GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Lecture

Models for heavy-ion collisions: (Part 4): transport models

SS2024: ‚Dynamical models for relativistic heavy-ion collisions'

2. Quantum field theory
➔ Kadanoff-Baym dynamics

From weakly to strongly interacting systems

In-medium effects (on hadronic or partonic levels!) = changes of particle properties in the hot and dense medium

Example: hadronic medium - vector mesons, strange mesons

QGP – ‚dressing' of partons

Many-body theory:

Strong interaction ➔ large width = short life-time

➔ broad spectral function ➔ quantum object

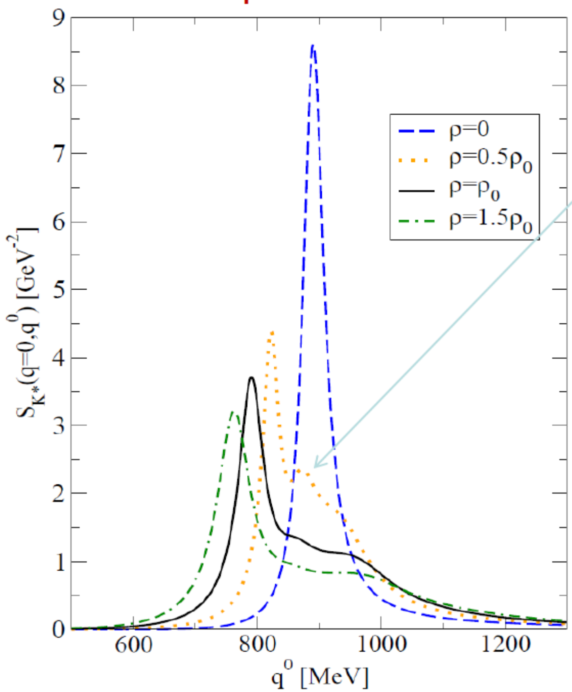
- **How to describe the dynamics of broad strongly interacting quantum states in transport theory?**

❑ **semi-classical BUU**

first order gradient expansion of quantum Kadanoff-Baym equations

❑ **generalized transport equations based on Kadanoff-Baym dynamics**

Kbar* spectral function



Λ(1783)N^{-1} and Σ(1830)N^{-1} exitations

Barcelona / Valencia group

Dynamical description of strongly interacting systems

- **Semi-classical on-shell BUU:** applies for small collisional width, i.e. for a weakly interacting systems of particles

How to describe strongly interacting systems?!

- **Quantum field theory →**

Kadanoff-Baym dynamics for **resummed** single-particle Green functions $S^<$ (= $G^<$)

$$\hat{S}_{0x}^{-1} \, S_{xy}^{<} = \Sigma_{xz}^{ret} \odot S_{zy}^{<} + \Sigma_{xz}^{<} \odot S_{zy}^{adv}$$

(1962)

Integration over the intermediate spacetime

Green functions $S^<$ / self-energies Σ:

$$iS_{xy}^{<} = \eta \langle \{ \Phi^{+}(y) \Phi(x) \} \rangle$$

$$iS_{xy}^{>} = \langle \{ \Phi(y) \Phi^{+}(x) \} \rangle$$

$$iS_{xy}^{c} = \langle T^{c} \{ \Phi(x) \Phi^{+}(y) \} \rangle \quad -causal$$

$$iS_{xy}^{a} = \langle T^{a} \{ \Phi(x) \Phi^{+}(y) \} \rangle \quad -anticausal$$

$$S_{xy}^{ret} = S_{xy}^{c} - S_{xy}^{<} = S_{xy}^{>} - S_{xy}^{a} \quad -retarded$$

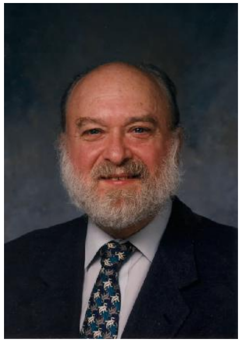
$$S_{xy}^{adv} = S_{xy}^{c} - S_{xy}^{>} = S_{xy}^{<} - S_{xy}^{a} \quad -advanced$$

$$\eta = \pm 1 (bosons \, / \, fermions)$$

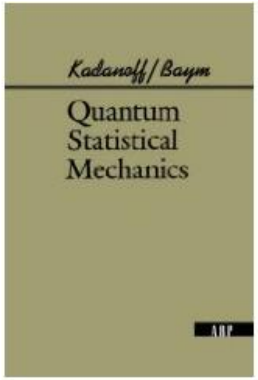
$$T^{a} (T^{c}) - (anti-)time-ordering \ operator$$

$$\hat{S}_{0x}^{-1} \equiv -(\partial_{x}^{\mu} \partial_{\mu}^{x} + M_{0}^{2}) \quad boson$$

$$= i\gamma^{\mu} \partial_{\mu}^{x} - M \quad fermion$$



Leo Kadanoff



Gordon Baym

Heisenberg picture

- **Relativistic formulations of the many-body problem are described within covariant field theory.**

The fields themselves are distributions in space-time $x = (t, \mathbf{x})$ ➔
from Schrödinger picture → Heisenberg picture:

- **In the Heisenberg picture the time evolutions of the system is described by time-dependent operators that are evolved with the help of the unitary time-evolution operator *U* (*t, t'*) which follows**

$$i\frac{\partial \hat{U}(t,t_0)}{\partial t} = \hat{H}(t)\hat{U}(t,t_0) \qquad (1)$$

Schrödinger operator of the system

Eq. (1) has the formal solution:

$$\hat{U}(t,t_0) = T\left(\exp\left[-i\int_{t_0}^{t} \mathrm{dz}\ \hat{\mathrm{H}}(\mathrm{z})\right]\right) = \sum_{n=0}^{\infty} \frac{T[-i\int_{t_0}^{t} \mathrm{dz}\ \hat{\mathrm{H}}(\mathrm{z})]^{\mathrm{n}}}{n!} \qquad (2)$$

Dyson series

If *H* doesn't depend on time: $\hat{U}(t,t_0) = e^{-i\hat{H}(t-t_0)}$

$$\Psi(x,t) = \hat{U}(t,t_0=0)\Psi(x,t_0=0)$$

Time evolution operator in Heisenberg picture

- The time evolution of any operator O in the Heisenberg picture from time t_0 to t is given by

$$\hat{O}_H(t) = \hat{U}^+(t,t_o)\,\hat{O}\,\hat{U}(t,t_o)$$

(3)

