Production of light nuclei in relativistic HIC via rate equations (arxiv:2108.13151)

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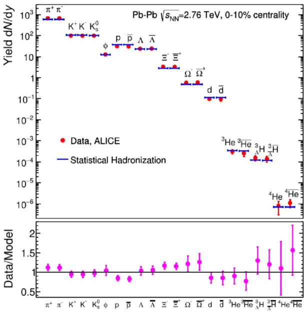
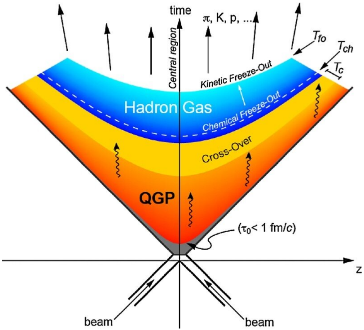
Introduction

- at LHC, the ALICE collaboration measured the yields of light nuclei (Jaroslav Adam et al., Phys. Rev. C, 93(2):024917, 2016)



(a) Hadron abundances and statistical hadronization model predictions (A. Andronic et.al., Nature 561, 321 (2018))



(b) Space-time diagram of a HIC (P. Braun-Munzinger et.al., Nucl. Phys. A, 987:144201, 2019)

Introduction

- the binding energies of light nuclei are much smaller then temperature of the environment
- the nucleosynthesis in heavy-ion collisions can be described by the Saha equation (Volodymyr Vovchenko et. al., Phys. Lett. B, 800:135131, 2020)
- we use the principle of detailed balance to construct rate equations for the light nuclei
- the important reactions are of the following type

$$\frac{dN_A}{dt} = \frac{\left\langle \sigma_{A+X \to a \cdot N + X} v_{rel} \right\rangle}{V} N_X (-N_A + R \cdot N_N^a)$$

$$R = \frac{N_A^{equ} N_X^{equ}}{N_N^{a^{equ}}}$$

Introduction

- As an example, consider $\rho \leftrightarrow \pi + \pi$

$$\frac{\mathrm{d}N_\rho}{\mathrm{d}t} = -\Gamma_{\rho\to 2\pi} N_\rho + \frac{\left\langle \sigma_{\pi+\pi\to\rho} v_{rel} \right\rangle}{V} N_\pi^2$$
$$\frac{\mathrm{d}N_\pi}{\mathrm{d}t} = 2\Gamma_{\rho\to 2\pi} N_\rho - 2\frac{\left\langle \sigma_{\pi+\pi\to\rho} v_{rel} \right\rangle}{V} N_\pi^2$$

- in equilibrium, the lhs is zero, thus we have

$$\frac{\left\langle \sigma_{\pi+\pi\to\rho} v_{rel} \right\rangle}{V} = \Gamma_{\rho\to 2\pi} \frac{N_\rho^{equ}}{N_\pi^{equ\,2}}$$

- by introducing fugacities $\lambda_i = e^{\frac{\mu_i(T)}{T}} = \frac{N_i(T)}{N_i^{equ}(T)}$, we finaly get

$$\frac{\mathrm{d}\lambda_\rho}{\mathrm{d}t} = -\Gamma_{\rho\to 2\pi}(\lambda_\rho + \lambda_\pi^2)$$
$$\frac{\mathrm{d}\lambda_\pi}{\mathrm{d}t} = 2\Gamma_{\rho\to 2\pi}\frac{N_\rho^{equ}}{N_\pi^{equ}}(\lambda_\rho - \lambda_\pi^2)$$

Introduction

- we first have to determine the averaged cross sections, the volume and the multiplicities in chemical equilibrium in dependence of T
- particles: nucleons, the light nuclei and their corresponding anti-particles, π, ρ, ω, K, K^*, Δ, Λ, Σ, Ξ and Ω
- the catalysing particles X are just π and K, because they will have the largest contribution (large abundances and cross sections)

