

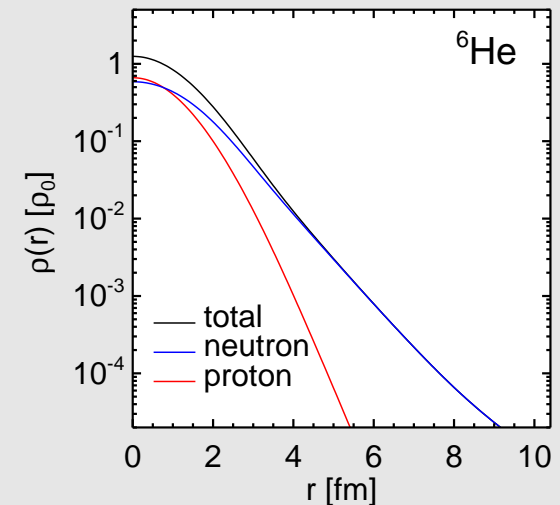
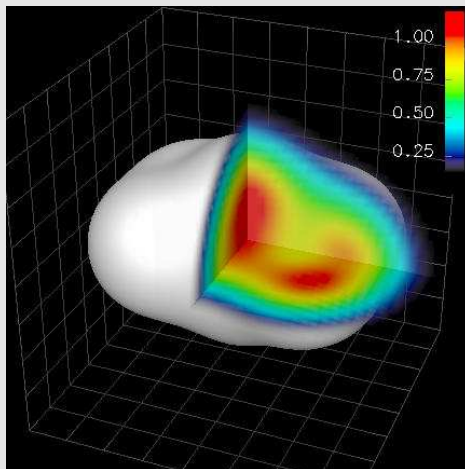
# Von Schalen, Clustern und Halos

- Neue Konzepte zur Lösung des nuklearen Vielteilchenproblems -

Hans Feldmeier  
GSI Darmstadt

Thomas Neff  
MSU East Lansing

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TU Darmstadt

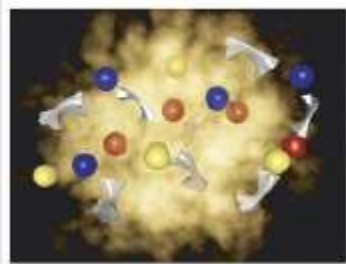
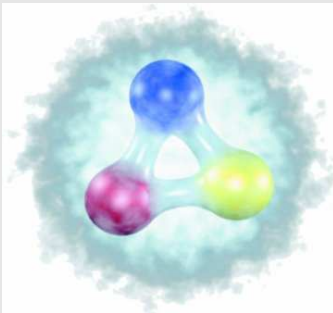
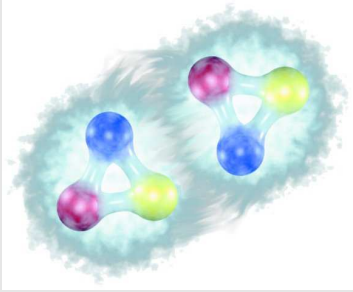
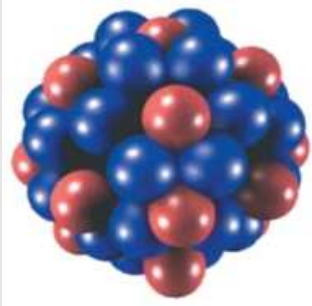


# Von der QCD zur Kernstruktur

mehr Auflösung / fundamentaler

Quantum Chromo Dynamics

Kernstruktur



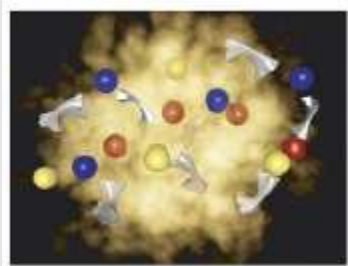
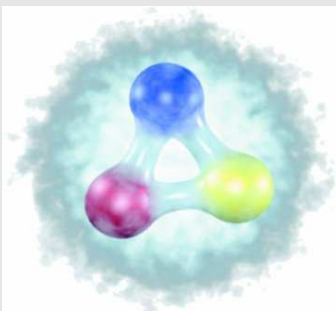
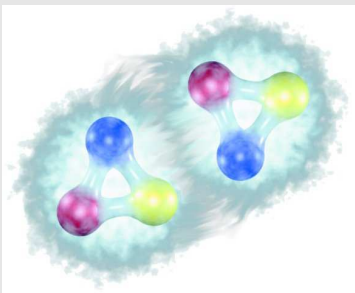
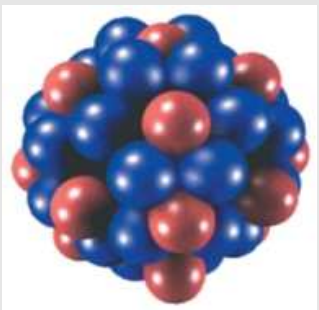
- Atomkerne
- Wenig-Nukleonen-Systeme
- Nukleon-Nukleon-Potential
- Hadronenstruktur
- Quarks & Gluonen
- Deconfinement

# Von der QCD zur Kernstruktur

mehr Auflösung / fundamentaler

Quantum Chromo Dynamics

Kernstruktur



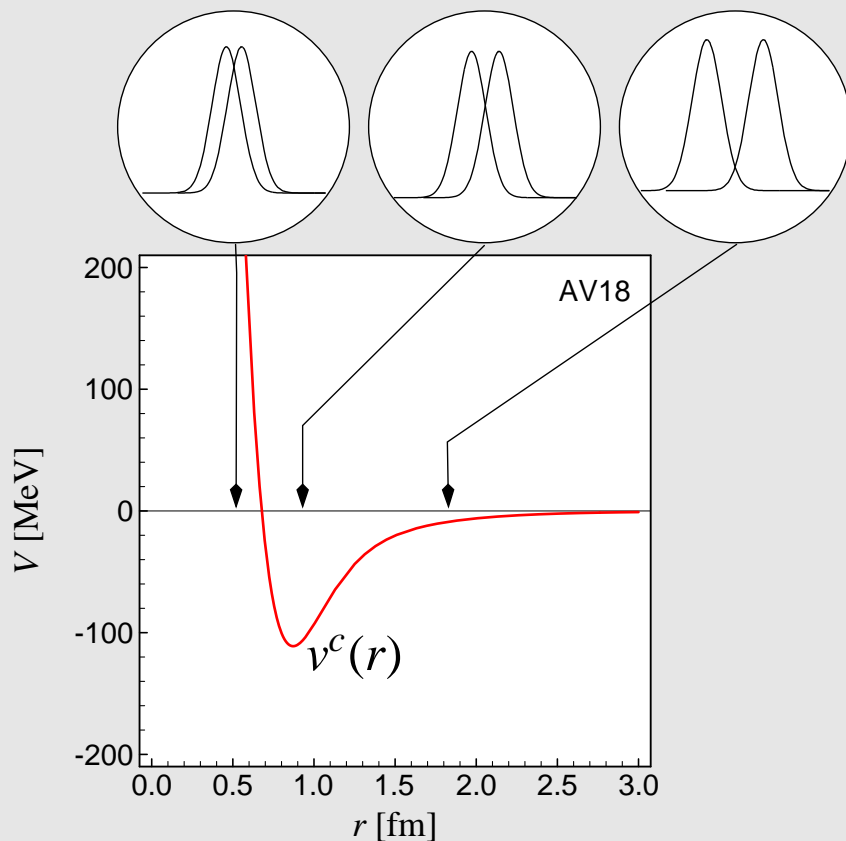
*löse*  
das wechselwirkende nukleare  
Vielteilchenproblem

*konstruiere*  
realistische Nukleon-Nukleon  
Wechselwirkung aus der QCD

# Potential und Größe des Protons

Proton Ladungsradius  $\sqrt{\langle r^2 \rangle_e} \approx 0.86$  fm

## Proton Ladungsverteilung und $S=0, T=1$ Potential



- Proton Durchmesser nicht klein gegen Reichweite der Wechselwirkung
- Halbdichte-Überlapp bei maximaler Anziehung, Ausläufer berühren sich noch bei mittlerem NN-Abstand in Kernmaterie
- $V_{NN}$  nicht elementar ähnlich wie Atom-Atom-Potential
- erwartet Dreikörperkräfte

# Realistic Nucleon-Nucleon Interaction

## Realistic NN Potential

- describes phase shifts and deuteron properties
- central, spin-orbit, tensor, momentum dependent

$$\tilde{V}_{NN}(\mathbf{r}_{12}, \mathbf{p}_{12}, \sigma_1, \sigma_2, \tau_1, \tau_2)$$

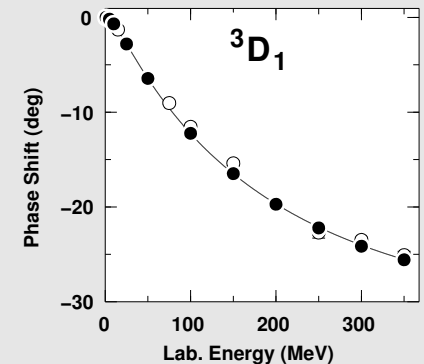
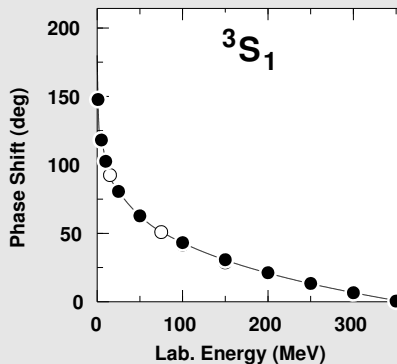
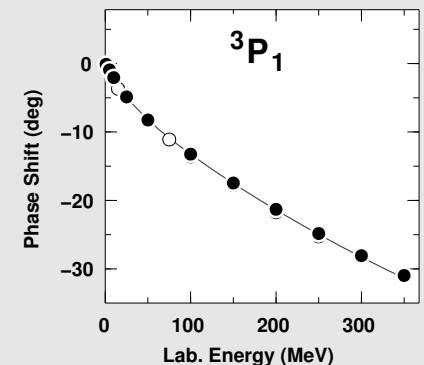
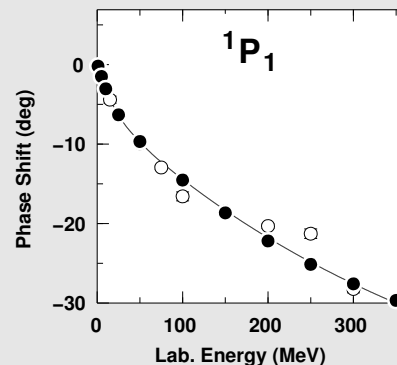
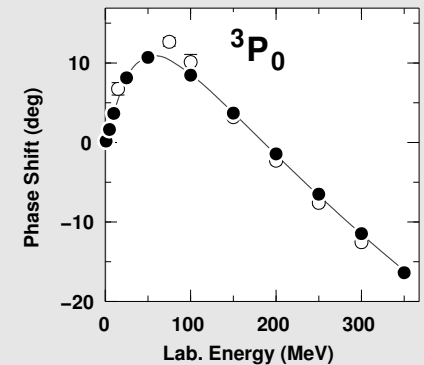
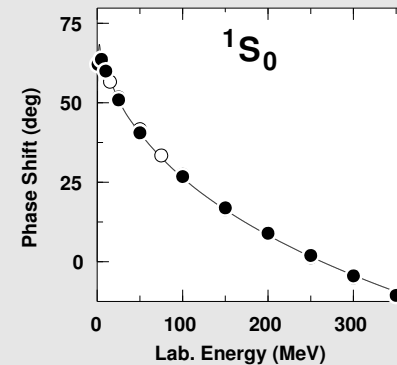
different realistic NN interactions describe scattering data

e.g.

- chiral effective field theory (pion exchange & contact terms)
- Bonn potential (pion & heavier meson exchange)
- Argonne potential (pion exchange & phenomenological short range terms)

## NN scattering data

$2S+1L_J$



# Ab Initio Nuclear Structure Calculations

describe basic properties of nuclear many-body system

in terms of a **realistic** nucleon-nucleon interaction  $\tilde{H}$  and a many-body state  $|\hat{\Psi}\rangle$

$\langle \mathbf{r}_1, \sigma_1, \tau_1; \mathbf{r}_2, \sigma_2, \tau_2; \dots; \mathbf{r}_A, \sigma_A, \tau_A | \hat{\Psi} \rangle$

**Aim**

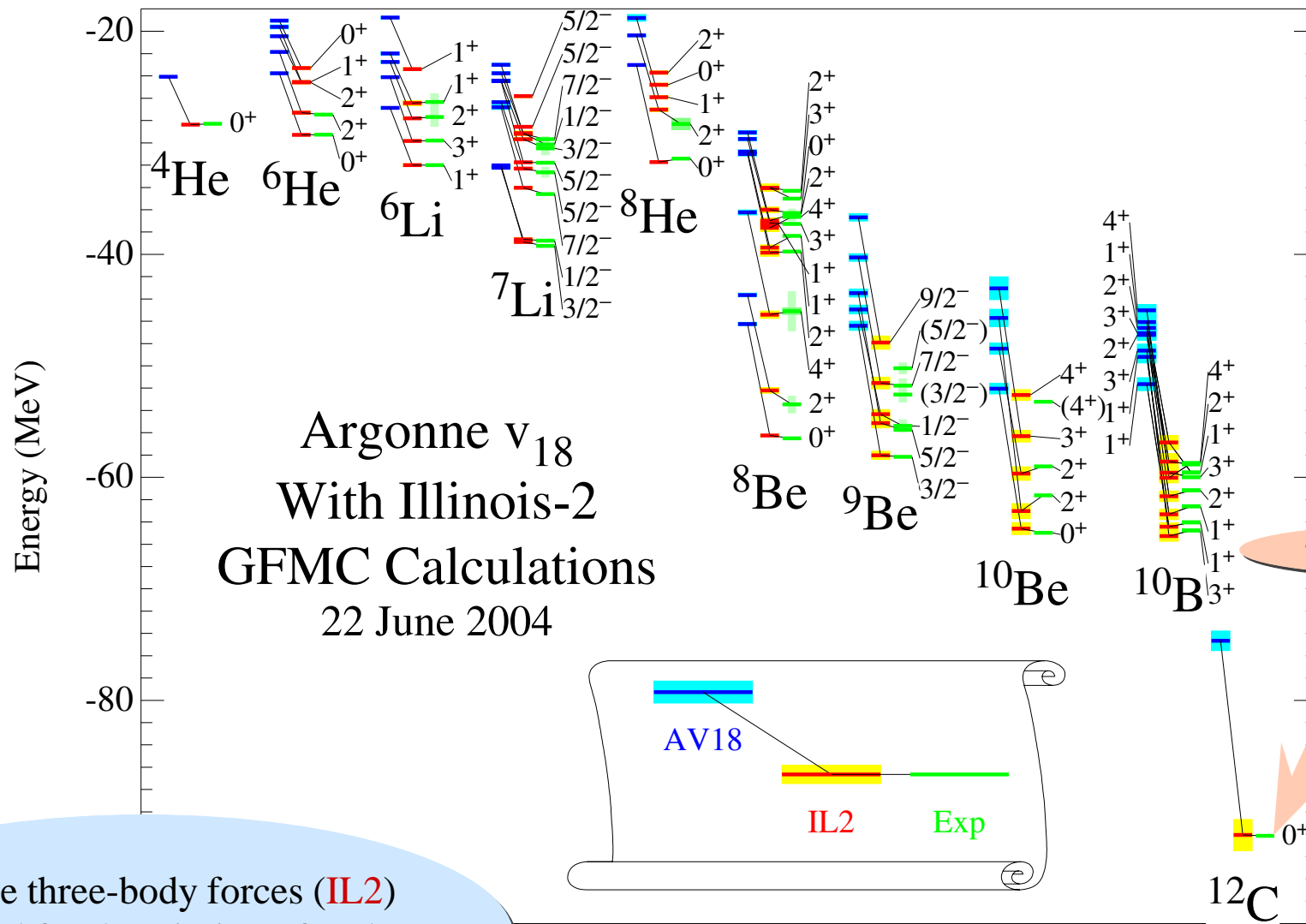
→ solve

$$\tilde{H} |\hat{\Psi}_n\rangle = E_n |\hat{\Psi}_n\rangle$$

by means of Green Function Monte Carlo Method (GFMC)



# GFMC calculations ("exact" numerical solution)



genuine three-body forces (IL2)  
needed for description of real  
nuclei

$^{12}\text{C}$  results are preliminary.

# Ab Initio Kernstruktur

GFMC Ergebnis:

$$\langle \mathbf{r}_1, \sigma_1, \tau_1; \mathbf{r}_2, \sigma_2, \tau_2; \dots; \mathbf{r}_A, \sigma_A, \tau_A | \hat{\Psi}_n \rangle$$

ist sehr kompliziert für  
realistische Nukleon-Nukleon-Potentiale.

**WARUM?**

# Inhalt

✓ Wo liegt das Problem?

- Unitary Correlation Operator Method  
realistischer → korrelierter Hamiltonian

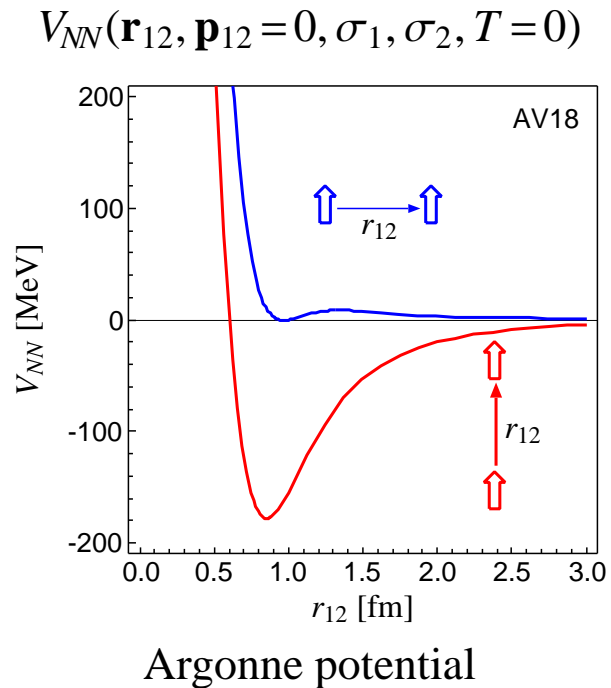
- Anwendungen:

No-Core Schalenmodell

Fermionische Molekular Dynamik

- Zusammenfassung und Ausblick

# Realistic $NN$ -potential

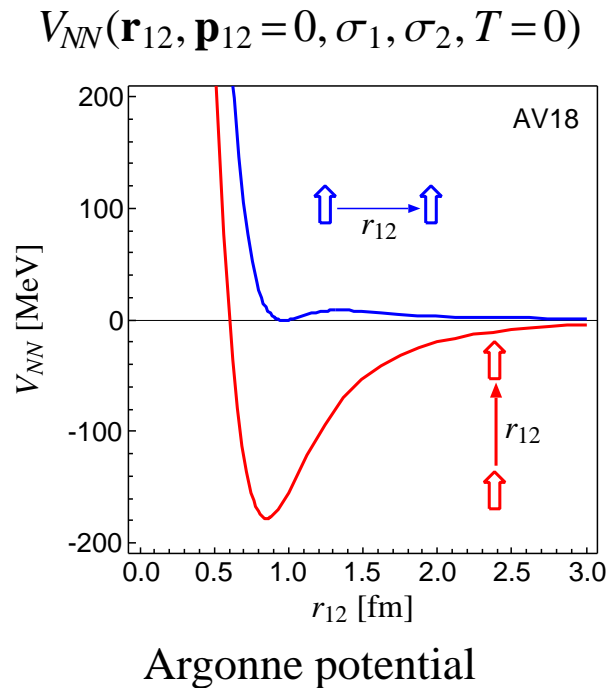


- $V_{NN}$  repulsive at small distances
  - ➔ strong short-range central correlations
  - nucleons cannot get closer than  $\approx 0.6$  fm
- $V_{NN}$  depends strongly on orientation of  $\sigma_1, \sigma_2$  with respect to  $\mathbf{r}_{12}$ 
  - ➔ tensor correlations
  - protons and neutrons want to align their spins with  $\mathbf{r}_{12}$

## Problem:

Slater determinants **cannot** describe these correlations

# Realistic $NN$ -potential



- $\tilde{V}_{NN}$  repulsive at small distances
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  - ➔ tensor correlations  
protons and neutrons want to align their spins with  $\mathbf{r}_{12}$

## Problem:

Slater determinants **cannot** describe these correlations

**Solution:** include short-range correlations by unitary transformation (**UCOM**)

$$|\hat{\Psi}\rangle = \tilde{C} |\Psi\rangle = \tilde{C}_\Omega \tilde{C}_r |\Psi\rangle$$

