Contents

- **Introduction**
- **Phenomenological equation of state**

  a) Hadron resonance Gas (HG)

  b) Quark-Gluon Plasma (QGP)
- **3d ideal hydro simulations**

  a) dynamical trajectories of matter

  b) collective flow

  c) particle spectra
- **Properties of the initial state**
- **Conclusions**

Introduction: hydrodynamic modeling of nuclear collisions

Ideal hydrodynamics assumes solving differential equations

$$\partial_\nu T^{\mu\nu} = 0, \quad \partial_\mu J_B^\mu \equiv \partial_\mu (n u^\mu) = 0, \quad (\mu, \nu = 0,1,2,3).$$

expressing local energy-momentum and baryon number conservation, where

$$T^{\mu\nu} = (\varepsilon + p) u^\mu u^\nu - p g^{\mu\nu}$$

Is the energy-momentum tensor of the ideal fluid: ε is the nergy density, P-pressure and u-collective 4-velocity.

These equations should be supplemented by

a) **equation of state (EOS)** of the fluid

$$P = P(\varepsilon, n)$$

b) **initial conditions: n present calculations we start from two cold nuclei approaching each other.**

The nuclei are stabilized by the mean field and have realistic (Woods-Saxon) density distributions.

Most famous hydro models

1+1-d models: Landau, 1953 – full stopping of produced fluid in Lorenz-contracted volume; Bjorken, 1983 – partial transparency of colliding nuclei, delayed formation of produced fluid at proper time;

;

2+1-d models (transverse hydro + Bjorken longitudinal expansion)**:**
Kolb, Sollfrank & Heinz, 1999; Teaney, Lauret & Shuryak, 2001; Hirano, 2002;

Full 3+1d models (starting with cold nuclei): Harlow, Amsden & Nix, 1976; Stoecker, Maruhn & Greiner,1979; Rischke et al, 1995; Hama et al. 2005;

Multi-fluid models: Amsden et al, 1978; Clare & Strottman, 1986; Mishustin, Russkikh & Satarov, 1988; Brachmann et al, 2000; Ivanov, Russkikh & Toneev, 2006;

Hydro-kinetic models: Bass&Dumitru, 2000; Teaney et al. 2002001, Petersen, Steinheimer, Bleicher at al. 2008.

EOS1: HG with excluded volume correction

Satarov, Dmitriev&Mishustin: Phys. Atom. Nucl. 72 (2009) 1390

$$P = \sum_{i=\text{hadrons}} P_i^{id}(\mu_i - P v_i, T)$$

P_i^{id} – pressure of ideal gas

$v_i = v \sim (0.5\text{—}2)\ \text{fm}^3$ excluded volume

Excluded volume correction following Rischke, Gorenstein, Stöcker, Greiner, Z. Phys. C51 (1991) 485

μ_S is determined from the net strangeness neutrality

$$n_S = 0$$

Chemical potential for species i

$$\mu_i = \mu_B B_i + \mu_S S_i$$

Baryonic charge (B_i) Strangeness (S_i)

Hadronic species included: all known hadrons with m ≤ 2 GeV, apart of $f_0(600)$

$$i = \begin{cases} M = \pi, \rho, \omega, \ldots, K, \overline{K}, \ldots (bosons) \rightarrow i \le N_B = 59 \\ B = N, \Delta, \Lambda, \Sigma, \ldots (fermions) \rightarrow i \le N_F = 41 \\ \overline{B} = \overline{N}, \overline{\Delta}, \overline{\Lambda}, \overline{\Sigma}, \ldots (fermions) \rightarrow i \le N_F = 41 \end{cases}$$

This set Is very similar to THERMUS : Wheaton&Cleymans, hep-ph/0407174)

EOS2: Quark-Gluon phase within the Bag model

$$P_Q(\mu,T) = (\tilde{N}_g + \frac{21}{2}\tilde{N}_f)\frac{\pi^2}{90}T^4 + \tilde{N}_f(\frac{T^2\mu^2}{18} + \frac{\mu^4}{324\pi^2}) +$$
$$+\frac{1-\xi}{\pi^2}\int_{m_s}^{\infty} dE(E^2 - m_s^2)^{3/2}\left\{\left[e^{\frac{E-\mu_s}{T}}+1\right]^{-1} + \left[e^{\frac{E+\mu_s}{T}}+1\right]^{-1}\right\} - B$$

$$\tilde{N}_g = 16(1-0.8\xi)$$

$$N_f = 2(1-\xi)$$

perturbative correction $(\xi \sim \alpha_s)$

ξ, B, m_s – parameters of the model

$\xi=0.2$ extracted from lattice data

m_s = 150 MeV

$B^{1/4}$=230 MeV/fm^3

⇓

$T_c(n=0)$=165 MeV

for u,d quarks

$$\mu_q = \frac{\mu}{3}$$

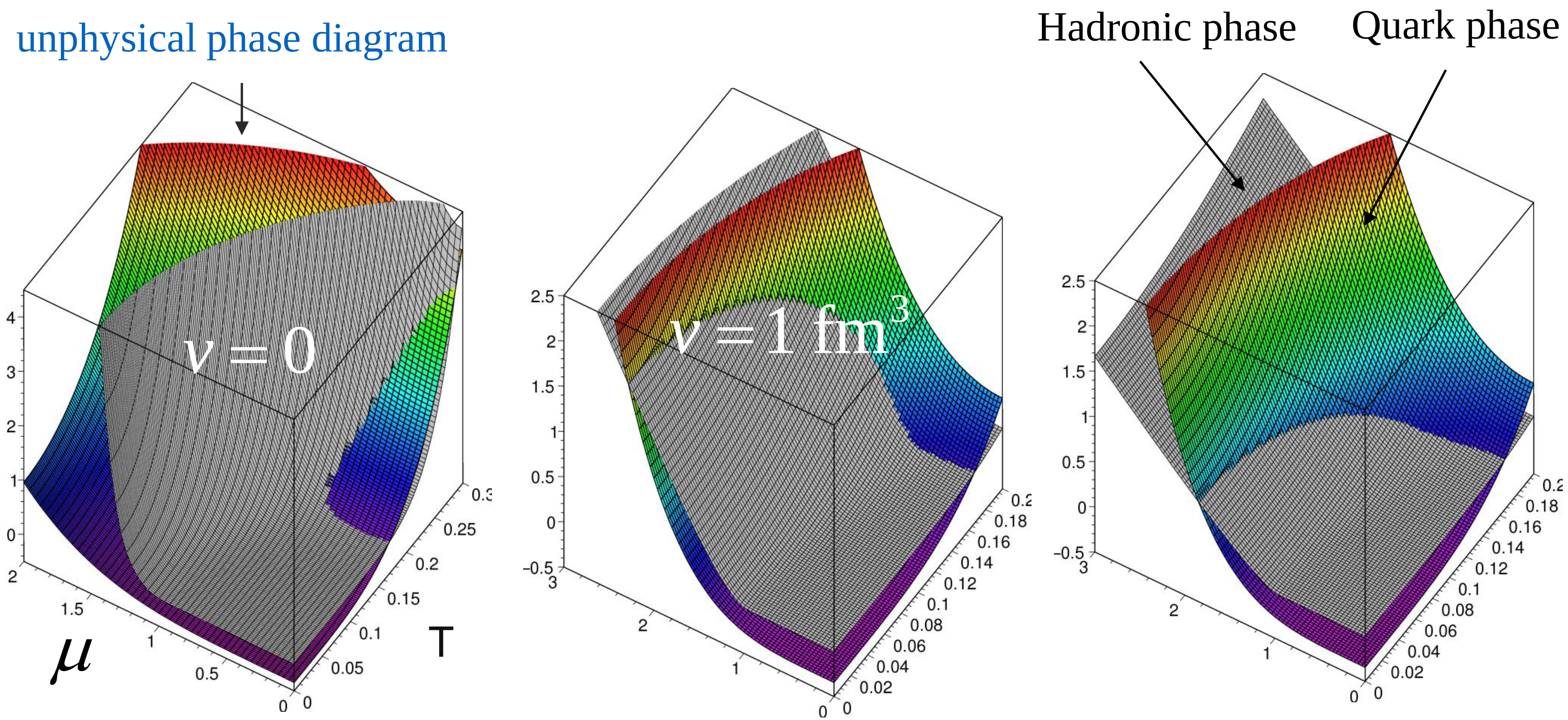
for s quarks

$$\mu_s = \frac{\mu}{3} - \mu_S$$

Phase transition HG-QGP

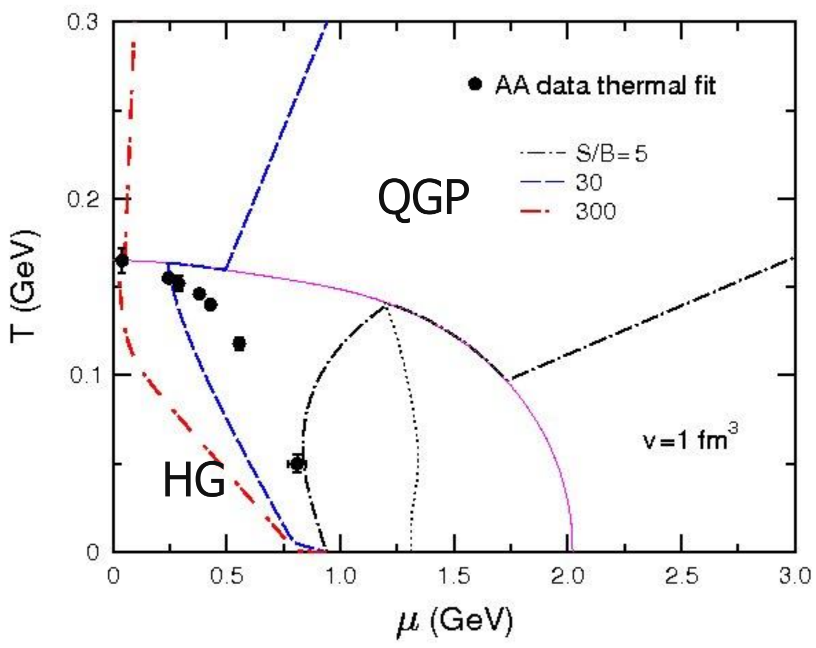
Gibbs criterion for phase transition: $P_H(\mu_B, T) = P_Q(\mu_B, T)$

