GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Lecture

Models for heavy-ion collisions: (Part 5): transport models

SS2024: ‚Dynamical models for relativistic heavy-ion collisions'

2. Quantum field theory

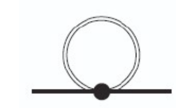
➔ Kadanoff-Baym dynamics

➔ generalized off-shell transport equations

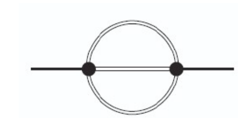
Reminder: Lecture 6

Kadanoff-Baym equations of motion for $G^<$

1) $$-\left[\partial_\mu^x \partial_x^\mu + m^2\right] G^{\gtrless}(x,y) = \underline{\Sigma^\delta(x)\, G^{\gtrless}(x,y)}$$ **potential term**



interaction term
$$\left\{ \begin{aligned} &+ \int_{t_0}^{x_0} dz_0 \int d^d z \; \left[\Sigma^>(x,z) - \Sigma^<(x,z)\right] G^{\gtrless}(z,y) \\ &- \int_{t_0}^{y_0} dz_0 \int d^d z \; \Sigma^{\gtrless}(x,z) \left[G^>(z,y) - G^<(z,y)\right], \end{aligned} \right.$$



(20)

2) $$-\left[\partial_\mu^y \partial_y^\mu + m^2\right] G^{\gtrless}(x,y) = \underline{\Sigma^\delta(y)\, G^{\gtrless}(x,y)}$$

d: dimension of space

$$\left\{ \begin{aligned} &+ \int_{t_0}^{x_0} dz_0 \int d^d z \; \left[G^>(x,z) - G^<(x,z)\right] \Sigma^{\gtrless}(z,y) \\ &- \int_{t_0}^{y_0} dz_0 \int d^d z \; G^{\gtrless}(x,z) \left[\Sigma^>(z,y) - \Sigma^<(z,y)\right], \end{aligned} \right.$$

Kadanoff-Baym equations include:

- **the influence of the mean-field on the particle propagation generated by the tadpole diagram**
- **as well as scattering processes as inherent in the sunset diagram.**

Wigner transformation of the Kadanoff-Baym equation

- do **Wigner transformation** of the Kadanoff-Baym equation

$$\boxed{F_{XP} = \int d^4(x-y)\ e^{iP_\mu(x^\mu - y^\mu)}\ F_{xy}}$$

For any function F_{XY} with X=(*x*+y)/2 – space-time coordinate, P – 4-momentum

Convolution integrals convert under Wigner transformation as

$$\int d^4(x-y)\ e^{iP_\mu(x^\mu - y^\mu)}\ F_{1,xz} \odot F_{2,zy} = e^{-i\Diamond}\ F_{1,PX}\ F_{2,PX}$$

an infinite series in the differential operator $\Diamond$

Operator $\Diamond$ is a 4-dimentional generalizaton of the Poisson-bracket:

$$\Diamond\{F_1\}\{F_2\} := \frac{1}{2}\left(\frac{\partial F_1}{\partial X_\mu}\frac{\partial F_2}{\partial P^\mu} - \frac{\partial F_1}{\partial P_\mu}\frac{\partial F_2}{\partial X^\mu}\right)$$

- consider only contribution up to first order in the gradients

  = a standard approximation of kinetic theory which is justified if the gradients in the mean spacial coordinate X are small

From Kadanoff-Baym equations to transport equations

- separate all retarded and advanced quantities – Geen functions and self- energies – into real and imaginary parts:

$$S_{XP}^{ret,adv} = Re S_{XP}^{ret} \mp \frac{i}{2} A_{XP} , \qquad \Sigma_{XP}^{ret,adv} = Re \Sigma_{XP}^{ret} \mp \frac{i}{2} \Gamma_{XP}$$

The imaginary part of the retarded propagator is given by the normalized spectral function A_{XP}:

$$A_{XP} = i \left[S_{XP}^{ret} - S_{XP}^{adv} \right] = -2 \, Im \, S_{XP}^{ret}$$

$$\int \frac{dP_0^2}{4\pi} A_{XP} = 1$$

The imaginary part of the selfenergy corresponds to the width Γ_{XP}; then from Dyson-Schwinger equation:

$$Re S_{XP}^{ret} = \frac{P^2 - M_0^2 - Re \Sigma_{XP}^{ret}}{\Gamma_{XP}} A_{XP}$$

algebraic solution

The spectral function A_{XP} in first order gradient expansion (for bosons) :

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - Re \Sigma_{XP}^{ret})^2 + \Gamma_{XP}^2/4}$$

The real part of the retarded propagator in first order gradient expansion :

$$Re S_{XP}^{ret} = \frac{P^2 - M_0^2 - Re \Sigma_{XP}^{ret}}{(P^2 - M_0^2 - Re \Sigma_{XP}^{ret})^2 + \Gamma_{XP}^2/4}$$

A_{XP} and $Re\Sigma_{XP}^{ret}$ in first order gradient expansion depend ONLY on Σ_{XP}^{ret} !

From Kadanoff-Baym equations to generalized transport equations

After the first order gradient expansion of the Wigner transformed Kadanoff-Baym equations and separation into the real and imaginary parts one gets:

Generalized transport equations (GTE):

drift term | Vlasov term | backflow term | collision term = 'gain' - 'loss' term

$$\diamond\{P^2 - M_0^2 - Re\Sigma^{ret}_{XP}\}\{S^<_{XP}\} - \boxed{\diamond\{\Sigma^<_{XP}\}\{ReS^{ret}_{XP}\}} = \frac{i}{2}[\Sigma^>_{XP}S^<_{XP} - \Sigma^<_{XP}S^>_{XP}]$$

Backflow term incorporates the off-shell behavior in the particle propagation
! vanishes in the quasiparticle limit $A_{XP} \rightarrow \delta(p^2-M^2)$

- GTE: Propagation of the Green's function $iS^<_{XP}=A_{XP}N_{XP}$, which carries information not only on the number of particles (N_{XP}), but also on their properties, interactions and correlations (via A_{XP})

- Spectral function:

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - Re\Sigma^{ret}_{XP})^2 + \Gamma^2_{XP}/4}$$

$\Gamma_{XP} = -Im\,\Sigma^{ret}_{XP} = 2p_0\Gamma$ – 'width' of spectral function
= reaction rate of particle (at space-time position X)

