Lecture
The Quark model

WS2020/21: 'Physics of Strongly Interacting Matter'
The **quark model** is a classification scheme for hadrons in terms of their **valence quarks** — the quarks and antiquarks which give rise to the quantum numbers of the hadrons.

The quark model in its modern form was developed by **Murray Gell-Mann** - American physicist who received the **1969 Nobel Prize** in physics for his work on the theory of elementary particles.

* QM - independently proposed by **George Zweig**

Hadrons are not 'fundamental', but they are built from 'valence quarks', i.e. quarks and antiquarks, which give the quantum numbers of the hadrons.

\[
| \text{Baryon} \rangle = |qqq\rangle \quad | \text{Meson} \rangle = |q\bar{q}\rangle
\]

\( q = \text{quarks}, \ \bar{q} = \text{antiquarks} \)
The quark quantum numbers:

- **flavor (6):** u (up-), d (down-), s (strange-), c (charm-), t (top-), b (bottom-) quarks
  
- **anti-flavor for anti-quarks** \( \bar{q} : \bar{u}, \bar{d}, \bar{s}, \bar{c}, \bar{t}, \bar{b} \)

- **charge:** \( Q = -1/3, +2/3 \) (u: 2/3, d: -1/3, s: -1/3, c: 2/3, t: 2/3, b: -1/3 )

- **baryon number:** \( B=1/3 \) - as baryons are made out of three quarks

- **spin:** \( s=1/2 \) - quarks are the fermions!

- **strangeness:** \( S_s = -1, \ S_{\bar{s}} = 1, \ S_q = 0 \) for \( q = u, d, c, t, b \) (and \( \bar{q} \))

- **charm:** \( C_c = 1, \ C_{\bar{c}} = -1, \ C_q = 0 \) for \( q = u, d, s, t, b \) (and \( \bar{q} \))

- **bottomness:** \( B_b = -1, \ B_{\bar{b}} = 1, \ B_q = 0 \) for \( q = u, d, s, c, t \) (and \( \bar{q} \))

- **topness:** \( T_t = 1, \ T_{\bar{t}} = -1, \ T_q = 0 \) for \( q = u, d, s, c, b \) (and \( \bar{q} \))
The quark quantum numbers:

**hypercharge:** \( Y = B + S + C + B + T \)  

\( (= \text{baryon charge} + \text{strangeness} + \text{charm} + \text{bottomness} + \text{topness}) \)  

- \( I_3 \) (or \( I_z \) or \( T_3 \)) - 3‘d component of isospin

**charge** (Gell-Mann–Nishijima formula):

\[ Q = I_3 + Y/2 \]  

\( (= 3‘d \text{ component of isospin} + \text{hypercharge}/2) \)
# Quark quantum numbers

<table>
<thead>
<tr>
<th>Property</th>
<th>Quark</th>
<th>( d )</th>
<th>( u )</th>
<th>( s )</th>
<th>( c )</th>
<th>( b )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q – electric charge</td>
<td></td>
<td>(-\frac{1}{3})</td>
<td>(\frac{2}{3})</td>
<td>(-\frac{1}{3})</td>
<td>(\frac{2}{3})</td>
<td>(-\frac{1}{3})</td>
<td>(\frac{2}{3})</td>
</tr>
<tr>
<td>I – isospin</td>
<td></td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{2})</td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
</tr>
<tr>
<td>(I_z) – isospin (z)-component</td>
<td></td>
<td>(-\frac{1}{2})</td>
<td>(\frac{1}{2})</td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
</tr>
<tr>
<td>S – strangeness</td>
<td></td>
<td>(0)</td>
<td>(0)</td>
<td>(-1)</td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
</tr>
<tr>
<td>C – charm</td>
<td></td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
<td>(1)</td>
<td>(0)</td>
<td>(0)</td>
</tr>
<tr>
<td>B – bottomness</td>
<td></td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
<td>(-1)</td>
<td>(0)</td>
</tr>
<tr>
<td>T – topness</td>
<td></td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
<td>(1)</td>
</tr>
</tbody>
</table>
The quark model is the follow-up to the **Eightfold Way** classification scheme (proposed by Murray Gell-Mann and Yuval Ne'eman, 1961)

The **Eightfold Way** may be understood as a consequence of **flavor symmetries** between various kinds of quarks. Since the strong nuclear force affects quarks the same way regardless of their flavor, replacing one flavor of a quark with another in a hadron should not change its mass very much. Mathematically, this replacement may be described by **elements of the SU(3) group**.