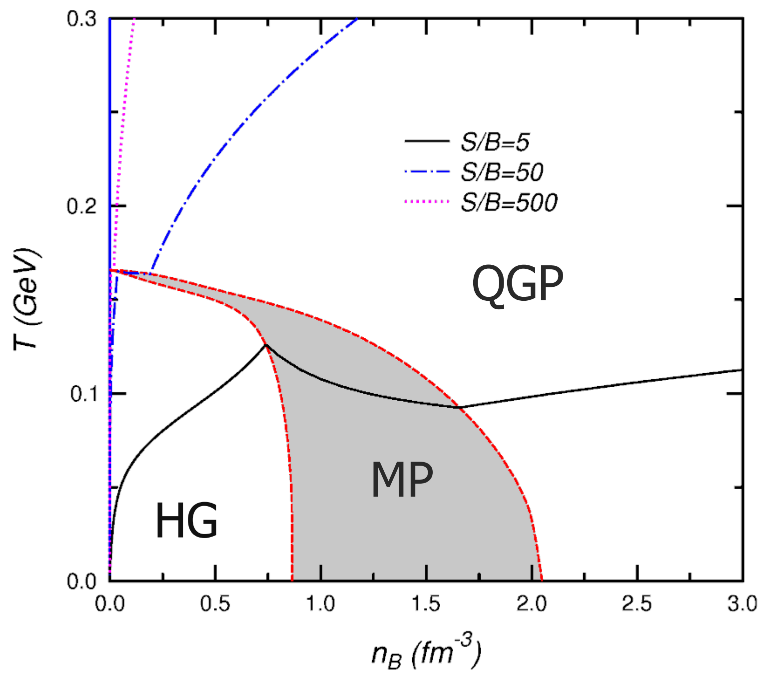
Compare pressures of two phases as functions of T, μ



Finite size of hadrons (v~1 fm^3) is crucial for PT!

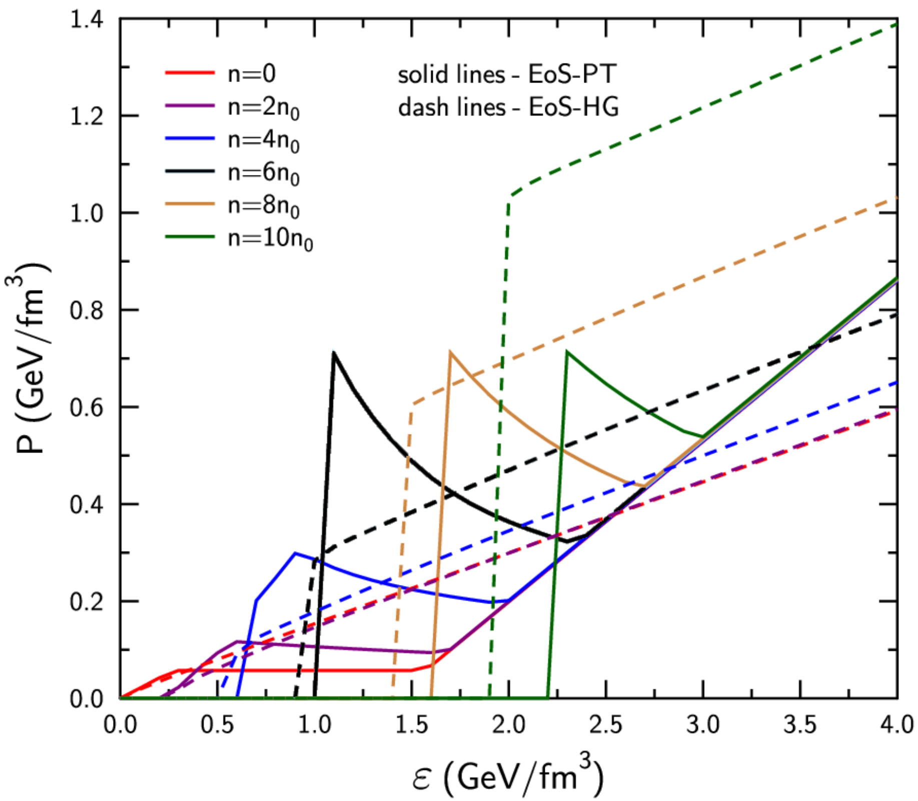
Adiabatic trajectories in T-mu anf T-n planes





Temperature increases at transition from quarks to hadrons. This is different compared to chiral models like LσM or NJL models