- $\tilde{C}_r$  central correlator shifts nucleons out of repulsive core
- $\tilde{C}_\Omega$  tensor correlator aligns spins along  $\mathbf{r}_{12}$

# Unitary Correlation Operator Method (UCOM)

## Correlation Operator

introduce short-range correlations by means of a unitary transformation with respect to the relative coordinates of all pairs

$$\underline{\mathcal{C}} = \exp[-i\underline{G}] = \exp\left[-i \sum_{i<j} \underline{g}_{ij}\right] \quad (1)$$

$$\begin{aligned} \underline{G}^\dagger &= \underline{G} \\ \underline{\mathcal{C}}^\dagger \underline{\mathcal{C}} &= 1 \end{aligned}$$

## Correlated States

$$|\hat{\Psi}\rangle = \underline{\mathcal{C}} |\Psi\rangle$$

## Correlated Operators

$$\hat{\mathcal{O}} = \underline{\mathcal{C}}^{-1} \underline{\mathcal{O}} \underline{\mathcal{C}}$$

$$\langle \hat{\Psi} | \underline{\mathcal{O}} | \hat{\Psi}' \rangle = \langle \Psi | \underline{\mathcal{C}}^{-1} \underline{\mathcal{O}} \underline{\mathcal{C}} | \Psi' \rangle = \langle \Psi | \hat{\mathcal{O}} | \Psi' \rangle$$

# Central and Tensor Correlators

$$\tilde{C} = \tilde{C}_\Omega \tilde{C}_r$$

## Central Correlator $\tilde{C}_r$

- radial distance-dependent shift in the relative coordinate of a nucleon pair

$$g_r = \frac{1}{2} [s(\tilde{r}) q_r + q_r s(\tilde{r})]$$

$$q_r = \frac{1}{2} \left[ \frac{\tilde{\mathbf{r}}}{\tilde{r}} \cdot \tilde{\mathbf{q}} + \tilde{\mathbf{q}} \cdot \frac{\tilde{\mathbf{r}}}{\tilde{r}} \right]$$

## Tensor Correlator $\tilde{C}_\Omega$

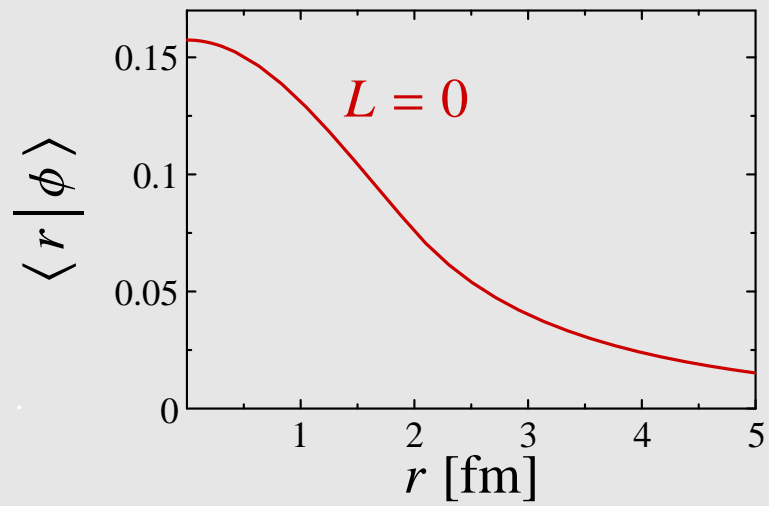
- angular shift depending on the orientation of spin and relative coordinate of a nucleon pair

$$g_\Omega = \frac{3}{2} \vartheta(\tilde{r}) [(\tilde{\sigma}_1 \cdot \tilde{\mathbf{q}}_\Omega)(\tilde{\sigma}_2 \cdot \tilde{\mathbf{r}}) + (\tilde{\mathbf{r}} \leftrightarrow \tilde{\mathbf{q}}_\Omega)]$$

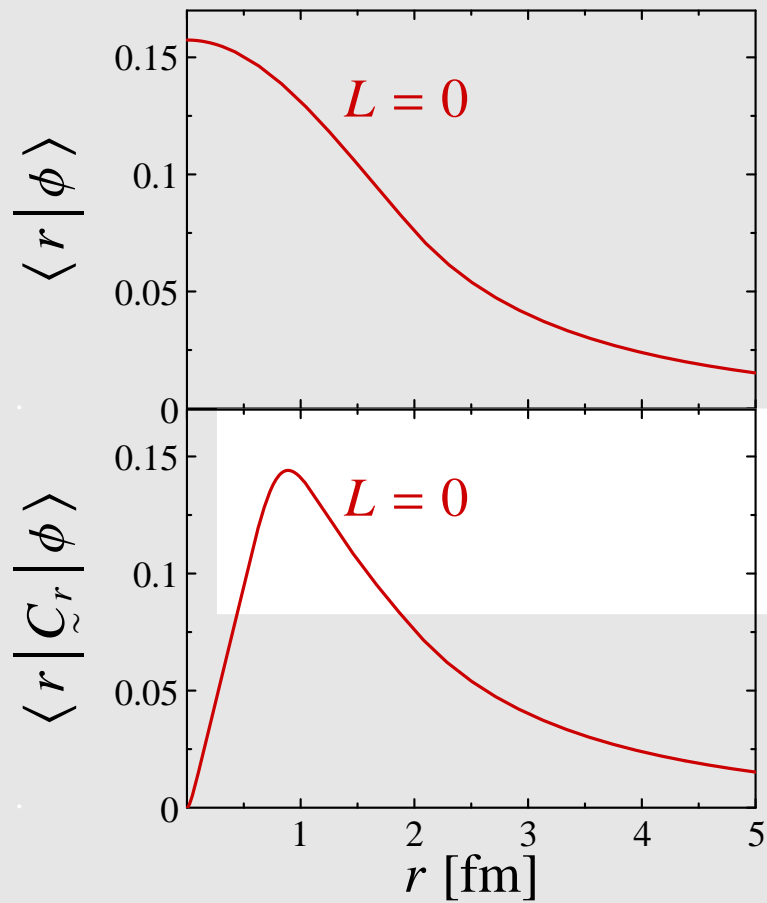
$$\tilde{\mathbf{q}}_\Omega = \tilde{\mathbf{q}} - \frac{\tilde{\mathbf{r}}}{\tilde{r}} q_r$$

$s(r)$  and  $\vartheta(r)$   
for given potential determined  
in the two-body system

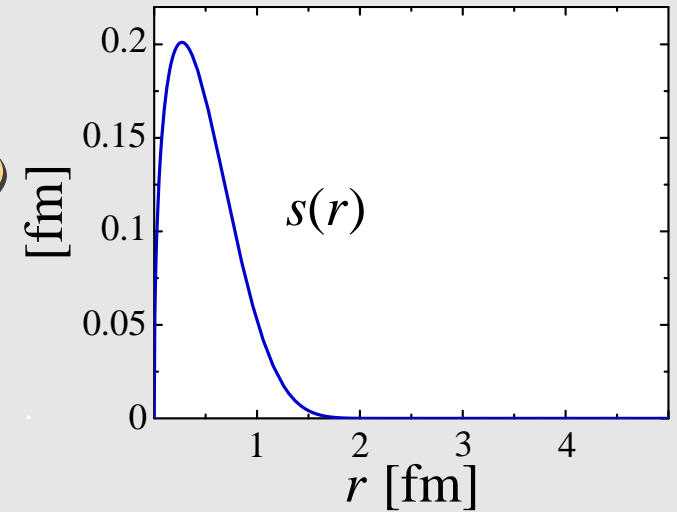
# Correlated States: The Deuteron



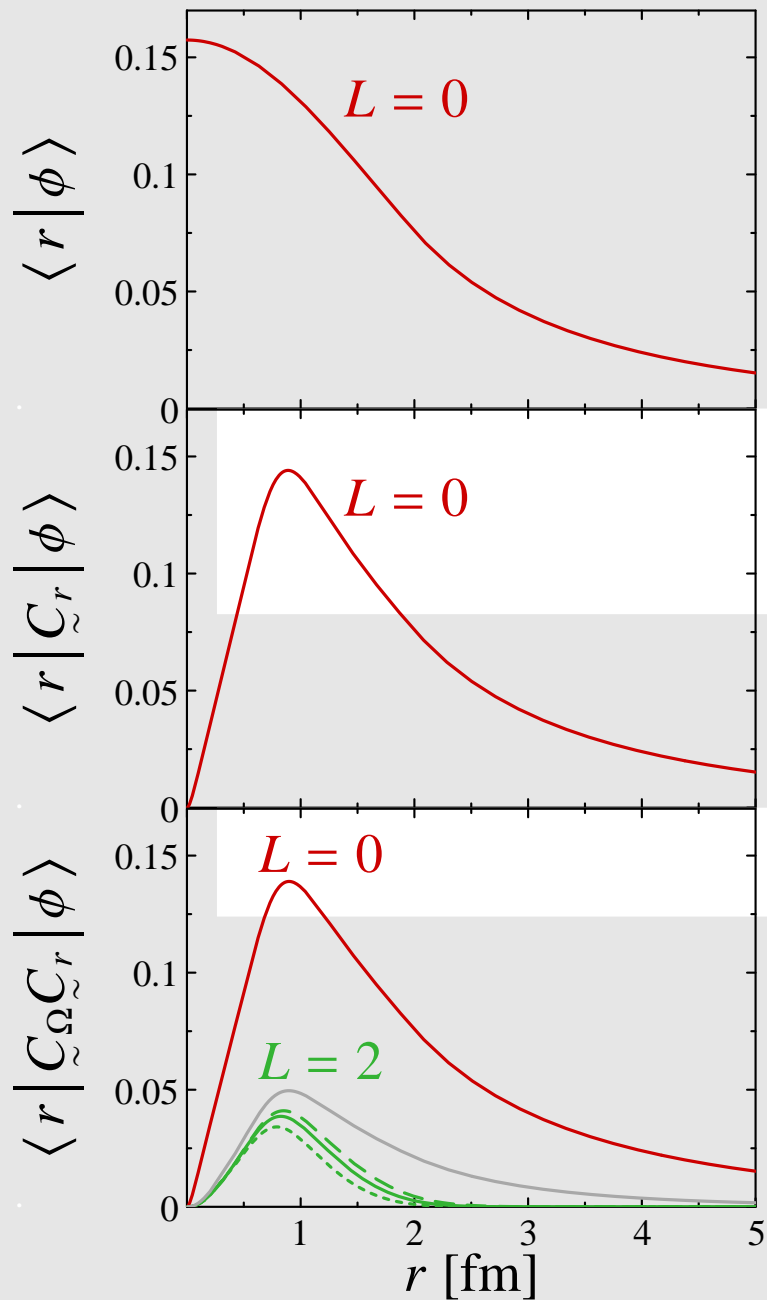
# Correlated States: The Deuteron



central  
correlations



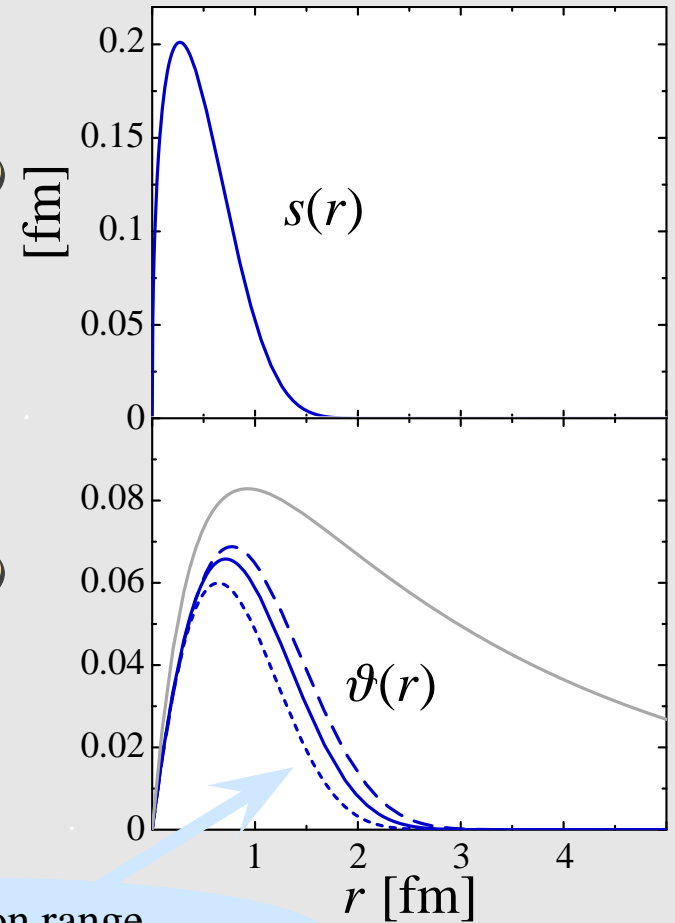
# Correlated States: The Deuteron



central correlations

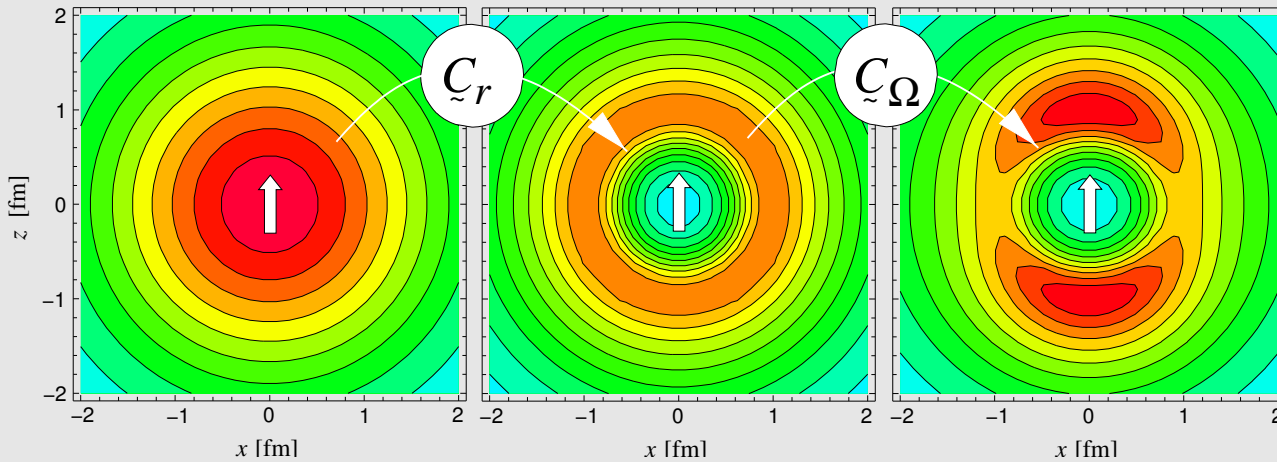
tensor correlations

constraint on range of tensor correlator



# Correlations in Nuclei

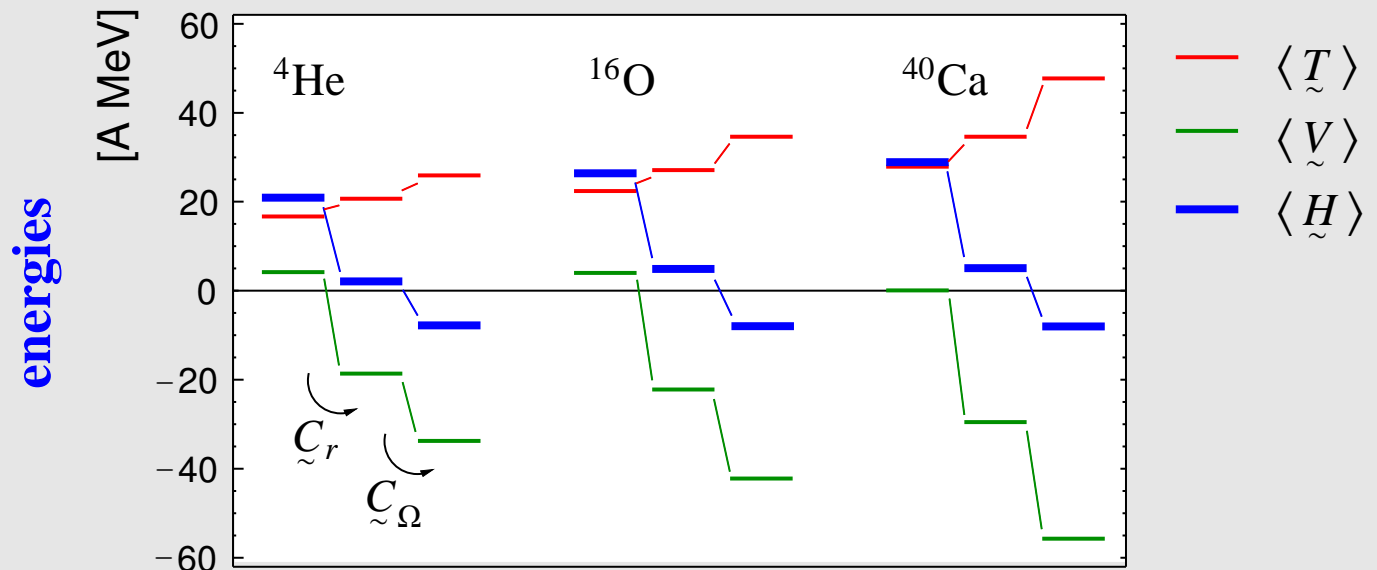
two-body densities



$$\rho_{S,T}^{(2)}(\mathbf{r}_1 - \mathbf{r}_2) \quad S = 1, M_S = 1, T = 0$$

- radial correlator shifts density out of the repulsive core
- tensor correlator aligns density with spin orientation

- both radial and tensor correlations are essential for binding



# Correlated Interaction – $V_{\text{UCOM}}$

$$\hat{H}_{\sim} = \tilde{T} + \tilde{V}_{\text{UCOM}} + \tilde{V}_{\text{UCOM}}^{[3]} + \dots$$

- **closed operator expression** for the correlated interaction  $\tilde{V}_{\text{UCOM}}$  in two-body approximation
- correlated interaction and original NN-potential are **phase shift equivalent** by construction
- unitary transformation results in a **pre-diagonalisation** of Hamiltonian

