GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Lecture

Models for heavy-ion collisions: (Part 2): transport models – BUU

SS2024: ‚Dynamical models for relativistic heavy-ion collisions‘

Vlasov equation-of-motion

Vlasov equation

- free propagation of particles in the self-generated HF mean-field potential:

$$\frac{\partial}{\partial t} f(\vec{r},\vec{p},t) + \frac{\vec{p}}{m}\vec{\nabla}_{\vec{r}}\, f(\vec{r},\vec{p},t) - \vec{\nabla}_{\vec{r}} U(\vec{r},t)\, \vec{\nabla}_{\vec{p}} f(\vec{r},\vec{p},t) = 0$$

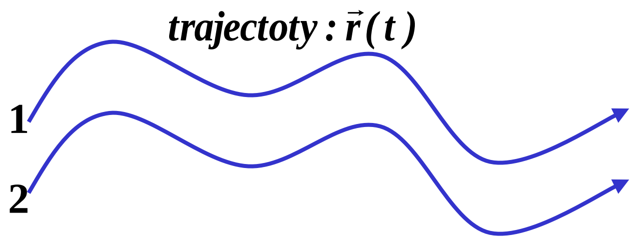
Here *U* is a self-consistent potential associated with *f* phase-space distribution:

$$U(\vec{r},t) = \frac{1}{(2\pi\hbar)^3}\int d^3r' d^3p\, V(\vec{r}-\vec{r}\,',t)\, f(\vec{r}\,',\vec{p},t)$$

➔ **Classical equations of motion :**

$$\dot{\vec{r}} = \frac{d\vec{r}}{dt} = \frac{\vec{p}}{m}$$

$$\dot{\vec{p}} = \frac{d\vec{p}}{dt} = -\vec{\nabla}_{\vec{r}} U(\vec{r},t)$$

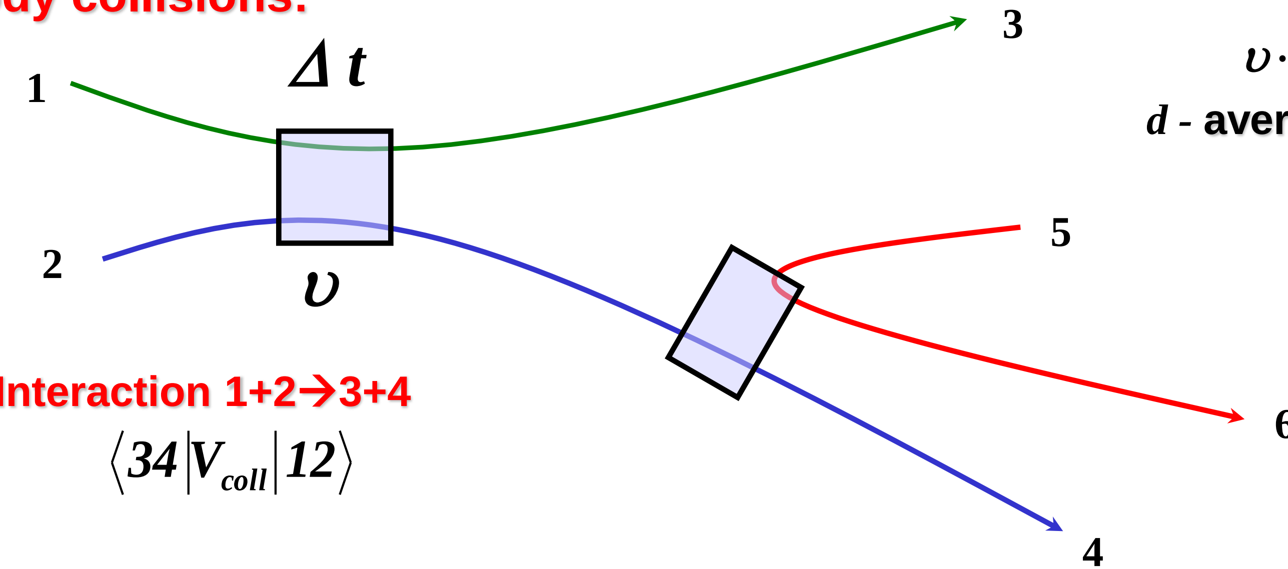


From the Vlasov equation of motion to Boltzmann-Uehling-Uhlenbeck equation (BUU) – collision term

Dynamical transport models with collisions

➔ In order to describe the collisions between the individual(!) particles, one has to go **beyond the mean-field level !** (See Part 2: Correlation dynamics)

add **2-body collisions:**



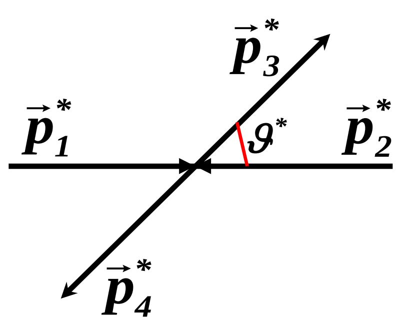
$\upsilon \cdot \Delta t << \langle d \rangle$

d - average distance

Interaction 1+2→3+4

$\langle 34 | V_{coll} | 12 \rangle$

In cms: $\vec{p}_1^{\,*} + \vec{p}_2^{\,*} = \vec{p}_3^{\,*} + \vec{p}_4^{\,*} = 0$



$(\vec{r}_1, \vec{p}_1)\ (\vec{r}_2, \vec{p}_2) \rightarrow (\vec{r}_3, \vec{p}_3)\ (\vec{r}_4, \vec{p}_4)$

- If the phase-space around $(\vec{r}_3, \vec{p}_3)$ and $(\vec{r}_4, \vec{p}_4)$ is essentially empty then the scattering is allowed,
- if the states are filled → Pauli suppression = Pauli principle

BUU (VUU) equation

Boltzmann (Vlasov)-Uehling-Uhlenbeck equation (NON-relativistic formulation!)
- free propagation of particles in the self-generated HF mean-field potential with an on-shell collision term:

$$\frac{d}{dt} f(\vec{r},\vec{p},t) \equiv \frac{\partial}{\partial t} f(\vec{r},\vec{p},t) + \frac{\vec{p}}{m}\vec{\nabla}_{\vec{r}}\ f(\vec{r},\vec{p},t) - \vec{\nabla}_{\vec{r}} U(\vec{r},t)\ \vec{\nabla}_{\vec{p}} f(\vec{r},\vec{p},t) = \left(\frac{\partial f}{\partial t}\right)_{coll}$$

Collision integral for 1+2→3+4 (let's consider fermions) :

Probability including Pauli blocking of fermions

$$I_{coll} \equiv \left(\frac{\partial f}{\partial t}\right)_{coll} \Rightarrow \frac{1}{((2\pi)^3)^3} \int d^3p_2\, d^3p_3\, d^3p_4\ \cdot w(1+2\rightarrow 3+4)\cdot P$$

$$\times (2\pi)^3 \delta^3(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4)\ (2\pi)\delta\left(\frac{\vec{p}_1^{\,2}}{2m_1}+\frac{\vec{p}_2^{\,2}}{2m_2}-\frac{\vec{p}_3^{\,2}}{2m_3}-\frac{\vec{p}_4^{\,2}}{2m_4}\right)$$

Transition probability for *1+2→3+4:* $w(1+2\rightarrow 3+4) \Rightarrow \upsilon_{12}\cdot \frac{d^3\sigma}{d^3q}$

where $\upsilon_{12} = \frac{\hbar}{m}|\vec{p}_1-\vec{p}_2|$ **- relative velocity of the colliding nucleons**

$\frac{d^3\sigma}{d^3q}$ **- differential cross section, *q* – momentum transfer** $\vec{q}=\vec{p}_1-\vec{p}_3$

BUU: Collision integral

$$I_{coll} = \frac{4}{(2\pi)^3}\int d^3p_2\, d^3p_3 \int d\Omega\ |\upsilon_{12}|\,\delta^3(\vec{p}_1+\vec{p}_2-\vec{p}_3-\vec{p}_4)\cdot\frac{d\sigma}{d\Omega}(1+2\rightarrow 3+4)\cdot P$$

Probability including Pauli blocking of fermions:

$$P = f(\vec{r},\vec{p}_3,t)\,f(\vec{r},\vec{p}_4,t)\,\left[1-f(\vec{r},\vec{p}_1,t)\right]\left[1-f(\vec{r},\vec{p}_2,t)\right]$$
$$-\,f(\vec{r},\vec{p}_1,t)\,f(\vec{r},\vec{p}_2,t)\,\left[1-f(\vec{r},\vec{p}_3,t)\right]\left[1-f(\vec{r},\vec{p}_4,t)\right]$$
$$\equiv \underbrace{f_3 f_4\underline{(1-f_1)(1-f_2)}}_{\text{Gain}} - \underbrace{f_1 f_2\underline{(1-f_3)(1-f_4)}}_{\text{Loss}}$$

Gain term
3+4→1+2

Loss term
1+2→3+4

Pauli blocking factors for fermions *

For particle 1 and 2:
Collision term = Gain term – Loss term

$$I_{coll} = G - L$$

***Note: for bosons – enhancement factor *1+f* (where *f<<1*);**
often one neglects bose enhancement for HIC, i.e. *1+f →1*