4-dimentional generalizaton of the Poisson-bracket:

$$\diamond\{F_1\}\{F_2\} := \frac{1}{2}\left(\frac{\partial F_1}{\partial X_\mu}\frac{\partial F_2}{\partial P^\mu} - \frac{\partial F_1}{\partial P_\mu}\frac{\partial F_2}{\partial X^\mu}\right)$$

- Life time $\tau = \frac{\hbar c}{\Gamma}$

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From Kadanoff-Baym equations to transport equations

After the first order gradient expansion of the Wigner transformed Kadanoff-Baym equations and separation into the real and imaginary parts one gets:

1. Generalized transport equations:

drift term — Vlasov — backflow term — collision term = 'loss' term - 'gain'

$$\underbrace{\diamond \{ P^2 - M_0^2}_{\text{drift term}} - \underbrace{Re\Sigma_{XP}^{ret} \} \{ S_{XP}^{<} \}}_{\text{Vlasov}} - \underbrace{\diamond \{ \Sigma_{XP}^{<} \} \{ ReS_{XP}^{ret} \}}_{\text{backflow term}} = \frac{i}{2} [\underbrace{\Sigma_{XP}^{>} S_{XP}^{<}}_{\text{'loss' term}} - \underbrace{\Sigma_{XP}^{<} S_{XP}^{>}}_{\text{'gain'}}]$$

Backflow term incorporates the off-shell behavior in the particle propagation
! vanishes in the quasiparticle limit

2. Generalized mass-shell equations:

$$[P^2 - M_0^2 - Re\Sigma_{XP}^{ret}] S_{XP}^{<} - \Sigma_{XP}^{<} ReS_{XP}^{ret} = \frac{1}{2} \diamond \{ \Sigma_{XP}^{<} \} \{ A_{XP} \} - \frac{1}{2} \diamond \{ \Gamma_{XP} \} \{ S_{XP}^{<} \}$$

! Eqs. (1) and (2) are not fully consistent → differ by higher order gradient terms since (1) contains ReS_{XP}^{ret} in the backflow term

4-dimentional generalizaton of the Poisson-bracket:

$$\diamond \{ F_1 \} \{ F_2 \} := \frac{1}{2} \left(\frac{\partial F_1}{\partial X_\mu} \frac{\partial F_2}{\partial P^\mu} - \frac{\partial F_1}{\partial P_\mu} \frac{\partial F_2}{\partial X^\mu} \right)$$

From Kadanoff-Baym equations to transport equations

The Botermans-Malfliet solution (1990):

- separate spectral information from occupation density:

$$i\,S^{<}_{XP} \;=\; N_{XP}\,A_{XP}\,, \qquad i\,S^{>}_{XP} \;=\; (\,1 \,+\, N_{XP}\,)\,A_{XP}$$

$$i\,\Sigma^{<}_{XP} \;=\; N^{\Sigma}_{XP}\,\Gamma_{XP}\,, \qquad i\,\Sigma^{>}_{XP} \;=\; (\,1 \,+\, N^{\Sigma}_{XP}\,)\,\Gamma_{XP}$$

N -number distribution
A - spectral function
Γ- width of spectral function = reaction rate of particle (at phase-space position XP)

Greens function $S^{<}$ characterizes the number of particles (*N*) and their properties (*A* – spectral function)

- rewrite $\Sigma^{<}_{XP} \;=\; -\,i\,\Gamma_{XP}\,N^{\Sigma}_{XP} \;=\; -\,i\,\underline{\Gamma_{XP}\,N_{XP}} \;+\; C_{XP}$ non-equilibrium corrections

with $C_{XP} \;=\; -\,i\,\underline{\Gamma_{XP}}\,(\,N^{\Sigma}_{XP} \,-\, \underline{N_{XP}}\,) \;=\; i\,(\,\Sigma^{<}_{XP}\,S^{>}_{XP} \,-\, \Sigma^{>}_{XP}\,S^{<}_{XP}\,)\,A^{-1}_{XP}$

→ ‚correction term' = collision term $/A_{XP}$
of 2nd gradient orders → have to be omitted for consistency !

→ as a consequence: $N^{\Sigma} \to N\,, \quad \Sigma^{<} \to S^{<}\,\Gamma/A$

→ **Generalized transport equations can be written:**

$$\boxed{\begin{aligned} &A_{XP}\,\Gamma_{XP}\Big[\diamond\,\{\,P^2 - M_0^2 - Re\Sigma^{ret}_{XP}\,\}\,\{\,S^{<}_{XP}\,\} \,-\, \frac{1}{\Gamma_{XP}}\,\diamond\,\{\,\Gamma_{XP}\,\}\,\{\,(\,P^2 - M_0^2 - Re\Sigma^{ret}_{XP}\,)\,S^{<}_{XP}\,\}\Big] \\ &\qquad = \; i\,[\,\Sigma^{>}_{XP}\,S^{<}_{XP} \,-\, \Sigma^{<}_{XP}\,S^{>}_{XP}\,] \end{aligned}}$$

! now consistent in gradient order with the mass-shell equation ! → used in PHSD

General testparticle off-shell equations of motion

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- Employ testparticle Ansatz for the real valued quantity $i\, S^<_{XP}$ -

$$\boxed{F_{XP} = A_{XP} N_{XP} = i\, S^<_{XP} \sim \sum_{i=1}^{N} \delta^{(3)}(\vec{X} - \vec{X}_i(t))\ \delta^{(3)}(\vec{P} - \vec{P}_i(t))\ \delta(P_0 - \epsilon_i(t))}$$

insert in generalized transport equations and determine equations of motion !

➔ General testparticle ‚Cassing-Juchem off-shell equations of motion'
for the time-like particles:

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[2\,\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)} \right],$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} Re\Sigma^{ret}_i + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)} \right],$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial Re\Sigma^{ret}_{(i)}}{\partial t} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t} \right],$$

with $F_{(i)} \equiv F(t, \vec{X}_i(t), \vec{P}_i(t), \epsilon_i(t))$

$$C_{(i)} = \frac{1}{2\epsilon_i} \left[\frac{\partial}{\partial \epsilon_i} Re\Sigma^{ret}_{(i)} + \frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial}{\partial \epsilon_i} \Gamma_{(i)} \right]$$

Note: the common factor $1/(1-C_{(i)})$ can be absorbed in an ‚eigentime' of particle (i) !

