Bjorken flow attractors with transverse dynamics

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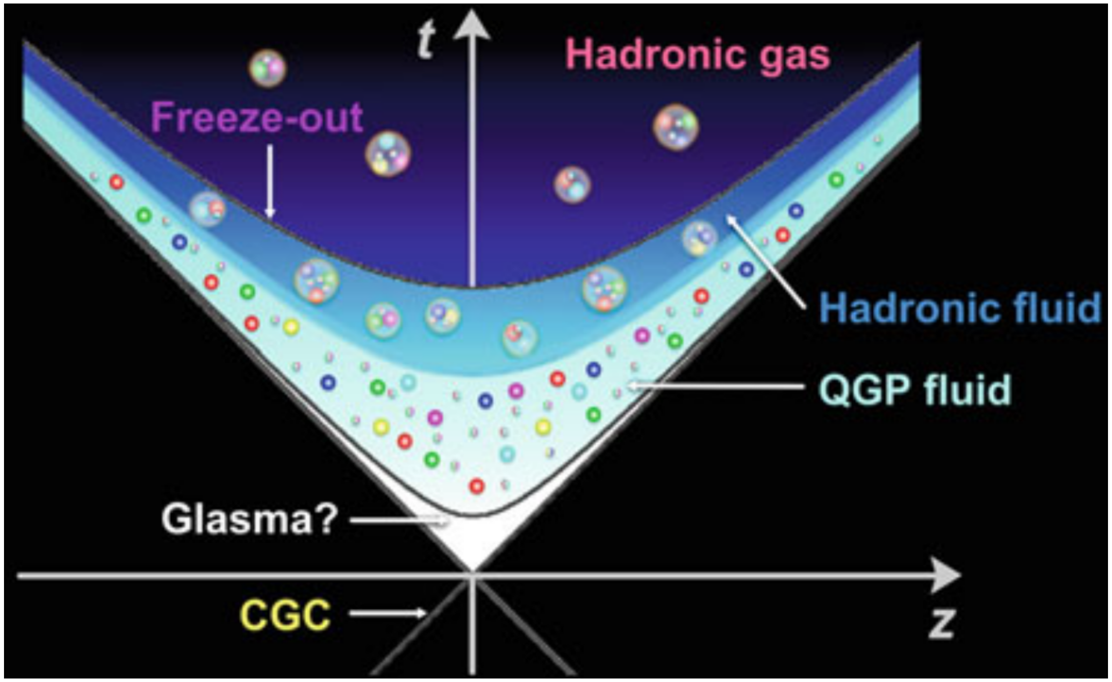
Outline

Bjorken model

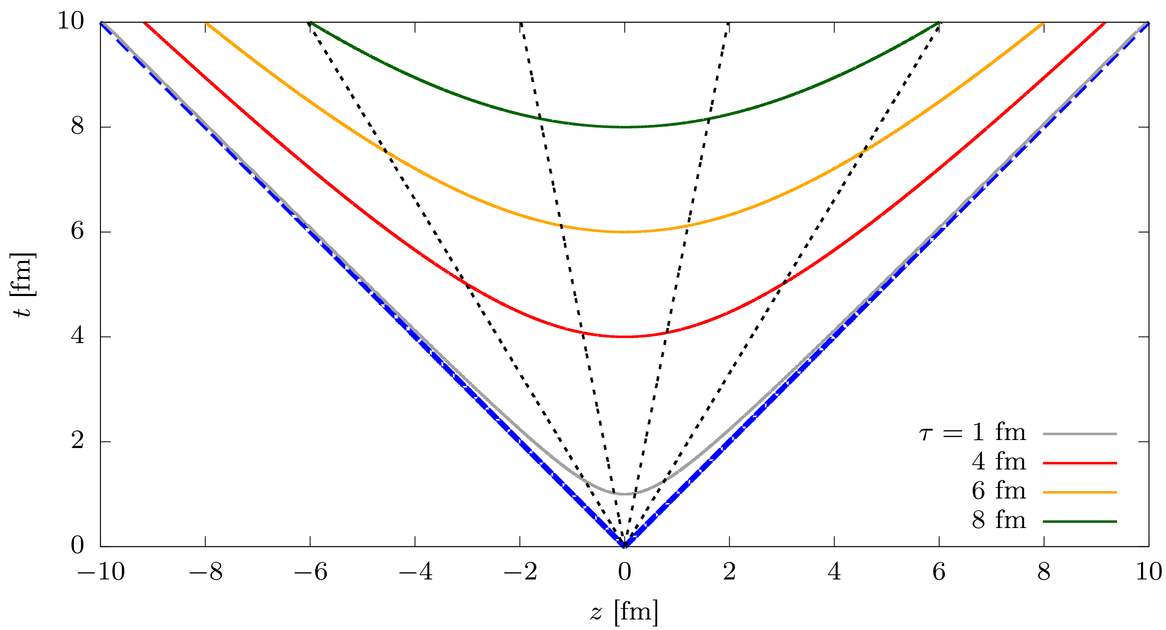


[A. Monnai, PhD Thesis (Tokyo, 2014)]

Nuclear collision model:

- Initial state: Colour glass condensate
- Early stage: Glasma?
- Onset of QGP
- Hadronisation
- Freeze-out

Longitudinal boost invariance



- Introducing proper (Bjorken) time τ and rapidity η_s:

$$\tau = \sqrt{t^2 - z^2}, \qquad \eta_s = \frac{1}{2} \ln \frac{t+z}{t-z} = \text{arctanh} \frac{\text{z}}{\text{t}}, \tag{1}$$

a Lorentz boost along z with velocity $V = \tanh \alpha$ gives:

$$\tau' = \tau, \qquad \eta_s' = \eta_s - \alpha. \tag{2}$$

- Longitudinal boost invariance $\Rightarrow T^{\mu\nu}$ cannot depend on η_s.

$0+1$-D Bjorken flow

- In the Landau frame ($T^\mu{}_\nu u^\nu = e u^\mu$), we have

$$T^{\mu\nu} = (e+p)\, u^\mu u^\nu - p g^{\mu\nu} + \pi^{\mu\nu}. \tag{3}$$

- Under boost invariance and ignoring transverse expansion,

$$u^\mu \partial_\mu = \tfrac{t}{\tau}\partial_t + \tfrac{z}{\tau}\partial_z = \partial_\tau. \tag{4}$$

- By construction, $\pi^\mu{}_\mu = 0$, $\pi^{\mu\nu} = \pi^{\nu\mu}$ and $\pi^{\mu\nu} u_\nu = 0$, such that

$$\pi^\mu{}_\nu = \mathrm{diag}\left(0, \tfrac{\pi}{2}, \tfrac{\pi}{2}, -\pi\right). \tag{5}$$

- In this case, $T^\mu{}_\nu = \mathrm{diag}(e, -\mathcal{P}_T, -\mathcal{P}_T, -\mathcal{P}_L)$, where

$$\mathcal{P}_T = p - \frac{\pi}{2}, \qquad \mathcal{P}_L = p + \pi. \tag{6}$$

- Due to the attractor, information about initial $\chi = \mathcal{P}_L/\mathcal{P}_T$ is lost at finite time.