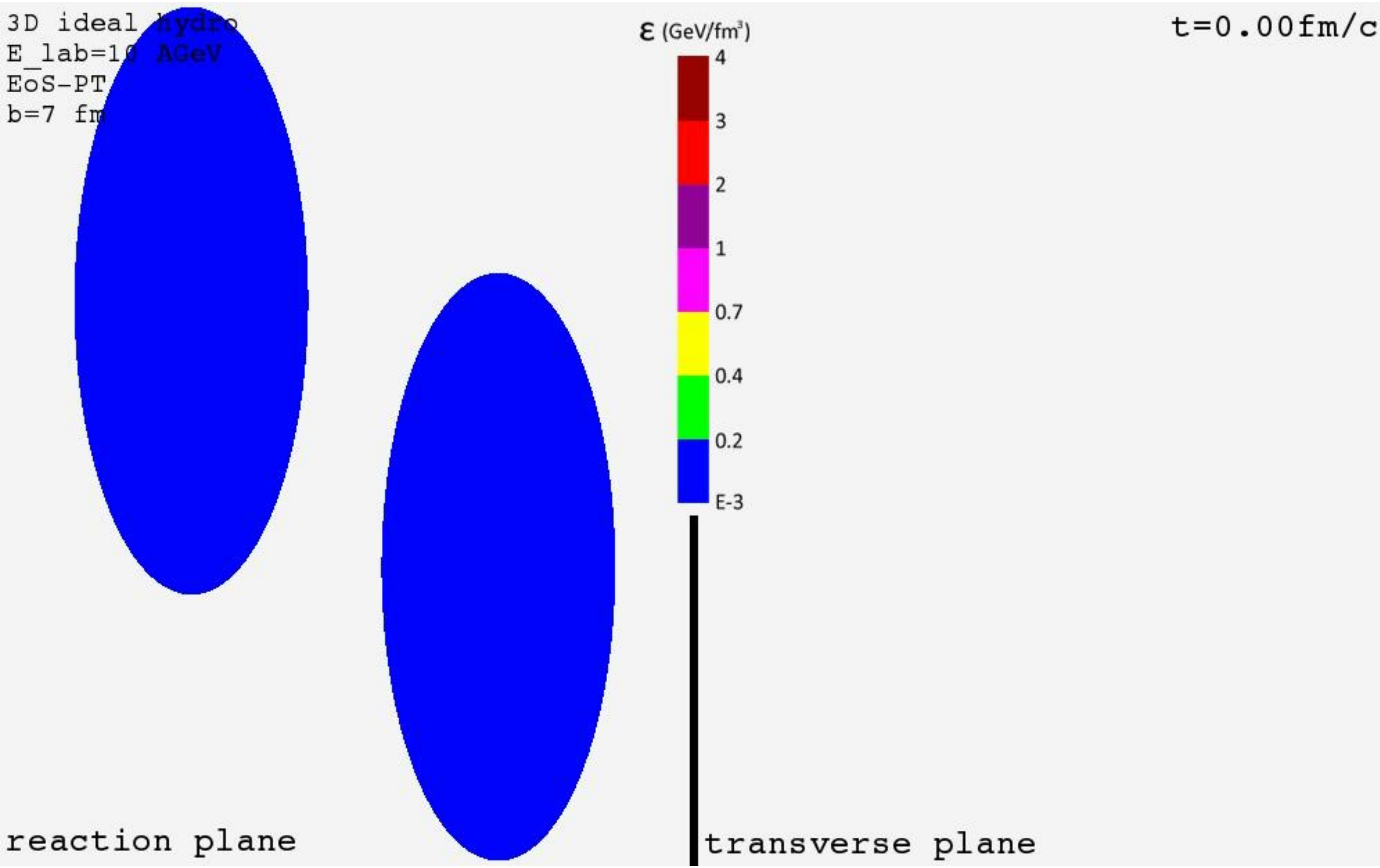
Pressure for EoS-HG and EoS-PT



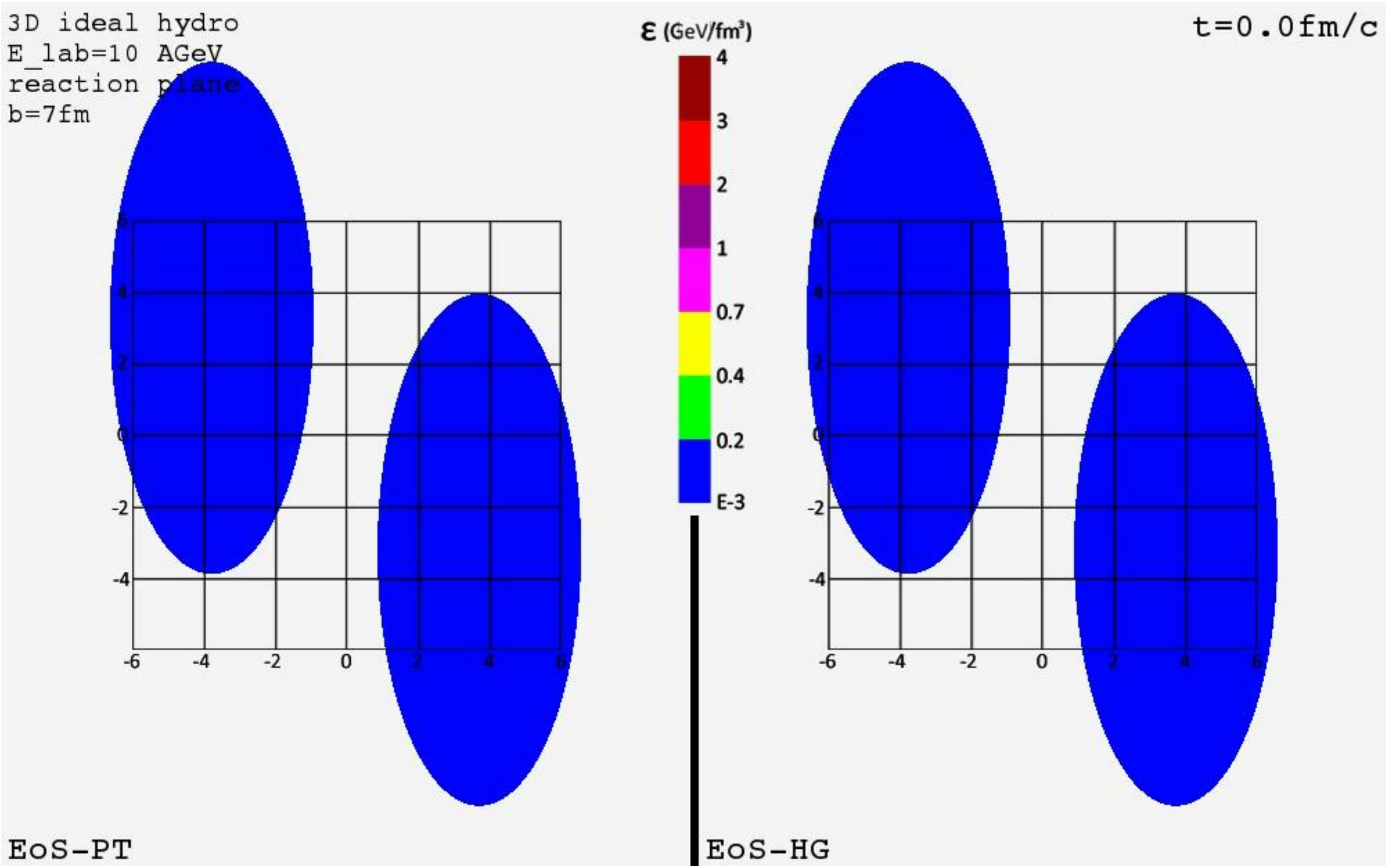
➡ **In principle, EoS-PT is "softer " than EoS-HG**

➡ **but in some density intervals P_EoS-PT > P_EoS-HG (mixed phase effect)**

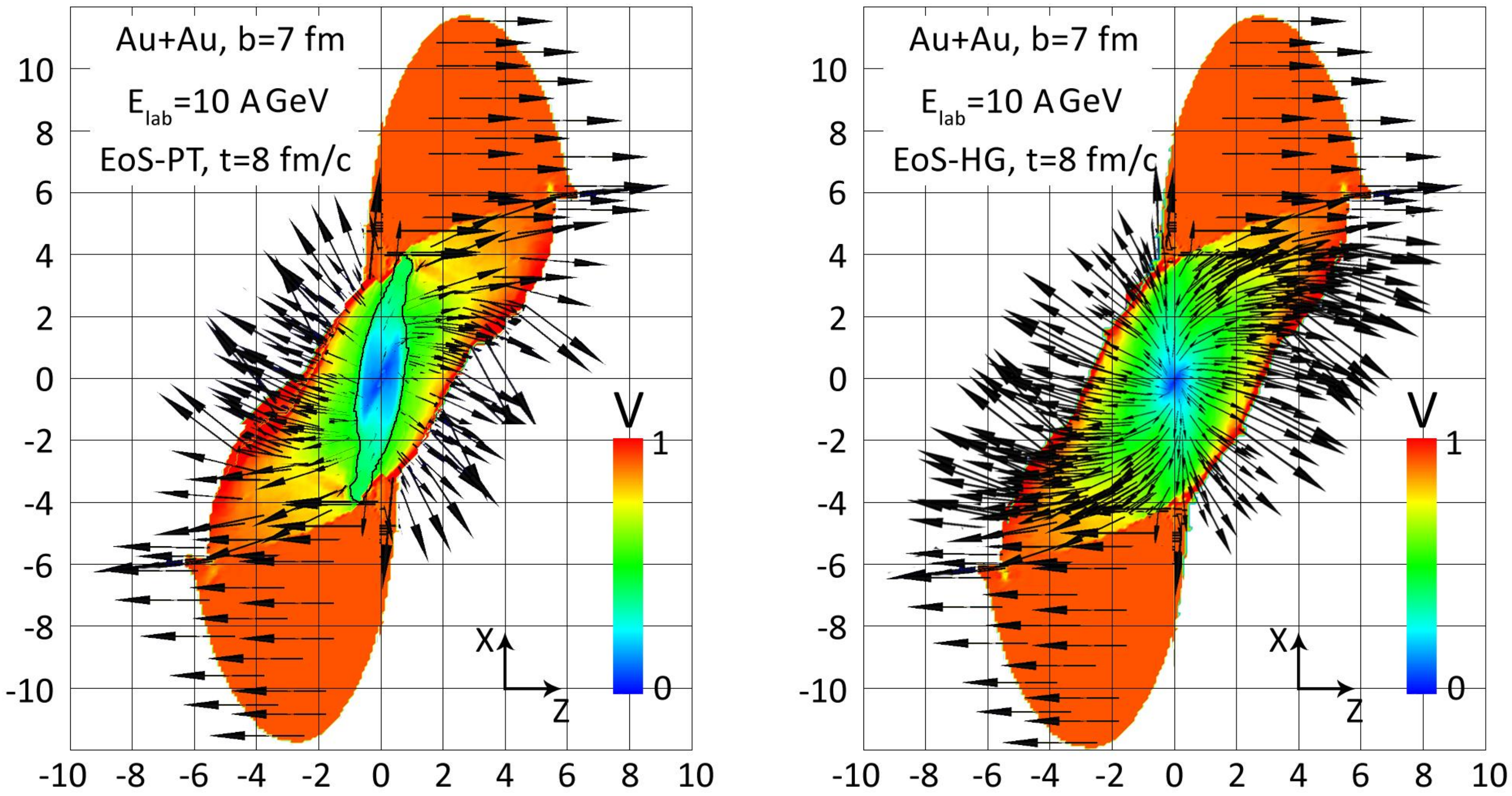
Peripheral Au+Au collision (EoS-PT)