Thermal averaged cross sections

- average over Boltzmann distribution:

$$\left\langle \sigma_{A+X\to a\cdot N+X} v_{rel} \right\rangle = \frac{\iint \frac{d\vec{p}_A^3}{(2\pi)^3}\frac{d\vec{p}_X^3}{(2\pi)^3} e^{-(E_A+E_X)/T}\sigma(p_{lab})v_{rel}(\vec{p}_A,\vec{p}_X)}{\iint \frac{d\vec{p}_A^3}{(2\pi)^3}\frac{d\vec{p}_X^3}{(2\pi)^3} e^{-(E_A+E_X)/T}}$$

- the known cross sections are taken from the PDG (Particle Data Group and P A et. al., Progress of Theoretical and Experimental Physics, 2020(8), 082020)
- we are interested in the case were the nuclei are splited in their nucleonic constituents $\rightarrow$ inelastic cross sections

Thermal averaged cross sections

- as an example the results for $\pi^+ + d$ scattering:

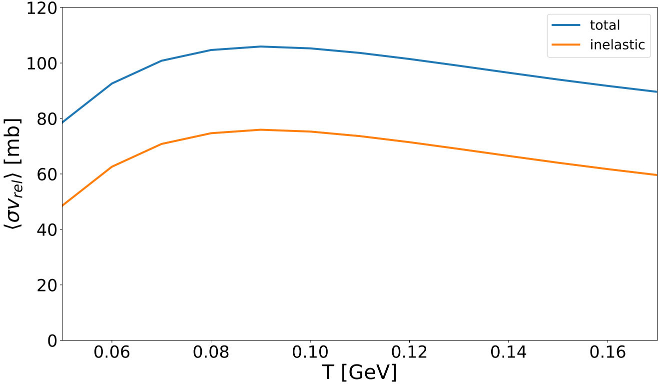


Figure: Total (blue) and inelastic (orange) thermal cross section for $\pi^+ + d$ scattering as function of the tempertaure T.

Thermal Model and Saha equation

- it is usefull to consider a simplified (analytical) example
- system is dominated by effectively massles pions
- relation between T and V (isentropic expansion): $V \propto T^{-3}$
- for all particles without the pions the non-relativistic approximation is used:

$$N_i(T) \approx g_i \left(\frac{m_i T}{2\pi} \right)^{\frac{3}{2}} e^{-m_i/T} \lambda_i V$$

- here λ_i are the fugacities for μ_i
- a simplified expression for the μ_i's by using $N_i(T_c) = N_i(T)$ and $\mu_i(T_c) = 0$ (Volodymyr Vovchenko et. al., Phys. Lett. B, 800:135131, 2020):

$$\mu_i(T) = \frac{3}{2} T \, ln\left(\frac{T}{T_c} \right) + m_i(1 - \frac{T}{T_c})$$

Thermal Model and Saha equation

- now we are able to calculate the normalised ratio $\frac{N_A(T)}{N_A(T_c)}$

$$
\begin{aligned}
\frac{N_A(T)}{N_A(T_c)} &= \left(\frac{T}{T_c}\right)^{\frac{3}{2}} e^{-m_A(\frac{1}{T}-\frac{1}{T_c})} e^{\frac{\mu_A(T)}{T}} \frac{V(T)}{V_c} \\
&= \left(\frac{T}{T_c}\right)^{\frac{3}{2}} e^{-m_A(\frac{1}{T}-\frac{1}{T_c})} e^{a\cdot(\frac{3}{2}\,ln\left(\frac{T}{T_c}\right)+m_N(\frac{1}{T}-\frac{1}{T_c}))} \frac{T_c^3}{T^3} \\
&= \left(\frac{T}{T_c}\right)^{\frac{3}{2}(a-1)} e^{(a\cdot m_N-m_A)(\frac{1}{T}-\frac{1}{T_c})} \\
&= \left(\frac{T}{T_c}\right)^{\frac{3}{2}(a-1)} e^{B_A(\frac{1}{T}-\frac{1}{T_c})}
\end{aligned}
$$