MIS attractor

[M. Heller, M. Spalinski, PRL **115** (2015) 072501]

- For Bjorken flow, $\nabla_\mu T^{\mu\nu} = 0$ reduces to

$$\tau \partial_\tau e + e + p + \pi = 0. \tag{7}$$

- Taking the ultrarelativistic limit ($e = 3p$) & parton gas ($\mu = 0$ and $e = aT^4$) approximations, MIS hydro gives [A. Jaiswal, PRC **87** (2013) 051901(R)]

$$\frac{\partial \pi}{\partial \tau} = -\frac{\pi}{\tau_R} - \beta_\pi \frac{4}{3\tau} - \lambda \frac{\pi}{\tau}, \qquad \beta_\pi = \frac{\eta}{\tau_R}, \qquad \tau_R = \frac{5\eta/s}{T}, \qquad \lambda = \frac{38}{21}. \tag{8}$$

- Denoting $\chi' = d\chi/d\tilde{w}$ with $\tilde{w} = \frac{\tau T}{4\pi\eta/s}$, we have

$$\frac{3+\chi}{8\pi}\frac{d\chi}{d\tilde{w}} = \frac{(1-\chi)(2+\chi)^2}{15} - \frac{2+\chi}{4\pi\tilde{w}}\frac{6}{35}\left(1 + \frac{23\chi}{3} + \frac{2\chi^2}{3}\right), \tag{9}$$

which is of first order and depends only on $\tilde{w}$, together with $\tilde{w}_0$ and χ_0.

MIS attractor

[M. Heller, M. Spalinski, PRL **115** (2015) 072501]

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- Solutions at large and small $\tilde{w}$,

$$\chi(\tilde{w} \gg 1) = 1 - \frac{2}{\pi\tilde{w}} + \frac{6}{7\pi^2\tilde{w}^2} + O(\tilde{w}^{-3}), \qquad \leftarrow \text{divergent ;)}$$
$$\chi(\tilde{w} \ll 1) = \chi_\infty + b\tilde{w}, \qquad \chi_\infty \simeq -0.132, \qquad b \simeq 0.863, \tag{10}$$

... are independent of $\tilde{w}_0, \chi_0$.

- Initial conditions can be taken into account only via the transseries solution:

$$\chi(\tilde{w}) = \sum_{m=0}^{\infty} c^m \Omega^m(\tilde{w}) X_m(\tilde{w}), \qquad X_m(\tilde{w}) = \sum_{n=0}^{\infty} X_{m,n}\tilde{w}^{-n}, \tag{11}$$

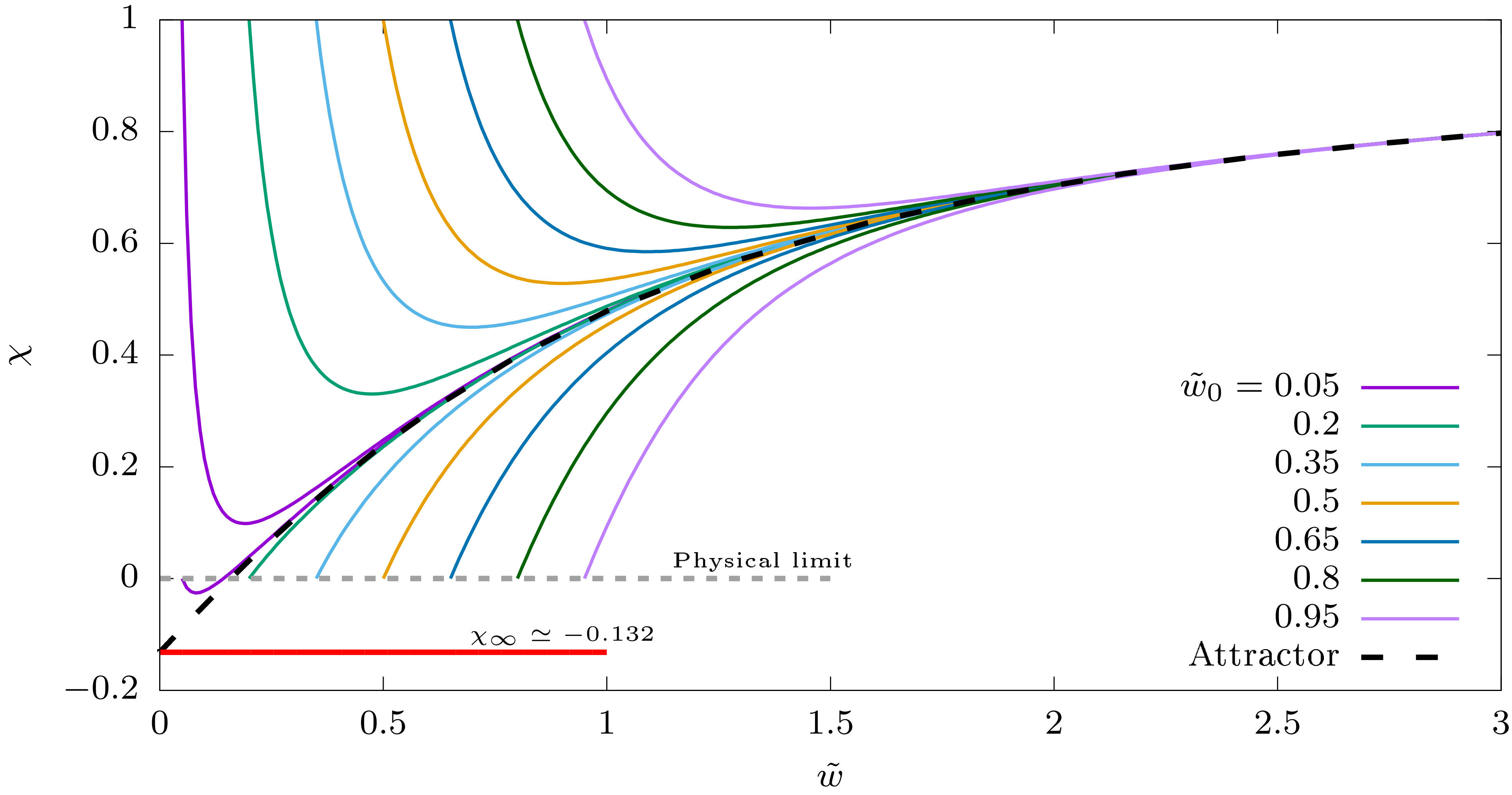
where c is a constant, while

$$\Omega(\tilde{w}) = \tilde{w}^{-\gamma} e^{-\xi_0 \tilde{w}}, \qquad \gamma = \frac{18}{35}, \qquad \xi_0 = \frac{6\pi}{5}. \tag{12}$$

- The constants $X_{m,n}$ are independent of χ_0, $\tilde{w}_0$:

$$X_{0,0} = 1, \qquad X_{0,1} = -\frac{2}{\pi}, \qquad X_{0,2} = \frac{6}{7\pi^2},$$
$$X_{1,0} = 1, \qquad X_{1,1} = -\frac{3}{10\pi}, \qquad X_{1,2} = \frac{2657}{12600\pi^2},$$
$$X_{2,0} = \frac{5}{12}, \qquad X_{2,1} = -\frac{5}{24\pi}, \qquad X_{2,2} = \frac{349}{1080\pi^2}. \tag{13}$$

MIS attractors in Bjorken flow



- Regularity at $\tilde{w} = 0$ selects the attractor.
- Solutions initialised at various $\tilde{w}_0$ decay towards the attractor.
- Hydro casually gives negative χ.