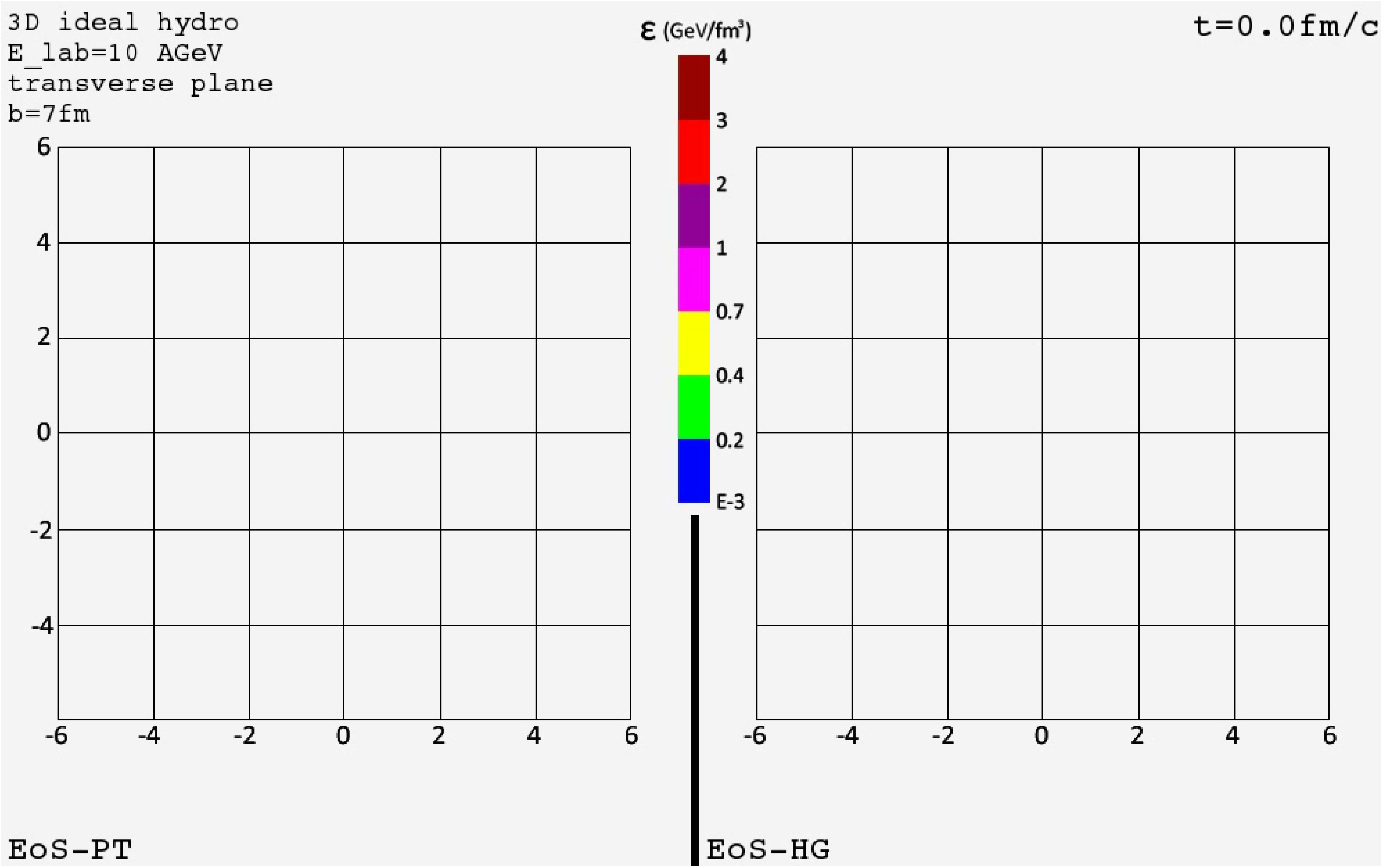
Peripheral Au+Au collision (PT vs HG)



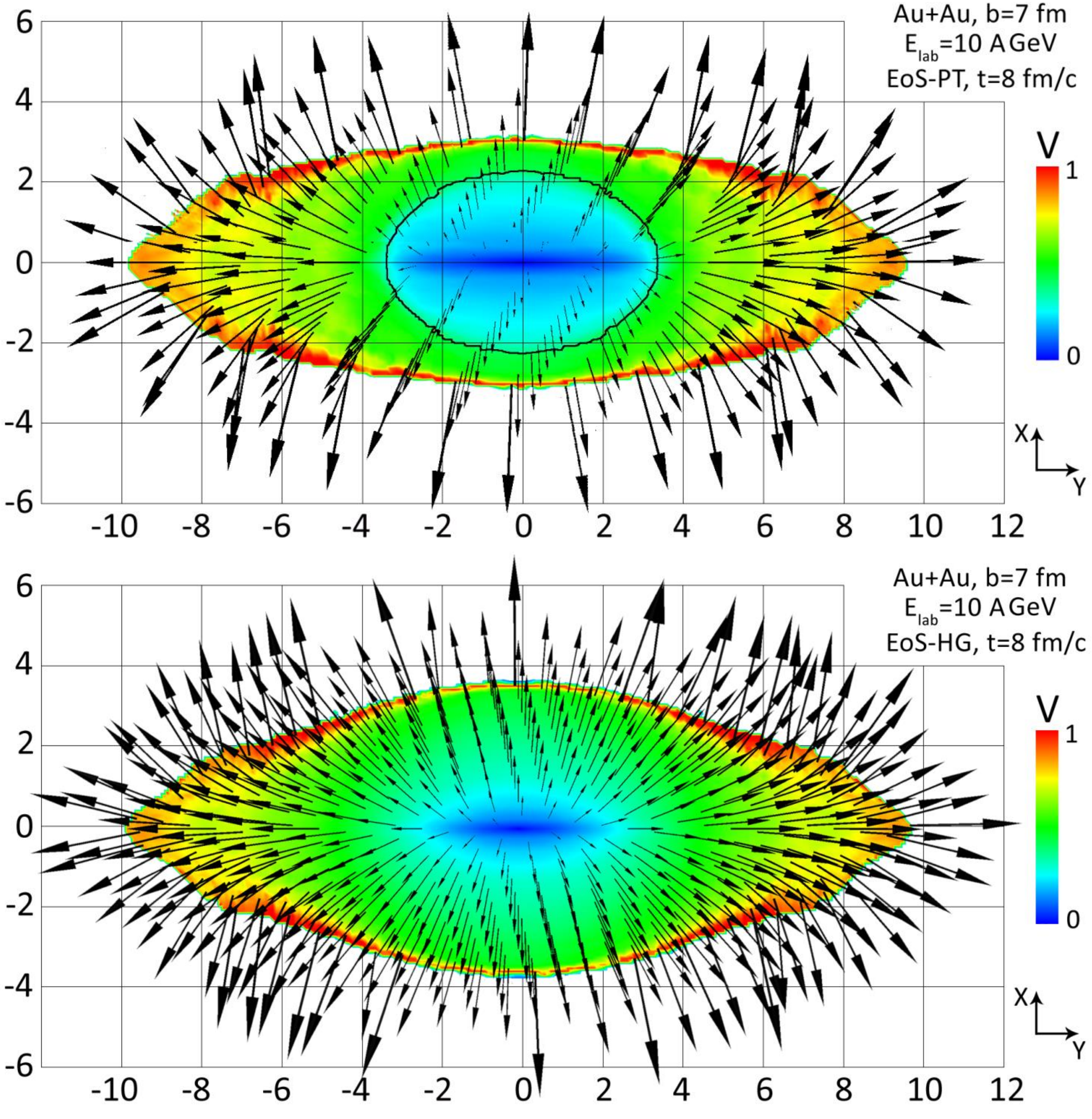
Velocity fields in reaction plane



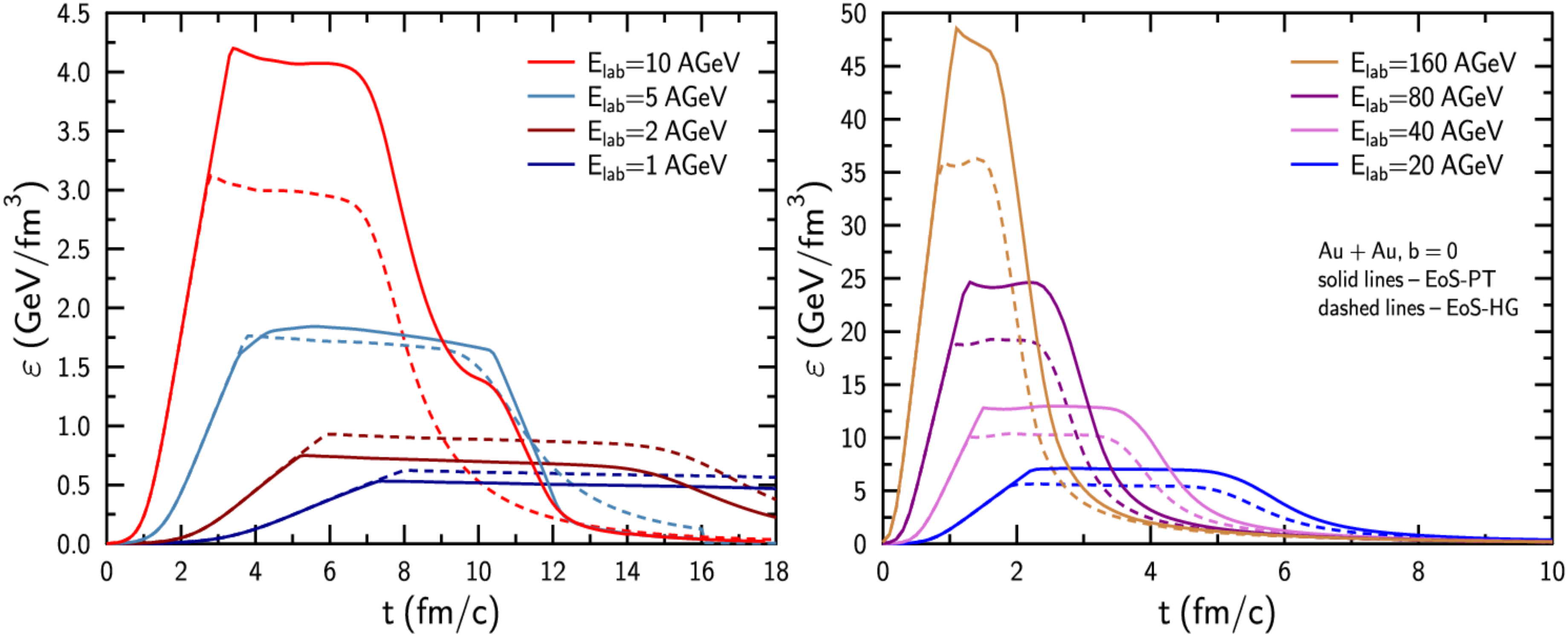
Peripheral Au+Au collision (PT vs HG)