If H doesn't depend on time: $\hat{U}(t,t_0) = e^{-i\hat{H}(t-t_0)}$

$$\hat{O}_H(t) = e^{iH(t-t_0)}\hat{O}\;e^{-iH(t-t_0)}$$

Schrödinger picture → **Heisenberg picture:**

$\Psi(x,t)$ $\Psi(x,t_0=0)$

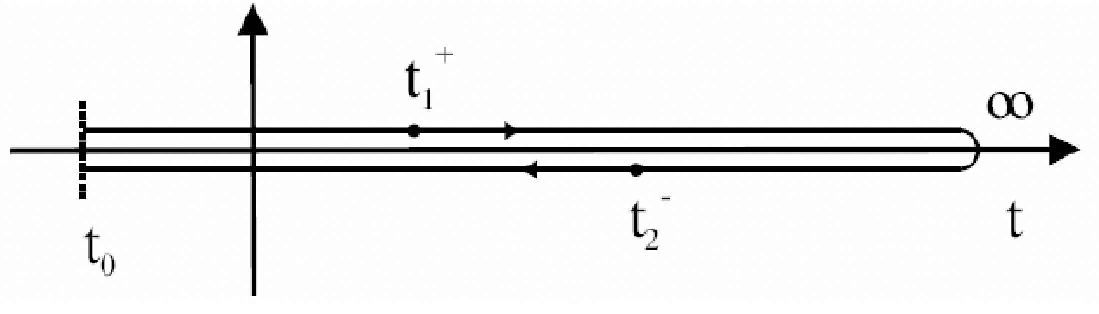
$\hat{O}$ $\hat{O}_H(t) = \hat{U}^+(t,t_o)\,\hat{O}\,\hat{U}(t,t_o)$

Expectation value in Heisenberg picture

- If the initial state is given by some density matrix ρ, which may be a pure or mixed state
- then the time evolution of expectation value $O(t)$ of the operator O in the Heisenberg picture from time t_0 to t is given by

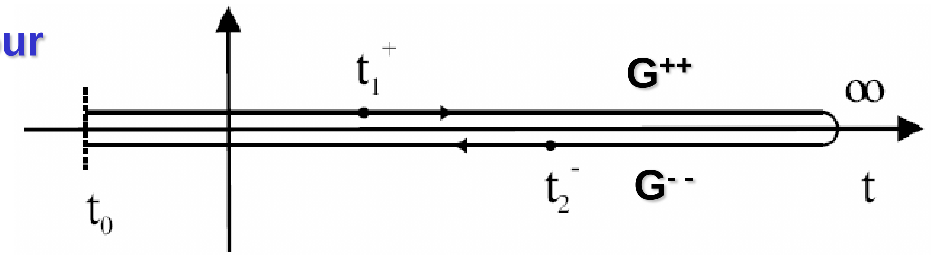
$$O(t) = \langle \hat{O}_H(t) \rangle = \mathrm{Tr}\left(\hat{\rho}\,\hat{\mathrm{O}}_{\mathrm{H}}(\mathrm{t})\right) = \mathrm{Tr}\left(\hat{\rho}\,\hat{\mathrm{U}}(\mathrm{t}_0,\mathrm{t})\hat{\mathrm{O}}\,\hat{\mathrm{U}}(\mathrm{t},\mathrm{t}_0)\right) = \mathrm{Tr}\left(\hat{\rho}\,\hat{\mathrm{U}}^{\dagger}(\mathrm{t},\mathrm{t}_0)\hat{\mathrm{O}}\,\hat{\mathrm{U}}(\mathrm{t},\mathrm{t}_0)\right) \quad (4)$$

This implies that first the system is evolved from t_0 to t and then backward from t to t_0. This may be expressed as a time integral along the Keldysh-Contour



Two-point functions on the Keldysh contour

Real-time (Keldysh-) Contour
in the Heisenberg picture



Consider: Interacting field theory for spinless massive scalar bosons ➔ scalar field $\phi(x)$

- **Green functions: elementary degrees of freedom** $x = (t_x, \vec{x}), \quad y = (t_y, \vec{y})$

Causal: $iG^c(x,y) = iG^{++}(x,y) = \langle\, \hat{T}^c(\phi(x)\phi(y))\,\rangle$ — t_x and t_y on upper part; $t_x > t_y$

Small: $iG^<(x,y) = iG^{+-}(x,y) = \langle \phi(y)\phi(x) \rangle$ — t_x on upper; t_y on lower part

Large: $iG^>(x,y) = iG^{-+}(x,y) = \langle \phi(x)\phi(y) \rangle$ — t_x on lower; t_y on upper part

Anticausal: $iG^a(x,y) = iG^{--}(x,y) = \langle\, \hat{T}^a(\phi(x)\phi(y))\,\rangle$ — t_x and t_y on lower part; $t_y > t_x$

T^c / T^a denote time ordering on the upper/lower branch of the real-time contour

In matrix notation:

$$G(x,y) = \begin{matrix} & + & - \\ + & G^c(x,y) & G^<(x,y) \\ - & G^>(x,y) & G^a(x,y) \end{matrix} \qquad (5)$$

Green functions on contour

- Relation to the one-body density matrix ρ :

(6) $$\rho(\mathrm{x},\mathrm{x}';t) = -iG^<(\mathrm{x},\mathrm{x}';t,t)$$ ← $$G^<(\mathrm{x},\mathrm{x}';t) = \int_{-\infty}^{\infty} d(\tau-\tau')\; G^<(\mathrm{x},\mathrm{x}';\tau,\tau')$$

$t=(\tau+\tau')/2$

- Two-point functions *F* on the closed-time-path (CTP) generally can be expressed by retarded (R) and advanced (A) components as

(7)
$$\begin{aligned} F^{\mathrm{R}}(x,y) &= F^c(x,y) - F^<(x,y) = F^>(x,y) - F^a(x,y) \\ F^{\mathrm{A}}(x,y) &= F^c(x,y) - F^>(x,y) = F^<(x,y) - F^a(x,y) \end{aligned}$$

Note:
only two Green functions are independent!

