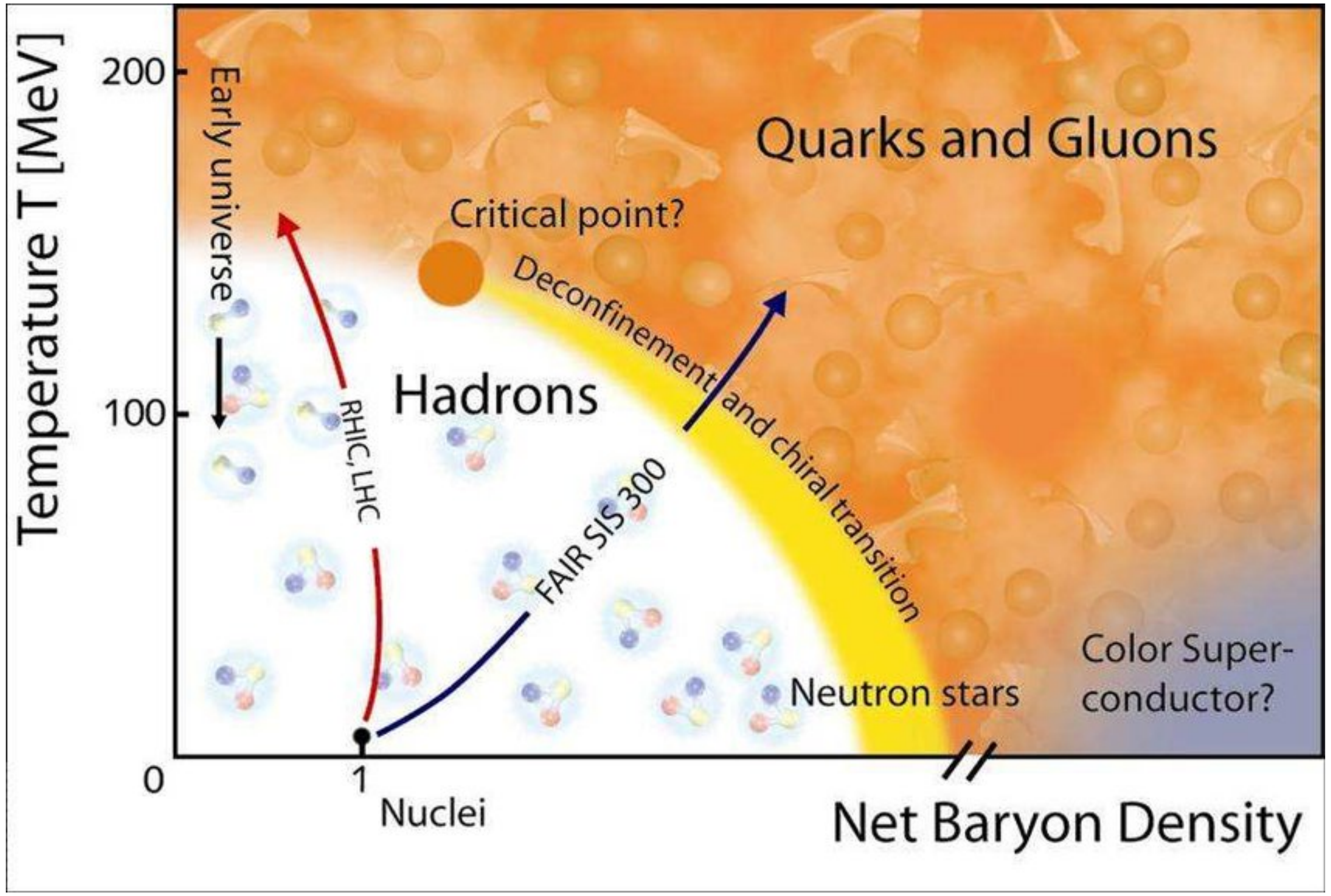
- Motivation
- Hadron resonance gas in partial chemical equilibrium
- Derivation of proton number fluctuations
- Results
- Summary

Motivation

Motivation

QCD Phase Diagram and Critical Point

- Search for phase-transition and critical point in QCD phase diagram
- Theoretical calculations of net-Baryon number fluctuations show a strong scaling with the correlation lengths
  Y. Hatta , et al., PRD 67, 014028 (2003)
- Higher order cumulants are sensitive to the CP where the correlation lengths diverges
  Y. Hatta , et al., PRD 67, 014028 (2003)

