Effects of Momentum Cuts on Cumulants of Conserved Charges

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Ref) F.Karsch, KM, K.Redlich, arXiv:1508.02614

Contents / Questions

1. **What causes kinematic momentum cuts dependence of cumulants?**
   - Fluctuations in an ideal pion gas
2. **What is the role of the above mechanism in a multi-component system?**
   - Results of M/σ^2 and $\kappa\sigma^2$ in HRG
3. **Can we compare theory and data with kinematic cuts?**
   - Scaling relation btw. p_t cut and finite V

Caveat : All in equilibrium regime

Electric Charge Fluctuations

Pion $m_\pi \sim T$: Bose statistics

$$p(T,\mu_Q) = -T\int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \underbrace{\ln(1-e^{-(E_p-\mu_Q)/T})}_{\pi^+} + \underbrace{\ln(1-e^{-(E_p+\mu_Q)/T})}_{\pi^-}$$

⇩

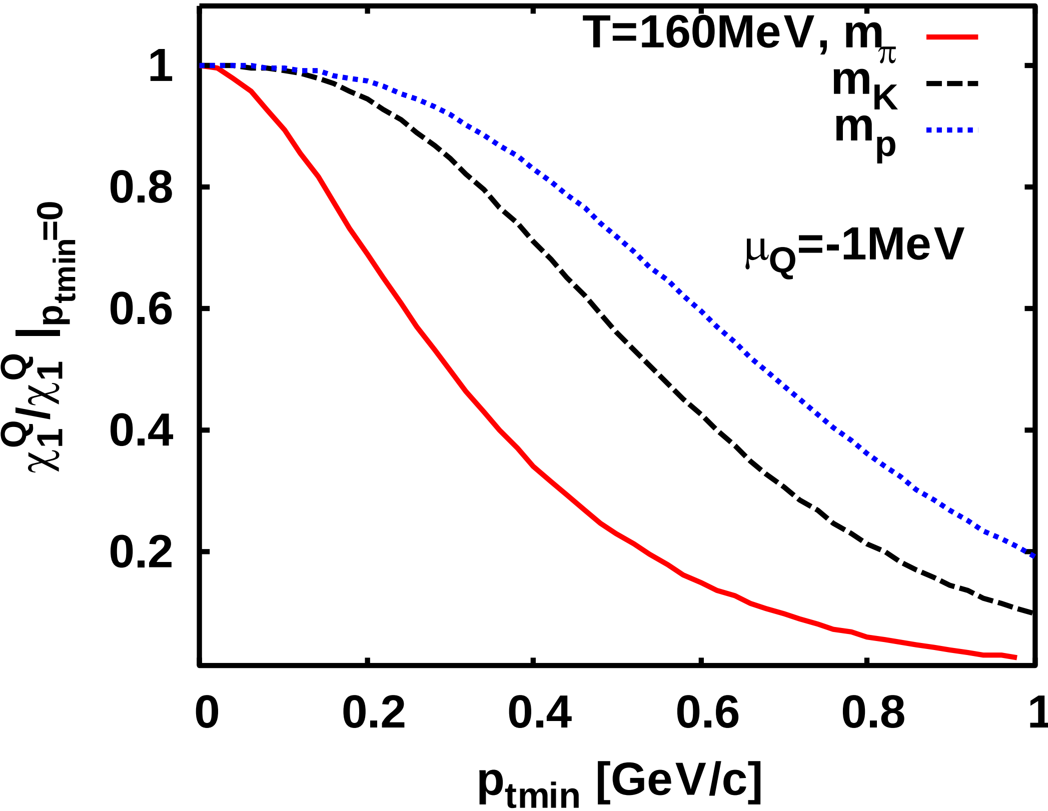
$$\chi_n^Q = \begin{cases} \dfrac{m^2}{\pi^2 T^2}\displaystyle\sum_{k=1}^{\infty} k^{n-2}\hat{K}_2(km/T)\cosh(k\mu_Q/T), & n = \text{even} \\ \dfrac{m^2}{\pi^2 T^2}\displaystyle\sum_{k=1}^{\infty} k^{n-2}\hat{K}_2(km/T)\sinh(k\mu_Q/T), & n = \text{odd} \end{cases}$$

$$\hat{K}_2(km/T) = \frac{k}{2m^2T}\int_{\eta_{\min}}^{\eta_{\max}} d\eta \int_{p_{t_{\min}}}^{p_{t_{\max}}} dp_t\, p_t |\boldsymbol{p}| e^{-kE_p/T}$$
$$= K_2(km/T) \quad (\eta_{\min} = -\infty, \eta_{\max} = \infty, p_{t_{\min}} = 0, \text{and } p_{t_{\max}} = \infty)$$

