Quarkonia production in AA collisions... How can we preserve quantum mechanics?

Pol B Gossiaux, SUBATECH (NANTES)

STRONG Creta

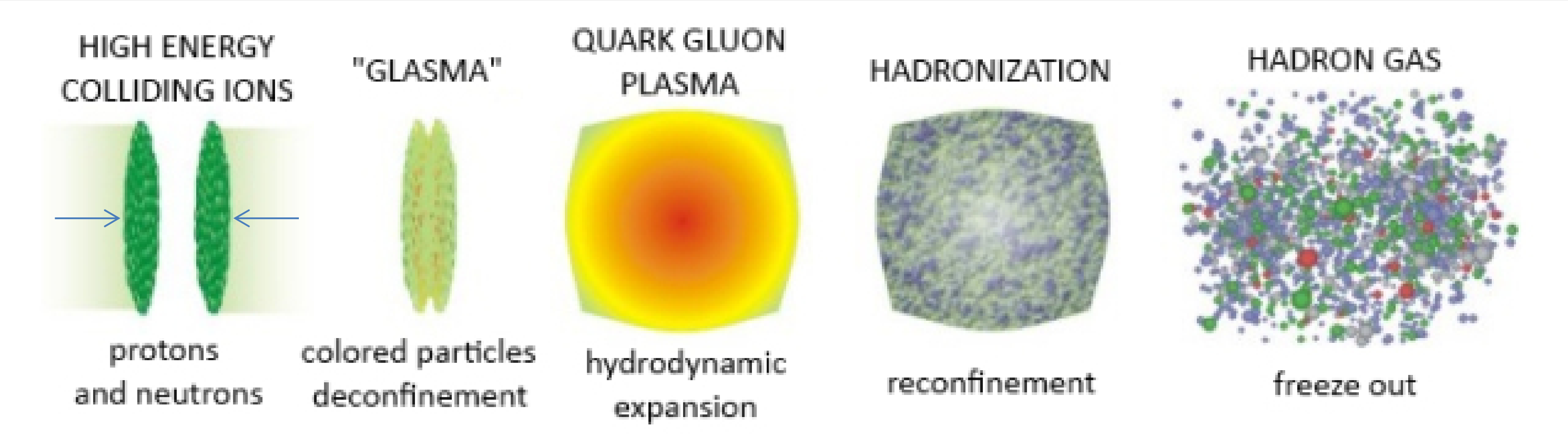
1. **Background and Motivation**
2. **One specific Open Quantum System Scheme**
3. **Results from Nantes Team**

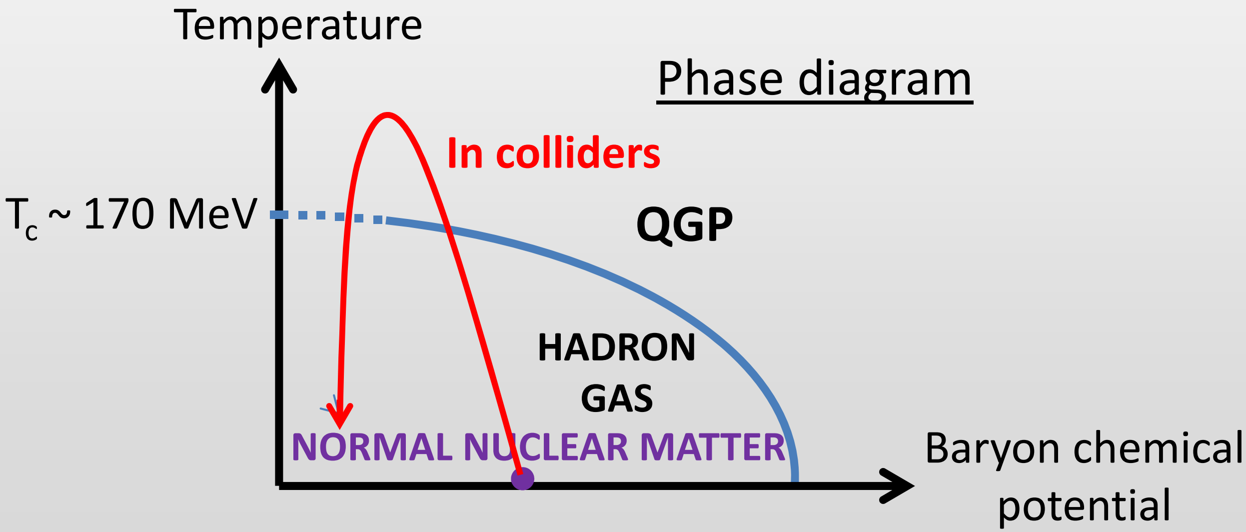
With Joerg Aichelin, Denys Yen Arrebato Villar, **Stéphane Delorme**, Thierry Gousset & Roland Katz

and Pays de la Loire

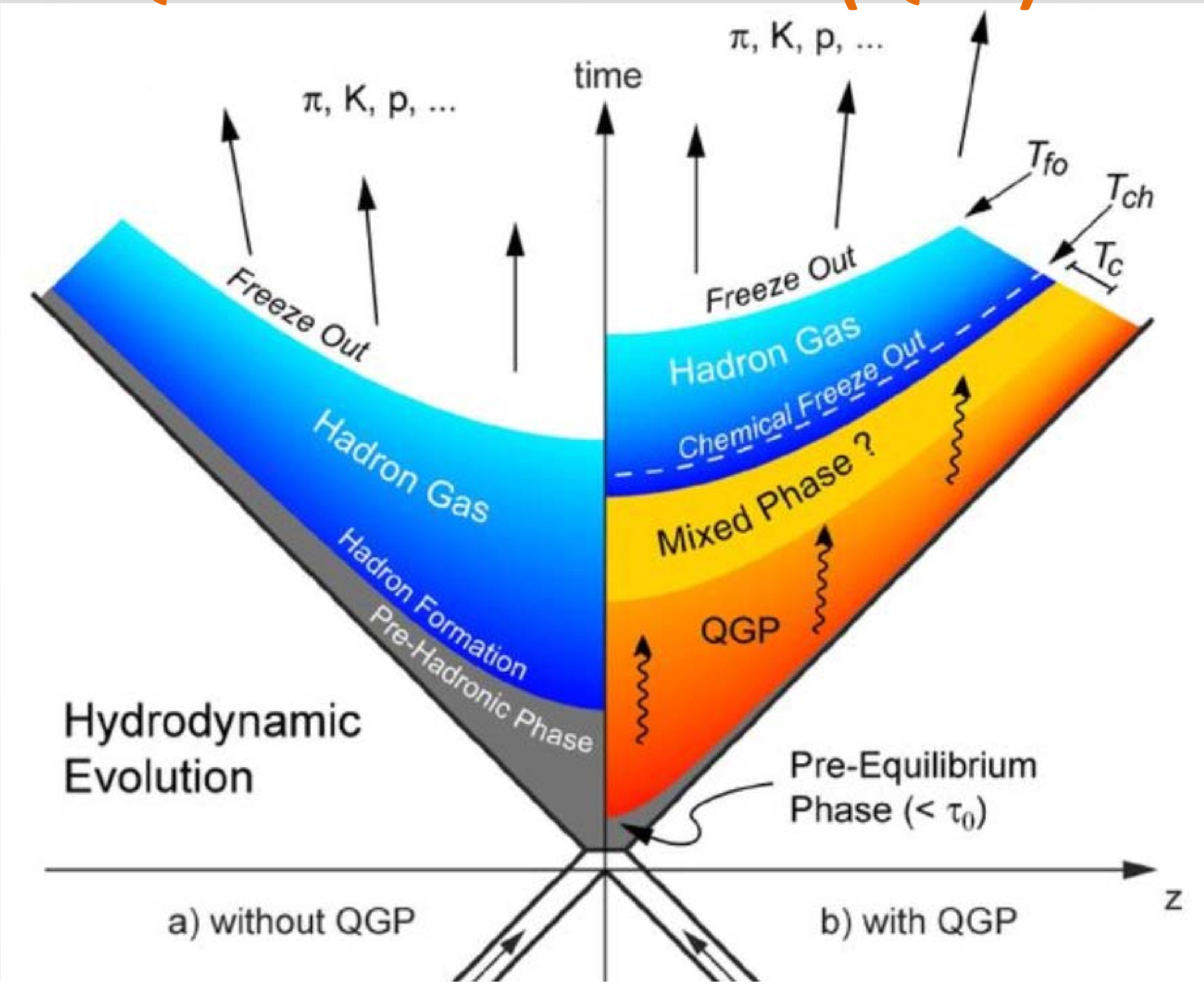
The Quark Gluon Plasma (QGP) in AA

How ? => By colliding heavy ions at very high energies !





The Quark Gluon Plasma (QGP) in AA



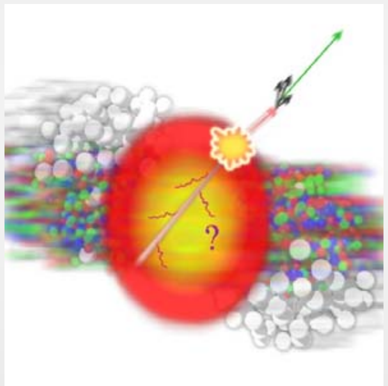
« Hard » probes

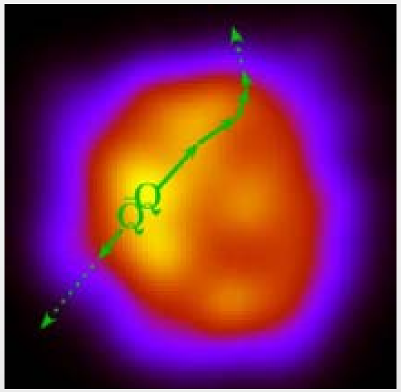
To study the medium properties before the freeze out «horizon»…

Deconfined ? Density and T ? Transport properties ? …

… one can analyse the « tomography » of the medium as seen by the hard probes (⇔ incomplete thermalisation)

High p_T partons quenching





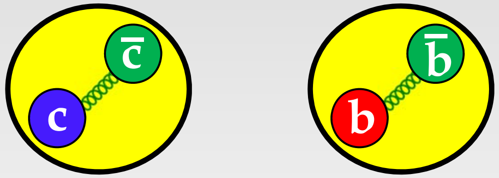
Massive quarks diffusion

Why hard probes ?

- ✓ Produced only in early pQCD processes before the GQP medium
- ✓ Do not flow hydrodynamically but propagate/interact inside the medium via other processes sensitive to its properties
- ✓ Less sensitive to hadronic stages

Quarkonia: Φ

Quarks			
	u up	*c* charm	*t* top
	d down	*s* strange	*b* bottom



CHARMONIA BOTTOMONIA

QUARKONIA

Various (more or less) tightly bound energy states

CHARMONIA

J/ψ : m = 3.096 GeV/c

χ_{cJ} **:** m ≈ 3.5 GeV/c

ψ' : m = 3.686 GeV/c

BOTTOMONIA

ϒ(1S) : m = 9.460 GeV/c

χ_{bJ} **:** m ≈ 9.9 GeV/c

ϒ(2S) : m = 10.023 GeV/c

ϒ(3S) : m = 10.355 GeV/c

Quarkonia suppression as HP

Expected medium effects : the « Quarkonia suppression »

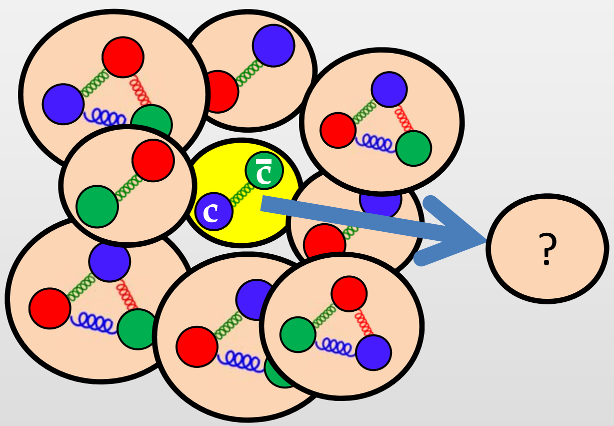
Smaller amount of quarkonia produced in heavy ion collisions
per binary nucleon collision as compared to pp collisions.

Quantified with the *nuclear modfication factor:*

$$R_{\rm AA}(p_{\rm T},\eta) = \frac{dN^{\rm AA}/d^2p_{\rm T}d\eta}{\langle N_{\rm coll}\rangle\, dN^{\rm pp}/d^2p_{\rm T}d\eta}$$

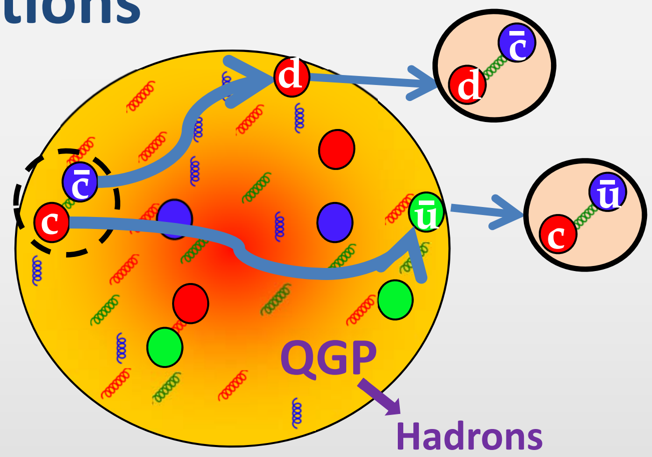
Different contributions

In hadronic phases:



« Normal » suppression (~ small)
From Cold Nuclear Matter effects

In QGP:

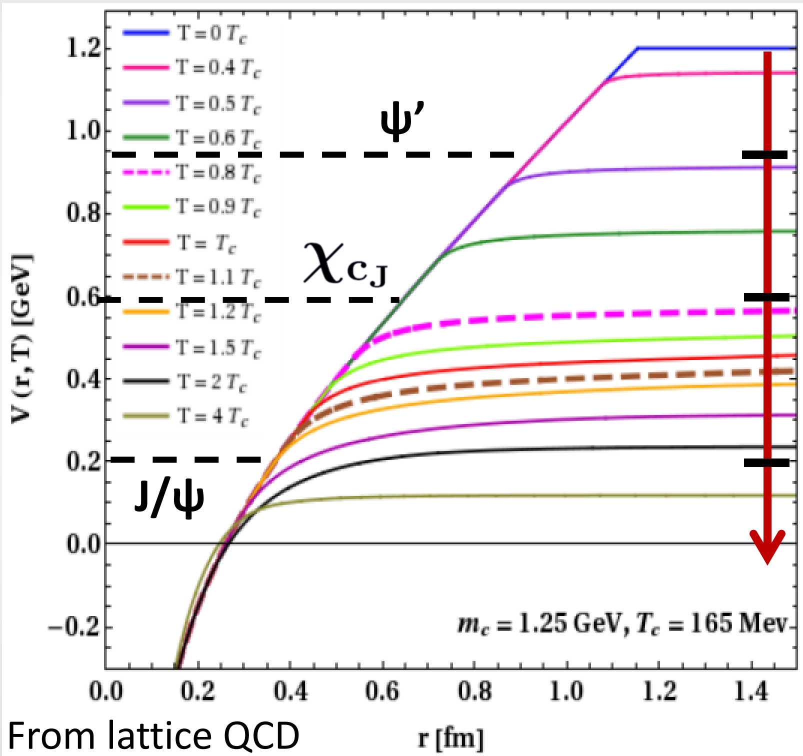


« Abnormal » suppression
from color screening and collisions with the medium partons

Historical models

Sequential suppression (Matsui and Satz)

$Q\bar{Q}$ color potential



T_{QGP}=0

$T_{diss}(\psi')$

$T_{diss}(\chi_{cJ})$

$T_{diss}(J/\psi)$

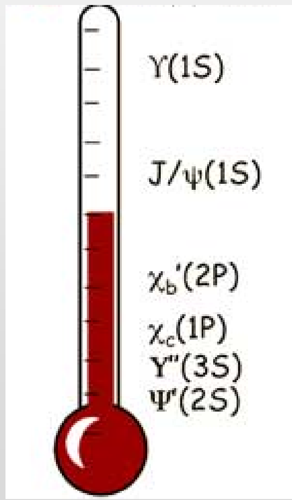
T_{QGP}->∞

T ↗ => screening ↗ => progressive states melting

« all or nothing »:

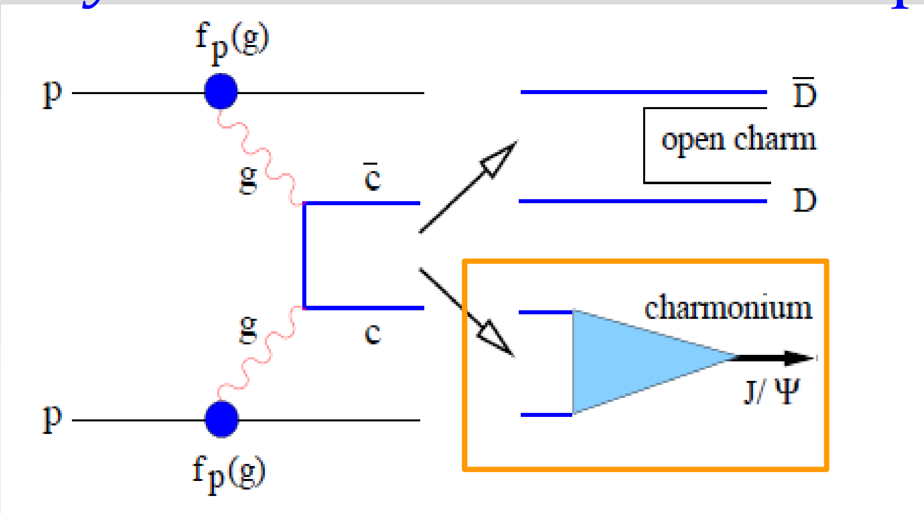
- If $T_{early\ QGP} > T_{diss}$ => the state is not produced
- If $T_{early\ QGP} < T_{diss}$ => the state is produced like in pp

=> *Quarkonia as early QGP thermometer*



T. Matsui and H. Satz, Phys. Lett. B 178 (1986) 416

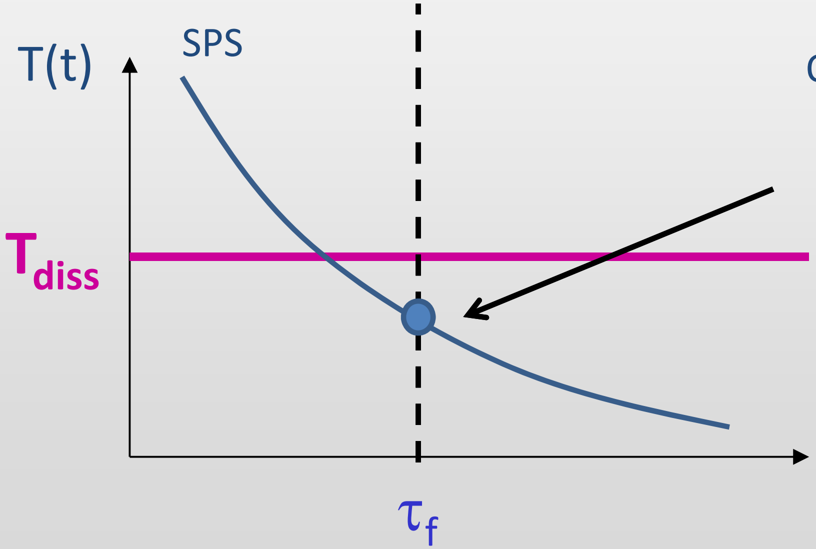
Dynamical version of the sequential suppression scenario



a) In vacuum: Quarkonia are formed after some "formation time" τ_f (typically the Heisenberg time), usually assumed to be independent of the surrounding medium

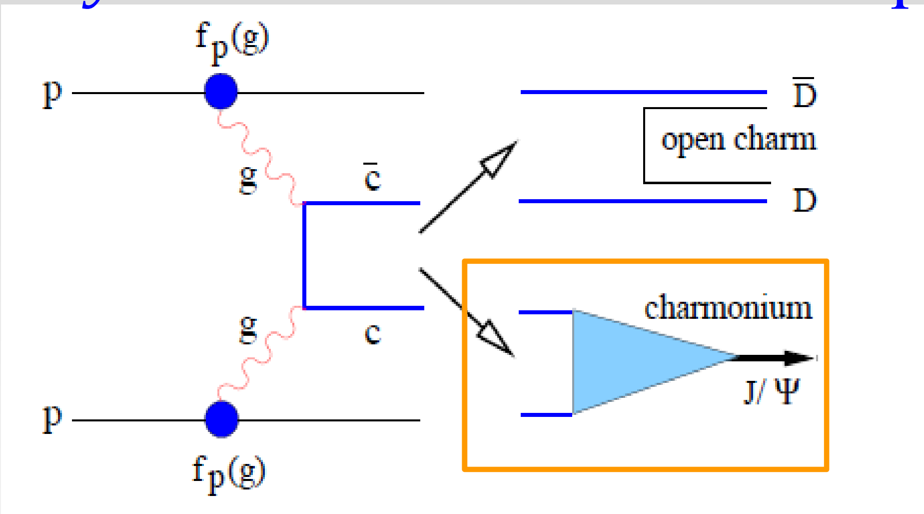
Standard folklore of sequential suppression: b.1) If $T(\tau_f,x_0)<T_{diss}$ the quarkonia is indeed created (as in vacuum)

Local temperature in the medium



Quarkonia state formed as in the vacuum

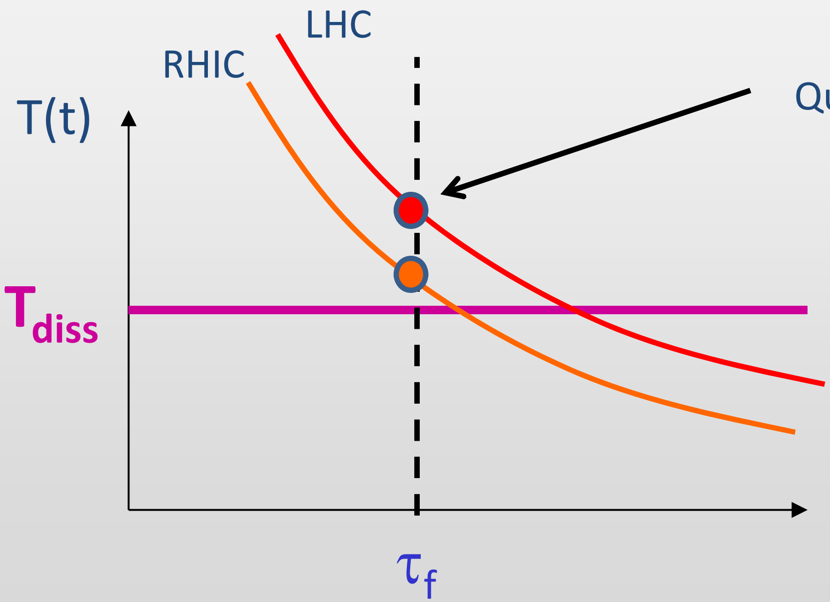
Dynamical version of the sequential suppression scenario



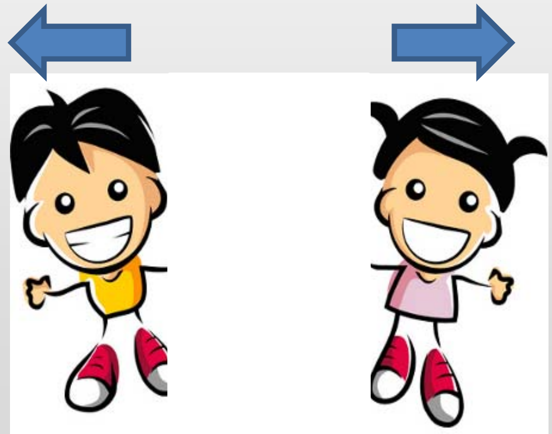
a) In vacuum: Quarkonia are formed after some "formation time" τ_f (typically the Heisenberg time), usually assumed to be independent of the surrounding medium

Standard folklore of sequential suppression: b.2) If $T(\tau_f,x_0)>T_{diss}$ the quarkonia is NOT created (Q-Qbar pair is "lost" for quarkonia production)

Local temperature in the medium



Quarkonia state "suppressed"