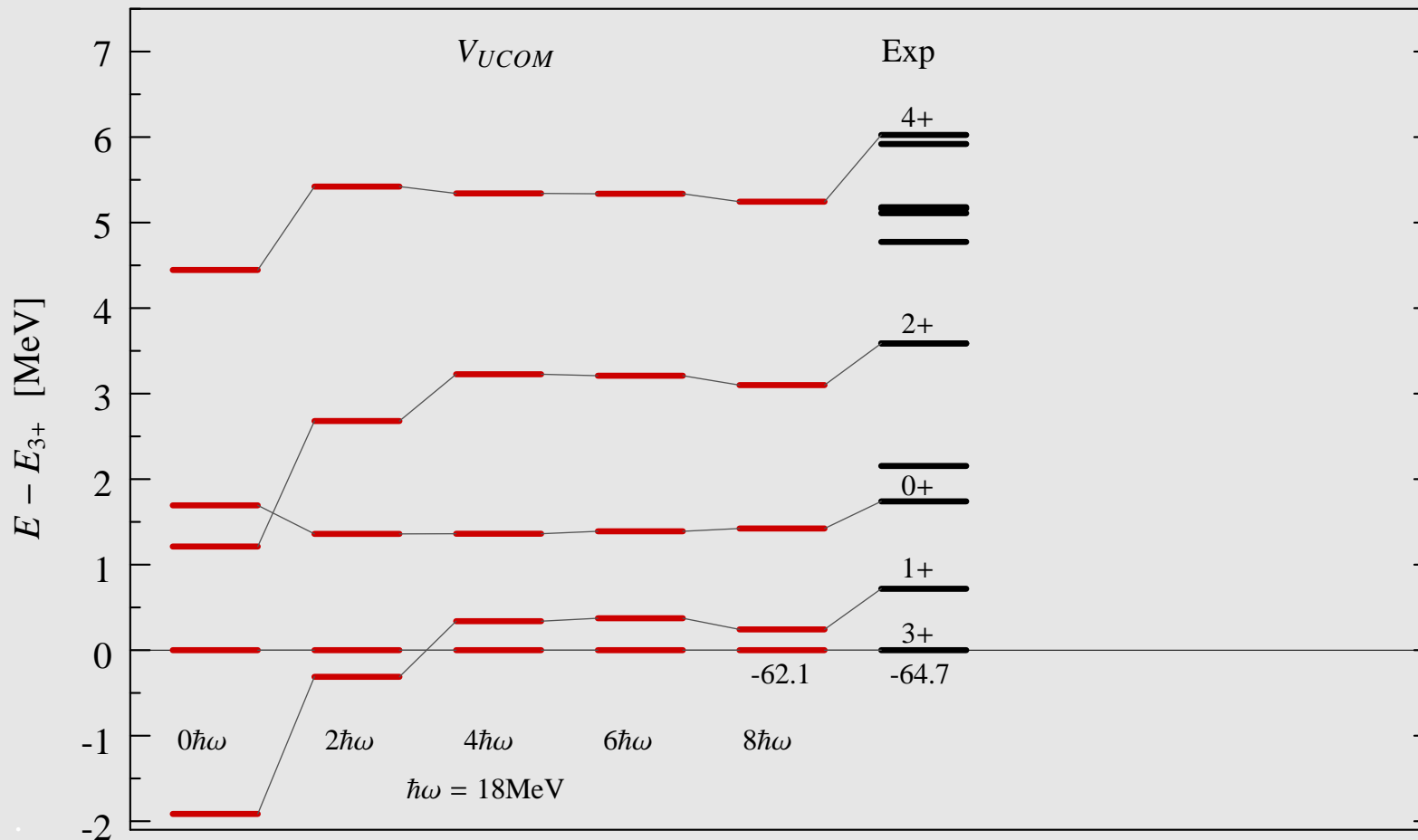
# Anwendung 1

## No-Core Shell Model



# Benchmarking $V_{UCOM}$ for $^{10}\text{B}$ :

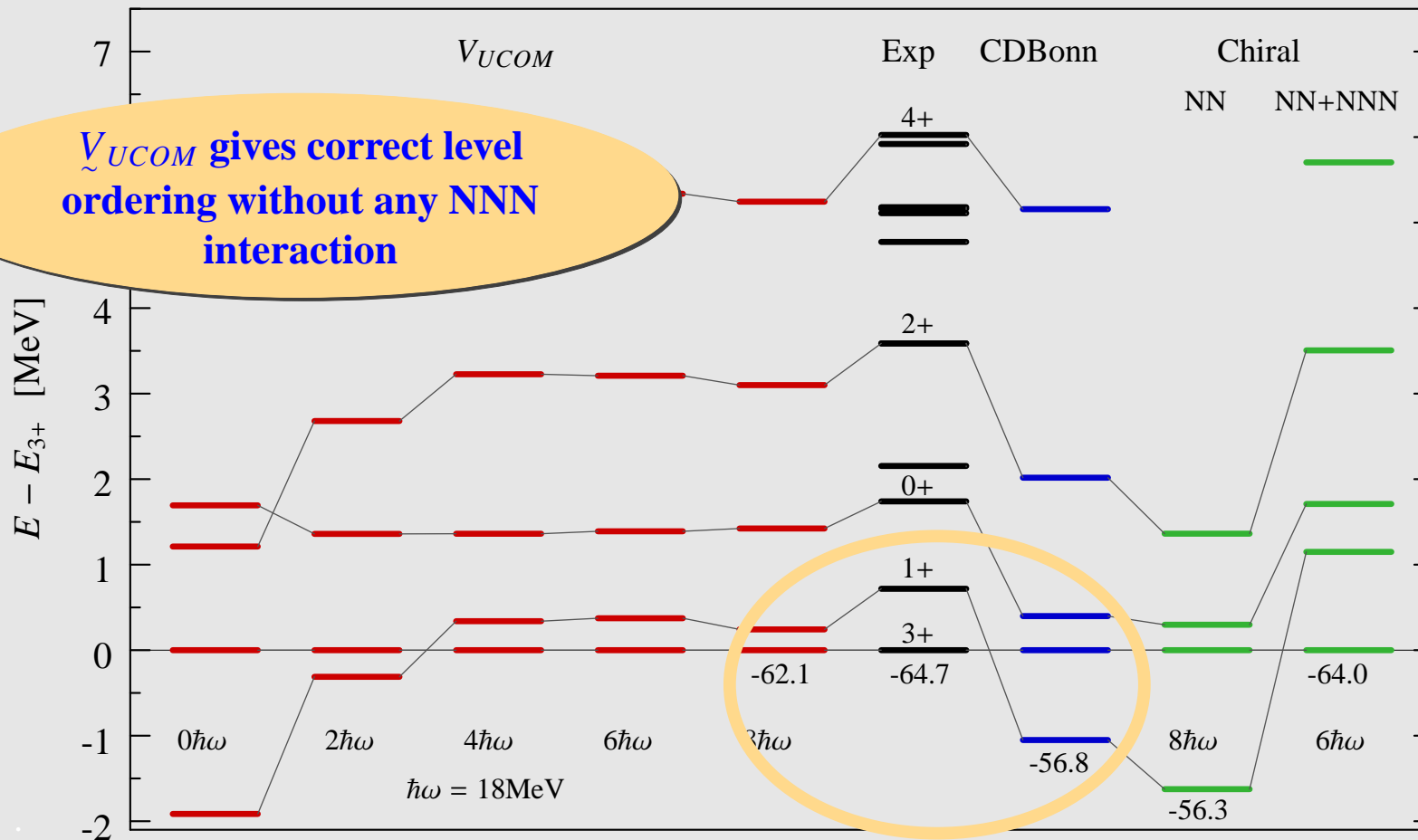
- large-scale NCSM calculations throughout the p-shell in progress



calculations by Petr Navrátil – preliminary

# Benchmarking $V_{UCOM}$ for $^{10}\text{B}$ :

- large-scale NCSM calculations throughout the p-shell in progress



calculations by Petr Navrátil – preliminary

## Anwendung 2

# Fermionic Molecular Dynamics



# Hilbert Space: Fermionic Molecular Dynamics

## Fermionic

Slater determinant

$$|Q\rangle = \mathcal{A}\left(|q_1\rangle \otimes \cdots \otimes |q_A\rangle\right)$$

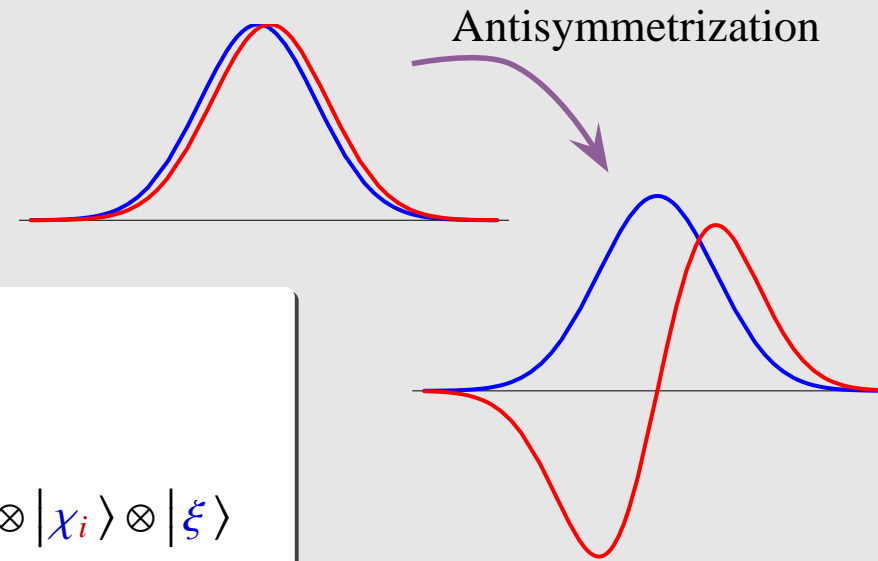
➔ antisymmetrized A-body state

## Molecular

single-particle states

$$\langle \mathbf{x} | q \rangle = \sum_i c_i \exp\left\{-\frac{(\mathbf{x} - \mathbf{b}_i)^2}{2a_i}\right\} \otimes |\chi_i\rangle \otimes |\xi\rangle$$

➔ Gaussian wave-packets in phase-space,  
spin is free, isospin is fixed



➔ Hilbert space contains  
shell-model, clusters, halos

## Dynamics in Hilbert space

spanned by one or several non-orthogonal  $|Q^{(a)}\rangle$

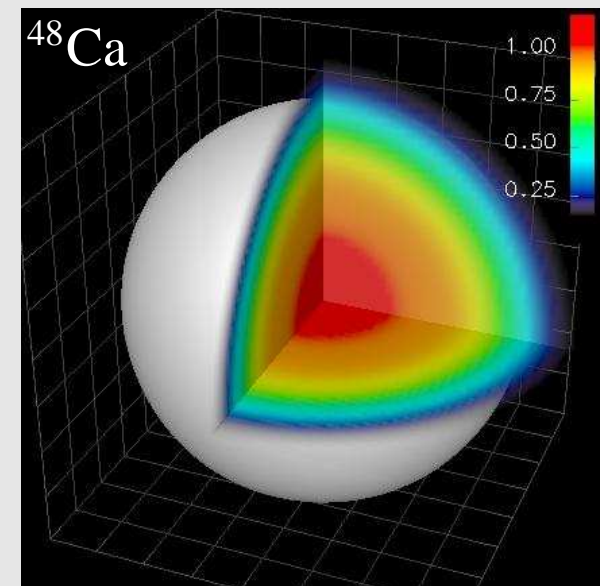
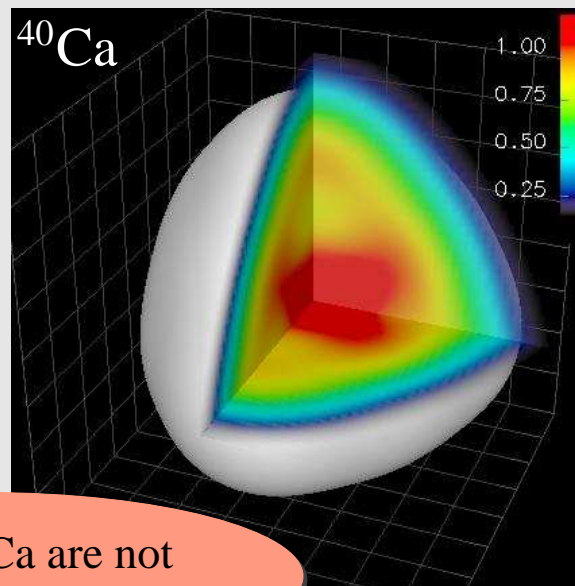
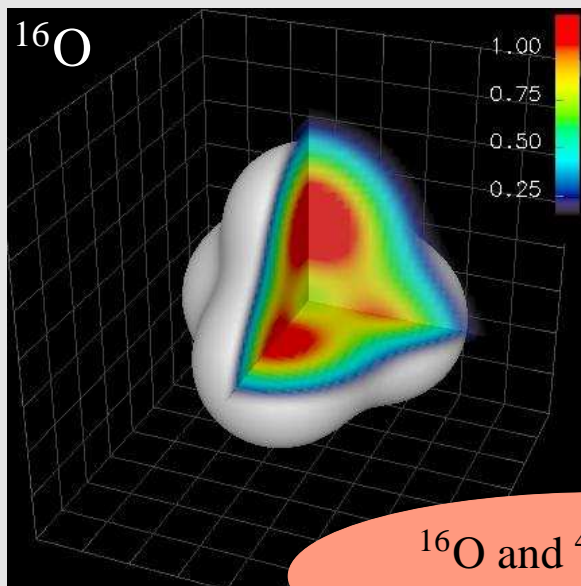
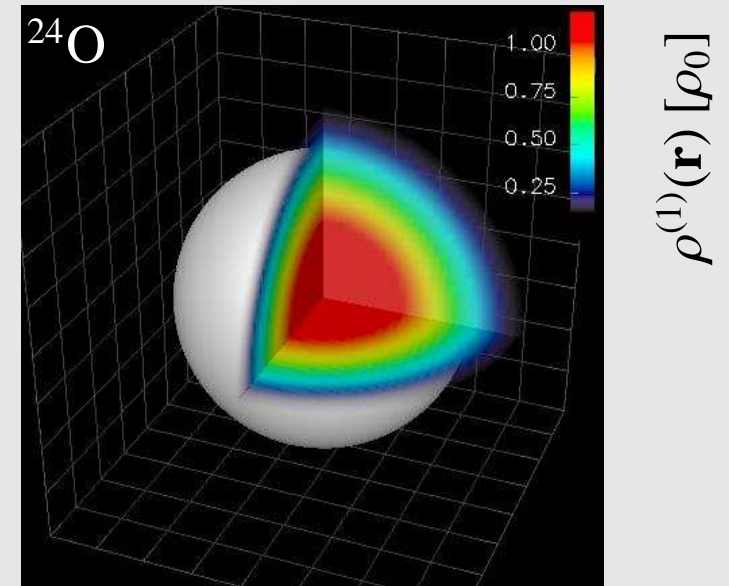
$$|\Psi\rangle = \sum_a \psi_a |Q^{(a)}\rangle$$

variational principle →  $Q^{(a)} = \{q_\nu^{(a)}, \nu = 1 \cdots A\}$ ,  $\psi_a$

# ● Effective Correction to the Interaction

## Effective two-body interaction

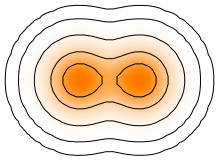
- ➔ correlated two-body interaction  $\hat{H} = \tilde{C}^\dagger H \tilde{C}$  is lacking three-body forces
- ➔ instead of three-body force use additional **momentum-dependent** and **spin-orbit** two-body correction term
- ➔ fit correction term to binding energies and radii of “closed-shell” nuclei
- ➔ altogether a **15%** correction to the *ab-initio* two-body potential



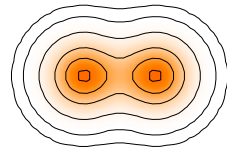
<sup>16</sup>O and <sup>40</sup>Ca are not “closed shell” nuclei !

# FMD - Projection, Variation after Proj., Multiconfiguration

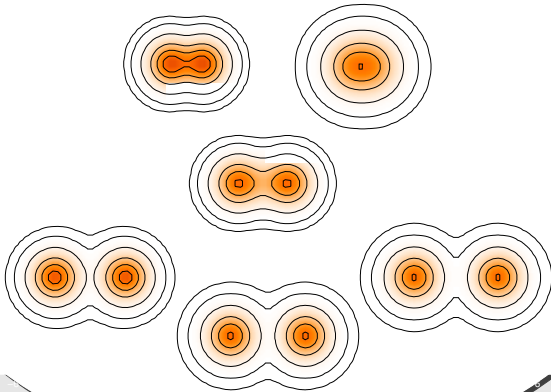
PAV



VAP

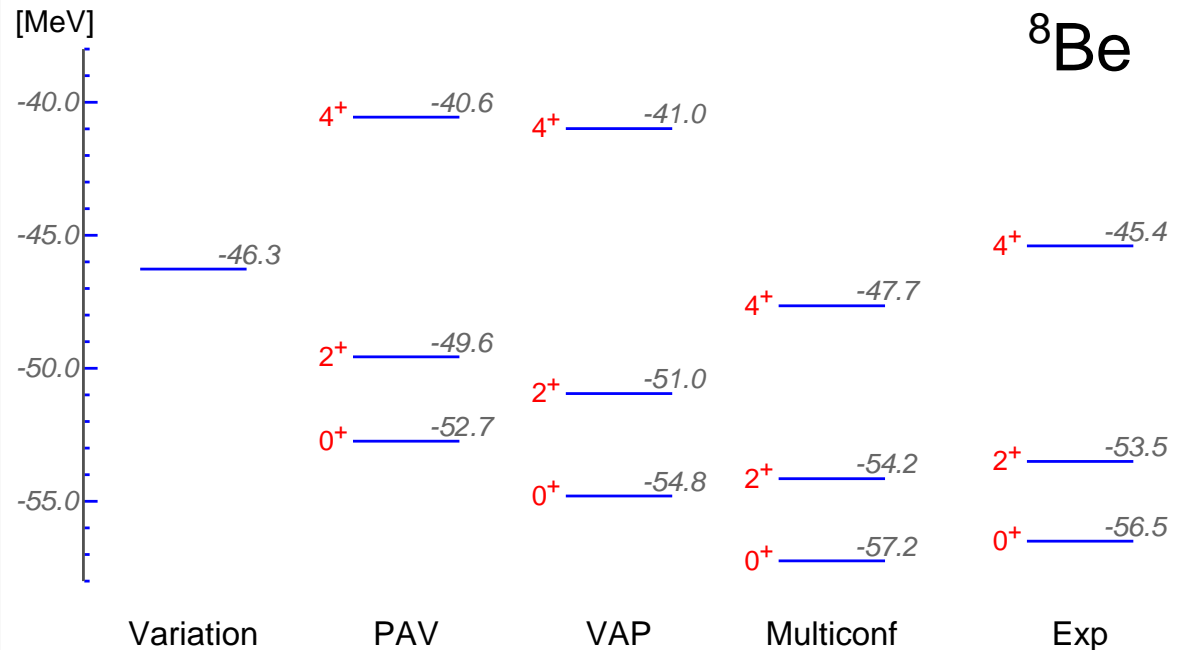


Multiconfig



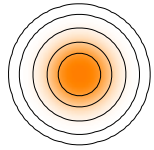
Radius and Quadrupole Moment  
as Generator Coordinates

	$r_{charge}$ [fm]	$Q$ [fm <sup>2</sup> ]	$B(E2)$ [ $e^2\text{fm}^4$ ]
PAV	2.39	-6.25	9.31
VAP	2.49	-8.02	15.36
Multiconfig	2.74	-11.88	30.39



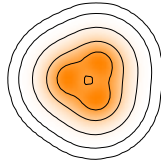
# FMD - Variation, PAV $\pi$ , Multiconfig.

V/PAV

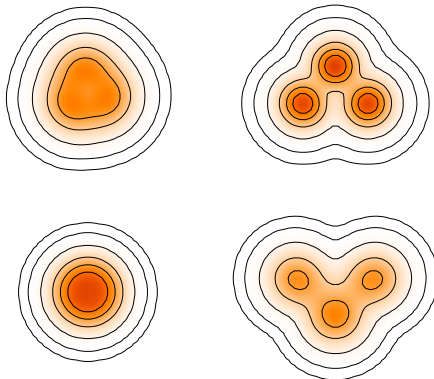


$^{12}\text{C}$

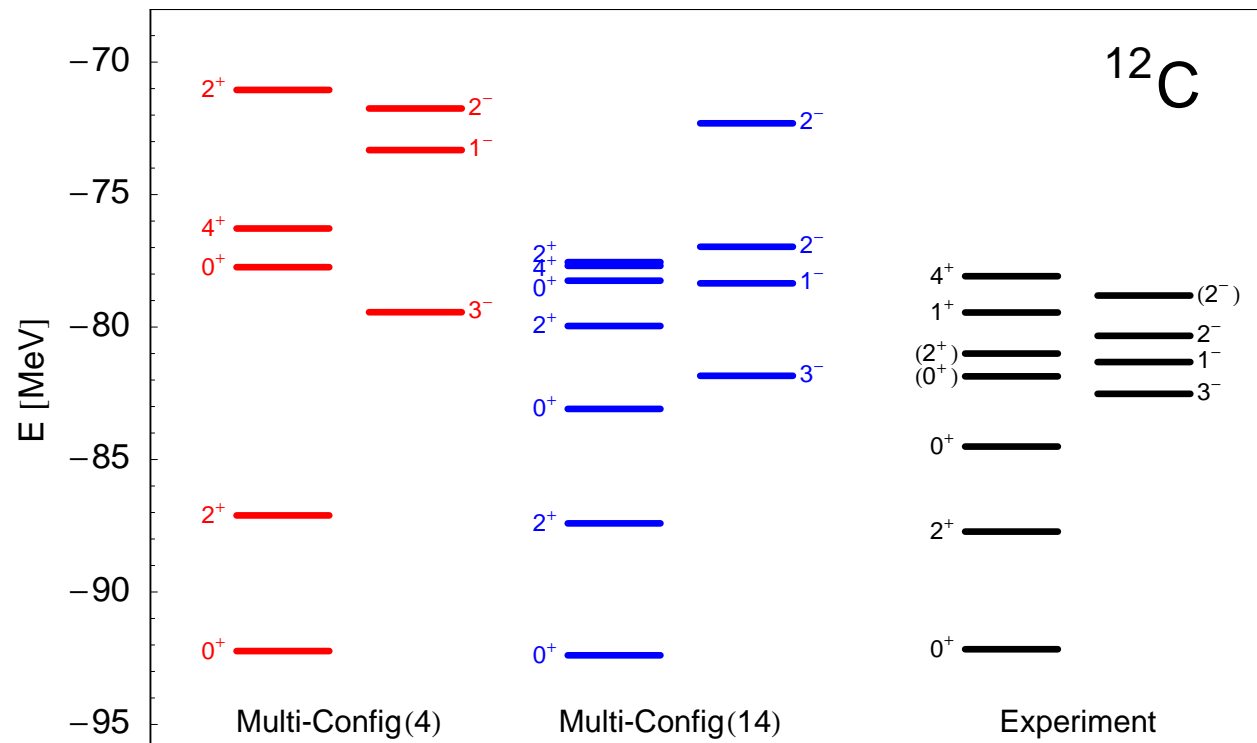
PAV $\pi$



Multiconfig(4)