= Multicomponent Boltzmann gas pressure with mass *km*, charge *k*, and degeneracy k^{n-4}

P. Braun-Munzinger et al., NPA'12

Number (χ_1)



} Same m/T(=3)

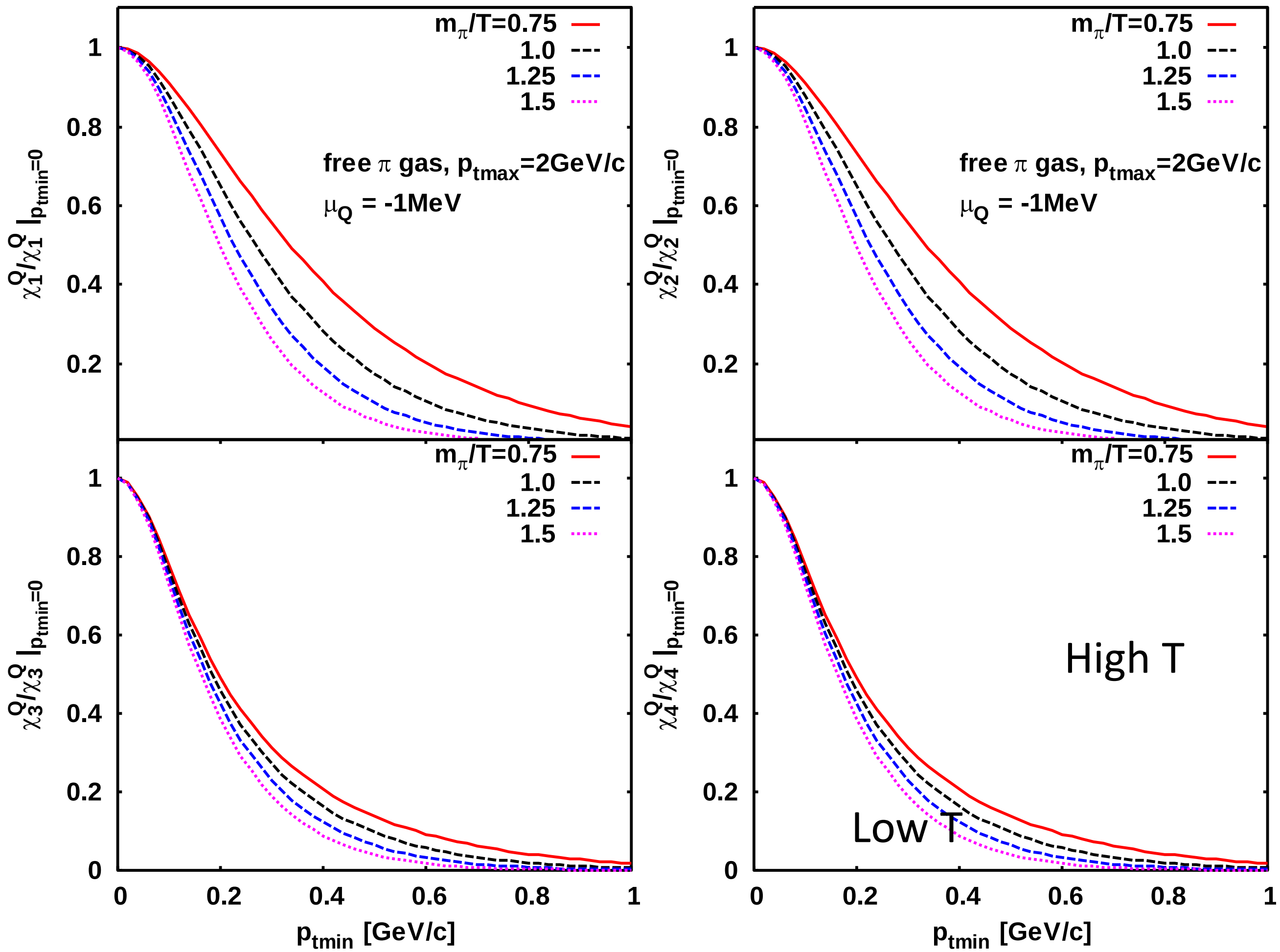
Mass : change in pt spectrum shape

Temperature : effective cut

Only quantitative difference btw Boltzmann and Bose

$$\hat{K}_2(km/T) = \frac{1}{2}\left(\frac{T}{km}\right)^2 \int d\eta \int_{kp_{t_{\min}}/T}^{kp_{t_{\max}}/T} dx x^2 \cosh\eta\, e^{-\sqrt{x^2\cosh^2\eta+(km/T)^2}}$$