Limiting cases

- **$\Gamma(X,P) = \Gamma(X)$ - width depends only on space-time X:**

  P = $(P_0, \vec{P})$

  use M^2 as an independent variable $M^2 = P^2 - Re\Sigma^{ret}$

  and fix P_0 by $P_0^2 = \vec{P}^2 + M^2 + Re\Sigma^{ret}_{X\vec{P}M^2}$ **=>**

  follows:

$$\boxed{\frac{dM_i^2}{dt} = \frac{M_i^2 - M_0^2}{\Gamma_{(i)}} \frac{d\Gamma_{(i)}}{dt}}$$

i.e. the deviation of M_i^2 from the pole mass (squared) M_0^2 scales with Γ_i !

On-shell limit

- $\Gamma(X,P) \rightarrow 0$

$$A_{XP} = \frac{\Gamma_{XP}}{(P^2 - M_0^2 - Re\Sigma^{ret}_{XP})^2 + \Gamma^2_{XP}/4}$$

quasiparticle approximation :

$$A_{XP} = 2\, p\, \delta(P^2 - M_0^2)$$

- **$\Gamma(X,P)$ such that**

$$\nabla_X \Gamma = 0 \quad \text{and} \quad \nabla_P \Gamma = 0$$

E.g.: Γ = const

$\Gamma = \Gamma_{vacuum}$ (M)

$\Leftrightarrow$

‚Vacuum' spectral function with constant or mass dependent width Γ:
i.e. spectral function A_{XP} does NOT change the shape (and pole position) during propagation through the medium

$\Downarrow$

Backflow term - which incorporates the off-shell behavior in the particle propagation - vanishes !

$$\frac{d\vec{X}_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[2\,\vec{P}_i + \vec{\nabla}_{P_i} Re\Sigma^{ret}_{(i)} + \cancel{\frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{P_i} \Gamma_{(i)}} \right],$$

$$\frac{d\vec{P}_i}{dt} = -\frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\vec{\nabla}_{X_i} Re\Sigma^{ret}_i + \cancel{\frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \vec{\nabla}_{X_i} \Gamma_{(i)}} \right],$$

$$\frac{d\epsilon_i}{dt} = \frac{1}{1 - C_{(i)}} \frac{1}{2\epsilon_i} \left[\frac{\partial Re\Sigma^{ret}_{(i)}}{\partial t} + \cancel{\frac{\epsilon_i^2 - \vec{P}_i^2 - M_0^2 - Re\Sigma^{ret}_{(i)}}{\Gamma_{(i)}} \frac{\partial \Gamma_{(i)}}{\partial t}} \right],$$

$\Rightarrow$ **Hamiltons equation of motion (independent on Γ) $\rightarrow$ BUU limit !**

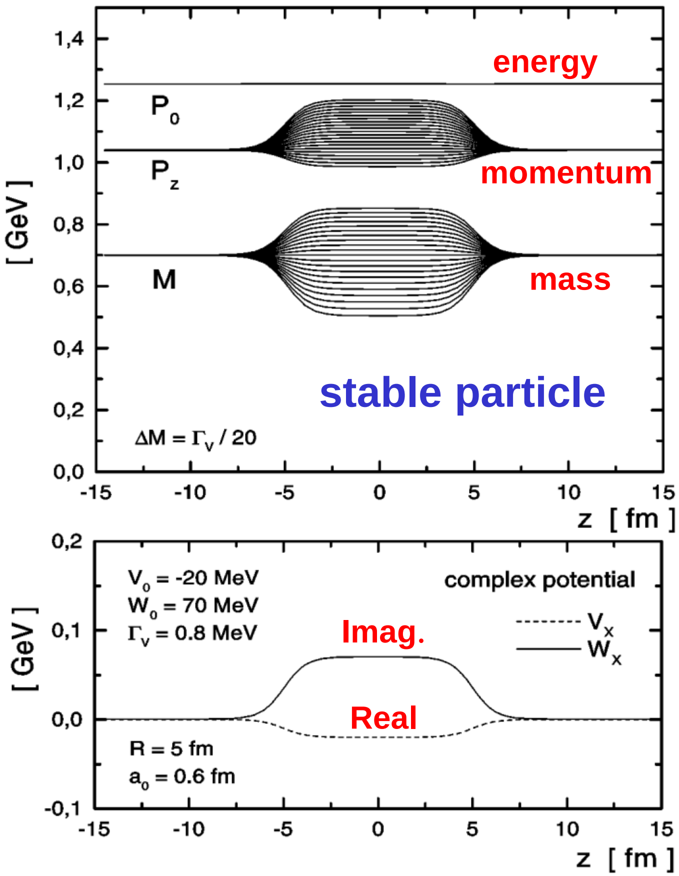
W. Cassing, Eur. Phys. J. ST 168 (2009) 3

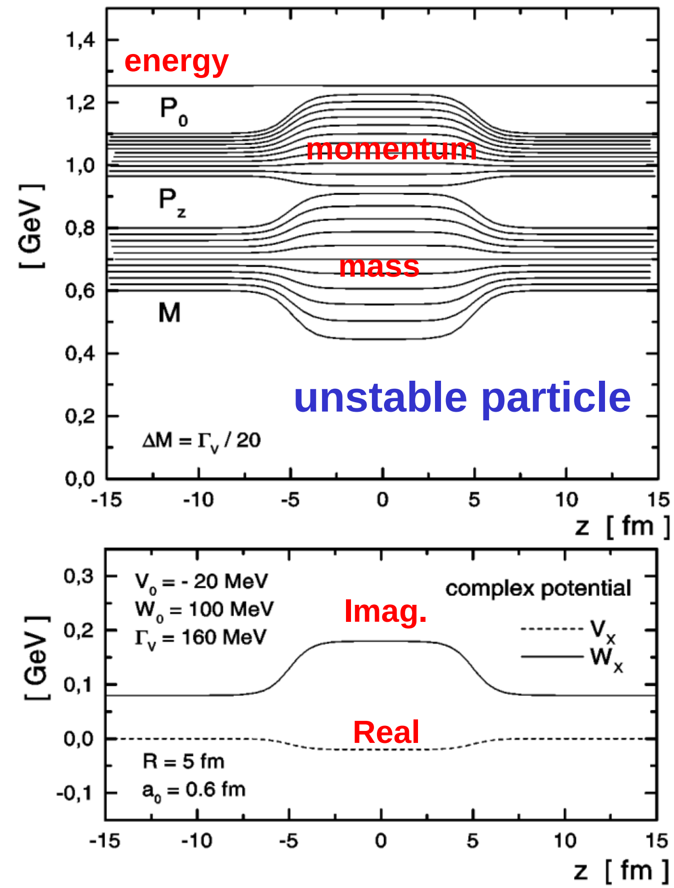
Model cases

Propagation of stable (left) and unstable (right) particles in complex potential with real part (atractive) and strong negative imaginary part.

