Non-equilibrium Dynamics of the Chiral/Deconfinement Phase Transition

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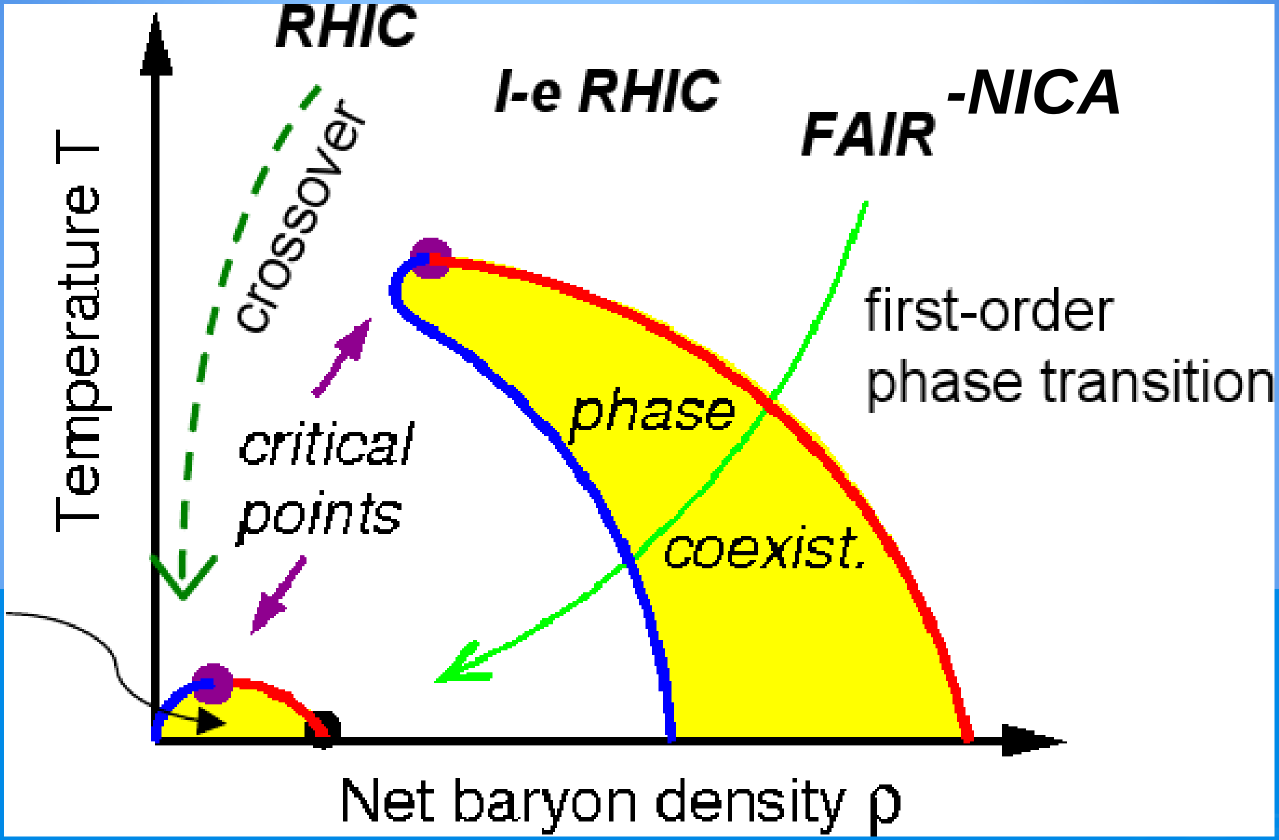
Phase diagram of strongly-interacting matter



Such a phase diagram is still a beautiful dream! We hope that future FAIR-NICA experiments will help to establish what is the reality.

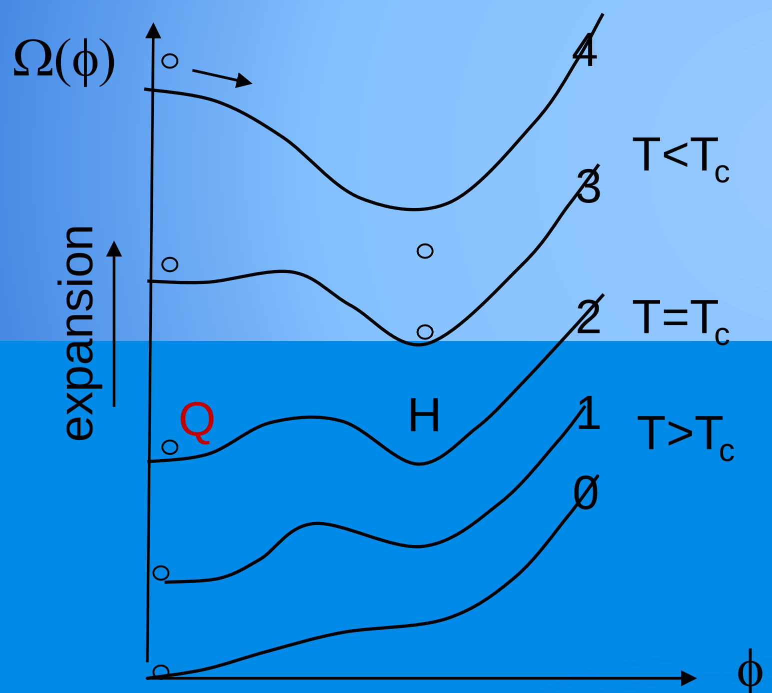
Effects of fast dynamics

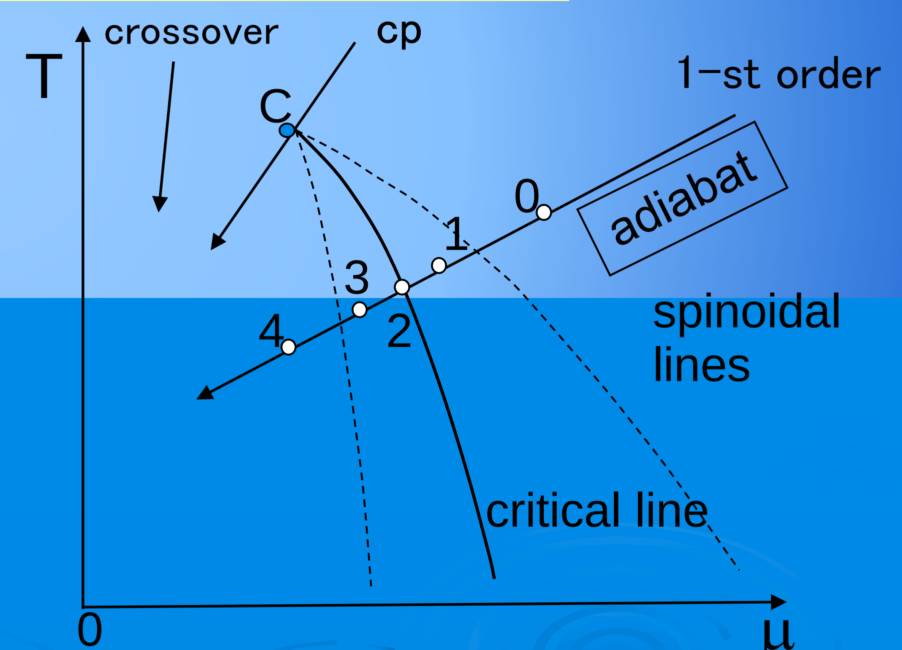
Effective thermodynamic potential for a 1st order transition

$$\Omega(\phi;T,\mu)=\Omega_0(T,\mu)+\frac{a}{2}\phi^2+\frac{b}{4}\phi^4+\frac{c}{6}\phi^6$$

a,b,c are functions of T and μ

Equilibrium ϕ is determined by $\frac{\partial\Omega}{\partial\phi}=0\Rightarrow P=-\Omega(\phi_{eq})$