Velocity fields in transverse plane



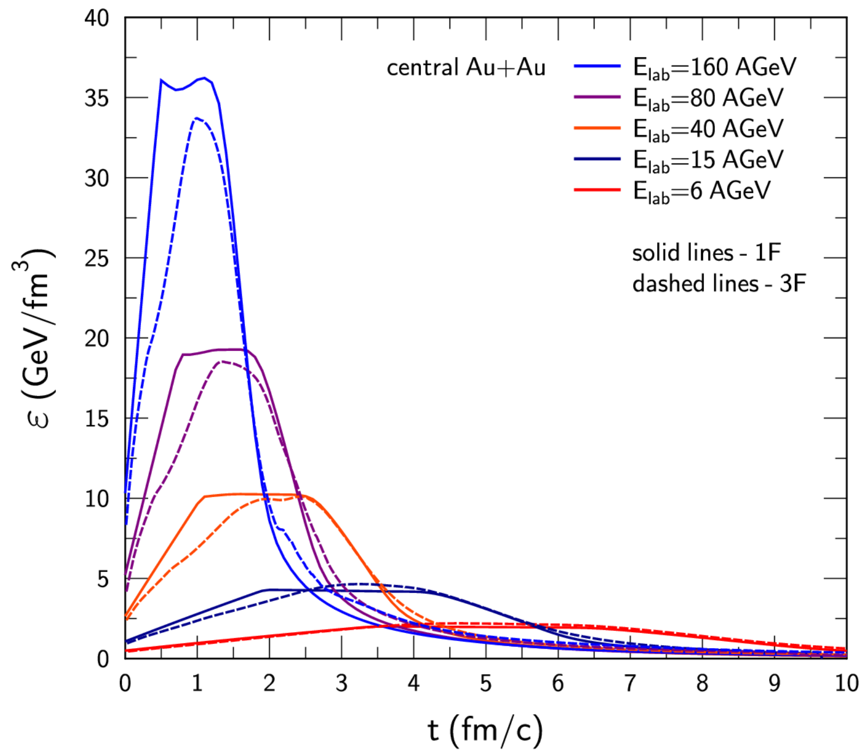
Energy density and baryon density in central box



Energy densities > 2 GeV/fm^3 appear at Elab >5 AGeV during the time interval less than 5 fm/c.

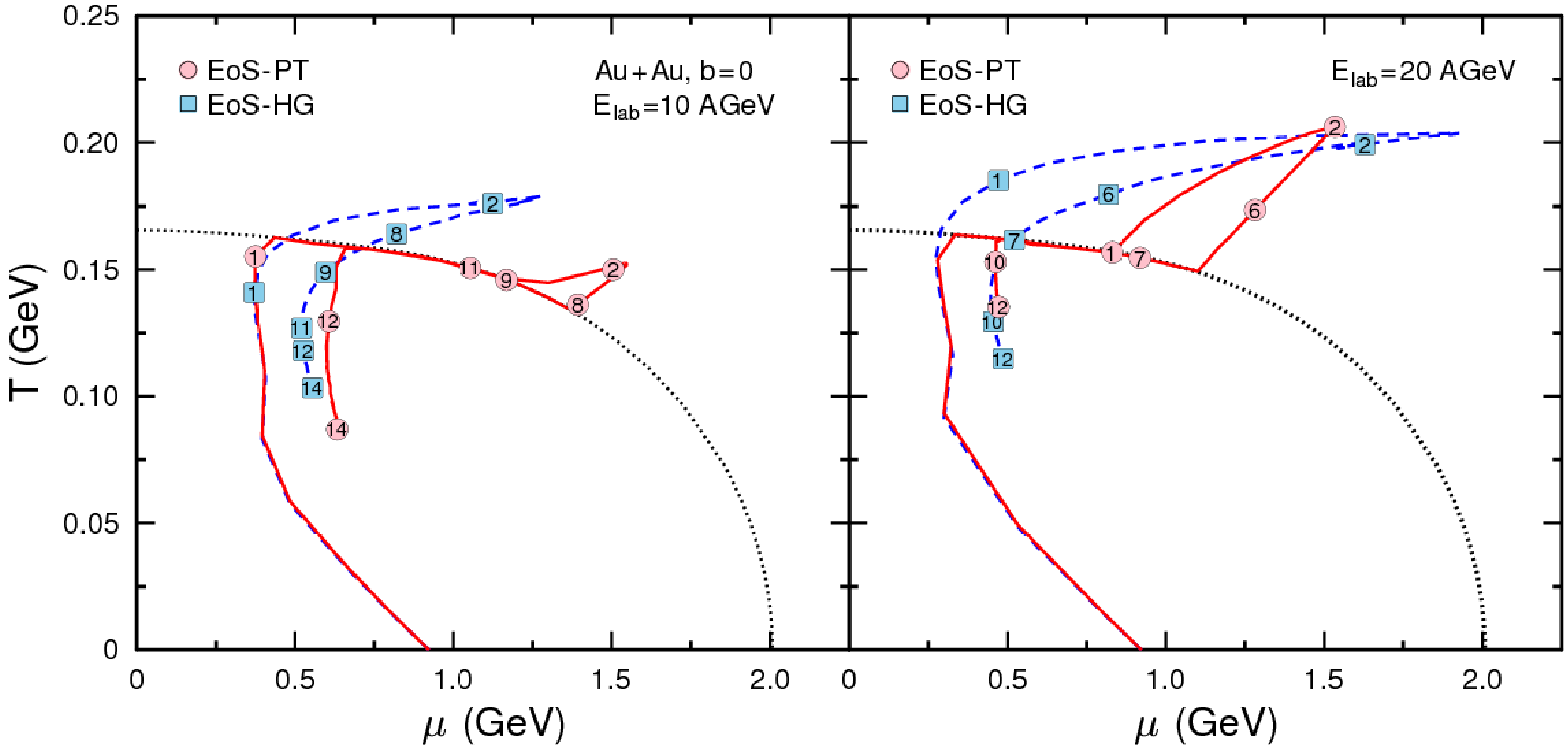
Baryon densities>10 n_0 are reached at E_{lab}>10 AGeV!

Comparison of 1–fluid and 3–fluid* models



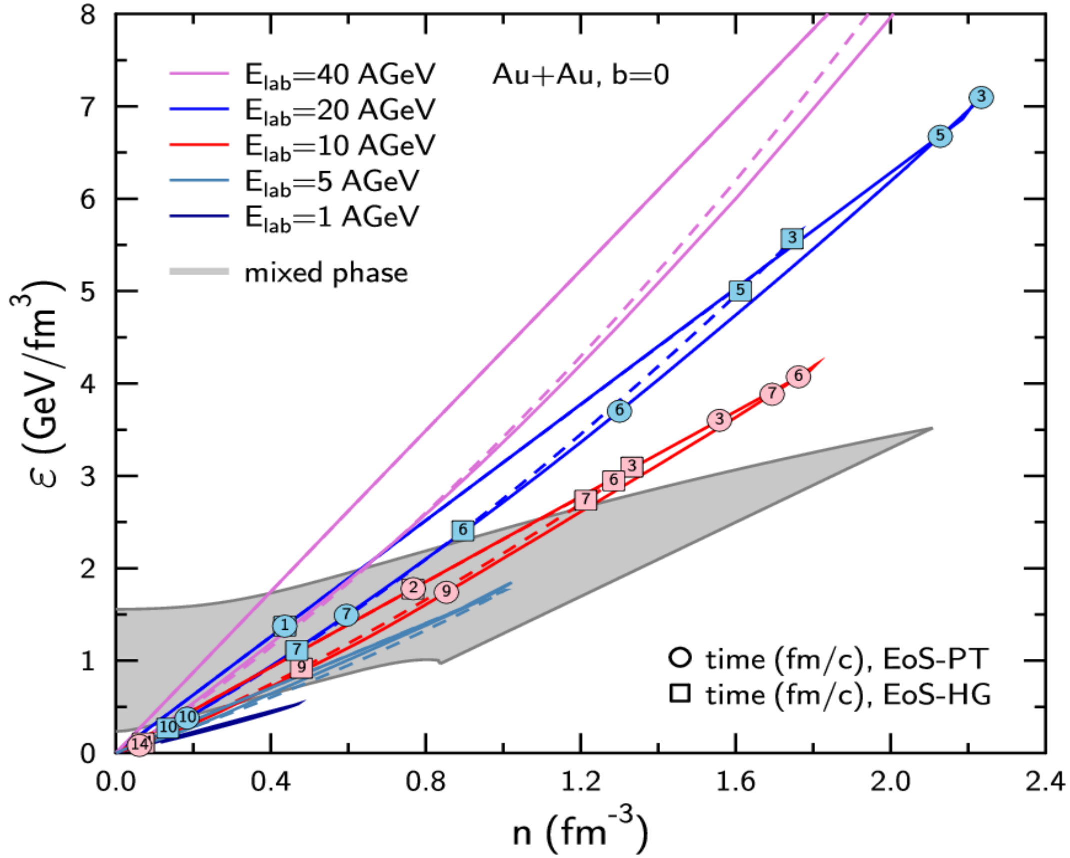
Transparency effects are rather week at E_{lab}<15 AGeV (in central collisions), at higher energies they are noticeable only at very early times, less than 2 fm/c

Dynamical trajectories of matter in central cell 1



In the equilibrium scenario the final state is not sensitive to the phase transition.
Non-equilibrium effects may help to see it!