Numerical methods

- Hydro is solved using vSHASTA. [E. Molnar, H. Niemi, D. H. Rischke, EPJC **65** (2010) 615]
  - SHArp and Smooth Transport Algorithm. [J. P. Boris, D. L. Book, JCP **11** (1973) 38]
  - Israel-Stewart theory. [W. Israel, J. M. Stewart, Ann. Phys. **118** (1979) 341]

$$\tau_R \frac{\partial \pi}{\partial \tau} + \pi = -\frac{4\eta}{3\tau} - \frac{38\tau_R}{21\tau}\pi$$

- The Boltzmann eq. is solved using BAMPS. [Z. Xu, C. Greiner, PRC **71** (2005) 064901]
  - Boltzmann Approach to Multi-Parton Scattering.
  - Test-particle based approach.
  - Monte Carlo sampling for collisions.

$$k^\mu \frac{\partial f}{\partial x^\mu} - \Gamma^i{}_{\mu\nu} k^\mu k^\nu \frac{\partial f}{\partial k^i} = C[f]$$

- RTA is solved using RLB. [V. E. Ambruș, R. Blaga, PRC **98** (2018) 035201]
  - Relativistic Lattice Boltzmann method.
  - AW approximation for collision term. [J. Anderson, H. Witting, Physica **74** (1974) 466]
  - Vielbeins for curvilinear coords. [C. Y. Cardall *et al.*, PRD **88** (2013) 023011]

$$C[f] = -\frac{k \cdot u}{\tau_R}(f - f^{(\mathrm{eq})}).$$

(Nearly)-conformal fluids

- Forcing $\mu = 0$ implies that n is not conserved.
- BAMPS automatically conserves $n \Rightarrow$ nearly-conformal fluids are considered with

$$\frac{g}{(2\pi)^3} e^{\mu/T} = \frac{n}{8\pi T^3}, \qquad \mu_0 = 0. \tag{14}$$

- $\partial_\mu N^\mu = 0 \Rightarrow n(\tau) = n_0 \tau_0 / \tau$.
- Now $e = 3nT \neq aT^4$, but we consider $\eta/s = \mathrm{const.}$, such that

$$\tau_R = \frac{5\eta}{4p} = \frac{5\eta}{sT}\left(1 - \frac{1}{4}\ln\lambda\right), \qquad \lambda = e^{\mu/T} = \frac{n\pi^2}{gT^3}. \tag{15}$$

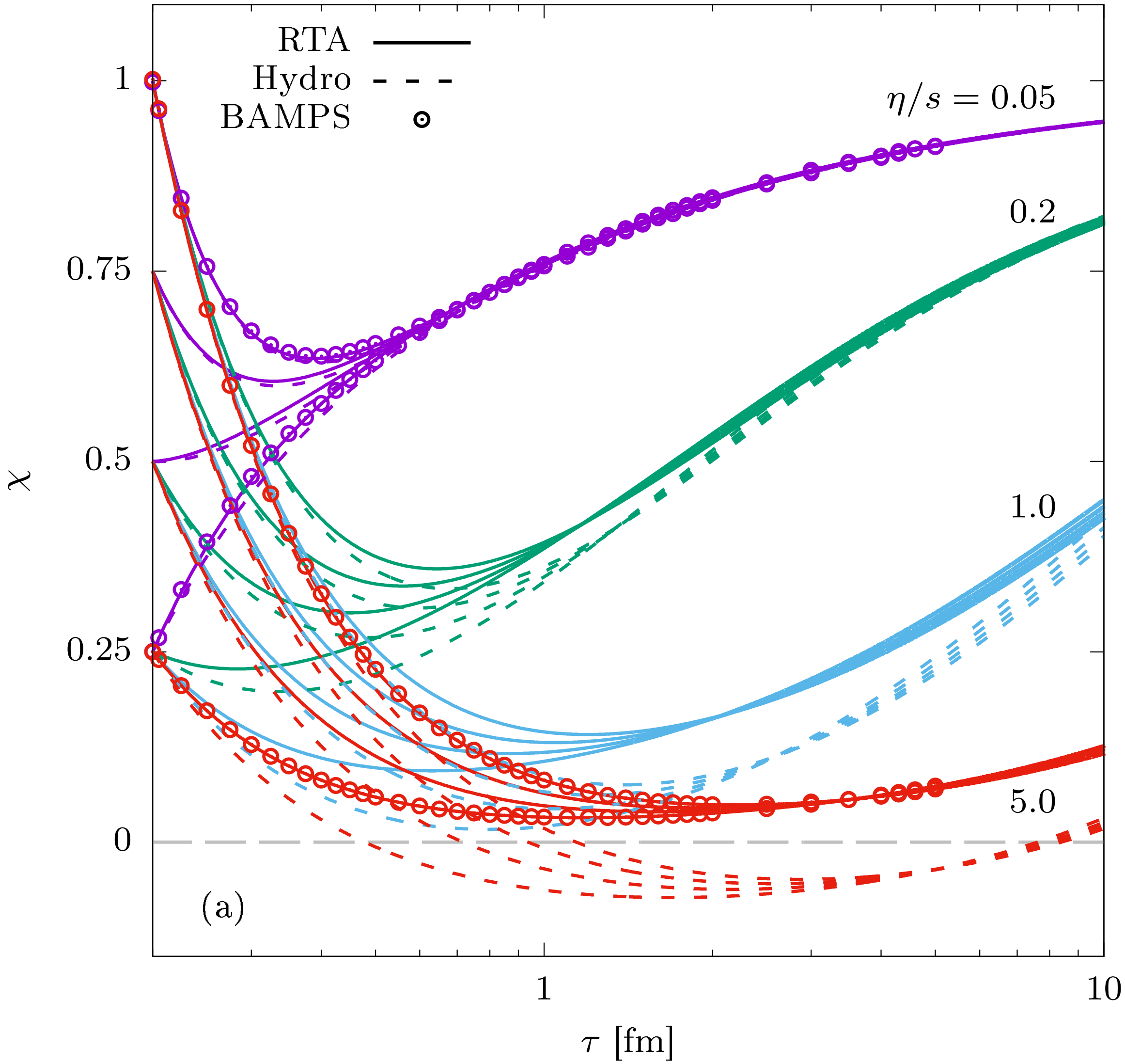
- New "nearly conformal" $\tilde{w}_{\rm nc}$ required:

$$\begin{aligned} \mu = 0: \qquad & \tilde{w} = \frac{5\tau}{4\pi\tau_R} = \frac{\tau T}{4\pi\eta/s}, \\ \mu \neq 0: \qquad & \tilde{w}_{\rm nc} = \frac{5\tau}{4\pi\tau_R} = \frac{\tau T}{4\pi\eta/s}\left[1 + \ln\left(\frac{\tau P^{3/4}}{\tau_0 P_0^{3/4}}\right)\right]^{-1}. \end{aligned} \tag{16}$$