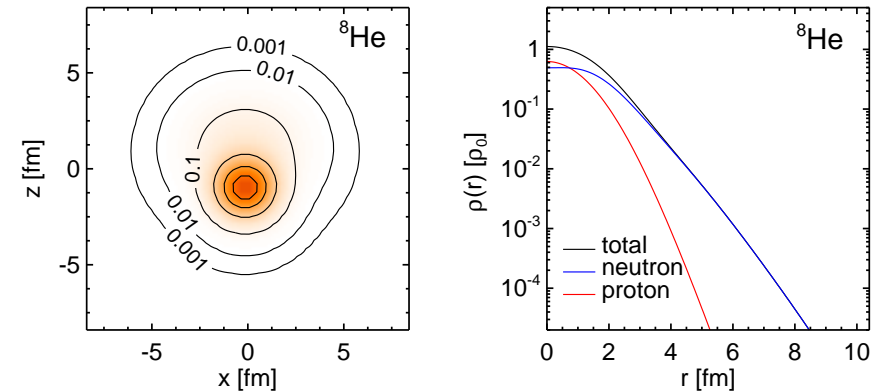
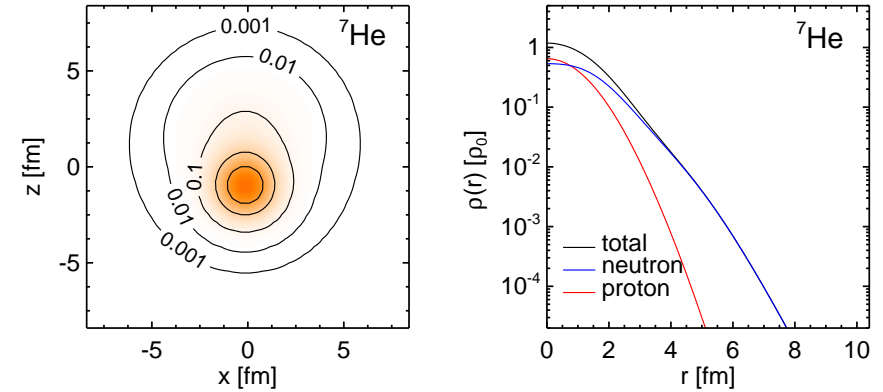
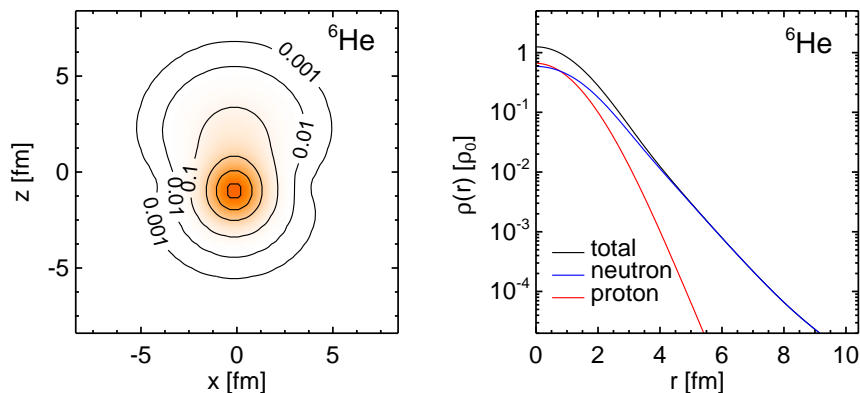
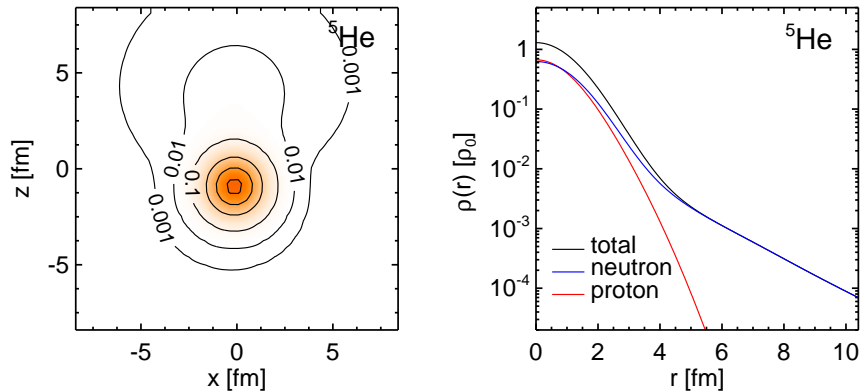
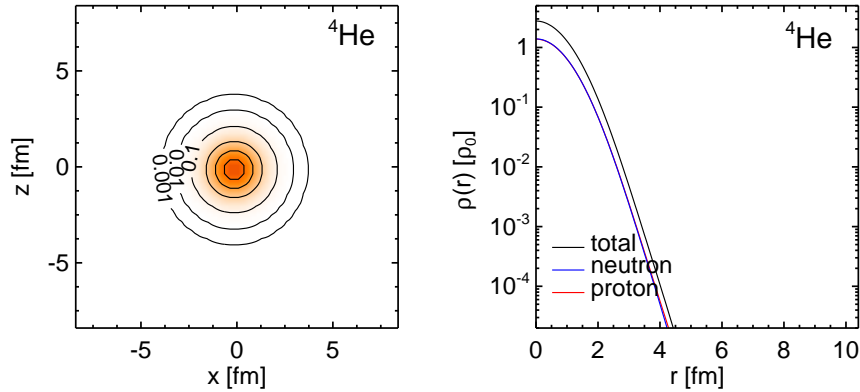


	$E$ [MeV]	$r_{charge}$ [fm]	$B(E2)$ [ $e^2\text{fm}^4$ ]
V/PAV	-81.4	2.36	-
PAV $\pi$	-88.5	2.51	36.3
Multiconfig(4)	-92.2	2.52	42.8
Multiconfig(14)	-92.4	2.52	42.9
Exp	-92.2	2.47	$39.7 \pm 3.3$



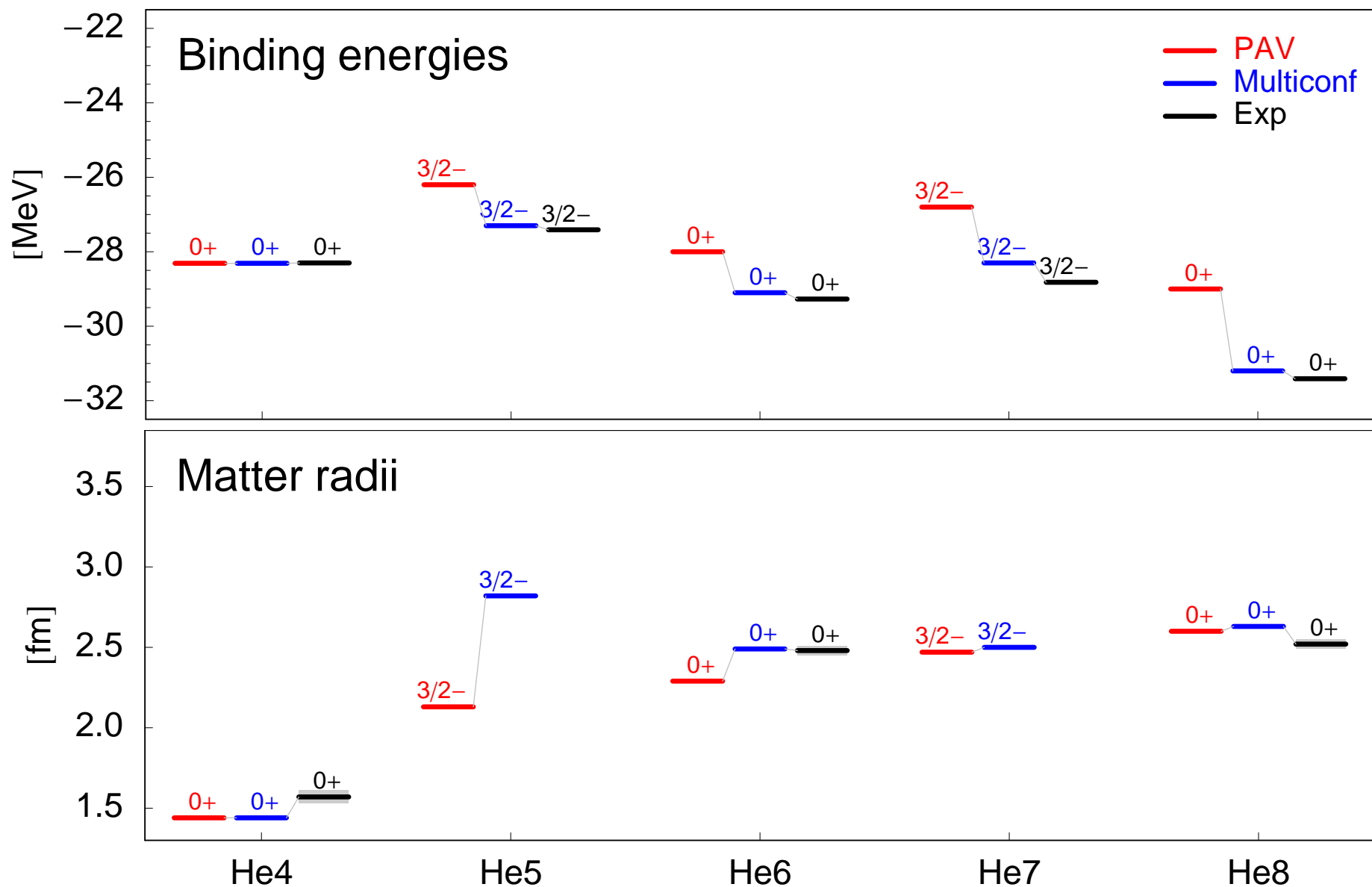


# Helium Isotopes

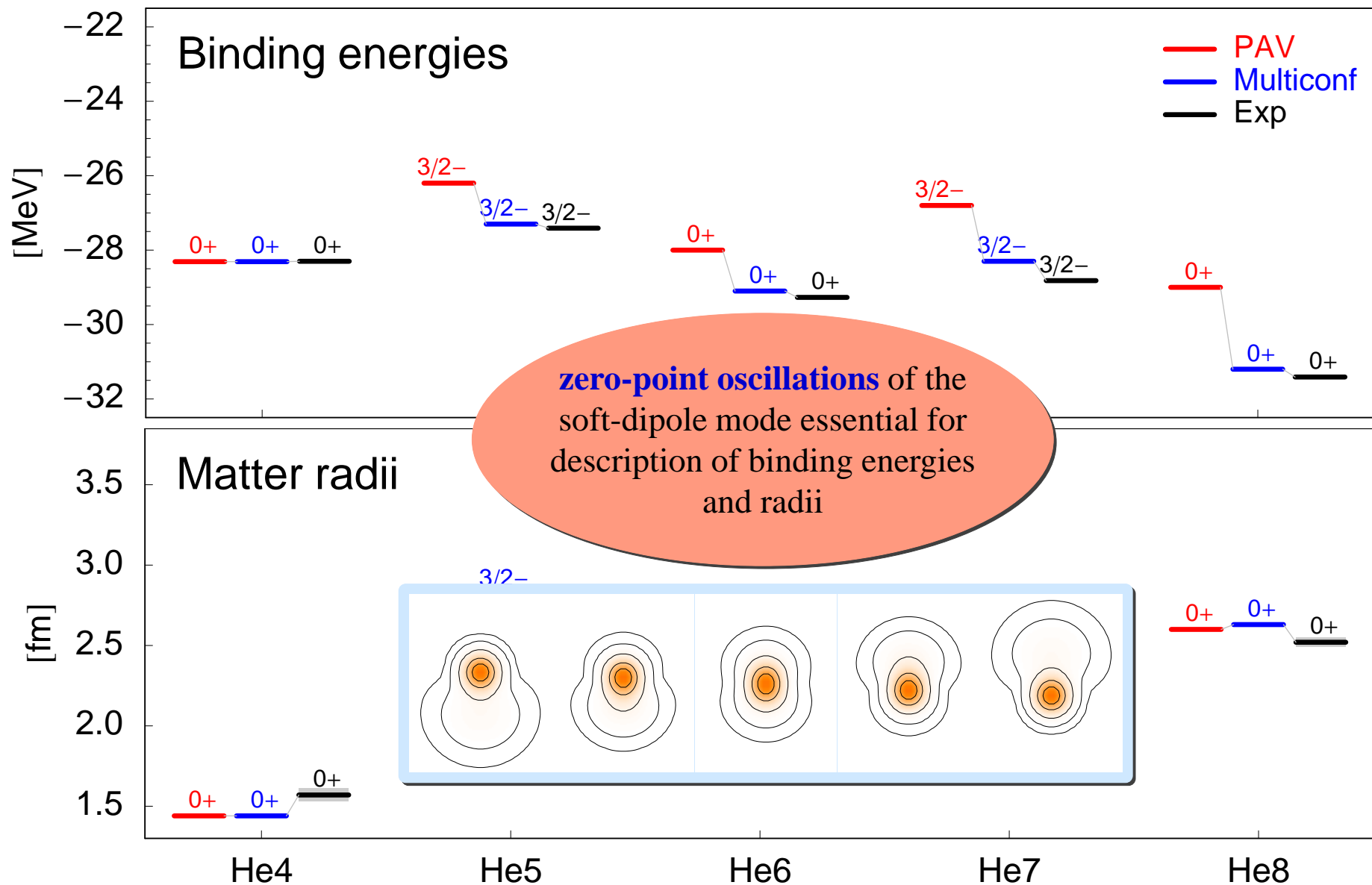


**soft-dipole mode**  
neutrons are oscillating  
against  $\alpha$ -core

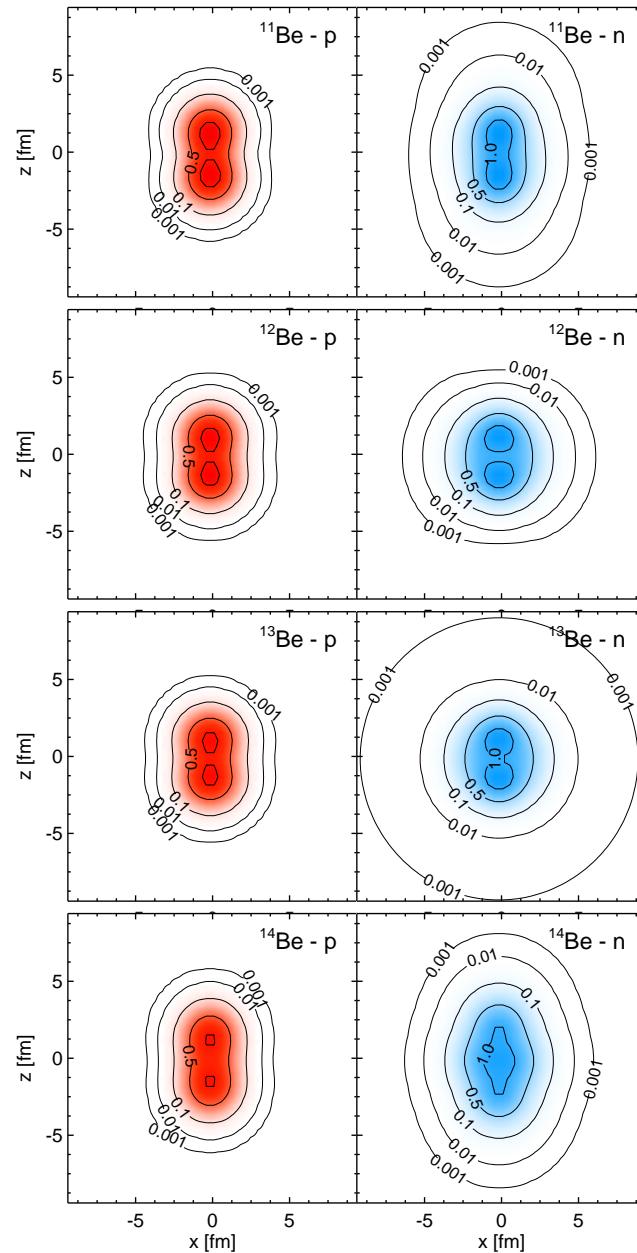
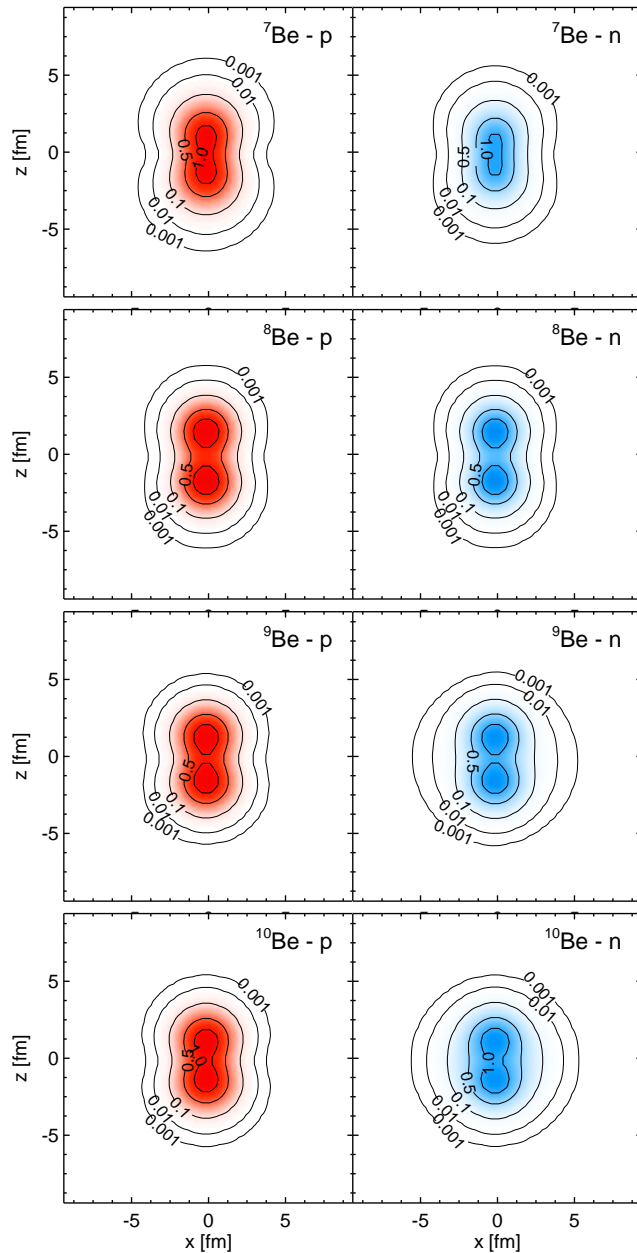
# Helium Isotopes