giving in particular the relation

(8) $$F^{\mathrm{R}}(x,y) - F^{\mathrm{A}}(x,y) = F^>(x,y) - F^<(x,y)$$

Note that the advanced and retarded components of the Green functions contain only spectral and no statistical information (see below)

Dyson-Schwinger equation on the contour

❑ **Dyson-Schwinger equation** **(follows from Schrödinger eq.):**

$$G(x,y) = G_0(x,y) + G_0(x,y)\Sigma(x,y)G(x,y) \tag{9}$$

Dyson-Schwinger equation on the closed-time-path reads in matrix form:

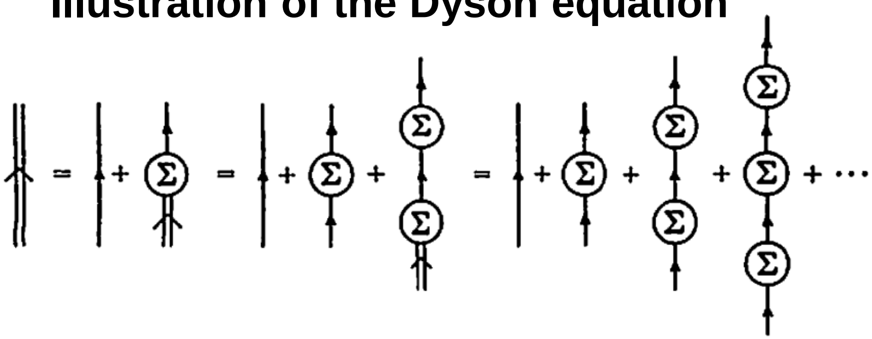
$$\begin{pmatrix} G^c(x,y) & G^<(x,y) \\ G^>(x,y) & G^a(x,y) \end{pmatrix} = \begin{pmatrix} G_0^c(x,y) & G_0^<(x,y) \\ G_0^>(x,y) & G_0^a(x,y) \end{pmatrix} +$$
$$\begin{pmatrix} G_0^c(x,x') & G_0^<(x,x') \\ G_0^>(x,x') & G_0^a(x,x') \end{pmatrix} \odot \begin{pmatrix} \Sigma^c(x',y') & -\Sigma^<(x',y') \\ -\Sigma^>(x',y') & \Sigma^a(x',y') \end{pmatrix} \odot \begin{pmatrix} G^c(y',y) & G^<(y',y) \\ G^>(y',y) & G^a(y',y) \end{pmatrix} \tag{10}$$

Free propagator for Bose case:

$$\hat{G}_{0x}^{-1} = -(\partial_\mu^x \partial_x^\mu + m^2)$$

$$\hat{G}_{0x}^{-1} G_0^{R/A}(x,y) = \delta(x-y)$$

Illustration of the Dyson equation



$\odot$ **means convolution integral over the closed time-path**

Towards the Kadanoff-Baym equations

For Bose case the free propagator is defined via the negative inverse Klein-Gordon operator in space-time representation

$$\hat{G}_{0x}^{-1} = -(\partial_\mu^x \partial_x^\mu + m^2) \qquad (11)$$

which is a solution of the Klein-Gordon equation in the following sense:

$$\boxed{\hat{G}_{0x}^{-1} G_0^{R/A}(x,y) = \delta(x-y)}$$

$$\hat{G}_{0x}^{-1} \begin{pmatrix} G_0^c(x,y) & G_0^<(x,y) \\ G_0^>(x,y) & G_0^a(x,y) \end{pmatrix} = \delta(\mathbf{x}-\mathbf{y}) \begin{pmatrix} \delta(x_0-y_0) & 0 \\ 0 & -\delta(x_0-y_0) \end{pmatrix} \qquad (12)$$

Free Green function $G_0(x,y)$

$$= \delta(\mathbf{x}-\mathbf{y})\delta_p(x_0-y_0)$$

$$x = (x^0, \mathbf{x})$$
$$y = (y^0, \mathbf{y})$$

with δ_p denoting the δ-function on the closed time path (CTP).
In (11) m denotes the bare mass of the scalar field.

The Kadanoff-Baym equations

To derive the **Kadanoff-Baym equations** one multiplies Dyson-Schwinger eq. (10) with 1) G_{0x}^{-1} and 2) with G_{0y}^{-1}

This gives **four equations** for $G^<$, $G^>$ (for propagation in x or in y) which can be written in the form:

1) (10)*G_{0x}^{-1} ➔ propagation of Green functions in variable x

(11)

$$-(\partial_\mu^x \partial_x^\mu + m^2) G^{R/A}(x,y) = \delta(x-y) + \Sigma^{R/A}(x,x') \odot G^{R/A}(x',y)$$

$$-(\partial_\mu^x \partial_x^\mu + m^2) G^<(x,y) = \Sigma^R(x,x') \odot G^<(x',y) + \Sigma^<(x,x') \odot G^A(x',y)$$

$$-(\partial_\mu^x \partial_x^\mu + m^2) G^>(x,y) = \Sigma^R(x,x') \odot G^>(x',y) + \Sigma^>(x,x') \odot G^A(x',y)$$

2) (10)*G_{0y}^{-1} ➔ propagation of Green functions in variable y
(similar to (11) → adjoint eqs.)

➔ **Kadanoff-Baym equations:**
provide nonequilibrium time evolution of quantum system in terms of 2-point Green functions

L. P. Kadanoff, G. Baym, ‚*Quantum Statistical Mechanics*‘, Benjamin, 1962

Derivation of the selfenergy

Effective action Γ :

Yu. Ivanov, J. Knoll, D. Voskresensky, NPA657 (1999) 413

$$\Gamma[G] = \Gamma^0 + \frac{i}{2}\left[\ln(1 - \odot_p G_0 \odot_p \Sigma) + \odot_p G \odot_p \Sigma\right] + \Phi[G] \quad (15)$$

Resummed propagators with self-generated mean-field

Γ^0 – ‚free' part of action (kinetic + mass terms), G_0 - free propagator,
$\odot_p$ means convolution integral over the closed time-path

$\Phi(G)$ is the ‚interaction part' = sum of all connected nPI diagrams built up by the full $G(x,y)$