Consider **u, d, s** quarks:

- then the quarks lie in the fundamental representation, 3 (called the **triplet**) of the **flavour group** SU(3): [3]

The **antiquarks** lie in the complex conjugate representation 3 - anti-triplet: [3]
Quark quantum numbers

triplet in SU(3)$_\text{flavor}$ group: [3]

anti-triplet in SU(3)$_\text{flavor}$ group: [\bar{3}]

Y=2(Q-T$_3$)

E.g. u-quark: Q=+2/3, T$_3$=+1/2 $\Rightarrow$ Y=1/3
The quark quantum numbers:

- **Collor 3**: red, green and blue \(\rightarrow\) triplet in \(SU(3)_{\text{collor}}\) group: [3]

- **Anticollor 3**: antired, antigreen and antiblue \(\rightarrow\) anti-triplet in \(SU(3)_{\text{collor}}\) group [\(\bar{3}\)]

- The quark colors (red, green, blue) combine to be **colorless**
- The quark anticolors (antired, antigreen, antiblue) also combine to be **colorless**

All hadrons \(\rightarrow\) color neutral \(=\) color singlet in the \(SU(3)_{\text{collor}}\) group

**History**: The quantum number 'color' has been introduced (idea from Greenberg, 1964) to describe the state \(\Delta^{++}(uuu)\) \((Q=+2, J=3/2)\), discovered by Fermi in 1951 as \(\pi^+p\) resonance: \(\Delta^{++}(uuu) \rightarrow p(uud) + \pi^+(\bar{d}u)\)

The state \(\Delta^{++}(u \uparrow u \uparrow u \uparrow)\) with all parallel spins (to achieve \(J=3/2\)) is forbidden according to the Fermi statistics (without color)!

Concept of color charge – finalized in 1973: William Bardeen, Harald Fritzsch, and Murray Gell-Mann
Quark quantum numbers

**The current quark masses:**
The current quark mass is also called as the mass of the 'naked' ('bare') quark defined as the constituent quark cores (constituent quarks with no covering) of a valence quark.

- **current masses of the quarks**
  
  \[
  \begin{align*}
  m_u &= 1.8 - 2.8 \text{ MeV/c}^2 \\
  m_d &= 4.3 - 5.2 \text{ MeV/c}^2 \\
  m_s &= 92 - 104 \text{ MeV/c}^2 \\
  m_c &= 1.3 - 1.5 \text{ GeV/c}^2 \\
  m_b &= 4.2 - 4.7 \text{ GeV/c}^2 \\
  m_t &= 156 - 176 \text{ GeV/c}^2
  \end{align*}
  \]

**Note:** the constituent quark mass is the mass of a 'dressed' current quark, i.e. for quarks surrounded by a cloud of virtual quarks and gluons:

\[
M_{u(d)}^* \sim 350 \text{ MeV/c}^2
\]
Building Blocks of Matter

Fermionen

Periodensystem

Leptonen
- \( \tau \)
- \( \mu \)
- \( e \)

Quarks
- \( t \)
- \( b \)
- \( c \)
- \( s \)
- \( d \)
- \( u \)

Electro Darstellung

- \( u \)
- \( d \)
- \( s \)
- \( c \)
- \( t \)
- \( b \)
- \( e \)
- \( \mu \)
- \( \tau \)

Generation

Farbe

Elektrische Ladung

m_{q,L} [MeV]