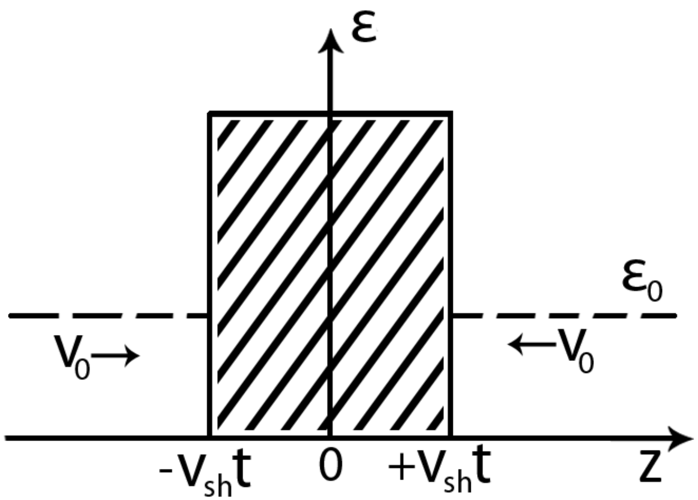
Dynamical trajectories of matter in central cell 2

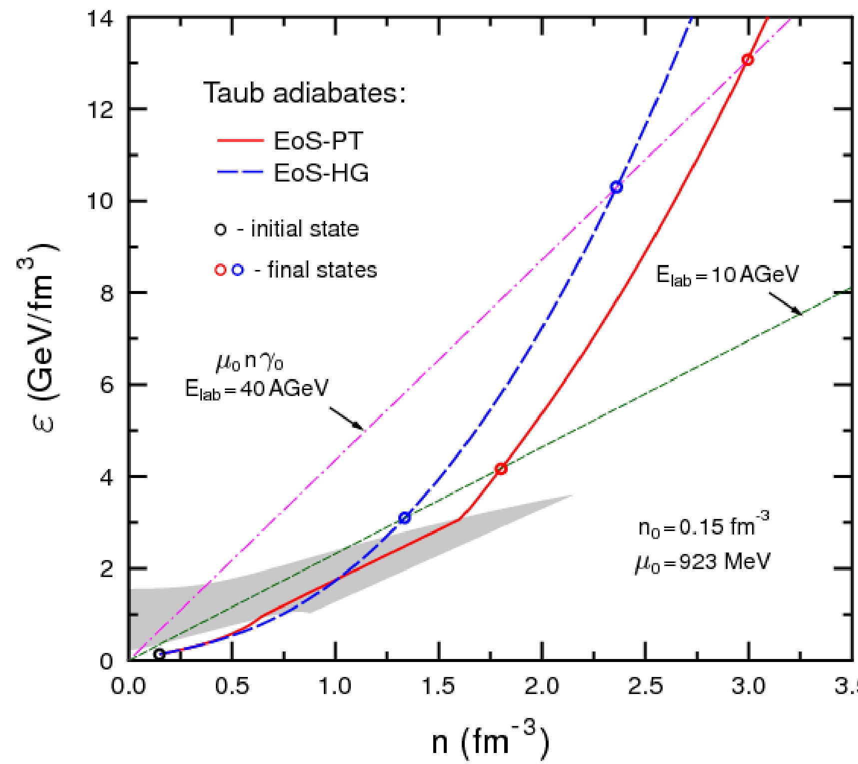


The passage time through the mixed-phase region is very short, only about 3 fm/c: non-equilibrium effects must be important!

Simple picture of Initial stae:1D shock wave

collision of two slabs





$T^{0z}; T^{zz}; nU^z$ continuous in the shock front rest frame

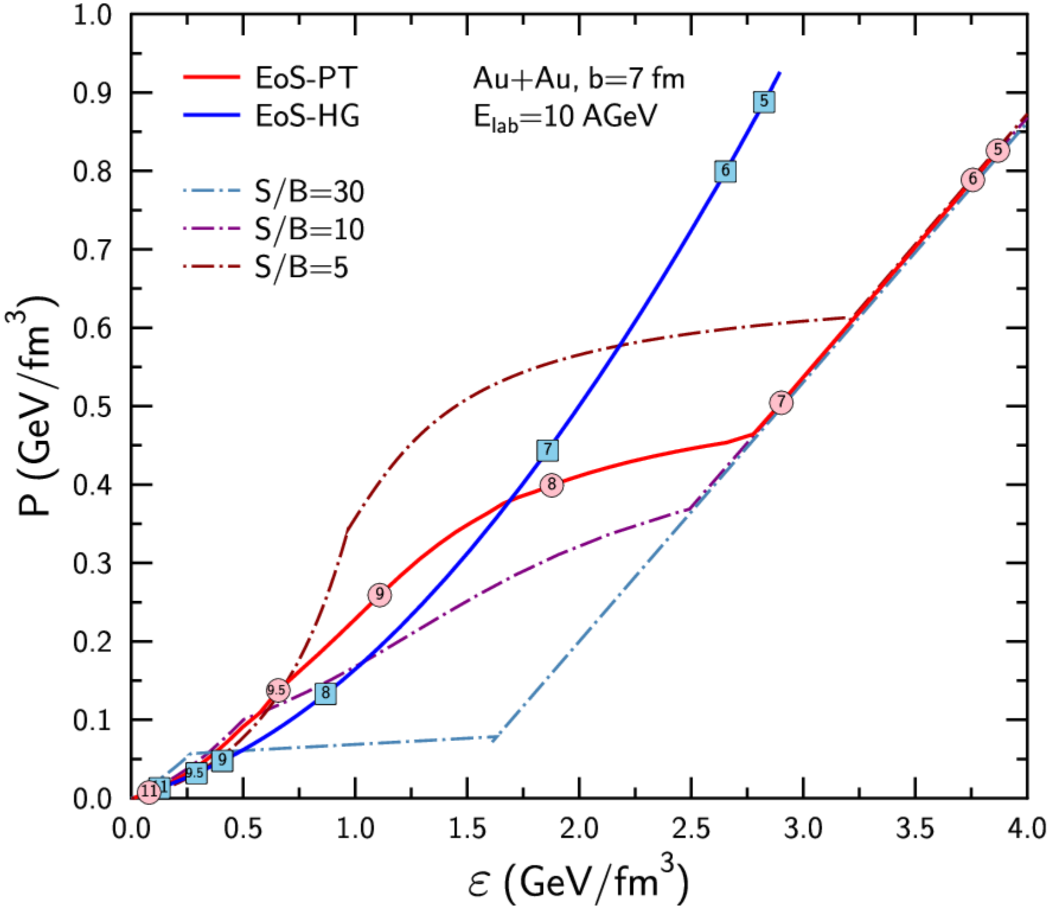
⇒ Taub adiabate

$$\varepsilon_0 (P + \varepsilon_0)\, n^2 = \varepsilon (P + \varepsilon)\, n_0^2$$

$$P = P(\varepsilon; n) \quad P_0 = 0 \quad \varepsilon_0 = \mu_0 n_0$$

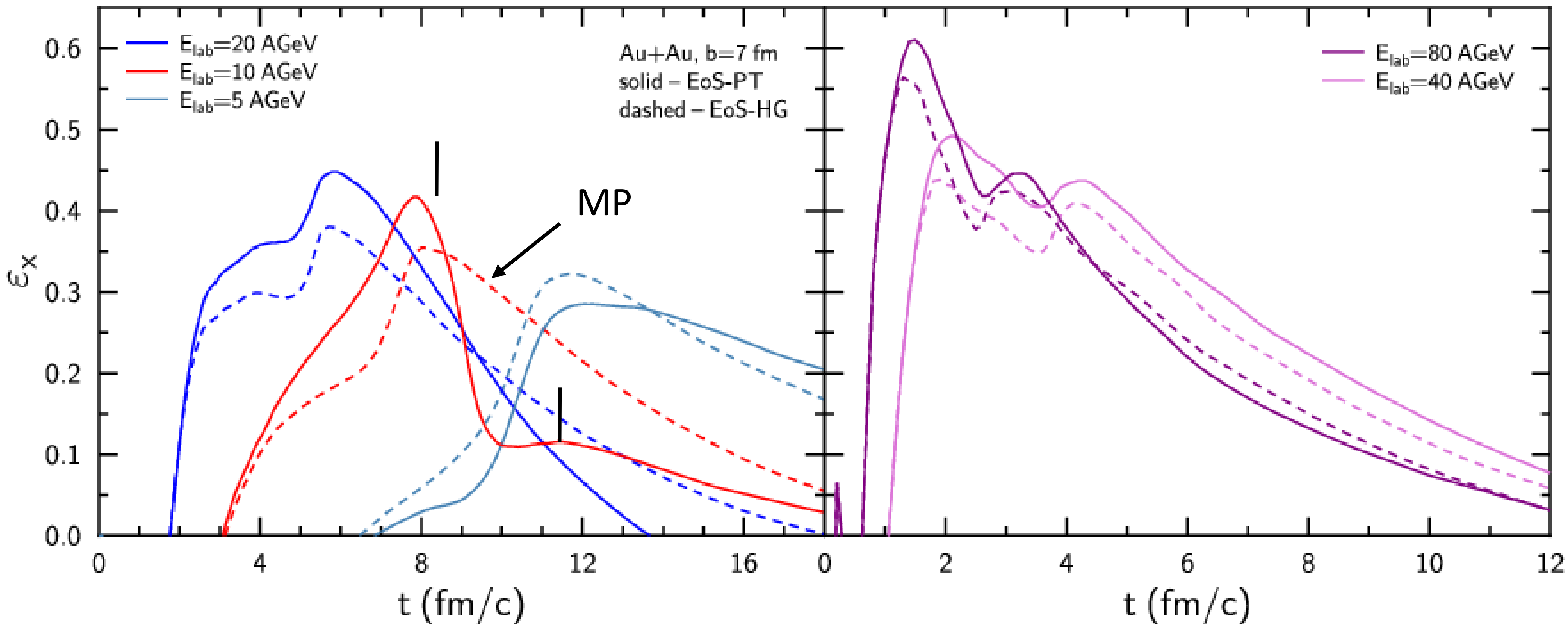
$$\frac{\varepsilon(n;T)}{n} = \gamma_0 \frac{\varepsilon_0}{n_0}$$ stopping condition