Collision integral for system in equilibrium

Consider fermion gas in equilibrium - described by Fermi-Dirac distribution:

$$n_F(\epsilon) = (1 + \exp((\epsilon - \mu)/T))^{-1}$$

T – temperature,
μ – baryon chemical potential

Collision interal of fermion system:

$$I(\mathrm{p}_1, \mathrm{p}_1; t) = \frac{g}{(2\pi)^3} \int d^3p_2 \int d\Omega \; v_{12} \frac{d\sigma}{d\Omega}(\mathrm{p}_1 + \mathrm{p}_2, \mathrm{p}_2 - \mathrm{p}_4)$$
$$\times \{n(\mathrm{p}_3; t) n(\mathrm{p}_4; t) \bar{n}(\mathrm{p}_1; t) \bar{n}(\mathrm{p}_2; t) - n(\mathrm{p}_1; t) n(\mathrm{p}_2; t) \bar{n}(\mathrm{p}_3; t) \bar{n}(\mathrm{p}_4; t)\}$$

$$\epsilon = \mathrm{p}^2/(2m), \; \bar{n}(\mathrm{p}; t) = 1 - n(\mathrm{p}; t) \qquad n = n_F(\epsilon) = n_F(\mathrm{p}^2/(2m))$$

In equilibrium collision term = 0

→ **Gain term = Loss term**

$$\boxed{I(\mathrm{p}_1, \mathrm{p}_1; t) = 0}$$

i.e. number of forward and backward reactions in the system is the same

→ **$n_F(\varepsilon)$ is stationary solution of $I(p_1,p_2;t)=0$**

Collision integral for system in equilibrium

To show that $I(\mathbf{p}_1, \mathbf{p}_1; t) = 0$ **we have to demonstrate that**

$$n(\mathbf{p}_3)n(\mathbf{p}_4)\bar{n}(\mathbf{p}_1)\bar{n}(\mathbf{p}_2) = n(\mathbf{p}_1)n(\mathbf{p}_2)\bar{n}(\mathbf{p}_3)\bar{n}(\mathbf{p}_4)$$

Consider

$$1 - n_F(\epsilon) = 1 - \frac{1}{1+\exp((\epsilon-\mu)/T)} = \frac{1-(1+\exp((\epsilon-\mu)/T)}{1+\exp((\epsilon-\mu)/T)} = -\frac{\exp((\epsilon-\mu)/T)}{1+\exp((\epsilon-\mu)/T)}$$

$$\frac{\exp((\epsilon_1-\mu)/T)}{(1+\exp((\epsilon_1-\mu)/T))}\frac{\exp((\epsilon_2-\mu)/T)}{(1+\exp((\epsilon_2-\mu)/T))}\frac{1}{(1+\exp((\epsilon_3-\mu)/T))}\frac{1}{(1+\exp((\epsilon_4-\mu)/T))} =$$

$$\frac{\exp((\epsilon_3-\mu)/T)}{(1+\exp((\epsilon_3-\mu)/T))}\frac{\exp((\epsilon_4-\mu)/T)}{(1+\exp((\epsilon_4-\mu)/T))}\frac{1}{(1+\exp((\epsilon_1-\mu)/T))}\frac{1}{(1+\exp((\epsilon_2-\mu)/T))}$$

Since the denominators on both sides are the same one has to proof only

$$\exp((\epsilon_1-\mu)/T)\exp((\epsilon_2-\mu)/T) = \exp((\epsilon_3-\mu)/T)\ \exp((\epsilon_4-\mu)/T)$$

$$\exp((\epsilon_1+\epsilon_2)/T) = \exp((\epsilon_3+\epsilon_4)/T)$$

Since due to energy conservation $\delta(\epsilon_1 + \epsilon_2 - \epsilon_3 - \epsilon_4)$ we have $\epsilon_1 + \epsilon_2 = \epsilon_3 + \epsilon_4$, what proofs that $n_F(\epsilon)$ is a stationary solution of collision integral for system in equilibrium

➔ Transport equations have a correct thermodynamic limit!

Dynamical transport model: collision terms

- BUU eq. for different particles of type *i=1,…n*

$$Df_i \equiv \frac{d}{dt} f_i = I_{coll}\left[f_1, f_2, ..., f_n\right] \qquad (20)$$

Drift term=Vlasov eq. collision term

$$i: \quad Baryons: \; p, n, \Delta(1232), N(1440), N(1535), ..., \Lambda, \Sigma, \Sigma^*, \Xi, \Omega; \; \Lambda_C$$

$$Mesons: \; \pi, \eta, K, \bar{K}, \rho, \omega, K^*, \eta', \phi, a_1, ..., D, \bar{D}, J/\Psi, \Psi', ...$$

➔ coupled set of BUU equations for different particles of type *i=1,…n*

$$\begin{cases} Df_N = I_{coll}\left[f_N, f_\Delta, f_{N(1440)}, ..., f_\pi, f_\rho, ...\right] \\ Df_\Delta = I_{coll}\left[f_N, f_\Delta, f_{N(1440)}, ..., f_\pi, f_\rho, ...\right] \\ ... \\ Df_\pi = I_{coll}\left[f_N, f_\Delta, f_{N(1440)}, ..., f_\pi, f_\rho, ...\right] \\ ... \end{cases}$$

E.g., Nucleon transport in N,π,Δ system : $Df_N = I_{coll}$

(only 1←→2, 2←→2 reactions indicated here)

$$\frac{\partial f_N}{\partial t} + \vec{v}\cdot\frac{\partial f_N}{\partial r} - \nabla_r U_N \cdot \frac{\partial f_N}{\partial p} = I_{NN\to NN} + I_{N\Delta\to N\Delta} + I_{N\pi\to N\pi} + I_{NN\to N\Delta} + I_{NN\to\Delta\Delta} + I_{N\Delta\to\Delta\Delta} + I_{N\Delta\to NN} + I_{N\pi\to\Delta}$$

$$= \frac{\pi}{\hbar}\frac{g_1 g_2}{4}\frac{(2\pi)^2(\hbar c)^4}{\mu_{NN}c^2\cdot\mu_{NN}c^2}\int\frac{d^3p_2}{(2\pi\hbar)^3}\int\frac{d^3p_1'}{(2\pi\hbar)^3}\int\frac{d^3p_2'}{(2\pi\hbar)^3}\frac{d\sigma}{d\Omega}(p_1'-p_1)\cdot(2\pi\hbar)^3\delta^3(p_1+p_2-p_1'-p_2')\delta(\varepsilon_1+\varepsilon_2-\varepsilon_1'-\varepsilon_2')\cdot$$
$$\left[\frac{p_1'-p_2'}{p_1-p_2}f_N(p_1')f_N(p_2')(1-f_N(p_1))(1-f_N(p_2)) - \frac{p_1-p_2}{p_1'-p_2'}f_N(p_1)f_N(p_2)(1-f_N(p_1'))(1-f_N(p_2'))\right]$$

$$+ \frac{\pi}{\hbar}\frac{g_1 g_2}{4}\frac{(2\pi)^2(\hbar c)^4}{\mu_{N\Delta}c^2\cdot\mu_{N\Delta}c^2}\int\frac{d^3p_2}{(2\pi\hbar)^3}\int\frac{d^3p_1'}{(2\pi\hbar)^3}\int\frac{d^3p_2'}{(2\pi\hbar)^3}\frac{d\sigma}{d\Omega}(p_1'-p_1)\cdot(2\pi\hbar)^3\delta^3(p_1+p_2-p_1'-p_2')\delta(\varepsilon_1+\varepsilon_2-\varepsilon_1'-\varepsilon_2')\cdot$$
$$\left[\frac{p_1'-p_2'}{p_1-p_2}f_N(p_1')f_\Delta(p_2')(1-f_N(p_1))(1-f_\Delta(p_2)) - \frac{p_1-p_2}{p_1'-p_2'}N(p_1)f_\Delta(p_2)(1-f_N(p_1'))(1-f_\Delta(p_2'))\right]$$

$$+ \frac{\pi}{\hbar}\frac{g_1 g_2}{4}\frac{(2\pi)^2(\hbar c)^4}{\mu_{N\pi}c^2\cdot\mu_{N\pi}c^2}\int\frac{d^3p_2}{(2\pi\hbar)^3}\int\frac{d^3p_1'}{(2\pi\hbar)^3}\int\frac{d^3p_2'}{(2\pi\hbar)^3}\frac{d\sigma}{d\Omega}(p_1'-p_1)\cdot(2\pi\hbar)^3\delta^3(p_1+p_2-p_1'-p_2')\delta(\varepsilon_1+\varepsilon_2-\varepsilon_1'-\varepsilon_2')\cdot$$
$$\left[\frac{p_1'-p_2'}{p_1-p_2}f_N(p_1')f_\pi(p_2')(1-f_N(p_1))(1+f_\pi(p_2)) - \frac{p_1-p_2}{p_1'-p_2'}N(p_1)f_\pi(p_2)(1-f_N(p_1'))(1+f_\pi(p_2'))\right]$$