Used approximation: Two-particle irreducible (2PI) diagrams

- **Define selfenergy Σ by the variation of Γ [G]**

$$\delta\Gamma = 0 = \frac{i}{2}\Sigma\,\delta G - \frac{i}{2}\frac{G_0}{1 - G_0\Sigma}\delta\Sigma + \frac{i}{2}G\,\delta\Sigma + \delta\Phi \quad (16)$$

$$= \frac{i}{2}\Sigma\,\delta G - \frac{i}{2}\underbrace{\frac{1}{G_0^{-1} - \Sigma}}_{=G}\delta\Sigma + \frac{i}{2}G\,\delta\Sigma + \delta\Phi = \frac{i}{2}\Sigma\,\delta G + \delta\Phi$$

$$\Rightarrow \quad \Sigma = 2i\frac{\delta\Phi}{\delta G} = 2\frac{\delta\Phi}{\delta(-iG)}$$

➔ The selfenergy Σ are obtained by opening of a propagator line in the irreducible diagrams Φ

Example: scalar theory with self-interactions

Φ^4 – theory: the interacting field theory for **spinless massive scalar bosons** provides a **‚theoretical laboratory‘** for testing approximation schemes

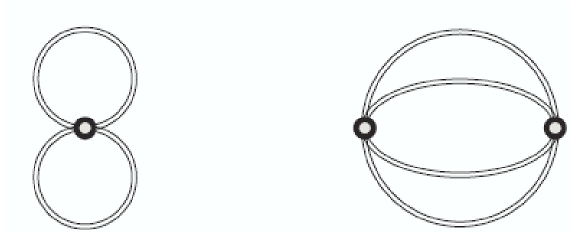
❑ **Lagrangian density:**

$$\mathcal{L}(x) = \frac{1}{2}\partial_\mu^x \phi(x) \partial_x^\mu \phi(x) - \frac{1}{2} m^2 \phi(x)^2 - \frac{\lambda}{4!}\phi^4(x) \qquad (17)$$

$\phi(x)$ – real scalar field
λ – is a coupling constant

❑ **Φ(G) :** the sum of all closed **2PI** diagrams built up by the full G(x,y):

Φ(G) up to 3-loop order;
~ 2nd order in λ (i.e. 2PI)



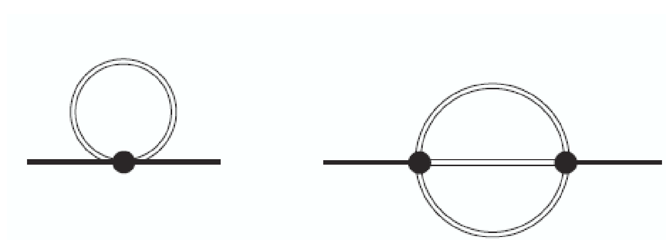
d+1: *d*=dimension of space
(d=3 or 2) + 1(=time)

$$i\Phi = \frac{i\lambda}{8}\int_p d^{d+1}x \; G(x,x)^2 - \frac{\lambda^2}{48}\int_p d^{d+1}x \int_p d^{d+1}y \; G(x,y)^4 \qquad (18)$$

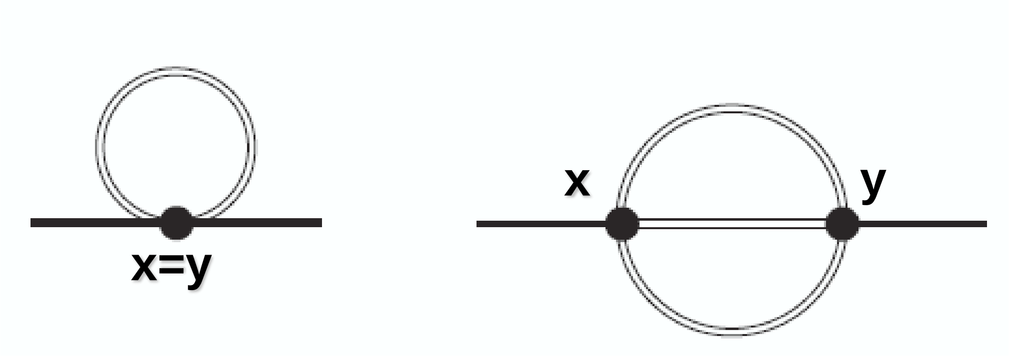
From (16) ➔ **self-energies** are defined by the variation of Φ w.r.t G(y,x):

$$\Sigma(x,y) = 2i\,\frac{\delta\Phi}{\delta G(y,x)}$$

➔ **Cut a line and stretch:**



2PI self-energies in Φ^4 - theory



$$\Sigma(x,y) = \Sigma^\delta(x)\,\delta_p^{(d+1)}(x-y) + \Theta_p(x_0-y_0)\,\Sigma^>(x,y) + \Theta_p(y_0-x_0)\,\Sigma^<(x,y) \quad (19)$$

Local in space and time part: tadpole

Nonlocal part: sunset

$$\Sigma^\delta(x) = \frac{\lambda}{2}\, i\, G^<(x,x)$$

$$\Sigma^{\gtrless}(x,y) = -\frac{\lambda^2}{6}\, G^{\gtrless}(x,y)\, G^{\gtrless}(x,y)\, G^{\lessgtr}(y,x) = -\frac{\lambda^2}{6}\left[G^{\gtrless}(x,y)\right]^3$$

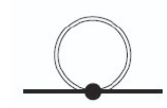
local ‚potential' term (~λ)
leads to the generation of an effective mass for the field quanta

interaction term (~ λ^2)

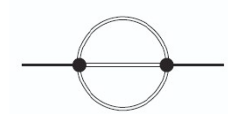
Kadanoff-Baym equations of motion for $G^<$

1) $$-\left[\partial_\mu^x \partial_x^\mu + m^2\right] G^{\gtrless}(x,y) = \underline{\Sigma^\delta(x)\, G^{\gtrless}(x,y)}$$ **potential term**

interaction term
$$+\int_{t_0}^{x_0} dz_0 \int d^d z \; \left[\Sigma^>(x,z) - \Sigma^<(x,z)\right] G^{\gtrless}(z,y)$$
$$-\int_{t_0}^{y_0} dz_0 \int d^d z \; \Sigma^{\gtrless}(x,z) \left[G^>(z,y) - G^<(z,y)\right],$$



tadpole diagram



sunset diagram

2) $$-\left[\partial_\mu^y \partial_y^\mu + m^2\right] G^{\gtrless}(x,y) = \underline{\Sigma^\delta(y)\, G^{\gtrless}(x,y)}$$

d: dimension of space

$$+\int_{t_0}^{x_0} dz_0 \int d^d z \; \left[G^>(x,z) - G^<(x,z)\right] \Sigma^{\gtrless}(z,y)$$
$$-\int_{t_0}^{y_0} dz_0 \int d^d z \; G^{\gtrless}(x,z) \left[\Sigma^>(z,y) - \Sigma^<(z,y)\right], \qquad (20)$$

Kadanoff-Baym equations include:
- the influence of the mean-field on the particle propagation generated by the tadpole diagram
- as well as scattering processes as inherent in the sunset diagram.