[MeV]
Hadrons in the Quark model

Gell-Mann (1964): **Hadrons are not, fundamental', but they are built from, valence quarks',**

\[ | \text{Baryon} \rangle = | \text{qqq} \rangle \quad | \text{Meson} \rangle = | \text{qq} \rangle \]  

**Baryon charge:**  
\[ B_B = 1 \quad B_m = 0 \]

**Constraints** to build hadrons from quarks:
- strong color interaction (red, green, blue)
- confinement
- quarks must form color-neutral hadrons

**State function for baryons (fermions) — antisymmetric** under interchange of two quarks

\[ \Psi_A \equiv |qqq\rangle_A = [ |\text{color}\rangle \otimes |\text{space}\rangle \otimes |\text{spin}\rangle \otimes |\text{flavor}\rangle ]_A \]  

Since all baryons are color neutral, the color part of \( \Psi_A \) must be antisymmetric, i.e. a \( \text{SU}(3) \) \text{color singlet} (c.f. \( \Delta^+ (u \uparrow u \uparrow u \uparrow) \) state \( J=S=3/2, L=0 \):  
\[ [ |\text{space}\rangle_s \otimes |\text{spin}\rangle_s \otimes |\text{flavor}\rangle_s ]_s \) )

\[ \Psi_A \equiv |qqq\rangle_A = |\text{color}\rangle_A \otimes [ |\text{space}\rangle \otimes |\text{spin}\rangle \otimes |\text{flavor}\rangle ]_s \]  

symmetric

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Hadrons in Quark model

Possible states $\Psi_A$:

$$\Psi_A = |\text{color}\rangle_A \otimes [ |\text{space}\rangle_S \otimes |\text{spin}\rangle_A \otimes |\text{flavor}\rangle_A ]_S$$ (6)

$$[ |\text{space}\rangle_S \otimes |\text{spin}\rangle_S \otimes |\text{flavor}\rangle_S ]_S$$ (7)

or a linear combination of (6) and (7):

$$\Psi_A = \alpha |\text{color}\rangle_A \otimes [ |\text{space}\rangle_S \otimes |\text{spin}\rangle_A \otimes |\text{flavor}\rangle_A ]_S$$

$$+ \beta |\text{color}\rangle_A \otimes [ |\text{space}\rangle_S \otimes |\text{spin}\rangle_S \otimes |\text{flavor}\rangle_S ]_S$$ (8)

where $\alpha^2 + \beta^2 = 1$

Consider flavor space (u,d,s quarks) $\rightarrow$ SU(3)$_{\text{flavor}}$ group

Possible states: $|\text{flavor}\rangle$:

- (6) – antisymmetric
- (7) – symmetric
- (8) – mixed symmetry
Mesons in the Quark model

\[ |\text{Meson} \rangle = |qq^- \rangle \]

Quark triplet in SU(3)_{flavor} group: [3]

Anti-quark anti-triplet in SU(3)_{flavor} group: [\bar{3}]

From group theory: the nine states (nonet) made out of a pair can be decomposed into the trivial representation, 1 (called the singlet), and the adjoint representation, 8 (called the octet).

\[ [3] \otimes [\bar{3}] = [8] \oplus [1] \]

octet + singlet
Mesons in the Quark model

3 states: $Y=0$, $I_3=0$

linear combination of $uar{u}$, $dar{d}$, $sar{s}$

$A = \sqrt{\frac{1}{2}} (uar{u} - dar{d}) \Rightarrow \pi^0$, $B = \sqrt{\frac{1}{6}} (uar{u} + dar{d} - 2sar{s}) \Rightarrow \eta$

$C = \sqrt{\frac{1}{3}} (uar{u} + dar{d} + sar{s}) \Rightarrow \eta'$
Mesons in the Quark model

Classification of mesons:

❖ **Quantum numbers:**
  • spin $S$
  • orbital angular momentum $L$
  • total angular momentum $J = L + S$