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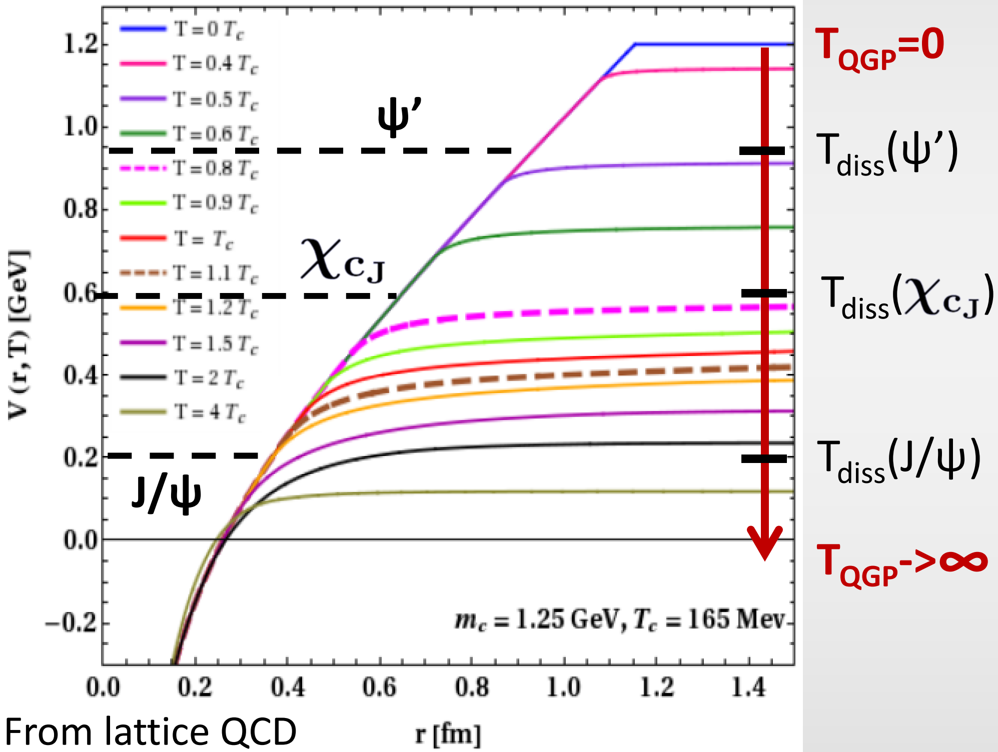


From lattice QCD

T ↗ => screening ↗ => progressive states melting

« all or nothing »:

- If $T_{early\ QGP} > T_{diss}$ => the state is not produced
- If $T_{early\ QGP} < T_{diss}$ => the state is produced like in pp

=> *Quarkonia as early QGP thermometer*

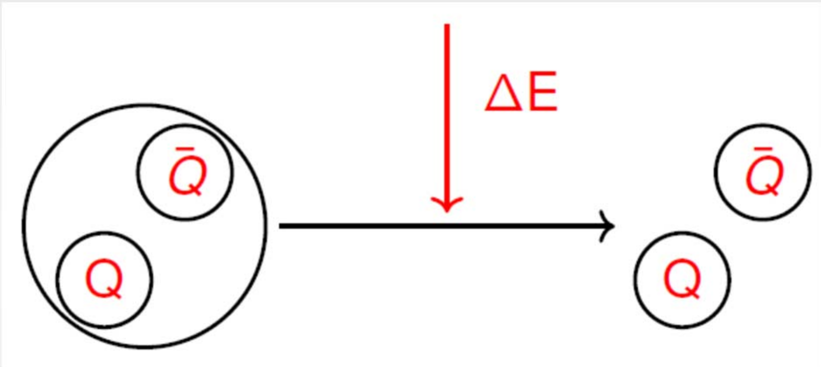
BUT:

- Everything at the BEGINNING in quasi-stationnary medium
- Adiabatic evolution
- No interactions with partons
- Questionnable T_{diss}

T. Matsui and H. Satz, Phys. Lett. B 178 (1986) 416

Alternate suppression mechanisms

- Collisions with medium partons (gluo-dissociaton, q – quarkonia quasi elastic scattering)



- => pair dissociation => **Suppression**

- ⇔ loss of probability of the quarkonia … Often described by some imaginary potential in modern approaches (see 2nd part)

Historical models

Suppression $N_{J/\psi} = \beta N_{J/\psi}(t=0)$

Precise β-value: depends on the dynamics of the system / physical assumptions

Recombination (Braun Munzinger, Stachel, Gorenstein,…)

5 October 2000

PHYSICS LETTERS B

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(Non) thermal aspects of charmonium production and a new look at J/ψ suppression ☆

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Abstract

To investigate a recent proposal that J/ψ production in ultra-relativistic nuclear collisions is of thermal origin we have reanalyzed the data from the NA38/50 Collaboration within a thermal model including charm. Comparison of the calculated with measured yields demonstrates the non-thermal origin of hidden charm production at SPS energy. However, the ratio ψ′/(J/ψ) exhibits, in central nucleus-nucleus collisions, thermal features which lead us to a new interpretation of open charm and charmonium production at SPS energy. Implications for RHIC and LHC energy measurements will be discussed.

$$N_{J/\psi} = \alpha \frac{N_{c\bar{c}}^2}{N_{\text{charged}}}$$

Precise α-value: depends on the dynamics of the system / physical assumptions

Statistical hadronisation… complete dissociation and loss of memory of quarkonia in the QGP (=> no more hard probe *per se*.)

Strong hypothesis : everything happens to be thermalized just before freeze out… Not relevant for high p_T quarkonia

Schematic summary of approaches

Initial quasi stationary
Sequential Suppression
assumption
(Matsui & Satz 86)

Final quasi "instantaneous"*
Statistical Hadronisation
assumption
(Andronic, Braun-Munzinger & Stachel)

* Personal interpretation

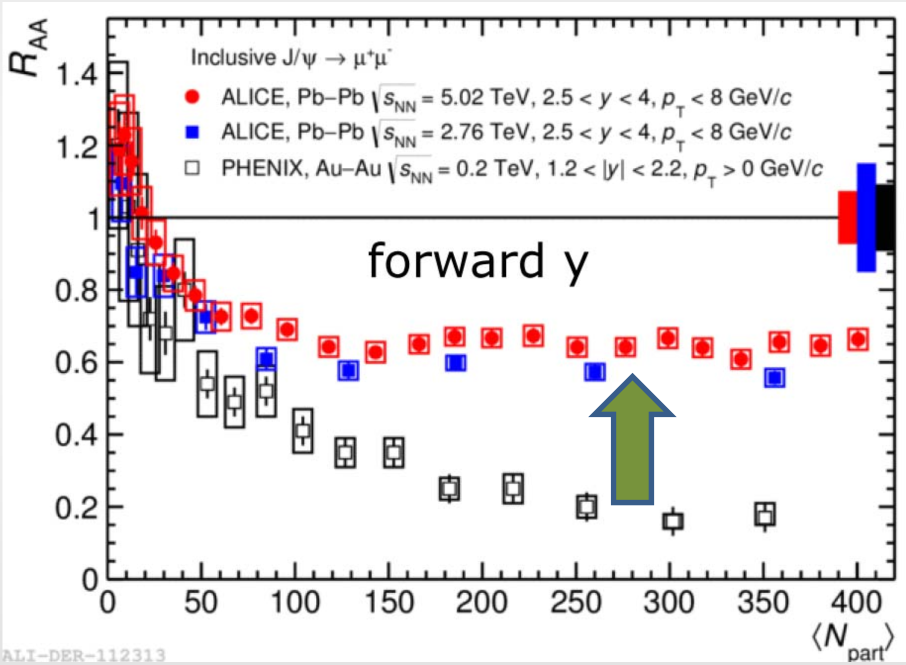
$$N_{J/\psi} = \beta N_{J/\psi}(t=0)$$

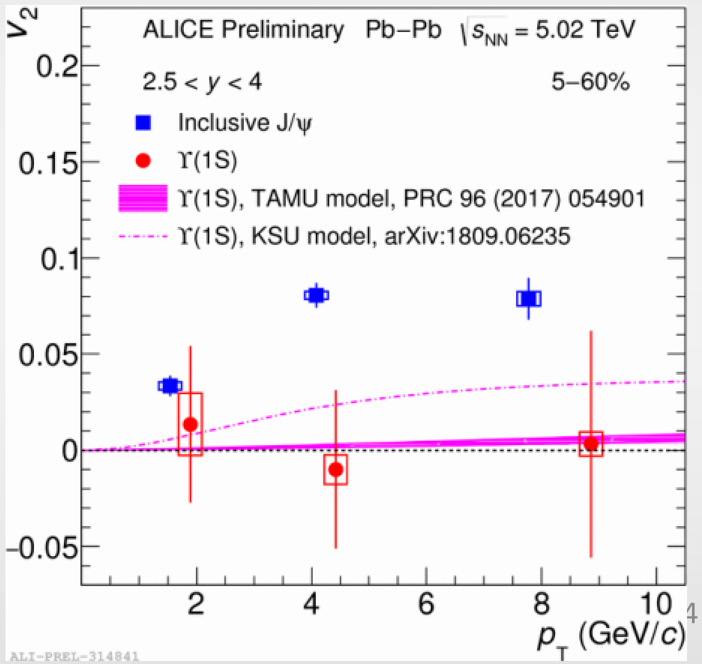
$$N_{J/\psi} = \alpha \frac{N_{c\bar{c}}^2}{N_{\text{charged}}}$$

Looking at recent data

Strong Hints for some charm recombination as:

- **Less suppression passing from RHIC -> LHC collider (larger T)**
- **J/ψ benefit from medium (elliptical) flow …**





- **Upsilon states, on the other hand, do not seem to « flow »**

Historical models

In medium Recombination (…Thews, Rafelski, Schroedter, 2000)

PHYSICAL REVIEW C, VOLUME 63, 054905

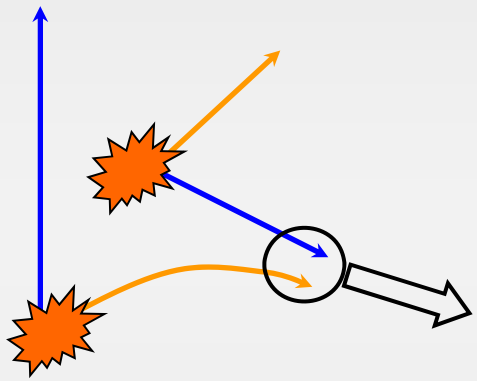
Enhanced J/ψ production in deconfined quark matter

Robert L. Thews, Martin Schroedter, and Johann Rafelski
Department of Physics, University of Arizona, Tucson, Arizona 85721
(Received 29 August 2000; published 23 April 2001)

In high energy heavy ion collisions at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven and the Large Hadron Collider at CERN, each central event will contain multiple pairs of heavy quarks. If a region of deconfined quarks and gluons is formed, a mechanism for additional formation of heavy quarkonium bound states will be activated. This is a result of the mobility of heavy quarks in the deconfined region, such that bound states can be formed from a quark and an antiquark that were originally produced in separate incoherent interactions. Model estimates of this effect for J/ψ production at RHIC indicate that significant enhancements are to be expected. Experimental observation of such enhanced production would provide evidence for deconfinement unlikely to be compatible with competing scenarios.

DOI: 10.1103/PhysRevC.63.054905 PACS number(s): 12.38.Mh, 25.75.−q, 14.40.Gx

Assume the existence of bound states inside QGP.



Kinetic rate equations $$\frac{dN_{J/\psi}(\tau)}{d\tau} = \frac{\lambda_{\mathrm{F}}(\tau)}{V(\tau)} N_c N_{\bar{c}} - \lambda_{\mathrm{D}}(\tau)\rho_{\mathrm{g}}(\tau) N_{J/\psi}(\tau)$$

Inholds both suppression and (re) combinaison… one of the first dynamical model.

Initial quasi stationary
Sequential Suppression
assumption
(Matsui & Satz 86)

Final quasi "instantaneous"*
Statistical Hadronisation
assumption
(Andronic, Braun-Munzinger & Stachel)

???

Dynamical Models (implicit hope to measure T above Tc); often relies on some hypothesis not fully justified:

- (In medium) dissociation cross-section (how valid in dense medium ?)
- Prob survival $= \exp\left(-\int_{t_0}^{t_{\rm fin}} \Gamma(T(t))dt\right)$
- …

???

Nowadays, all state-of-the art dynamical models include both suppression and recombinaison, although not fully consistently

A central quantity: the decay rate Γ

Several approaches

pQCD view (Bhanot & Peskin), later on consolidated by NRQCD (Brambilla & Vairo)

$$\Phi + g \to Q + \bar{Q}$$

Dissociation cross section σ

$$\Gamma_\Phi(T) = \langle \sigma n_g \rangle_T$$

QFT/Lattice QCD

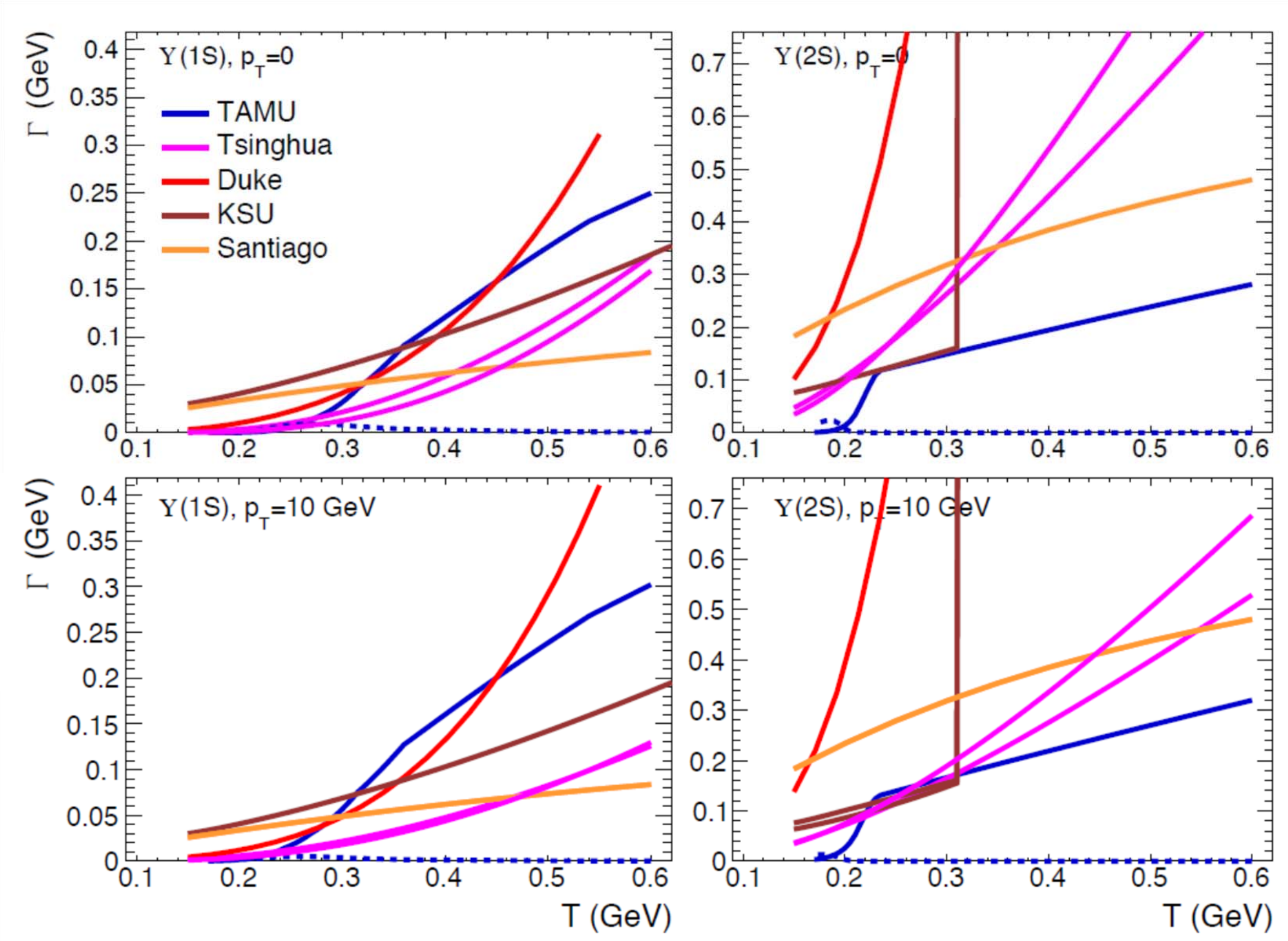
Time correlator

$$\mathcal{C}_>(t,\vec{r}) \approx \langle \psi(t,\tfrac{\vec{r}}{2})\bar{\psi}(t,-\tfrac{\vec{r}}{2})\psi(0,0)\bar{\psi}(0,0)\rangle$$

Satisfies Schroedinger equation with imaginary potential iW . Breakthrough by Laine et al. (2006)

$$\Gamma_\Phi(T) = -2\langle \Phi | W | \Phi \rangle$$

A central quantity,… but disputed

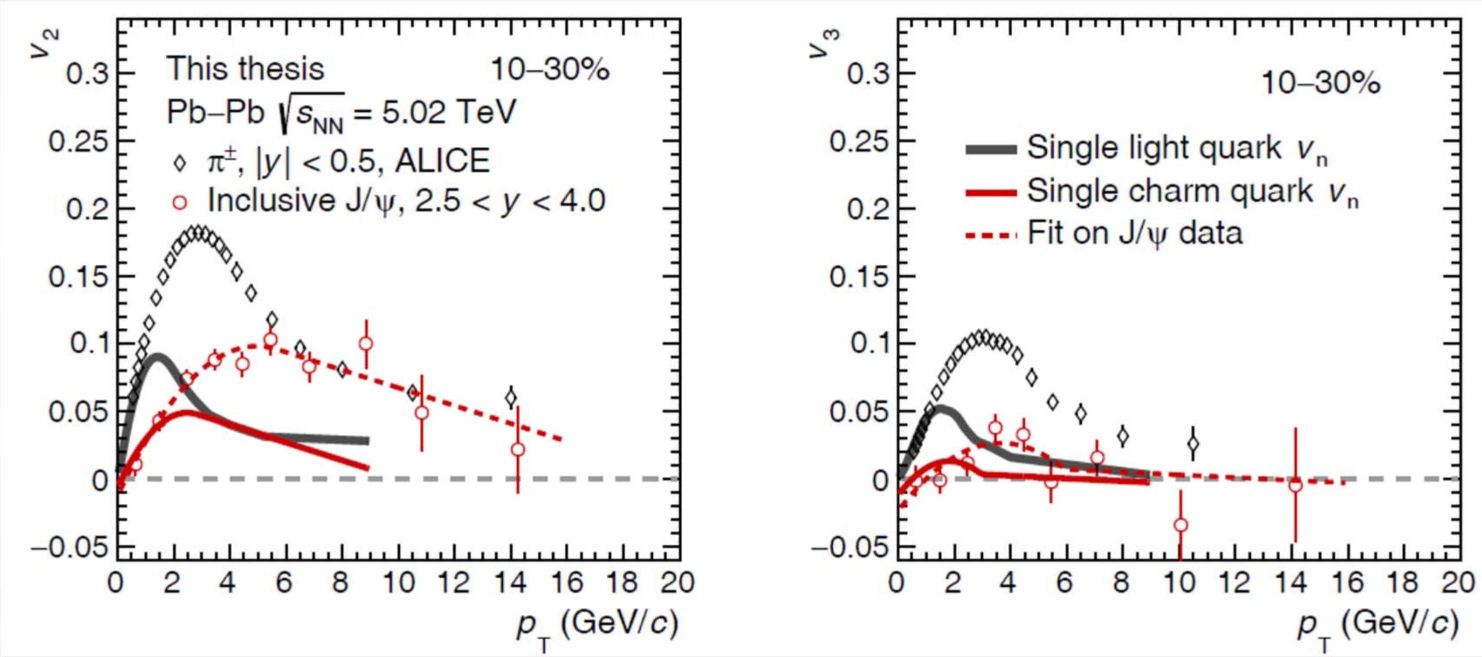


EMMI RRTF on QUARKONIA (Dec 2019)

Looking at recent data

Coalescence explains it all ?

- v_2 & $v_3(\pi)$ => v_2 & $v_3(q)$ (reverse engineering)
- v_2 & $v_3(J/\psi$ fit) => v_2 & $v_3(c)$ (reverse engineering)

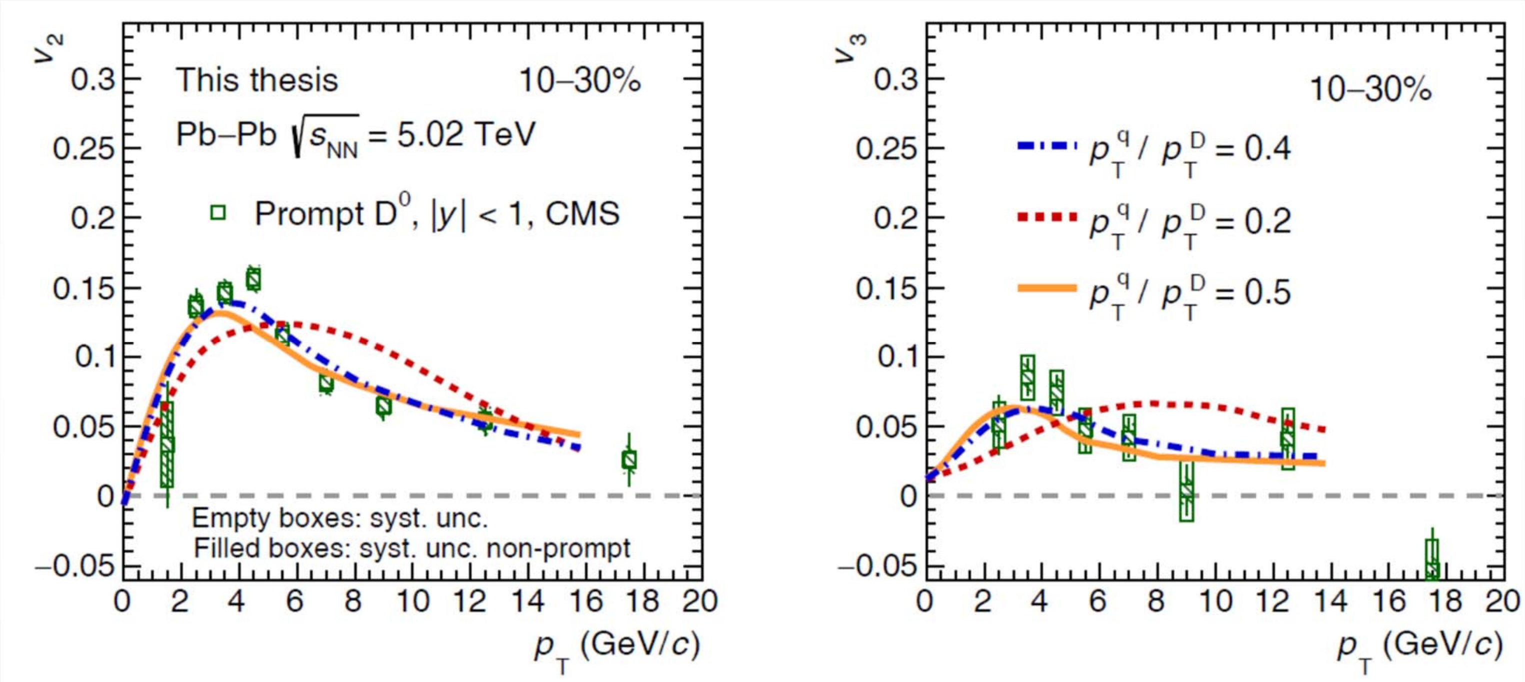


Shreyasi Acharya et al. (ALICE) JHEP, 10:141, 2020.

Looking at recent data

Coalescence explains it all ?