KB equations for Φ^4–theory for homogeneous system

- do **Wigner transformation** of the Kadanoff-Baym equations:

$$F_{XP} = \int d^4(x-y)\ e^{iP_\mu(x^\mu - y^\mu)}\ F_{xy}$$

For any function F_{XY} with X=(*x*+y)/2 – space-time coordinate, P – 4-momentum

- **Example:** Solution of KB for the case of Φ^4 – theory for homogeneous system (no X dependence):

➔ Wigner transformed KB:

$$\partial_{t_1}^2 G^<(\mathbf{p},t_1,t_2) = -[\mathbf{p}^2 + m^2 + \tilde{\Sigma}^\delta(t_1)]\, G^<(\mathbf{p},t_1,t_2)$$

$$- \int_{t_0}^{t_1} dt'\ [\Sigma^>(\mathbf{p},t_1,t') - \Sigma^<(\mathbf{p},t_1,t')]\ G^<(\mathbf{p},t',t_2)$$

$$+ \int_{t_0}^{t_2} dt'\ \Sigma^<(\mathbf{p},t_1,t')\ [G^>(\mathbf{p},t',t_2) - G^<(\mathbf{p},t',t_2)]$$

Collision term

$$= -[\mathbf{p}^2 + m^2 + \tilde{\Sigma}^\delta(t_1)]\, G^<(\mathbf{p},t_1,t_2) + I_1^<(\mathbf{p},t_1,t_2),$$

Self-energies in two-time, momentum space (p; t; t_0) representation:

$$\tilde{\Sigma}^\delta(t) = \frac{\lambda}{2} \int \frac{d^dp}{(2\pi)^d}\ i\,G^<(\mathbf{p},t,t)\,,$$

$$\Sigma^{\lessgtr}(\mathbf{p},t,t') = -\frac{\lambda^2}{6} \int \frac{d^dq}{(2\pi)^d} \int \frac{d^dr}{(2\pi)^d}\ G^{\lessgtr}(\mathbf{q},t,t')\ G^{\lessgtr}(\mathbf{r},t,t')\ G^{\gtrless}(\mathbf{q}+\mathbf{r}-\mathbf{p},t',t)\,.$$

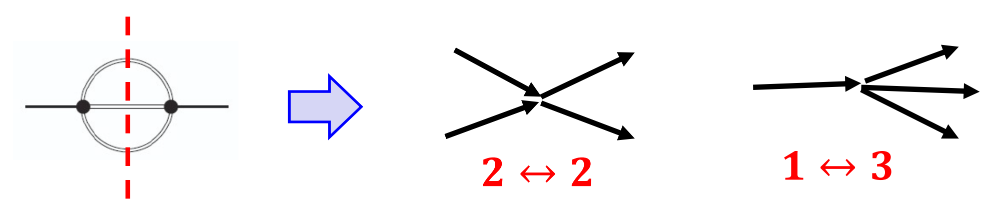
$$= -\frac{\lambda^2}{6} \int \frac{d^dq}{(2\pi)^d} \int \frac{d^dr}{(2\pi)^d}\ G^{\lessgtr}(\mathbf{q},t,t')\ G^{\lessgtr}(\mathbf{r},t,t')\ G^{\lessgtr}(\mathbf{p}-\mathbf{q}-\mathbf{r},t,t')\,.$$

KB equations for Φ⁴–theory for homogeneous system

Collision term:

$$
\begin{aligned}
I_1^<(\mathbf{p},t_1,t_2) = \\
&+\int_{t_0}^{t_1} dt' \, \frac{\lambda^2}{6} \int \frac{d^d q}{(2\pi)^d} \int \frac{d^d r}{(2\pi)^d} \; G^>(\mathbf{q},t_1,t') \; G^>(\mathbf{r},t_1,t') \; G^<(\mathbf{q}+\mathbf{r}-\mathbf{p},t',t_1) \; G^<(\mathbf{p},t',t_2) \\
&-\int_{t_0}^{t_2} dt' \, \frac{\lambda^2}{6} \int \frac{d^d q}{(2\pi)^d} \int \frac{d^d r}{(2\pi)^d} \; G^<(\mathbf{q},t_1,t') \; G^<(\mathbf{r},t_1,t') \; G^>(\mathbf{q}+\mathbf{r}-\mathbf{p},t',t_1) \; G^>(\mathbf{p},t',t_2) \\
&+\int_{t_0}^{t_2} dt' \, \frac{\lambda^2}{6} \int \frac{d^d q}{(2\pi)^d} \int \frac{d^d r}{(2\pi)^d} \; G^<(\mathbf{q},t_1,t') \; G^<(\mathbf{r},t_1,t') \; G^>(\mathbf{q}+\mathbf{r}-\mathbf{p},t',t_1) \; G^<(\mathbf{p},t',t_2) \\
&-\int_{t_0}^{t_1} dt' \, \frac{\lambda^2}{6} \int \frac{d^d q}{(2\pi)^d} \int \frac{d^d r}{(2\pi)^d} \; G^<(\mathbf{q},t_1,t') \; G^<(\mathbf{r},t_1,t') \; G^>(\mathbf{q}+\mathbf{r}-\mathbf{p},t',t_1) \; G^<(\mathbf{p},t',t_2),
\end{aligned}
$$

! KB collision term apart from 2 ↔ 2 processes also involves 1 ↔ 3 processes which are not allowed by energy conservation in an on-shell collision term for massive particles!