❖ **Properties with respect to Poincare transformation:**

1) **continuos transformation** ➔ Lorentz boost (3 parameters: $\beta$)

$$ U_B \sim e^{i\beta\hat{a}} $$

Casimir operator (invariant under transformation): $M^2 = p_\mu p^\mu$

2) **rotations** (3 parameters: Euler angle $\phi$):

$$ U_R \sim e^{i\phi\hat{J}} $$

Casimir operator: $J^2$

3) **space-time shifts** (4 parameters: $a_\mu$)

$$ U_{st} \sim e^{i\alpha_\mu x^\mu} $$

$x'_{\mu} \rightarrow x_{\mu} + a_{\mu}$
Mesons in the Quark model

Classification of mesons:

❖ **Discrete operators:**

4) **parity transformation:** flip in sign of the spacial coordinate \( \vec{r} \mapsto -\vec{r} \)

\[ P = (-1)^L + 1 \]

eigenvalue \( P = \pm 1 \)

5) **time reversal:** \( t \mapsto -t \)

eigenvalue \( T = \pm 1 \)

6) **charge conjugation:** \( C = -C \) \((C\text{-parity applies only to neutral systems})\)

\[ C = (-1)^L + s \]

\( C \text{- parity: eigenvalue } C = \pm 1 \)

⇒ **General PCT theorem:** \( P \cdot C \cdot T = 1 \)

due to the fact that discrete transformations correspond to the U(1) group they are multiplicative

Properties of the distinguishable (not continuum!) particles are defined by

\[ M^2 (\text{or } M), \quad J^2 (\text{or } J), \quad P, \ C \]
Mesons in the Quark model

Classification of mesons:

the mesons are classified in $J^{PC}$ multiplets

1) $L=0$ states: $J=0$ or 1, i.e. $J=S$

\[ P = (-1)^L + 1 = -1 \]

\[ C = (-1)^L + S = (-1)^S = \begin{cases} +1 & \text{for } S=0 \\ -1 & \text{for } S=1 \end{cases} \]

\[ J^{PC} = \begin{cases} 0^{++} \text{ - pseudoscalar states} \\ 1^{--} \text{ - vector states} \end{cases} \]

2) $L=1$ states - orbital excitations; $P = (-1)^L + 1 = +1$

\[ \bar{J} = \bar{L} + \bar{S} \]

\[ \begin{cases} S=-1 \ J=0 & J^{PC} = 0^{++} \text{ - scalar states} \\ S=0 \ J=1 & 1^{++} \text{ - axial vectors} \\ S=1 \ J=2 & 1^{+-} \text{ - axial vectors} \end{cases} \]

\[ 1^{++} \text{ - tensor} \]

\[ |L - S| \leq J \leq L + S \]
Mesons in the Quark model

Classification used in PDG (Particle Data Group)
C-parity applies only to neutral systems

\[ I^G (J^{PC}) \text{ or } I^G (J^P) \text{ multiplets} \]

G-parity is a multiplicative quantum number that results from the generalization of C-parity to multiplets of particles, i.e. to all charged states (since strong interaction doesn’t depend on electric charge!)

\[ G = C (-1)^I \]

I – isospin of particles

Since \( C = (-1)^L + S \)

\[ G = C (-1)^{L+S} + I \]

\[ \pi^0 \]  
\[ \pi^\pm \]

\( I^G(J^{PC}) = 1^-(0^-) \)

\( I^G(J^P) = 1^-(0^-) \)

Citation: P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020)
## Mesons in the Quark model