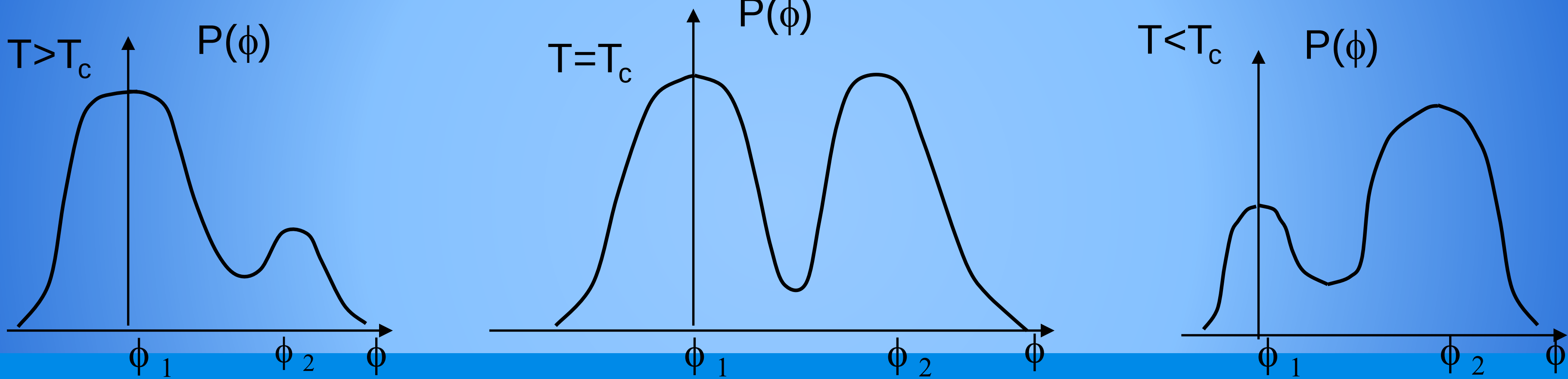
In rapidly expanding system 1-st order transition is delayed until the barrier between two competing phases disappears - spinodal decomposition

I. Mishustin, Phys. Rev. Lett. 82 (1999) 4779; Nucl. Phys. A681 (2001) 56

Equilibrium fluctuations of order parameter in 1st order phase transition

Probability distribution for fluctuations

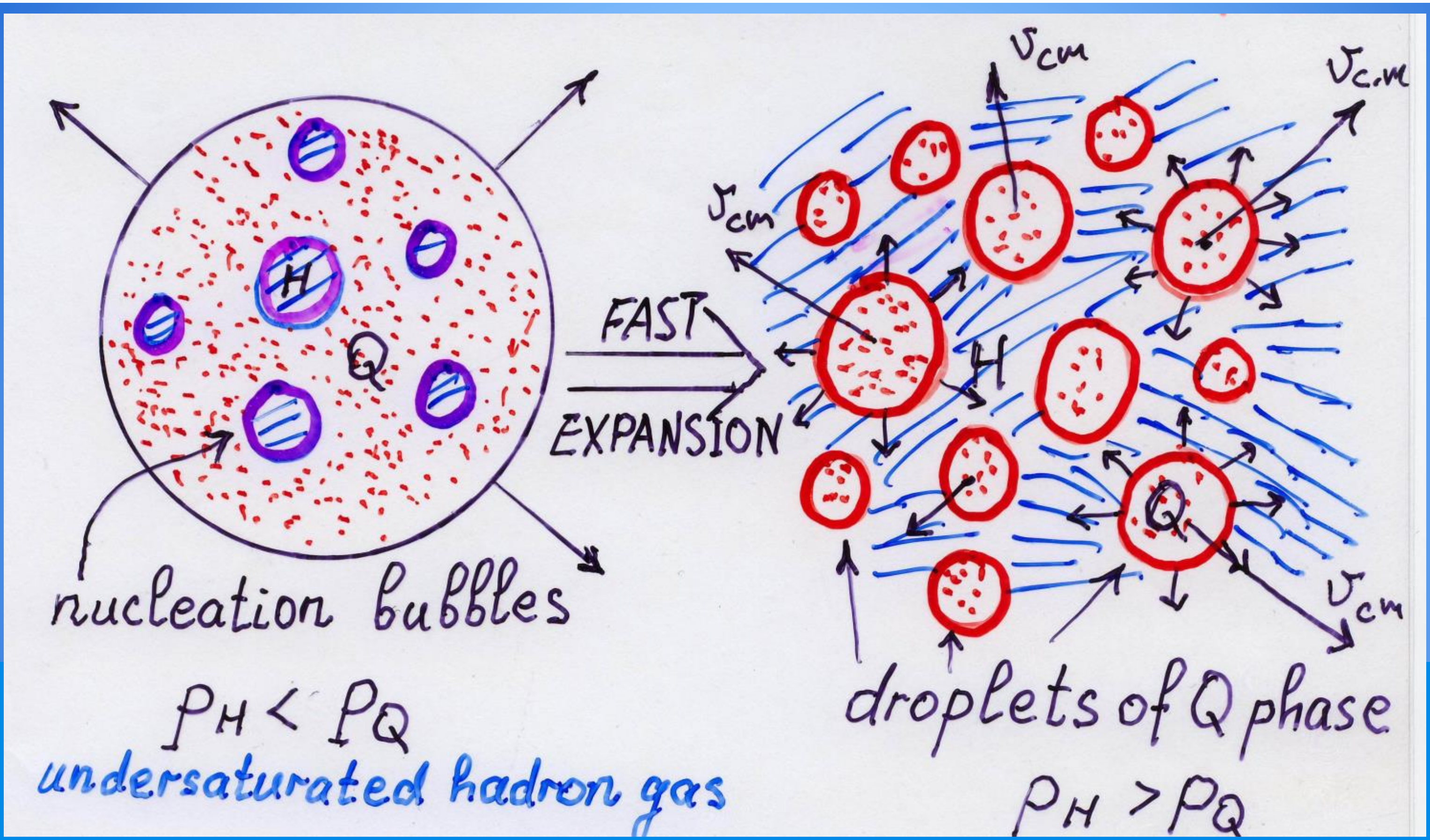
$$P(\delta\phi) \propto \exp\left(-\frac{\Delta\Omega(\delta\phi)V}{T}\right), \quad \delta\phi = \phi - \langle\phi\rangle$$



Phase I — Mixed phase — Phase II

- **In an equilibrated system fluctuations of the order parameter, i.e. Polyakov loop, should demonstrate bi-modal distributions (lattice calculations?);**
- **In a rapidly evolving system these fluctuations will be out of equilibrium;**
- **During supercooling process strong fluctuations may develop in the form of droplets** of a metastable phase.

Rapid expansion through a 1st order phase transition



The system is trapped in a metastable state until it enters the spinodal instability region, when Q phase becomes unstable and splits into droplets

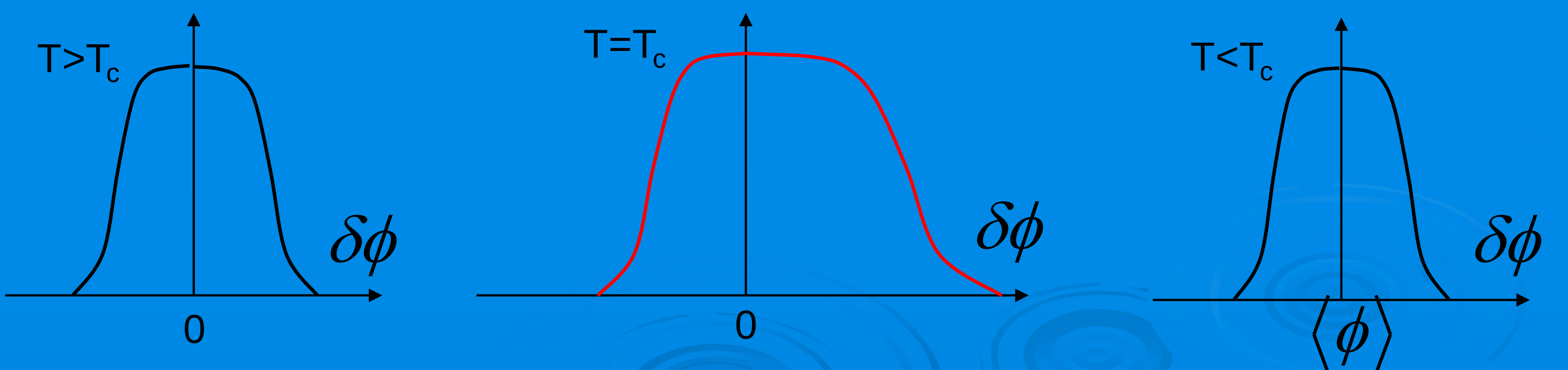
Csernai&Mishustin, 1995; Mishustin, 1999; Rafelski et al. 2000; Randrup, 2003; Peach&Stoecker, 2003; Stephanov, 2005, 2009; Steinheimer&Randrup 2013; Nahrgang, Herold, Mishustin, Bleicher, 2013-p.t.; Liang, Li, Song, 2016-p.t.. …

Evolution of equilibrium fluctuations in 2nd order phase transition

$$\Omega(\phi) = \frac{1}{2}a(T)\phi^2 + \frac{1}{2}b(\nabla\phi)^2 + \frac{\lambda}{4}\phi^4, \quad a(T) = a_0(T - T_c)$$