- here we introduced the binding energy of a nucleus $B_A = a \cdot m_N - m_A$

Thermal Model and Saha equation

- this result is different to the standard thermal model result

$$\left.\frac{N_A(T)}{N_A(T_c)}\right|_{stand.} = \left(\frac{T}{T_c}\right)^{\frac{3}{2}} e^{-m_A(\frac{1}{T}-\frac{1}{T_c})}$$

- the major difference is clearly the value in the exponential: $2\,\mathrm{MeV} \approx B_A \ll m_A \approx 1000\,\mathrm{MeV}$
- we see, that the exponential behaviour is strongly weakened

Thermal Model and Saha equation

- to gain the full solution (HRG in PCE) we need to consider also the contributions of the other particles (Volodymyr Vovchenko et. al., Phys. Lett. B, 800:135131, 2020)

$$S_{eff}(T_c) = V \sum_{j \in all\ particles} s_j(T, \tilde{\mu}_j, \mu_B, \mu_S)$$

$$N_{i\ eff}(T_c) = V \sum_{j \in all\ particles} \langle n_i \rangle_j \, n_j(T, \tilde{\mu}_j, \mu_B, \mu_S)$$

$$B_{eff}(T_c) = V \sum_{j \in all\ particles} B_j \, n_j(T, \tilde{\mu}_j, \mu_B, \mu_S)$$

$$0 = V \sum_{j \in all\ particles} S_j \, n_j(T, \tilde{\mu}_j, \mu_B, \mu_S)$$

Thermal Model and Saha equation

- $\langle n_i \rangle_j$ is the averaged number of stable hadrons i which came from the decay(-chain) of hadron j
- the chemical potentials are given as

$$\tilde{\mu}_j = \sum_{i \in \text{stable}} \langle n_i \rangle_j \mu_i \,; \;\; j \in \text{all particles}$$

- by solving the set of non-linear equations we will get $V(T)$, $\mu_i(T)$, $\mu_B(T)$ and $\mu_S(T)$
- all relativistic Boltzmann particles have their "normal" degeneracy factors with exception the Δ-baryon ($g_\Delta^{\mathrm{eff}} = 2g_\Delta$)
- fit model to experimental data (ALICE $0-10\%$ central Pb-Pb (2,72 TeV) e.g. J. Adam et. a., Physics Letters B, 754:360372, 2016) to obtain: $V(T_c) = 4017.5\,\mathrm{fm}^3$, $\mu_B(T_c) = 2.98\,\mathrm{MeV}$ and $\mu_S(T_c) = 0.39\,\mathrm{MeV}$ at $T_c = 155\,\mathrm{MeV}$

Thermal Model and Saha equation

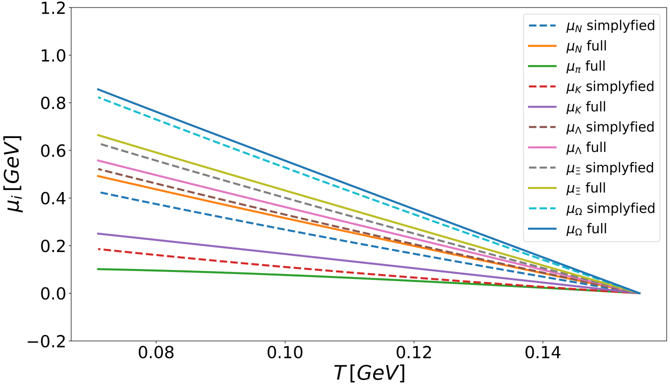


Figure: The μ_i's of the as stable considered hadrons in dependence of T.