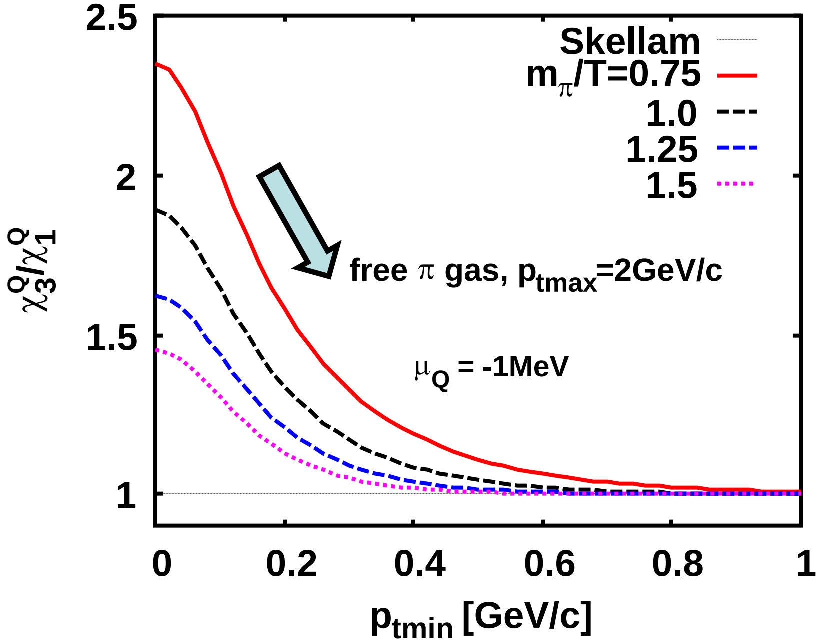
Low p_t Cut Effects on Cumulants

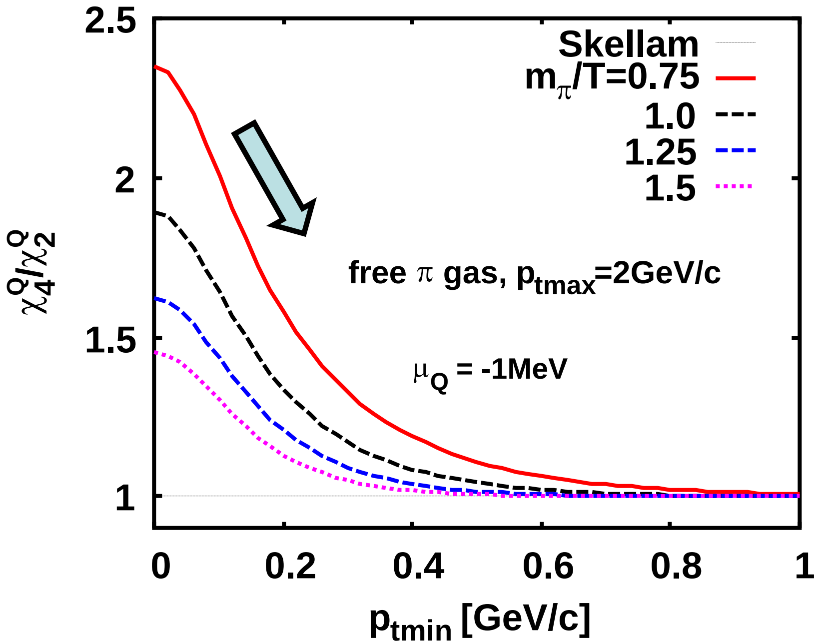


Higher order cumulants are more affected by low pt cut: QS effect

Classical description gives the same ptmin dependence for all cumulants

Low p_t Cut Effects on Cumulant Ratios





Substantial decrease from p_{tmin}=0 to p_{tmin} = 0.2 (STAR) or 0.3 (PHENIX) GeV

Look like $\chi_3/\chi_1 \sim \chi_4/\chi_2$?