- $v_n(q)$ & $v_n(c)$ + relative weights of masses (momenta) => $v_n(D)$



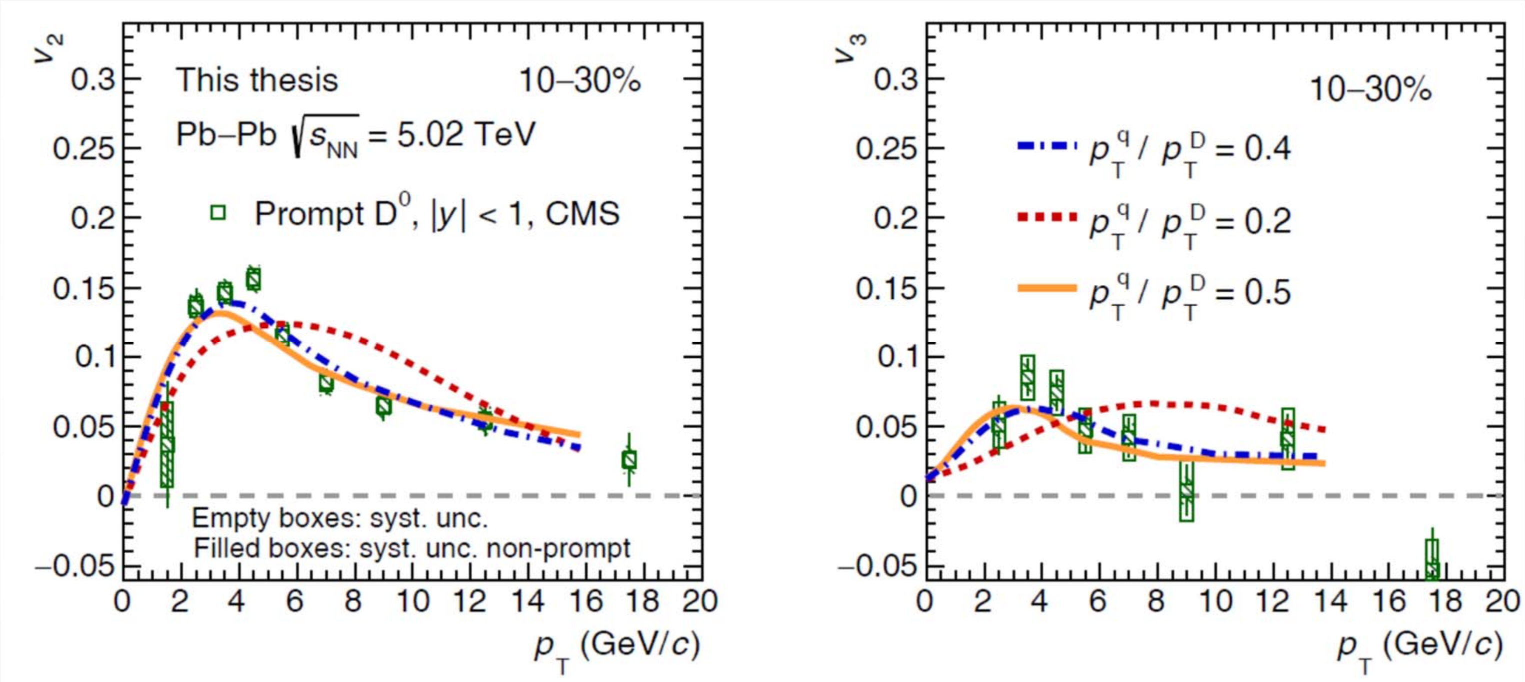
Shreyasi Acharya et al. (ALICE) JHEP, 10:141, 2020.

- Good global agreement for $p_T^q/p_T^D = 0.4 \Leftrightarrow m_q \approx 0.7 – 0.8$ GeV
- Either … you consider that this is way too high => discard the plausibility of coalescence approach

Looking at recent data

Coalescence explains it all ?

- $v_n(q)$ & $v_n(c)$ + relative weights of masses (momenta) => $v_n(D)$



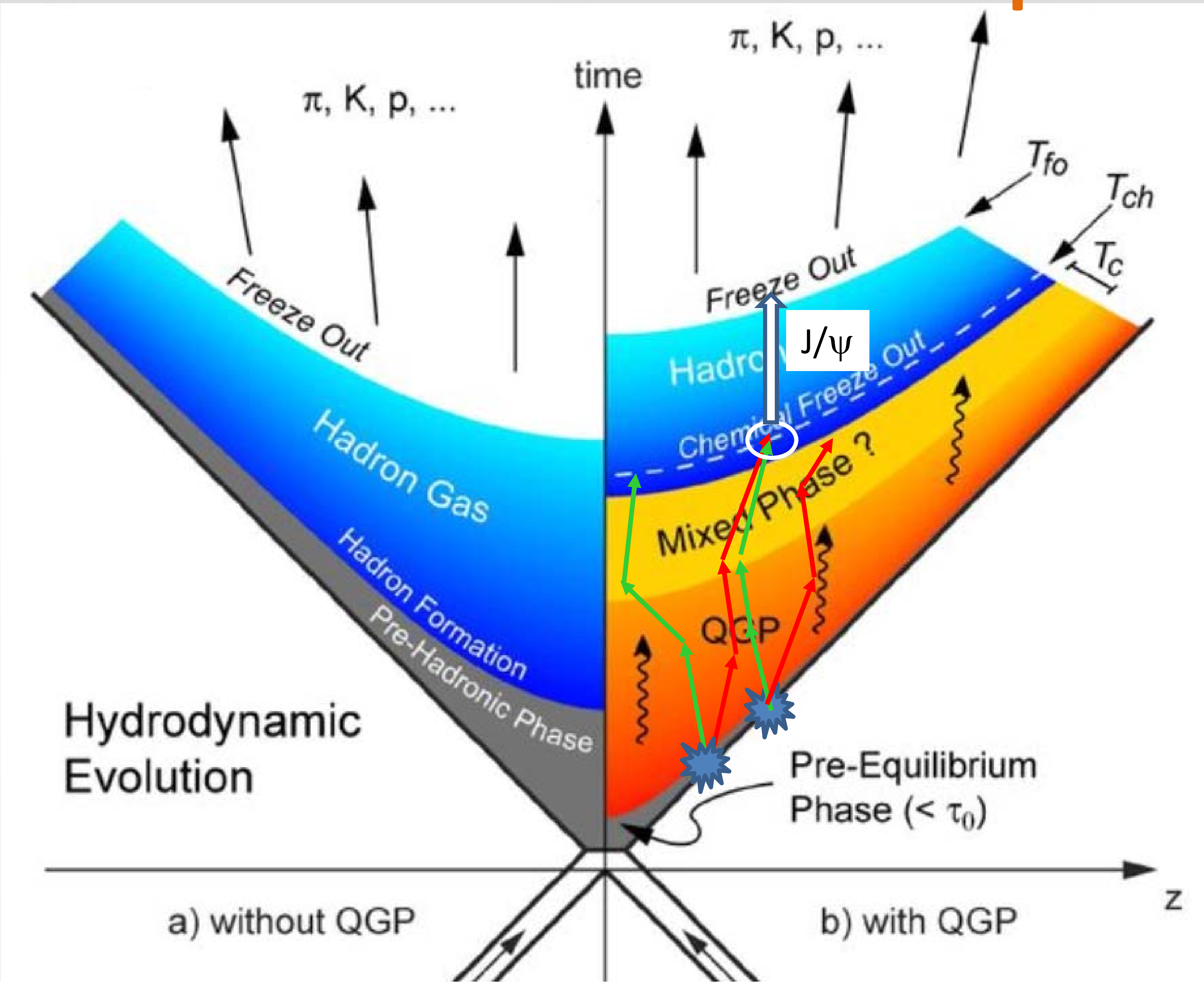
Shreyasi Acharya et al. (ALICE) JHEP, 10:141, 2020.

- Good global agreement for $p_T^q/p_T^D = 0.4 \Leftrightarrow m_q \approx 0.7 - 0.8$ GeV
- Or you consider such light-quark masses are achievable close to T_c => coalescence is indeed a good scheme to understand both charmonia and D mesons flows…

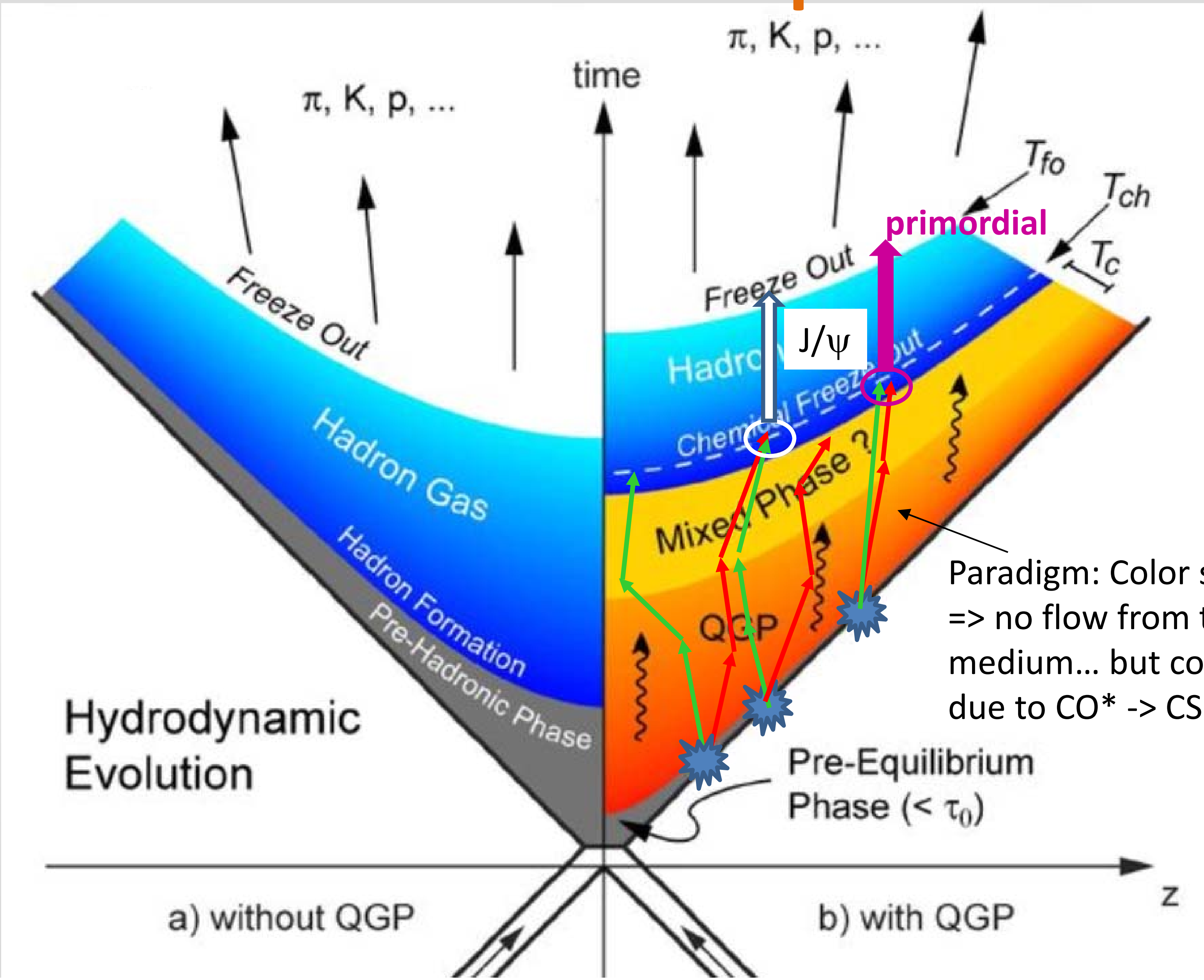
However, no attempt to explain $R_{AA}(p_T)$

Disappearance of all $c - \bar{c}$ correlations before FO

Charmonia in the coalescence picture



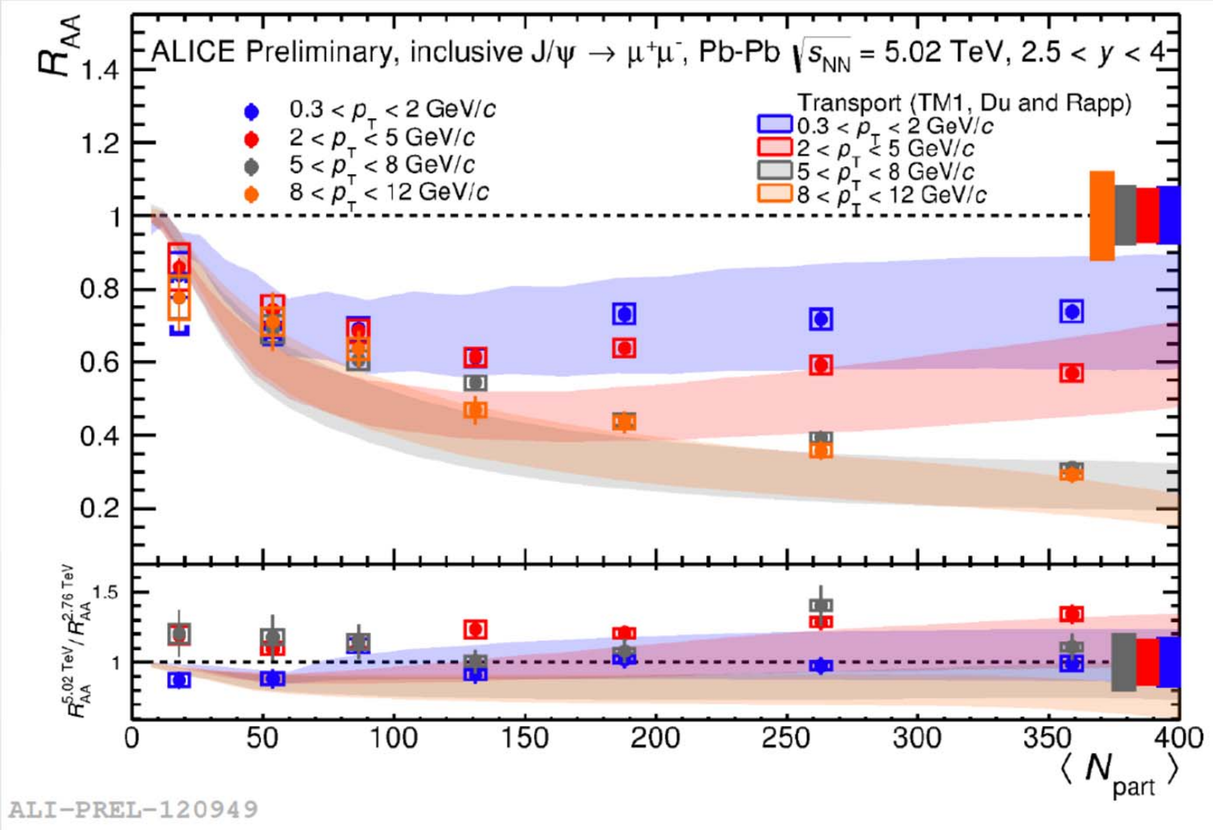
Charmonia in the transport models



Paradigm: Color singlet => no flow from the medium… but could be due to CO* -> CS

Looking at recent data

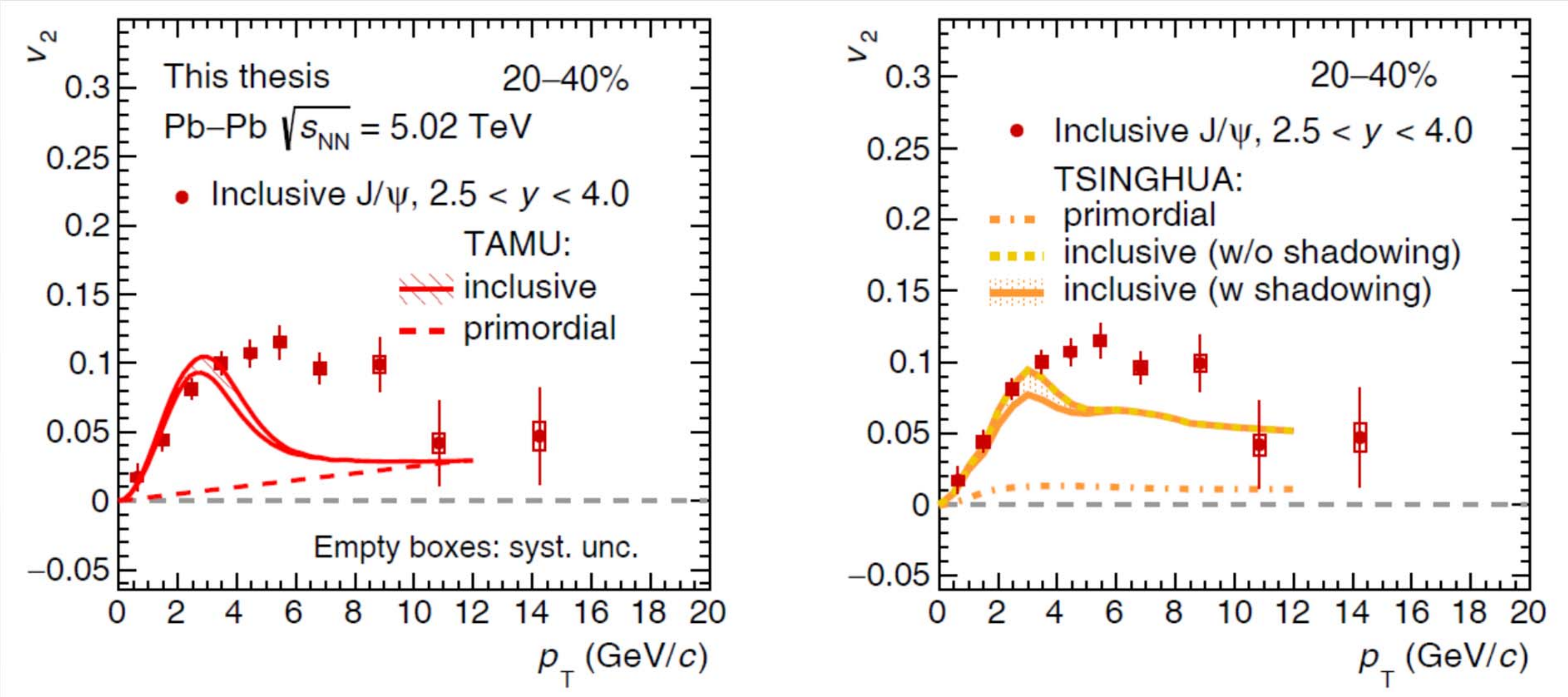
Transport theories



- In transport theory, primordial component is mandatory to reproduce the absolute production as a function of centrality & p_T class

Looking at recent data

Transport theories



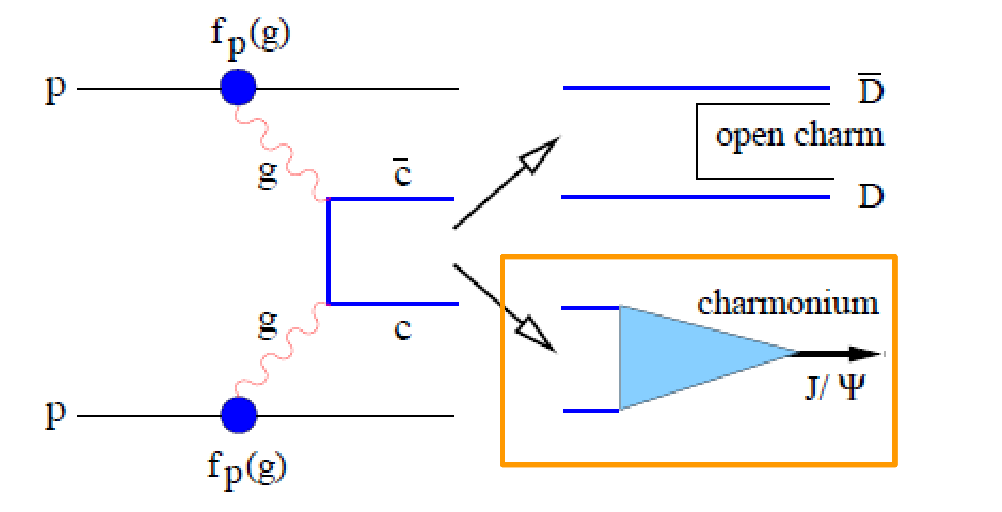
- Good agreement for low p_T, where J/ψ formation proceeds through recombination at FO
- Disagreement from intermediate p_T on, where primordial production start having a large weight (crucial for the $R_{AA}(p_T)$)

Motivations

- Need to revisit how robustly we understand the survival of primordial component
- Possible role of singlet <-> octet transitions

Quantum coherence

Once upon a time



Open heavy flavor and quarkonia assumed to be uncorrelated

Formed after some "formation time" τ_f (typically the Heisenberg time), usually assumed to be independent of the surrounding medium

Common belief in QGP community:

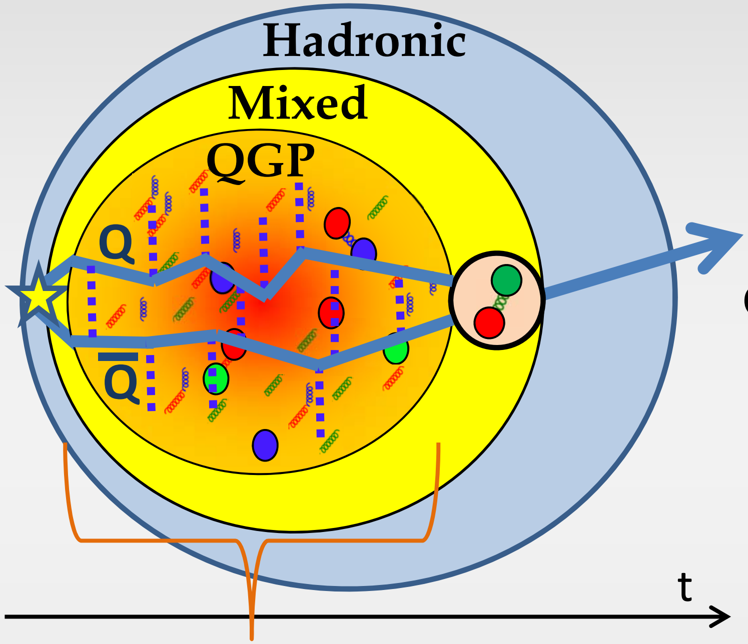
Quarkonia initially « formed » in QGP and survive with an *individual* survival probability

$$S(t) = e^{-\int_0^t \Gamma(T(t'))dt'}$$

or

$$S(t) = e^{-\int_{\tau_f}^t \Gamma(T(t'))dt'}$$

Quantum coherence



Quarkonium

?

Open mesons

only resolved at the end of the evolution

Beware of quantum coherences during the whole evolution !

~~Very complicated QFT problem at finite T(t) !!!~~

How to proceed ?

Dating back to Blaizot & Ollitrault, Thews, Cugnon and Gossiaux; early 90's

Motivations

- Need to revisit how robustly we understand the survival of primordial component
- Possible role of singlet <-> octet transitions
- J/ψ are *quantum* bound states => need for a formalism that preserves *quantum* properties… and continuous transitions between bound and unbound states

New motto: QQ real-time dynamics with the Open Quantum System framework

Consider:

color screening, (non-)dissociative interactions and QGP dynamics

INNER DYNAMICS OF EACH $Q\bar{Q}$ PAIR

A dynamical and continuous picture of the dissociation, recombination, transitions between states, and energy exchanges with the QGP

+ (possibly)

$Q\bar{Q}$ PAIRS EVOLUTION IN A VERY DYNAMIC QGP

Realistic t-dependent background:

Monte-Carlo event generator with initial fluctuations

=> *Quarkonia as QGP continuous thermometers*

Motivations

- Need to revisit how robustly we understand the survival of primordial component
- Possible role of singlet <-> octet transitions
- J/ψ are *quantum* bound states => need for a formalism that preserves *quantum* properties… and continuous transitions between bound and unbound states
- Even with the tools and methods of the OQS, such prerequisite may be extremely difficult to achieve for the case of the recombination of many c-cbar pairs.