$$+ \frac{\pi}{\hbar}\frac{g_1 g_2}{4}\frac{(2\pi)^2(\hbar c)^4}{\mu_{NN}c^2\cdot\mu_{N\Delta}c^2}\int\frac{d^3p_2}{(2\pi\hbar)^3}\int\frac{d^3p_1'}{(2\pi\hbar)^3}\int\frac{d^3p_2'}{(2\pi\hbar)^3}\frac{d\sigma}{d\Omega}(p_1'-p_1)\cdot(2\pi\hbar)^3\delta^3(p_1+p_2-p_1'-p_2')\delta(\varepsilon_1+\varepsilon_2-\varepsilon_1'-\varepsilon_2')\cdot$$
$$\left[\frac{p_1'-p_2'}{p_1-p_2}f_N(p_1')f_\Delta(p_2')(1-f_N(p_1))(1-f_N(p_2)) - \frac{p_1-p_2}{p_1'-p_2'}N(p_1)f_N(p_2)(1-f_N(p_1'))(1-f_\Delta(p_2'))\right]$$

$$+ \frac{\pi}{\hbar}\frac{g_1 g_2}{4}\frac{(2\pi)^2(\hbar c)^4}{\mu_{NN}c^2\cdot\mu_{\Delta\Delta}c^2}\int\frac{d^3p_2}{(2\pi\hbar)^3}\int\frac{d^3p_1'}{(2\pi\hbar)^3}\int\frac{d^3p_2'}{(2\pi\hbar)^3}\frac{d\sigma}{d\Omega}(p_1'-p_1)\cdot(2\pi\hbar)^3\delta^3(p_1+p_2-p_1'-p_2')\delta(\varepsilon_1+\varepsilon_2-\varepsilon_1'-\varepsilon_2')\cdot$$
$$\left[\frac{p_1'-p_2'}{p_1-p_2}f_\Delta(p_1')f_\Delta(p_2')(1-f_N(p_1))(1-f_N(p_2)) - \frac{p_1-p_2}{p_1'-p_2'}N(p_1)f_N(p_2)(1-f_\Delta(p_1'))(1-f_\Delta(p_2'))\right]$$

$$+ \frac{\pi}{\hbar}\frac{g_1 g_2}{4}\frac{(2\pi)^2(\hbar c)^4}{\mu_{N\Delta}c^2\cdot\mu_{\Delta\Delta}c^2}\int\frac{d^3p_2}{(2\pi\hbar)^3}\int\frac{d^3p_1'}{(2\pi\hbar)^3}\int\frac{d^3p_2'}{(2\pi\hbar)^3}\frac{d\sigma}{d\Omega}(p_1'-p_1)\cdot(2\pi\hbar)^3\delta^3(p_1+p_2-p_1'-p_2')\delta(\varepsilon_1+\varepsilon_2-\varepsilon_1'-\varepsilon_2')\cdot$$
$$\left[\frac{p_1'-p_2'}{p_1-p_2}f_\Delta(p_1')f_\Delta(p_2')(1-f_N(p_1))(1-f_\Delta(p_2)) - \frac{p_1-p_2}{p_1'-p_2'}N(p_1)f_\Delta(p_2)(1-f_\Delta(p_1'))(1-f_\Delta(p_2'))\right]$$

$$+ \frac{\pi}{\hbar}\frac{g_1 g_2}{4}\frac{(2\pi)^2(\hbar c)^4}{\mu_{N\Delta}c^2\cdot\mu_{NN}c^2}\int\frac{d^3p_2}{(2\pi\hbar)^3}\int\frac{d^3p_1'}{(2\pi\hbar)^3}\int\frac{d^3p_2'}{(2\pi\hbar)^3}\frac{d\sigma}{d\Omega}(p_1'-p_1)\cdot(2\pi\hbar)^3\delta^3(p_1+p_2-p_1'-p_2')\delta(\varepsilon_1+\varepsilon_2-\varepsilon_1'-\varepsilon_2')\cdot$$
$$\left[\frac{p_1'-p_2'}{p_1-p_2}f_N(p_1')f_N(p_2')(1-f_N(p_1))(1-f_\Delta(p_2)) - \frac{p_1-p_2}{p_1'-p_2'}N(p_1)f_\Delta(p_2)(1-f_N(p_1'))(1-f_N(p_2'))\right]$$

$$+ \int\frac{d^3p_\pi}{(2\pi\hbar)^3}\int\frac{d^3p_\Delta}{(2\pi\hbar)^3}|\langle p_\Delta|T|p_N p_\pi\rangle|^2\cdot(2\pi\hbar)^3\delta^3(p_N+p_\pi-p_\Delta)\delta(\varepsilon_N+\varepsilon_\pi-\varepsilon_\Delta)\cdot$$
$$[f_\Delta(p_\Delta)(1+f_\pi(p_\pi))(1-f_N(p_N)) - f_N(p_N)f_\pi(p_\pi)(1-f_\Delta(p_\Delta))]$$

Full collision term consists of >10000 different particle combinations

- ➔ set of transport equations coupled via I_{coll} and mean field

Dynamical transport model: collision terms

Collision terms for *(N,Δ,π)* system: $\Delta \leftrightarrow \pi N$

* *Relativistic formulation*

Eq. for Δ

$$Df_\Delta = \sum_{\pi,N} \frac{g}{(2\pi)^3} \int \frac{d^3p_\pi}{E_\pi} \frac{d^3p_N}{E_N} |M_{\Delta\leftrightarrow\pi N}|^2 \cdot \delta^4(p_\pi + p_N - p_\Delta) \times \underbrace{f_\pi(p_\pi)\ f_N(p_N)(1 - f_\Delta(p_\Delta))}_{\pi N \to \Delta}$$

$$- \sum_{\pi,N} \frac{g}{(2\pi)^3} \int \frac{d^3p_\pi}{E_\pi} \frac{d^3p_N}{E_N} |M_{\Delta\leftrightarrow\pi N}|^2 \cdot \delta^4(p_\pi + p_N - p_\Delta) \times \underbrace{f_\Delta(p_\Delta)\ (1 - f_N(p_N))(1 + f_\pi(p_\pi))}_{\Delta \to \pi N}$$

$$= \text{Gain}\ (\pi N \to \Delta) - \text{Loss}\ (\Delta \to \pi N)$$

Δ production *Δ decay*

Eq. for π

$$Df_\pi = \sum_{N,\Delta} \frac{g}{(2\pi)^3} \int \frac{d^3p_\Delta}{E_\Delta} \frac{d^3p_N}{E_N} |M_{\Delta\leftrightarrow\pi N}|^2 \cdot \delta^4(p_\pi + p_N - p_\Delta) \times \underbrace{f_\Delta(p_\Delta)\ (1 + f_\pi(p_\pi))(1 - f_N(p_N))}_{\Delta \to \pi N}$$

$$- \sum_{\pi,N} \frac{g}{(2\pi)^3} \int \frac{d^3p_\Delta}{E_\Delta} \frac{d^3p_N}{E_N} |M_{\Delta\leftrightarrow\pi N}|^2 \cdot \delta^4(p_\pi + p_N - p_\Delta) \times \underbrace{f_\pi(p_\pi)\ f_N(p_N)(1 - f_\Delta(p_\Delta))}_{\pi N \to \Delta}$$

$$= \text{Gain}\ (\Delta \to \pi N) - \text{Loss}\ (\pi N \to \Delta)$$

π production *π absorbtion*

by Δ decay *by nucleon*

Dynamical transport model: possible interactions

Consider possible interactions for the sytem of *(N,R,m)*,
where *N*-nucleons, *R*- resonances, *m*-mesons

- **elastic collisions:**

Baryon-baryon (BB):

$$NN \to NN$$
$$NR \to NR$$
$$RR' \to RR'$$

meson-Baryon (mB)

$$mN \to mN$$
$$mR \to mR$$

meson-meson (mm)

$$m\ m' \to m\ m'$$

Detailed balance:

$$a+b \leftrightarrow c$$
$$a+b \leftrightarrow c+d$$

- **inelastic collisions:**

Baryon-baryon (BB):

$$NN \leftrightarrow NR$$
$$NR \leftrightarrow NR'$$
$$NN \leftrightarrow RR'$$
...
$$BB \to X$$

meson-Baryon (mB)

$$mN \leftrightarrow R$$
$$mR \leftrightarrow R'$$
$$mB \leftrightarrow m'B'$$
...
$$mB \to X$$

meson-meson (mm)

$$m\ m' \leftrightarrow \tilde{m}$$
$$m\ m' \leftrightarrow m''m'''$$
...
$$m\ m' \to X$$

X **- multi-particle state**

Elementary hadronic interactions

Consider all possible interactions – elastic and inelastic collisions - for the sytem of *(N,R,m),* where *N*-nucleons, *R*- resonances, *m*-mesons, and resonance decays

Low energy collisions:

- **binary 2←→2 and 2←→3(4) reactions**
- **1←→2 : formation and decay of baryonic and mesonic resonances**