Electric Charge Cumulants

$$\chi_n^Q = \begin{cases} \frac{m^2}{\pi^2 T^2} \sum_{k=1}^{\infty} k^{n-2} \hat{K}_2(km/T) \cosh(k\mu_Q/T), \; n = \text{even} \\ \frac{m^2}{\pi^2 T^2} \sum_{k=1}^{\infty} k^{n-2} \hat{K}_2(km/T) \sinh(k\mu_Q/T), \; n = \text{odd} \end{cases}$$

Leading order : Skellam distribution

***k*>1 : all positive contribution** ⇒ $\frac{\chi_{n+2}^Q}{\chi_n^Q} > 1$

Higher *n* : larger contribution from higher *k* terms

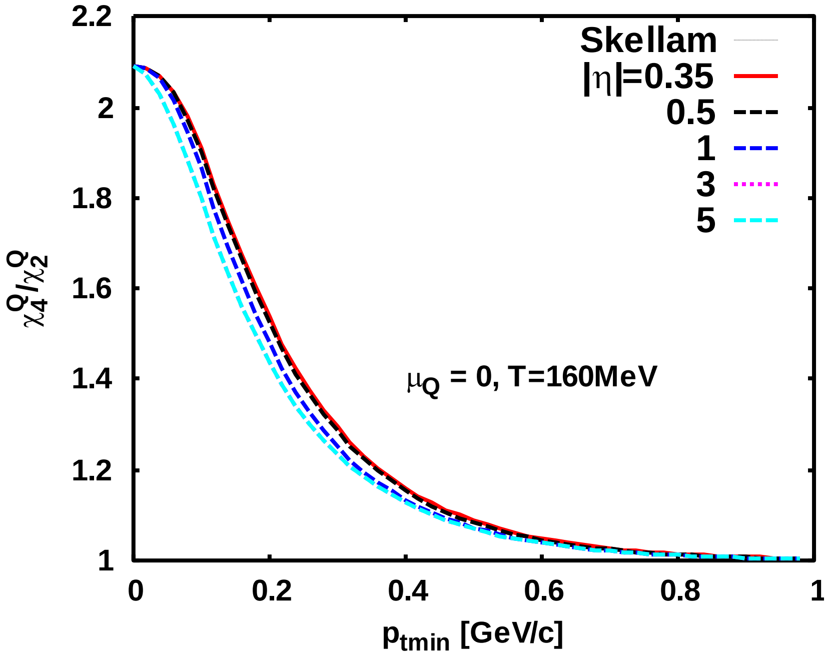
Momentum cut effects through $\hat{K}_2(km/T)$

General property for small μ_Q/T (< 1% accuracy for μ_Q@RHIC)

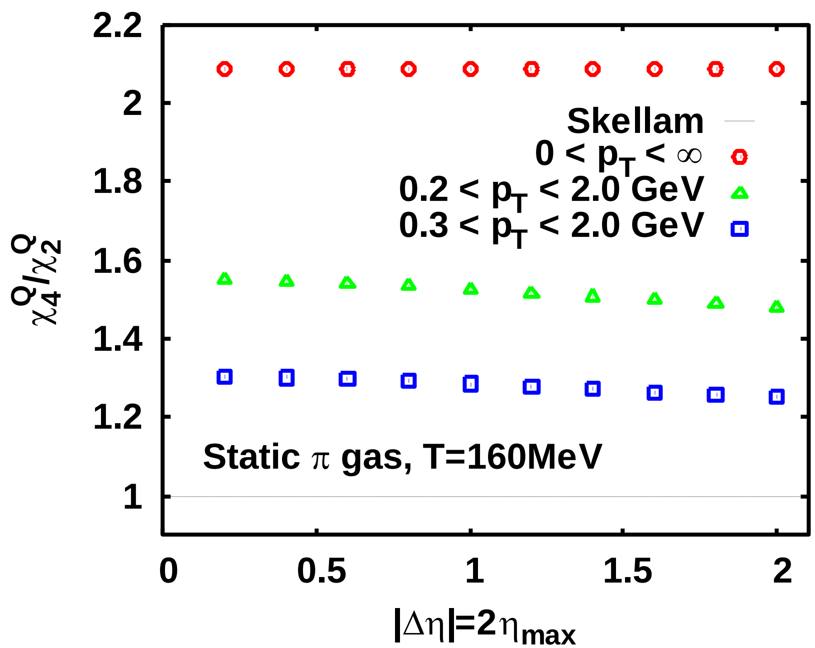
$$\frac{\chi_3^Q}{\chi_1^Q} \simeq \frac{\chi_4^Q}{\chi_2^Q}, \cdots, \frac{\chi_{2n+1}^Q}{\chi_{2n-1}^Q} \simeq \frac{\chi_{2n+2}^Q}{\chi_{2n}^Q} \qquad \frac{\chi_{2n-1}^Q}{\chi_{2n}^Q} = \frac{\sum_k k^{2n-3}\hat{K}_2(km/T) k\mu_Q/T}{\sum_k k^{2n-2}\hat{K}_2(km/T)} = \frac{\mu_Q}{T}$$