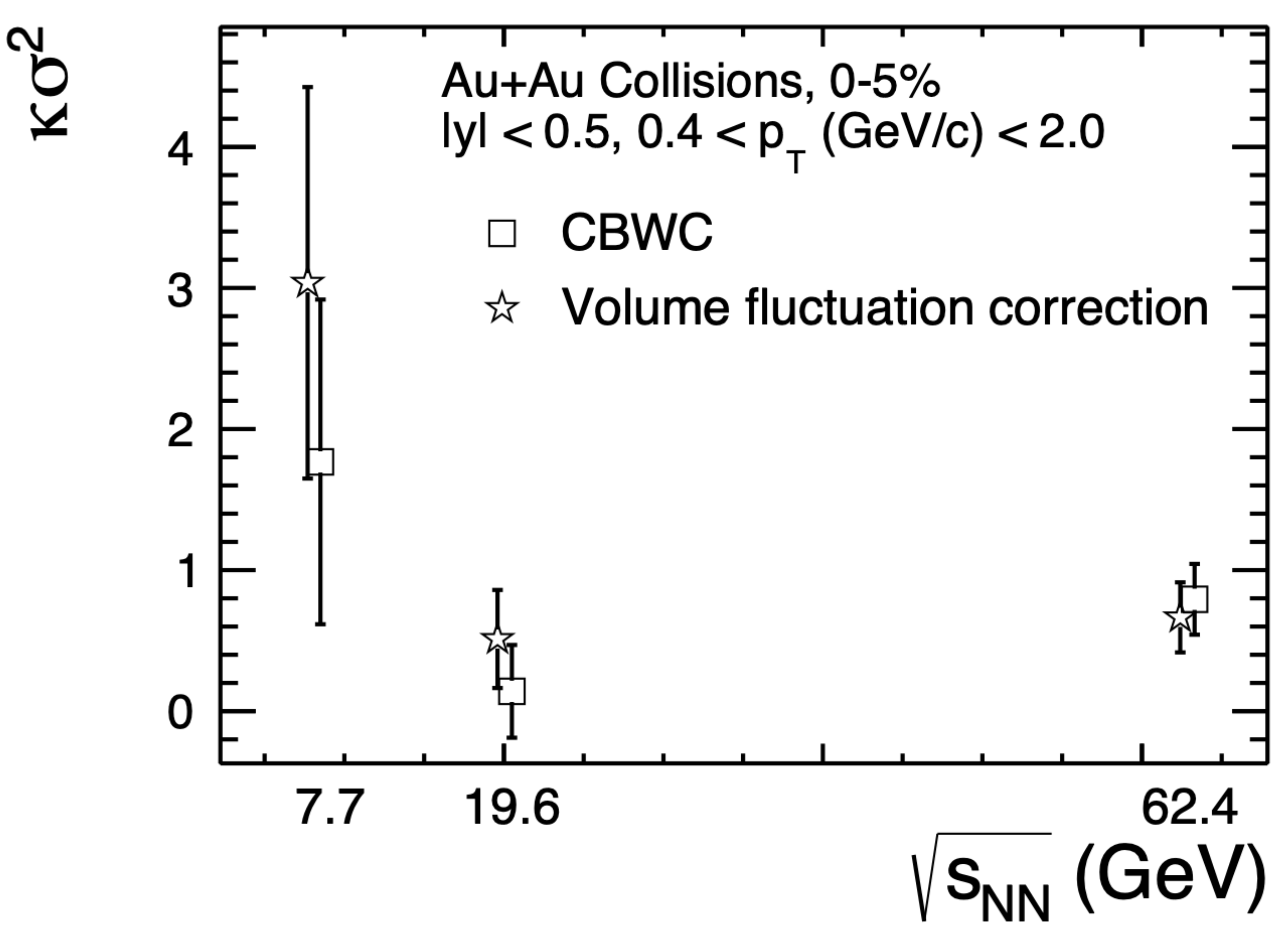


Picture: P. Bicudo, et al., arXiv: 1102.5531 [hep-lat]

Motivation

Connecting experiment and theory

Picture taken from M. Abdallah et al. [STAR], arXiv:2101.12413 [nucl-ex],
CBWC: centrality bin width correction



- STAR data show a strong increase of t net-p kurtosis at low energies
  M. Abdallah et al. [STAR], arXiv:2101.12413 [nucl-ex]

- Possible experimental impacts on the fluctuations:

  - Cluster production, e.g. d and t
    Z. Feckova,´ et al., PRC 92, no.6, 064908 (2015), T. Neidig, et al., arXiv: 2108.131

  - Detector efficiency and mass cuts
    S. He, et al., PLB 774, 623-629 (2017), A. Bazavov ,et al., PRD 101, no.7, 074502 (2020)

  - Volume might be too small for grand-canonical equilibrium
    H. Bebie, et al, NPB 378, 95-128 (1992), T. Hirano, et al., PRC 66, 054905 (2002), R. Rapp, PRC 66, 017901 (2002)

  - Net-p as proxy for net-B
    Y. Hatta, et al., PRL 91, M. Kitazawa ,et al., PRC 86, 024904 (2012)

$$\frac{\chi_4}{\chi_2} = \frac{\left\langle (\Delta N)^4 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c} = \kappa\sigma^2$$

Motivation

Impact of collision stages

- Chemical freeze-out: end of inelastic scattering
- Between the freeze-out stages: Assumption of Partial Chemical Equilibrium (PCE) effective numbers of long living hadrons fixed at the values they had at the chemical freeze-out
- Kinetic freeze-out: end of interaction
- Different freeze-out temperatures at chemical and kinetic FO
  I. Melo et al, JPG 47, no.4, 045107 (2020)
- How is the temperature dependence of the net-p cumulants after chemical freeze-out?

Hadron resonance gas in partial chemical equilibrium

Hadron resonance gas in partial chemical equilibrium

Starting point

- HRG calculations for central Au+Au reactions at RHIC BES
- Grand-canonical HRG with „stable“ hadrons (pions, nucleons, kaons, …) and resonances
- Starting point: Strangeness neutrality and chemical freeze-out
- Partition sum:

$$\ln \mathcal{Z} = \sum_i (\pm 1) \frac{g_r V}{2\pi^2} \times \int_0^\infty dk\, k^2 \, \ln\left(1 \pm \gamma_s^{|S_i|} e^{\mu_i/T} e^{-\sqrt{m_i + k^2}/T}\right)$$

Starting values for different energies

$\sqrt{s_{NN}}$ [GeV]	$T_{\rm fo}$ [MeV]	μ_B [MeV]	μ_S [MeV]	γ_s
7.7	144.3	398.2	89.5	0.95
11.5	149.4	287.3	64.5	0.92
14.5	151.6	264.0	58.1	0.94
19.6	153.9	187.9	43.2	0.96
27.0	155.0	144.4	33.5	0.98
39.0	156.4	103.2	24.5	0.94
54.4	160.0	83.0	18.7	0.94
62.4	160.3	69.8	16.7	0.86
200	164.3	28.4	5.6	0.93

Data: L. Adamczyk et al. [STAR], PRC 96, no.4, 044904 (2017)
and M. Abdallah et al. [STAR], arXiv:2101.12413 [nucl-ex]