# Helium Isotopes

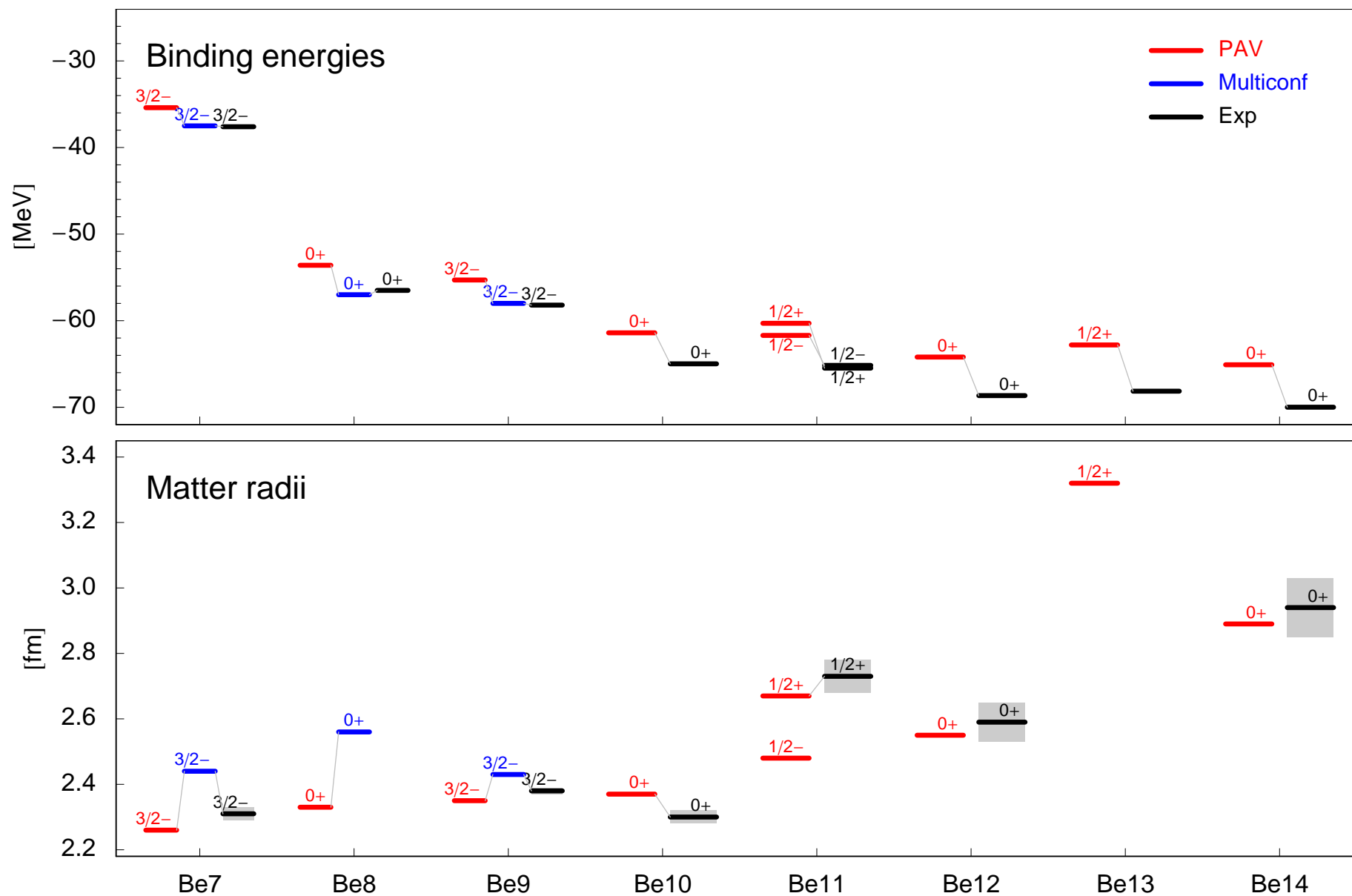


# Beryllium Isotopes



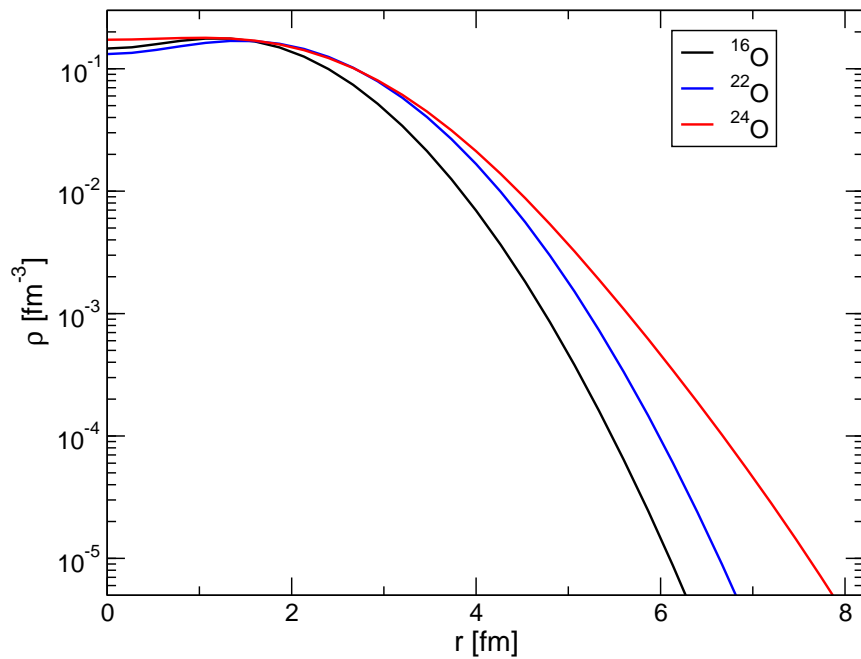
cluster structure  
changes with  
addition of neutrons

# Beryllium Isotopes

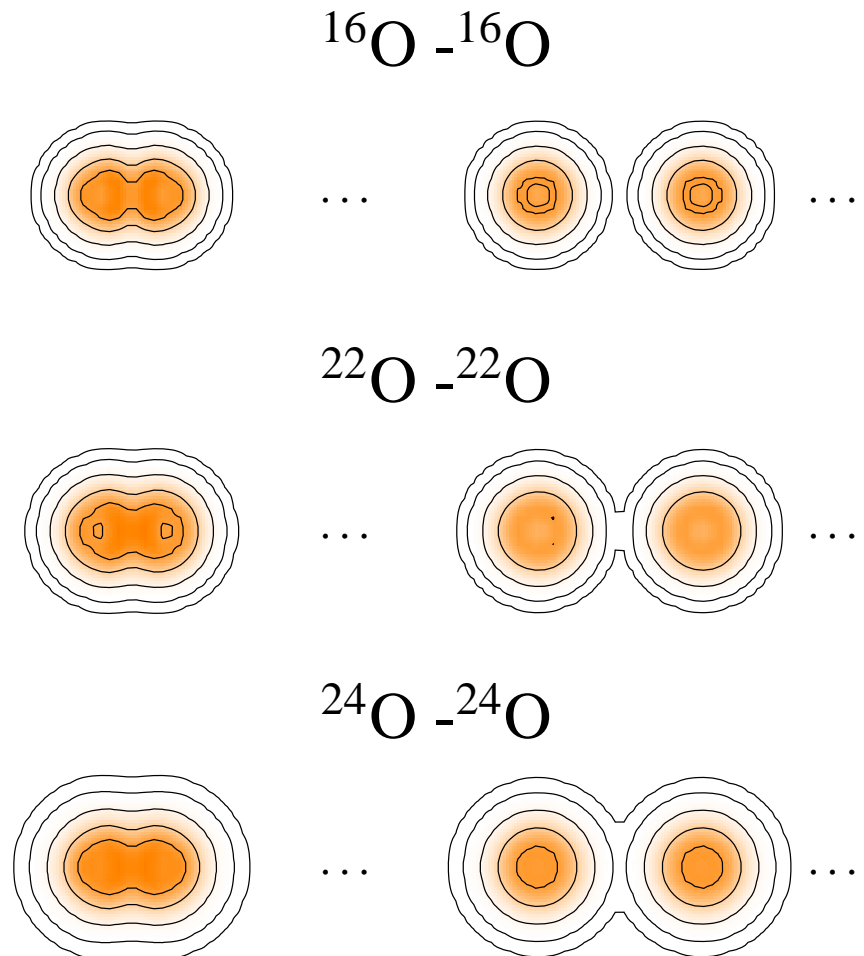


# Mikroskopische Kern-Kern-Potentiale

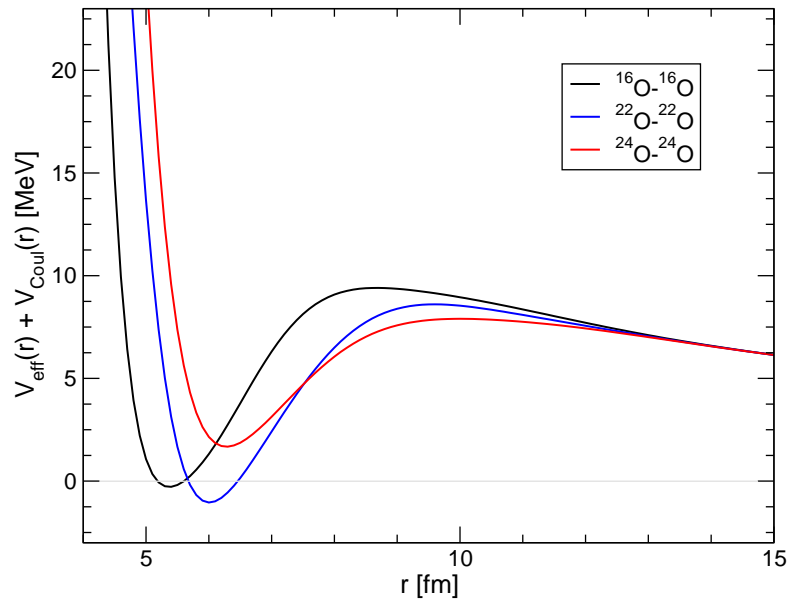
- Vielteilchenzustände:  
Fermionische Molekulardynamik (FMD)
- Effektive  $NN$ -Wechselwirkung:  
abgeleitet von der realistischen  
Argonne-V18  $NN$ -Wechselwirkung  
(UCOM)



Massendichten



# Potentiale & Astrophysikalische S-Faktoren

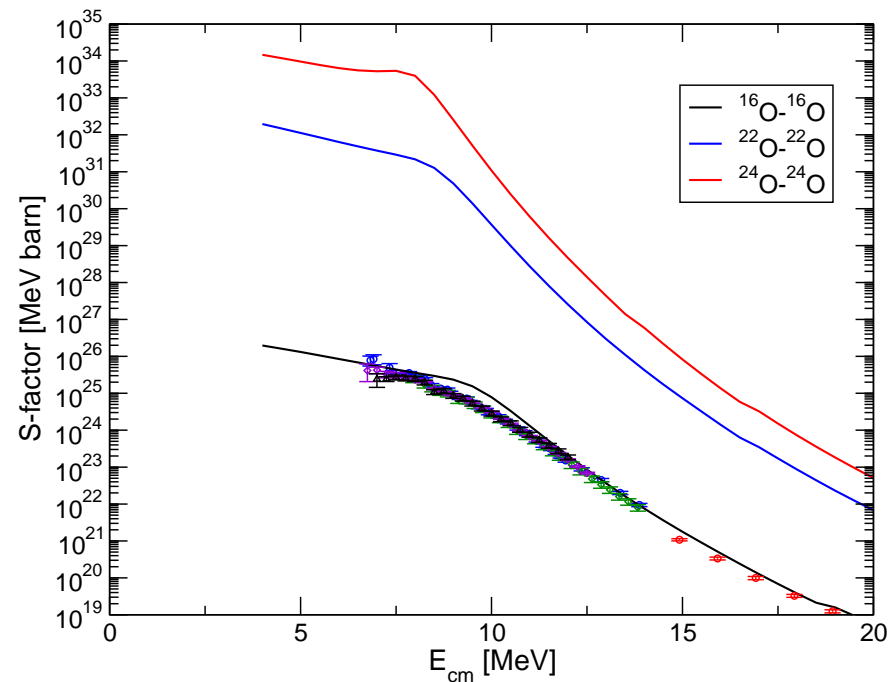


Mikroskopisch berechnete  
Kern-Kern-Potentiale

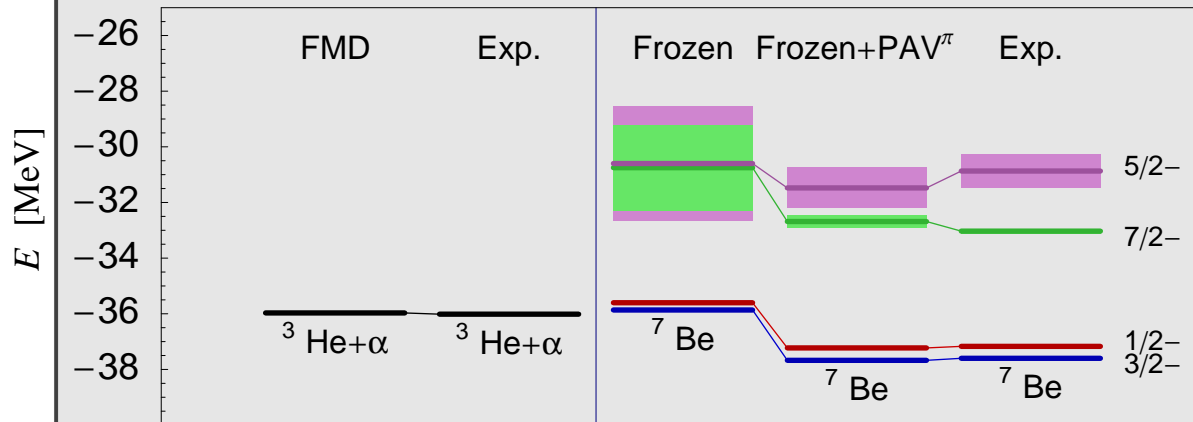
Astrophysikalischer S-Faktor

$$S(E) = \sigma_{\text{fusion}}(E) E e^{2\pi\eta}$$

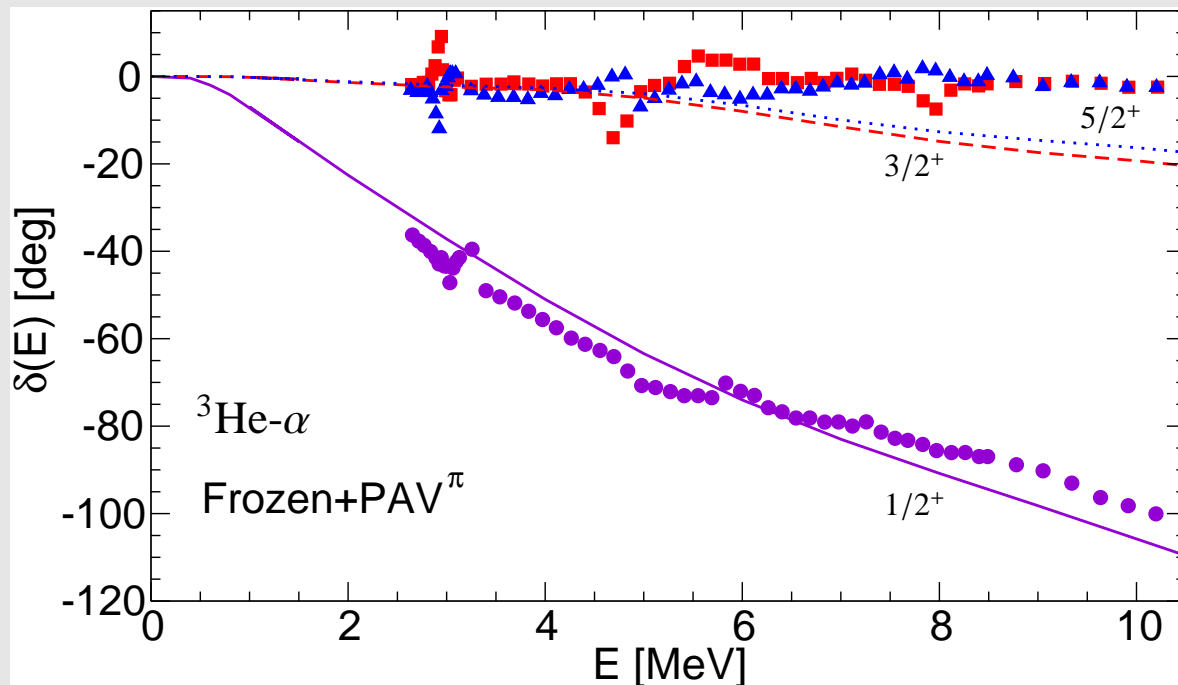
$\eta$ =Sommerfeld-Parameter



# Outlook: Resonances & Scattering in FMD



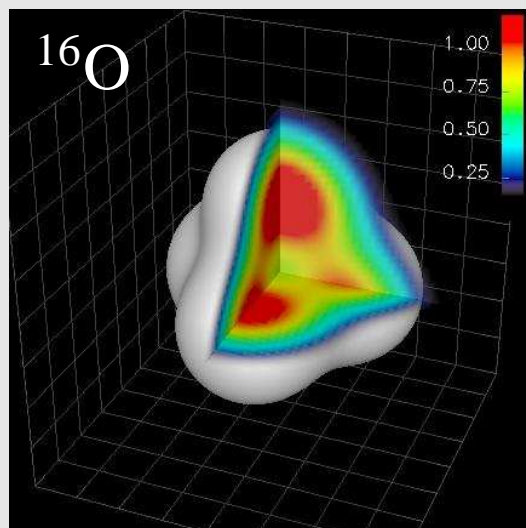
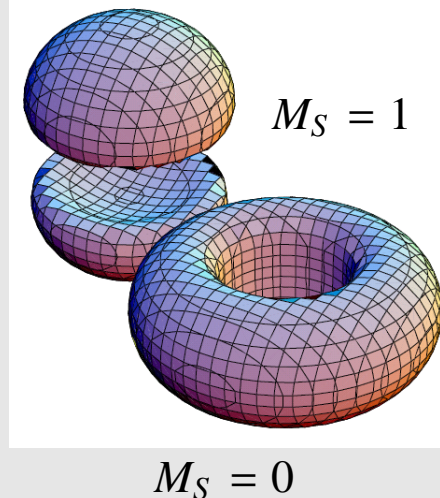
- collective coordinate representation as tool for the description of continuum states in FMD



first steps towards fully microscopic and consistent description of **structure and reactions**

# Zusammenfassung und Ausblick

- Neue Ära der Kernstruktur**
- ➔ **Unitary Correlation Operator Method** beschreibt kurzreichweitige radiale und tensorielle Korrelationen
  - ➔ **UCOM** erzeugt ab-initio korrelierte Wechselwirkung  $\hat{H}$  für Vielteilchenmethoden wie HF, Schalenmodell, FMD
  - ➔ Observablen müssen und können auch korreliert werden
  - ➔ no-core Schalenmodell Rechnungen für leichte Kerne
  - ➔ **FMD** Rechnungen mit demselben  $\hat{H}^{C2} + \hat{H}^{corr}$  für  $3 \leq A \leq 60$
  - ➔ Ein mikroskopisches Modell für:  
Bindungsenergien, Radien, Spektren, Übergänge,  
Kontinuum, Resonanzen, Reaktionen



## Ausblick

- ➔ "ab initio" Idee weiter verfolgen → Vorhersagekraft
- ➔ Vorhersagen für viele exotische leichte Kerne (vor Messung)
- ➔ Korrelierte Übergänge: M1,  $\beta$ -Zerfall, quenching
- ➔ Ab-initio korrelierte WW in HF, RPA oder ähnlichem für  $A > 60$
- ➔ Langreichweitige Tensorkorr. – 3-Teilchenkräfte
- ➔ ...

# Epilogue

- **thanks to my group & my collaborators**
- A. Cribeiro, K. Langanke  
Gesellschaft für Schwerionenforschung (GSI)
- T. Neff  
NSCL, Michigan State University
- R. Roth, H. Hergert, N. Paar, P. Papakonstantinou, A. Zapp  
Institut für Kernphysik, TU Darmstadt



supported by the DFG through SFB 634  
“Nuclear Structure, Nuclear Astrophysics and Fundamental Experiments...”