### isospin

<table>
<thead>
<tr>
<th>$L$</th>
<th>$S$</th>
<th>$J^{PC}$</th>
<th>$I = 1$</th>
<th>$I = 1/2$</th>
<th>$I = 0$</th>
<th>$m$ [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L = 0$</td>
<td>$S = 0$</td>
<td>$0^{-+}$</td>
<td>$\pi$</td>
<td>$K$</td>
<td>$\eta, \eta'$</td>
<td>140 ($m_{\pi}$) − 500</td>
</tr>
<tr>
<td></td>
<td>$S = 1$</td>
<td>$1^{--}$</td>
<td>$\rho$</td>
<td>$K^*$</td>
<td>$\omega, \phi$</td>
<td>~ 800</td>
</tr>
<tr>
<td>$L = 1$</td>
<td>$S = 0$</td>
<td>$1^{+-}$</td>
<td>$B$</td>
<td>$Q_2$</td>
<td>$H$</td>
<td>1250</td>
</tr>
<tr>
<td></td>
<td>$S = 1$</td>
<td>$2^{++}$</td>
<td>$A_2$</td>
<td>$K'^*$</td>
<td>$f, f'$</td>
<td>1400</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$1^{++}$</td>
<td>$A_1$</td>
<td>$Q_1$</td>
<td>$D$</td>
<td>1300</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$0^{++}$</td>
<td>$\delta$</td>
<td>$\kappa$</td>
<td>$\epsilon, S^*$</td>
<td>1150</td>
</tr>
</tbody>
</table>
**Mesons in the Quark model**

### $J^{PC} = 0^{--}$ - Pseudoscalar nonet (L=0, S=0)

<table>
<thead>
<tr>
<th>particle</th>
<th>symbol</th>
<th>Mass (MeV)</th>
<th>$I_{s}$</th>
<th>$I_{z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+$</td>
<td>$u\bar{d}$</td>
<td>140</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\pi^-$</td>
<td>$d\bar{u}$</td>
<td>135</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>$\pi^0$</td>
<td>$(u\bar{u} - d\bar{d})/\sqrt{2}$</td>
<td>135</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$K^+$</td>
<td>$u\bar{s}$</td>
<td>494</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>$K^-$</td>
<td>$s\bar{u}$</td>
<td>494</td>
<td>1/2</td>
<td>-1/2</td>
</tr>
<tr>
<td>$K^0$</td>
<td>$d\bar{s}$</td>
<td>$\approx$</td>
<td>1/2</td>
<td>-1/2</td>
</tr>
<tr>
<td>$\bar{K}^0$</td>
<td>$s\bar{d}$</td>
<td>$\approx$</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$(u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$</td>
<td>548</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\eta'$</td>
<td>$(u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$</td>
<td>958</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### $J^{PC} = 1^{--}$ - Vector nonet (L=0, S=1)

<table>
<thead>
<tr>
<th>particle</th>
<th>symbol</th>
<th>Mass (MeV)</th>
<th>$I_{s}$</th>
<th>$I_{z}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho^+$</td>
<td>$u\bar{d}$</td>
<td>770</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$\rho^-$</td>
<td>$d\bar{u}$</td>
<td>$\approx$</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>$\rho^0$</td>
<td>$(u\bar{u} - d\bar{d})/\sqrt{2}$</td>
<td>$\approx$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$K^{*+}$</td>
<td>$u\bar{s}$</td>
<td>892</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>$K^{*-}$</td>
<td>$s\bar{u}$</td>
<td>$\approx$</td>
<td>1/2</td>
<td>-1/2</td>
</tr>
<tr>
<td>$K^{*0}$</td>
<td>$d\bar{s}$</td>
<td>$\approx$</td>
<td>1/2</td>
<td>-1/2</td>
</tr>
<tr>
<td>$\bar{K}^{*0}$</td>
<td>$s\bar{d}$</td>
<td>$\approx$</td>
<td>1/2</td>
<td>1/2</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$(u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$</td>
<td>782</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$(u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$</td>
<td>1020</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Baryons in the Quark model

\[ |\text{Baryon} \rangle = |qqq \rangle \]

**Quark triplet in SU(3)_{flavor} group:** \([3]\)

**Eqs. (4-8): state function for baryons** – **antisymmetric** under interchange of two quarks:

\[ \Psi_A \equiv |qqq \rangle_A = [|\text{color} \rangle \otimes |\text{space} \rangle \otimes |\text{spin} \rangle \otimes |\text{flavor} \rangle \rangle_A \]

where \(|\text{flavor} \rangle\) state can be symmetric (S), antisymmetric (A) or mixed symmetry (M)