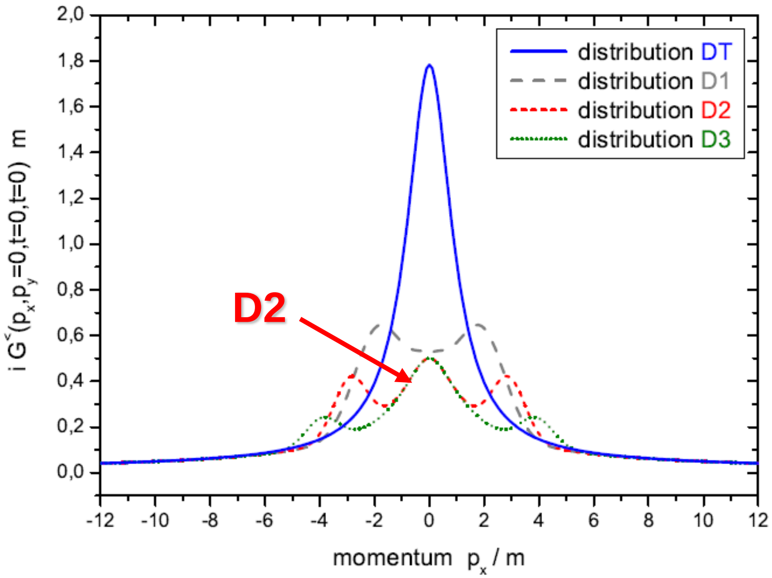
Solutions of KB equations for Φ^4 – theory for homogeneous system

- Set initial conditions:

Example: set 4 different initial distributions DT, D1, D2, D3 that are all characterized by the same energy density

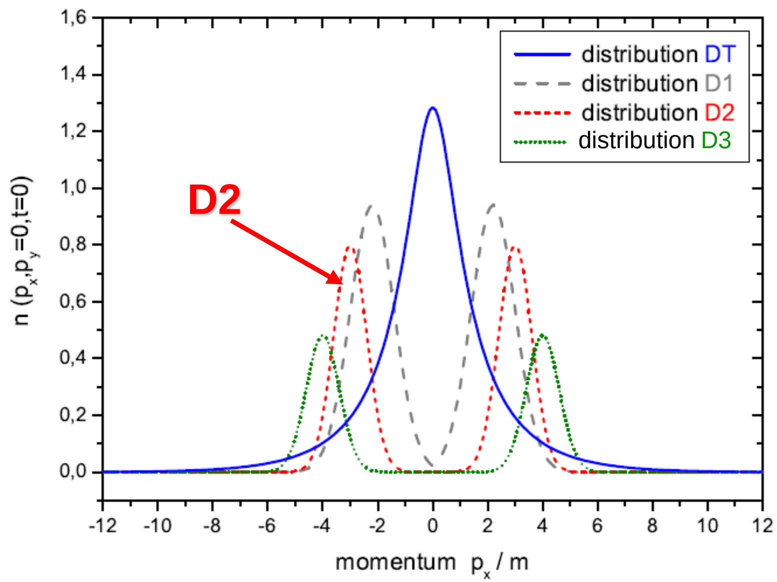
➔ for large times ($t \to \infty$) all initial distributions should lead to the same equilibrium final state