Thermal Model and Saha equation

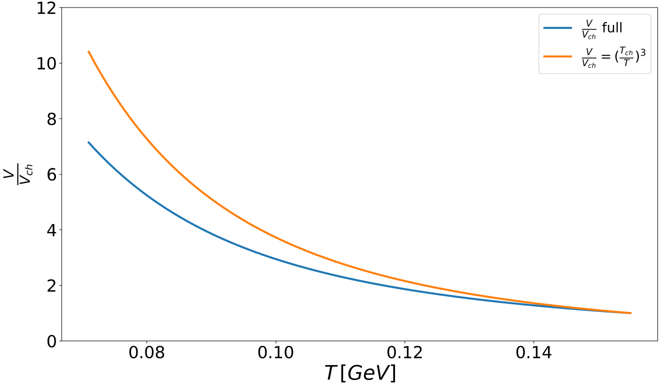


Figure: The volume ratio in dependence of T.

Solving the rate equations

$$\begin{aligned}\frac{\mathrm{d}N_N}{\mathrm{d}t} &= 2\tilde{\alpha}_{D+\pi\to 2N+\pi} N_\pi (N_D - R_{01}\, N_N^2) + 3\tilde{\alpha}_{T+\pi\to 3N+\pi} N_\pi (N_T - R_{02}\, N_N^3) \\ &+ 3\tilde{\alpha}_{\mathrm{He}^3+\pi\to 3N+\pi} N_\pi (N_{\mathrm{He}^3} - R_{03}\, N_N^3) + 4\tilde{\alpha}_{\mathrm{He}^4+\pi\to 4N+\pi} N_\pi (N_{\mathrm{He}^4} - R_{04}\, N_N^4) \\ &+ \tilde{\alpha}_{\Delta\to N+\pi} (N_\Delta - R_{05}\, N_N N_\pi) + 2\tilde{\alpha}_{D+K/\overline{K}\to 2N+K/\overline{K}} N_{K/\overline{K}} (N_D - R_{01}\, N_N^2) \\ &+ 3\tilde{\alpha}_{T+K/\overline{K}\to 3N+K/\overline{K}} N_{K/\overline{K}} (N_T - R_{02}\, N_N^3) + 3\tilde{\alpha}_{\mathrm{He}^3+K/\overline{K}\to 3N+K/\overline{K}} N_{K/\overline{K}} (N_{\mathrm{He}^3} - R_{03}\, N_N^3) \\ &+ 4\tilde{\alpha}_{\mathrm{He}^4+K/\overline{K}\to 4N+K/\overline{K}} N_{K/\overline{K}} (N_{\mathrm{He}^4} - R_{04}\, N_N^4) \end{aligned} \tag{3.7}$$

$$\frac{\mathrm{d}N_D}{\mathrm{d}t} = \tilde{\alpha}_{D+\pi\to 2N+\pi} N_\pi (-N_D + R_{01}\, N_N^2) + \tilde{\alpha}_{D+K/\overline{K}\to 2N+K/\overline{K}} N_{K/\overline{K}} (-N_D + R_{01}\, N_N^2) \tag{3.8}$$

$$\frac{\mathrm{d}N_T}{\mathrm{d}t} = \tilde{\alpha}_{T+\pi\to 3N+\pi} N_\pi (-N_T + R_{02}\, N_N^3) + \tilde{\alpha}_{T+K/\overline{K}\to 3N+K/\overline{K}} N_{K/\overline{K}} (-N_T + R_{02}\, N_N^3) \tag{3.9}$$

$$\frac{\mathrm{d}N_{\mathrm{He}^3}}{\mathrm{d}t} = \tilde{\alpha}_{\mathrm{He}^3+\pi\to 3N+\pi} N_\pi (-N_{\mathrm{He}^3} + R_{03}\, N_N^3) + \tilde{\alpha}_{\mathrm{He}^3+K/\overline{K}\to 3N+K/\overline{K}} N_{K/\overline{K}} (-N_{\mathrm{He}^3} + R_{03}\, N_N^3) \tag{3.10}$$

$$\frac{\mathrm{d}N_{\mathrm{He}^4}}{\mathrm{d}t} = \tilde{\alpha}_{\mathrm{He}^4+\pi\to 4N+\pi} N_\pi (-N_{\mathrm{He}^4} + R_{04}\, N_N^4) + \tilde{\alpha}_{\mathrm{He}^4+K/\overline{K}\to 4N+K/\overline{K}} N_{K/\overline{K}} (-N_{\mathrm{He}^4} + R_{04}\, N_N^4) \tag{3.11}$$