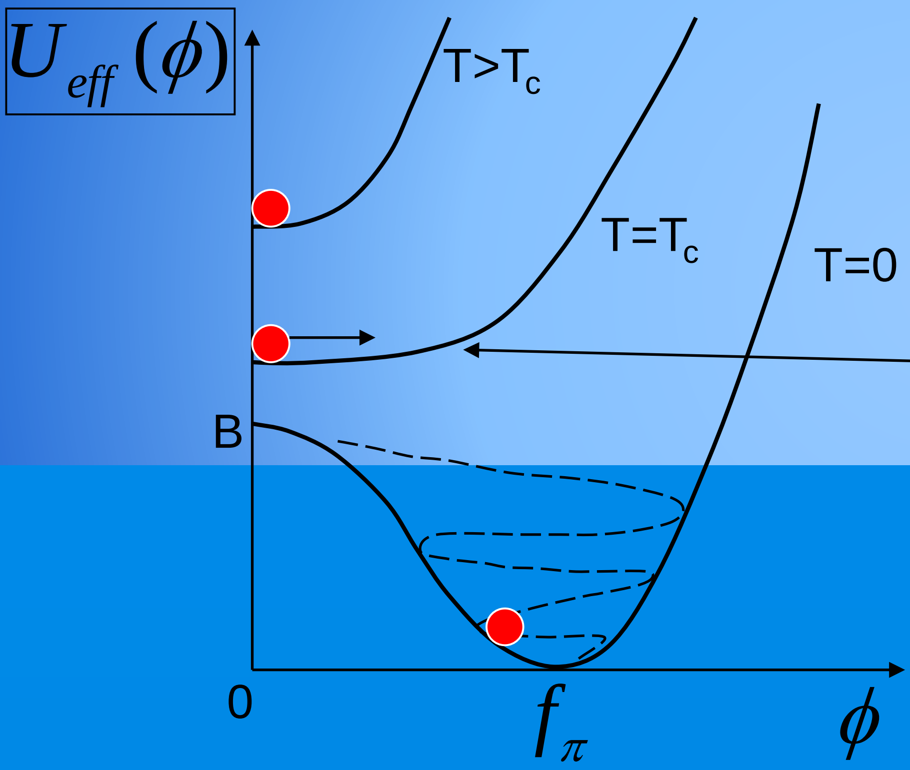
$$\langle\phi\rangle = \frac{a(T)}{\lambda}, \ T < T_c \ \text{and} \ \langle\phi\rangle = 0, \ T > T_c, \quad \delta\phi = \phi - \langle\phi\rangle$$

$$\text{Distribution of fluctuations} \quad P(\delta\phi) \sim \exp\left[-\frac{\Delta\Omega(\delta\phi)V}{T}\right]$$



In rapidly expanding system critical fluctuations have not sufficient time to develop

Critical slowing down in the 2nd order phase transition



Fluctuations of the order parameter evolve according to the relaxation equation

$$\frac{d\delta\phi}{dt} = -\gamma\frac{\partial\Omega}{\partial\phi} \approx -\frac{\delta\phi}{\tau_{\rm rel}}$$

In the vicinity of the critical point the relaxation time for the order parameter diverges - no restoring force

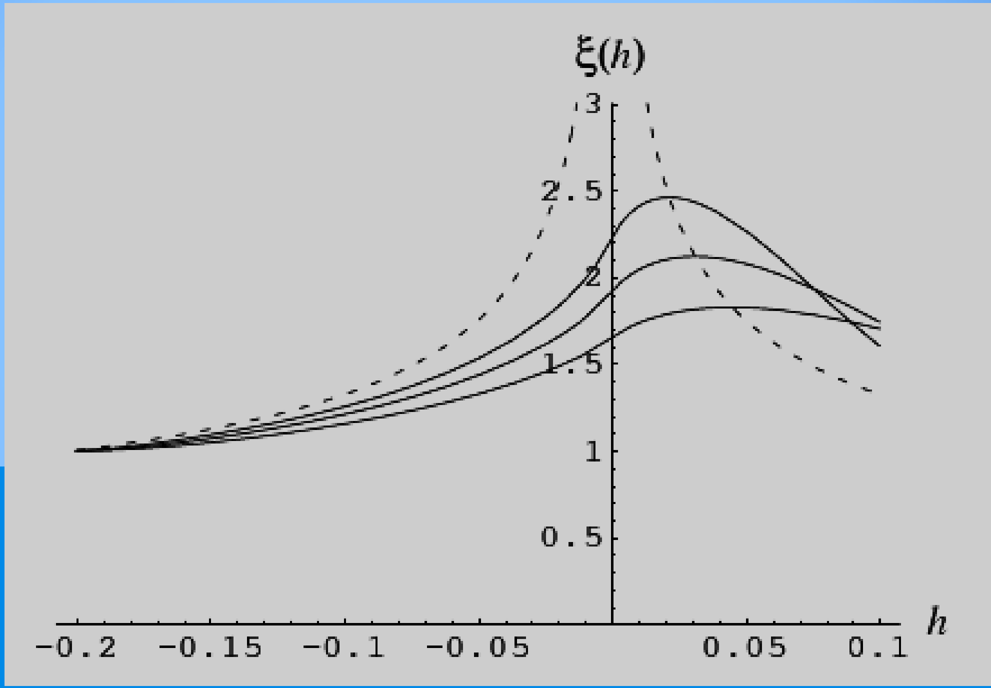
$$\tau_{\rm rel}(T) \sim \frac{1}{|T-T_c|^{\nu}} \to \infty, \quad \nu \sim 2$$

“Rolling down” from the top of the potential is similar to spinodal decomposition (Csernai&Mishustin 1995)

(Landau&Lifshitz, vol. X, Physical kinetics)

Critical slowing down 2

B. Berdnikov, K. Rajagopal, Phys. Rec. D61 (2000)



Critical fluctuations have not enough time to build up. One can expect only a factor 2 enhancement in the correlation length even for slow cooling rate, dT/dt=10 MeV/fm.

Simple model for chiral phase transition

Scavenius, Mocsy, Mishustin&Rischke, Phys. Rev. C64 (2001) 045202

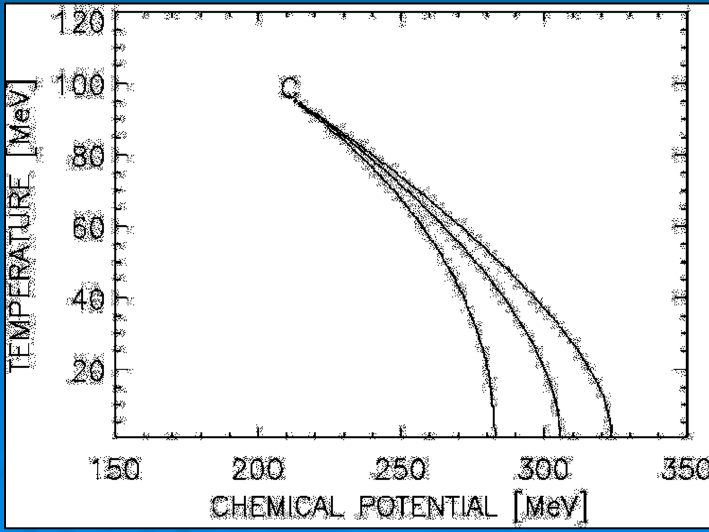
Linear sigma model (LσM) with constituent quarks

$$L = \bar{q}[i\gamma\partial - g(\sigma + i\gamma_5 \boldsymbol{\tau\pi})]q + \frac{1}{2}[\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \boldsymbol{\pi} \partial^\mu \boldsymbol{\pi}] - U(\sigma, \boldsymbol{\pi}),$$

$$U(\sigma, \vec{\pi}) = \frac{\lambda^2}{4}(\sigma^2 + \boldsymbol{\pi}^2 - v^2)^2 - H\sigma, \quad <\sigma>_{\rm vac} = f_\pi \to H = f_\pi m_\pi^2$$

Effective thermodynamic potential contains contributions of mean field σ and quark-antiquark fluid:

Phase diagram



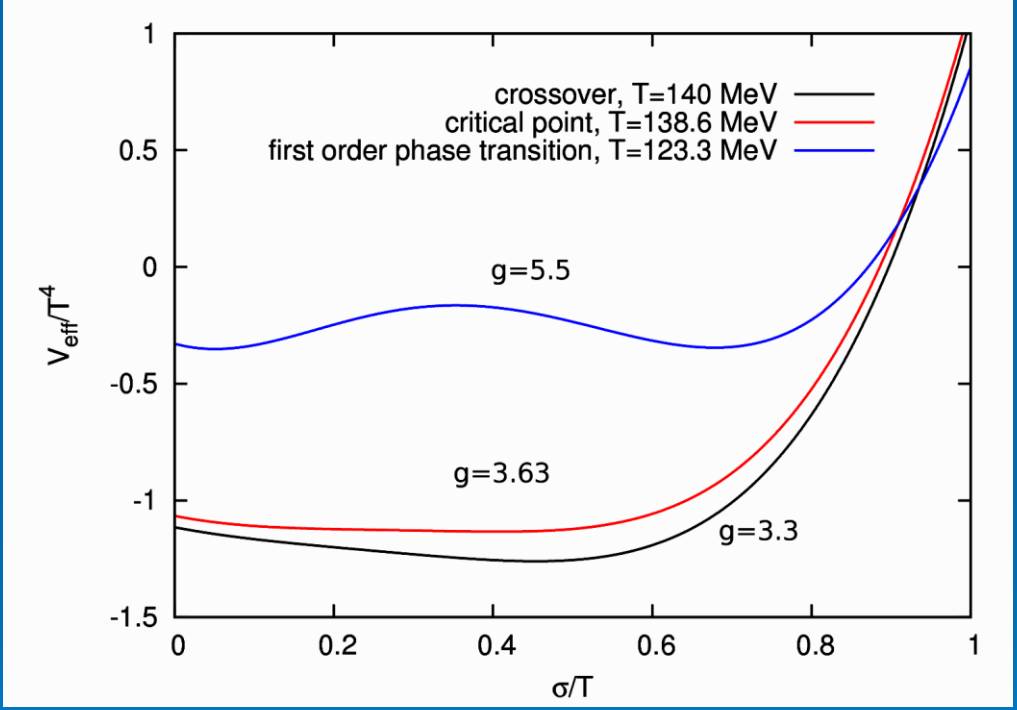
$$U_{\rm eff}(\sigma; T, \mu) = U(\sigma, \boldsymbol{\pi}) + \Omega_q(m; T, \mu)$$

$$m^2 = g^2(\sigma^2 + \boldsymbol{\pi}^2), \quad \boldsymbol{\pi} \approx 0$$

CO, 2nd and 1st order chiral transitions are obtained in T-μ plane.

Effective thermodynamic potential

$$\Omega_q(m;T,\mu) = -\nu_q T \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \left\{ \ln\left[1+\exp\left(\frac{\mu-\sqrt{m^2+p^2}}{T}\right)\right] + (\mu\to-\mu) \right\}, \quad \nu=2N_f N_c$$



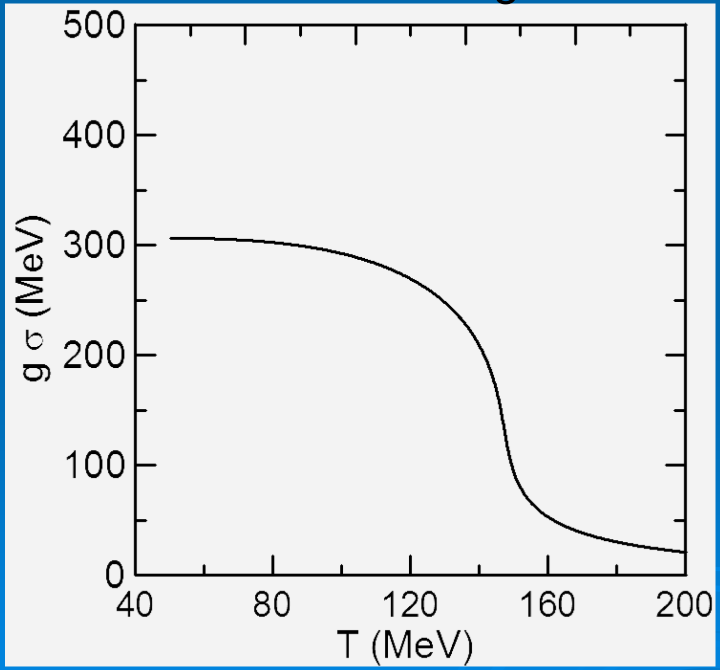
First we consider μ=0 system but tune the order of the chiral phase transition by changing the coupling g.

Equilibrium order parameter field

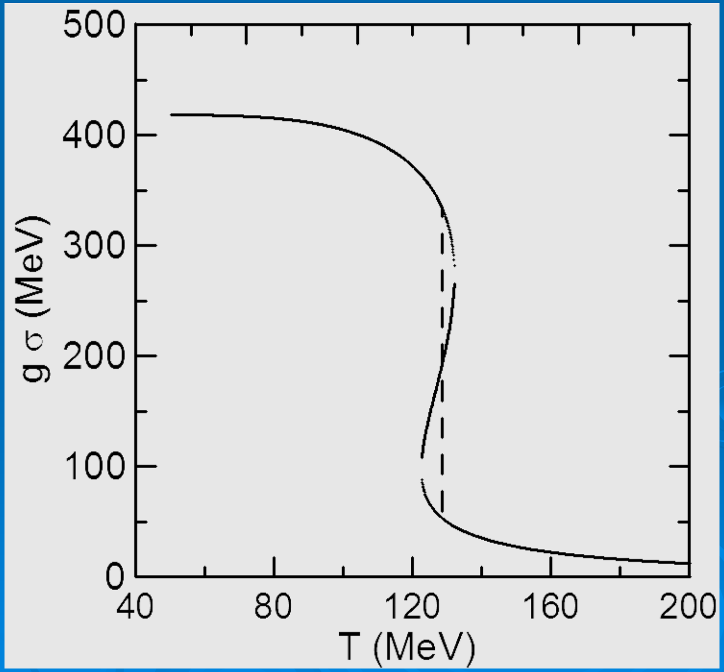
$$\lambda\left(\sigma^2 - v^2\right)\sigma + g\rho_s(\sigma) = 0,\ \rho_s(\sigma) = \frac{\partial \Omega_q(m;T,\mu)}{\partial \sigma}$$

$$m_\sigma^2(\tau) = \frac{\partial^2 U_{eff}}{\partial \sigma^2} = \lambda^2(3\sigma^2 - \sigma^2{}_0) + g\frac{\partial \rho_s}{\partial \sigma}$$

crossover g=3.3



1-st order g=4.5



Only 1 equilibrium solution at each T

3 solutions at 122 MeV<T<132 MeV
unstable states - spinodal instability

Non-equilibrium Chiral Fluid Dynamics

I.N. Mishustin, O. Scavenius, Phys. Rev. Lett. 83 (1999) 3134;
K. Paech, H. Stocker and A. Dumitru, Phys. Rev. C 68 (2003) 044907;
M. Nahrgang, C. Herold, S. Leupold, , C. Herold, M. Bleicher, Phys. Rev. C 84 (2011) 024912;
M. Nahrgang, C. Herold, S. Leupold, I. Mishustin, M. Bleicher, J. Phys. G40 055108.

Fluid is formed by constituent quarks and antiquarks which interact with the chiral field via quark effective mass $m = g\sigma$

CFD equations are obtained from the energy momentum conservation for the coupled system fluid+field

$$\partial_\nu (T_{\text{fluid}}^{\mu\nu} + T_{\text{field}}^{\mu\nu}) = 0 \Rightarrow \partial_\nu T_{\text{fluid}}^{\mu\nu} = -\partial_\mu T_{\text{field}}^{\mu\nu} \equiv S^\nu$$

$$S^\nu = -(\partial^2\sigma + \frac{\partial U_{\text{eff}}}{\partial\sigma})\partial^\nu\sigma = (g\rho_s + \eta\partial_t\sigma)\partial^\nu\sigma$$

We solve generalized e. o. m. with friction (η) and noise (ξ):

$$\partial_\mu\partial^\mu\sigma + \frac{\partial U_{\text{eff}}}{\partial\sigma} + g < \bar{q}q > + \eta\partial_t\sigma = \xi$$

Langevin equation for the order parameter

$$< \xi(t,\vec{r}) >= 0, \quad < \xi(t,r)\xi(t',r') >= \frac{1}{V} m_\sigma \eta \delta(t-t')\delta(r-r')\coth\left(\frac{m_\sigma}{2T}\right)$$

Calculation of damping term

T.Biro and C. Greiner, PRL, 79. 3138 (1997)