$$Re\Sigma^{ret} - \frac{i}{2}\Gamma \;=\; \frac{V(P_0,\vec{P})}{1+\exp\{(|\vec{r}|-R)/a_0\}} \;-\; i\left(\frac{W(P_0,\vec{P})}{1+\exp\{(|\vec{r}|-R)/a_0\}} + \frac{\Gamma_V}{2}\right)$$

$$V(P_0,\vec{P}) \;=C_V\,\frac{\Lambda_V^2}{\Lambda_V^2-(P_0^2-\vec{P}^2)}\,, \qquad W(P_0,\vec{P}) \;=C_W\,\frac{\Lambda_W^2}{\Lambda_W^2-(P_0^2-\vec{P}^2)}$$





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Remarks on mean-field potential in off-shell transport models

- **Many-body theory:** Interacting relativistic particles have a complex self-energy:

$$\boxed{\Sigma^{ret}_{XP} = Re\,\Sigma^{ret}_{XP} + i\,Im\,\Sigma^{ret}_{XP}}$$

The neg. imaginary part $\Gamma_{XP} = -Im\,\Sigma^{ret}_{XP} = 2p_0\Gamma$ is related via $\Gamma = \Gamma_{coll} + \Gamma_{dec}$ to the inverse livetime of the particle $\tau \sim 1/\Gamma$.

- The collision width Γ_{coll} is determined from the loss term of the collision integral I_{coll}

$$\boxed{-I_{coll}(loss) = \Gamma_{coll}(X,\vec{P},M^2)N_{X\vec{P}\,M^2}}$$

- By dispersion relation (Kramers–Kronig relation) we get a contribution to the real part of self-energy:

$$\boxed{Re\,\Sigma^{ret}_{XP}(p_0) = P\int_0^\infty dq\,\frac{Im\,\Sigma^{ret}_{XP}(q)}{(q-p_0)}}$$

which gives a mean-field potential U_{XP} via: $\boxed{Re\,\Sigma^{ret}_{XP}(p_0) = 2p_0 U_{XP}}$

➔ The complex self-energy relates in a self-consistent way to the self-generated mean-field potential and collision width (inverse lifetime)

‚On-shell' transport models

Basic concept of the ‚on-shell' transport models (VUU, BUU, QMD, SMASH etc.):

1) Transport equations = first order gradient expansion of the Wigner transformed Kadanoff-Baym equations
2) Quasiparticle approximation or/and vacuum spectral functions :
$A(X,P) = 2\,p\,\delta(p^2-M^2)$ $\qquad A_{vacuum}(M)$

- For each particle species i ($i = N, R, Y, \pi, \rho, K, \ldots$) the phase-space density f_i follows the BUU transport equations

$$\left(\frac{\partial}{\partial t} + \left(\nabla_{\vec{p}} U\right) \nabla_{\vec{r}} - \left(\nabla_{\vec{r}} U\right) \nabla_{\vec{p}}\right) f_i(\vec{r}, \vec{p}, t) = I_{coll}(f_1, f_2, \ldots, f_M)$$

- with collision terms I_{coll} describing elastic and inelastic hadronic reactions:
  baryon-baryon, meson-baryon, meson-meson, formation and decay of baryonic and mesonic resonances, string formation and decay (for inclusive particle production: BB → X , mB →X, mm →X, X =many particles)
- with propagation of particles in self-generated mean-field potential
  $U(p,\rho) \sim Re(\Sigma^{ret})/2p_0$

- Numerical realization – solution of classical equations of motion + Monte-Carlo simulations for test-particle interactions

Problems in the treatment of short-lived resonances in the on-shell semi-classical transport models

Problem:

dynamical changes of spectral function by propagation through the medium are NOT included in the ‚on-shell‘ semi-classical transport equations !

$\Rightarrow$ the resonance spectral function can be changed only due to explicit collisions with other particles in ‚on-shell‘ semi-classical transport models !

Reason for the problem:

backflow term* is missing in the explicit ‚on-shell‘ dynamical equations since this backflow term vanishes in the on-shell limit, however, does NOT vanish in the off-shell limit (i.e. becomes very important for the dynamics of broad resonances)!

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* **Generalized transport equations**

$$\underbrace{\diamond\{P^2 - M_0^2}_{\text{drift term}} - \underbrace{Re\Sigma_{XP}^{ret}\}\{S_{XP}^<}_{\text{Vlasov term}}\} - \underbrace{\diamond\{\Sigma_{XP}^<\}\{ReS_{XP}^{ret}\}}_{\text{backflow term}}$$

$$= \frac{i}{2}\,[\underbrace{\Sigma_{XP}^> S_{XP}^<}_{\text{‚loss‘ term}} - \underbrace{\Sigma_{XP}^< S_{XP}^>}_{\text{‚gain‘ term}}],$$

Operator <> - 4-dimentional generalizaton of the Poisson-bracket

$$\diamond\{F_1\}\{F_2\} := \frac{1}{2}\left(\frac{\partial F_1}{\partial X_\mu}\frac{\partial F_2}{\partial P^\mu} - \frac{\partial F_1}{\partial P_\mu}\frac{\partial F_2}{\partial X^\mu}\right)$$

W. Cassing et al., NPA 665 (2000) 377

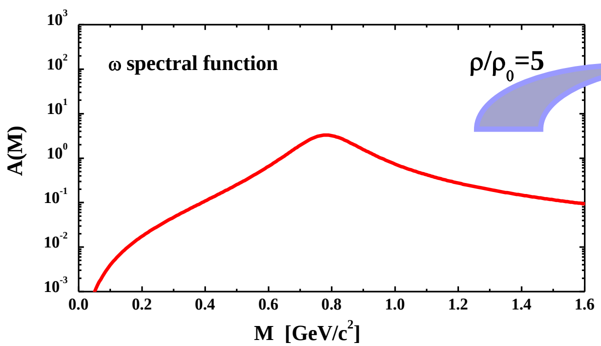
Short-lived resonances in semi-classical transport models