Final state of matter evolution is isentropic expansion of quark-gluon plasma,



and finally, free streaming of produced hadrons

Spatial anisotropy ε_x



- Larger drop of ε_x for EoS-PT (for Elab = 10 AGeV)

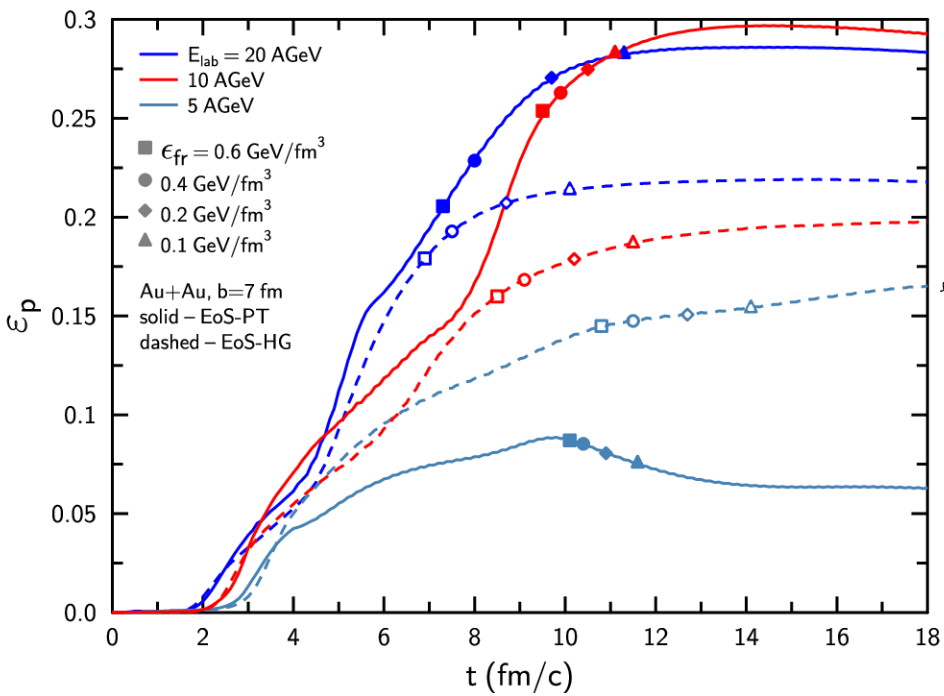
$$\varepsilon_x = \frac{\int dxdy\,(y^2 - x^2)\,\epsilon}{\int dxdy\,(y^2 + x^2)\,\epsilon}$$

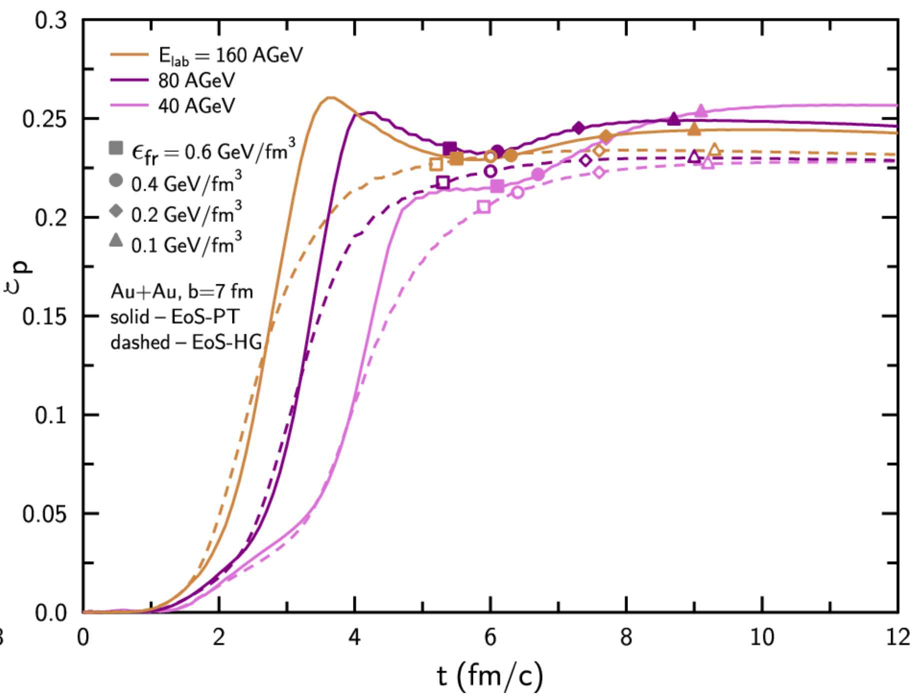
Momentum anisotropy

$$''_p = \frac{R\, dxdy\,(T^{xx} - T^{yy})}{R\, dxdy\,(T^{xx} + T^{yy})}$$

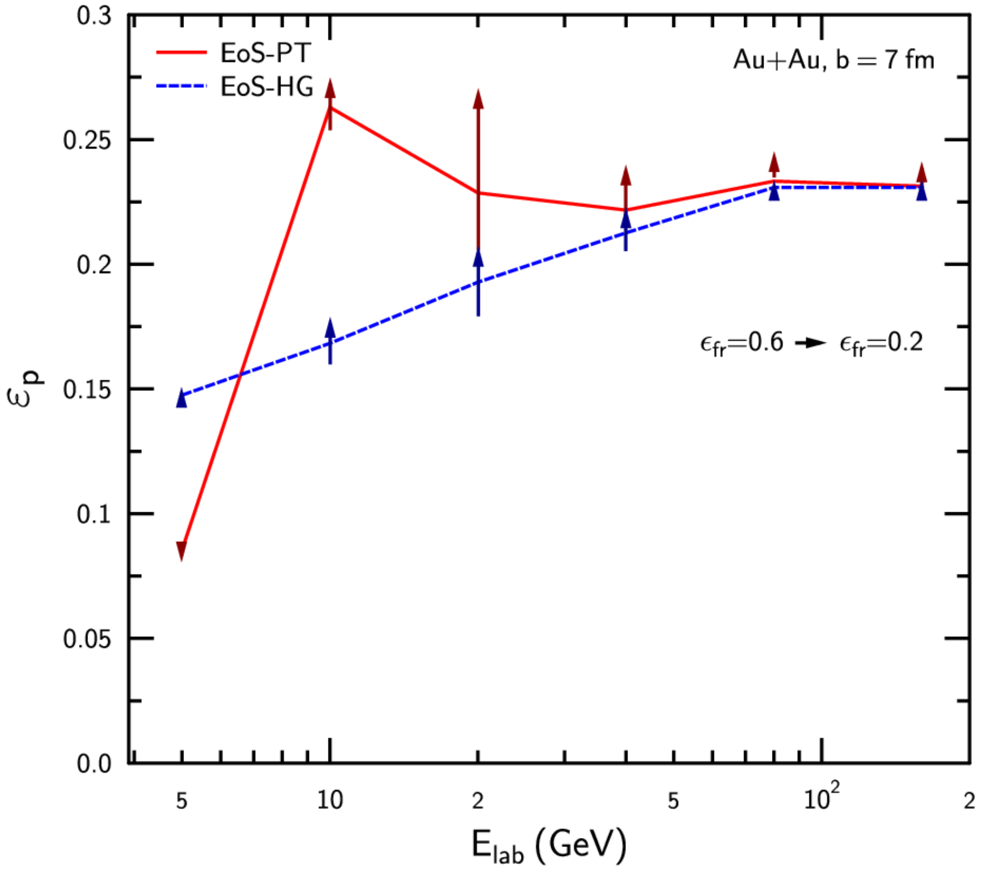
$$T^{xx} = ('' + P)\,°^2 v_x^2 + P$$

$$T^{yy} = ('' + P)\,°^2 v_y^2 + P$$





Excitation function of elliptic flow



The peak at Elab=10 AGeV is correlated with the longest time spent in the mixed phase