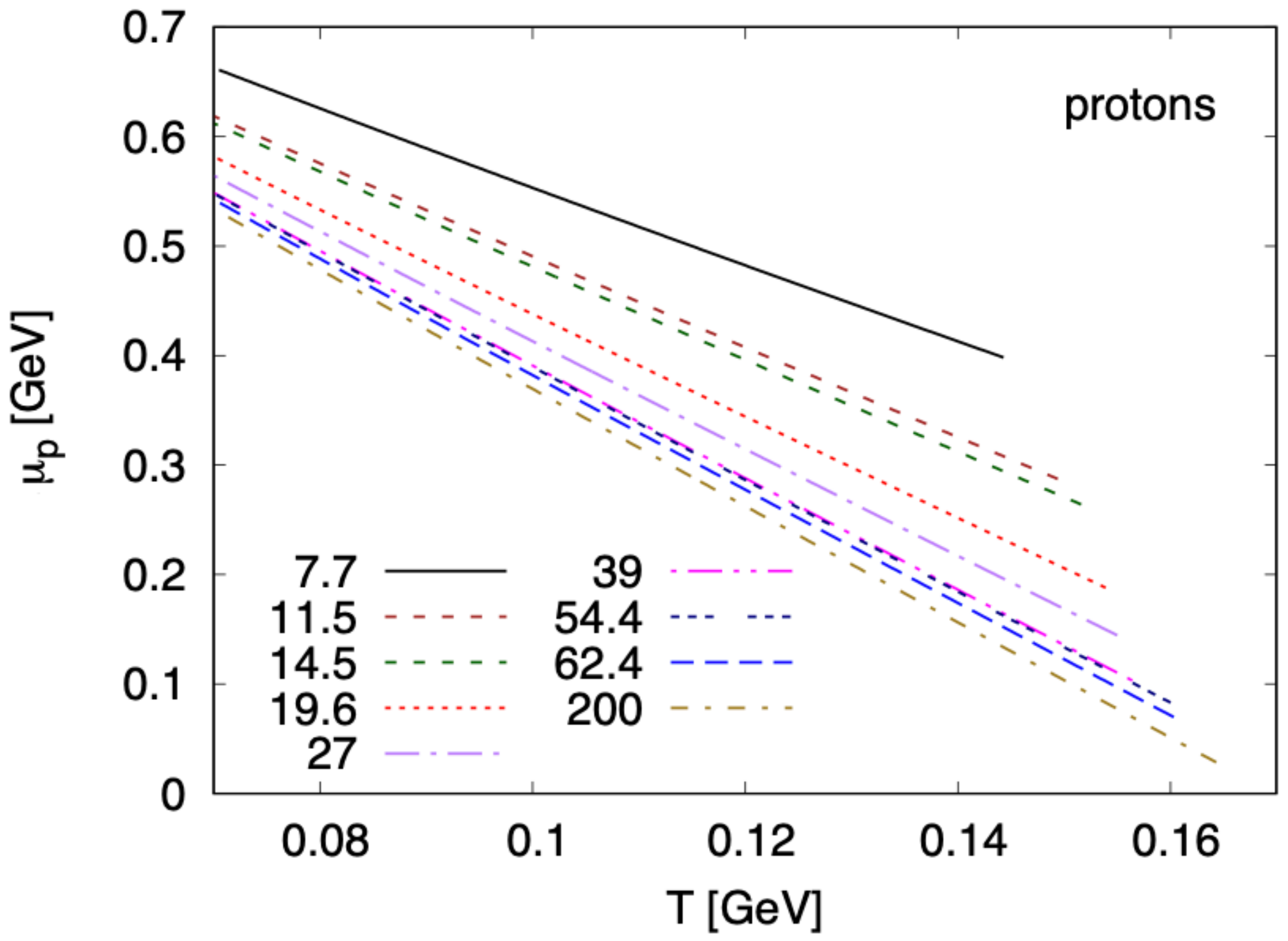
Hadron resonance gas in partial chemical equilibrium