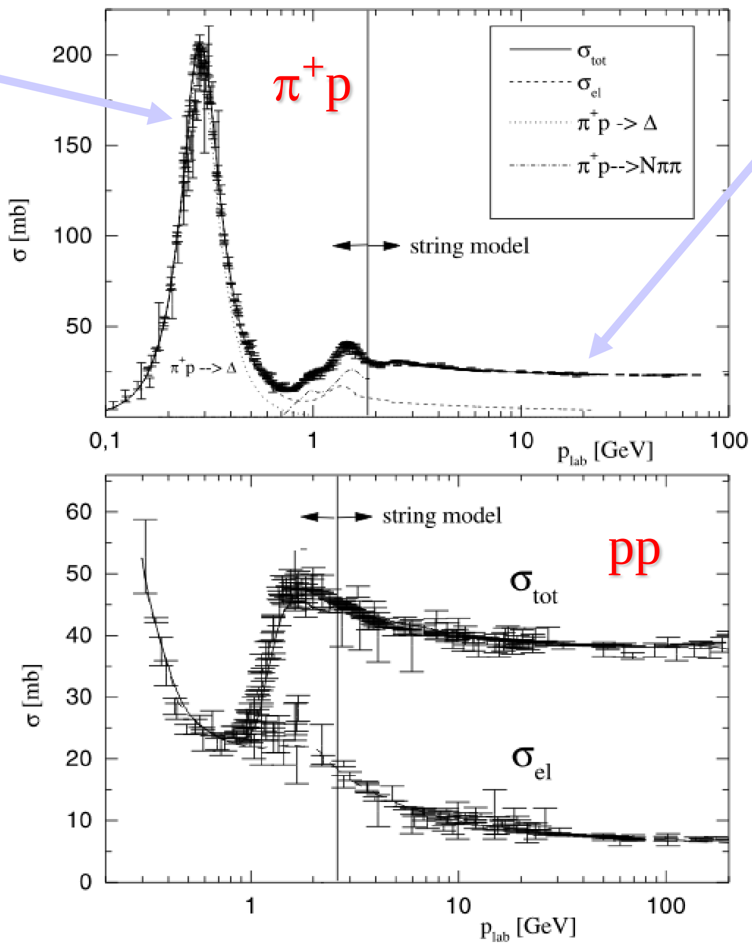
BB ←→ B´B´
BB ←→ B´B´m
mB ←→ m´B´
mB ←→ B´
mm ←→ m´m´
mm ←→ m´ . . .

Baryons:

B = p, n, Δ(1232), N(1440), N(1535), ...

Mesons:

M = π, η, ρ, ω, φ, ...



High energy collisions:

(above $s^{1/2}$~2.5 GeV)

Inclusive particle production:

BB→X , mB→X, mm→X

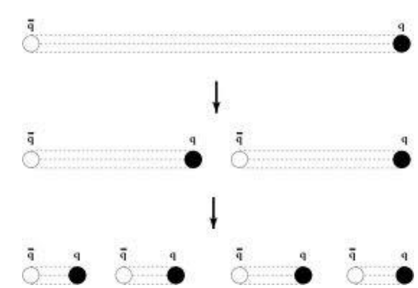
X =many particles

described by

string formation and decay

(string = excited color singlet states *q-qq*, *q-qbar*)

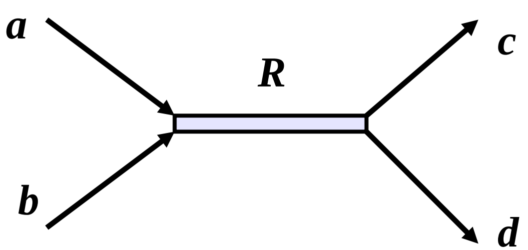
using LUND string model



Elementary reactions with resonances

- Consider the reaction $a + b \to R \to c + d$

intermediate resonance



Cross section:

$$d\sigma_{ab\to cd} = \frac{(2\pi)^4}{4p_a\sqrt{s}} \delta^4(p_a + p_b - p_c - p_d)\,\overline{|M_{ab\to cd}|^2} \cdot \frac{1}{((2\pi)^3)^2} \frac{d^3p_c}{2E_c} \frac{d^3p_d}{2E_d}$$

Matrix element: $M_{ab\to cd} = M_{ab\to R} \cdot P_R \cdot M_{R\to cd}$

Propagator: $P_R = \dfrac{1}{s - M_R^2 + \Pi}$

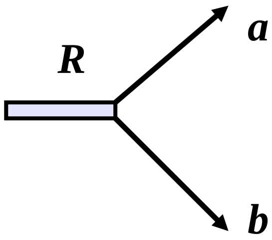
where self-energy $\Pi = i\sqrt{s}\;\Gamma_{tot}(s)$, $\quad \Gamma_{tot}(s) = \sum_j \Gamma_j$

total width

Elementary reactions with resonances

The **spin averaged/sumed matrix element squared** is

$$\overline{|M_{ab\to cd}|^2} = \overline{|M_{ab\to R}|^2}\cdot P_R^{\ 2}\cdot\overline{|M_{R\to cd}|^2}$$



Partial decay width:

$$\Gamma_{R\to ab}(s) = \frac{p_a}{8\pi s}\cdot\overline{|M_{R\to ab}|^2}$$

p_a – **momentum of *a* in the rest frame of *R* or cms *a+b***

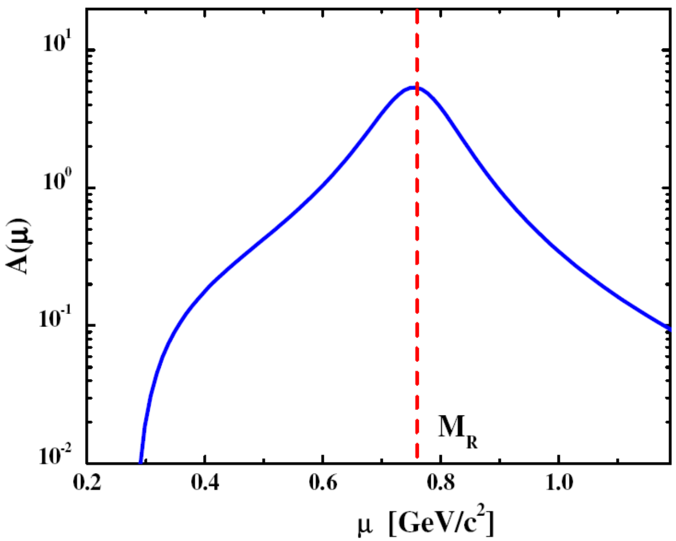
$$\sigma_{ab\to R\to cd}(s) = \frac{2J_R+1}{(2J_a+1)(2J_b+1)}\,\frac{4\pi}{p_a}\,\frac{s\Gamma_{R\to ab}(s)\cdot\Gamma_{R\to cd}(s)}{(s-M_R^2)^2+s\Gamma_{tot}^2(s)}$$

Spectral function

Production of resonance with effective mass μ ➔

- **spectral function = Breight-Wigner distribution**

$$A(\mu) = \frac{2}{\pi} \frac{\mu^2 \Gamma_{tot}(\mu)}{(\mu^2 - M_R^2)^2 + \mu^2 \Gamma_{tot}^2(\mu)}$$



Normalization condition: $\int_0^\infty d\mu \, A(\mu) = 1$

The total width = the sum over all partial channels:

$$\Gamma_{tot}(\mu) = \sum_j \Gamma_j(\mu)$$

- **Life time of resonance with mass μ:**

$$\tau(\mu) = \frac{\hbar c}{\Gamma_{tot}(\mu)}$$

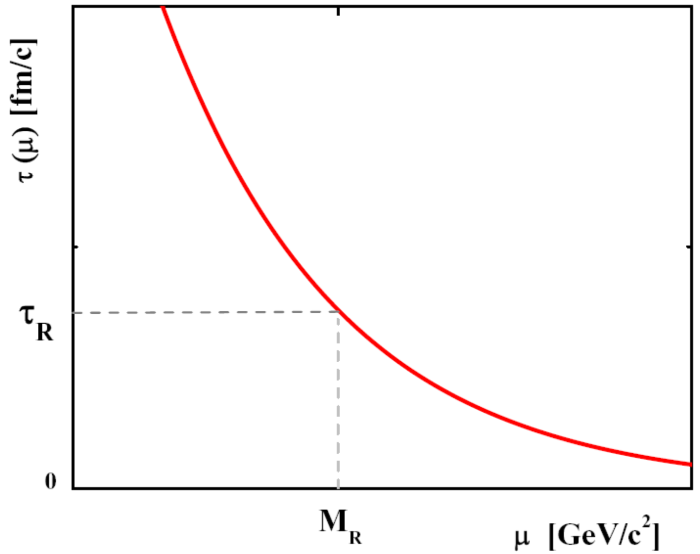
Note: Experimental life time ➔ with pole mass $\mu = M_R$

$$\tau_R = \frac{\hbar c}{\Gamma_{tot}(\mu = M_R)}$$

Decay rate

Decay rate: $\frac{dN}{dt} \sim -\frac{1}{\tau} N$

$\Rightarrow N(t) \sim e^{-\frac{t}{\tau}}$



pole mass

Total probability to decay $P_{decay} = e^{-\Gamma_{tot} \cdot t}$

Total probability to survive: $P = 1 - e^{-\Gamma_{tot} \cdot t}$

Branching ratio= probability to decay to channel j**:**

$$Br \equiv P_j = \frac{\Gamma_j(\mu)}{\Gamma_{tot}(\mu)}$$

Detailed balance

Detailed balance: $a + b \leftrightarrow c + d$

Note: DB is important to get the correct equilibrium properties

$$\sigma_{c+d\to a+b}(s) = \frac{(2J_a+1)(2J_b+1)}{(2J_c+1)(2J_d+1)} \frac{p_{ab}^2(s)}{p_{cd}^2(s)} \sigma_{a+b\to c+d}(s)$$

***J*- spin**

Momentum of particle *a* (or *b*) in cms:

$$p_{ab}(s) = \frac{\left[(s-(m_a+m_b)^2)\,(s-(m_a-m_b)^2)\right]^{1/2}}{2\sqrt{s}}$$