Attractor with hydro, RTA and BAMPS

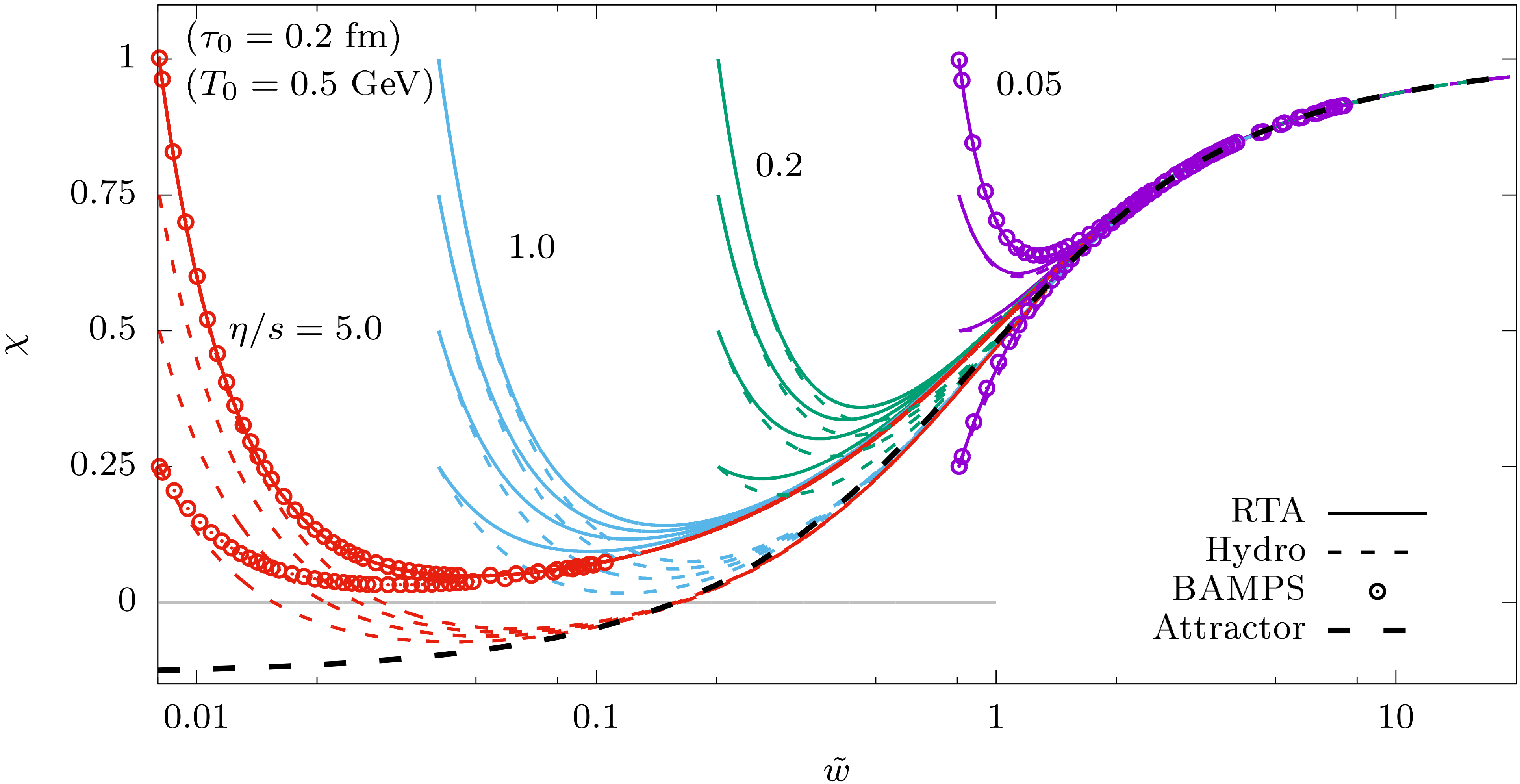
$(\tau_0 = 0.2 \text{ fm})$
$(T_0 = 0.5 \text{ GeV})$

$$\chi = \frac{\mathcal{P}_L}{\mathcal{P}_T}$$



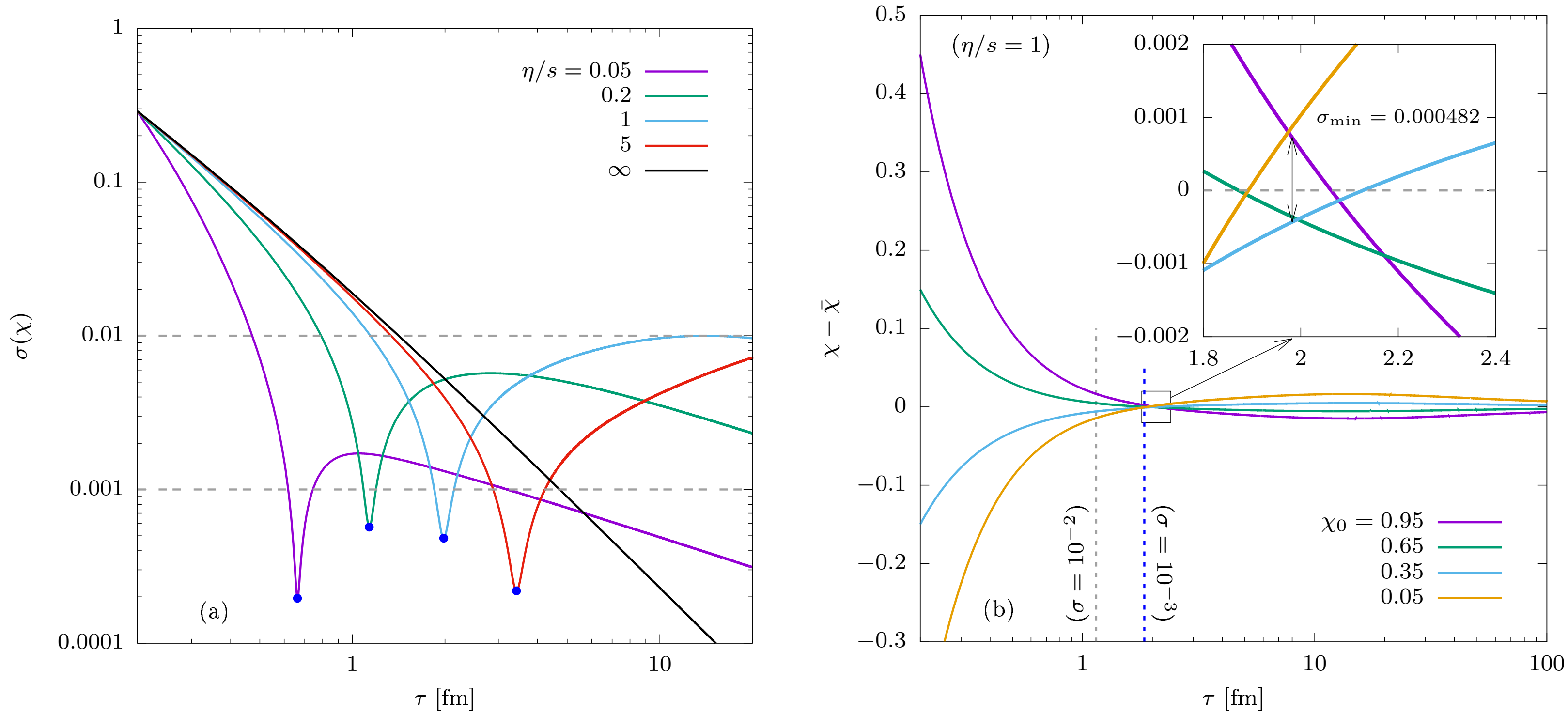
- The curves with different χ_0 converge.
- Overshooting because $T(\tau)$ depends on χ_0.