# How to improve ?

## Projection After Variation (PAV)

- mean-field may break symmetries of Hamiltonian
- restore reflection and rotational symmetry by parity and angular-momentum projection  $P_{MK}^{J^\pi}$

$$\sum_{K'} \langle Q | \tilde{H} P_{KK'}^{J^\pi} | Q \rangle \cdot c_{K'} = E_K^{J^\pi} \sum_{K'} \langle Q | P_{KK'}^{J^\pi} | Q \rangle \cdot c_{K'}$$

## Variation After Projection (VAP)

- effect of projection can be large
- perform VAP applying **constraints** on radius, dipole moment, quadrupole moment or octupole moment and minimize the energy in the projected energy surface

## Multiconfiguration Calculations

- diagonalize Hamiltonian in a set of projected intrinsic states

$$\left\{ P_{KK'}^{J^\pi} | Q^{(a)} \rangle, \quad a = 1, \dots, N \right\}$$

# Unitary Correlation Operator Method, UCOM

## Two-Body Correlations

➔ two-body generator

$$\underline{\underline{C}} = e^{-i\underline{\underline{G}}}, \quad \underline{\underline{G}} = \underline{\underline{G}}^\dagger = \sum_{i < j} \underline{\underline{g}}_{ij}$$

## Cluster Expansion

correlated operators  $\hat{\underline{\underline{A}}} = \underline{\underline{C}}^\dagger \underline{\underline{A}} \underline{\underline{C}}$  are no longer operators with definite particle number

➔ decompose correlated operator into irreducible  $k$ -body operators

$$\hat{\underline{\underline{A}}} = \underline{\underline{C}}^\dagger \underline{\underline{A}} \underline{\underline{C}} = \hat{\underline{\underline{A}}}^{[1]} + \hat{\underline{\underline{A}}}^{[2]} + \hat{\underline{\underline{A}}}^{[3]} + \dots$$

## Two-Body Approximation

$$\hat{\underline{\underline{T}}}^{C2} = \hat{\underline{\underline{T}}}^{[1]} + \hat{\underline{\underline{T}}}^{[2]}, \quad \hat{\underline{\underline{V}}}^{C2} = \hat{\underline{\underline{V}}}^{[2]}$$

✗ correlation range should be smaller than mean distance of nucleons (to avoid 3-body terms)

## Correlator $\underline{\underline{C}}$

should conserve translational, rotational and Galilei invariance  
short ranged

# Unitary Correlation Operator Method, UCOM

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## Two-Body Approximation

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✗ correlation range should be smaller than mean distance of nucleons (to avoid 3-body terms)

## Correlator $\tilde{C}$

should conserve translational, rotational and Galilei invariance  
short ranged

## Spin-Isospin Dependence

nuclear interaction strongly depends on spin and isospin

$$\tilde{V} = \sum_{S,T} \tilde{v}_{ST} \tilde{\Pi}_{ST}$$

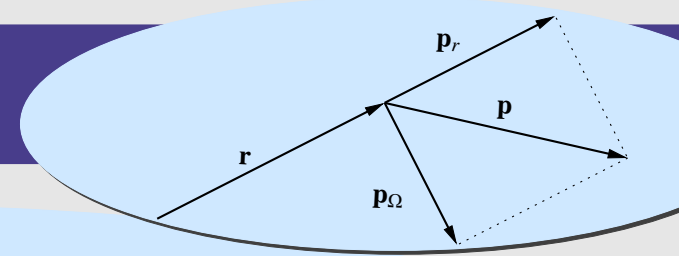
➔ different correlations in the respective channels

$$\tilde{g} = \sum_{S,T} \tilde{g}_{ST} \tilde{\Pi}_{ST}$$

➔ correlated interaction in two-body space

$$\hat{\tilde{V}} = \sum_{S,T} (e^{ig_{ST}} \tilde{v}_{ST} e^{-ig_{ST}}) \tilde{\Pi}_{ST}$$

# Radial and Tensor Correlations



$$\tilde{C} = \tilde{C}_\Omega \tilde{C}_r$$

$$= e^{-i\tilde{G}_\Omega} e^{-i\tilde{G}_r}$$

$$\mathbf{p} = \mathbf{p}_r + \mathbf{p}_\Omega$$

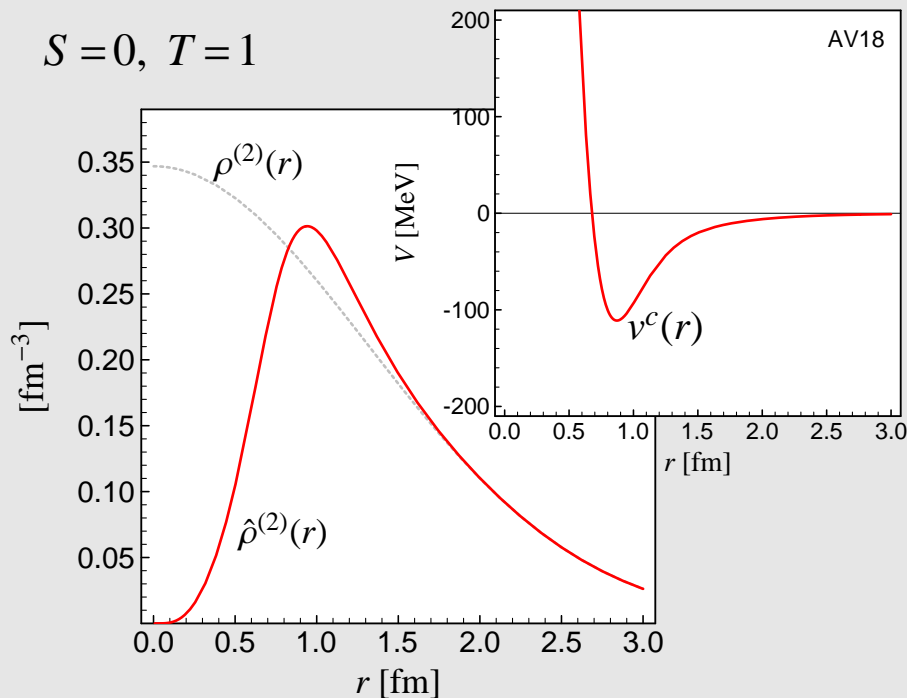
$$\mathbf{p}_r = \frac{1}{2} \left\{ \frac{\mathbf{r}}{r} (\mathbf{r} \cdot \mathbf{p}) + (\mathbf{p} \cdot \frac{\mathbf{r}}{r}) \frac{\mathbf{r}}{r} \right\}, \quad \mathbf{p}_\Omega = \frac{1}{2r} \left\{ \mathbf{l} \times \frac{\mathbf{r}}{r} - \frac{\mathbf{r}}{r} \times \mathbf{l} \right\}$$

## Radial Correlator

$$\tilde{G}_r = \frac{1}{2} \{ p_r s(r) + s(r) p_r \}$$

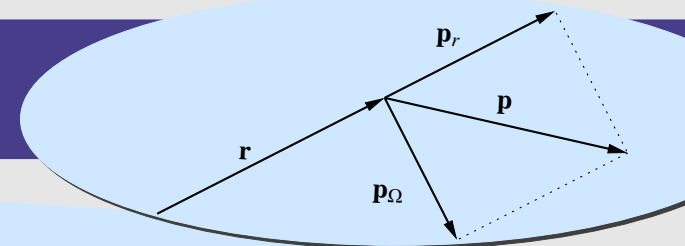
→ probability density shifted out of the repulsive core

$S = 0, T = 1$



Manfred Ristig Z.Physik **199** (1967) 325

# Radial and Tensor Correlations



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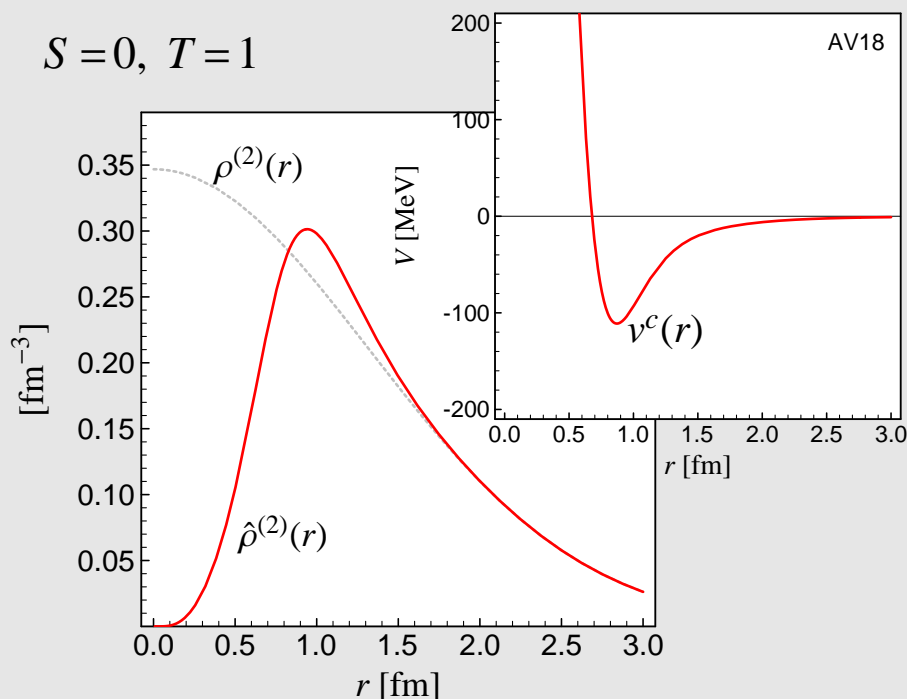
$$\mathbf{p}_r = \frac{1}{2} \left\{ \frac{\mathbf{r}}{r} (\frac{\mathbf{r}}{r} \mathbf{p}) + (\mathbf{p} \frac{\mathbf{r}}{r}) \frac{\mathbf{r}}{r} \right\}, \quad \mathbf{p}_\Omega = \frac{1}{2r} \left\{ \mathbf{l} \times \frac{\mathbf{r}}{r} - \frac{\mathbf{r}}{r} \times \mathbf{l} \right\}$$

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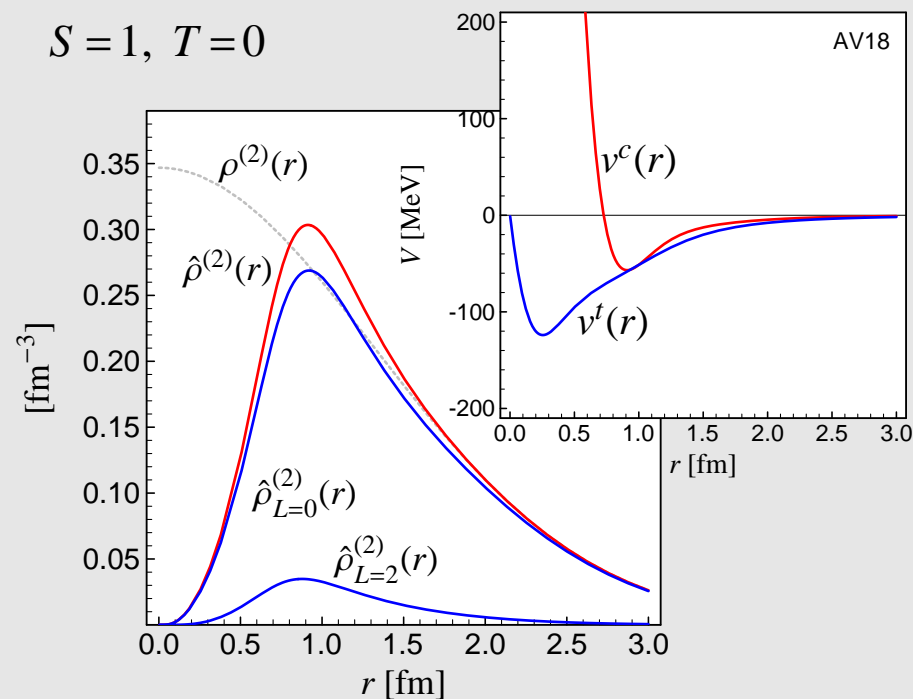
Manfred Ristig Z.Physik **199** (1967) 325

## Tensor Correlations

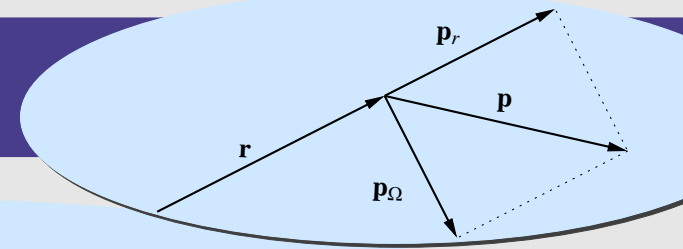
$$\tilde{G}_\Omega = \vartheta(r) \frac{3}{2} \{ (\tilde{\sigma}_1 \cdot \tilde{\mathbf{p}}_\Omega) (\tilde{\sigma}_2 \cdot \tilde{\mathbf{r}}) + (\tilde{\sigma}_1 \cdot \tilde{\mathbf{r}}) (\tilde{\sigma}_2 \cdot \tilde{\mathbf{p}}_\Omega) \}$$

→ tensor force admixes other angular momenta

$S = 1, T = 0$



# Radial and Tensor Correlations



$$\tilde{C} = \tilde{C}_\Omega \tilde{C}_r$$

$$= e^{-i\tilde{G}_\Omega} e^{-i\tilde{G}_r}$$

$$\mathbf{p} = \mathbf{p}_r + \mathbf{p}_\Omega$$

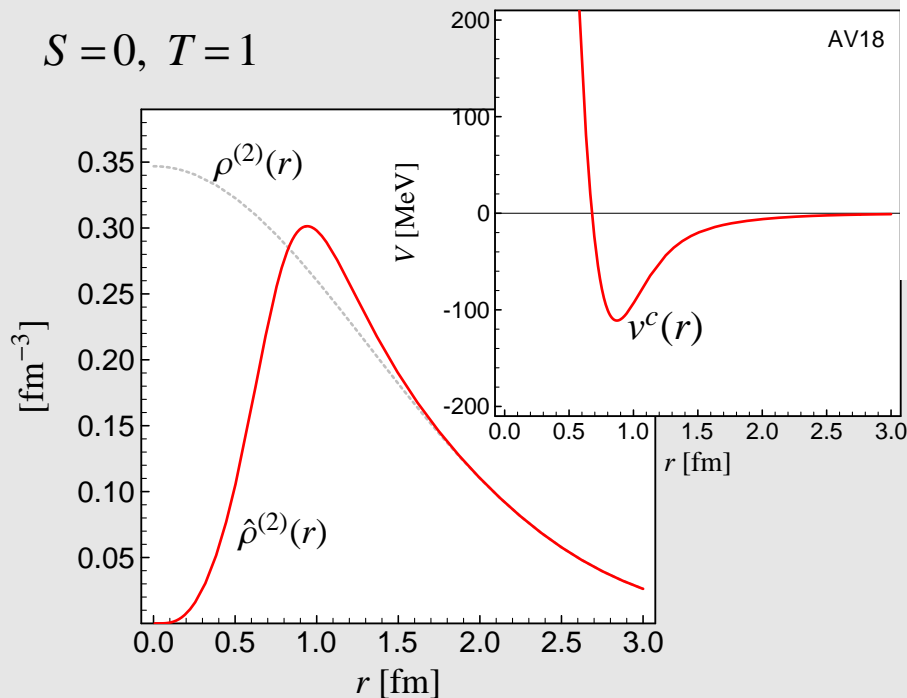
$$\mathbf{p}_r = \frac{1}{2} \left\{ \frac{\mathbf{r}}{r} (\frac{\mathbf{r}}{r} \mathbf{p}) + (\mathbf{p} \frac{\mathbf{r}}{r}) \frac{\mathbf{r}}{r} \right\}, \quad \mathbf{p}_\Omega = \frac{1}{2r} \left\{ \mathbf{l} \times \frac{\mathbf{r}}{r} - \frac{\mathbf{r}}{r} \times \mathbf{l} \right\}$$

## Radial Correlator

$$\tilde{G}_r = \frac{1}{2} \{ p_r s(r) + s(r) p_r \}$$

→ probability density shifted out of the repulsive core

$S = 0, T = 1$

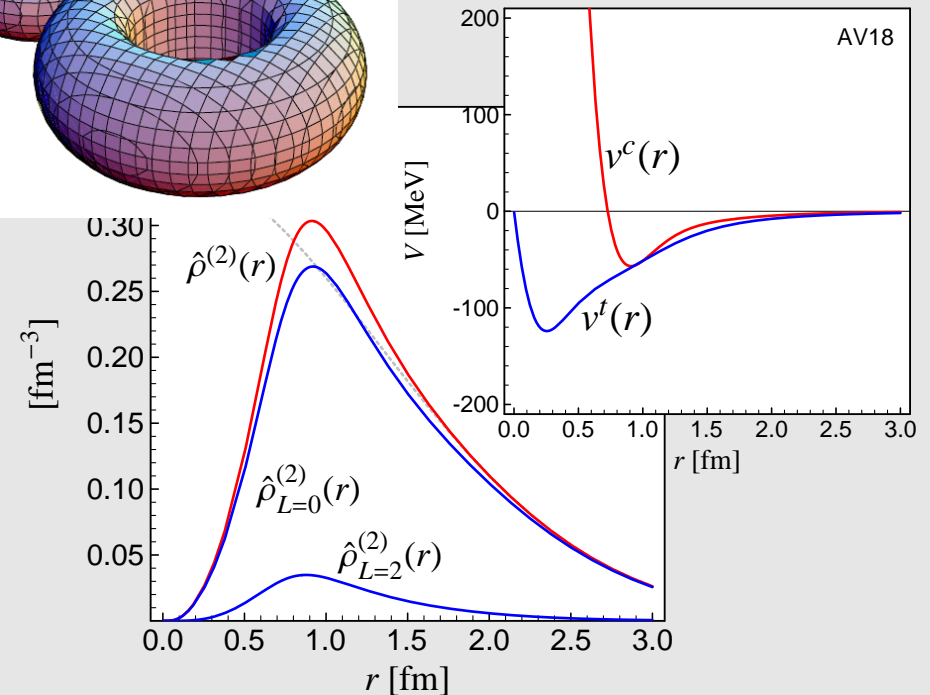
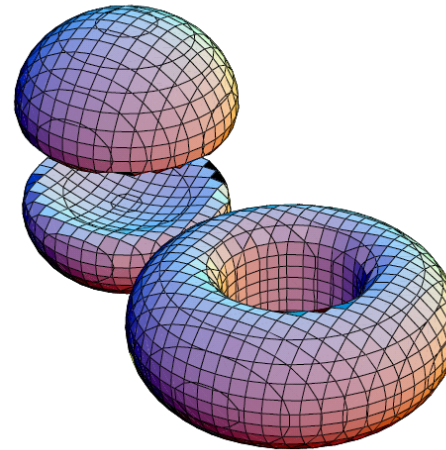


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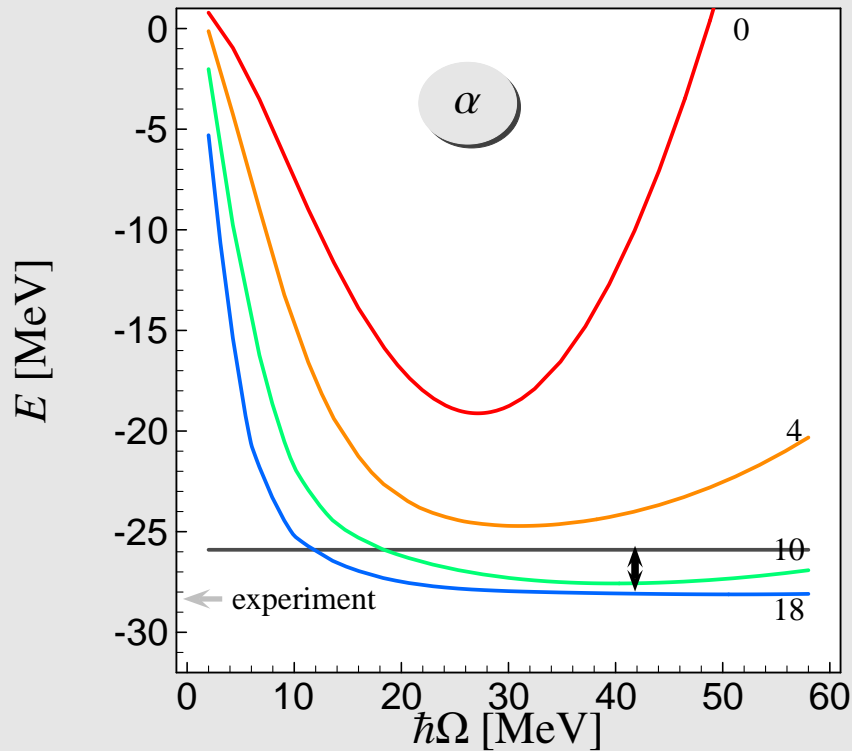
## Tensor Correlations

$$\tilde{G}_\Omega = \frac{1}{2} \{ (\sigma_2 \cdot \mathbf{r})(\sigma_1 \cdot \mathbf{p}_\Omega) + (\sigma_1 \cdot \mathbf{r})(\sigma_2 \cdot \mathbf{p}_\Omega) \}$$

mixes other angular momenta



$^4\text{He}$



test of two-body approximation

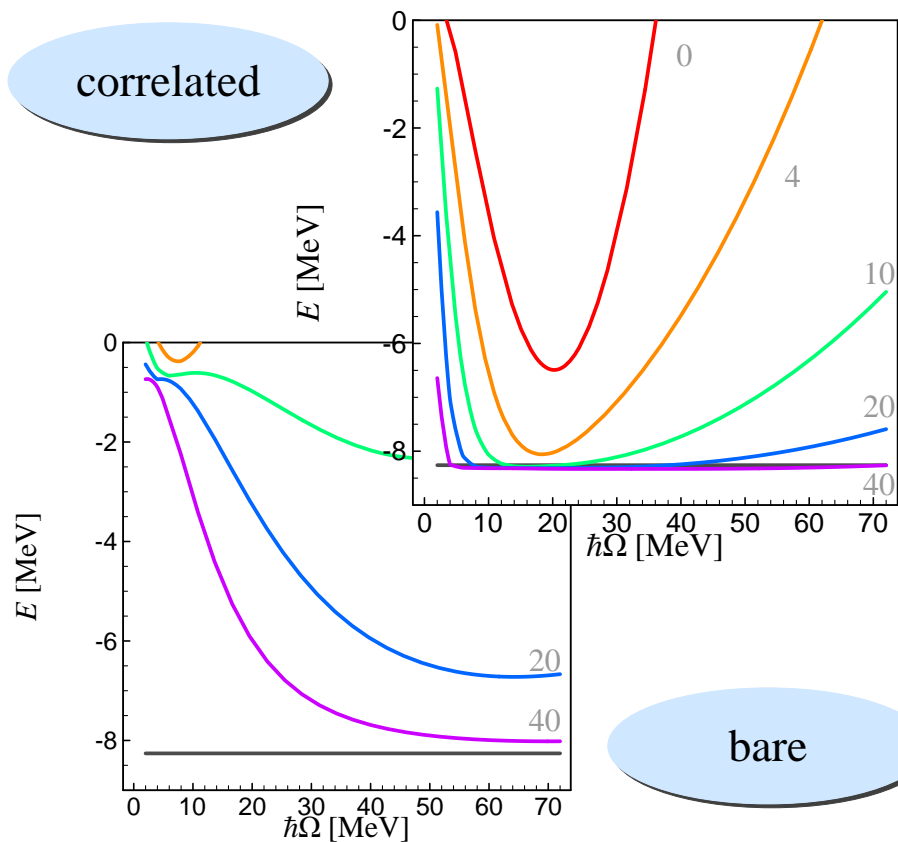
Quasi-exact calculations for light nuclei possible

exact result from PRC64 (2001) 044001

- use no-core shell model code from Petr Navratil (LLNL)