M. Nahrgang, S. Leupold, C. Herold, M. Bleicher, PRC 84, 024912 (2011)

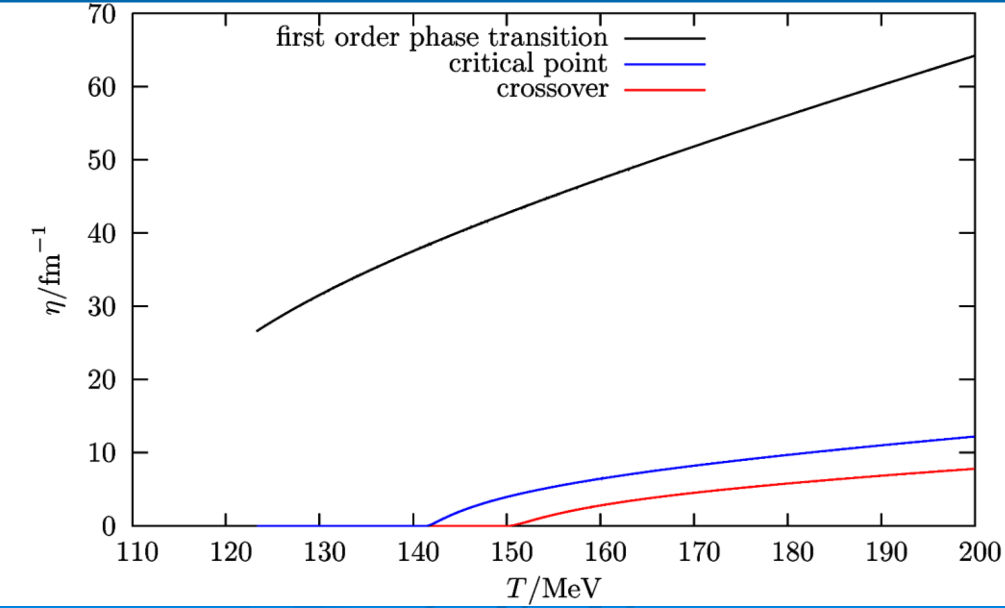
The damping is associated with the processes:

$$\sigma \to \bar{q}q, \ \sigma \to \pi\pi$$

It has been calculated using 2PI effective action

$$\eta = g^2 \frac{\nu_q}{\pi m_\sigma^2}\left[1 - 2n_F\left(\frac{m_\sigma}{2}\right)\right]\left(\frac{m_\sigma^2}{4} - m_q^2\right)^{3/2}$$

Around Tc the damping is due to the pion modes, η=2.2/fm

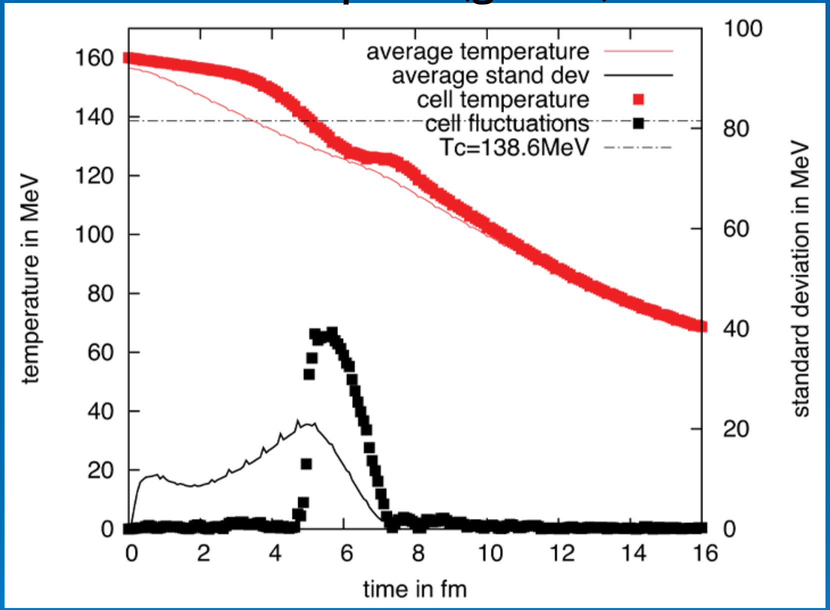


Dynamic simulations: Bjorken-like expansion

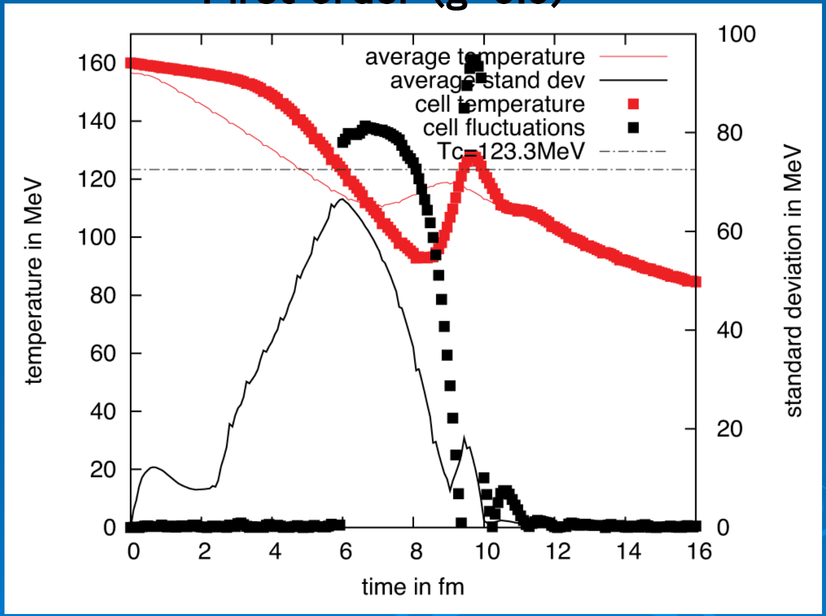
Initial state: cylinder of length L in z direction, with ellipsoidal cross section in x–y direction

$$\text{At } t=0:\ v(z)=\frac{2z}{L}0.2c,\ \ -\frac{L}{2}<z<\frac{L}{2};\ \ v_x=v_y=0;\ T=160\ MeV$$

Critical point (g=3.63)



First order (g=5.5)



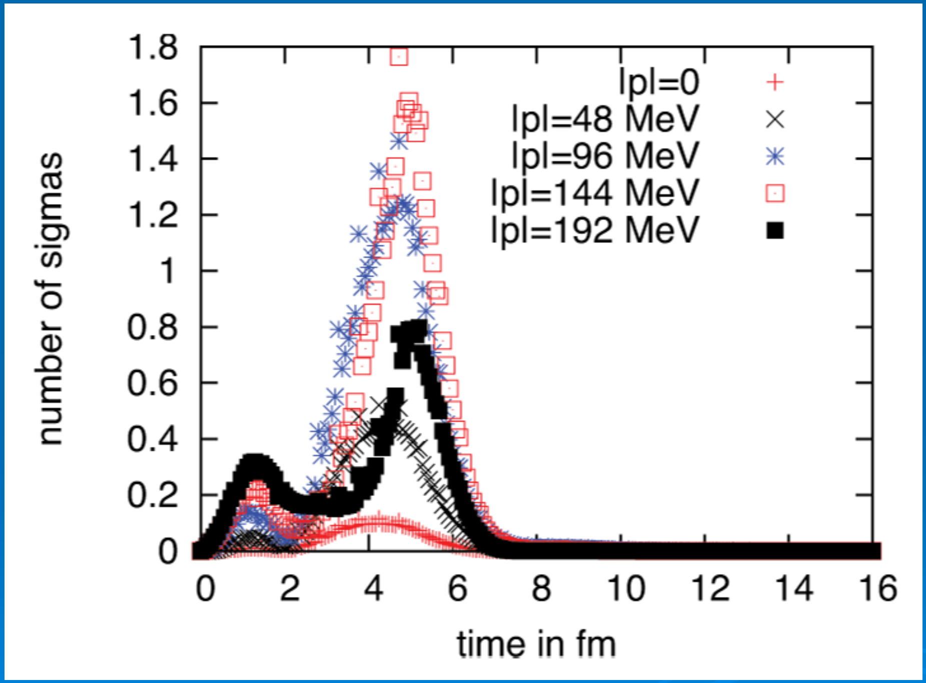
Mean values and standard deviation of T for the whole system and for a central cell (1 fm^3) are shown as a function of time.

Supercooling and reheating effects are clearly seen in the 1–st order transition, fluctuations become especially strong after 4 fm/c.