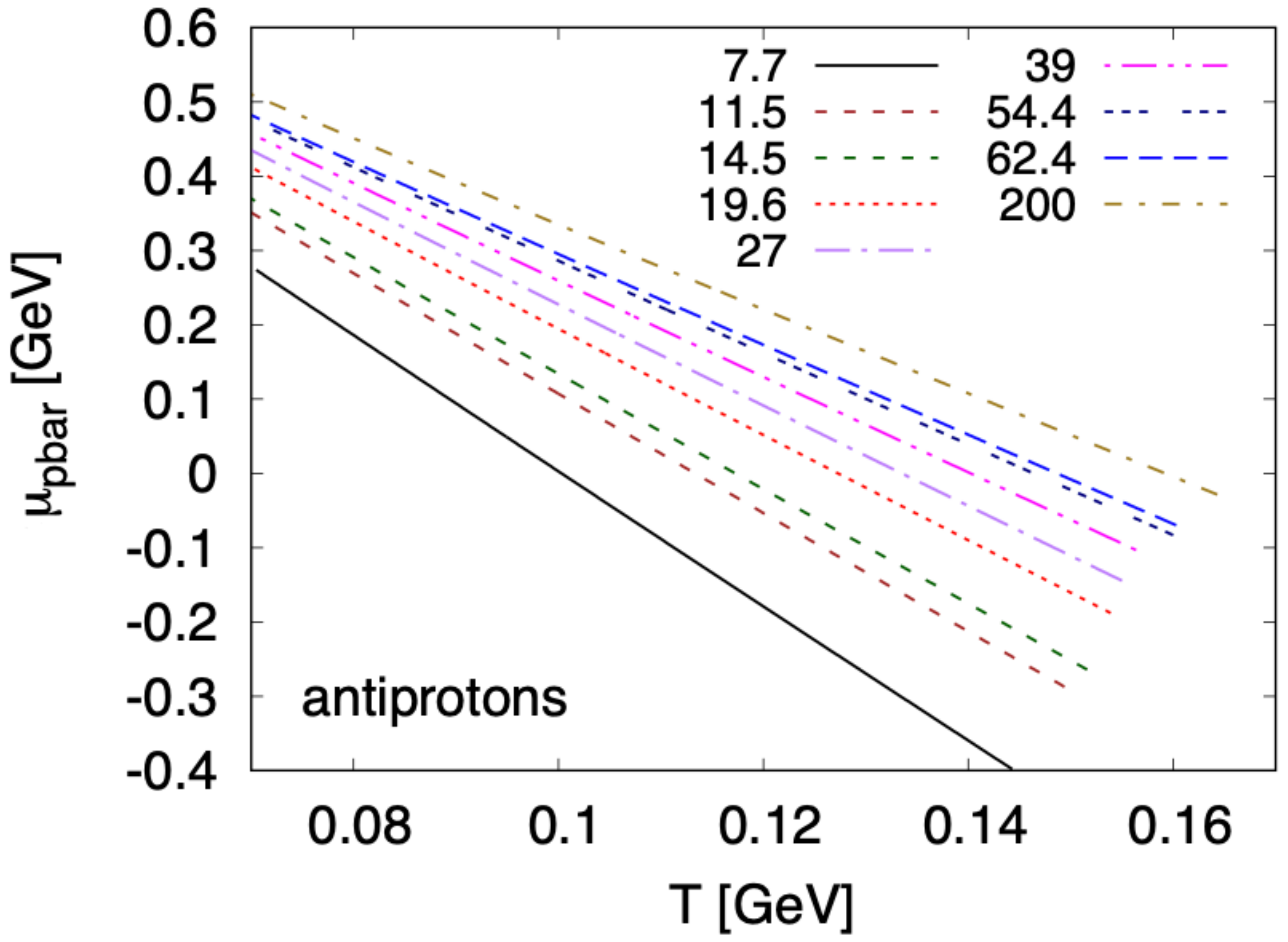
Temperature evolution

- Partition sum: $$\ln \mathcal{Z} = \sum_R \frac{g_r V}{2\pi^2} m_R^2 T \sum_{j=1}^{\infty} \frac{(\mp 1)^{j-1}}{j^2} e^{j\mu_R/T} K_2\left(\frac{jm_R}{T}\right)$$
- Each stable species has its own chemical potential
- PCE condition: $$\frac{\langle N_h^{\mathrm{eff}}(T)\rangle_c}{S(T)} = \left.\frac{\langle N_h^{\mathrm{eff}}(T)\rangle_c}{S(T)}\right|_{T=T_{\mathrm{fo}}}$$
- with Entropy: $$S = \sum_R \frac{VP_R + E_R - \langle N_R\rangle_c \mu_R}{T}$$
- Resonance potential: $$\mu_R = \sum_h p_{R\to h}\mu_h$$
- Different impacts on the proton number: Direct proton production and resonance decays into protons (sometimes different decay channels into p)

M. Nahrgang, et al., EPJ C 75, no.12, 573 (2015)

Temperature dependence of the proton and anti-proton chemical potentials:





Derivation of proton number cumulants

Derivation of proton number fluctuations

Generating function

- Assuming binomial probability to produce N protons by the decay of Resonance R:

$$P(N) = \sum_{N_R=N}^{\infty} P_R(N_R)P(N;N_R) \quad \text{with} \quad P(N;N_R) = \binom{N_R}{N} p_R^N (1-p_R)^{N_R-N}$$

- Plugging this in the generating function for a fixed resonance number:

$$K_b(i\xi) = \ln\left\{\sum_{N=0}^{N_R} e^{i\xi N}\binom{N_R}{N} p_R^N (1-p_R)^{N_R-N}\right\} = \ln\left(e^{i\xi}p_R + (1-p_R)\right)^{N_R}$$

- Final generating function with sum over the resonance number:

$$K(i\xi) = \ln\left\{\sum_{N_R=0}^{\infty}\sum_{N=0}^{N_R} e^{i\xi N} P_R(N_R)P(N;N_R)\right\} = \ln\left\{\sum_{N_R=0}^{\infty} P_R(N_R)\sum_{N=0}^{N_R} e^{i\xi N} P(N;N_R)\right\}.$$

- Second sum is known: $K(i\xi) = \sum_R \ln\left\{\sum_{N_R=0}^{\infty} P_R(N_R)\left(e^{i\xi}p_R + (1-p_R)\right)^{N_R}\right\}$

Derivation of proton number cumulants

Derivatives of generating function

- Cumulant of order l: $\left\langle (\Delta N)^l \right\rangle_c = \left. \frac{\mathrm{d}^l K(i\xi)}{\mathrm{d}(i\xi)^l} \right|_{\xi=0}$
- First to forth order proton number cumulant:

$$\langle N_p \rangle_c = \sum_R p_R \langle N_R \rangle_c$$

$$\left\langle (\Delta N_p)^2 \right\rangle_c = \sum_R \left[p_R^2 \left\langle (\Delta N_R)^2 \right\rangle_c + p_R(1-p_R) \langle N_R \rangle_c \right]$$

$$\left\langle (\Delta N_p)^3 \right\rangle_c = \sum_R \left[p_R^3 \left\langle (\Delta N_R)^3 \right\rangle_c + 3p_R^2(1-p_R) \left\langle (\Delta N_R)^2 \right\rangle_c + p_R(1-p_R)(1-2p_R) \langle N_R \rangle_c \right]$$

$$\left\langle (\Delta N_p)^4 \right\rangle_c = \sum_R \Big[p_R^4 \left\langle (\Delta N_R)^4 \right\rangle_c + 6p_R^3(1-p_R) \left\langle (\Delta N_R)^3 \right\rangle_c + p_R^2(1-p_R)(7-11p_R) \left\langle (\Delta N_R)^2 \right\rangle_c$$

- with: $\left\langle (\Delta N_R)^l \right\rangle_c = \frac{\partial^l \ln \mathcal{Z}_R}{\partial(\mu_R/T)^l} = \frac{g_R V}{2\pi^2} m_R^2 T \quad \times \sum_{i=1}^{\infty} (\mp 1)^{j-1} j^{l-2} e^{j\mu_R/T} K_2\left(\frac{j m_R}{T}\right)$
- Net-p cumulant: $\left\langle (\Delta N_{p-\bar{p}})^l \right\rangle_c = \left\langle (\Delta N_p)^l \right\rangle_c + (-1)^l \left\langle (\Delta N_{\bar{p}})^l \right\rangle_c$

Derivation of proton number cumulants

Cumulant ratios

- The cumulant ratios are given as:

$$\frac{\chi_2}{\chi_1} = \frac{\left\langle (\Delta N)^2 \right\rangle_c}{\langle N \rangle_c} = \frac{\sigma^2}{M} \qquad \frac{\chi_3}{\chi_2} = \frac{\left\langle (\Delta N)^3 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c} = S\sigma$$

$$\frac{\chi_4}{\chi_2} = \frac{\left\langle (\Delta N)^4 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c} = \kappa\sigma^2 \qquad \frac{\chi_5}{\chi_1} = \frac{\left\langle (\Delta N)^5 \right\rangle_c}{\langle N \rangle_c} = \frac{S^H\sigma^5}{M}$$

$$\frac{\chi_5}{\chi_2} = \frac{\left\langle (\Delta N)^5 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c} = S^H\sigma^3 \qquad \frac{\chi_6}{\chi_2} = \frac{\left\langle (\Delta N)^6 \right\rangle_c}{\langle N \rangle_c} = \kappa^H\sigma^4$$

with:

$$\sigma^2 = \left\langle (\Delta N)^2 \right\rangle_c$$

$$S = \frac{\left\langle (\Delta N)^3 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c^{3/2}}$$

$$\kappa = \frac{\left\langle (\Delta N)^4 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c^{2}}$$

$$S^H = \frac{\left\langle (\Delta N)^5 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c^{5/2}}$$

$$\kappa^H = \frac{\left\langle (\Delta N)^6 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c^{3}}$$