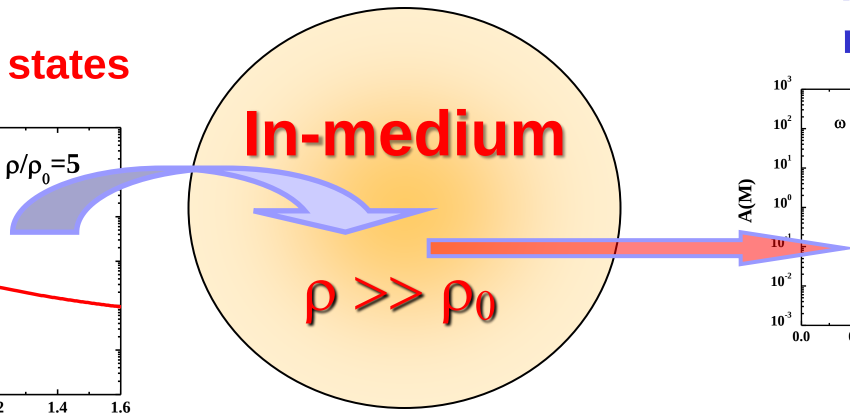
Spectral function:

$$A(M,p,\rho) = \frac{2}{\pi}\,\frac{M^2\Gamma_{tot}(M,p,\rho)}{(M^2 - M_0^2 - \mathrm{Re}\Sigma^{ret}) + \left(M\Gamma_{tot}(M,p,\rho)\right)^2},$$

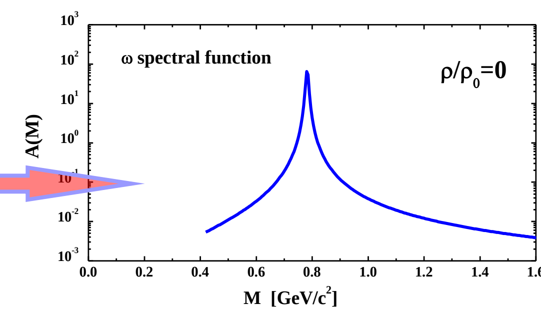
width $\Gamma \sim -\mathrm{Im}\,\Sigma^{ret}/M$

In-medium:
production of broad states

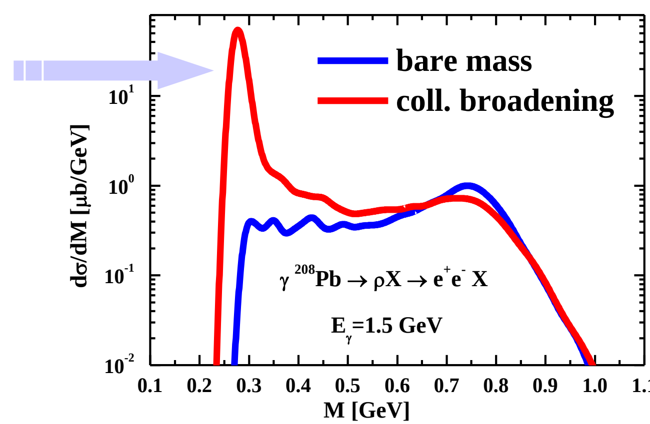




Vacuum ($\rho = 0$)
narrow states



Example :
ρ-meson propagation through the medium within on-shell BUU model
Problem: broad in-medium spectral function does not become on-shell in vacuum in ‚on-shell' transport models!



BUU: M. Effenberger et al, PRC60 (1999)

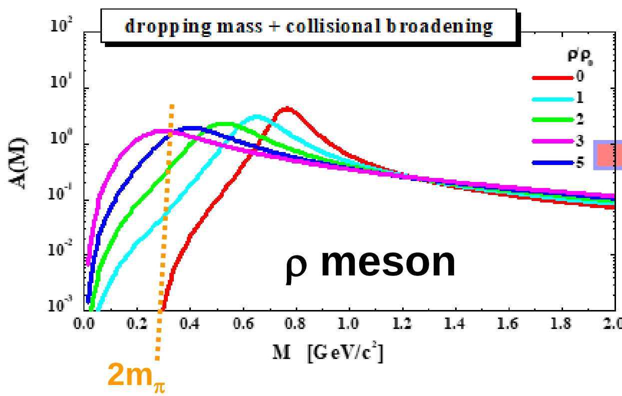
In-medium
$\rho \gg \rho_0$

Off-shell vs. on-shell transport dynamics

Time evolution of the mass distribution of ρ and ω mesons for central C+C collisions (b=1 fm) at 2 A GeV for dropping mass + collisional broadening scenario

$$A(M,p,\rho) = \frac{2}{\pi}\frac{M^2\Gamma_{tot}(M,p,\rho)}{(M^2 - M_0^2 - Re\Sigma^{ret}) + \left(M\Gamma_{tot}(M,p,\rho)\right)^2},$$