(Nearly)-conformal fluids



- Conformal attractor retains validity.
- Problem: τ [fm] is difficult to extract from $\tilde{w}$.
- Instead focus on quantities w.r.t. τ.

Hydrodynamisation time $\delta\tau_H$



- Memory loss can be characterised by $\sigma(\chi) = \left[\int_0^1 d\chi_0 \, (\chi - \overline{\chi})^2\right]^{1/2}$ at fixed τ.
- Due to overshooting, first minimum $\sigma_{\min} > 0$.
- Hydrodynamisation time scale given by:

$$\delta\tau_H^{\sigma_{\rm th}} = \frac{\tau_H^{\sigma_{\rm th}}}{\tau_0} - 1, \qquad \sigma(\tau_H^{\sigma_{\rm th}}) = \max(\sigma_{\min}, \sigma_{\rm th}). \tag{17}$$

Hydrodynamisation time $\delta\tau_H$: Limits

- At large $\tilde{w}_0$, the conformal solution can be approximated by

$$\chi(\tilde{w}) = X_0(\tilde{w}) + c\Omega(\tilde{w})X_1(\tilde{w}) + \dots \simeq 1 - (1-\chi_0)e^{-\frac{2\xi_0}{3}\tilde{w}_0\delta\tau} + \dots. \quad (18)$$

- The standard deviation is simply $\sigma(\chi) \simeq \sigma(\chi_0)e^{-\frac{2\xi_0}{3}\tilde{w}_0\delta\tau}$, such that

$$\delta\tau_H^{\sigma_{\rm th}} = \frac{5\eta/s}{\tau_0 T_0}\ln\left[\frac{\sigma(\chi_0)}{\sigma_{\rm th}}\right], \qquad \sigma(\chi_0) = \frac{1}{\sqrt{12}}. \quad (19)$$

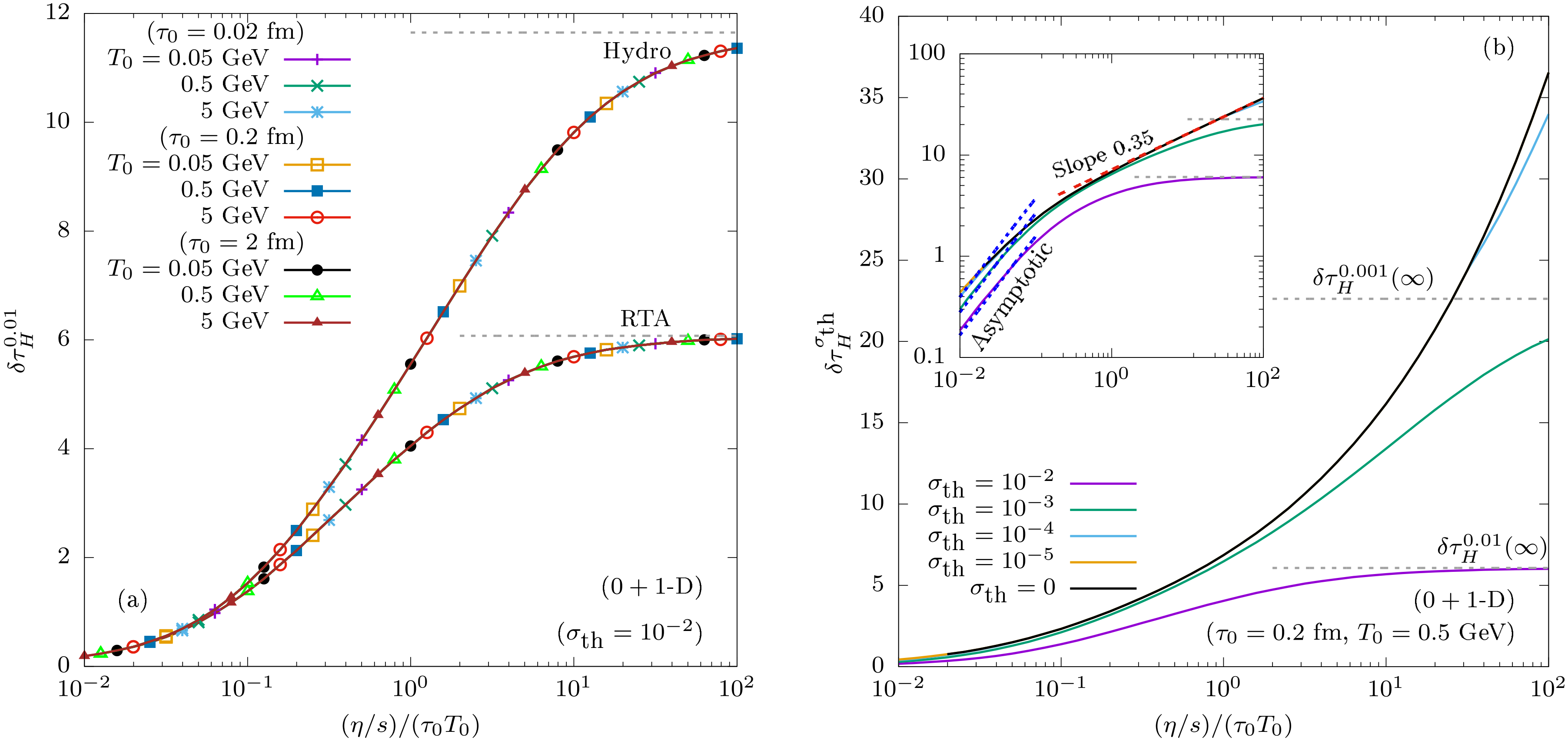
- In the FS limit ($\tilde{w} \to 0$), both hydro and RTA can be solved exactly:

$$\sigma(\chi) \sim \tau^{-2\gamma}, \qquad \gamma_{\rm hydro} \simeq 0.642, \qquad \gamma_{\rm RTA} = 1. \quad (20)$$