Results

Results

Net-p fluctuations

- Chemical fo to kinetic fo: from right to left

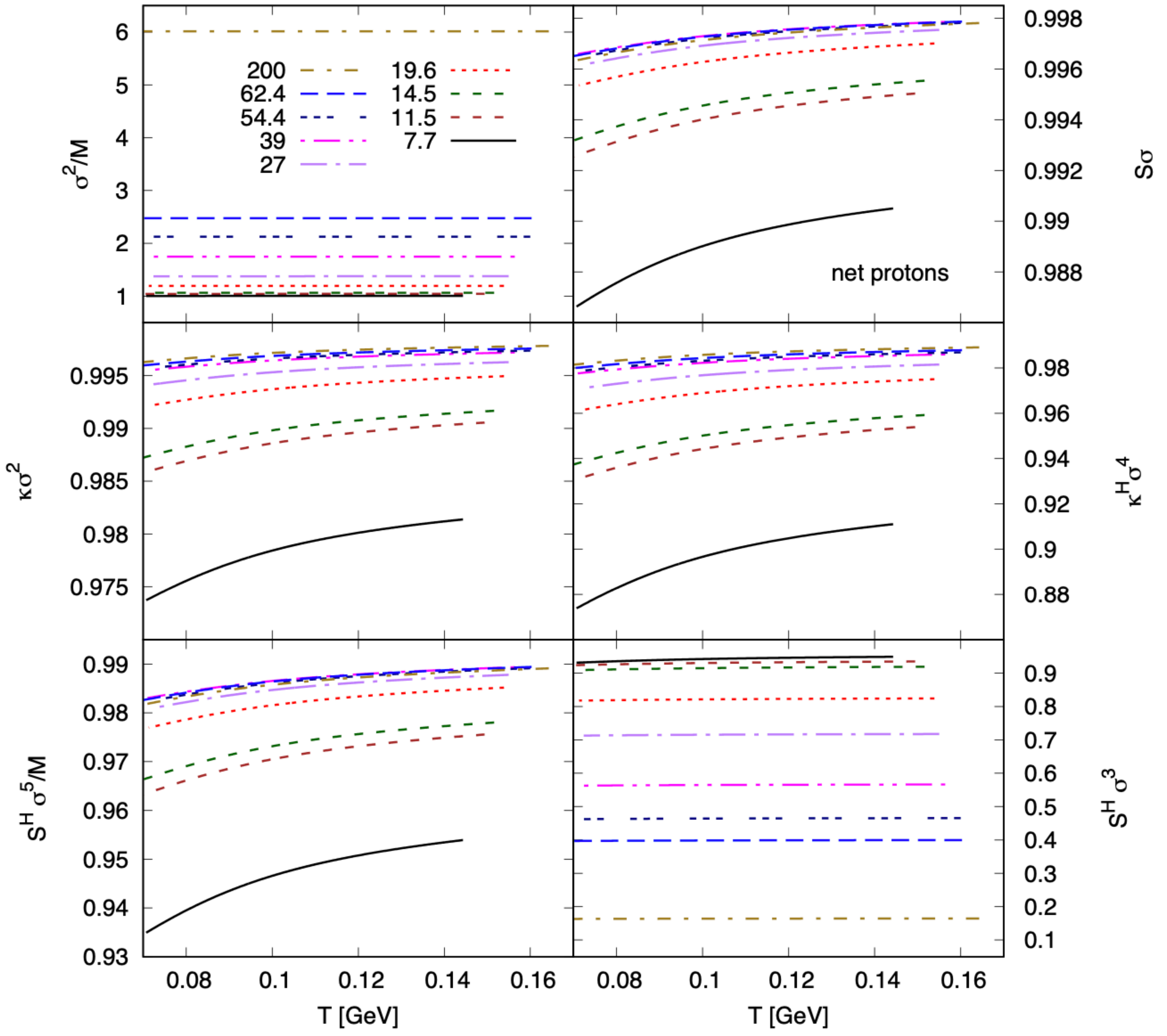
To remember:

$$\frac{\chi_2}{\chi_1} = \frac{\left\langle (\Delta N)^2 \right\rangle_c}{\langle N \rangle_c} = \frac{\sigma^2}{M} \qquad \frac{\chi_3}{\chi_2} = \frac{\left\langle (\Delta N)^3 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c} = S\sigma$$

$$\frac{\chi_4}{\chi_2} = \frac{\left\langle (\Delta N)^4 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c} = \kappa\sigma^2 \qquad \frac{\chi_5}{\chi_1} = \frac{\left\langle (\Delta N)^5 \right\rangle_c}{\langle N \rangle_c} = \frac{S^H\sigma^5}{M}$$

$$\frac{\chi_5}{\chi_2} = \frac{\left\langle (\Delta N)^5 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c} = S^H\sigma^3 \qquad \frac{\chi_6}{\chi_2} = \frac{\left\langle (\Delta N)^6 \right\rangle_c}{\langle N \rangle_c} = \kappa^H\sigma^4$$

Net-p number cumulant-ratios as function of temperature for central Au+Au reactions at various beam-energies



Results

Explanation:

- Flatness of the curves:
  let us assume, that all resonance cumulants are the same as the mean:

- $\left\langle (\Delta N_p)^l \right\rangle_c = \sum_R p_R \langle N_R \rangle_c = \langle N_p \rangle_c$

  Remember: mean is fixed, other cumulants show weak temperature dependence

- Difference in even-to-even and even-to-odd ratios:

$$\left\langle (\Delta N_{p-\bar{p}})^l \right\rangle_c = \left\langle (\Delta N_p)^l \right\rangle_c + (-1)^l \left\langle (\Delta N_{\bar{p}})^l \right\rangle_c$$