τ_E: **environment correlation time**

$$\tau_E \sim \frac{1}{T}$$

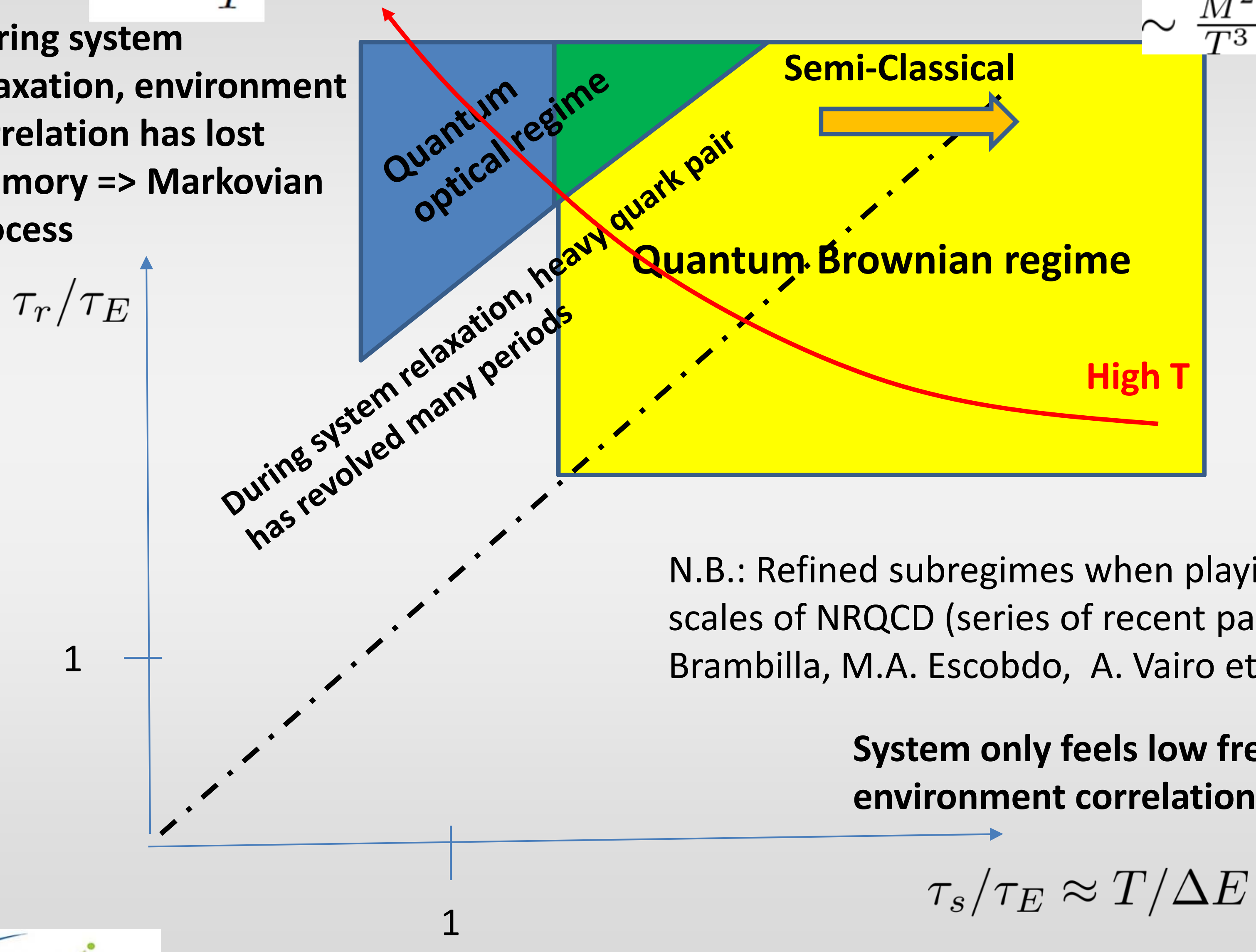
τ_S: **system intrinsic time scale**

$$\tau_S \approx \frac{1}{Mv^2} \approx \frac{1}{\Delta E}$$

τ_R: **system relax time**

$$\tau_r \sim \frac{1}{\Gamma} \approx \frac{1}{\alpha T (m_D^2 \langle r^2 \rangle)} \sim \frac{M^2}{T^3} \gg \tau_E$$

During system relaxation, environment correlation has lost memory => Markovian process



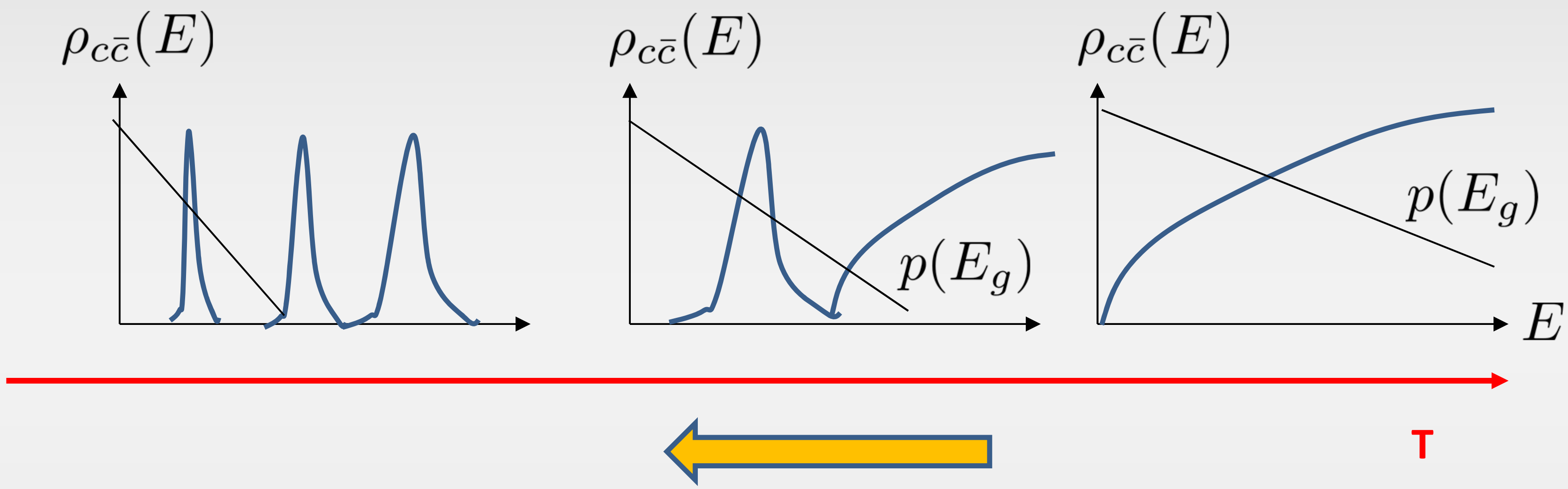
$$\frac{\tau_r}{\tau_E} \sim \frac{M^2}{T^2}$$

N.B.: Refined subregimes when playing with the scales of NRQCD (series of recent papers by N. Brambilla, M.A. Escobdo, A. Vairo et al)

System only feels low frequency part of environment correlation

Not clear all states goes from one regime to the other at the same T

Several regimes / effects



Time

Gluo-dissociation of well identified levels by scarce “high-energy” gluons (dilute medium => cross section ok)

Multiple scattering on quasi free states

Well identified formalisms (Quantum Master Equation, Boltzmann transport, Stochastic equations,…) in well identified regimes, but continuous evolution and no unique framework continuously applicable (to my knowledge)

Our current investigations

- Schroedinger Langevin Equation (presented at previous NED/TURIC workshops)

- Quantum Master Equation of Blaizot - Escobedo (with Stéphane Delorme, Roland Katz and Thierry Gousset)

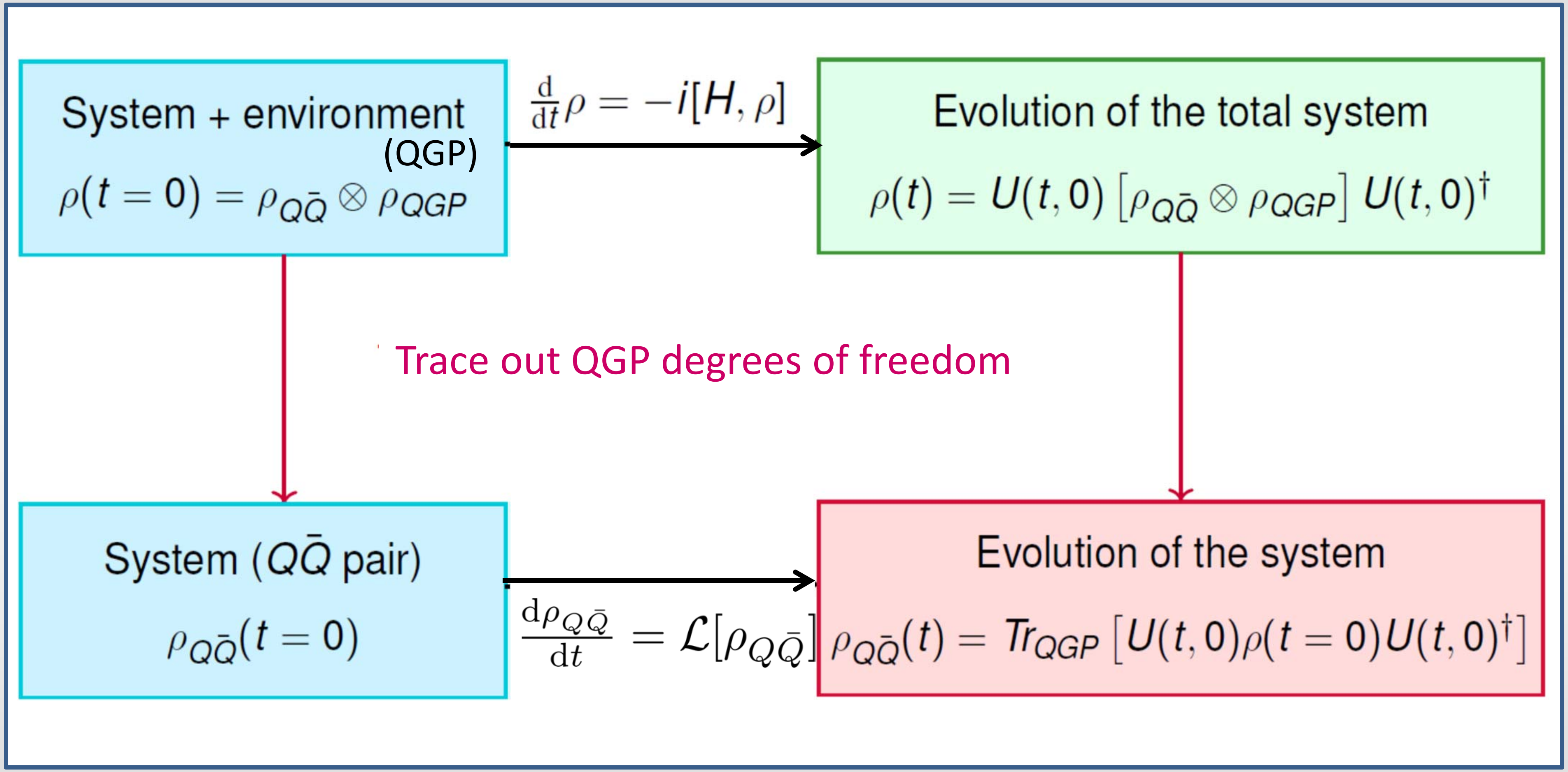
Jean-Paul Blaizot and Miguel Angel Escobedo, JHEP06 (2018) 034

- Remler density matrix approach (with Denys Yen Arrebato Villar and Joerg Aichelin)

E.A. Remler, ANNALS OF PHYSICS 136, 293-316 (1981)

Both formalisms able to deal with the dynamical recombination

Open Quantum System Formalism



However, $\mathcal{L}[\cdot]$ is generically a non local operator in time

Linblad Equation

Case of a Markovian time-evolution (τ_E smallest scale) => Lindblad equation (local in time)

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}(t)\right] + \sum_i \gamma_i \left[L_i \rho_{Q\bar{Q}}(t) L_i^\dagger - \frac{1}{2}\left\{L_i L_i^\dagger, \rho_{Q\bar{Q}}(t)\right\}\right]$$

$H_{Q\bar{Q}}$: $\{Q, \bar{Q}\}$ kinetics + screened potential V

L_i : Collapse operators (or dissipators), depend on the properties of the medium

3 important conservation properties :

$\rho_{Q\bar{Q}}^\dagger = \rho_{Q\bar{Q}}$	$\mathrm{Tr}[\rho_{Q\bar{Q}}] = 1$	$\langle\varphi\|\rho_{Q\bar{Q}}\|\varphi\rangle > 0, \forall\|\varphi\rangle$
(Hermiticity)	(Unitarity)	(Positivity)

Case of the B-E Quantum Master Equation

$$i\frac{d\mathcal{D}}{dt} = [\mathcal{H}, \mathcal{D}] \xrightarrow[\text{representation}]{\text{Interaction}} i\frac{d\mathcal{D}^I(t)}{dt} = [\mathcal{H}_1(t), \mathcal{D}^I(t)]$$

$$\mathcal{D}^I(t) = \mathcal{U}_I(t,t_0)\mathcal{D}(t_0)\mathcal{U}_I^\dagger(t,t_0)$$

$$\mathcal{H} = \mathcal{H}_Q + \mathcal{H}_1 + \mathcal{H}_{pl}$$

Coulomb gauge

Free Quark Hamiltonian

Plasma Hamiltonian

Quark-Plasma Interactions…

$$H_1 = -g\int_{\boldsymbol{r}} A_0^a(\boldsymbol{r})\, n^a(\boldsymbol{r})$$

No magnetic term (NR)

color charge density of the heavy particles

… treated as a perturbation

Average over plasma d.o.f + rapid environment hypothesis

Generic Linblad – like QME on $\mathcal{D}_Q$

$$\frac{\mathrm{d}\mathcal{D}_Q^I(t)}{\mathrm{d}t} = -\int_{t_0}^t \mathrm{d}t' \int_{\boldsymbol{x}\boldsymbol{x}'} \big([n^a(t,\boldsymbol{x}), n^a(t',\boldsymbol{x}')\mathcal{D}_Q^I(t_0)]\Delta^>(t-t',\boldsymbol{x}-\boldsymbol{x}') + [\mathcal{D}_Q^I(t_0)n^a(t',\boldsymbol{x}'), n^a(t,\boldsymbol{x})]\Delta^<(t-t',\boldsymbol{x}-\boldsymbol{x}')\big)$$

$\Delta^>, \Delta^<$ Time ordered HTL gluon propagators

Case of the B-E Quantum Master Equation

$$\frac{\mathrm{d}\mathcal{D}_Q^I(t)}{\mathrm{d}t} = -\int_{t_0}^{t} \mathrm{d}t' \int_{\boldsymbol{x}\boldsymbol{x}'} \Big([n^a(t,\boldsymbol{x}), n^a(t',\boldsymbol{x}')\mathcal{D}_Q^I(t_0)]\Delta^>(t-t',\boldsymbol{x}-\boldsymbol{x}') + [\mathcal{D}_Q^I(t_0)n^a(t',\boldsymbol{x}'), n^a(t,\boldsymbol{x})]\Delta^<(t-t',\boldsymbol{x}-\boldsymbol{x}')\Big)$$

Not local in time

Next hypothesis/simplification : large relaxation time ($\tau_R >> \tau_E$).

=> Slow evolution of D_Q over QGP correlation time τ_E

$$\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} + i[H_Q, \mathcal{D}_Q(t)] = -\int_{\boldsymbol{x}\boldsymbol{x}'} \int_0^{t-t_0} \mathrm{d}\tau \, [n^a_{\boldsymbol{x}}, U_Q(\tau) n^a_{\boldsymbol{x}'} U_Q^\dagger(\tau)\mathcal{D}_Q(t)] \, \Delta^>(\tau; \boldsymbol{x}-\boldsymbol{x}')) - \int_{\boldsymbol{x}\boldsymbol{x}'} \int_0^{t-t_0} \mathrm{d}\tau \, [\mathcal{D}_Q(t) U_Q(\tau) n^a_{\boldsymbol{x}'} U_Q^\dagger(\tau), n^a_{\boldsymbol{x}}] \, \Delta^<(\tau; \boldsymbol{x}-\boldsymbol{x}').$$

Free quarks

Local in time !

Case of the B-E Quantum Master Equation

Local in time !

$$\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} + i[H_Q, \mathcal{D}_Q(t)] = -\int_{\boldsymbol{x}\boldsymbol{x}'}\int_0^{t-t_0} \mathrm{d}\tau\, [n^a_{\boldsymbol{x}}, U_Q(\tau) n^a_{\boldsymbol{x}'} U_Q^\dagger(\tau)\mathcal{D}_Q(t)]\, \Delta^>(\tau; \boldsymbol{x}-\boldsymbol{x}'))$$
$$-\int_{\boldsymbol{x}\boldsymbol{x}'}\int_0^{t-t_0} \mathrm{d}\tau\, [\mathcal{D}_Q(t) U_Q(\tau) n^a_{\boldsymbol{x}'} U_Q^\dagger(\tau), n^a_{\boldsymbol{x}}]\, \Delta^<(\tau; \boldsymbol{x}-\boldsymbol{x}').$$

Further hypothesis/simplification : the response of the plasma to the perturbation caused by the heavy quarks is fast compared to the characteristic time scales of the heavy quark ($\tau_S >> \tau_E \Leftrightarrow T >> m_Q\, \alpha_s^2$).

High Temperature regime

=> Series expansion of U_Q around τ=0

$$\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} + i[H_Q, \mathcal{D}_Q(t)] \approx -\int_{\boldsymbol{x}\boldsymbol{x}'} [n^a_{\boldsymbol{x}}, n^a_{\boldsymbol{x}'}\mathcal{D}_Q] \int_0^\infty \mathrm{d}\tau \Delta^>(\tau; \boldsymbol{x}-\boldsymbol{x}'))$$
$$-\int_{\boldsymbol{x}\boldsymbol{x}'} [\mathcal{D}_Q n^a_{\boldsymbol{x}'}, n^a_{\boldsymbol{x}}] \int_0^\infty \mathrm{d}\tau\, \Delta^<(\tau; \boldsymbol{x}-\boldsymbol{x}')$$
$$+\int_{\boldsymbol{x}\boldsymbol{x}'} [n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] \int_0^\infty \mathrm{d}\tau\, \tau\, \Delta^>(\tau; \boldsymbol{x}-\boldsymbol{x}'))$$
$$+\int_{\boldsymbol{x}\boldsymbol{x}'} [\mathcal{D}_Q \dot{n}^a_{\boldsymbol{x}'}, n^a_{\boldsymbol{x}}] \int_0^\infty \mathrm{d}\tau\, \tau\, \Delta^<(\tau; \boldsymbol{x}-\boldsymbol{x}').$$

B-E Quantum Master Equation

$$
\begin{aligned}
\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} + i[H_Q, \mathcal{D}_Q(t)] \approx &- \int_{\boldsymbol{x}\boldsymbol{x}'} [n^a_{\boldsymbol{x}}, n^a_{\boldsymbol{x}'}\mathcal{D}_Q] \int_0^\infty \mathrm{d}\tau \Delta^>(\tau; \boldsymbol{x}-\boldsymbol{x}')) \\
&- \int_{\boldsymbol{x}\boldsymbol{x}'} [\mathcal{D}_Q n^a_{\boldsymbol{x}'}, n^a_{\boldsymbol{x}}] \int_0^\infty \mathrm{d}\tau\, \Delta^<(\tau; \boldsymbol{x}-\boldsymbol{x}') \\
&+ \int_{\boldsymbol{x}\boldsymbol{x}'} [n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] \int_0^\infty \mathrm{d}\tau\, \tau\, \Delta^>(\tau; \boldsymbol{x}-\boldsymbol{x}')) \\
&+ \int_{\boldsymbol{x}\boldsymbol{x}'} [\mathcal{D}_Q \dot{n}^a_{\boldsymbol{x}'}, n^a_{\boldsymbol{x}}] \int_0^\infty \mathrm{d}\tau\, \tau\, \Delta^<(\tau; \boldsymbol{x}-\boldsymbol{x}').
\end{aligned}
$$

Time integrals involve only the 0-frequency part of the HTL propagators, i.e. the real and imaginary potentials, leading to :

$$
\begin{aligned}
\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} + i[H_Q, \mathcal{D}_Q(t)] \approx &-\frac{i}{2} \int_{\boldsymbol{x}\boldsymbol{x}'} V(\boldsymbol{x}-\boldsymbol{x}')[n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q], \\
&+\frac{1}{2} \int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}') \left(\{n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2 n^a_{\boldsymbol{x}} \mathcal{D}_Q n^a_{\boldsymbol{x}'}\right) \\
&+\frac{i}{4T} \int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}') \left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q \dot{n}^a_{\boldsymbol{x}'}]\right)
\end{aligned}
$$

From there on, possibility to use lQCD potentials instead of HTL ones.

B-E Quantum Master Equation

Series expansion in τ_E/τ_S

Compact form: $\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} = \mathcal{L}\,\mathcal{D}_Q$ with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \cdots$

$$\mathcal{L}_0\,\mathcal{D}_Q \equiv -i[H_Q, \mathcal{D}_Q],$$

$$\mathcal{L}_1\,\mathcal{D}_Q \equiv -\frac{i}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} V(\boldsymbol{x}-\boldsymbol{x}')[n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q],$$

Mean field hamiltonian

$$\mathcal{L}_2\,\mathcal{D}_Q \equiv \frac{1}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left(\{n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2n^a_{\boldsymbol{x}}\mathcal{D}_Q n^a_{\boldsymbol{x}'}\right),$$

Fluctuations, Linblad form

$$\mathcal{L}_3\,\mathcal{D}_Q \equiv \frac{i}{4T}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q\dot{n}^a_{\boldsymbol{x}'}]\right)$$

Friction

N.B. : Friction is NOT of the Linbladian form => the evolution breaks positivity.

Positivity and Linblad form can be restored at the price **of extra subleading terms*** :

$$\left\{\left(n^a_{\mathbf{x}} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\left(n^a_{\mathbf{x}'} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right), \mathcal{D}_{Q\bar{Q}}\right\} - 2\left(n^a_{\mathbf{x}} + \frac{i}{4T}\dot{n}^a_{\mathbf{x}}\right)\mathcal{D}_{Q\bar{Q}}\left(n^a_{\mathbf{x}'} - \frac{i}{4T}\dot{n}^a_{\mathbf{x}'}\right)$$

$\mathcal{L}_2$

* As well as another time discretization

B-E Quantum Master Equation

Series expansion in τ_E/τ_S

Compact form: $\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} = \mathcal{L}\,\mathcal{D}_Q$ with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \cdots$

$$\mathcal{L}_0\,\mathcal{D}_Q \equiv -i[H_Q, \mathcal{D}_Q],$$

$$\mathcal{L}_1\,\mathcal{D}_Q \equiv -\frac{i}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} V(\boldsymbol{x}-\boldsymbol{x}')[n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q],$$

Mean field hamiltonian

$$\mathcal{L}_2\,\mathcal{D}_Q \equiv \frac{1}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left(\{n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2 n^a_{\boldsymbol{x}} \mathcal{D}_Q n^a_{\boldsymbol{x}'}\right),$$

Fluctuations, Linblad form

$$\mathcal{L}_3\,\mathcal{D}_Q \equiv \frac{i}{4T}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q \dot{n}^a_{\boldsymbol{x}'}]\right)$$

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$\mathcal{L}_3$

* As well as another time discretization

B-E Quantum Master Equation

Series expansion in τ_E/τ_S

Compact form: $\frac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} = \mathcal{L}\,\mathcal{D}_Q$ with $\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \cdots$

$$\mathcal{L}_0\,\mathcal{D}_Q \equiv -i[H_Q, \mathcal{D}_Q],$$

$$\mathcal{L}_1\,\mathcal{D}_Q \equiv -\frac{i}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} V(\boldsymbol{x}-\boldsymbol{x}')[n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q],$$

Mean field hamiltonian

$$\mathcal{L}_2\,\mathcal{D}_Q \equiv \frac{1}{2}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left(\{n^a_{\boldsymbol{x}} n^a_{\boldsymbol{x}'}, \mathcal{D}_Q\} - 2 n^a_{\boldsymbol{x}} \mathcal{D}_Q n^a_{\boldsymbol{x}'}\right),$$

Fluctuations, Linblad form

$$\mathcal{L}_3\,\mathcal{D}_Q \equiv \frac{i}{4T}\int_{\boldsymbol{x}\boldsymbol{x}'} W(\boldsymbol{x}-\boldsymbol{x}')\left([n^a_{\boldsymbol{x}}, \dot{n}^a_{\boldsymbol{x}'}\mathcal{D}_Q] + [n^a_{\boldsymbol{x}}, \mathcal{D}_Q \dot{n}^a_{\boldsymbol{x}'}]\right)$$

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$\mathcal{L}_4$

Application to QED and QCD for both cases of 1 body and 2 body densities…

B-E Quantum Master Equation: QED case

- For the relative motion (2 body):

$$\left.\begin{array}{l}\vec{s} = \vec{x}_1 - \vec{x}_2 \\ \vec{s}\,' = \vec{x}_1' - \vec{x}_2'\end{array}\right\} \quad \vec{r} = \frac{\vec{s}+\vec{s}\,'}{2} \quad \text{and} \quad \vec{y} = \vec{s} - \vec{s}\,'$$

- Near thermal equilibrium, Density operator is nearly diagonal => semi-classical expansion (power series in *y* up to 2nd order)

… However, we know from open heavy flavor analysis that it takes some finite relaxation time to reach this state

$$\frac{d}{dt}\mathcal{D}(r,y) = \mathcal{L}\mathcal{D}(r,y)$$

$$\begin{cases} \mathcal{L}_0 = \frac{2i\nabla_y \cdot \nabla_r}{M} \\ \mathcal{L}_1 = i\vec{y}\cdot\nabla V(r) \\ \mathcal{L}_2 = -\frac{1}{4}\vec{y}\cdot(\mathcal{H}(\vec{r}) + \mathcal{H}(0))\cdot\vec{y} \\ \mathcal{L}_3 = -\frac{1}{2MT}\vec{y}\cdot(\mathcal{H}(\vec{r}) + \mathcal{H}(0))\cdot\nabla_{\vec{y}} \end{cases}$$

$\mathcal{H}(\vec{r})$: Hessian matrix of im. pot. W

$$W(\vec{y}) = W(\vec{0}) + \frac{1}{2}\vec{y}\cdot\mathcal{H}(0)\cdot\vec{y}$$

- Wigner transform -> $\mathcal{D}(\vec{r},\vec{p})$ => $\{\vec{y},\nabla_y\} \to \{\nabla_p,\vec{p}\}$ Usual Fokker Planck eq.
- Easy MC implementation + generalization for N body system

B-E Quantum Master Equation: QCD case

singlet density matrix

$$\frac{d}{dt}\begin{pmatrix}\mathcal{D}_s\\ \mathcal{D}_o\end{pmatrix} = \mathcal{L}\begin{pmatrix}\mathcal{D}_s(\mathbf{r}_{rel}, \mathbf{r}'_{rel}, t)\\ \mathcal{D}_o(\mathbf{r}_{rel}, \mathbf{r}'_{rel}, t)\end{pmatrix}$$

octet density matrix

$$\mathcal{L} = \begin{pmatrix}\mathcal{L}_{ss} & \mathcal{L}_{so}\\ \mathcal{L}_{os} & \mathcal{L}_{oo}\end{pmatrix}$$

singlet-octet transitions

2 coupled color representations (singlet octet)

Alternate choice : $\begin{pmatrix}\mathcal{D}_0\\ \boxed{\mathcal{D}_8}\end{pmatrix}$ Off color-equilibrium component

With (infinite mass limit)

$$\mathcal{D}_8(r,t) \sim \mathcal{D}_8(r,0)e^{-N_c\Gamma(r)t} \to 0$$

Color equilibration

Still semi-classical approximation (power series in *y*).