Momentum of particle *c* (or *d*) in cms:

$$p_{cd}(s) = \frac{\left[(s-(m_c+m_d)^2)\,(s-(m_c-m_d)^2)\right]^{1/2}}{2\sqrt{s}}$$

Detailed balance on the level of 2←→n: treatment of multi-particle collisions in transport approaches

W. Cassing, NPA 700 (2002) 618

Generalized collision integral for *n* ←→ *m* reactions:

$$I_{coll} = \sum_n \sum_m I_{coll}[n \leftrightarrow m]$$

$$I^i_{coll}[n \leftrightarrow m] =$$
$$\frac{1}{2} N^m_n \sum_\nu \sum_\lambda \left(\frac{1}{(2\pi)^4}\right)^{n+m-1} \int \left(\prod_{j=2}^{n} d^4p_j \; A_j(x,p_j)\right) \left(\prod_{k=1}^{m} d^4p_k \; A_k(x,p_k)\right)$$
$$\times A_i(x,p) \; W_{n,m}(p,p_j;i,\nu \mid p_k;\lambda) \; (2\pi)^4 \; \delta^4(p^\mu + \sum_{j=2}^{n} p^\mu_j - \sum_{k=1}^{m} p^\mu_k)$$
$$\times [\tilde{f}_i(x,p) \prod_{k=1}^{m} f_k(x,p_k) \prod_{j=2}^{n} \tilde{f}_j(x,p_j) - f_i(x,p) \prod_{j=2}^{n} f_j(x,p_j) \prod_{k=1}^{m} \tilde{f}_k(x,p_k)].$$

$\tilde{f} = 1 + \eta f$ **is Pauli-blocking or Bose-enhancement factors; η=1 for bosons and η=-1 for fermions**

$W_{n,m}(p,p_j;i,\nu \mid p_k;\lambda)$ **is a transition probability**

A(x,p) - spectral function

Antibaryon production in heavy-ion reactions

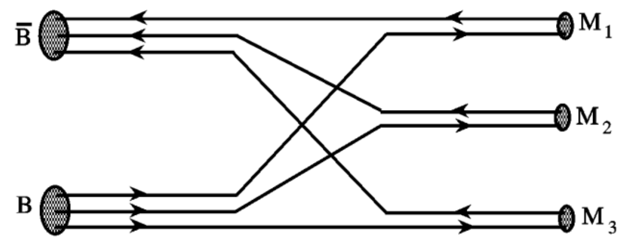
W. Cassing, NPA 700 (2002) 618
E. Seifert, W. Cassing, 1710.00665, 1801.07557

Multi-meson fusion reactions

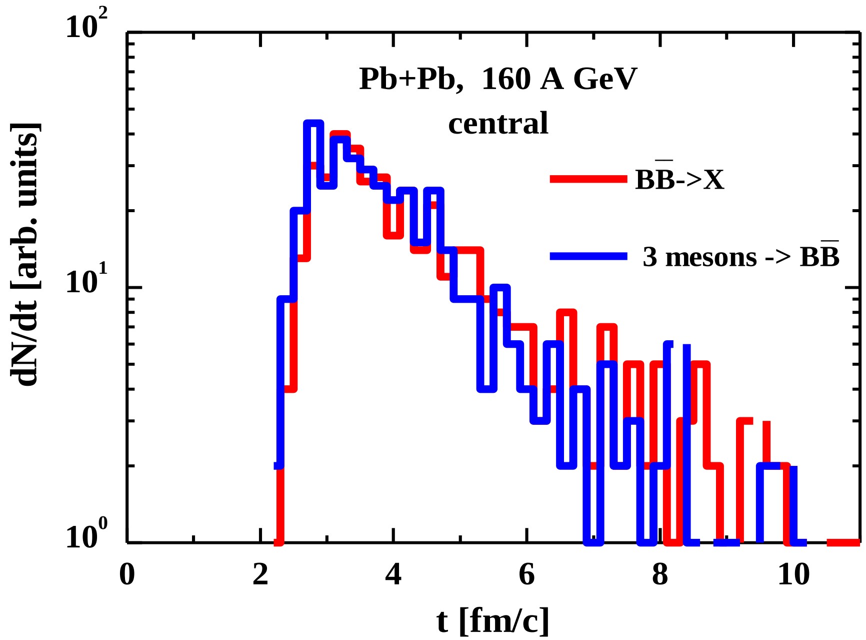
$m_1+m_2+...+m_n$ ←→ B+Bbar

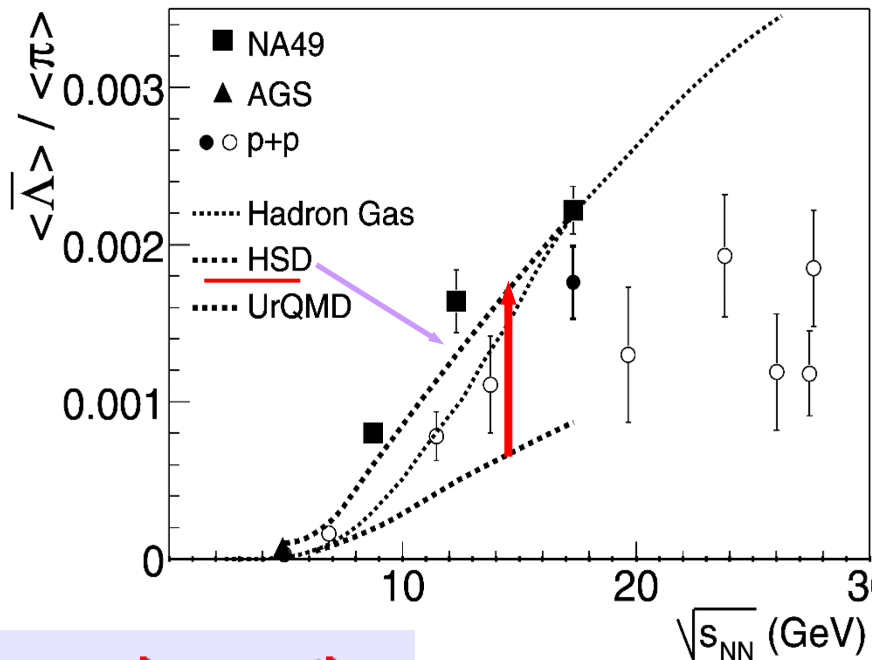
m=π,ρ,ω,.. B=p,Λ,Σ,Ω, (>2000 channels)

- ❑ important for anti-proton, anti-lambda, anti-Xi, anti-Omega dynamics !