- neglected 3-body correlated terms same order as genuine 3N interactions
- more investigations needed

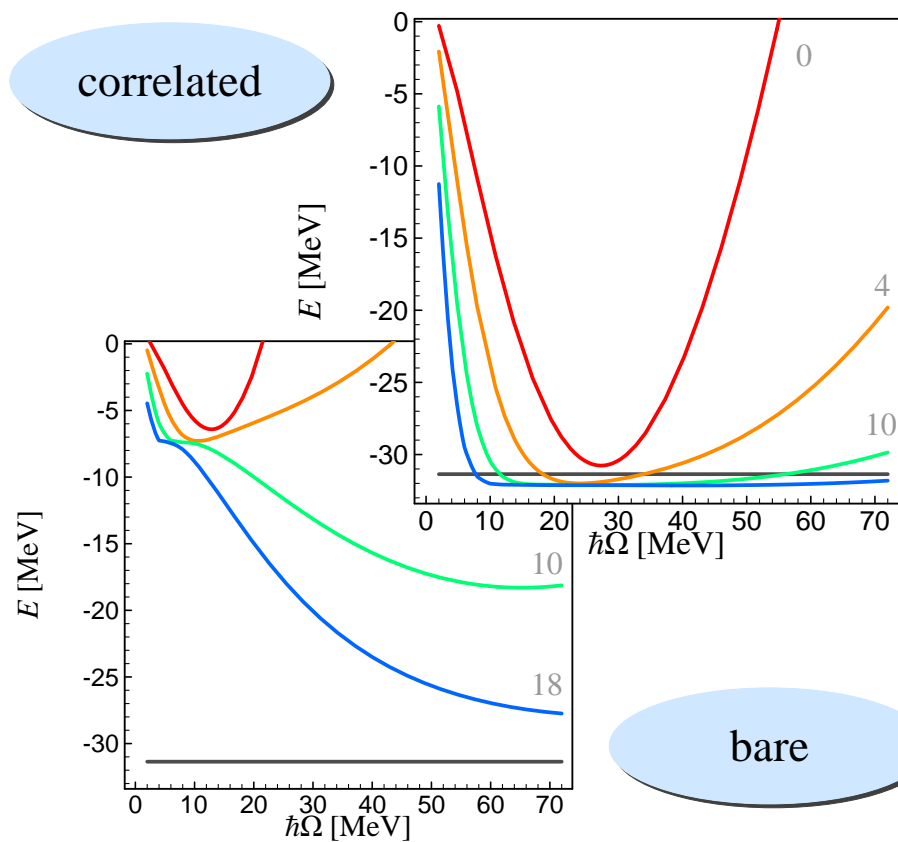
$^3\text{He}$



exact results from PRC52 (1995) 2885

only radial correlations

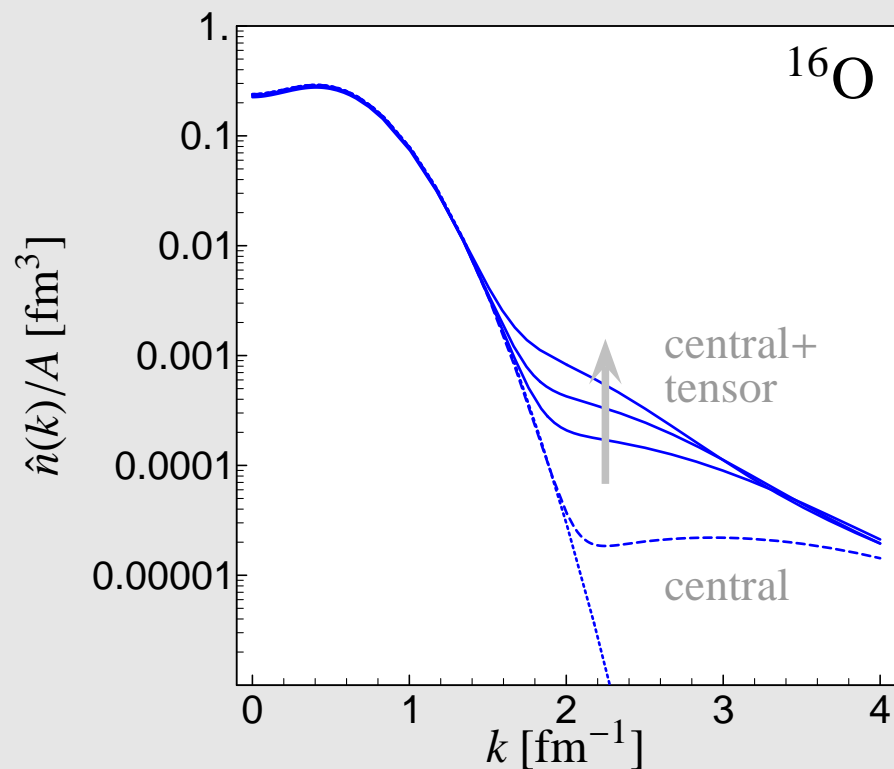
$^4\text{He}$



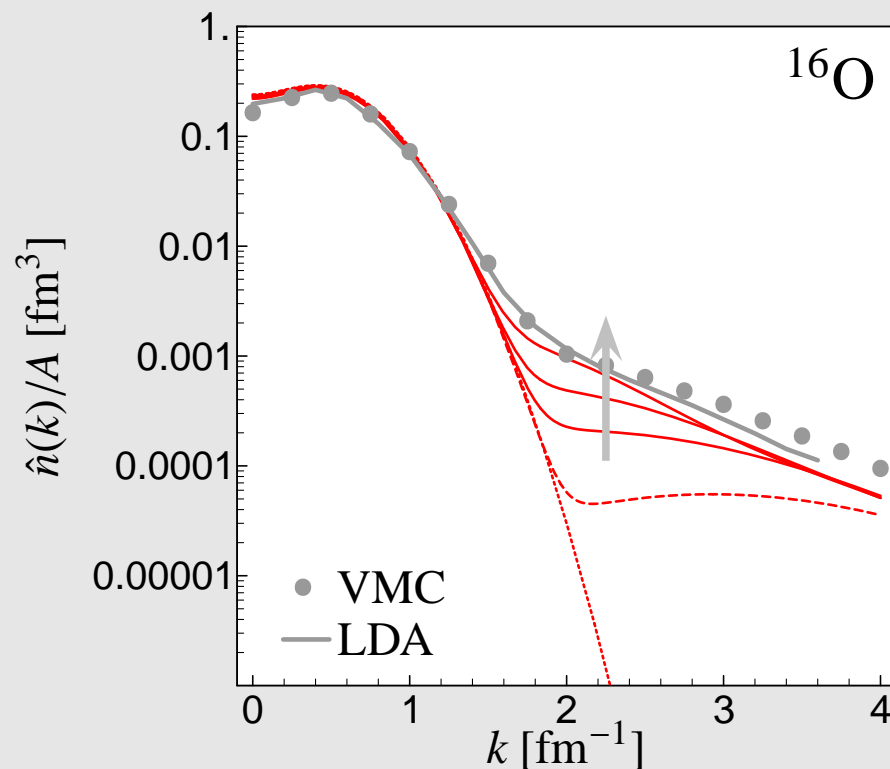
- use no-core shell model code from P tr Navratil (LLNL)

# Nucleon Momentum Distributions

## Bonn-A



## Argonne V18

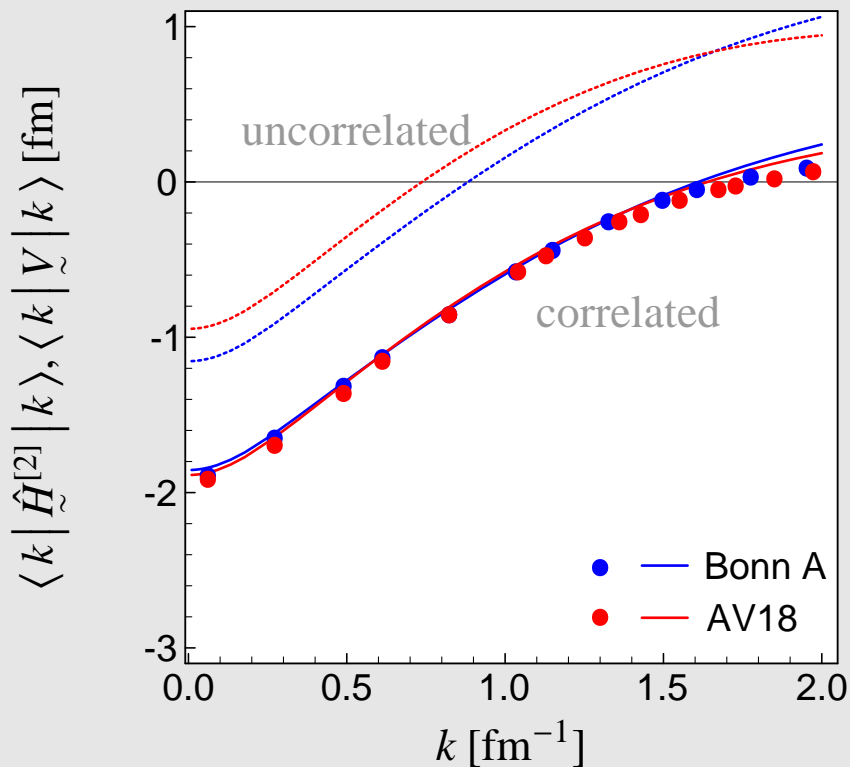


- correlations induce high-momentum components
- contributions of tensor correlations very big
- different correlator ranges relevant especially at the fermi surface

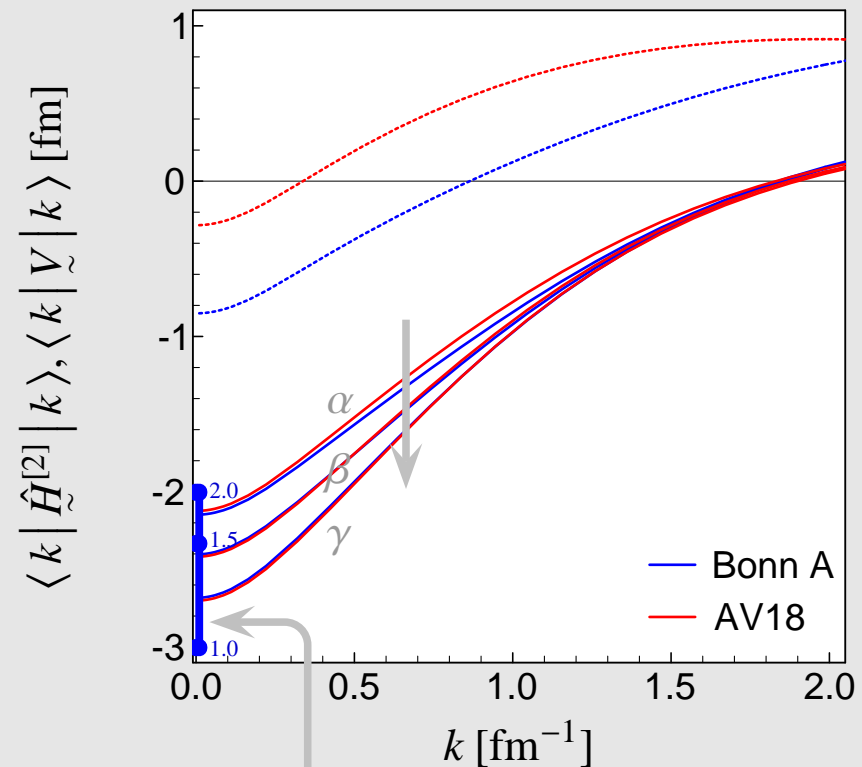
# Interaction in Momentum Space

$$\langle klm | \hat{H}_{\tilde{}}^{[2]} | k'l'm' \rangle = i^{l'-l} M \int d^3x Y_{lm}^*(\hat{\mathbf{x}}) j_l(kx) \langle \mathbf{x} | \hat{H}^{[2]} | \mathbf{x} \rangle j_{l'}(k'x) Y_{l'm'}(\hat{\mathbf{x}})$$

## $^1S_0$ channel



## $^3S_1$ channel

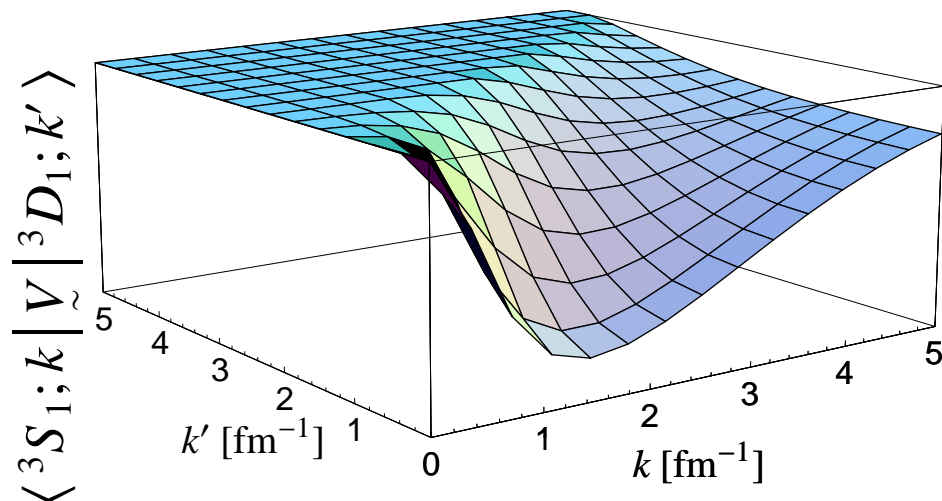
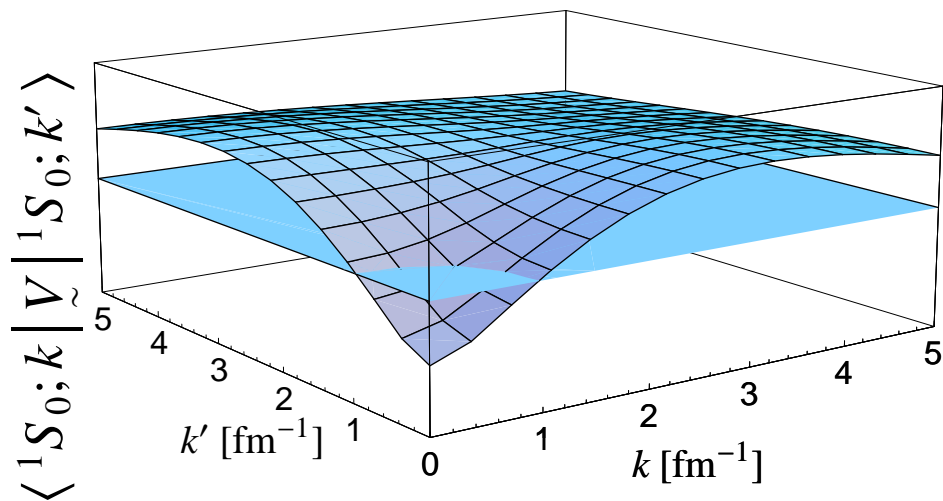


→ unique effective potential – identical to  $V_{\text{lowk}}$

Kuo, Schwenk, nucl-th/0108041

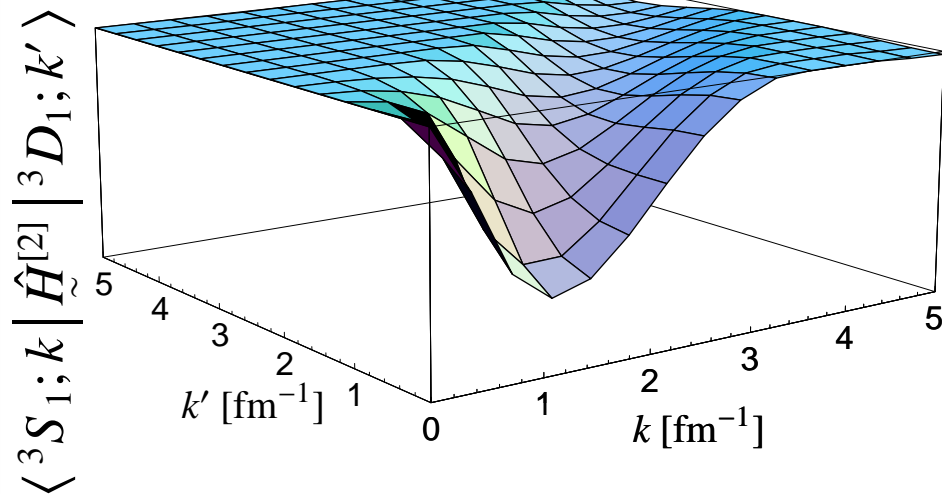
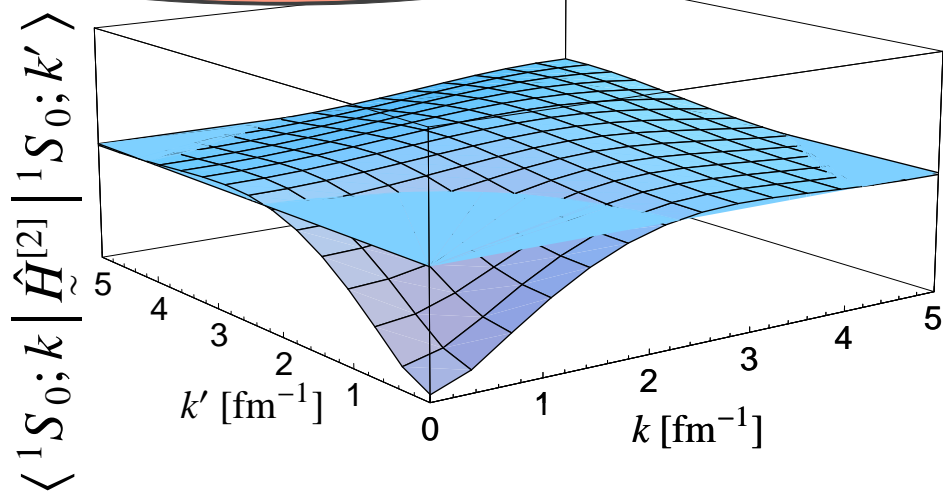
→  $V_{\text{lowk}}$  Cutoff  $\Lambda = 1.0 - 2.0 \text{ fm}^{-1}$

# Off-diagonal Matrix Elements



bare potential

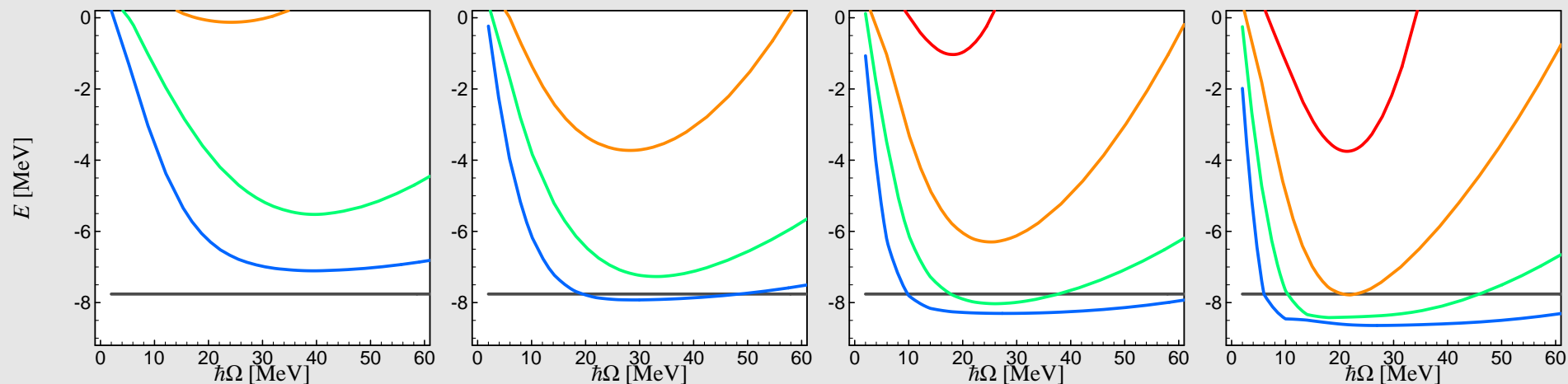
→ “pre-diagonalization”