Example of the $\mathcal{D}_s$ evolution (after SC expansion)

$$\begin{aligned}(D_s|\mathcal{L}|\mathcal{D}) = &\left(2i\frac{\nabla_{\boldsymbol{r}}\cdot\nabla_{\boldsymbol{y}}}{M} + i\frac{\nabla_{\mathcal{R}}\cdot\nabla_{\boldsymbol{Y}}}{2M} + iC_F\boldsymbol{y}\cdot\boldsymbol{\nabla}V(\boldsymbol{r})\right)D_s\\
&-2C_F\Gamma(\boldsymbol{r})(D_s - D_o)\\
&-\frac{C_F}{4}\left(\boldsymbol{y}\cdot\mathcal{H}(\boldsymbol{r})\cdot\boldsymbol{y}\,D_s + \boldsymbol{y}\cdot\mathcal{H}(0)\cdot\boldsymbol{y}\,D_o\right)\\
&-C_F\boldsymbol{Y}\cdot[\mathcal{H}(0)-\mathcal{H}(\boldsymbol{r})]\cdot\boldsymbol{Y}\,D_o\\
&+\frac{C_F}{2MT}\left[\nabla^2W(0)-\nabla^2W(\boldsymbol{r})-\boldsymbol{\nabla}W(\boldsymbol{r})\cdot\boldsymbol{\nabla}_{\boldsymbol{r}}\right](D_s - D_o)\\
&-\frac{C_F}{2MT}\left(\boldsymbol{y}\cdot\mathcal{H}(\boldsymbol{r})\cdot\boldsymbol{\nabla}_{\boldsymbol{y}}D_s + \boldsymbol{y}\cdot\mathcal{H}(0)\cdot\boldsymbol{\nabla}_{\boldsymbol{y}}D_o\right)\\
&-\frac{C_F}{2MT}\boldsymbol{Y}\cdot[\mathcal{H}(0)-\mathcal{H}(\boldsymbol{r})]\cdot\boldsymbol{\nabla}_{\boldsymbol{Y}}D_o.\end{aligned}$$

B-E Quantum Master Equation: QCD case

More complicated !

Strategy 1:

Evolution reduces to QED case

- Assume fast color equilibration
- => Dynamics studied near "maximal entropy state" ($\mathcal{D}_8 \approx 0$)

$$\partial_t \begin{pmatrix} D_0 \\ D_8 \end{pmatrix} = \begin{pmatrix} \mathcal{L}_0 + y a_{00}^{(1)} + y^2 a_{00}^{(2)} & y a_{08}^{(1)} + y^2 a_{08}^{(2)} \\ y a_{80}^{(1)} + y^2 a_{80}^{(2)} & \mathcal{L}_0 + a_{88}^{(0)} + y a_{88}^{(1)} + y^2 a_{88}^{(2)} \end{pmatrix}$$

- Color transitions treated in perturbative approach (small *y*)

$$\mathcal{L}' = \mathcal{L}_0 + y a_{00}^{(1)} + y^2 \left(a_{00}^{(2)} - \frac{a_{08}^{(1)} a_{80}^{(1)}}{a_{88}^{(0)}} \right)$$

$$a_{88}^{(0)} = -N_c \Gamma(\boldsymbol{r})$$

- Evolution of corresponding eigenstate:

$$\partial_t D_0' = \left(\frac{2i}{M} \boldsymbol{\nabla}_{\boldsymbol{r}} \cdot \boldsymbol{\nabla}_{\boldsymbol{y}} - \frac{C_F}{4} \boldsymbol{y} \cdot \mathcal{H}(0) \cdot \boldsymbol{y} - \frac{C_F (\boldsymbol{y} \cdot \boldsymbol{F}(\mathbf{r}))^2}{2 N_c^2 \Gamma(\mathbf{r})} - \frac{C_F}{2MT} \boldsymbol{y} \cdot \mathcal{H}(0) \cdot \boldsymbol{\nabla}_y \right) D_0' \equiv \mathcal{L}' D_0'.$$

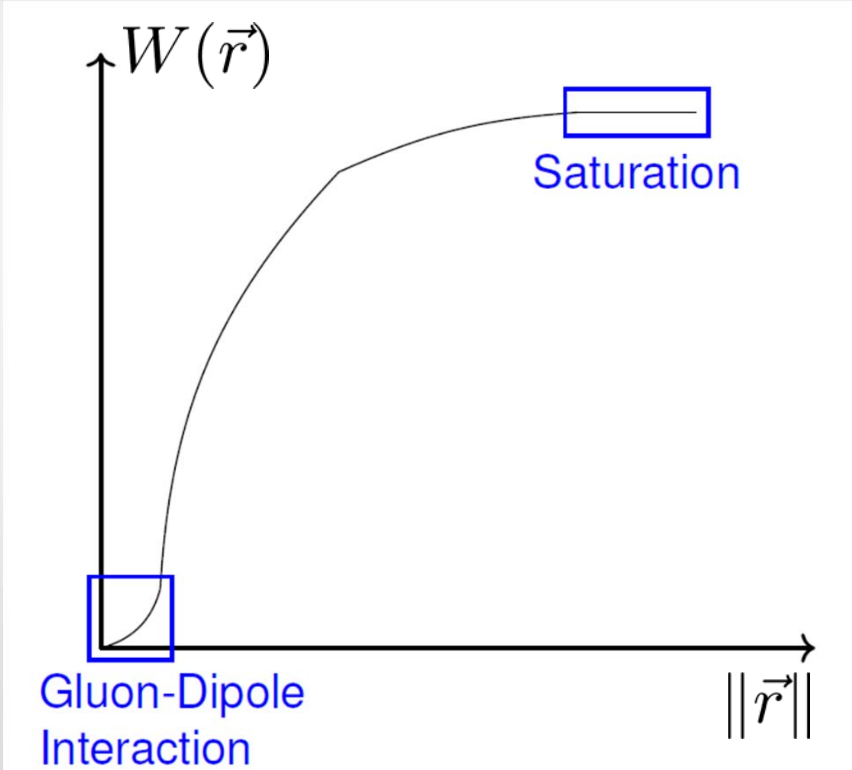
New kind of stochastic forces due to color exchanges

B-E Quantum Master Equation: QCD case

More complicated !

Strategy 1:

- Still Fokker Planck type of equation
- With however stochastic forces $\propto \frac{1}{\Gamma^{\frac{1}{2}}(\vec{r})}$
- $\Gamma(\vec{r}) := W(\vec{r}) - W(0)$
- Amplified force when the size of the pair becomes too small (rate of interaction -> 0 due to colour dipole nature)… incompatible with fast equilibration hypothesis.

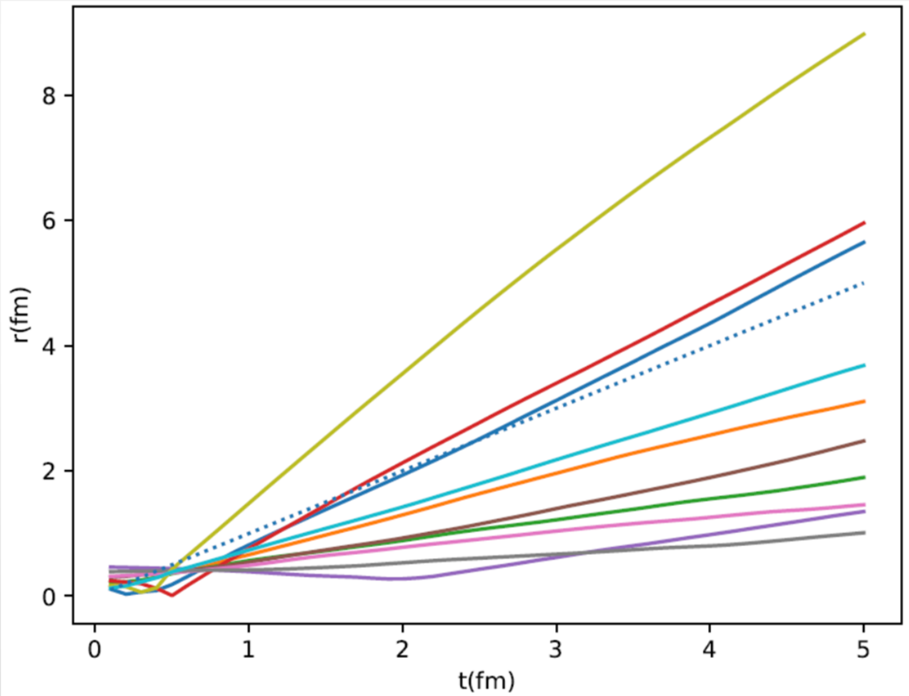


B-E Quantum Master Equation: QCD case

More complicated !

Strategy 1:

- Still Fokker Planck type of equation
- With however stochastic forces $\propto \frac{1}{\Gamma^{\frac{1}{2}}(\vec{r})}$
- $\Gamma(\vec{r}) := W(\vec{r}) - W(0)$
- Amplified force when the size of the pair becomes too small (rate of interaction -> 0 due to colour dipole nature)… incompatible with fast equilibration hypothesis.
- **Unphysical behaviour for ≈ 50% of the trajectories**



JP Blaizot : « Langevin dynamics not appropriate as color transitions break the physical picture of cumulative small deflections »

B-E Quantum Master Equation: QCD case

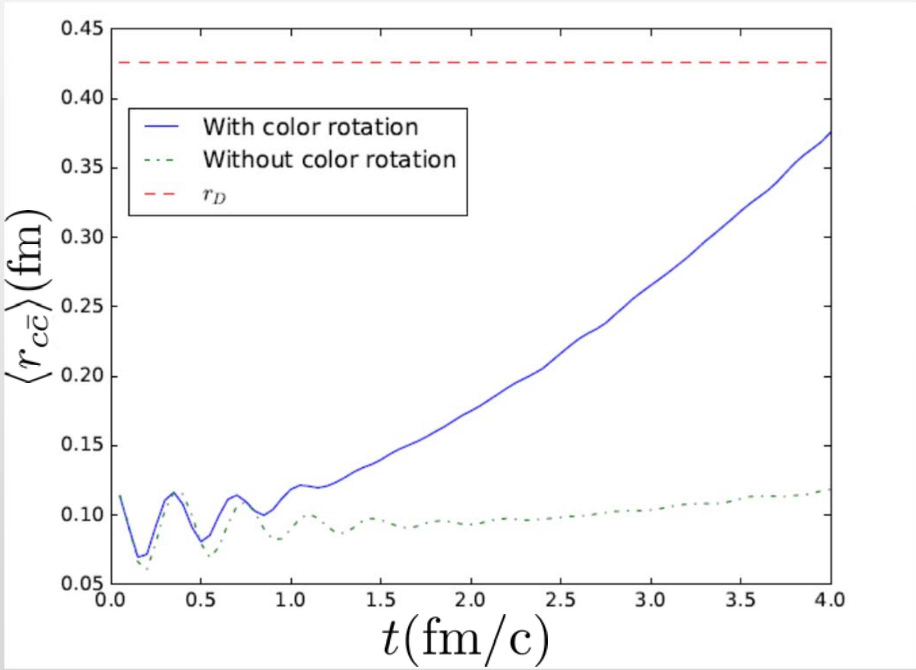
Strategy 2:

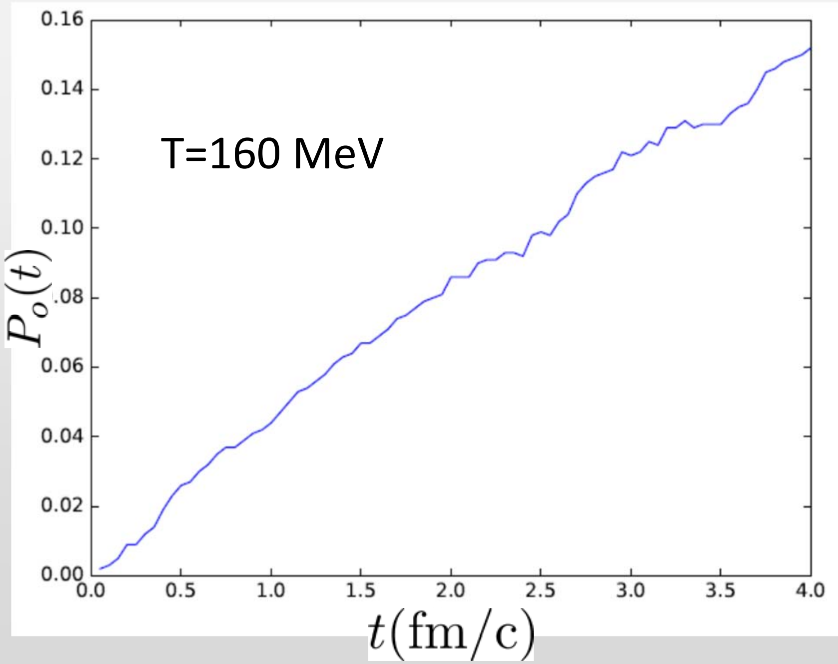
- Stay in the singlet – octet representation ; still assumes fast color equilibration
- After Wigner transform, most of the terms can be reshuffled under the form of a Boltzmann equation which can be solved with Monte Carlo techniques.

$$\left\{\partial_t + \boldsymbol{v}\cdot\boldsymbol{\nabla}_{\boldsymbol{r}} - C_F \boldsymbol{F}(\boldsymbol{r})\cdot\boldsymbol{\nabla}_{\boldsymbol{p}} - \frac{1}{2}\boldsymbol{\nabla}_{\boldsymbol{p}}\cdot\boldsymbol{\eta}_{\mathrm{s}}(\boldsymbol{r})\cdot\left(\boldsymbol{\nabla}_{\boldsymbol{p}} + \frac{\boldsymbol{v}}{T}\right)\right\} P_{\mathrm{s}} = -2C_F\Gamma(\mathbf{r})\left(P_{\mathrm{s}} - \frac{P_{\mathrm{o}}}{N_c^2-1}\right)$$

Octet <-> singlet transition

- However, residual terms which cannot be cast easily under this form





Our current project

Our Goal:

- Explicitly restore the Linbladian form and the positivity of BE equations => term $\mathcal{L}_4$
- Gain insight on the quarkonium dynamics inside the QGP by **solving exactly the B-E equations** for a single $c\bar{c}$ pair without performing the Semi-Classical approximation:
  - Evolution of the density matrix
  - Evolution of states probabilities over time
  - Singlet-octet transitions
  - Color relaxation time
  - …
- Comparison with the semi-classical approach for a various range of QGP temperatures (should be fine at large temperature… but down to ?)
- Possibly design improved algorithm for intermediate temperatures

Our current project

Adopted method:

- Trace out the global center of mass motion R (Heavy quarks => should not be deflected much) => equations on the relative coordinates only:

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle\vec{s}|\mathcal{D}_{c\bar{c}}|\vec{s}\,'\rangle = \int d^3R d^3R'\delta^{(3)}(\vec{R}-\vec{R}\,')\langle\vec{R}\vec{s}|\mathcal{D}_{c\bar{c}}|\vec{R}\,'\vec{s}\,'\rangle$$

$$\stackrel{?}{=} \mathcal{L}'[\langle\vec{s}|\mathcal{D}_{c\bar{c}}|\vec{s}\,'\rangle]$$

Both $\mathcal{L}_2$ and $\mathcal{L}_3$ can be reduced to $\mathcal{L}'_2$ and $\mathcal{L}'_3$. For $\mathcal{L}_4$ some terms can only be reduced at the price of assuming a state with a specific total momentum p_{tot}:

$$\langle\vec{R}\vec{s}|\mathcal{D}_{c\bar{c}}|\vec{R}\,'\vec{s}\,'\rangle = \langle\vec{s}|\mathcal{D}_{c\bar{c}}|\vec{s}\,'\rangle \times e^{i\vec{p}_{\rm tot}(\vec{R}-\vec{R}\,')}$$

➡ $\mathcal{L}'_{4,\vec{p}_{\rm tot}}$ Which might be good for phenomenology

- In a first approach, perform the study for a 1D reduced problem => reduced computational cost, although sufficient to gain insight

- Brute numerics for the residual equations (checking basic important properties + benchmarking on known solutions)

Positivity

- Equations for the QED-like plasma in 1D :

$$\frac{1}{\hbar}\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{D} = \frac{i}{M}(\hbar c)^2(\partial_s^2 - \partial_{s'}^2)\mathcal{D} - i[V(s) - V(s')]\mathcal{D}$$
$$+\left[2W(0) - W(s) - W(s') - 2W\left(\tfrac{s-s'}{2}\right) + 2W\left(\tfrac{s+s'}{2}\right)\right]\mathcal{D}$$
$$+\frac{(\hbar c)^2}{4MT}\left[2W''(0) - W''(s) - W''(s') - 2W''\left(\tfrac{s-s'}{2}\right) + 2W''\left(\tfrac{s+s'}{2}\right)\right]\mathcal{D}$$
$$-\frac{(\hbar c)^2}{4MT}\left[2W'(s)\partial_s + 2W'(s')\partial_{s'} + 2W'\left(\tfrac{s-s'}{2}\right)(\partial_s - \partial_{s'}) - 2W'\left(\tfrac{s+s'}{2}\right)(\partial_s + \partial_{s'})\right]\mathcal{D}$$
$$+\frac{(\hbar c)^4}{64M^2T^2}\left[2W''''(0) + W''''(s) + W''''(s') - 2W''''\left(\tfrac{s-s'}{2}\right) + 2W''''\left(\tfrac{s+s'}{2}\right)\right]\mathcal{D}$$
$$+\frac{(\hbar c)^4}{64M^2T^2}\left[4W'''(s)\partial_s + 4W'''(s')\partial_{s'} - 4W'''\left(\tfrac{s-s'}{2}\right)(\partial_s - \partial_{s'}) + 4W'''\left(\tfrac{s+s'}{2}\right)(\partial_s + \partial_{s'})\right]\mathcal{D}$$
$$+\frac{(\hbar c)^4}{64M^2T^2}\left[4W''(0)(\partial_s^2 + \partial_{s'}^2) + 4W''(s)\partial_s^2 + 4W''(s')\partial_{s'}^2 + 8W''\left(\tfrac{s-s'}{2}\right)\partial_s\partial_{s'} + 8W''\left(\tfrac{s+s'}{2}\right)\partial_s\partial_{s'}\right]\mathcal{D}$$
$$+\frac{(\hbar c)^4}{64M^2T^2}p_{\mathrm{tot}}^2\left[-2W''(0) + W''(s) + W''(s') + 2W''\left(\tfrac{s-s'}{2}\right) - 2W''\left(\tfrac{s+s'}{2}\right)\right]\mathcal{D}$$

$\mathcal{L}'_4$

- Indeed subleading in 1/T expansion
- No higher derivatives on D than the 2[nd] one => still a FP equation in the semi-classical limit.
- Higher derivatives of the imaginary potential W => possible UV divergences => need for some regularization.