**From group theory:** with three flavours, the decomposition in flavour is

\[
\]

\[
= ([6]_S \otimes [3]) \oplus ([\bar{3}] \otimes [3]) =
\]

\[
= [10]_S \oplus [8]_M \oplus [8]_M \oplus [1]_A
\]

The **decuplet** is **symmetric in flavour**, the **singlet** **antisymmetric** and the **two octets** have **mixed symmetry** (they are connected by a unitary transformation and thus describe the same states).

The **space and spin parts of the states** are then fixed once the orbital angular momentum is given.
Baryons in the Quark model

1) Combine first 2 quark triplets:

\[
[3] \otimes [3] = [6]_S \oplus [\bar{3}]_A
\]

2) Add a 3\textsuperscript{rd} quark:

\[
[3] \otimes [3] \otimes [3] = ([6]_S \oplus [\bar{3}]_A) \otimes [3] = [10]_S \oplus [8]_M \oplus [8]_M \oplus [1]_A
\]
Baryons in the Quark model

Octet [8]

Spin:

\[ J = S \]

\[ L = 0 \]

\[ J^P = \frac{1}{2}^+ \]

Decuplet [10]

Spin:

\[ J = S + L \]

\[ L = 1 \]

\[ J^P = \frac{3}{2}^+ \]
### Structure of known baryons

#### Ground states of Baryons

<table>
<thead>
<tr>
<th>$M$/MeV</th>
<th>$1/2^+$ $\Delta$</th>
<th>$3/2^+$ $\Sigma^*$</th>
<th>$1/2^+$ $\Lambda$</th>
<th>$1/2^+$ $\Sigma$</th>
<th>$3/2^+$ $\Xi$</th>
<th>$3/2^+$ $\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$I$</td>
<td>$1/2$</td>
<td>$3/2$</td>
<td>0</td>
<td>1</td>
<td>$1/2$</td>
<td>0</td>
</tr>
<tr>
<td>$S$</td>
<td>0</td>
<td>0</td>
<td>$-1$</td>
<td>$-1$</td>
<td>$-2$</td>
<td>$-3$</td>
</tr>
</tbody>
</table>

#### Excitation spectra

<table>
<thead>
<tr>
<th>$M$/MeV</th>
<th>$15/2^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>+</td>
</tr>
<tr>
<td>$I$</td>
<td>$1/2$</td>
</tr>
<tr>
<td>$S$</td>
<td>0</td>
</tr>
</tbody>
</table>
Now consider the basis states of mesons in 4 flavour $SU(4)_{\text{flavor}}$: $u, d, s, c$ quarks


SU(4) weight diagram showing the 16-plets for the pseudoscalar and vector mesons as a function of isospin $I$, charm $C$ and hypercharge $Y$. The nonets of light mesons occupy the central planes to which the ccbar states have been added.
Now consider the basis states of baryons in 4 flavour SU(4) flavour: \( u, d, s, c \) quarks

**SU(4) multiplets of baryons** made of \( u, d, s, \) and \( c \) quarks:

- the **20-plet** with an SU(3) octet  and the **20-plet** with an SU(3) decuplet.
If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" 1-3), we are tempted to look for some fundamental explanation of the situation. A highly promised approach is the purely dynamical "bootstrap" model for all the strongly interacting particles within which one may try to derive isotopic spin and strangeness conservation and broken eightfold symmetry from self-consistency alone 4). Of course, with only strong interactions, the orientation of the asymmetry in the unitary space cannot be specified; one hopes that in some way the selection of specific components of the F-spin by electromagnetism and the weak interactions determines the choice of isotopic spin and hypercharge directions.