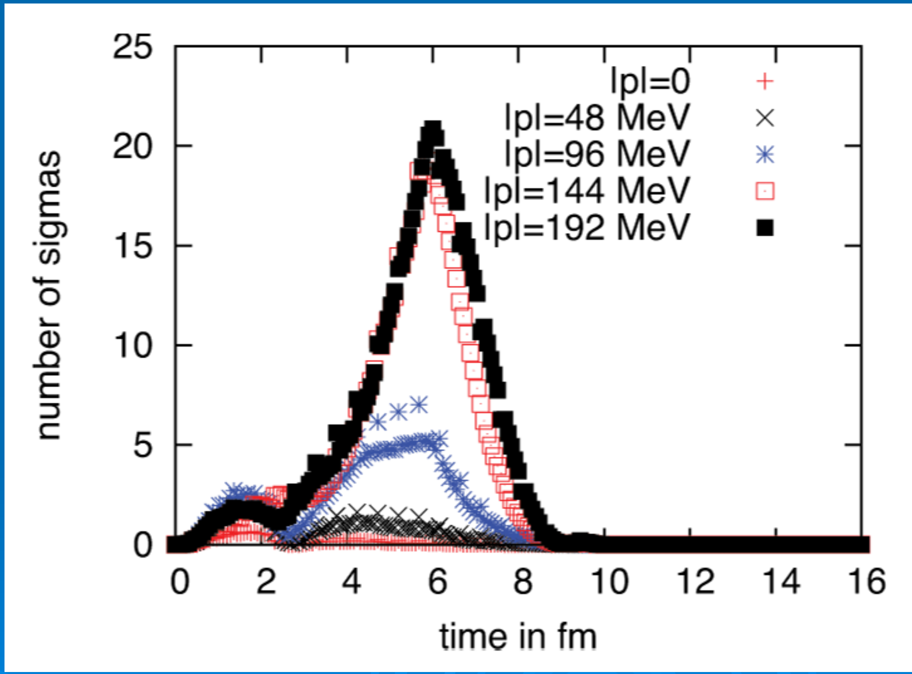
Sigma fluctuations in expanding fireball

$$\frac{dN_\sigma}{d^3k} = \frac{1}{(2\pi)^3}\frac{1}{2\omega_k}[\omega_k^2 |\sigma_k|^2 + |\dot{\sigma}_k|^2],\ \omega_k = \sqrt{m_\sigma^2 + k^2},\ \ m_\sigma^2 = \frac{\partial^2 U_{\mathrm{eff}}}{\partial \sigma^2}\Big|_{\sigma=\sigma_{\mathrm{eq}}}$$

Critical point (g=3.63)



First order (g=5.5)



Fluctuations are rather weak at critical point (left), but increase strongly at the 1st order transition (right) after 4 fm/c

Extension to finite baryon densities: Polyakov-Quark-Meson (PQM) model

C. Herold, M. Nahrgang, I. Mishustin, M. Bleicher, Nucl. Phys. A 925 (2014) 14;

- Include μ-dependence in Polyakov loop potential, (cf. Schäfer, Pawlowski, Wambach Fukushima)

$$\mathcal{U}(\ell, T, T_0)\ ,\quad T_0 \to T_0(\mu)$$

- Calculate grand canonical potential for finite chemical potential

$$\Omega_{q\bar{q}} = -2N_f T \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \left\{ \left(\ln\left[1 + 3\ell \mathrm{e}^{-\beta(E-\mu)} + 3\ell \mathrm{e}^{-2\beta(E-\mu)} + \mathrm{e}^{-3\beta(E-\mu)}\right] + (\mu \to -\mu) \right\} \right.$$

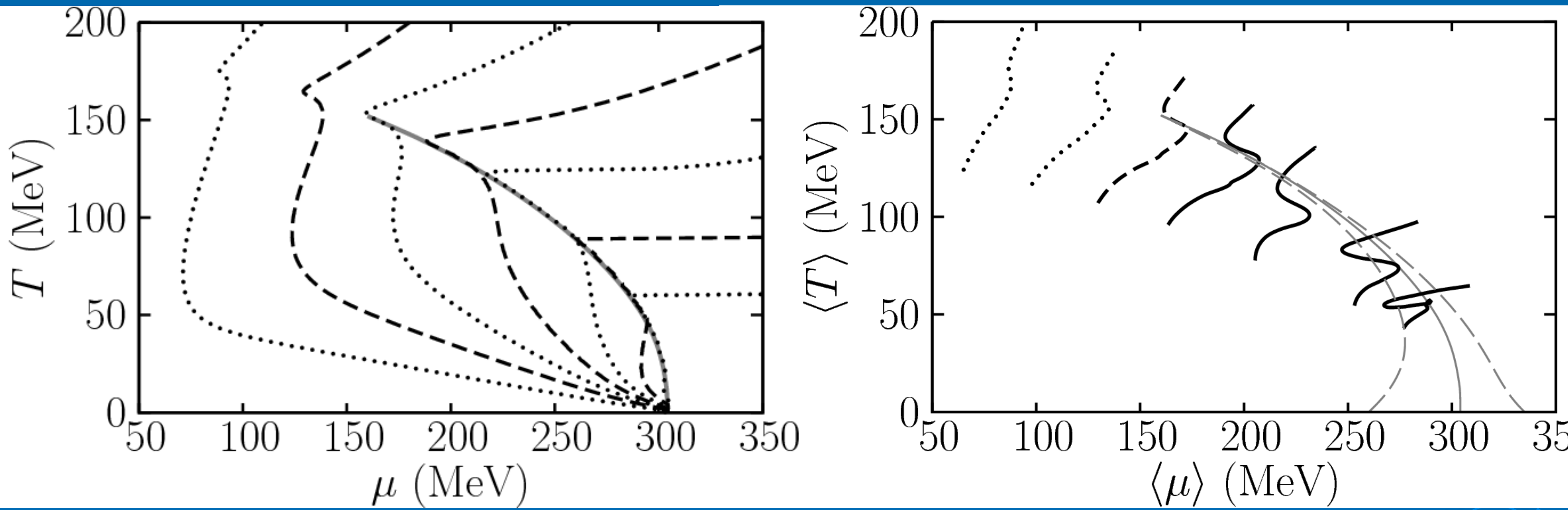
- Propagate (net) baryon density in the hydro sector

$$\partial_\mu n^\mu = 0\ ,\quad n^\mu = \rho u^\mu$$

Trajectories on the T-μ plane

CFD calculations are done for spherical fireball of R=4 fm

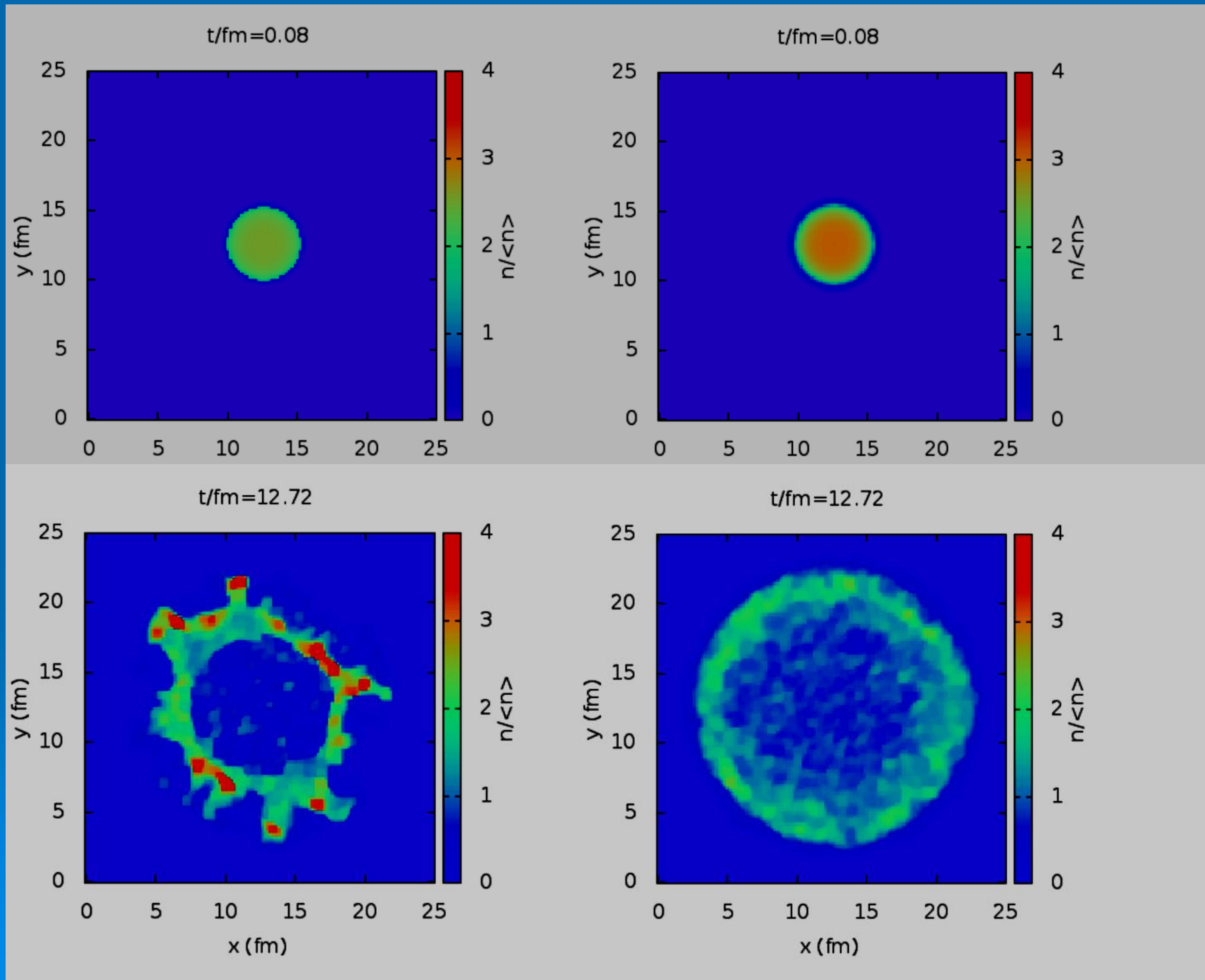
Isentropic expansion Hydrodynamic evolution