$$\begin{aligned}\frac{\mathrm{d}N_{\overline{N}}}{\mathrm{d}t} &= 2\tilde{\alpha}_{\overline{D}+\pi\to 2\overline{N}+\pi} N_\pi (N_{\overline{D}} - R_{06}\, N_{\overline{N}}^2) + 3\tilde{\alpha}_{\overline{T}+\pi\to 3\overline{N}+\pi} N_\pi (N_{\overline{T}} - R_{07}\, N_{\overline{N}}^3) \\ &+ 3\tilde{\alpha}_{\overline{\mathrm{He}^3}+\pi\to 3\overline{N}+\pi} N_\pi (N_{\overline{\mathrm{He}^3}} - R_{08}\, N_{\overline{N}}^3) + 4\tilde{\alpha}_{\overline{\mathrm{He}^4}+\pi\to 4\overline{N}+\pi} N_\pi (N_{\overline{\mathrm{He}^4}} - R_{09}\, N_{\overline{N}}^4) \\ &+ \tilde{\alpha}_{\overline{\Delta}\to \overline{N}+\pi} (N_{\overline{\Delta}} - R_{10}\, N_{\overline{N}} N_\pi) + 2\tilde{\alpha}_{\overline{D}+K/\overline{K}\to 2\overline{N}+K/\overline{K}} N_{K/\overline{K}} (N_{\overline{D}} - R_{06}\, N_{\overline{N}}^2) \\ &+ 3\tilde{\alpha}_{\overline{T}+K/\overline{K}\to 3\overline{N}+K/\overline{K}} N_{K/\overline{K}} (N_{\overline{T}} - R_{07}\, N_{\overline{N}}^3) + 3\tilde{\alpha}_{\overline{\mathrm{He}^3}+K/\overline{K}\to 3\overline{N}+K/\overline{K}} N_{K/\overline{K}} (N_{\overline{\mathrm{He}^3}} - R_{08}\, N_{\overline{N}}^3) \\ &+ 4\tilde{\alpha}_{\overline{\mathrm{He}^4}+K/\overline{K}\to 4\overline{N}+K/\overline{K}} N_{K/\overline{K}} (N_{\overline{\mathrm{He}^4}} - R_{09}\, N_{\overline{N}}^4) \end{aligned} \tag{3.12}$$

$$\frac{\mathrm{d}N_{\overline{D}}}{\mathrm{d}t} = \tilde{\alpha}_{\overline{D}+\pi\to 2\overline{N}+\pi} N_\pi (-N_{\overline{D}} + R_{06}\, N_{\overline{N}}^2) + \tilde{\alpha}_{\overline{D}+K/\overline{K}\to 2\overline{N}+K/\overline{K}} N_{K/\overline{K}} (-N_{\overline{D}} + R_{06}\, N_{\overline{N}}^2) \tag{3.13}$$

$$\frac{\mathrm{d}N_{\overline{T}}}{\mathrm{d}t} = \tilde{\alpha}_{\overline{T}+\pi\to 3\overline{N}+\pi} N_\pi (-N_{\overline{T}} + R_{07}\, N_{\overline{N}}^3) + \tilde{\alpha}_{\overline{T}+K/\overline{K}\to 3\overline{N}+K/\overline{K}} N_{K/\overline{K}} (-N_{\overline{T}} + R_{07}\, N_{\overline{N}}^3) \tag{3.14}$$

$$\frac{\mathrm{d}N_{\overline{\mathrm{He}^3}}}{\mathrm{d}t} = \tilde{\alpha}_{\overline{\mathrm{He}^3}+\pi\to 3\overline{N}+\pi} N_\pi (-N_{\overline{\mathrm{He}^3}} + R_{08}\, N_{\overline{N}}^3) + \tilde{\alpha}_{\overline{\mathrm{He}^3}+K/\overline{K}\to 3\overline{N}+K/\overline{K}} N_{K/\overline{K}} (-N_{\overline{\mathrm{He}^3}} + R_{08}\, N_{\overline{N}}^3) \tag{3.15}$$