width $\Gamma \sim -Im\,\Sigma^{ret}/M$

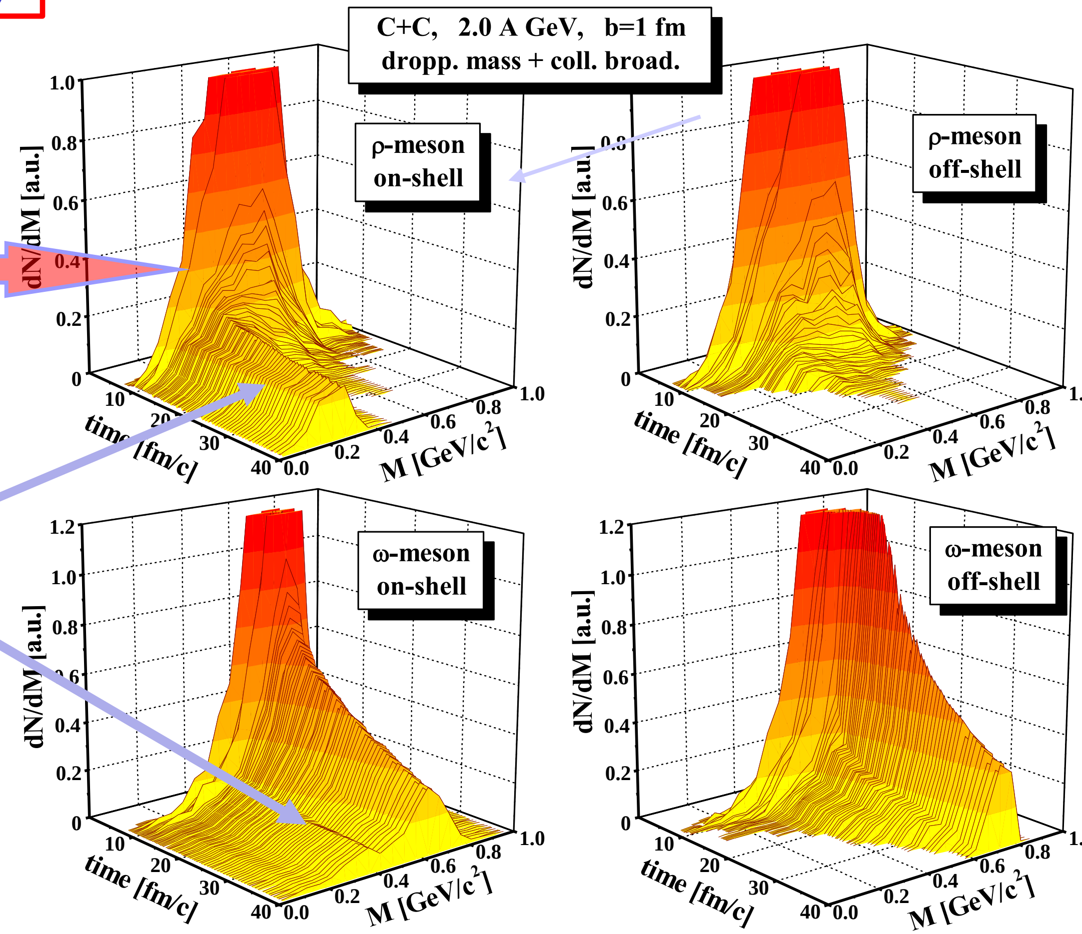


On-shell BUU:
low mass ρ and ω mesons live forever (and shine ‚fake' dileptons)!

The off-shell spectral function becomes on-shell in the vacuum dynamically by propagation through the medium!

On-shell

Off-shell



E.B. &W. Cassing, NPA 807 (2008) 214

Collision term in off-shell transport models

Collision term for reaction 1+2->3+4:

$$I_{coll}(X,\vec{P},M^2) = Tr_2 Tr_3 Tr_4 A(X,\vec{P},M^2) A(X,\vec{P}_2,M_2^2) A(X,\vec{P}_3,M_3^2) A(X,\vec{P}_4,M_4^2)$$

$$|G((\vec{P},M^2)+(\vec{P}_2,M_2^2) \rightarrow (\vec{P}_3,M_3^2)+(\vec{P}_4,M_4^2))|^2_{\mathcal{A},\mathcal{S}} \; \delta^{(4)}(P+P_2-P_3-P_4)$$

$$[\, N_{X\vec{P}_3M_3^2}\, N_{X\vec{P}_4M_4^2}\, \bar{f}_{X\vec{P}M^2}\, \bar{f}_{X\vec{P}_2M_2^2} \;-\; N_{X\vec{P}M^2}\, N_{X\vec{P}_2M_2^2}\, \bar{f}_{X\vec{P}_3M_3^2}\, \bar{f}_{X\vec{P}_4M_4^2} \,]$$

'gain' term 'loss' term

with $\bar{f}_{X\vec{P}M^2} = 1 + \eta\, N_{X\vec{P}M^2}$ and $\eta = \pm 1$ for bosons/fermions, respectively.

The trace over particles 2,3,4 reads explicitly

for fermions

$$Tr_2 = \sum_{\sigma_2,\tau_2} \frac{1}{(2\pi)^4} \int d^3P_2 \frac{dM_2^2}{2\sqrt{\vec{P}_2^2 + M_2^2}}$$

for bosons

$$Tr_2 = \sum_{\sigma_2,\tau_2} \frac{1}{(2\pi)^4} \int d^3P_2 \frac{dP_{0,2}^2}{2}$$

additional integration

The transport approach and the particle spectral functions are fully determined once the in-medium transition amplitudes G are known in their off-shell dependence!

In-medium transition rates: G-matrix approach

Need to know in-medium transition amplitudes G and their off-shell dependence

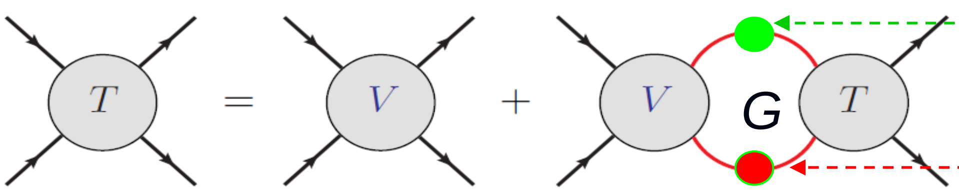
$$|G((\vec{P},M^2)+(\vec{P}_2,M_2^2)\rightarrow(\vec{P}_3,M_3^2)+(\vec{P}_4,M_4^2))|^2_{\mathcal{A},\mathcal{S}}$$

Coupled channel G-matrix approach

Transition probability :

$$\boxed{P_{1+2\rightarrow 3+4}(s)=\int d\cos(\theta)\ \frac{1}{(2s_1+1)(2s_2+1)}\sum_i\sum_\alpha\ G^\dagger G}$$

with G(p,ρ,T) - G-matrix from the solution of coupled-channel equations:



- **Meson selfenergy and spectral function**
- **Baryons: Pauli blocking and potential dressing**

- $\mathcal{T}_{ij}(\rho,T) = V_{ij} + V_{il}\ G_l(\rho,T)\ \mathcal{T}_{lj}(\rho,T)$

For strangeness:
D. Cabrera, L. Tolos, J. Aichelin, E.B., PRC C90 (2014) 055207; W. Cassing, L. Tolos, E.B., A. Ramos, NPA727 (2003) 59