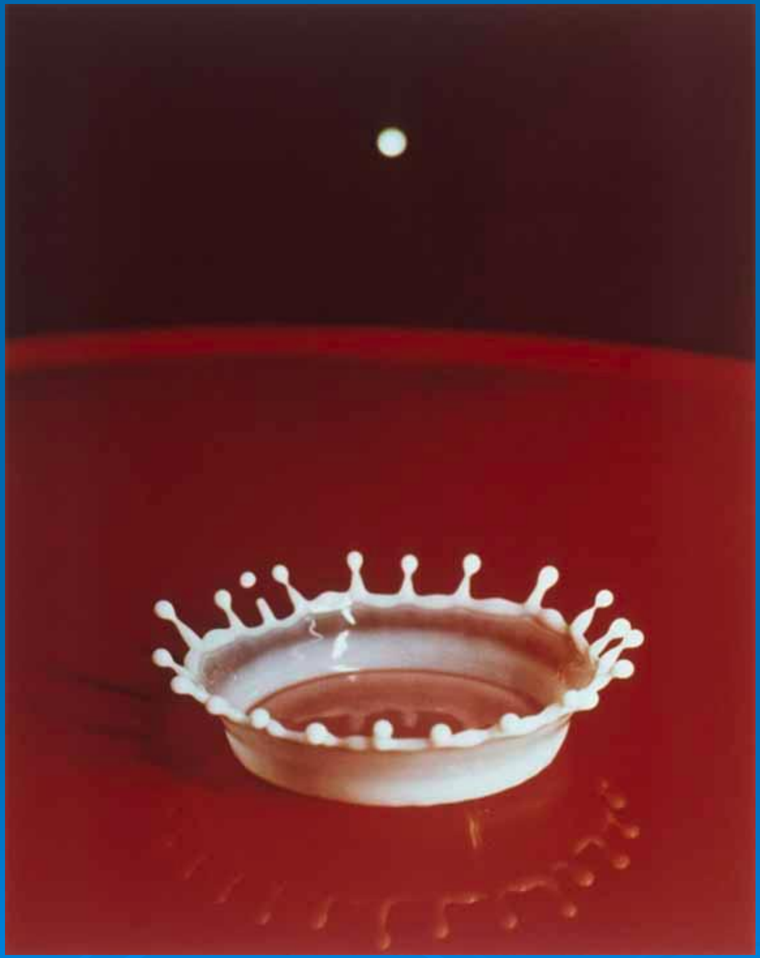
- **Trajectories are close to isentropes for crossover and CP;**
- **Non-equilibrium "back-bending" is clearly seen in FO case;**
- **In the case of strong FO transition (solid lines) the system is trapped in spinodal region for a significant time**

Dynamical droplet formation

First order | **Critical point**

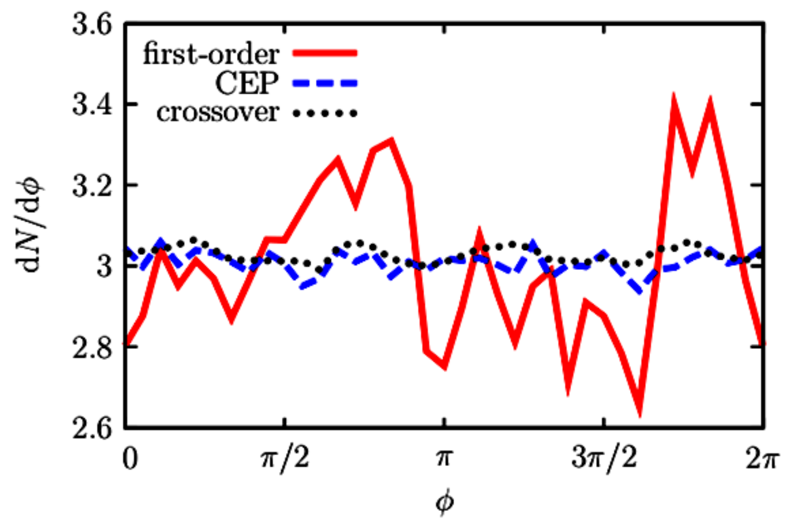


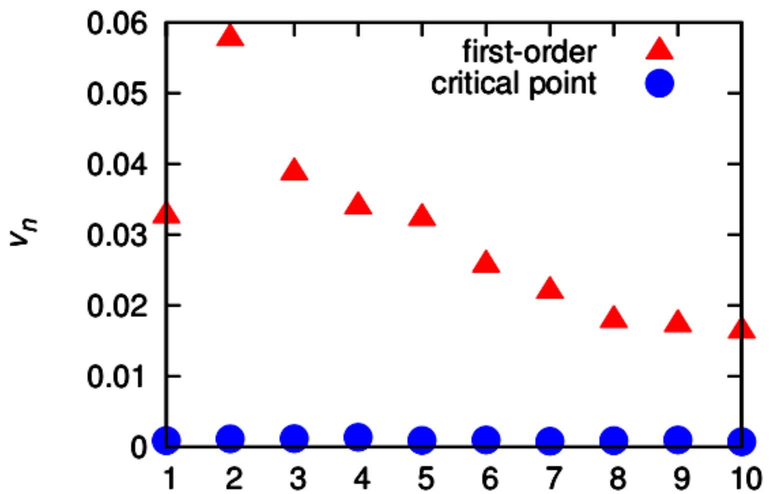
Splash of a milk drop



HEE-NC-57001

Observable signatures of high-density domains





Azimuthal fluctuations of net-B In single events: strong enhancement at first order PT

High harmonics of baryonic flow (averaged over many events):

$v_n = \langle \cos[n(\phi - \phi_n)] \rangle$

Father developments

- **In the previous calculations the EOS had a P=0 point at a finite baryon density (like the MIT bag model), that makes possible stable quark droplets**

- **It is interesting to see what happens in a more realistic case when quark droplets are unstable at zero pressure (J. Steinheimer et al, PRC 89 (2014) 034901)**

- **There exist several models which have such a property, in particular so called Quark-Hadron Model (S. Schramm et al.) or Quark-Dilaton Model (C. Sasaki et al.).**

SU(3) chiral quark-hadron (QH) model

V. Dexheimer, S. Schramm, Phys. Rev. C 81 (2010) 045201

Includes: a) 3 quarks (u,d,s) plus baryon octet,

b) scalar mesons (σ, ς), vector meson (ω)

c) Polyakov loop (l)