Even if we consider the scattering amplitudes of strongly interacting particles on the mass shell only and treat the matrix elements of the weak, electromagnetic, and gravitational interactions by means...
Exotic states

\[ |\text{Meson} \rangle = |qq \rangle + |qqqq \rangle + |qqg \rangle + ... \]

\[ |\text{Baryon} \rangle = |qqq \rangle + |qqqq q \rangle + |qqqg \rangle + ... \]

\[ |\text{Hybrid} \rangle = |qqg \rangle + ... \]

\[ |\text{Baryonium} \rangle = |qqqq \rangle + ... \]

\[ |\text{Glueball} \rangle = |gg \rangle + ... \]

Experimental evidence:

\[ \pi(1400) \]

\[ \sigma(600) \]

\[ f_{o}(1500) \]

\[ \parallel \]

very broad width

(200-300 MeV) => short lifetime < 1 fm/c

\[ |\text{Pentaquark} \rangle = |qqqq q \rangle + ... \]
Pentaquarks

„Flavour“-exotic state, e.g.

\[ |\Theta^+\rangle = |uudd\bar{s}\rangle \]

Decay:

\[ |uudd\bar{s}\rangle \rightarrow |uud\rangle + |us\rangle \]

\[ |\Theta^+\rangle \rightarrow |n\rangle + |K^+\rangle \]

\[ |uudd\bar{s}\rangle \rightarrow |uud\rangle + |ds\rangle \]

\[ |\Theta^+\rangle \rightarrow |p\rangle + |K^0\rangle \]

Very small life time (big width)?
Chiral Lagrangean:

\[ L_{\text{eff}} = -\bar{q} \left[ i\gamma^\mu \nabla_\mu - M \exp(i\gamma_5 \pi^a \lambda^a / f_\pi) \right] q \]

Pseudoscalar fields:\n\[ \pi^a = \{\pi, K, \eta\} \]

Chiral Quark-Soliton Model:

- solution of the Euler-Lagrange equation-of-motion => Solitons
- quantization of the soliton solutions under SU(3)_f

Predictions for the pentaquark state:

- Spin J=1/2, Parity P=positive: \( J^P = 1/2^+ \)
- Width \( \Gamma < 30 \text{ MeV} \)
- SU(3)_f - Antidecuplet
Predictions for the pentaquarks:

- Spin $J=1/2$, Parity $P=\text{positiv}$: $J^P = 1/2^+$
- Width $\Gamma < 15$ MeV
- SU(3)$_f$ - Antidecuplet + Oktet
  Oktet: Partner with $J^P = 3/2^+$
Other models of pentaquarks

A five-quark "bag"

A "meson-baryon molecule"
Positive experimental signals of $\Theta^+(1540)$
… but not seen by other experiments

CDF

BABAR

DELPHI

Delphi/LEP

ALEPH/LEP

E690/FermiLab

HyperCP

STAR/RHIC

HERA-B

PHENIX/RHIC
EXOTIC BARYONS

Minimum quark content: $\Theta^+ = uudd\bar{s}$, $\Phi^- = ssdd\bar{u}$, $\Phi^+ = ssuu\bar{d}$.

$\Theta(1540)^+$

$I(J^P) = 0(?)$

It is difficult to deny a place in the Summary Tables for a state that six experiments claim to have seen. Nevertheless, we believe it reasonable to have some reservations about the existence of this state on the basis of the present evidence.

Mass $m = 1539.2 \pm 1.6$ MeV
Full width $\Gamma = 0.90 \pm 0.30$ MeV

$NK$ is the only strong decay mode allowed for a strangeness $S=+1$ resonance of this mass.

$\Theta(1540)^+$ DECAY MODES

<table>
<thead>
<tr>
<th>Decay</th>
<th>Fraction ($\Gamma_i/\Gamma$)</th>
<th>$p$ (MeV/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$KN$</td>
<td>100%</td>
<td>270</td>
</tr>
</tbody>
</table>
In July 2015, the LHCb collaboration at CERN identified pentaquarks in the $\Lambda^0_{b} \to J/\psi K^- p$ channel.

- Two states, named $P^+_c(4380)$ and $P^+_c(4450)$