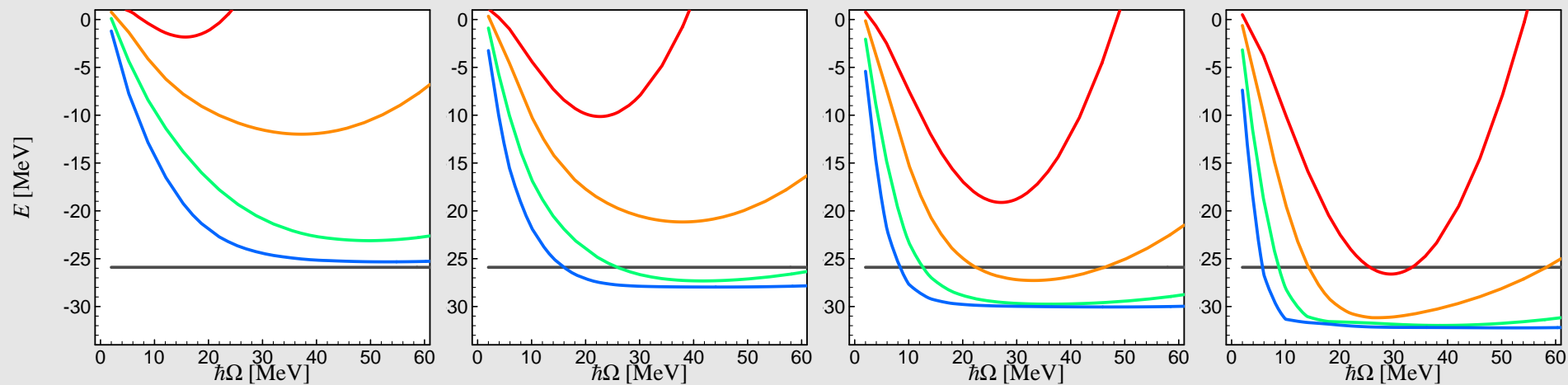
correlated interaction

Increasing range of tensor correlator

$^3\text{He}$

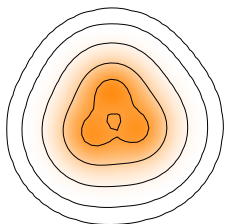


$^4\text{He}$



# $^{11}\text{B}$ ( $^3\text{He}, t$ ) $^{11}\text{C}$ – Gamov-Teller transitions

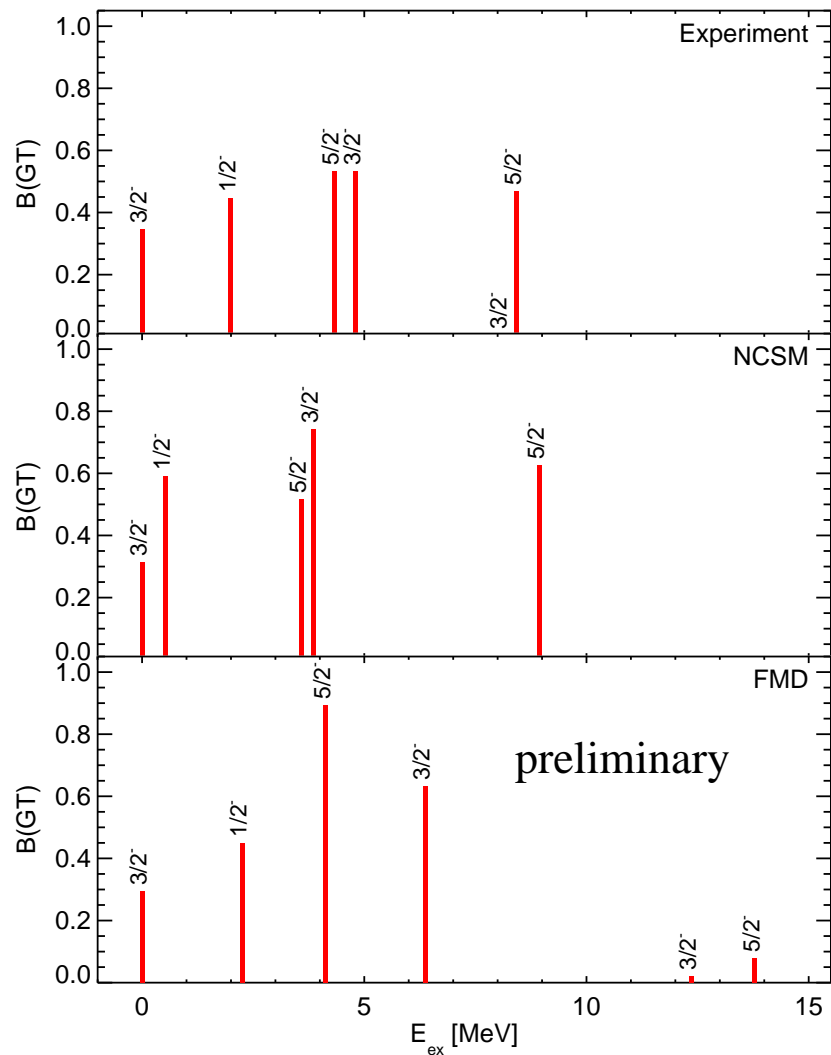
$^{11}\text{B}$



transition:  $\tilde{C}_{\Omega}^{-1} \sigma\tau_+ \tilde{C}_{\Omega}$   
 up to now only:  $\sigma\tau_+$

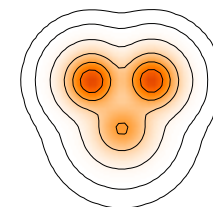
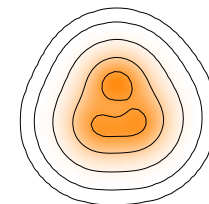
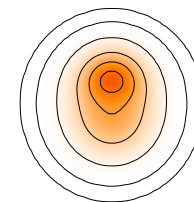
NCSM: Navrátil, Ormand  
 no core shell model with 3-body  
 force, PRC 68(2003)  
 third  $3/2^-$  missing

FMD with configuration mixing



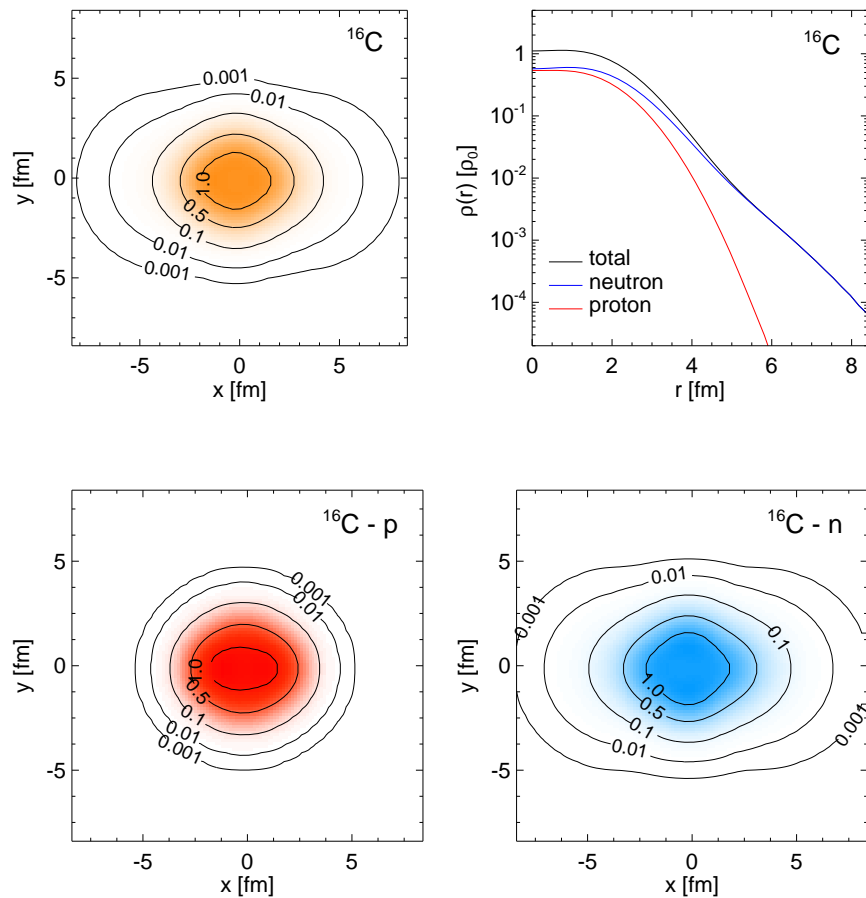
Exp.: Y. Fujita, P. von Brentano et al.  
 Phys. Rev. C 70, 011306(R) (2004)

$^{11}\text{C}$



# $^{16}\text{C}$ PAV only

## Variation



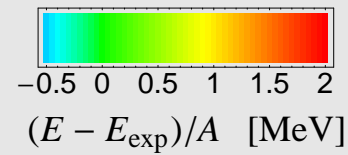
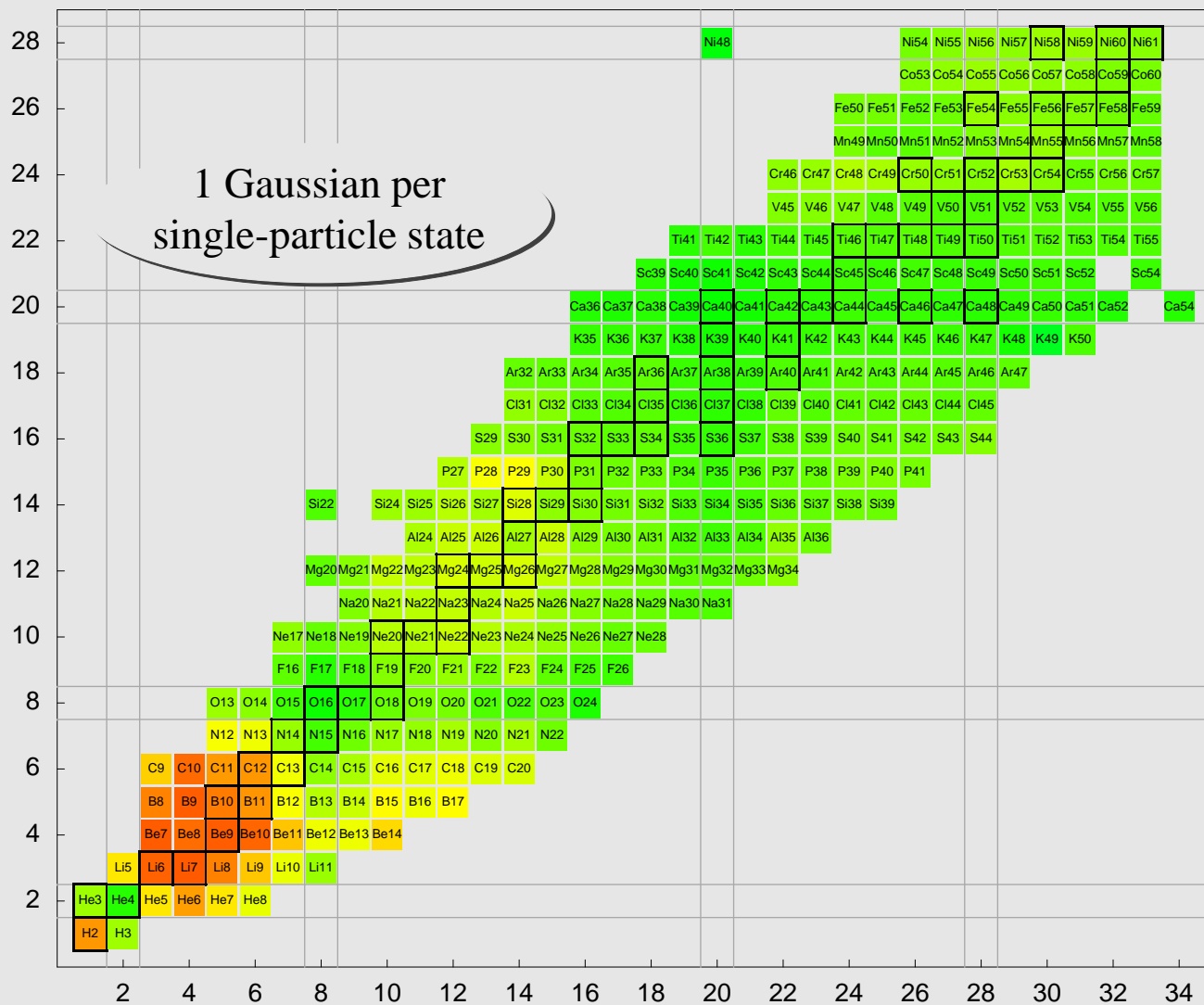
	$E_b$ [MeV]	$r_{charge}$ [fm]	$r_{matter}$ [fm]
PAV 1g	99.1	2.52	2.88
PAV	105.0	2.49	2.60
Exp	110.8		$2.70 \pm 0.03$
			$2.76 \pm 0.06$

	$E_{2^+}$ [MeV]	$B(E2)$ [ $e^2\text{fm}^4$ ]
PAV	1.29	4.6
Exp	1.77	$3.15 \pm 0.95$
Global Best Fit <sup>1</sup>	1.77	$82 \pm 14$

<sup>1</sup> Raman *et al*, Atomic Data and Nuclear Data Tables **78** (2001) 1

➔ calculated  $B(E2)$  consistent with anomalously long lifetime of  $2^+$  state measured at RIKEN

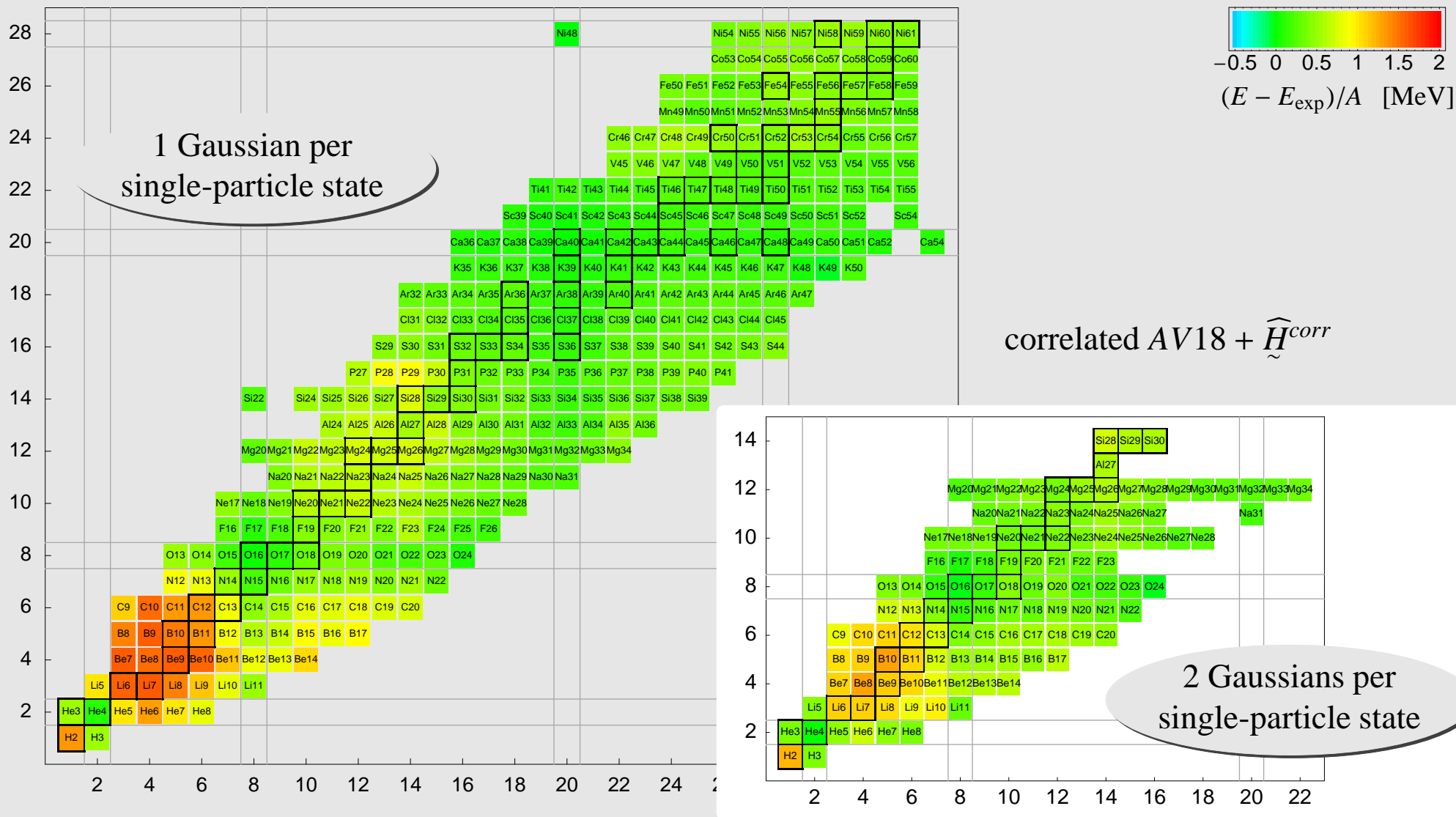
Imai *et al*, PRL in print



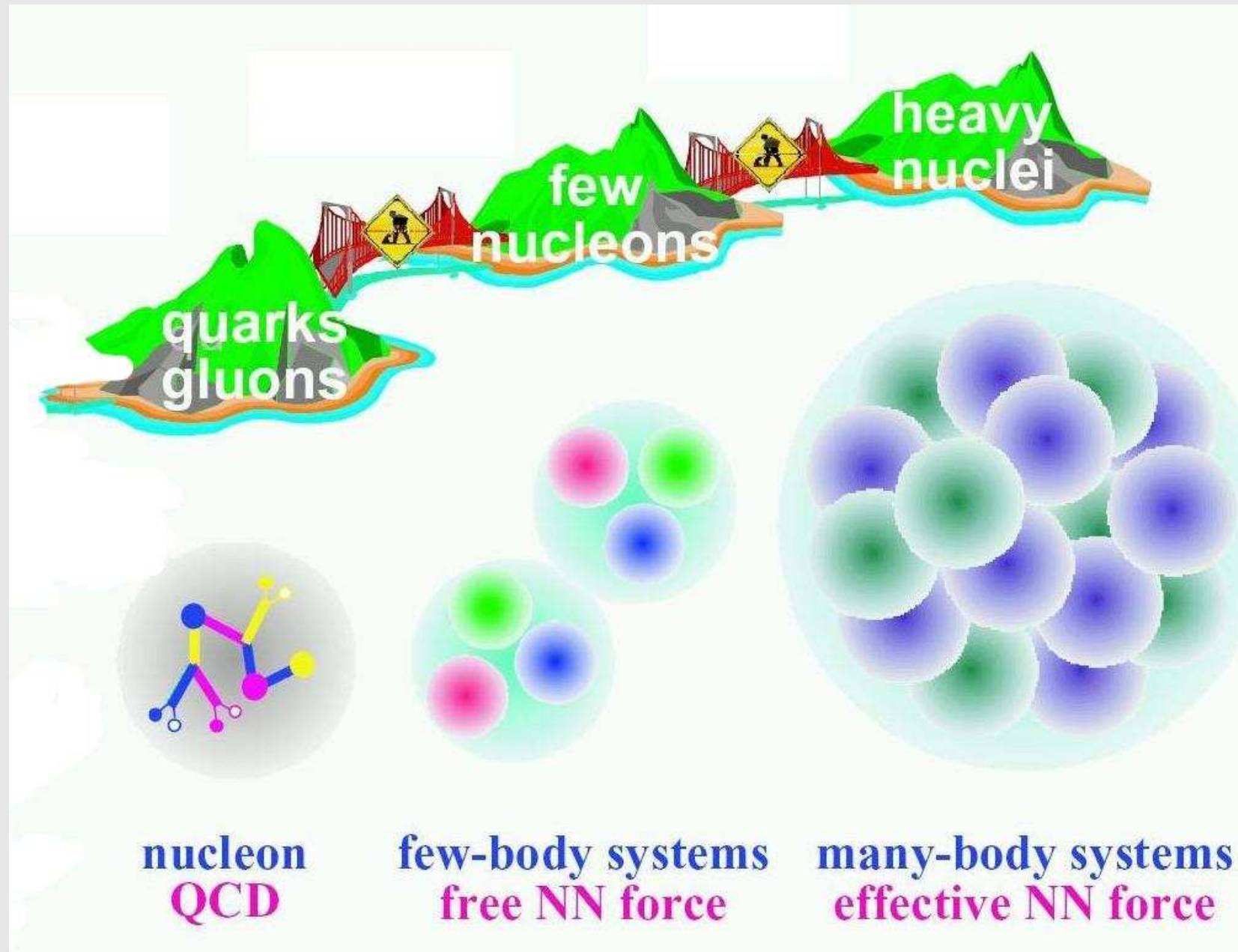
correlated  $AV18 + \widehat{H}^{corr}$

# FMD, Variation

# Nuclear Chart



# Nuclear Degrees of Freedom



# Nuclear Degrees of Freedom