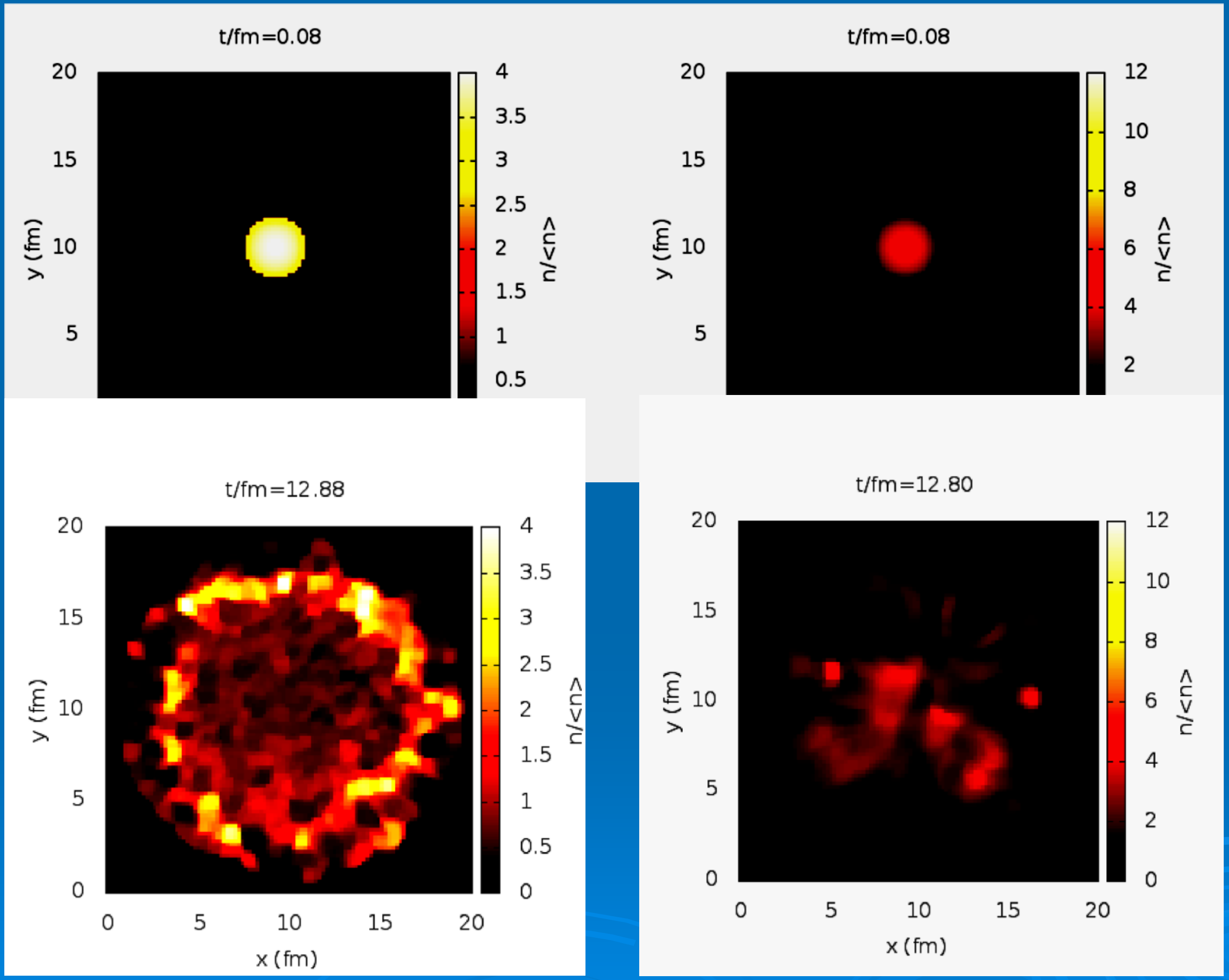
$$\mathcal{L} = \sum_i \overline{\psi}_i \left(i\gamma^\mu \partial_\mu - \gamma^0 g_{i\omega}\omega - M_i\right)\psi_i + \frac{1}{2}\left(\partial_\mu \sigma\right)^2 - U\left(\sigma, \zeta, \omega\right) - \mathcal{U}(\ell)$$

Effective masses:

$$\begin{aligned} M_q &= g_{q\sigma}\sigma + g_{q\zeta}\zeta + M_{0q} + g_{q\ell}(1-\ell) \\ M_B &= g_{B\sigma}\sigma + g_{B\zeta}\zeta + M_{0B} + g_{B\ell}\ell^2 \end{aligned}$$

PQM vs. QHM: domain formation

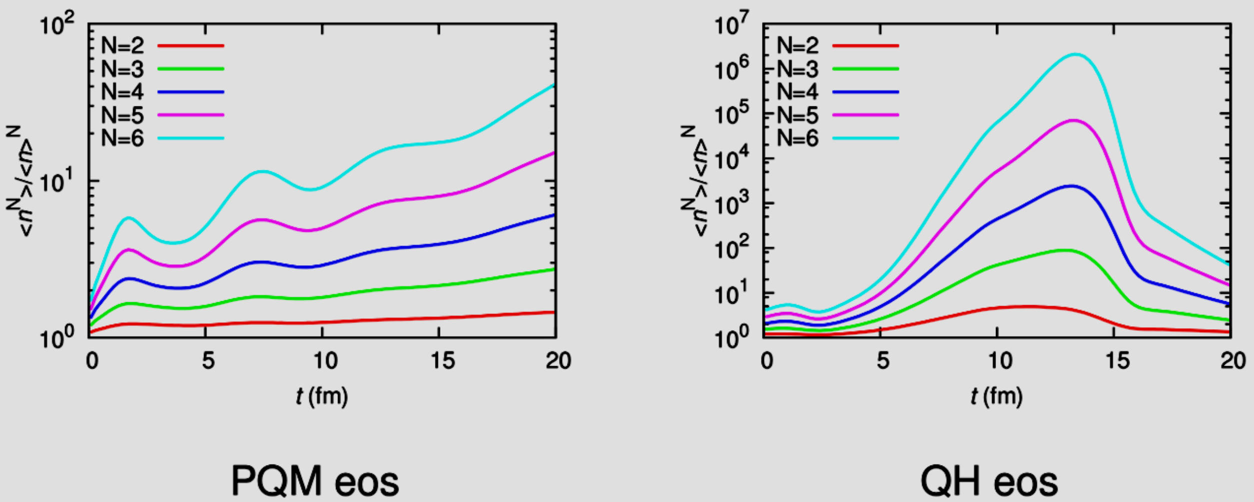
Herold, Limphirat, Kobodaj, Yan, Seam Pacific Conference 2014



QH predicts domains with much higher densities!

PQM vs. QHM: density moments

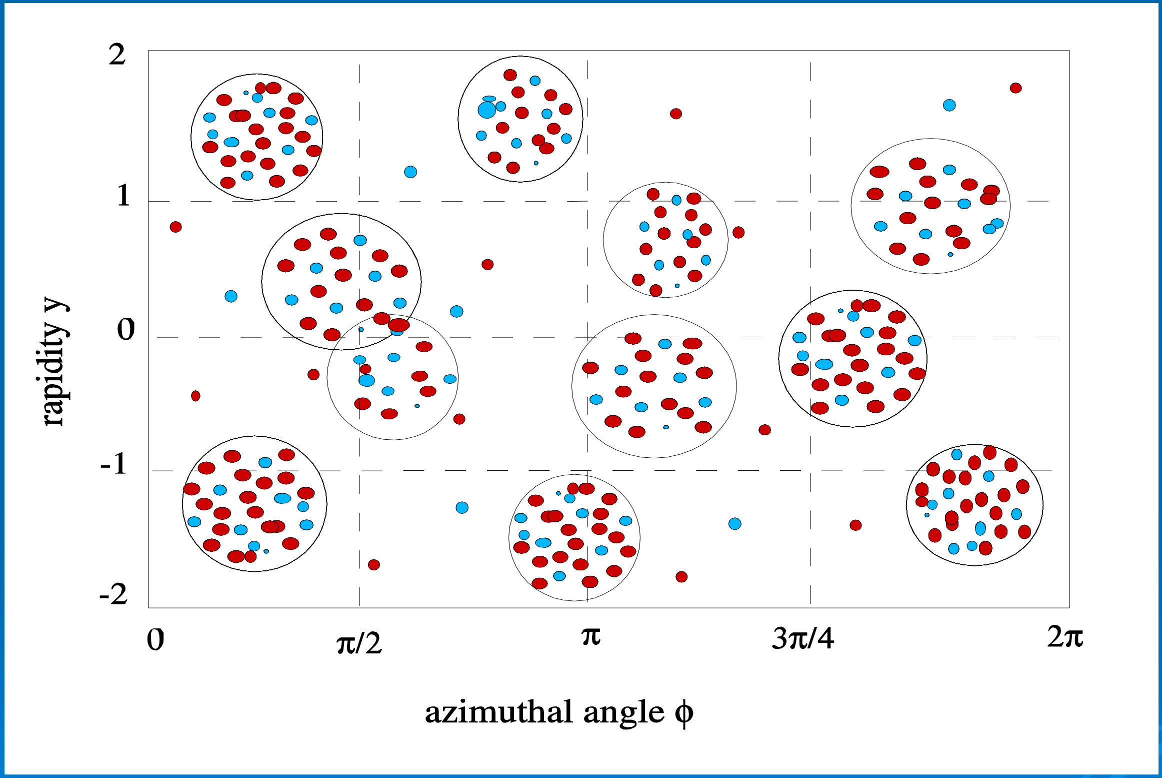
$$\langle n^N \rangle = \int \mathrm{d}^3x \, n(x)^N P_n(x) \qquad \text{with} \quad P_n(x) = \frac{n(x)}{\int \mathrm{d}^3x \, n(x)}$$



In PQM density contrast grows towards freeze-out stage, but in QHM it has a maximum at the intermediate dense stage. But strong clustering effect survives even at t>15 fm/c!

Experimental signatures of droplets

Look for bumpiness in distributions of net baryons in individual events, i. e. in azimuthal angle, rapidity, transverse momentum



The bumps correspond to the emission from individual domains.

Conclusions

- Phase transitions in relativistic heavy-ion collisions will most likely proceed out of equilibrium
- 2nd order phase transition (with CEP) is too weak to produce significant observable effects in fast dynamics
- Non-equilibrium effects in a1st order transition (spinodal decomposition, dynamical domain formation) may help to identify the chiral/deconfinement phase transition
- If QGP domains (droplets) survive until the freeze-out stage, they will show up by large non-statistical fluctuations of hadron multiplicities in phase space (in single events)
- Exotic objects like strangelets have a better chance to be formed in such a non-equilibrium scenario