Working notes on the experimental analysis of the cumulants of the net-proton number at LHC

I. Selyuzhenkov,[1] V. Skokov,[1] and collaborators[1]

[1] *GSI Helmholtzzentrum für Schwerionenforschung, D-64291 Darmstadt, Germany*

We address the problem of a reliable approach to the analysis of experimental data for extracting the cumulants of the net-proton number at LHC. We suggest a method, that allows to diminish systematic uncertainties attributable to the efficiency of proton/anti-proton measurements.

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INTRODUCTION

Theoretical expectations, model, lattice, freeze-out curve vs phase transition curve, speculations; phenomenology, negative sixth order cumulant, HRG

STAR and efficiency, problems, questions, uncertainties

MOMENTS OF THE NET-PROTON FLUCTUATIONS

The n-th moment of the net-proton fluctuations is defined by

$$\mu_n = \langle (\delta N_p - \delta N_{\bar{p}})^n \rangle, \tag{1}$$

where

$$\delta N = N - \langle N \rangle \tag{2}$$

and the operator $\langle \cdots \rangle$ denotes the following average

$$\langle f(N) \rangle = \sum_{N=-\infty}^{\infty} f(N) P(N). \tag{3}$$

Here $P(N)$ is the probability to observe N (net)particles. In experiment, the probability can be determined in event-by-event analysis. We note that direct measurements of the probability is impossible due to efficiency . . . (Ilya). The cumulants, however, can be measured using the following trick that diminishes side effects of measurements. Our first observation is that the following ratios are almost independent of the efficiency

$$\frac{\langle N_p^\alpha N_{\bar{p}}^\beta \rangle}{\langle N_p \rangle^\alpha \langle N_{\bar{p}} \rangle^\beta} \tag{4}$$

for any powers α and β. Second, the quantities $\langle N_p \rangle$ and $\langle N_{\bar{p}} \rangle$ can be calculated experimentally with reliable efficiency corrections. Therefore, the combination

$$(\langle N_p^\alpha N_{\bar{p}}^\beta \rangle)_{eff} = \frac{\langle N_p^\alpha N_{\bar{p}}^\beta \rangle}{\langle N_p \rangle^\alpha \langle N_{\bar{p}} \rangle^\beta} \cdot (\langle N_p \rangle^\alpha \langle N_{\bar{p}} \rangle^\beta)_{eff} \tag{5}$$

provides a simple way to . . . (Ilya).

The cumulants of the net-proton fluctuations can be reexpressed in terms of the moments according to

$$\chi_2 = \mu_2, \tag{6}$$
$$\chi_3 = \mu_3, \tag{7}$$
$$\chi_4 = \mu_4 - 3\mu_2^2, \tag{8}$$
$$\chi_5 = \mu_5 - 10\mu_3\mu_2, \tag{9}$$
$$\chi_6 = \mu_6 - 15\mu_4\mu_2 - 10\mu_3^2 + 30\mu_2^3. \tag{10}$$

In their turn, the moments are combinations of $(\langle N_p^\alpha N_{\bar{p}}^\beta \rangle)_{eff}$:

$$\mu_n = \sum_{i=0}^{n}\sum_{j=0}^{n} C_{ij}(n) m^{n-i-j}, \tag{11}$$

where the matrices C^n are given in Appendix. Here m is the mean net-proton number $m = \langle N_p - N_{\bar{p}} \rangle$. For LHC energies the mean net-proton number is negligible, as shown in Fig. Ilya. Therefore the above formulas for the moments are simplified to

$$\mu_n = \langle (N_p - N_{\bar{p}})^n \rangle = \sum_{i=0}^{n} (-1)^{n-i} \binom{n}{i} \langle N_p^i N_{\bar{p}}^{n-i} \rangle, \tag{12}$$