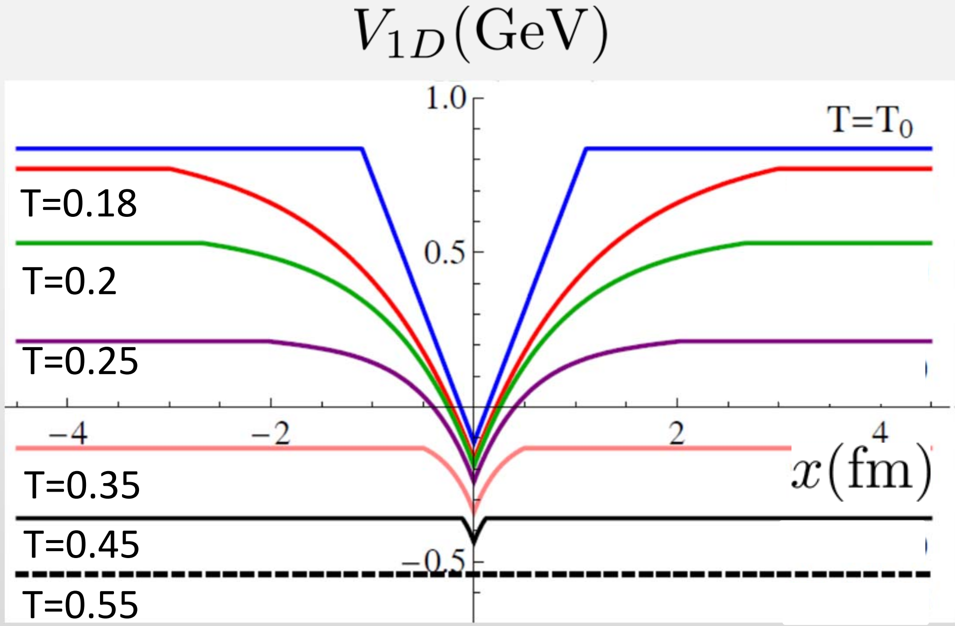
Further implementation features

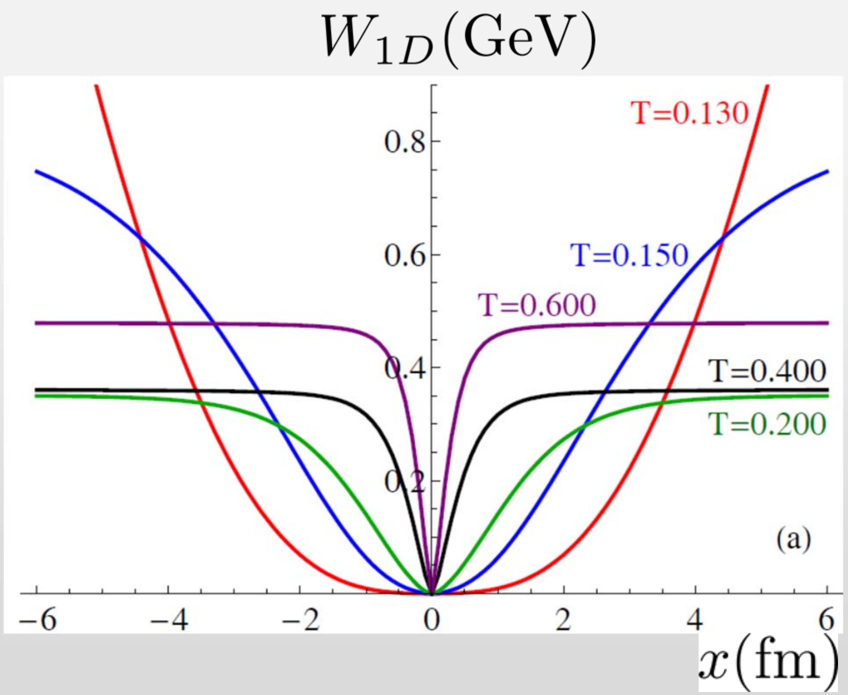
- 1D grid for both $s \in [-s_{\max}, +s_{\max}]$ and $s' \in [-s_{\max}, +s_{\max}]$

!!! Not the radial decomposition of $\mathcal{D}_{c\bar{c}}(\vec{s}, \vec{s}\,')$ which is more cumbersome

Even states will be considered as « S like » while odd states will be considered as « P like » states

Need to design a realistic 1D bona fide potential V + i W (based on 3d lQCD results, tuned to reproduce 3D mass spectra and decay widths)

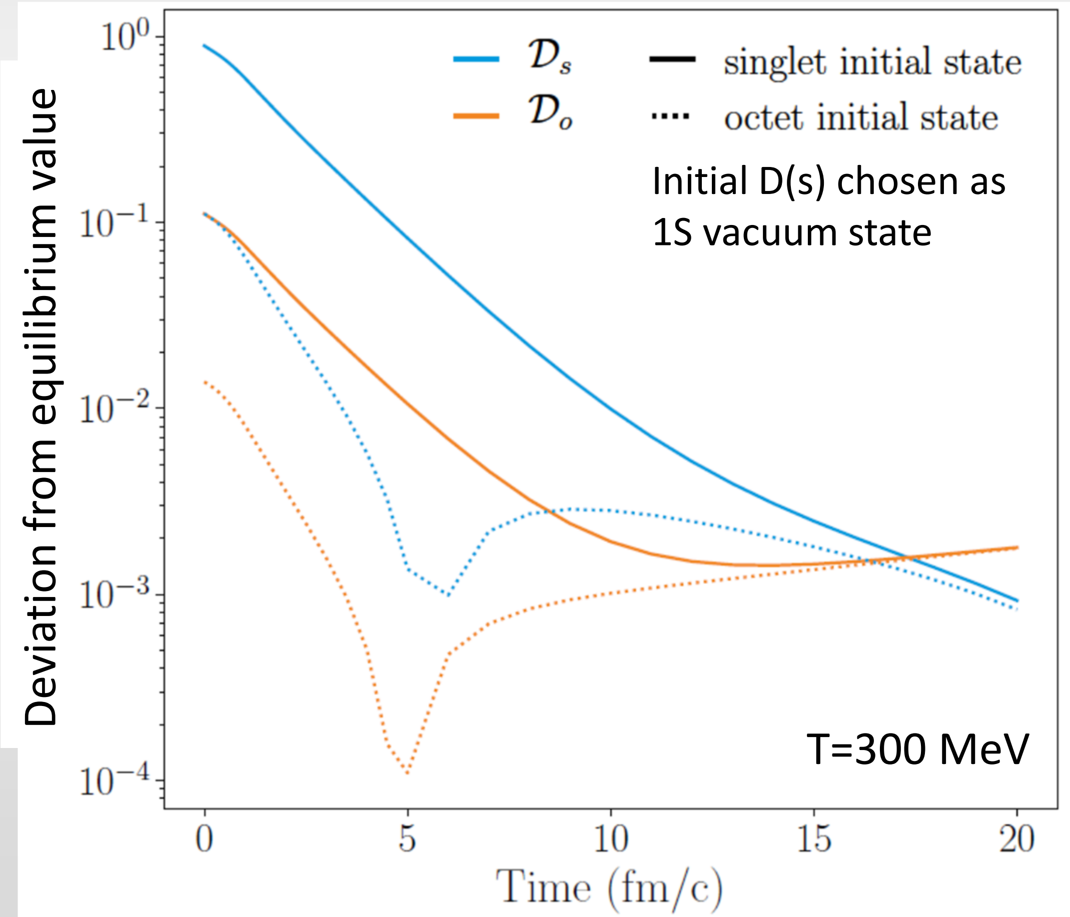




Results

Color Dynamics : Singlet – octet probabilities:

- Starting from singlet (———) or octets (- - - - -) states, one expects some equilibration / thermalisation -> asymptotic values : $D_s^{\rm eq} = D_o^{\rm eq} = \frac{1}{9} \quad (1+8) \times \frac{1}{9}$
- Study the deviations $|D_s - D_s^{\rm eq}|$ and $|D_o - D_o^{\rm eq}|$

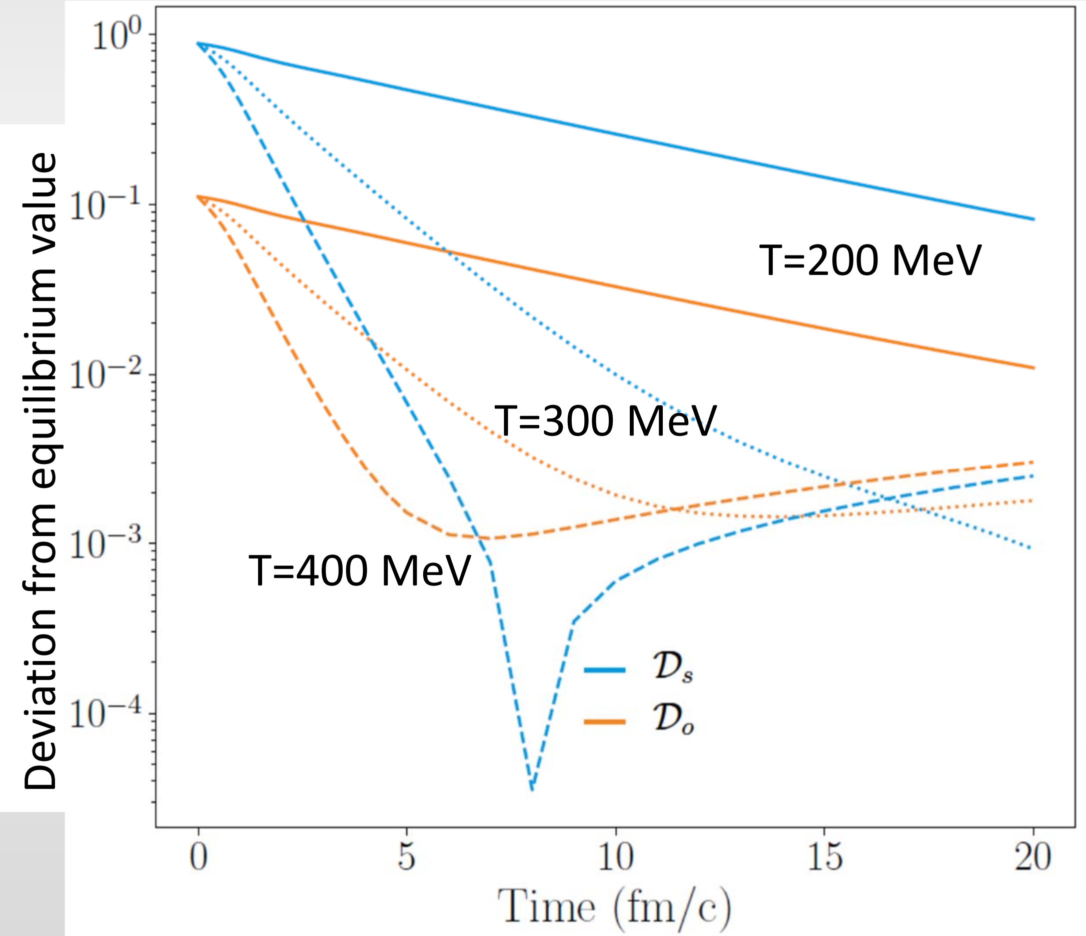


- At early times : Quasi exponential behaviour exp(-t/τ), with thermalisation time $\tau_o < \tau_s \approx 2$ fm/c
- At later time : Saturation possibly due to the grid size.
- Color appears to thermalize on time scales < QGP life time, but not instantaneoulsy.

Results

Color Dynamics : Singlet – octet probabilities:

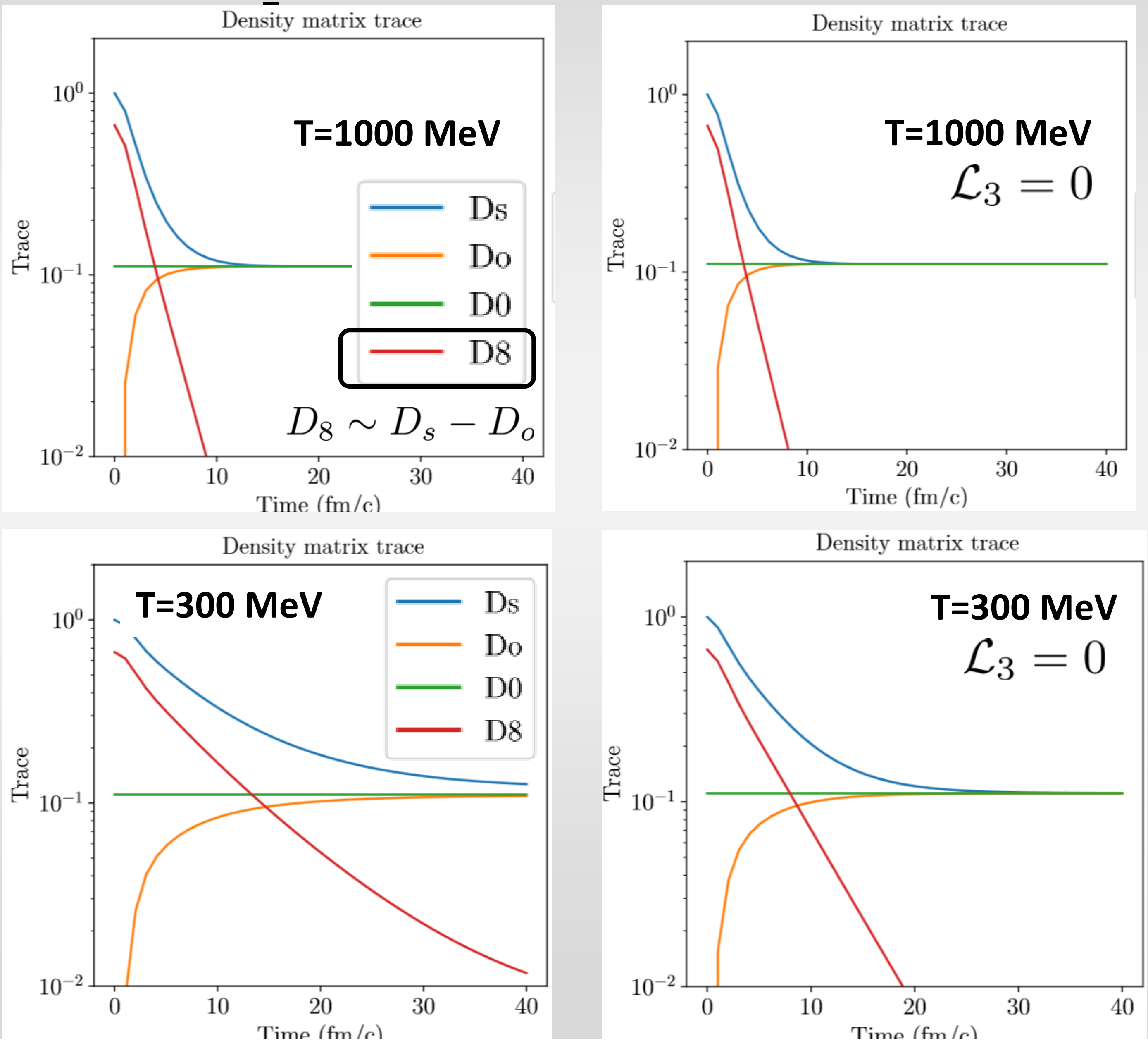
- Starting from singlet states with different QGP temperatures
- Study the deviations $|D_s - D_s^{eq}|$ and $|D_o - D_o^{eq}|$



- As expected, thermalisation time $\tau_{singlet}$ decreases for higher temperature.
- In concrete scenarios, might justify the « fast color equilibration » which later survive at smaller temperature.

Results

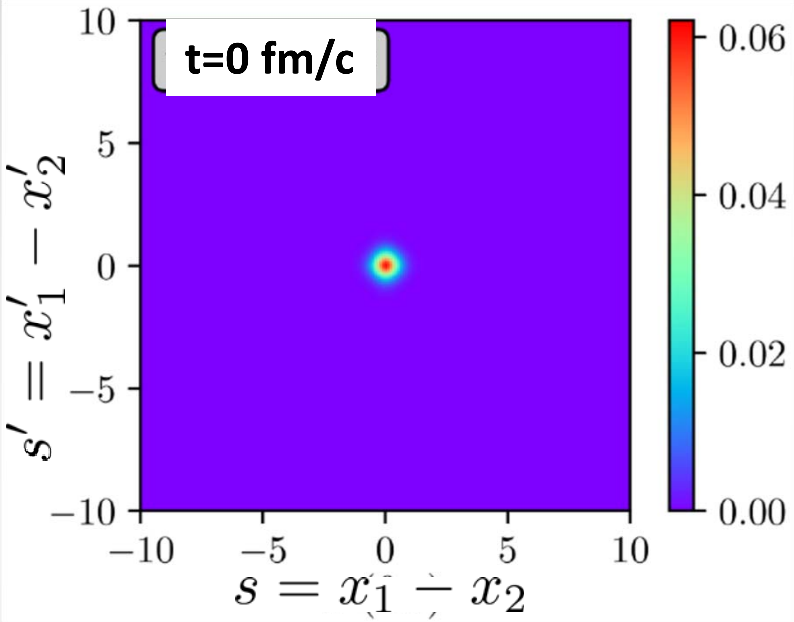
Role of the $\mathcal{L}_3$ term:

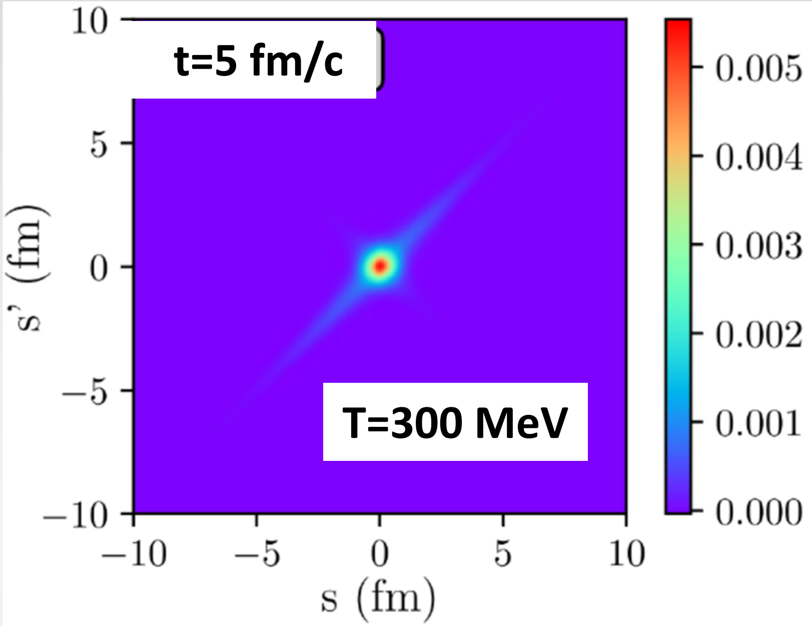


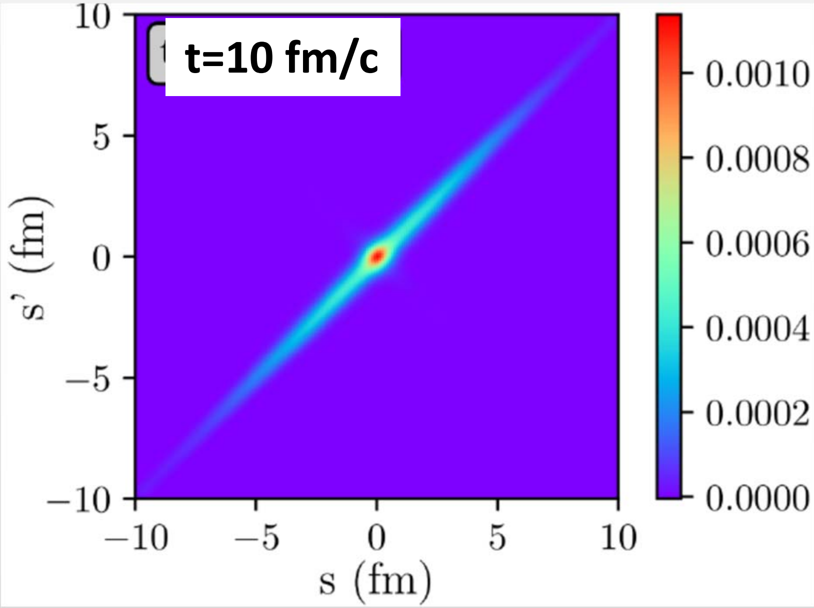
- **Early stage**: Friction ($\mathcal{L}_3$) terms are subleading at large T
- … at low T, they tend to delay the equilibration

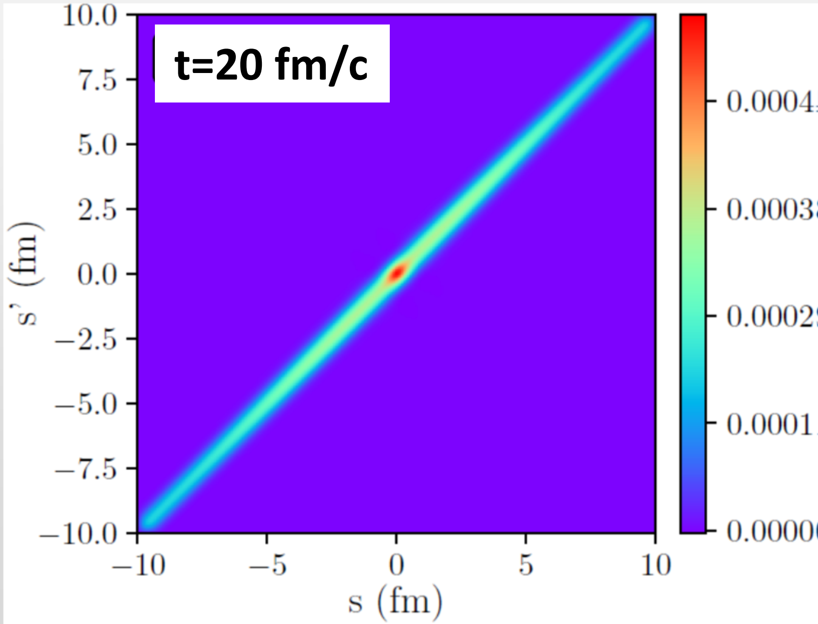
Results for Density matrix

1S singlet initial state:



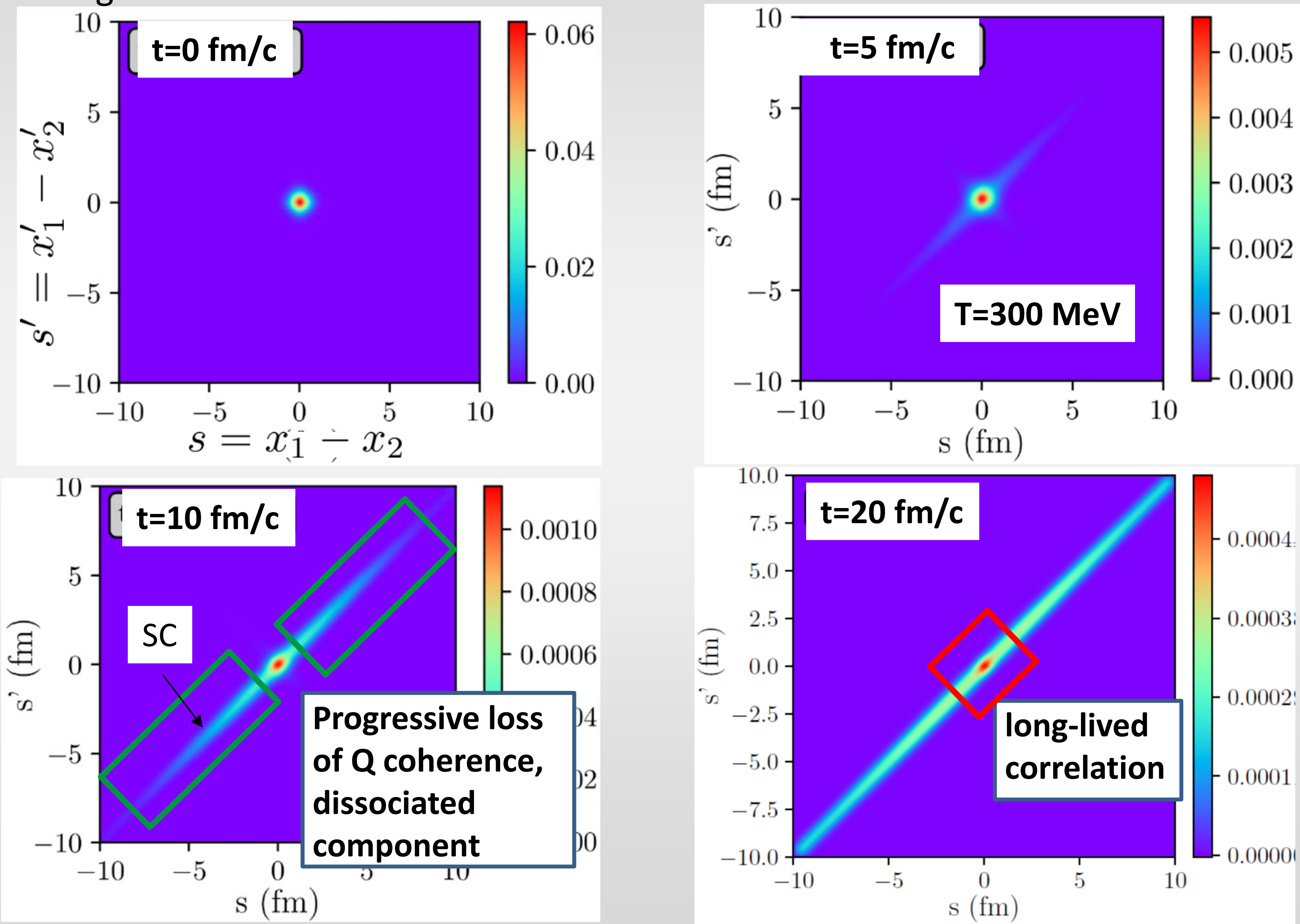






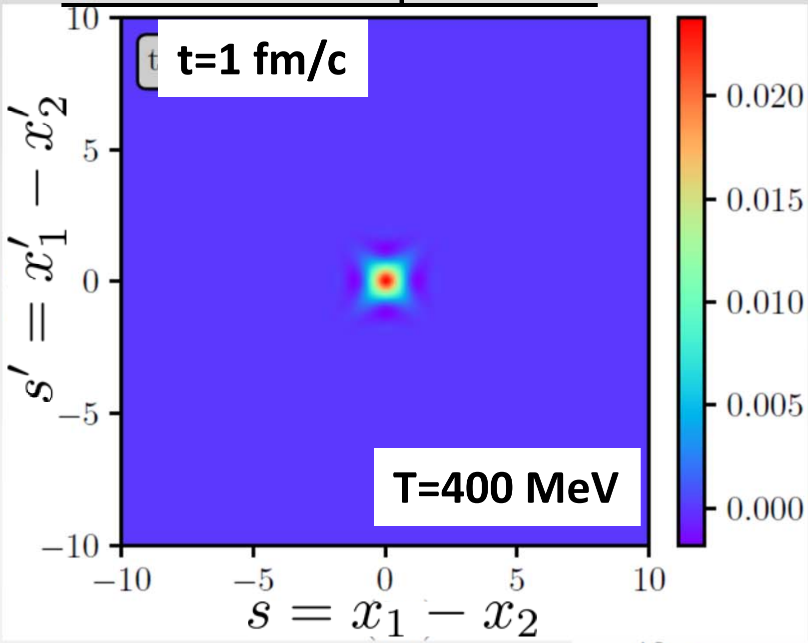
Results for Density matrix

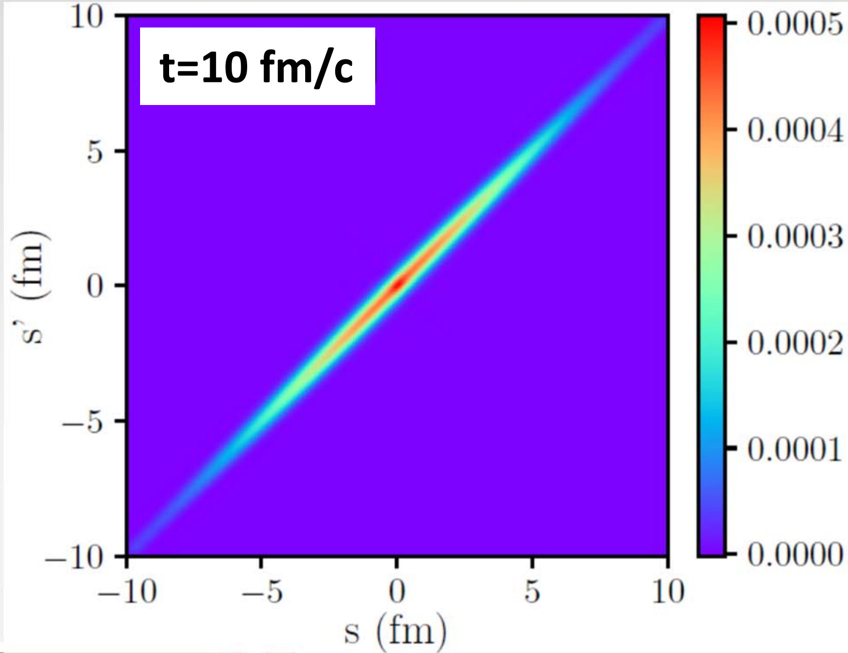
1S singlet initial state:

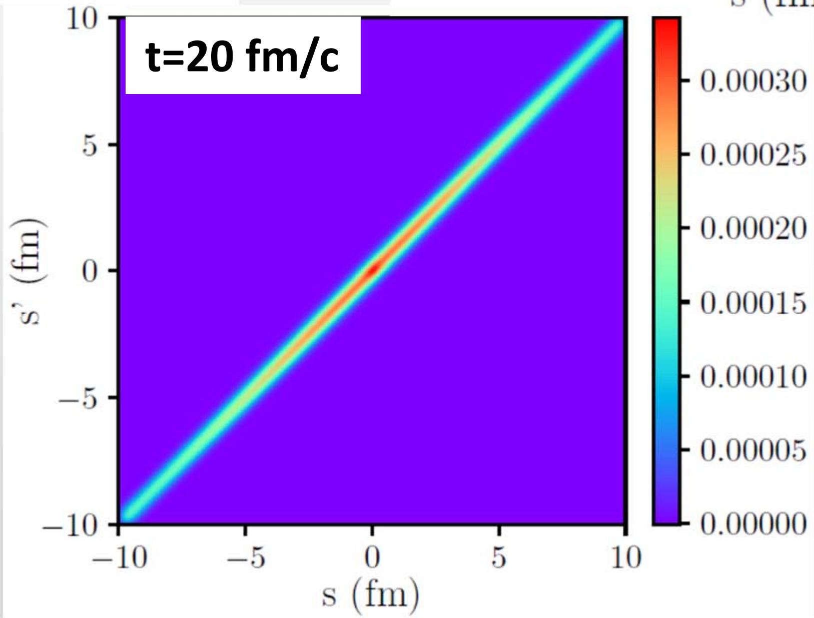


Results for Density matrix

Role of the temperature:



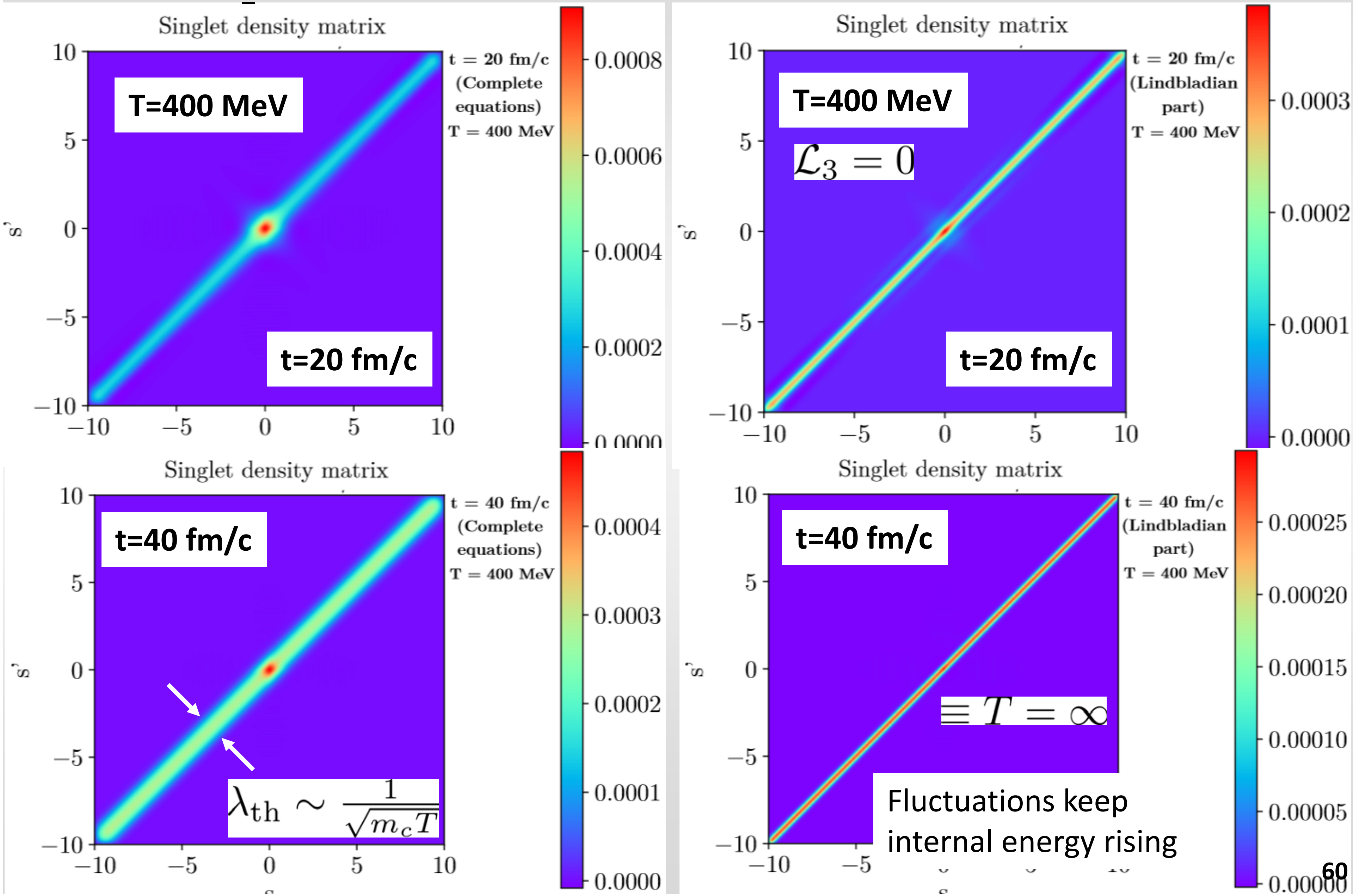




Larger T, faster decoherence

Results for Density matrix

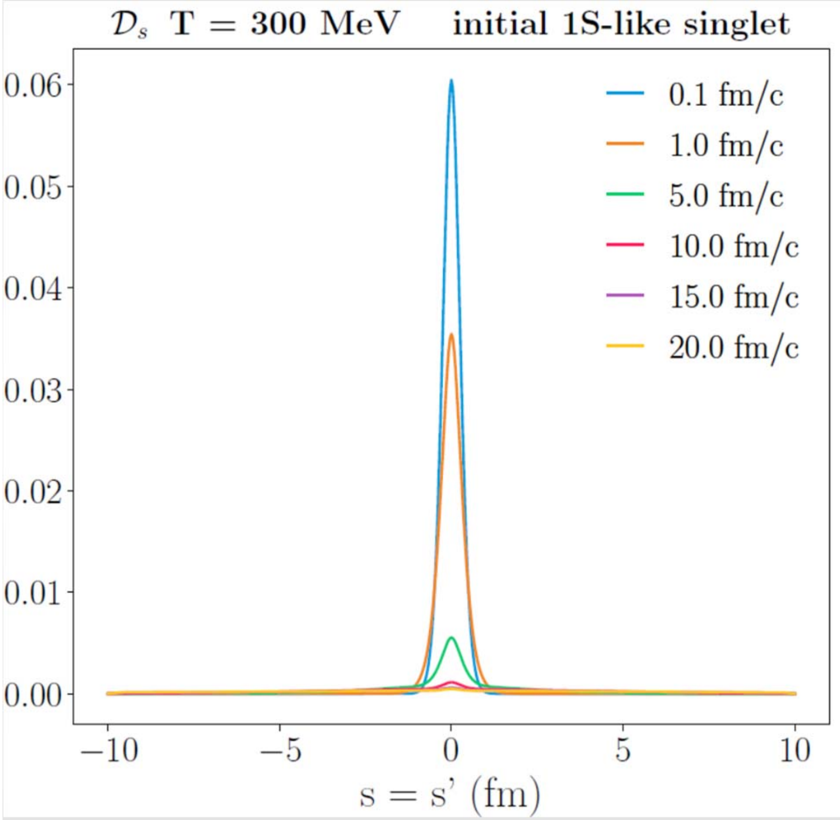
Role of the $\mathcal{L}_3$ term:

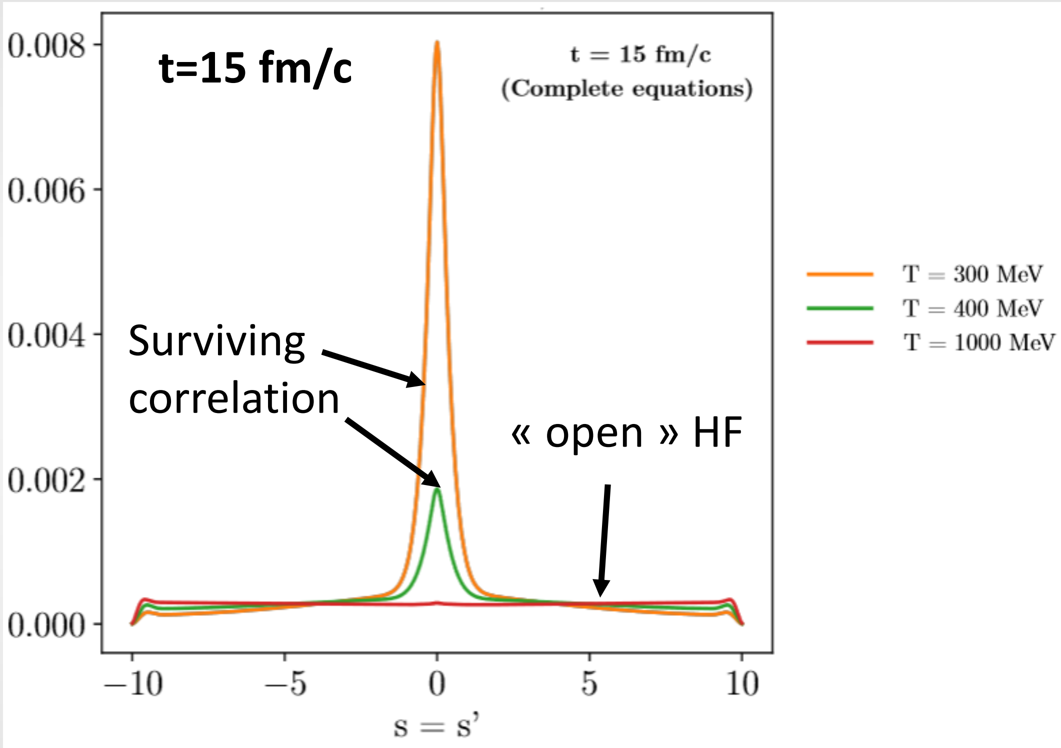


Fluctuations keep internal energy rising

Results for Density

$$\rho_s(s) = D_s(s, s' = s)$$



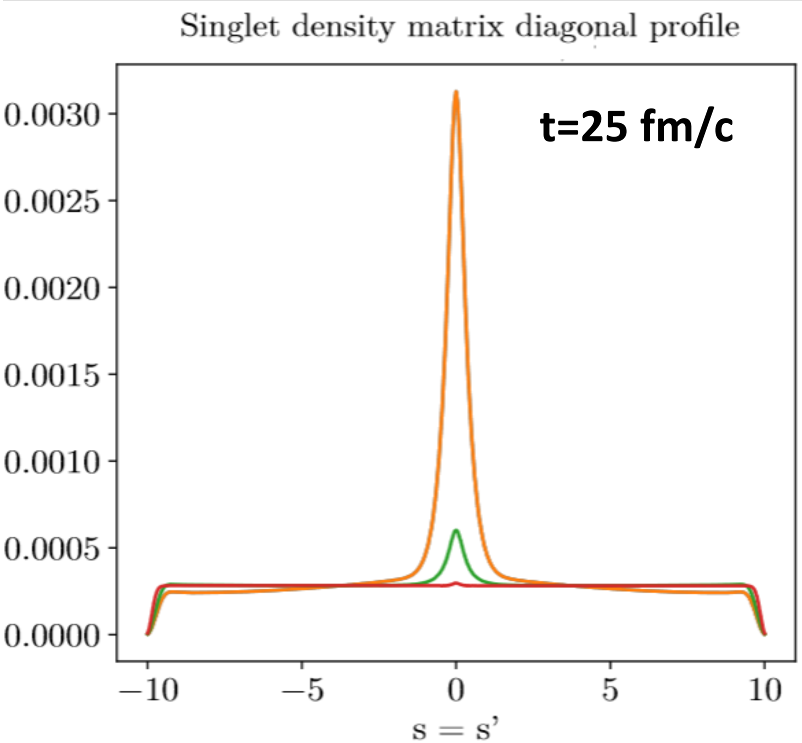


At a given T, increasing delocalisation with time

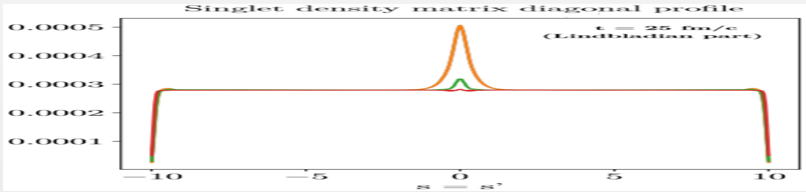
At a given time t, increasing delocalisation with T

Results for Density

Results $\rho(s)$:



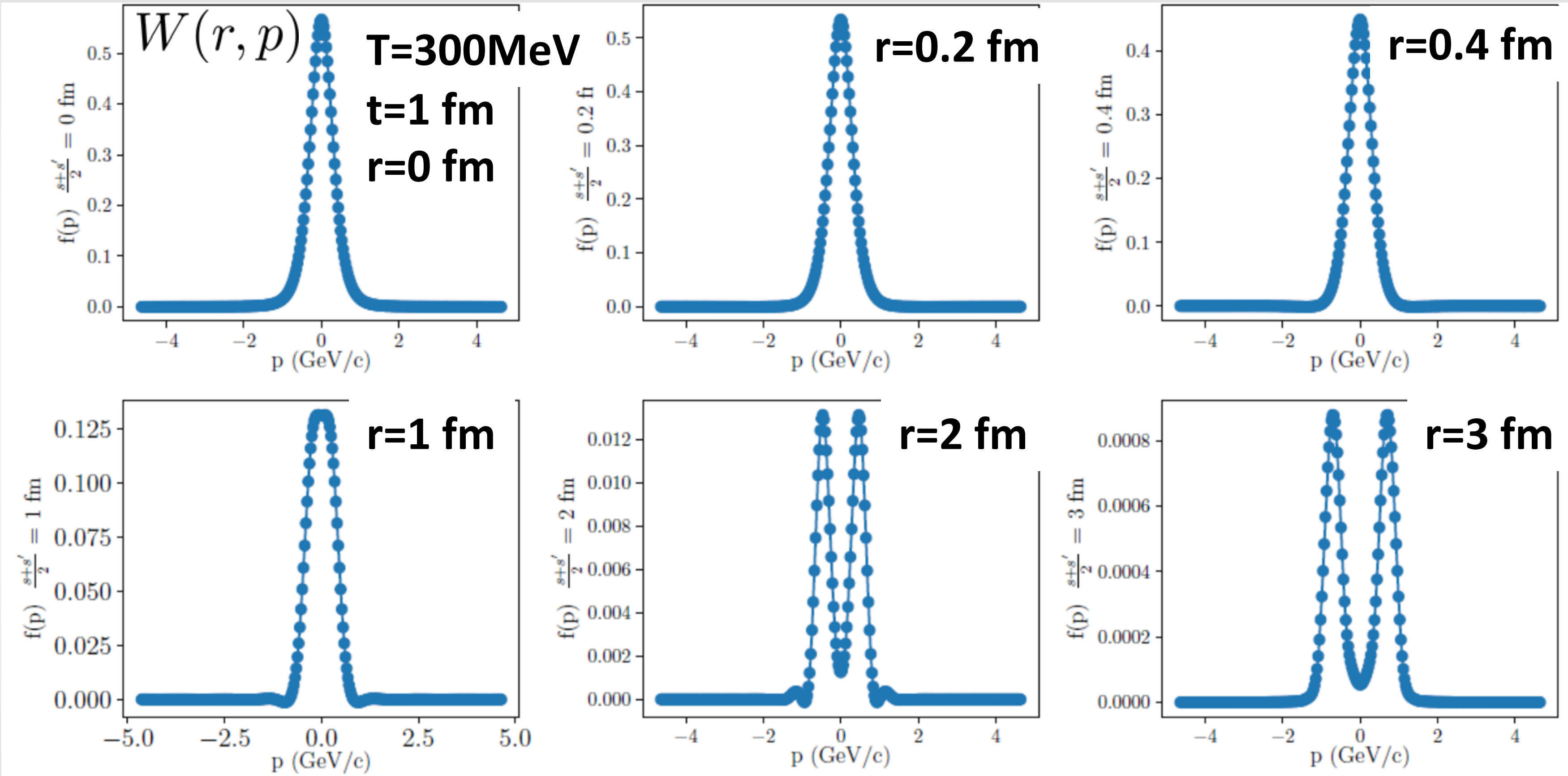
t=25 fm/c $\mathcal{L}_3 = 0$



Friction terms in $\mathcal{L}_3$ help the correlation to survive (feature already observed with SLE), hence the longer relaxation time.

Results for Density

Semi-classical analysis: computation of the discretized Wigner transform W(r,p) of D_s for different values of $r = \frac{s+s'}{2}$ (≡ position in a semi-classical approach)

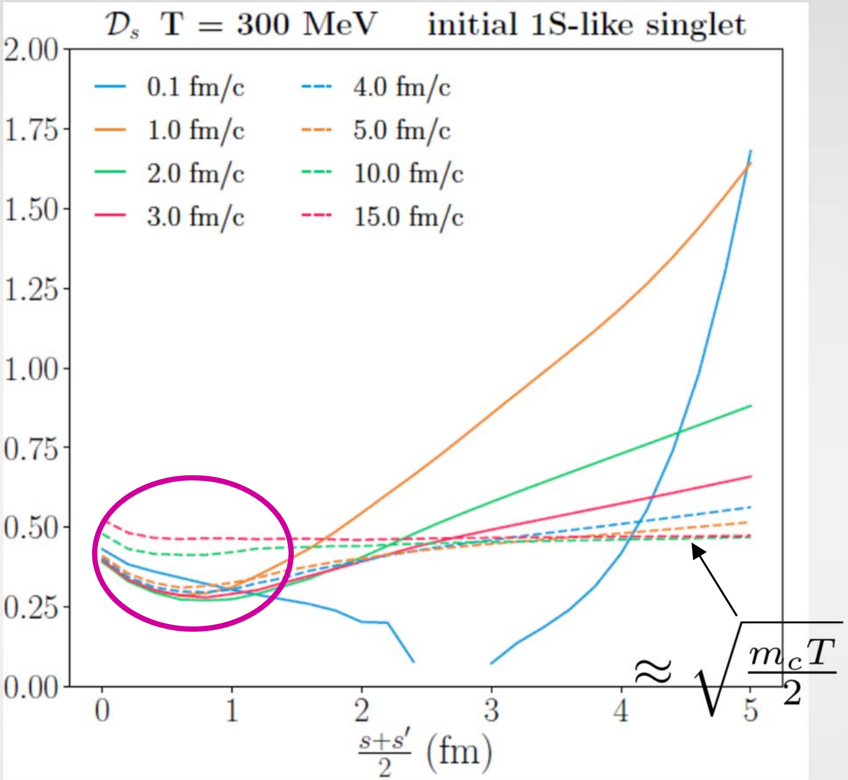


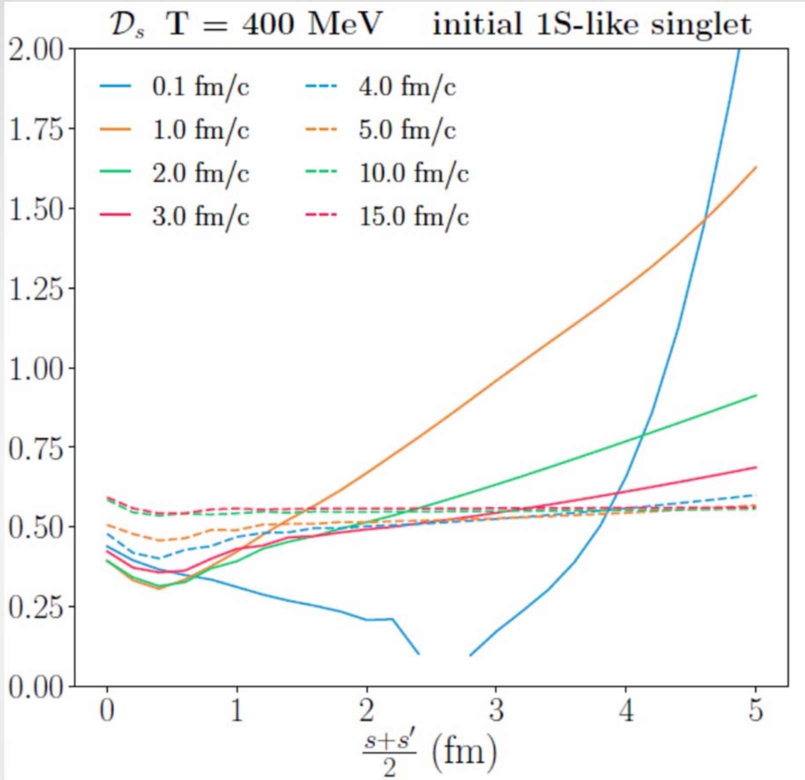
For a wide set of (t,r) : positive defined, Gaussian-like… However, some non Gaussian shapes observed as well

For large r – however supra luminous – some negative shoulders are observed.