$$\frac{\mathrm{d}N_{\overline{\mathrm{He}^4}}}{\mathrm{d}t} = \tilde{\alpha}_{\overline{\mathrm{He}^4}+\pi\to 4\overline{N}+\pi} N_\pi (-N_{\overline{\mathrm{He}^4}} + R_{09}\, N_{\overline{N}}^4) + \tilde{\alpha}_{\overline{\mathrm{He}^4}+K/\overline{K}\to 4\overline{N}+K/\overline{K}} N_{K/\overline{K}} (-N_{\overline{\mathrm{He}^4}} + R_{09}\, N_{\overline{N}}^4) \tag{3.16}$$

$$\frac{\mathrm{d}N_\Delta}{\mathrm{d}t} = \tilde{\alpha}_{\Delta\to N+\pi} (-N_\Delta + R_{05}\, N_N N_\pi) \tag{3.17}$$

$$\frac{\mathrm{d}N_{\overline{\Delta}}}{\mathrm{d}t} = \tilde{\alpha}_{\overline{\Delta}\to \overline{N}+\pi} (-N_{\overline{\Delta}} + R_{10}\, N_{\overline{N}} N_\pi) \tag{3.18}$$

$$\begin{aligned}\frac{\mathrm{d}N_\pi}{\mathrm{d}t} &= \tilde{\alpha}_{\Delta\to N+\pi} (N_\Delta - R_{05}\, N_N N_\pi) + \tilde{\alpha}_{\overline{\Delta}\to \overline{N}+\pi} (N_{\overline{\Delta}} - R_{10}\, N_{\overline{N}} N_\pi) \\ &+ 2\tilde{\alpha}_{\rho\to 2\pi} (N_\rho - R_{11}\, N_\pi^2) + 3\tilde{\alpha}_{\omega\to 3\pi} (N_\omega - R_{12}\, N_\pi^3) \end{aligned} \tag{3.19}$$

$$\frac{\mathrm{d}N_\rho}{\mathrm{d}t} = \tilde{\alpha}_{\rho\to 2\pi} (-N_\rho + R_{11}\, N_\pi^2) \tag{3.20}$$

$$\frac{\mathrm{d}N_\omega}{\mathrm{d}t} = \tilde{\alpha}_{\omega\to 3\pi} (-N_\omega + R_{12}\, N_\pi^3) \tag{3.21}$$

$$\frac{\mathrm{d}N_K}{\mathrm{d}t} = \tilde{\alpha}_{K^*\to K+\pi} (N_{K^*} - R_{13}\, N_\pi N_K) \tag{3.22}$$

$$\frac{\mathrm{d}N_{\overline{K}}}{\mathrm{d}t} = \tilde{\alpha}_{\overline{K}^*\to \overline{K}+\pi} (N_{\overline{K}^*} - R_{14}\, N_\pi N_{\overline{K}}) \tag{3.23}$$

$$\frac{\mathrm{d}N_{K^*}}{\mathrm{d}t} = \tilde{\alpha}_{K^*\to K+\pi} (-N_{K^*} + R_{13}\, N_\pi N_K) \tag{3.24}$$

$$\frac{\mathrm{d}N_{\overline{K}^*}}{\mathrm{d}t} = \tilde{\alpha}_{\overline{K}^*\to \overline{K}+\pi} (-N_{\overline{K}^*} + R_{14}\, N_\pi N_{\overline{K}}) \tag{3.25}$$

Solving the rate equations

- for all light nuclei up to He^4 rate equations has been implemented, but also the decays of ρ, ω, K^* and Δ has been considered
- we have just related the volume and temperature, but the system contains ODE's in time
- here we consider a parametrisation $V(t)$ (Yinghua Pan and Scott Pratt, Phys. Rev. C, 89(4):044911, 2014):

$$\frac{V(t)}{V_{ch}} = \frac{t}{t_{ch}} \frac{t_\perp^2 + t^2}{t_\perp^2 + t_{ch}^2};\ t_\perp = 6.5\frac{\mathrm{fm}}{c};\ t_{ch} = 9\frac{\mathrm{fm}}{c}$$