Brueckner theory

Transition rate for the process *1+2→3+4* in the medium follows from many-body Brueckner theory:

$$[G^+G]_{1+2\to 3+4}\;\delta^4(\Pi+\Pi_2-\Pi_3-\Pi_4)$$

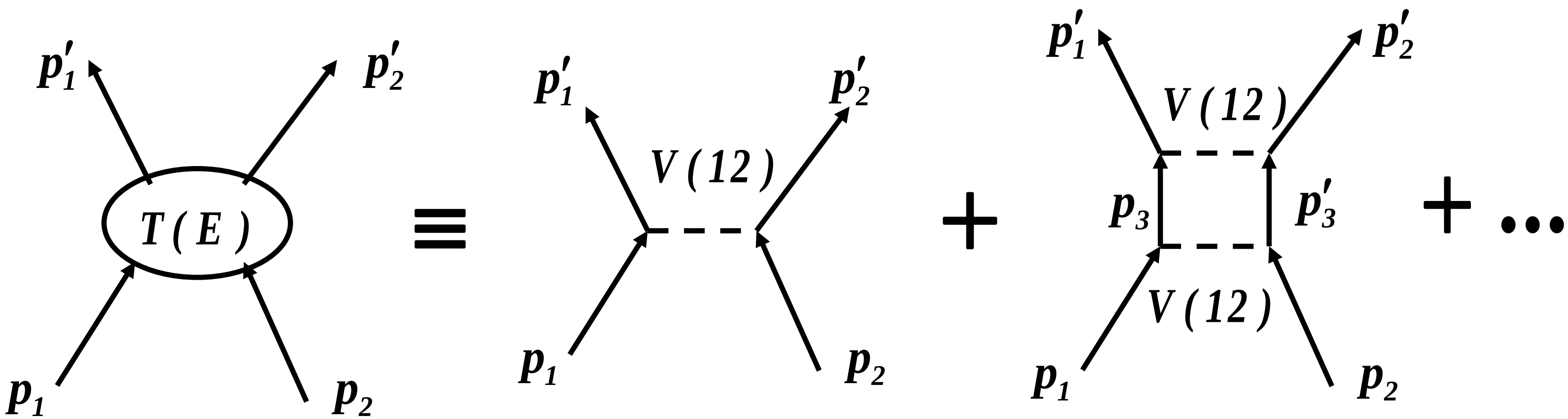
1) <u>2-body scattering in vacuum:</u>

Scattering amplitude:

$$T(E)=V+V\frac{1}{E-t(1)-t(2)+i\eta}T(E)$$

with the hamiltonian:

$$H=\sum_{i=1}^{A}t(i)+\frac{1}{2}\sum_{i<j}V(ij)$$



‚ladder‘ resummation

Brueckner theory

2) <u>2-body scattering in the medium</u>:

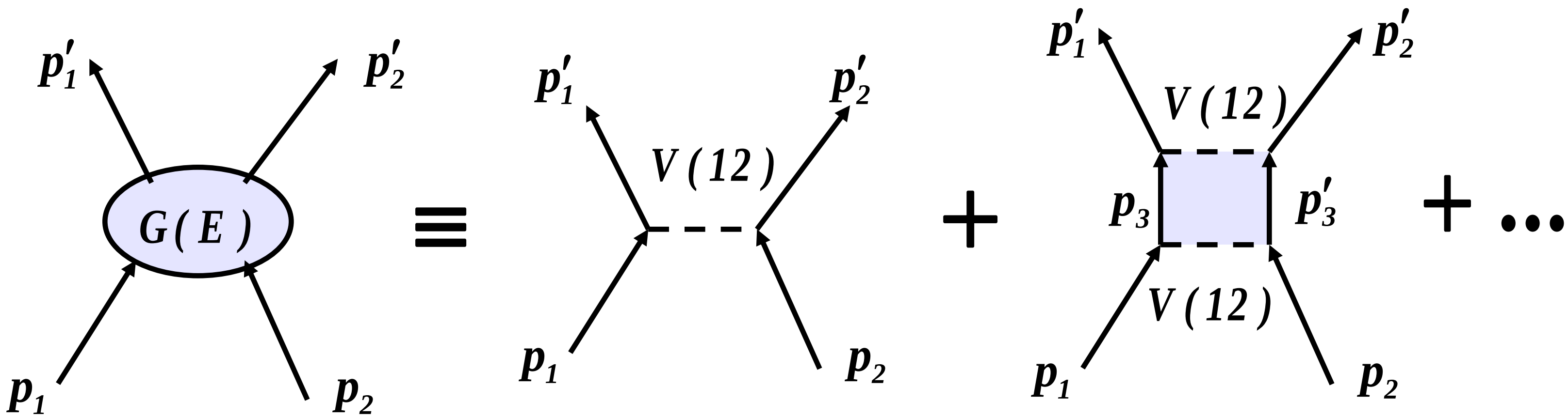
Scattering amplitude → from Brueckner theory:

$$\boxed{G(E) = V + V\frac{1}{E - h(1) - h(2) + i\eta}\underbrace{(1 - n_3 - n_3')}_{\text{Pauli-blocking}}G(E)}$$

n_3 – occupation number

with single-particle hamiltonian: $h(1) = t(1) + U^{MF}(1)$

Note: vacuum case : $h(1) = t(1)$ and $n_3 = n_3' = 0 \Rightarrow G-matrix \to T-matrix$



Propagation between scattering *V(12)* with mean field hamiltonian *h(1), h(2)*

! only allowed if intermediate states *3,3'* are not accupied !

Example: Transition probabilities for $\pi Y \leftrightarrow K^- p$ ($Y = \Lambda,\Sigma$)