Results for Density

<u>Semi-classical analysis</u>: Next compute the r.m.s. p : $\sqrt{\langle p^2 \rangle_W}$



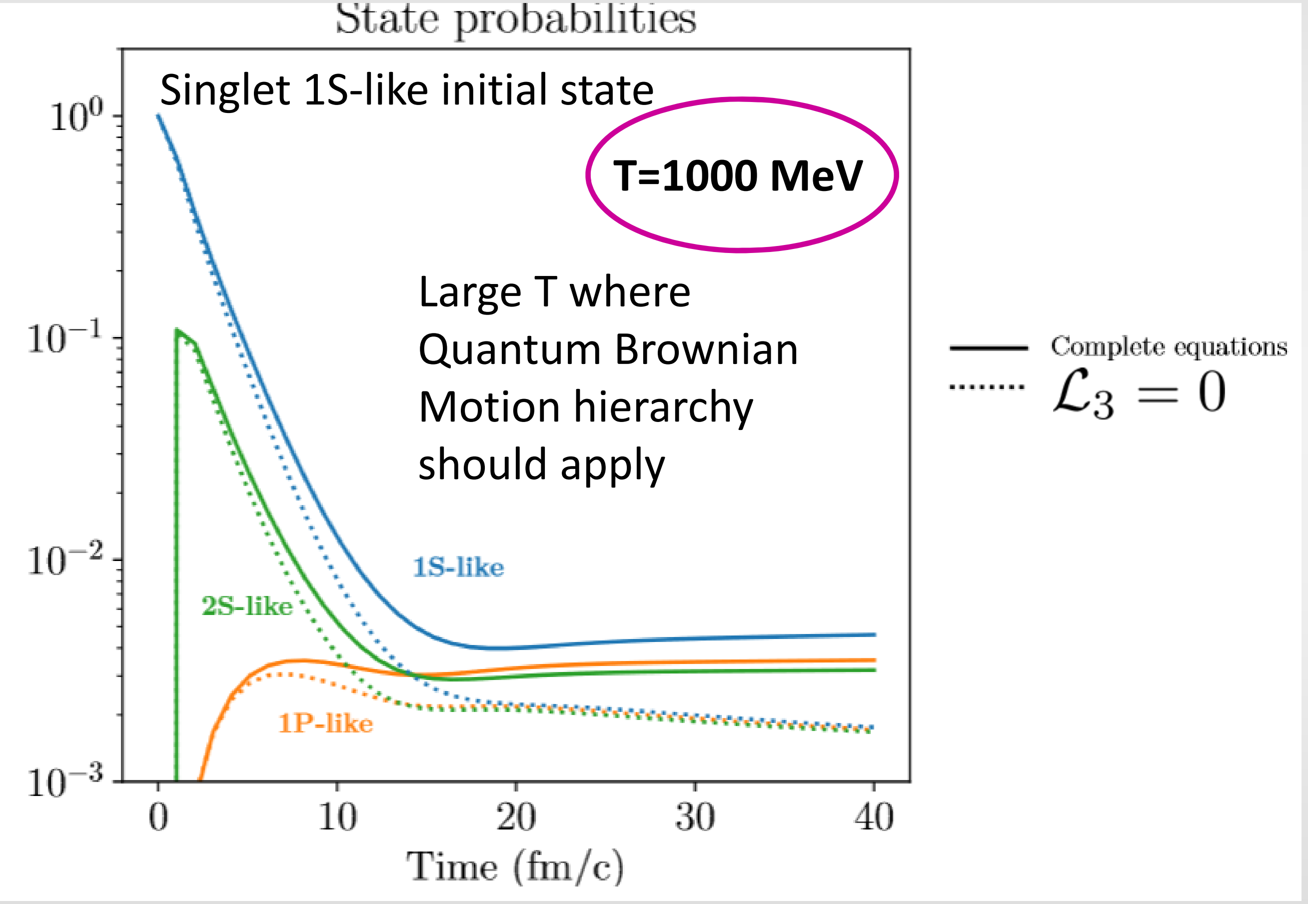


- At asymptotic times : convergence -> thermal value whatever $c\bar{c}$ distance
- At early times : some undefined $\sqrt{\langle p^2 \rangle_W}$ due to the negative shoulders. Genuine quantum effect, however at supra luminous separations
- For intermediate times : survival of the $c\bar{c}$ correlation at small distance, with r.m.s. p < thermal value (cold state need some time to heat up)… How realistic is it described by SC equations ? Under investigation.

Results for projection on vacuum states

!!! Vacuum states ≠ eigenstates at local T

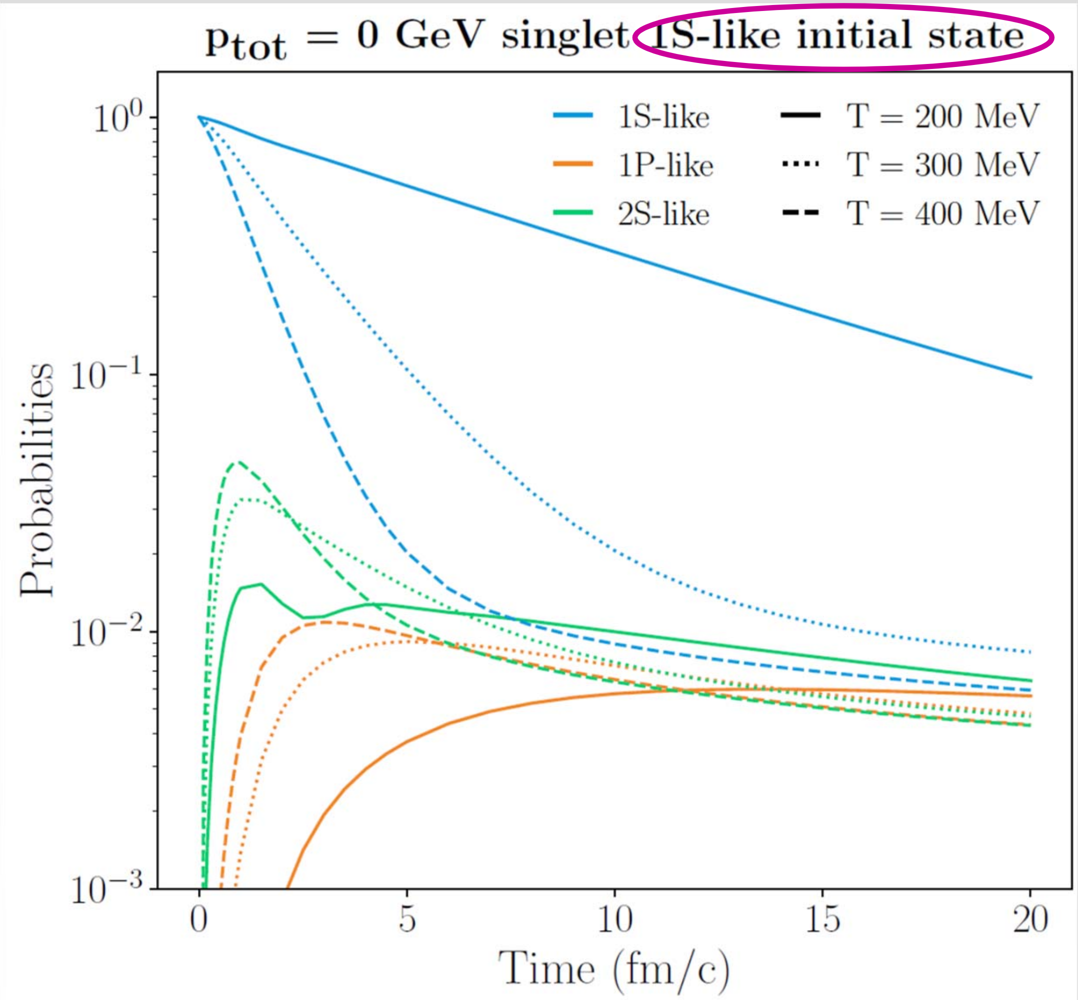
Gedanken experiment : instantaneous cooling down -> T=0 after t in QGP



- At small times, $\mathcal{L}_3 \ll \mathcal{L}_2$ fluctuations dominate… higher state repopulation
- At late times, $\mathcal{L}_3 \sim \mathcal{L}_2$ leading to asymptotic distribution of states. If $\mathcal{L}_3 = 0$, no dissipation => internal energy keeps rising.

Results for projection on vacuum states

For more « realistic » temperatures

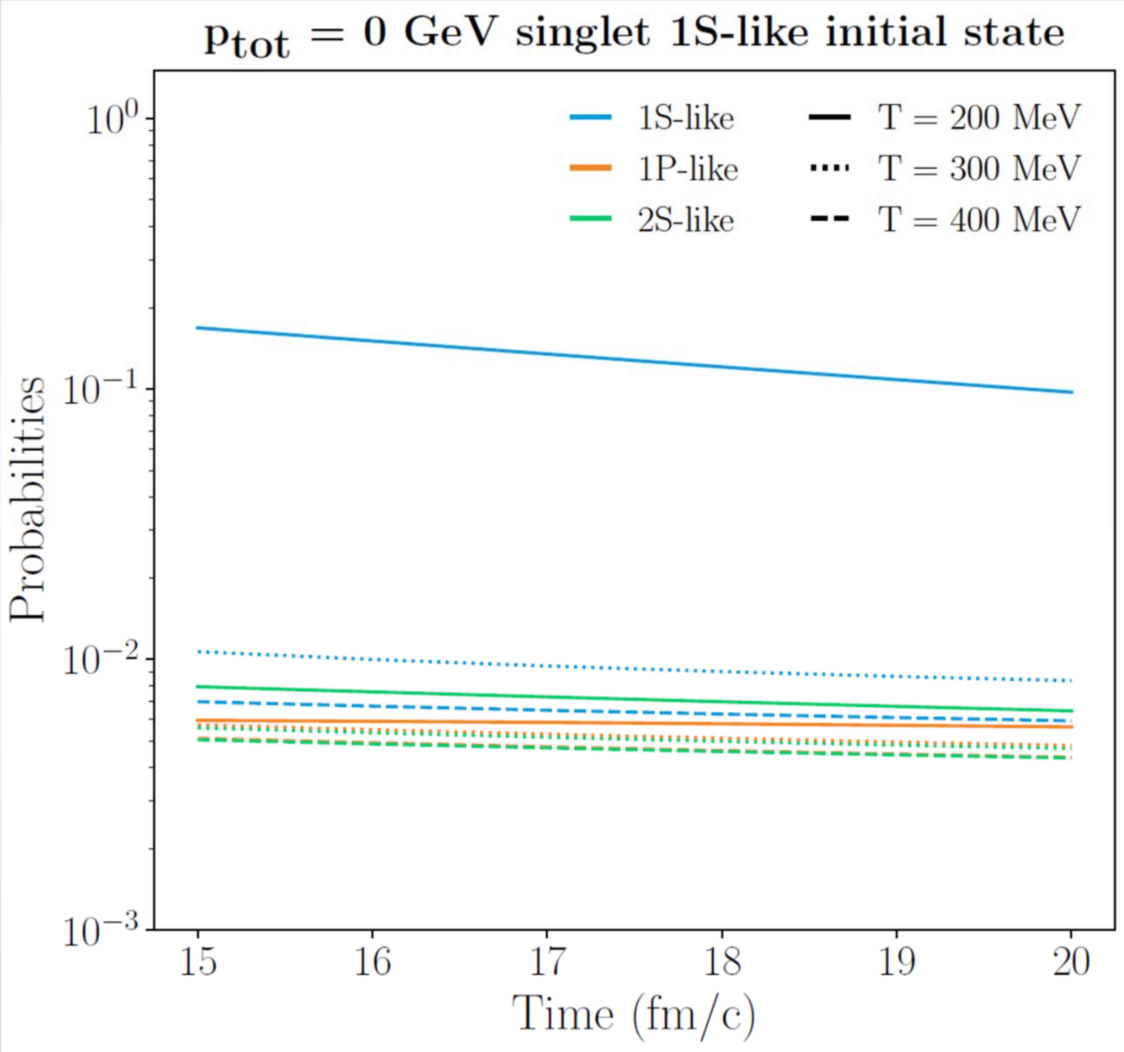


Pretty complex interplay between binding, diffusion and transitions between states

- ➢ Faster (and larger) suppression for larger QGP temperature
- ➢ Transient phase up to 5 fm/c : re-equilibration
- ➢ Common evolution (decrease) of all states at large times for T=300 and 400 MeV

Results for projection on vacuum states

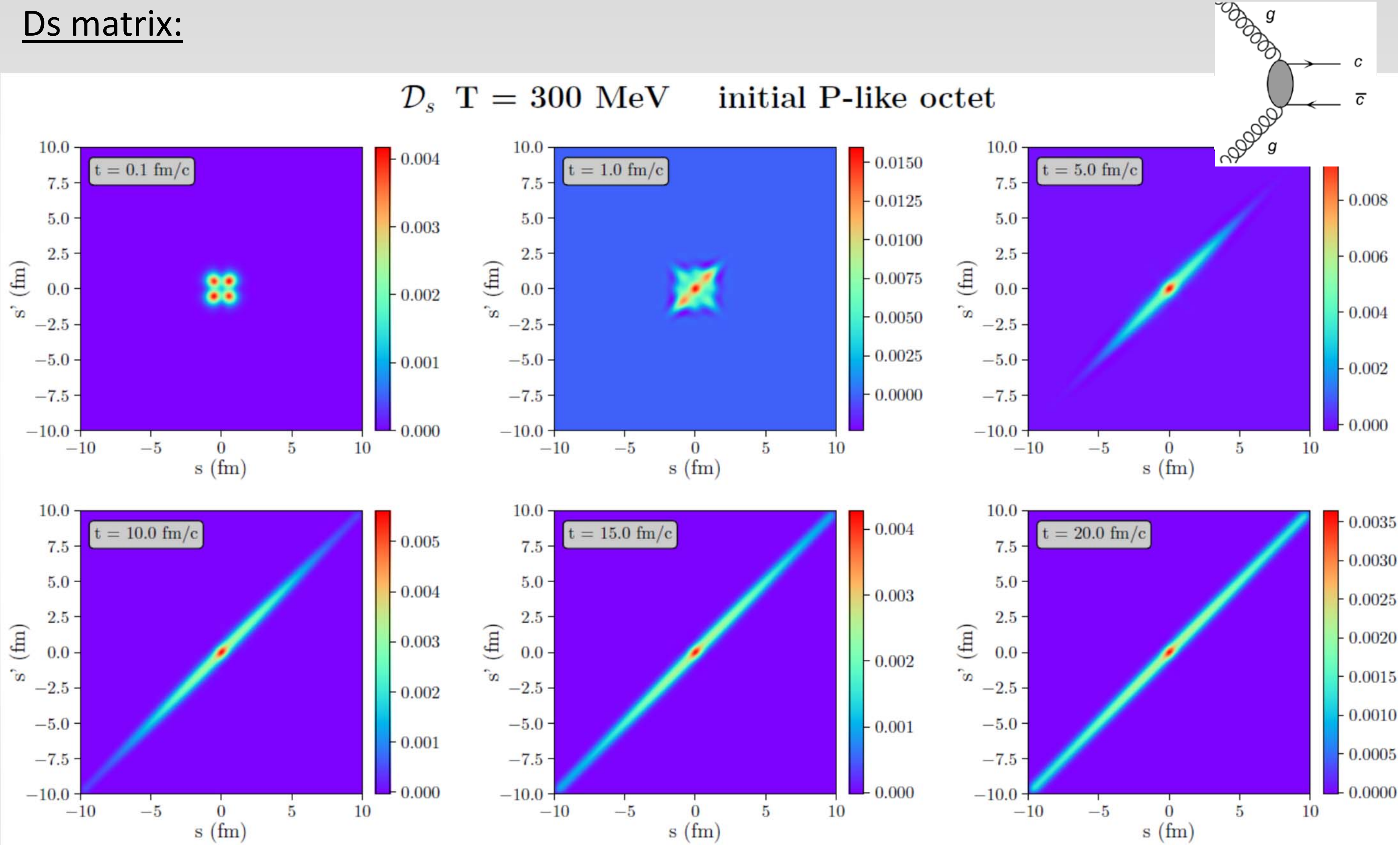
<u>For more « realistic » temperatures</u>



- Common hierarchy at late times… although usual detailled balance not preserved by the τ_E/τ_S expansion. To be investigated.

Results for more realistic octet Initial State

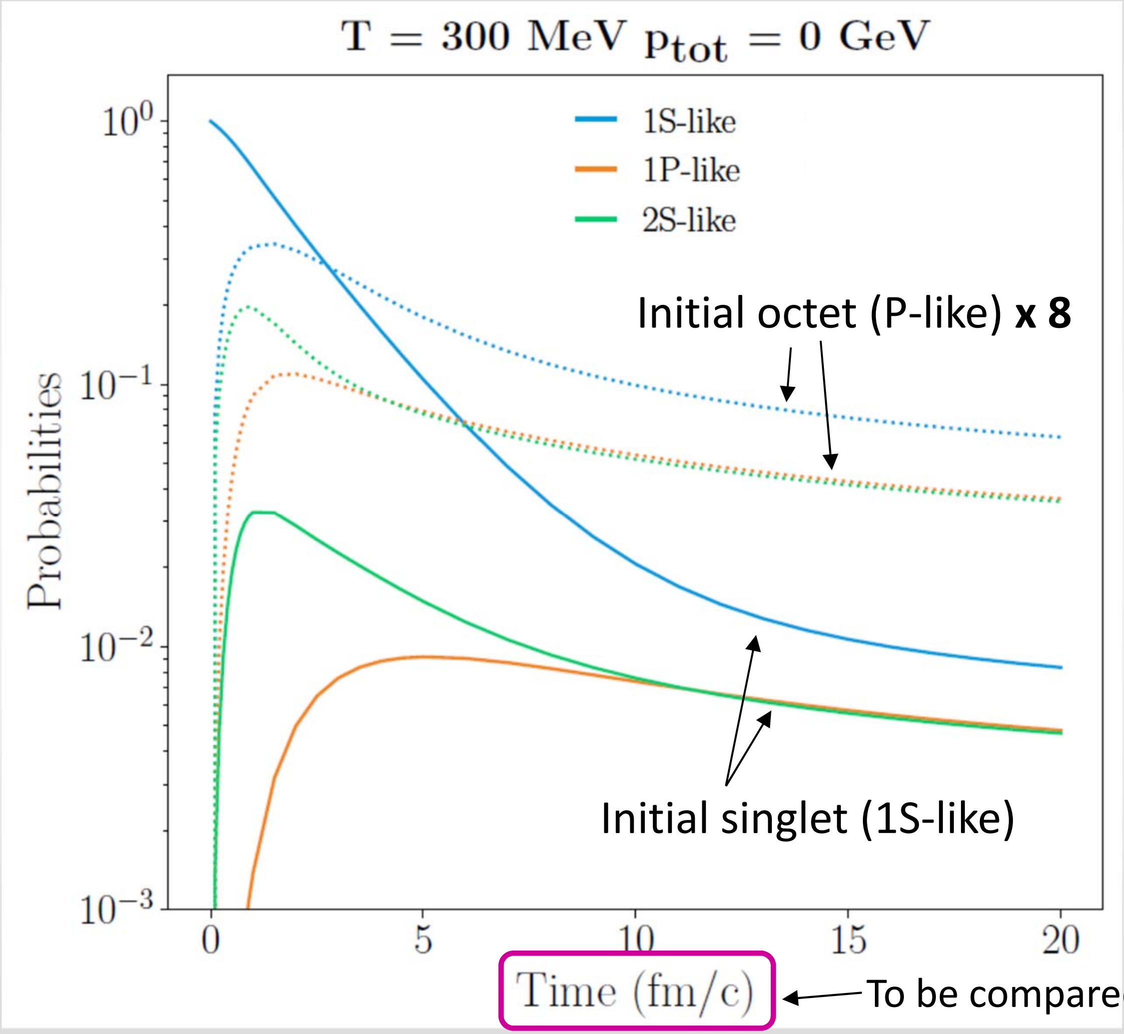
<u>Ds matrix:</u>

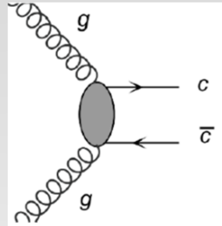


- Initial population of Ds shows a node at x=0 due to dipolar transitions… However, similar asymptotic behavior as for the singlet initial state.

Results for more realistic octet Initial State

<u>Projection on vacuum states:</u>





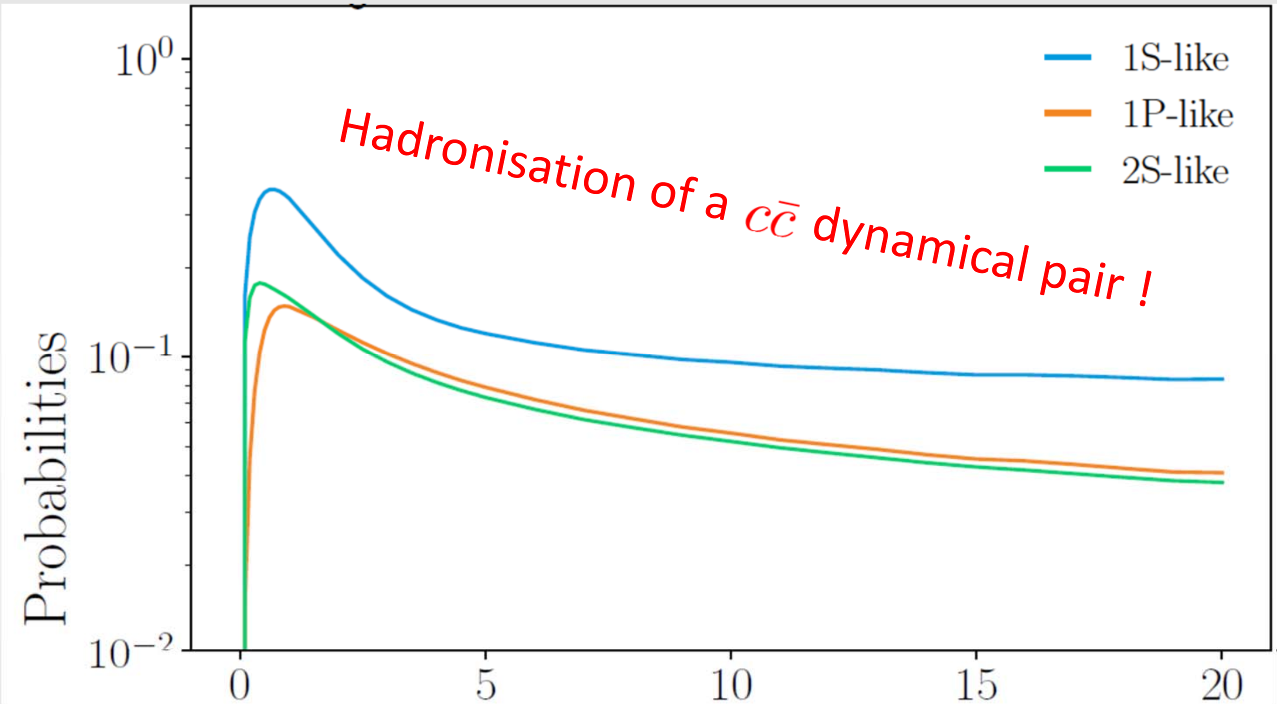
← To be compared with QGP lifetime or path length

- Rather fast initial population of singlet 1S due to color transitions induced by QGP degrees of freedom.
- Similar late time asymptotics (memory loss of the initial color state)

Results for more realistic octet Initial State

Now with cooling medium:

- Bjorken-like evolution of the temperature : $T(t) = T_0 \times \left(\frac{\tau_0}{t+\tau_0}\right)^{\frac{1}{3}}$ $T_0 = 600\,\mathrm{MeV}$, $\tau_0 = 1\,\mathrm{fm}$



- Bound state formation at **early times**… (rather opposite to the statistical hadronization picture… however not “exogeneous” pair => to be taken with a grain of salt)
- Even moderate *repopulation* of the ground state at late times… can be understood as the cooling of the level distribution.

Future steps

- Design the semi-classical expansion corresponding to our 1D model
- Compare the SC solutions to the exact solutions and better understand the range of applicability of SC expansion => possibly introduce quantum corrections
- Generalize to the 3D case + many $c\bar{c}$ pairs
- Implement the generalized algorithm in realistic event generator (EPOSHQ)

Conclusions and Perspectives

- 35-40 years after the concept of « quarkonia as hard probe », the field is now evolving in the direction of imbedding quantum features in the theoretical treatment !

- Still many challenges to solve and thus fantastic field for the young (and not so young) generation

- Semi-classical approximation have been used several times to describe quarkonium production… recent theoretical grounding in the quantum brownian regime by Blaizot and Escobdo

- Our recent contribution : exact solving of quantum master equation for the case of a single ccbar pair.

Subatech

Thank you !