Solving the rate equations

- first we want to look at the rates:

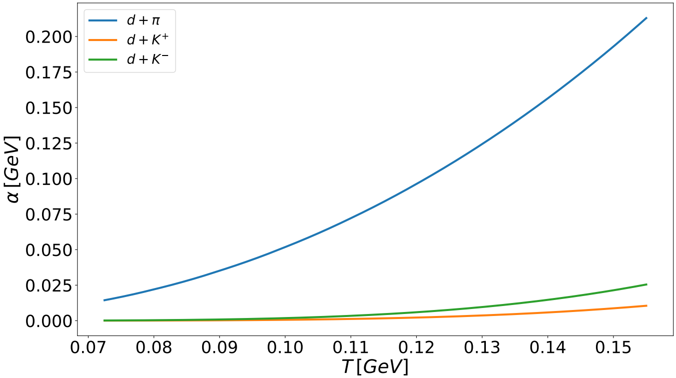


Figure: The rates $\alpha = \frac{\langle v_{rel}\sigma_{tot}\rangle N_X^{eq}}{V}$ in a fixed volume $V = 4000\,\mathrm{fm}^3$ for different temperatures (but fixed during the equilibration of the system) for different deuteron break up reactions

Solving the rate equations

- now we solve the full set of equations and look at the evolution of the deuteron number:

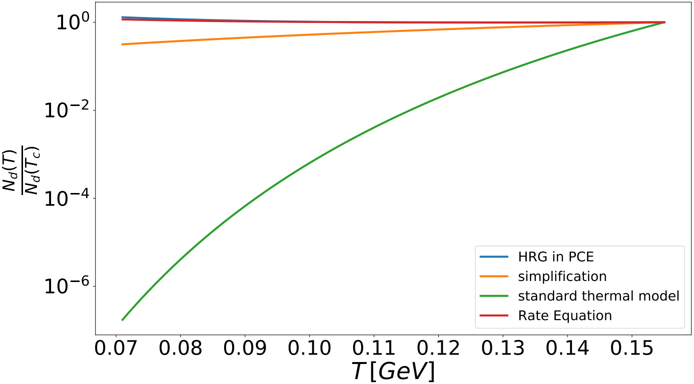


Figure: Normalised particle number of deuterons to the value at $T_c = 155\,\mathrm{MeV}$ for $g_\Delta^{\mathrm{eff}} = 2g_\Delta$

Solving the rate equations

- now we want to check how fast the system equilibrates when starting out of equilibrium:

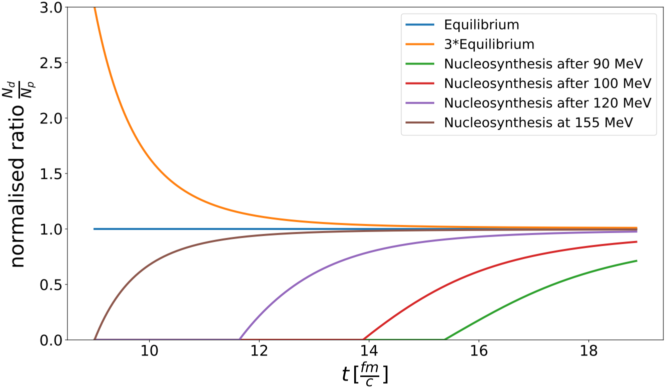


Figure: The ratio of deuterons to protons normalized to the same ratio at equilibrium for different initial conditions with $g_{\Delta}^{\mathrm{eff}} = 2g_{\Delta}$.