$$iG^{<}(\mathbf{p}, t=0, t=0)$$



$$2\omega_{\mathbf{p}} iG^{<}(\mathbf{p}, t=0, t=0) = 2n(\mathbf{p}, t=0) + 1$$

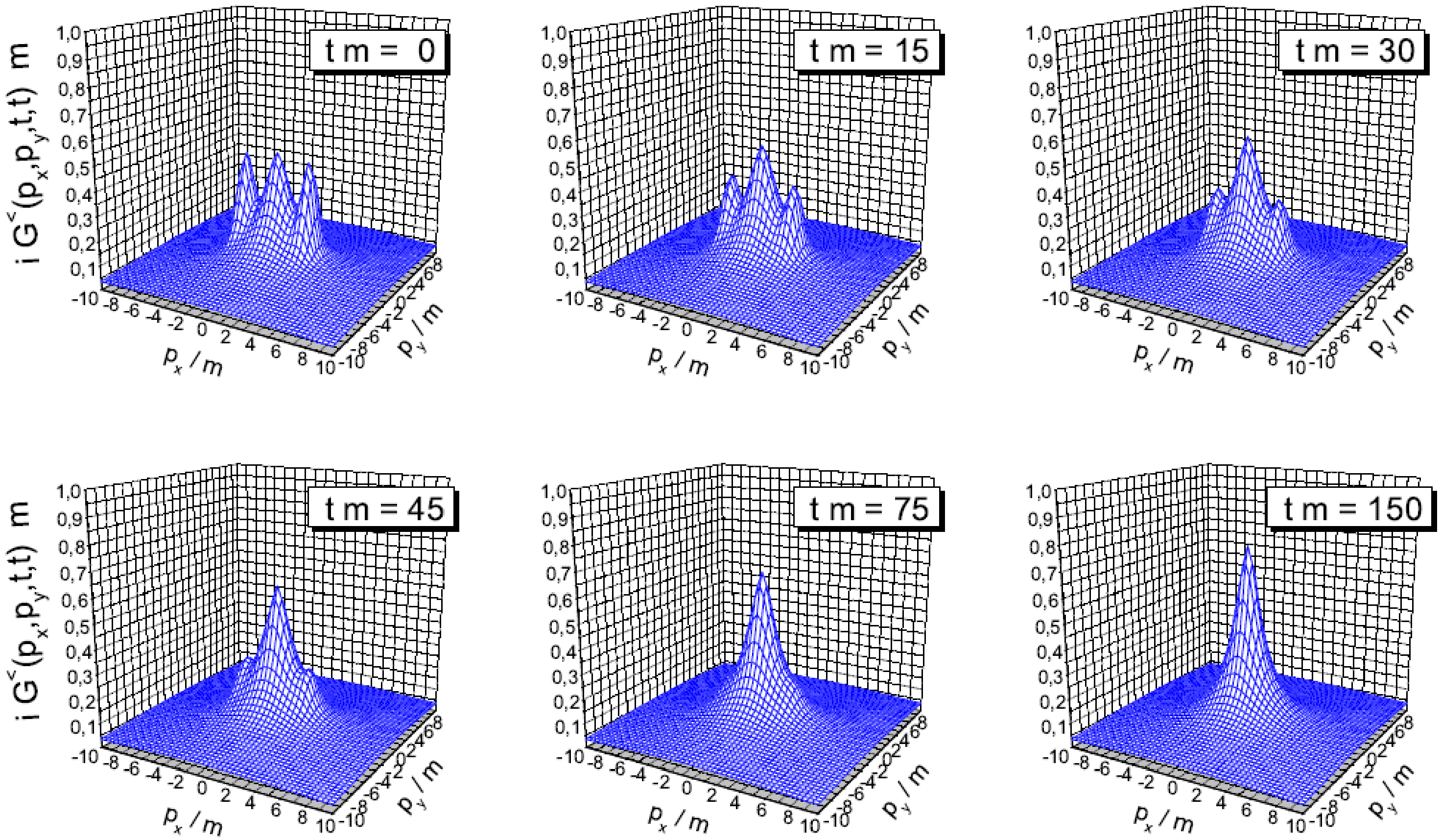
occupation density n



S. Juchem, W. Cassing, and C. Greiner, Phys. Rev. D 69 (2004) 025006; Nucl. Phys. A 743 (2004) 92

Solutions of KB equations for Φ^4 – theory

- Time evolution of the Green's function $iG^<(p_x; p_y; t; t)$ in momentum space for the initial distribution D2 for $\lambda/m=18$

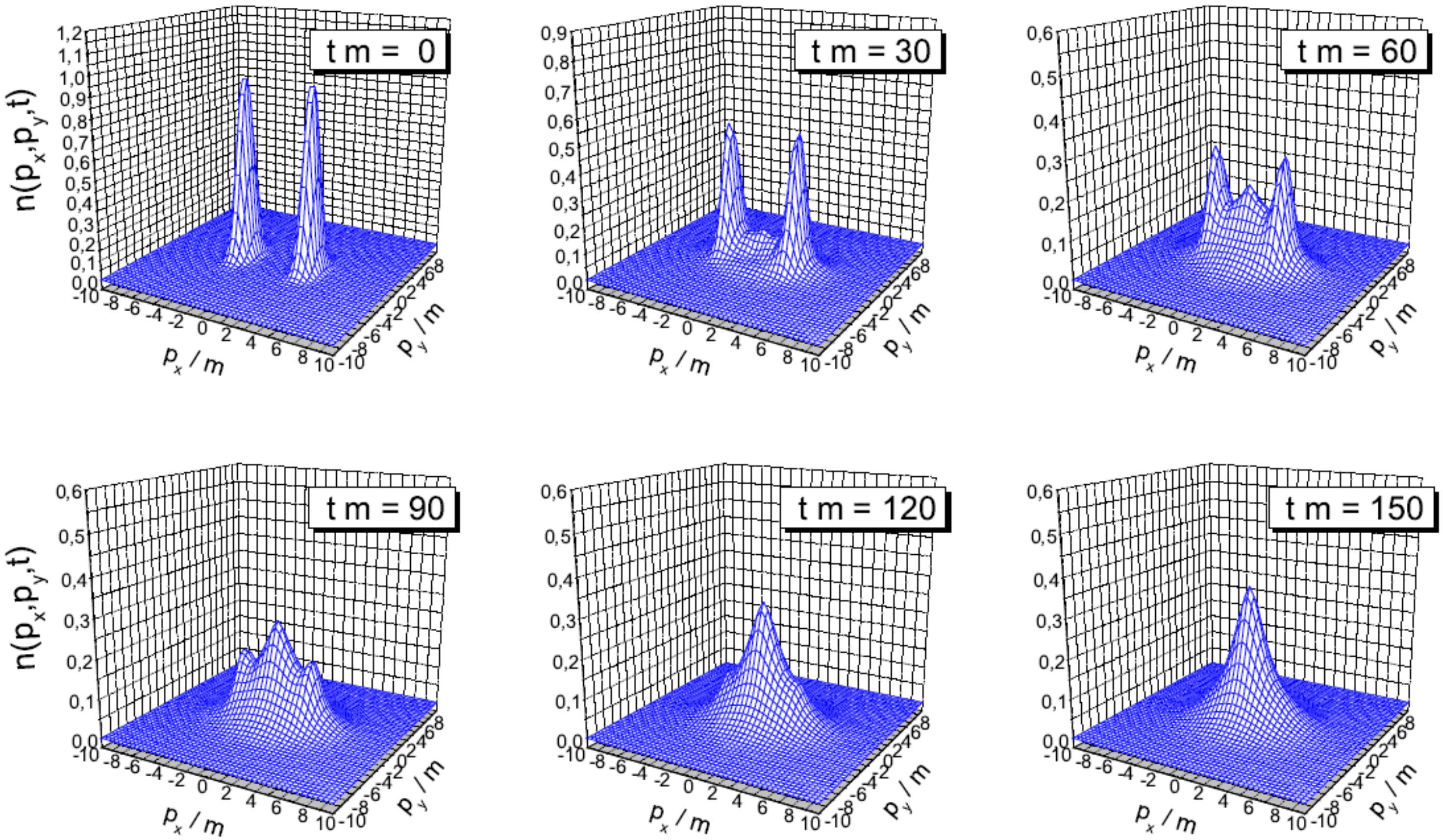


- t→∞ equilibrium final state

S. Juchem, W. Cassing, and C. Greiner, Phys. Rev. D 69 (2004) 025006; Nucl. Phys. A 743 (2004) 92

Solutions of KB equations for Φ^4 – theory

- **Time evolution of the occupation density $n(p_x; p_y; t)$ in momentum space for the initial distribution D2 for $\lambda/m=18$**