Results

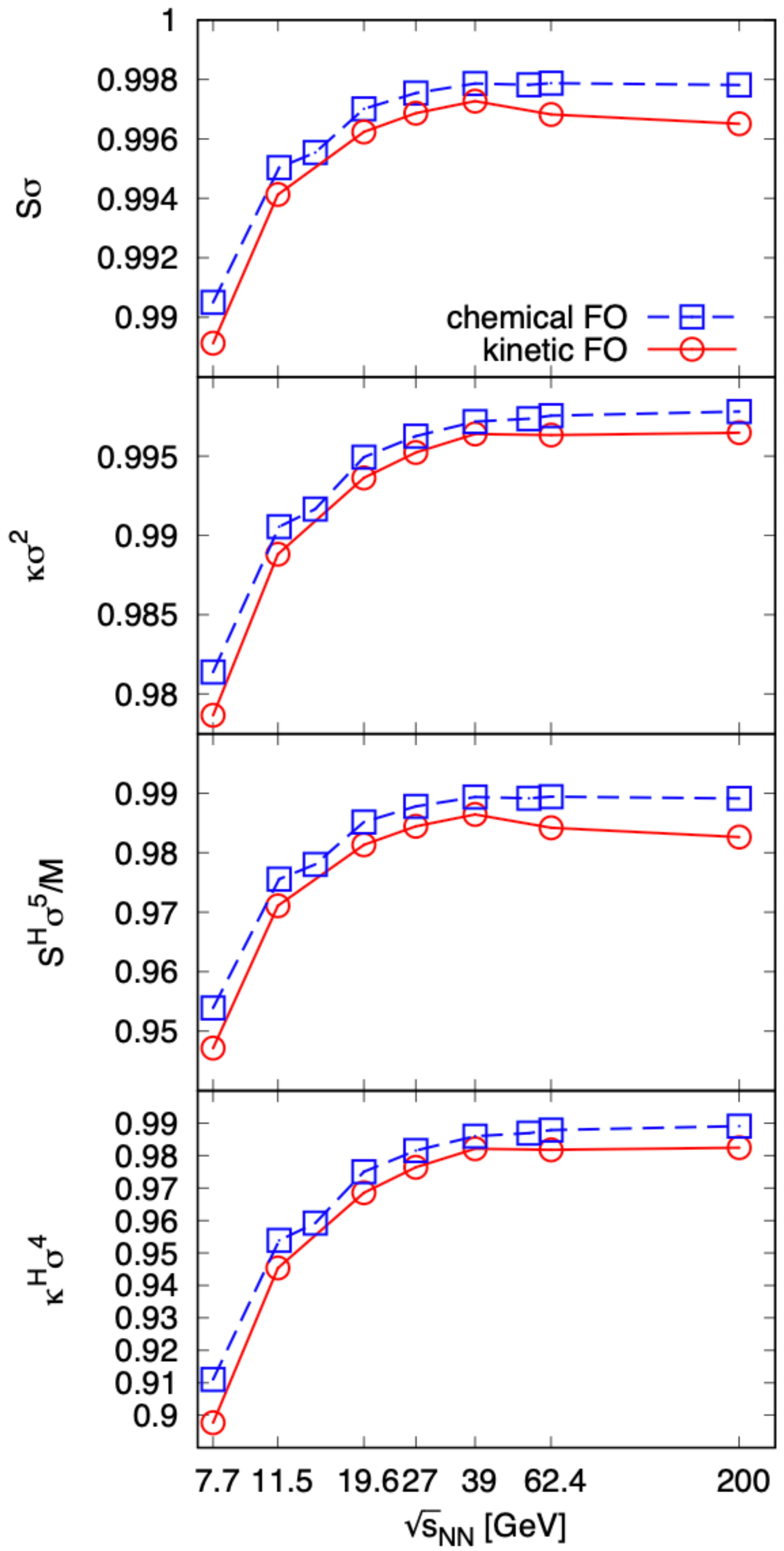
Energy dependence

- Difference between chemical and kinetic FO only small (biggest difference about 5%)
- Biggest differences for highest beam energy and higher order cumulant ratios

$$\frac{\chi_2}{\chi_1} = \frac{\left\langle (\Delta N)^2 \right\rangle_c}{\langle N \rangle_c} = \frac{\sigma^2}{M} \qquad \frac{\chi_3}{\chi_2} = \frac{\left\langle (\Delta N)^3 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c} = S\sigma$$

$$\frac{\chi_4}{\chi_2} = \frac{\left\langle (\Delta N)^4 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c} = \kappa\sigma^2 \qquad \frac{\chi_5}{\chi_1} = \frac{\left\langle (\Delta N)^5 \right\rangle_c}{\langle N \rangle_c} = \frac{S^H\sigma^5}{M}$$