Solving the rate equations

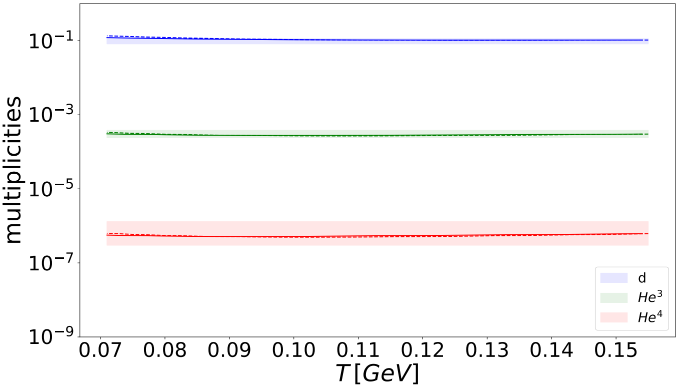


Figure: Solid lines represent the results of the rate equations, while dashed curves show the result of the HRG in PCE. The colored bands represent the experimental data (ALICE)

Effect of the $N + \overline{N} \rightleftharpoons 5\pi$ reaction

- a big advantage of the rate equation approach is the possibility of the annihilation of stable hadrons e.g. nucleons

$$\frac{dN_N}{dt} = \frac{\left\langle \sigma_{N+\overline{N}\rightleftharpoons 5\pi} v_{rel} \right\rangle}{V}(-N_N N_{\overline{N}} + R_{15}\, N_\pi^5)$$

$$\frac{dN_{\overline{N}}}{dt} = \frac{\left\langle \sigma_{N+\overline{N}\rightleftharpoons 5\pi} v_{rel} \right\rangle}{V}(-N_N N_{\overline{N}} + R_{15}\, N_\pi^5)$$

$$\frac{dN_\pi}{dt} = 5\frac{\left\langle \sigma_{N+\overline{N}\rightleftharpoons 5\pi} v_{rel} \right\rangle}{V}(N_N N_{\overline{N}} - R_{15}\, N_\pi^5)$$

$$R_{15} = \frac{N_N^{eq} N_{\overline{N}}^{eq}}{{N_\pi^{eq}}^5}$$

Effect of the $N + \overline{N} \rightleftharpoons 5\pi$ reaction

- the averaged cross section is about $50\,\mathrm{mb}$ for $p + \overline{p}$ scattering
- this type of reaction exlicitly violates the conservation of stable hadrons, but the net baryon number is still conserved

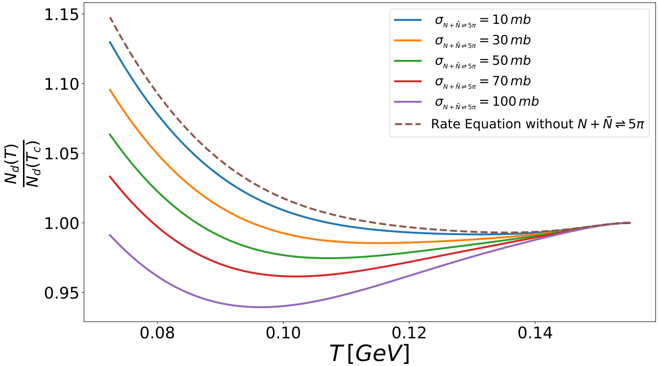


Figure: Normalised particle number of deuterons to the value at $T_c = 155\,\mathrm{MeV}$ and $g_{\Delta}^{\mathrm{eff}} = 2g_{\Delta}$.

Conclusions and Outlook

- both approaches are in great agreement with each other and also in the error range of the experimental data
- the same procedure could be done for RHIC or SPS energies
- the annihilation of nucleon and anti-nucleon into five pions only leads to a $4-5\%$ decrease in the effective nucleon number
- under-occupation in the nucleons leads to a suppression of the light nuclei

Conclusions and Outlook

- calculations also support earlier assumptions, that the nuclei do not need to be formed at the chemical freeze-out
- this approach neglects the formation time of the nuclei
- a quantum mechanical description of creation and decay of bound states (the nuclei) in an open thermal system (fireball) is needed