Hadronic spectra

$$E\frac{d^3N_i}{d^3p} = \frac{d^3N_i}{dy\,d^2p_T} = \frac{g_i}{(2\pi)^3}\int d\sigma_\mu p^\mu \left\{\exp\left(\frac{p_\nu U^\nu - \mu_i}{T}\right) \pm 1\right\}^{-1}$$

instantaneous freeze-out,
Cooper&Frye (1974)

isochronous (t=const)
freeze-out surface

$$d\sigma_\mu = d^3x \cdot \delta_{\mu;0}$$

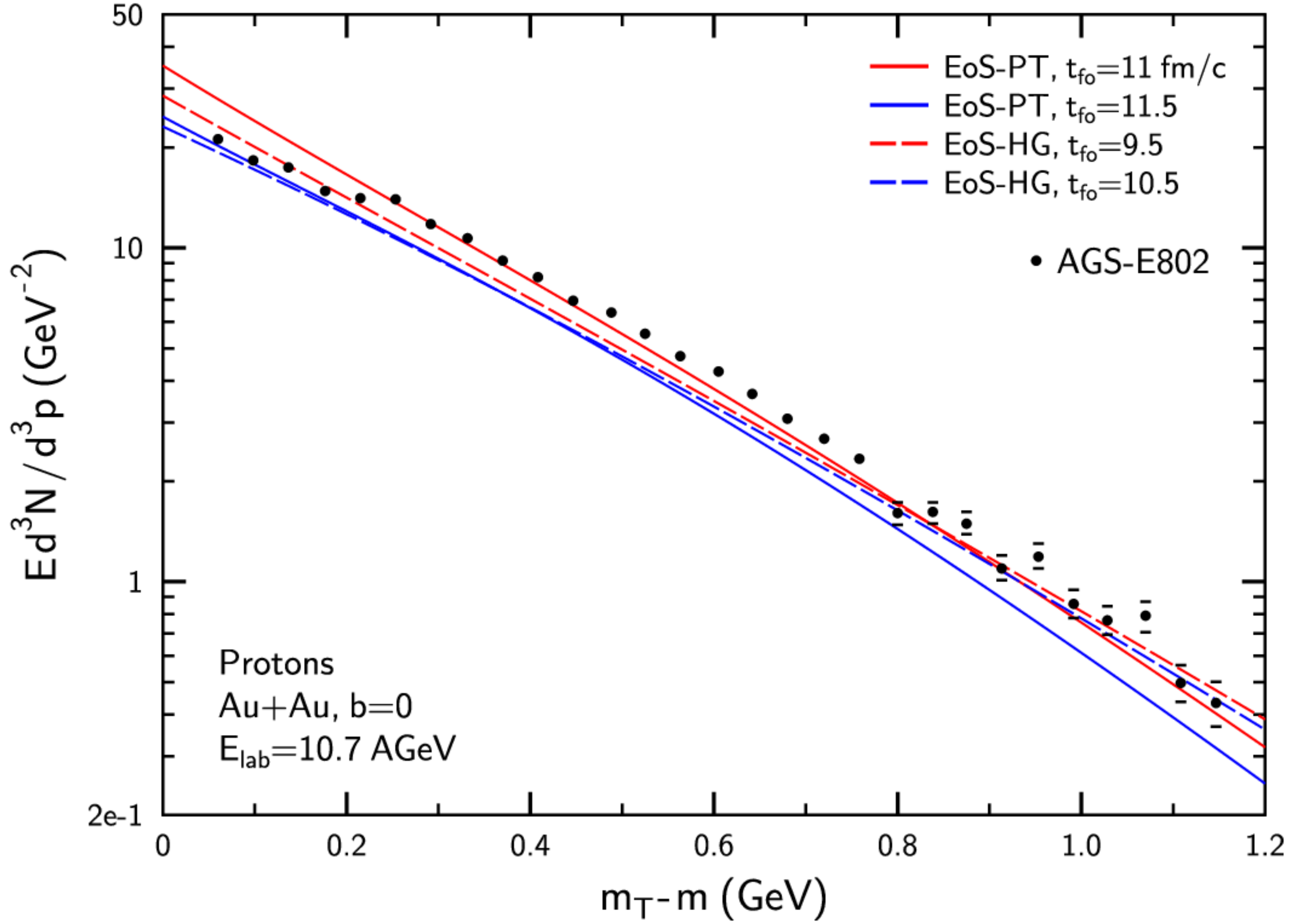
Contribution from resonance decays

$$E\frac{d^3N_{R\to iX}}{d^3p} = \frac{1}{4\pi q_0}\int d^3p_R \frac{d^3N_R}{d^3p_R}\,\delta\left(\frac{pp_R}{m_R} - E_0\right)$$

zero-width approximation

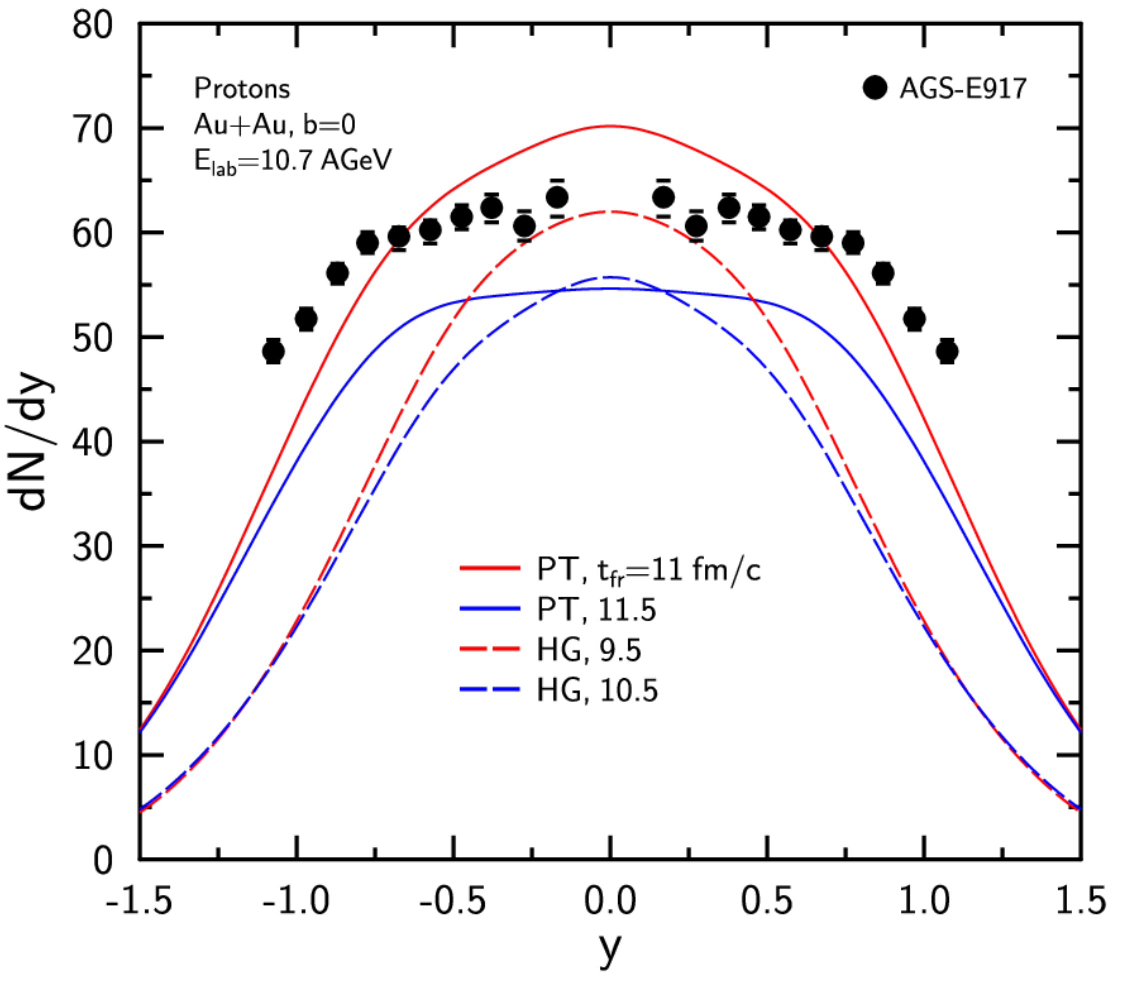
$$E_0 = \sqrt{m_i^2 + q_0^2} = \frac{m_R^2 + m_i^2 - m_X^2}{2m_R}$$

Pt spectra of produced hadrons



Low sensitivity to the EOS and freeze-out time

Proton rapidity distributions



Strong sensitivity to the EOS: more flat with PT

Summary

- **3D hydro calculations are important for understanding the dynamics of the matter evolution and physical conditions.**

- **Phase transition changes the intermediate-state dynamics but observables of the final state are not very sensitive to it.**

- **Calculations with EoS-PT as compared with EoS-HG show:**
  - higher momentum anisotropy
  - broader nucleon rapidity distributions

  } at Elab ~ 10-20 AGeV

- **Low energy program at RHIC and FAIR/NICA experiments may help to find traces of the deconfinement phase transition**

Outlook

- Incorporation of the realistic freeze-out effects: hybrid hydro-cascade approach a la Bleicher&Petersen
- Implementing non-equilibrium hadronization scenarios : explicit dynamics of the order parameter, fluctuations, critical slowing down
- Calculation of HBT radii
- Study of photon and dilepton emission
- Extension to higher energies by using fireball-like initial conditions