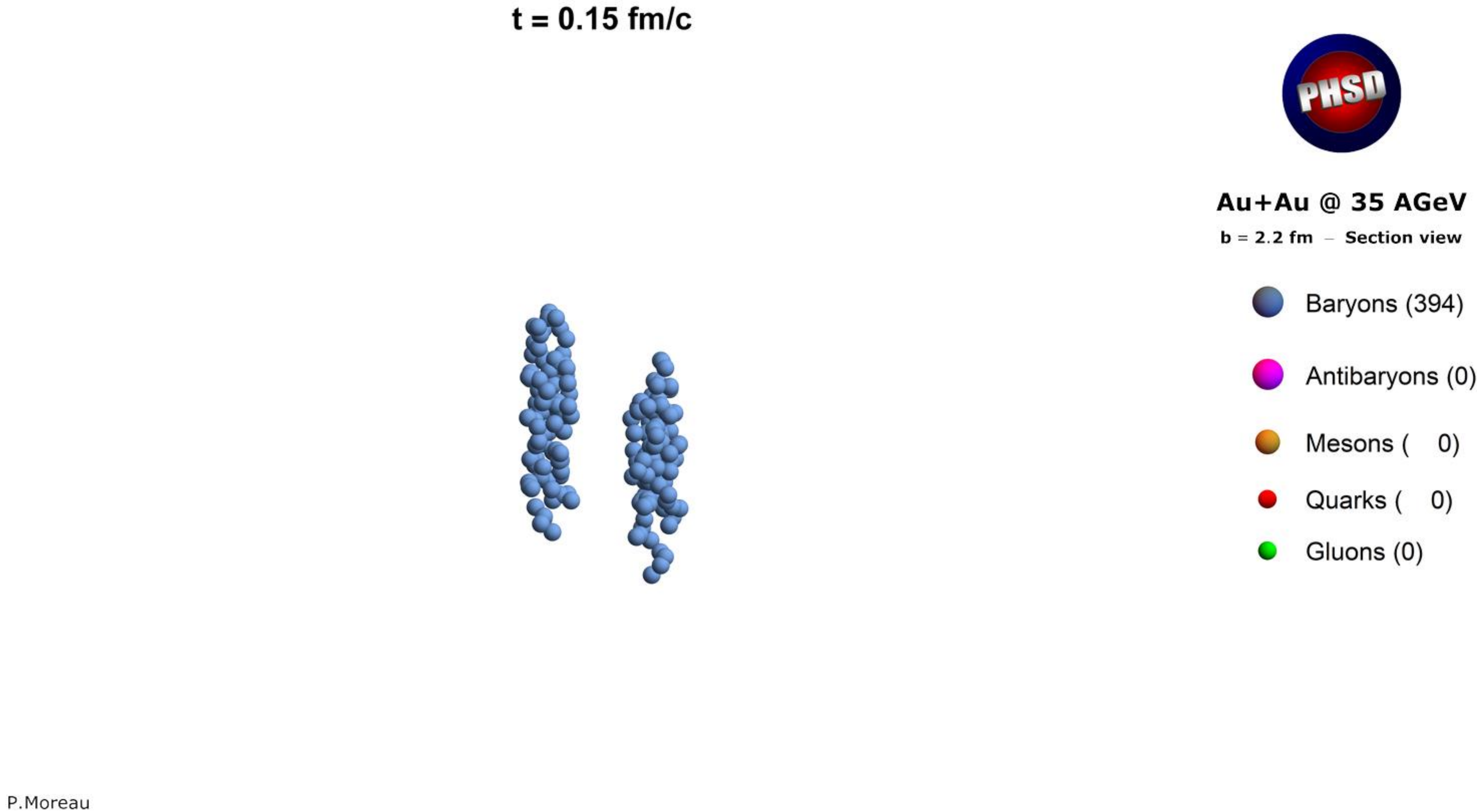
2←→3



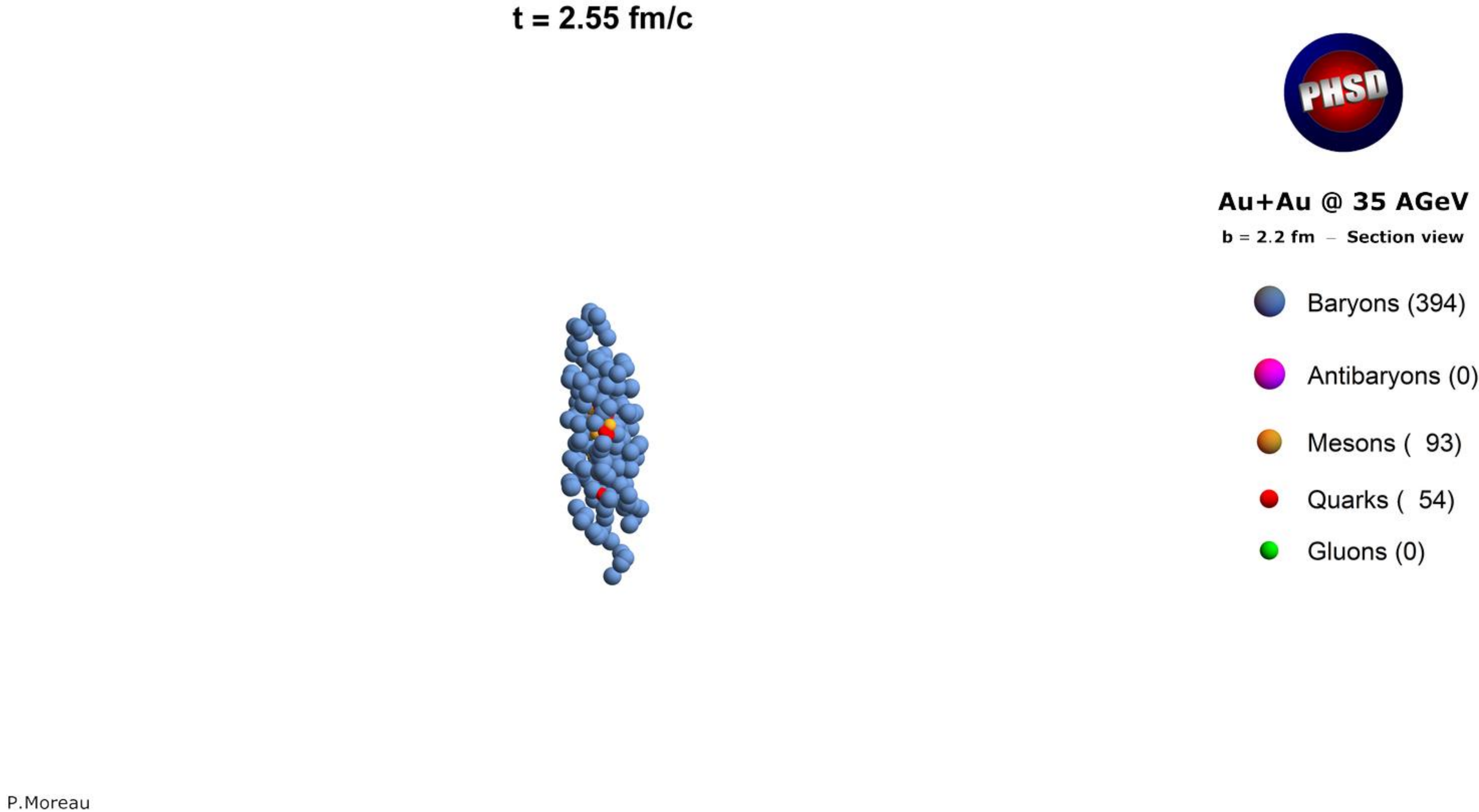


→ approximate equilibrium of annihilation and recreation

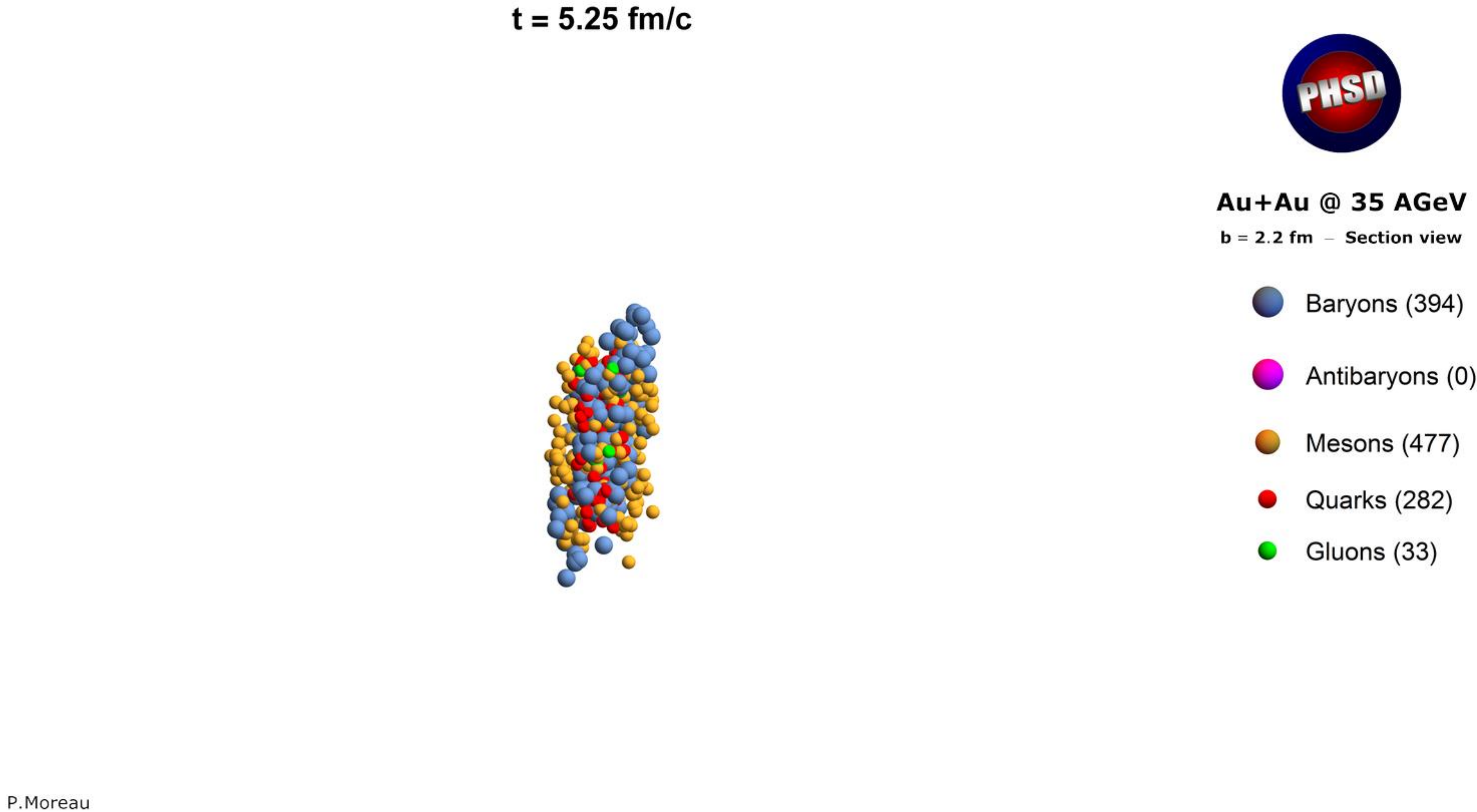
Stages of a collision in PHSD