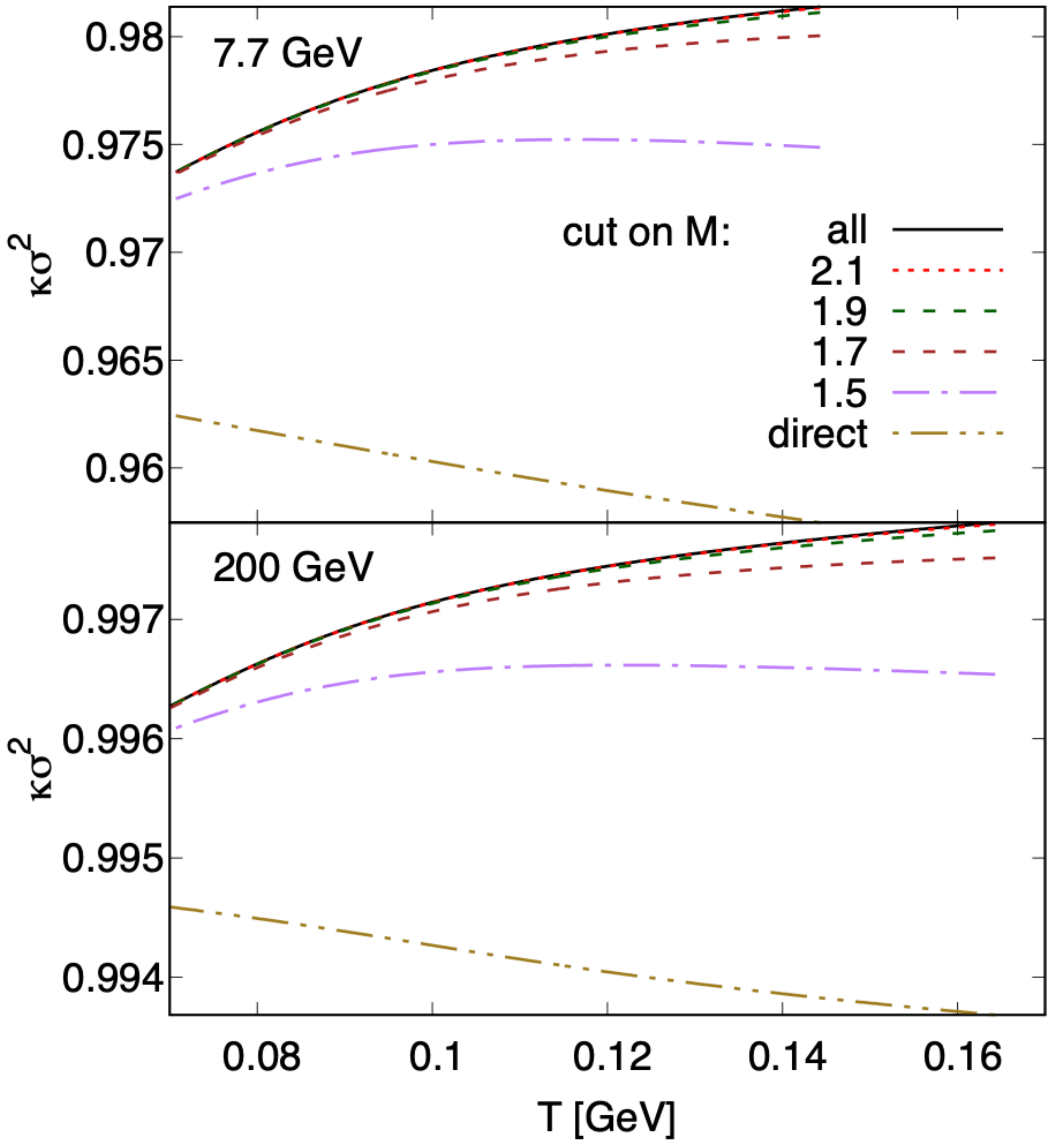
$$\frac{\chi_5}{\chi_2} = \frac{\left\langle (\Delta N)^5 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c} = S^H\sigma^3 \qquad \frac{\chi_6}{\chi_2} = \frac{\left\langle (\Delta N)^6 \right\rangle_c}{\langle N \rangle_c} = \kappa^H\sigma^4$$



Results

Impact of mass cuts

- Different mass cuts for the resonance masses applied to the HRG
- Impact is visible: The lower the allowed mass, the lower the kurtosis
- Effect similar for both energies
- Resonance decays are the strongest contributors to the the net-p cumulants



$$\frac{\chi_4}{\chi_2} = \frac{\left\langle (\Delta N)^4 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c} = \kappa\sigma^2$$

Results

Net-B fluctuations

- Formula:

$$\left\langle (\Delta B)^l \right\rangle_c = \sum_R B_R^l \left\langle (\Delta N_R)^l \right\rangle_c$$