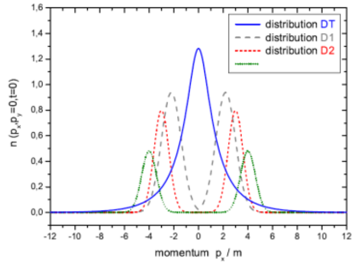
- **t→∞ equilibrium final state**

S. Juchem, W. Cassing, and C. Greiner, Phys. Rev. D 69 (2004) 025006; Nucl. Phys. A 743 (2004) 92

Boltzmann vs. Kadanoff-Baym dynamics

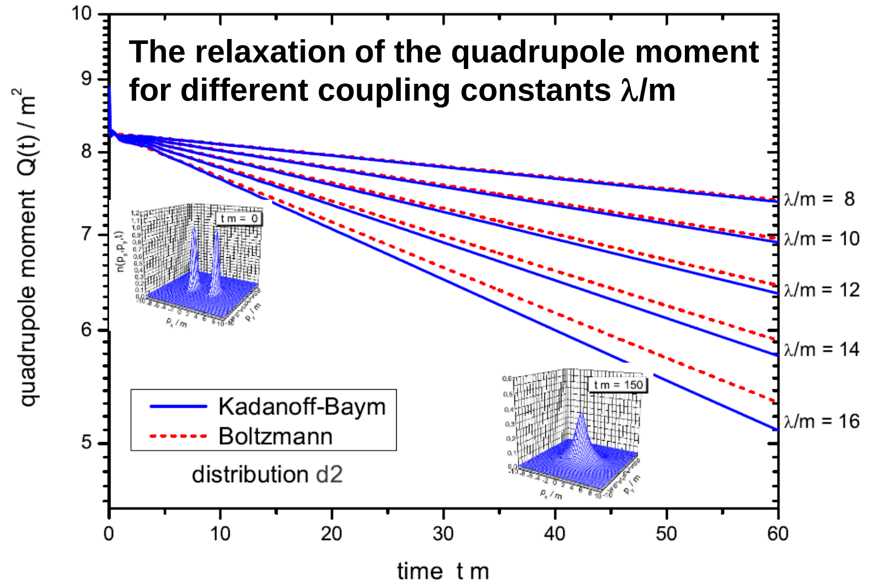
- **Example: Φ^4 – theory**

Initial distribution D1, D2



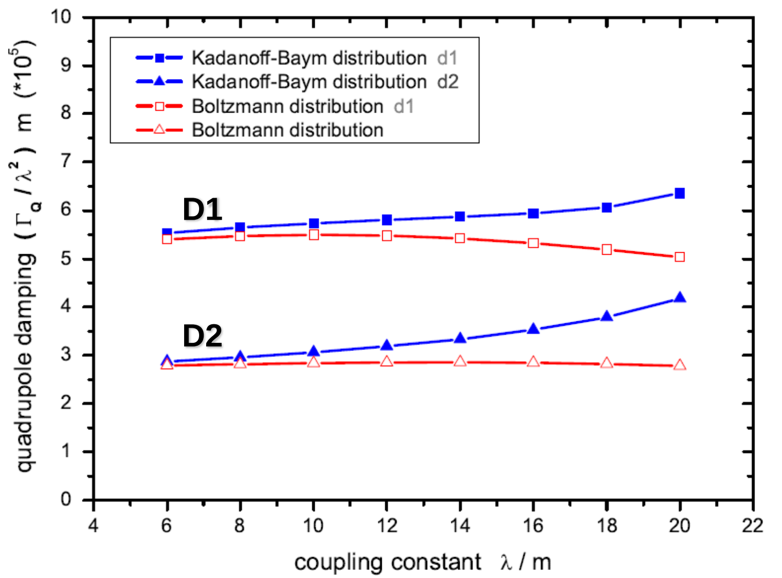
1) Consider quadrupole moment

$$Q(\tilde{t}) = \frac{\int \frac{d^dp}{(2\pi)^d} \; (p_x^2 - p_y^2) \; N(\mathbf{p}, \tilde{t})}{\int \frac{d^dp}{(2\pi)^d} \; N(\mathbf{p}, \tilde{t})}$$



2) The relaxation rate of the quadrupole moment vs. coupling constants λ/m

$$Q(\tilde{t}) \sim \exp\left(-\Gamma_Q \tilde{t}\right)$$



- **KB:** faster equilibration for larger coupling constant
- **Boltzmann:** works well for small coupling (on-shell states)

S. Juchem, W. Cassing, and C. Greiner, Phys. Rev. D 69 (2004) 025006; Nucl. Phys. A 743 (2004) 92

Advantages of Kadanoff-Baym dynamics vs Boltzmann

Kadanoff-Baym equations:

- propagate two-point Green functions
  $G^<(x,p) \rightarrow A(x,p) * N(x,p)$
  in 8 dimensions $x=(t,\vec{r})$ $p=(p_0,\vec{p})$
- $G^<$ carries information not only on the occupation number N_{XP}, but also on the particle properties, interactions and correlations via spectral function A_{XP}

Boltzmann equations

- propagate phase space distribution function $f(\vec{r},\vec{p},t)$ in 6+1 dimensions
- works well for small coupling = weakly interacting system,
  ➔ on-shell approach

- Applicable for strong coupling = strongly interaction system
- Includes memory effects (time integration) and off-shell transitions in collision term
- Dynamically generates a broad spectral function for strong coupling
- KB can be solved exactly for model cases as Φ^4 – theory
- KB can be solved in 1st order gradient expansion in terms of generalized transport equations (in test particle ansatz) for realistic systems of HICs

Useful literature

L. P. Kadanoff, G. Baym, ‚*Quantum Statistical Mechanics*‘, Benjamin, 1962

M. Bonitz, ‚*Quantum kinetic theory*‘, B.G. Teubner Stuttgart, 1998

W. Cassing and E.L. Bratkovskaya, ‘Hadronic and electromagnetic probes of hot and dense nuclear matter’, Phys. Reports 308 (1999) 65-233.
http://inspirehep.net/record/495619

S.J. Wang and W. Cassing, Annals Phys. 159 (1985) 328

S. Juchem, W. Cassing, and C. Greiner, Phys. Rev. D 69 (2004) 025006;
Nucl. Phys. A 743 (2004) 92

W. Cassing, Eur. Phys. J. ST 168 (2009) 3

W. Botermans and R. Malfliet, Phys. Rep. 198 (1990) 115

J. Berges, Phys.Rev.D7 (2006) 045022; AIP Conf. Proc. 739 (2005) 3

C.S. Fischer, J.Phys.G32 (2006) R253

W. Cassing, `Transport Theories for Strongly-Interacting Systems’,
Springer Nature: Lecture Notes in Physics 989, 2021; DOI: 10.1007/978-3-030-80295-0