Stages of a collision in PHSD



Stages of a collision in PHSD



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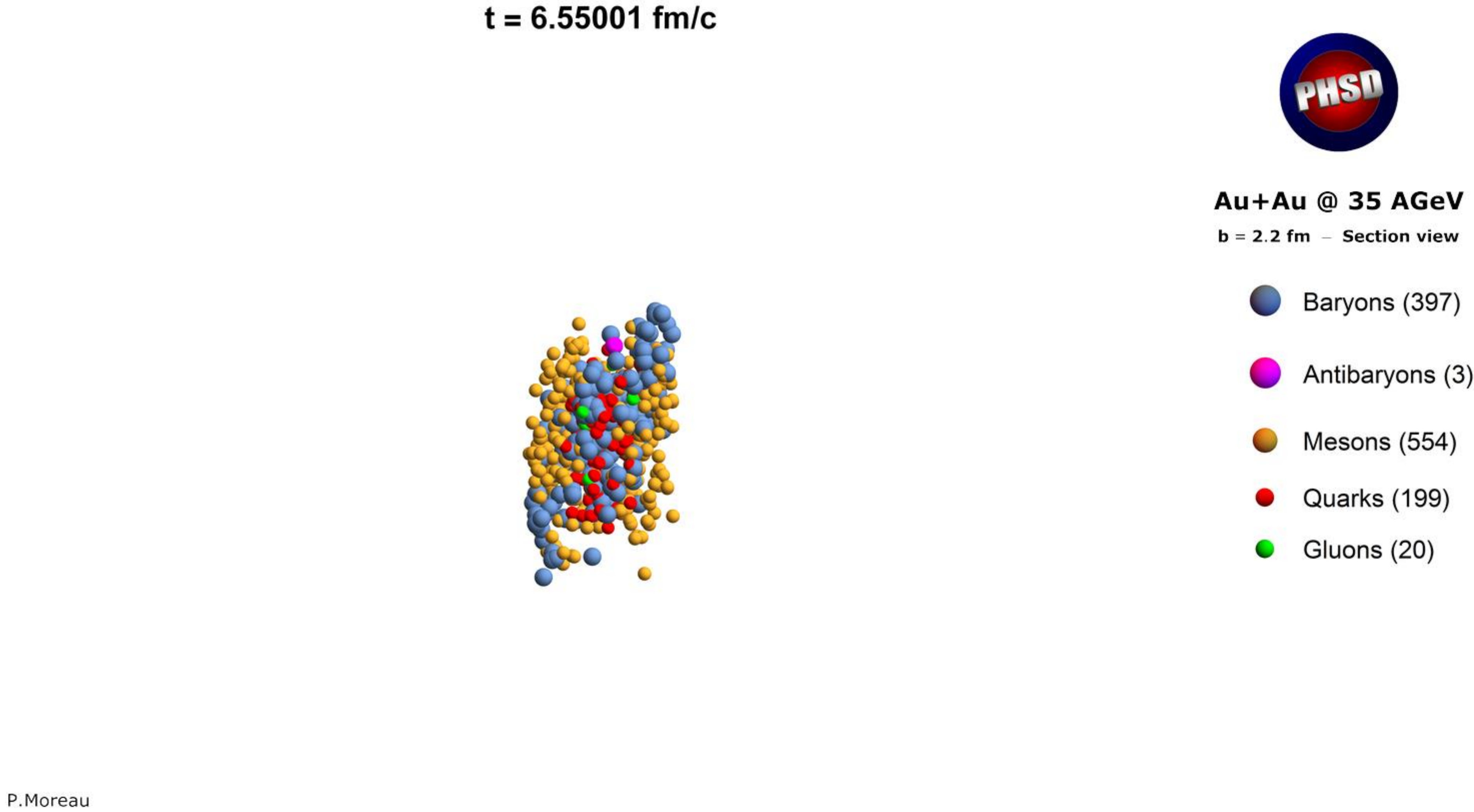

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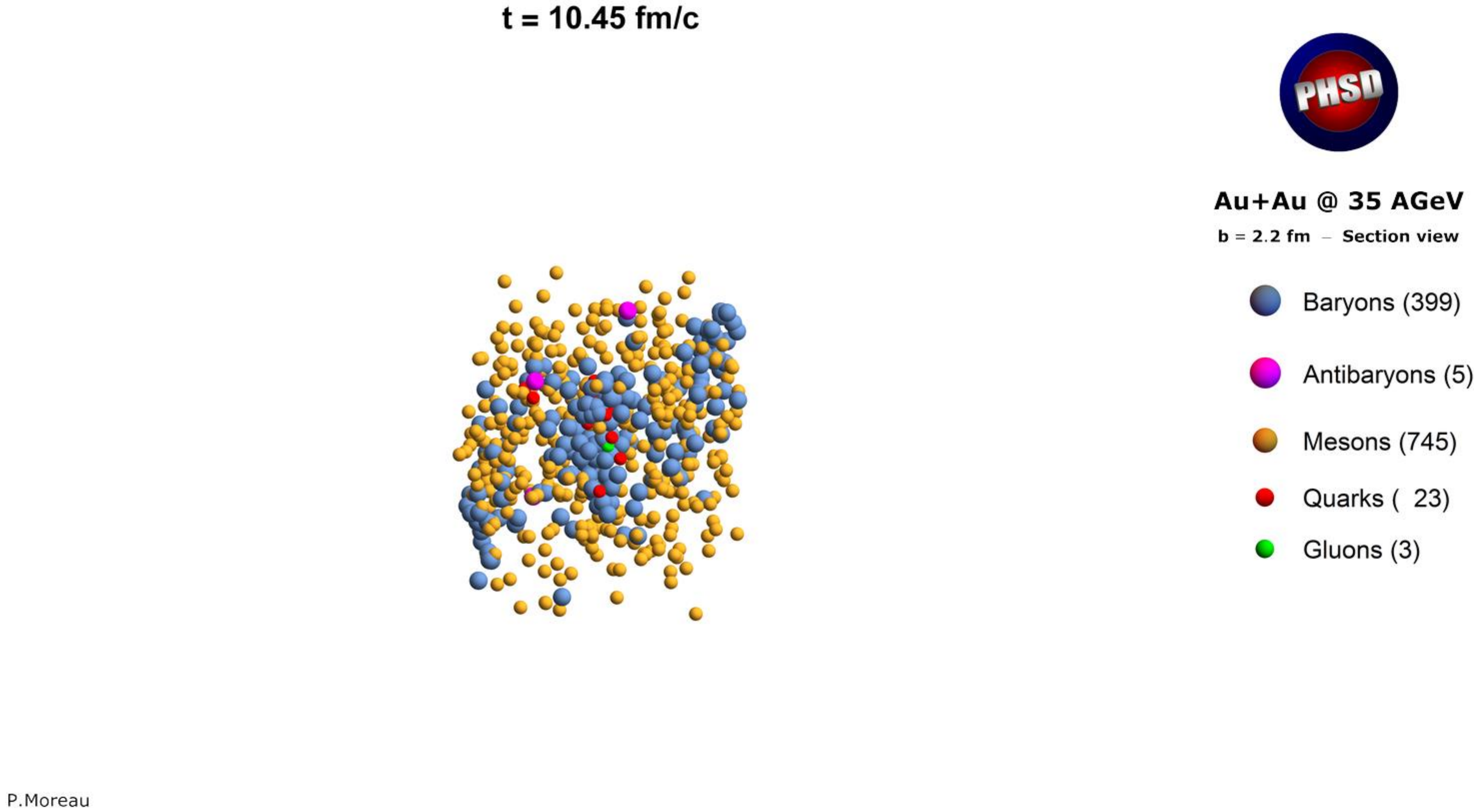