- Numerical computation gives:

	Hydro:	RTA:
$\sigma_{\rm th} = 10^{-2}$,	$\delta\tau_H^{0.01}(\infty) = 11.6492$,	$\delta\tau_H^{0.01}(\infty) = 6.07422$,
$\sigma_{\rm th} = 10^{-3}$,	$\delta\tau_H^{0.001}(\infty) = 74.785$,	$\delta\tau_H^{0.001}(\infty) = 22.6203$,
$\sigma_{\rm th} = 10^{-4}$,	$\delta\tau_H^{0.0001}(\infty) = 454.199$,	$\delta\tau_H^{0.0001}(\infty) = 75.0314$. (21)

Hydrodynamisation time $\delta\tau_H$: Scaling



- $\delta\tau_H^{\sigma_{\rm th}}$ is a universal function of $\tilde{w}_0^{-1} = (4\pi\eta s)/(\tau_0 T_0)$.
- Small η/s (large $\tilde{w}_0$) asymptotics confirmed.
- Large η/s (small $\tilde{w}_0$) behaviour fitted by $\delta\tau_H^0 \sim \tilde{w}_0^\alpha$, with $\alpha \simeq 0.35$.

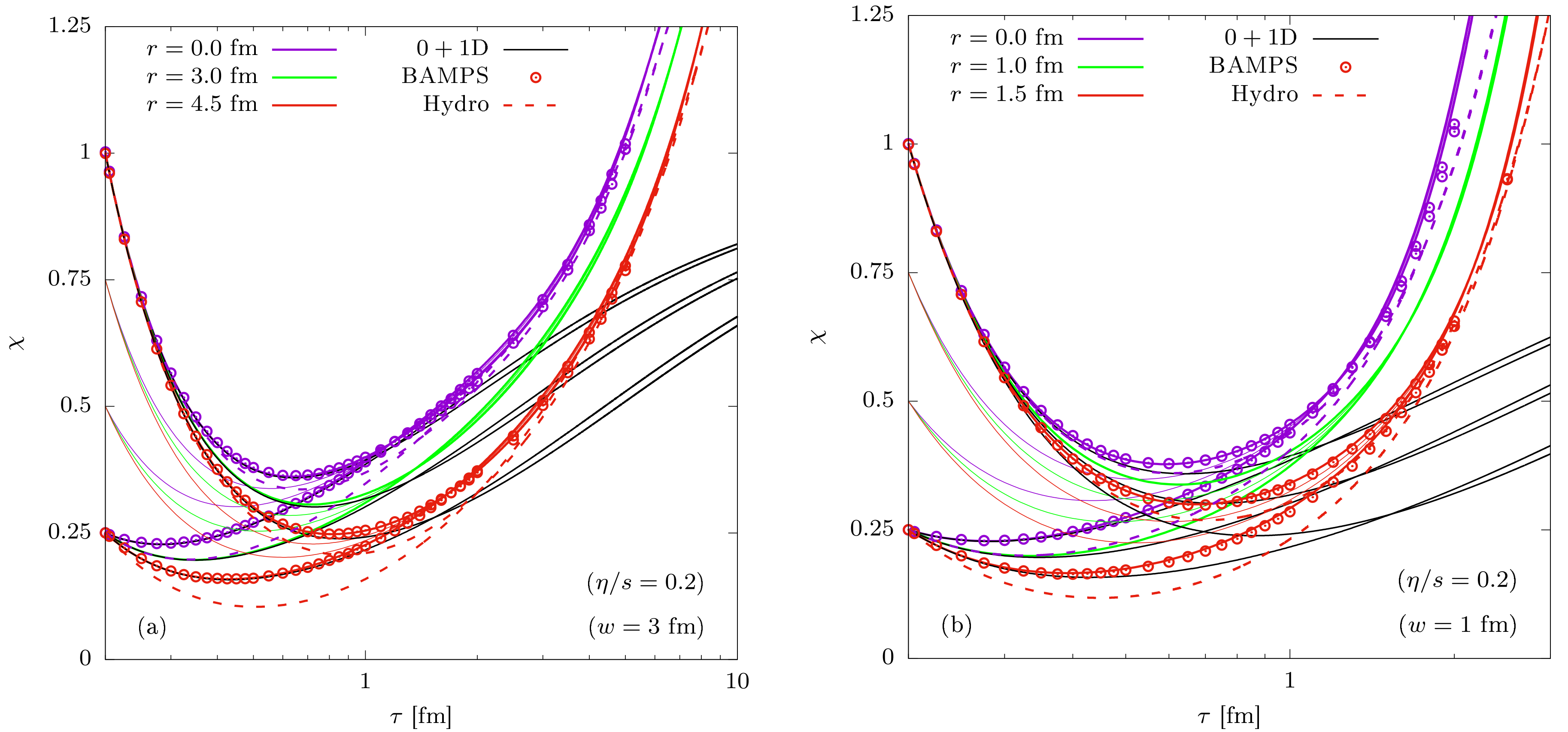
Initial setup

- Azimuthally symmetric, Gaussian initial state in the transverse plane:

$$n_0(r) = n_0(0)e^{-r^2/w^2}, \qquad T_0(r) = T_0(0)e^{-r^2/3w^2}, \tag{22}$$

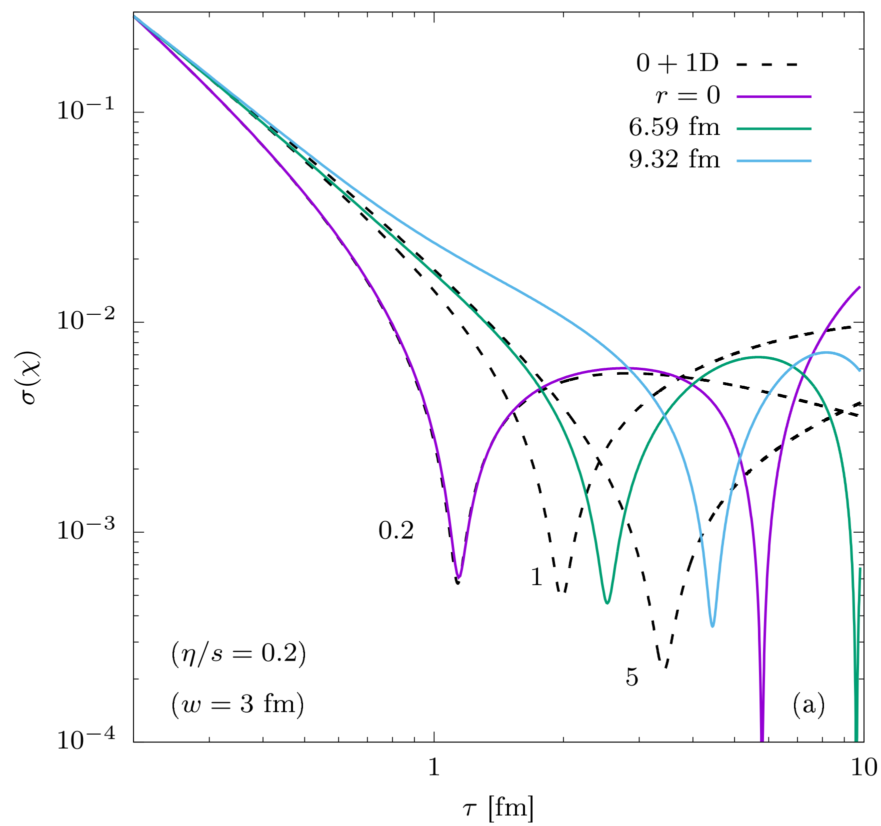
where $T_0(0) = 0.5$ GeV and $n_0(0) = gT_0^3(0)/\pi^2$.
- Discs at $r = w$ and $3w/2$ contain 74% and 95% of energy density, resp.
- $w = 3$ fm $(A + A)$ and 1 fm $(p + p)$.