- Similar behavior like for net-p

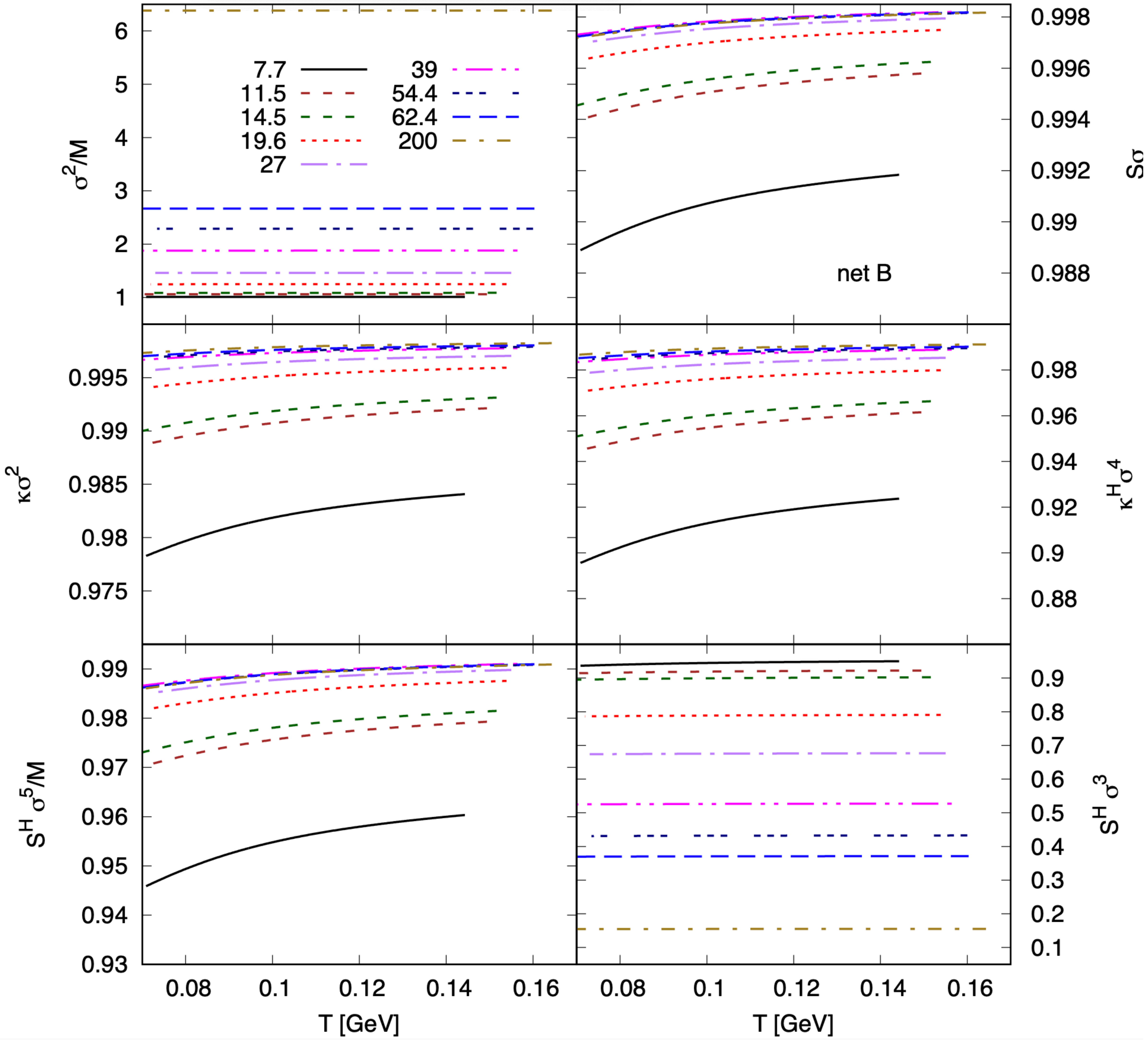
- Net-p is a good proxy

To remember:

$$\frac{\chi_2}{\chi_1} = \frac{\left\langle (\Delta N)^2 \right\rangle_c}{\langle N \rangle_c} = \frac{\sigma^2}{M} \qquad \frac{\chi_3}{\chi_2} = \frac{\left\langle (\Delta N)^3 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c} = S\sigma$$

$$\frac{\chi_4}{\chi_2} = \frac{\left\langle (\Delta N)^4 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c} = \kappa\sigma^2 \qquad \frac{\chi_5}{\chi_1} = \frac{\left\langle (\Delta N)^5 \right\rangle_c}{\langle N \rangle_c} = \frac{S^H\sigma^5}{M}$$

$$\frac{\chi_5}{\chi_2} = \frac{\left\langle (\Delta N)^5 \right\rangle_c}{\left\langle (\Delta N)^2 \right\rangle_c} = S^H\sigma^3 \qquad \frac{\chi_6}{\chi_2} = \frac{\left\langle (\Delta N)^6 \right\rangle_c}{\langle N \rangle_c} = \kappa^H\sigma^4$$



Summary

- Grand-canonical hadron resonance gas model in partial chemical equilibrium was used to study the temperature dependence of net-p and net-B fluctuations after chemical freeze-out up to kinetic freeze-out

- Only weak temperature dependence was observed, Net-p and net-B cumulant ratios were similar

- Mass cut of the resonances has a small impact on the results

- Effects like e.g. cluster production haven’t been included in the model

Backup

Generating function

$$K_b(i\xi) = \ln\left\{\sum_{N=0}^{N_R} e^{i\xi N}\binom{N_R}{N} p_R^N (1-p_R)^{N_R-N}\right\}$$
$$= \ln\left\{\sum_{N=0}^{N_R} \binom{N_R}{N} e^{i\xi N} p_R^N (1-p_R)^{N_R-N}\right\}$$
$$= \ln\left\{\sum_{N=0}^{N_R} \binom{N_R}{N} (e^{i\xi} p_R)^N (1-p_R)^{N_R-N}\right\}$$

$$\langle N_p \rangle_c = \sum_R p_R \langle N_R \rangle_c$$

$$\left\langle (\Delta N_p)^2 \right\rangle_c = \sum_R \left[p_R^2 \left\langle (\Delta N_R)^2 \right\rangle_c + p_R(1-p_R)\langle N_R \rangle_c \right],$$

$$\left\langle (\Delta N_p)^3 \right\rangle_c = \sum_R \left[p_R^3 \left\langle (\Delta N_R)^3 \right\rangle_c + 3p_R^2(1-p_R)\left\langle (\Delta N_R)^2 \right\rangle_c + p_R(1-p_R)(1-2p_R)\langle N_R \rangle_c \right],$$

$$\left\langle (\Delta N_p)^4 \right\rangle_c = \sum_R \Big[p_R^4 \left\langle (\Delta N_R)^4 \right\rangle_c + 6p_R^3(1-p_R)\left\langle (\Delta N_R)^3 \right\rangle_c + p_R^2(1-p_R)(7-11p_R)\left\langle (\Delta N_R)^2 \right\rangle_c$$
$$+ p_R(1-p_R)(1-6p_R+6p_R^2)\langle N_R \rangle_c \Big],$$

$$\left\langle (\Delta N_p)^5 \right\rangle_c = \sum_R \Big[p_R^5 \left\langle (\Delta N_R)^5 \right\rangle_c + 10p_R^4(1-p_R)\left\langle (\Delta N_R)^4 \right\rangle_c + 5p_R^3(1-p_R)(5-7p_R)\left\langle (\Delta N_R)^3 \right\rangle_c$$
$$+ 5p_R^2(1-p_R)(10p_R^2-12p_R+3)\left\langle (\Delta N_R)^2 \right\rangle_c$$
$$+ p_R(1-p_R)(1-2p_R)(12p_R^2-12p_R+1)\langle N_R \rangle_c \Big],$$

$$\left\langle (\Delta N_p)^6 \right\rangle_c = \sum_R \Big[p_R^6 \left\langle (\Delta N_R)^6 \right\rangle_c + 15p_R^5(1-p_R)\left\langle (\Delta N_R)^5 \right\rangle_c + 5p_R^4(1-p_R)(13-17p_R)\left\langle (\Delta N_R)^4 \right\rangle_c$$
$$+ 15p_R^3(1-p_R)(15p_R^2-20p_R+6)\left\langle (\Delta N_R)^3 \right\rangle_c$$
$$- p_R^2(1-p_R)(274p_R^3-476p_R^2+239p_R-31)\left\langle (\Delta N_R)^2 \right\rangle_c$$
$$+ p_R(1-p_R)(120p_R^4-240p_R^3+150p_R^2-30p_R+1)\langle N_R \rangle_c \Big],$$

Backup
HRG Properties

$$P_R = \frac{T \ln \mathcal{Z}_R}{V},$$
$$E_R = \frac{g_R V}{2\pi^2} \int_0^\infty dk\, k^2 \sqrt{k^2 + m_R^2}$$
$$\times \left(\exp\left(\frac{\sqrt{k^2 + m_R^2} - \mu_R}{T} \right) \pm 1 \right)^{-1}$$