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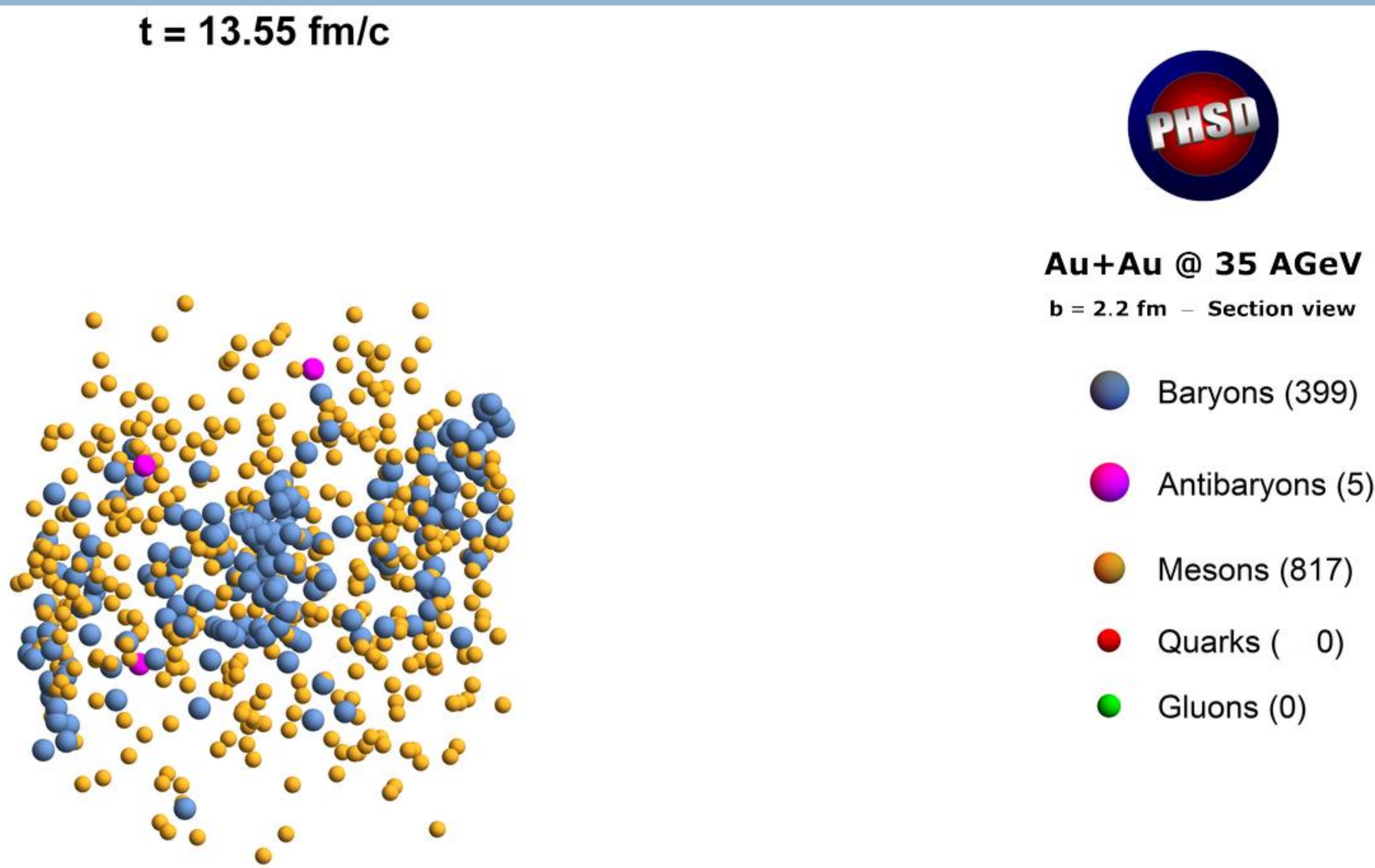
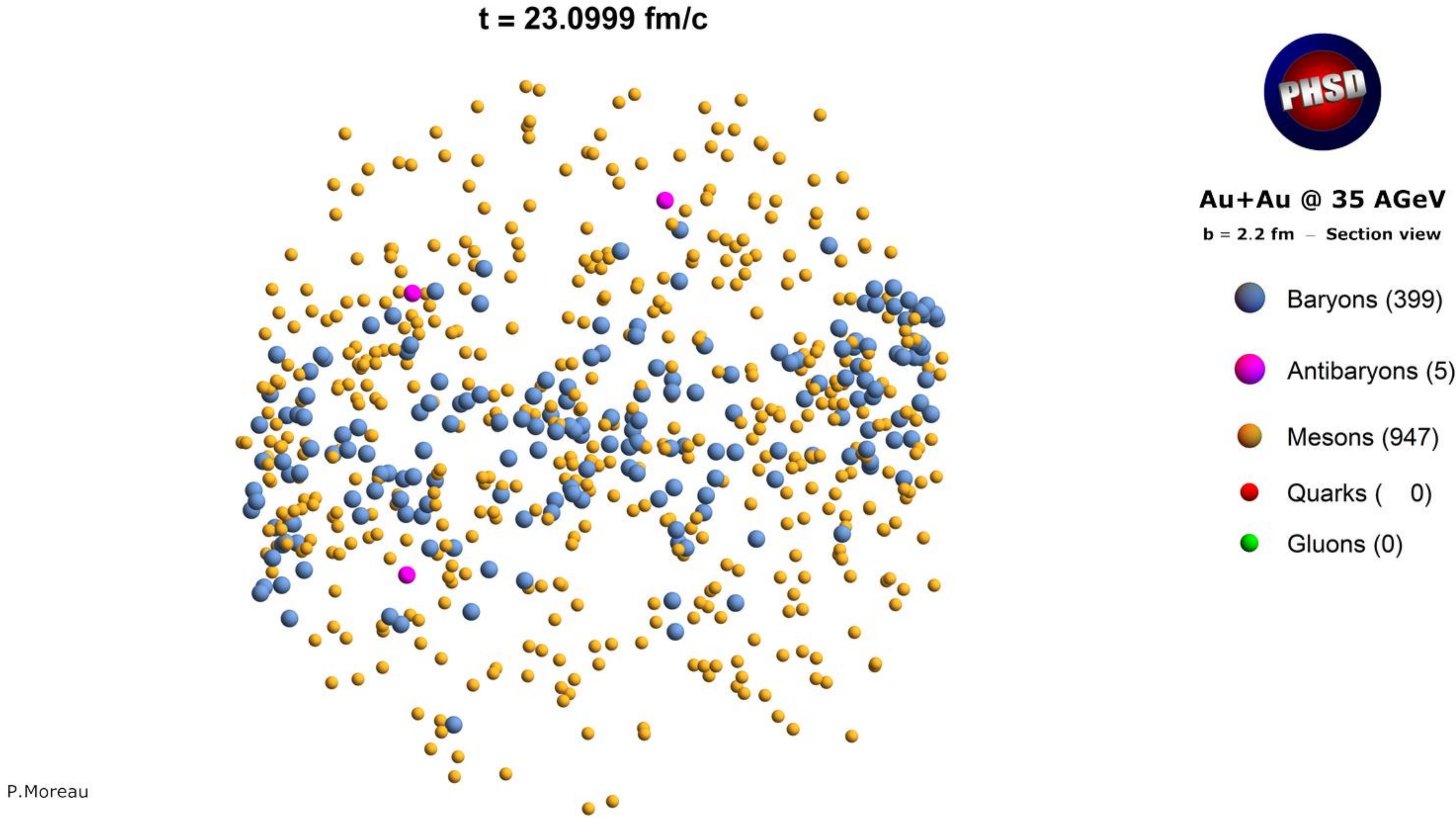


P.Moreau

Stages of a collision in PHSD



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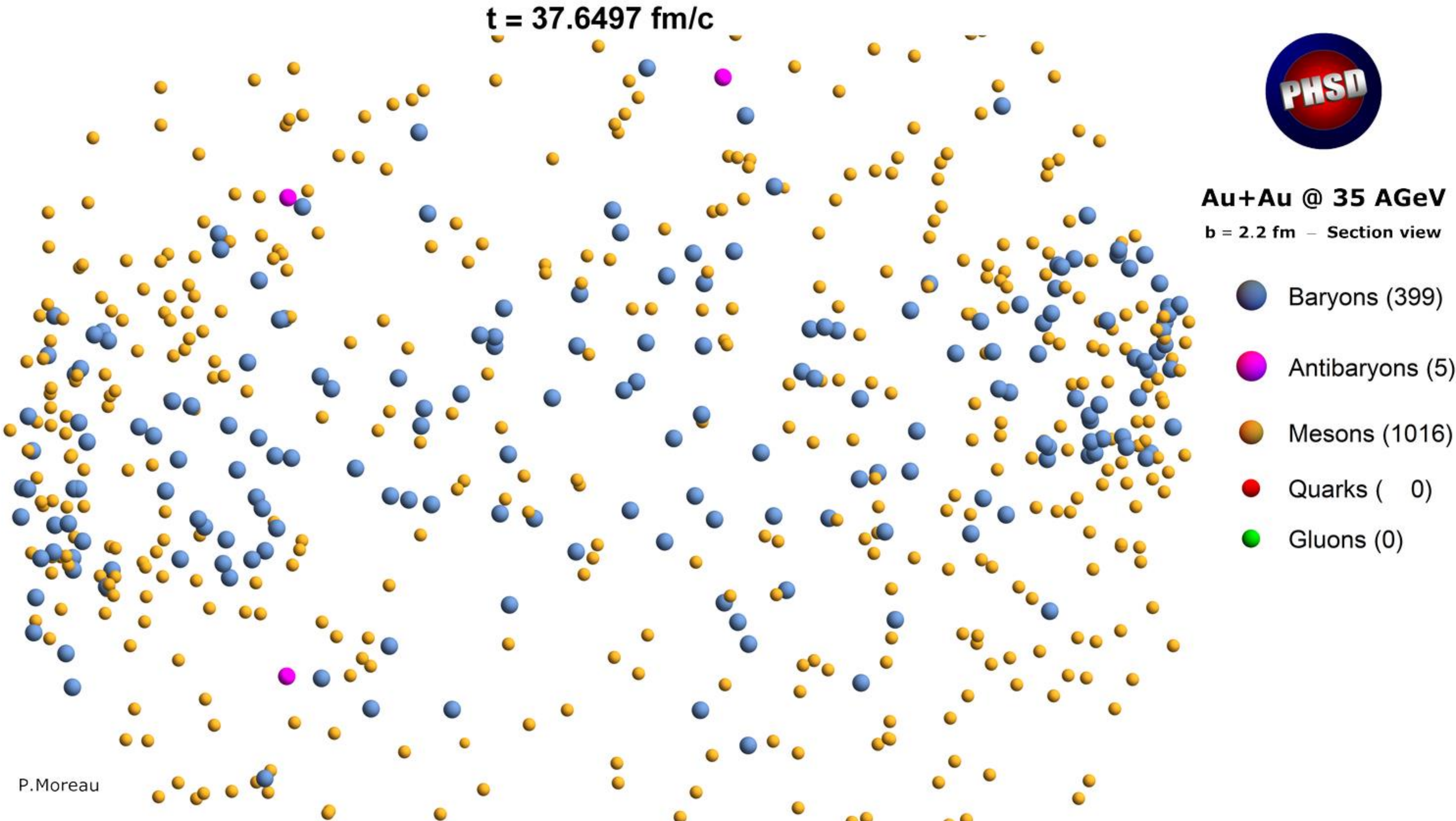

Useful literature

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