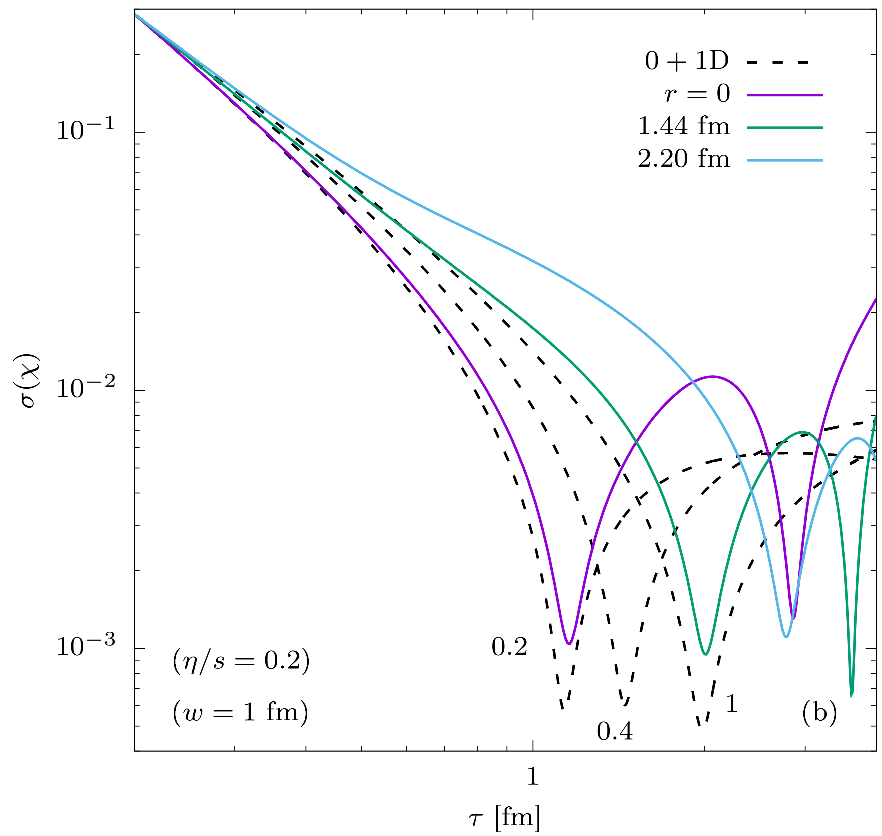
Hydro vs RTA vs BAMPS



- Local $\tilde{w}_0^{-1}(r) \equiv (4\pi\eta/s)/[\tau_0 T_0(r)]$ increases with $r \Rightarrow$ larger $\delta\tau_H$.
- At small τ, longitudinal expansion dominates $\Rightarrow \chi$ follows $0+1$D dynamics.
- At large τ, transverse expansion dominates $\Rightarrow \chi > 1$.

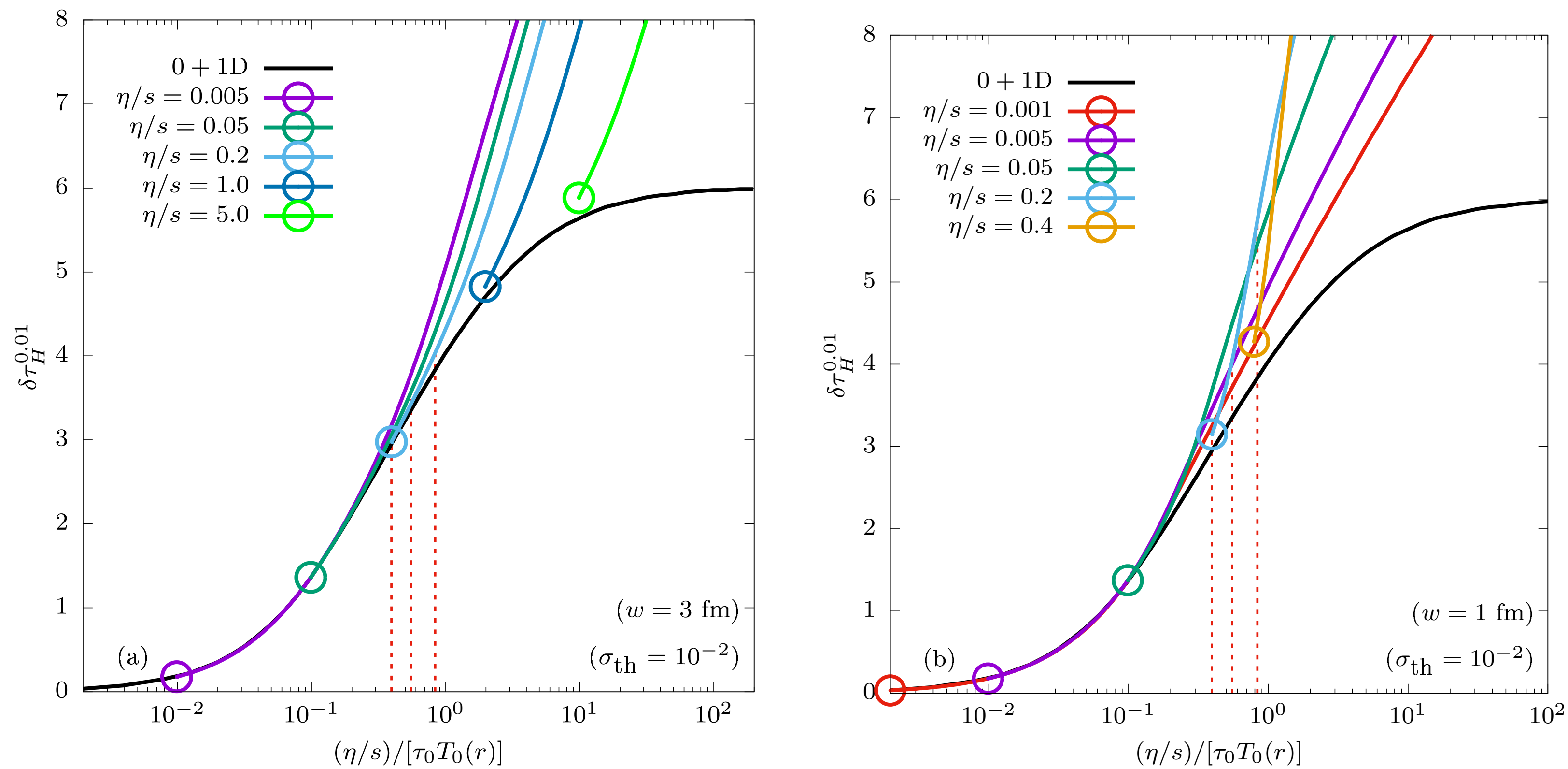
Standard deviation: comparison to $0+1D$





- Keeping $T_0 = 0.5$ GeV and $\tau_0 = 0.2$ fm, η/s can be varied in the $0+1D$ system to match the $\tilde{w}_0^{-1}$ at $r/w > 0$.
- Hydrodynamisation at the fireball centre is dominated by longitudinal expansion.
- At $r/w > 0$, hydrodynamisation is delayed by transverse expansion.
- Second minimum in σ indicates a transverse attractor.