where $\binom{n}{i} = \frac{n!}{i!(n-i)!}$. Also taking into account that $\langle N_p \rangle = \langle N_{\bar{p}} \rangle$ we can construct the ratios of cumulants that are independent of $(\langle N_p \rangle^\alpha \langle N_{\bar{p}} \rangle^\beta)_{eff} = (\langle N_p \rangle^{\alpha+\beta})_{eff}$. One of the possibilities is to consider the reduced cumulants defined by the ratios $\frac{\chi_n}{\chi_2^{n/2}}$, which, with help of Eqs. (10), can be easily expressed in terms of the reduced moments

$$\frac{\mu_n}{\mu_2^{n/2}} = \frac{\sum_{i=0}^{n} (-1)^{n-i} \binom{n}{i} \frac{\langle N_p^i N_{\bar{p}}^{n-i} \rangle}{\langle N_p \rangle^n}}{\left(\sum_{i=0}^{2} (-1)^{n-i} \binom{n}{i} \frac{\langle N_p^i N_{\bar{p}}^{n-i} \rangle}{\langle N_p \rangle^2} \right)^{n/2}} = \frac{\sum_{i=0}^{n} (-1)^{n-i} \binom{n}{i} \frac{\langle N_p^i N_{\bar{p}}^{n-i} \rangle}{\langle N_p \rangle^i \langle N_{\bar{p}} \rangle^{n-i}}}{\left(\sum_{i=0}^{2} (-1)^{n-i} \binom{n}{i} \frac{\langle N_p^i N_{\bar{p}}^{n-i} \rangle}{\langle N_p \rangle \langle N_{\bar{p}} \rangle} \right)^{n/2}}. \tag{13}$$

As can be seen from this expression, the reduced moments and the reduced cumulants do not depend on the efficiency.

CUMULANTS FROM SUBEVENTS

To avoid the effects of efficiency fluctuations one can use an approach similar to the anisotrpic flow measurement techniques and introduce subevents (random, momentum, rapidity, etc). For every cumulant order n one divides the set ov measured particles in the subset of similar (one can enforce equal) multiplicity $M_{p,i}$ and $M_{\bar{p},i}$:

$$\frac{\mu_n}{\mu_2^{n/2}} \sim \sum_{i=0}^{n} (-1)^{n-i} \binom{n}{i} \frac{\langle M_{p,1} M_{p,2} ... M_{p,i} \times M_{\bar{p},1} M_{\bar{p},2} M_{\bar{p},n-i} \rangle}{\langle M_p \rangle^i \langle M_{\bar{p}} \rangle^{n-i}} \Bigg/ \left(\sum_{i=0}^{2} (-1)^{n-i} \binom{n}{i} \frac{\langle M_{p,1} M_{p,i} \times M_{\bar{p},1} M_{\bar{p},n-i} \rangle}{\langle M_p \rangle \langle M_{\bar{p}} \rangle} \right)^{n/2}. \tag{14}$$

APPENDIX

The matricies C^n up to $n = 6$ are given by

$$C(2) = \begin{pmatrix} 1 & -2 & 1 \\ 2 & -2 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$C(3) = \begin{pmatrix} -1 & 3 & -3 & 1 \\ -3 & 6 & -3 & 0 \\ -3 & 3 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}$$

$$C(4) = \begin{pmatrix} 1 & -4 & 6 & -4 & 1 \\ 4 & -12 & 12 & -4 & 0 \\ 6 & -12 & 6 & 0 & 0 \\ 4 & -4 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C(5) = \begin{pmatrix} -1 & 5 & -10 & 10 & -5 & 1 \\ -5 & 20 & -30 & 20 & -5 & 0 \\ -10 & 30 & -30 & 10 & 0 & 0 \\ -10 & 20 & -10 & 0 & 0 & 0 \\ -5 & 5 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C(6) = \begin{pmatrix} 1 & -6 & 15 & -20 & 15 & -6 & 1 \\ 6 & -30 & 60 & -60 & 30 & -6 & 0 \\ 15 & -60 & 90 & -60 & 15 & 0 & 0 \\ 20 & -60 & 60 & -20 & 0 & 0 & 0 \\ 15 & -30 & 15 & 0 & 0 & 0 & 0 \\ 6 & -6 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$