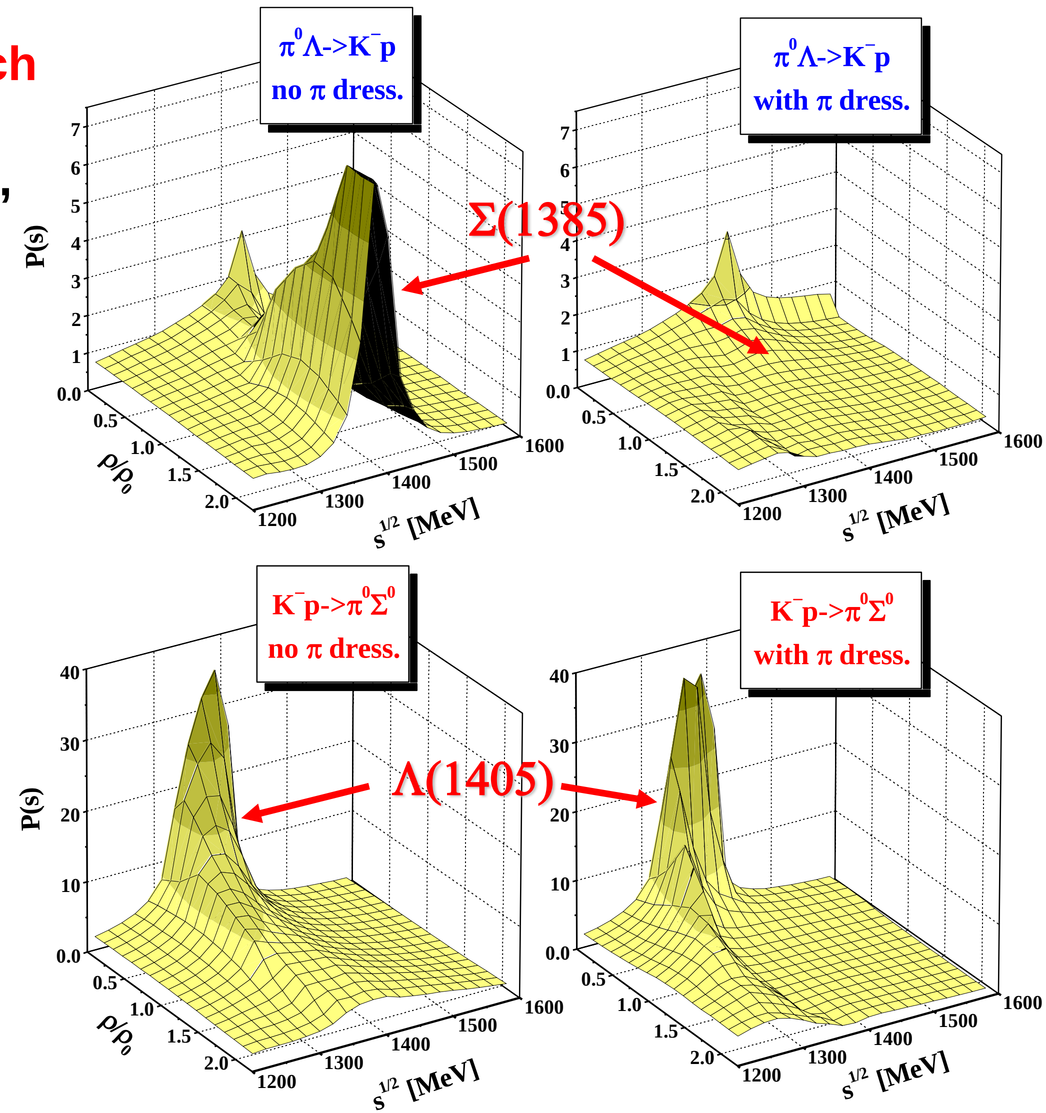
L. Tolos et al., NPA 690 (2001) 547

Coupled-channel G-matrix approach provides in-medium transition probabilities for different channels, e.g. $\pi Y \leftrightarrow K^- p$ ($Y = \Lambda,\Sigma$)

- **With pion dressing:** $\Lambda(1405)$ and $\Sigma(1385)$ melt away with baryon density

- **K^- absorption/production from πY collisions are strongly suppressed in the nuclear medium**

! πY is the dominant channel for K^- production in heavy-ion collisions !



W. Cassing, L. Tolos, E.L.B., A. Ramos, NPA 727 (2003) 59

KB dynamics for strongly interacting systems

In-medium effects (on hadronic or partonic levels!) = changes of particle properties in the hot and dense medium

Example: hadronic medium - vector mesons, strange mesons
QGP – ‚dressing' of partons

Many-body theory:
Strong interaction ➔ large width = short life-time
➔ broad spectral function ➔ quantum object

- **KB equations describe the dynamics of broad strongly interacting quantum states**
➔ transport theory for strongly interaction systems

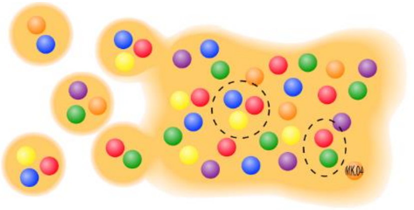
❑ **semi-classical BUU**

first order gradient expansion of quantum Kadanoff-Baym equations

❑ **generalized off-shell transport equations based on Kadanoff-Baym dynamics**

➔ Numerical realization: transport codes

Goal: microscopic transport description of the partonic and hadronic phase of HIC



Problems:

- ❑ How to model a QGP phase in line with lQCD data?
- ❑ How to solve the hadronization problem?

Ways to go:

pQCD based models:

- QGP phase: pQCD cascade
- hadronization: quark coalescence

➔ AMPT, HIJING, BAMPS

‚Hybrid‘ models:

- QGP phase: hydro with QGP EoS
- hadronic freeze-out: after burner - hadron-string transport model

➔ Hybrid-UrQMD

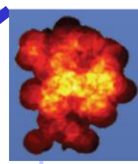
- microscopic transport description of the partonic and hadronic phase in terms of strongly interacting dynamical quasi-particles and off-shell hadrons

➔ PHSD

Dynamical models for HIC

Macroscopic

hydro-models:
- description of QGP and hadronic phase by hydrodynamical equations for fluid
- assumption of local equilibrium
- EoS with phase transition from QGP to HG
- initial conditions (e-b-e, fluctuating)



ideal
(Jyväskylä,SHASTA, TAMU, …)

viscous
(Romachkke,(2+1)D VISH2+1, (3+1)D MUSIC,…)

fireball models:
- no explicit dynamics: parametrized time evolution (TAMU)

‚Hybrid'
QGP phase: hydro with QGP EoS
- hadronic freeze-out: after burner - hadron-string transport model

(‚hybrid'-UrQMD, EPOS, …)

Microscopic

Non-equilibrium microscopic transport models – based on many-body theory

Hadron-string models
(UrQMD, IQMD, HSD, QGSM, SMASH …)

Partonic cascades
pQCD based
(Duke, BAMPS, …)

Parton-hadron models:
- QGP: pQCD based cascade
- massless q, g
- hadronization: coalescence

(AMPT, HIJING)

PHSD
- QGP: lQCD EoS
- massive quasi-particles (q and g with spectral functions) in self-generated mean-field
- dynamical hadronization
- HG: off-shell dynamics

(applicable for strongly interacting systems)

Useful literature

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