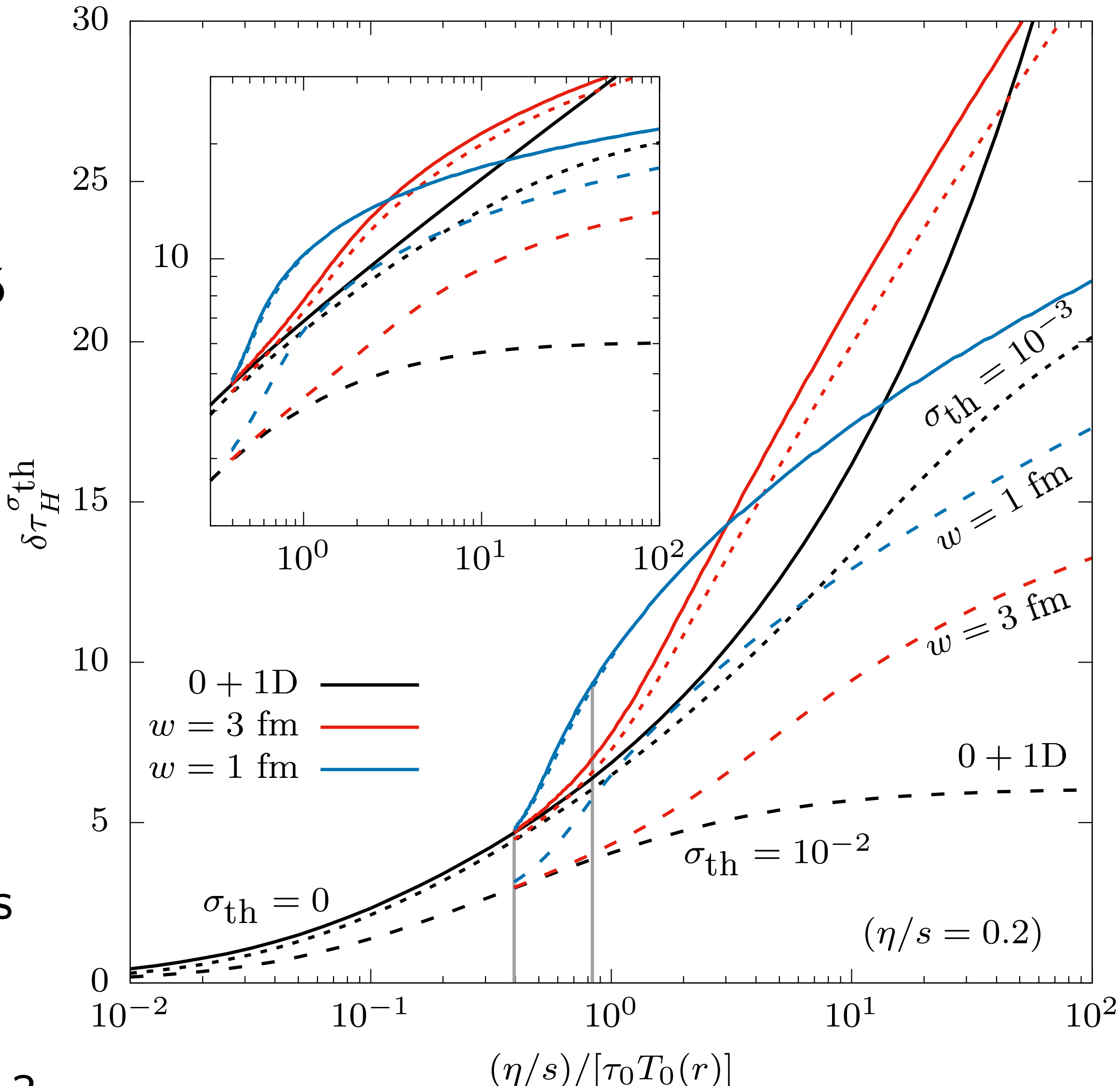
Hydrodynamisation time at $\sigma_{\rm th} = 0.01$



- For a simulation at η/s, the system covers $(\eta/s)/[\tau_0 T_0(r)]$ up to ∞.
- Small r: $\delta\tau_H$ well approximated by the $0+1$-D prediction.
- Large r: Transverse expansion kicks in earlier, delaying hydrodynamisation.
- $w = 3$ fm: $0+1$-D prediction robust in the region of physical interest ($\eta/s = 0.2$, $r < 3w/2$).
- $w = 1$ fm: Larger deviations, but $\delta\tau_H < 6$ for $0 < r < 3w/2$ and $\eta/s = 0.2$.

Hydrodynamisation time at $\eta/s = 0.2$

- $T_0 = 0.5$ GeV, $\eta/s = 0.2$.
- For $w = 3$ fm, hydrodynamisation for $0 \le r/w \le 1.5$ happens similar to $0 + 1$D.
- For $w = 1$ fm, $\delta\tau_H$ is close to $0 + 1$D prediction at $r = 0$, but deviates by $\sim 50\%$ at $r = 3w/2$.
- For large r/w, hydrodynamisation happens faster than for $0 + 1$D
  $\Rightarrow$ new attractor due to transverse expansion?



Conclusions

- Attractor solution for $\chi = \mathcal{P}_L/\mathcal{P}_T$ derived for conformal hydro and confirmed for nearly-conformal ideal gas.
- New measure $\delta\tau_H^{\sigma_{\rm th}} = (\tau_H^{\sigma_{\rm th}} - \tau_0)/\tau_0$ of "memory loss" scales with $\tilde{w}_0^{-1}(4\pi\eta/s)/(\tau_0 T_0)$ in $0+1$-D.
- $\delta\tau_H^{0.01} \lesssim 6$ for any parameters in $0+1$-D.
- For large systems ($w \gg \tau_0$), hydrodynamisation proceeds according to longitudinal dynamics for most of the fireball.
- Small systems present deviations w.r.t. $0+1$D prediction, which are not significant in the physically relevant parameter range.
- Results indicate a transverse attractor may exist at large τ/R.