Independent of momentum cut

Pseudorapidity Cut



No significant dependence on η cut



Difference coming from lower p_t cut

$$E_{\min} = \sqrt{p_{t\min}^2 \cosh^2 \eta(=0) + m^2}$$

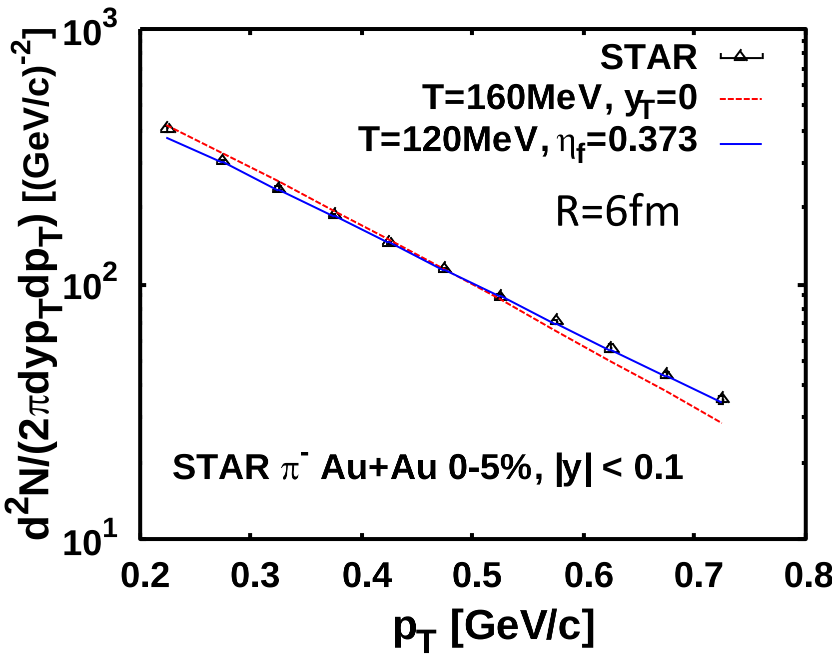
Lower cut induces effective mass heavier than m, thus approaching Skellam distribution by increasing ptmin.

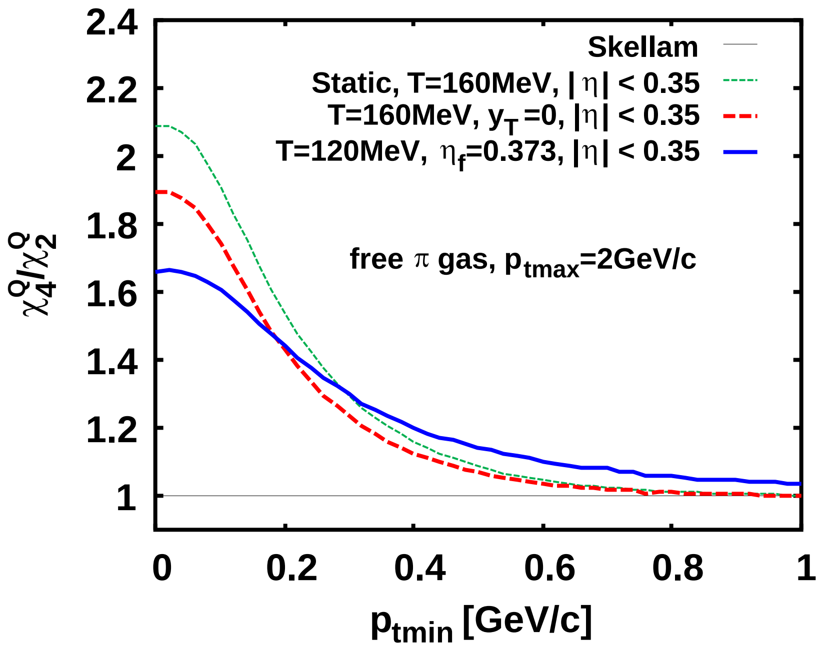
Cuts in $\eta_{\max}$ only affect high momentum particle contribution.

Effects of expansion

$$\chi_n^Q \propto \frac{\partial^{n-1} N}{\partial^{n-1}\mu_Q} \qquad N = \int \frac{d^3p}{(2\pi)^3} \int d^4x \frac{m_T \cosh(y-\eta_s)}{E_p} n_B(u\cdot p, T, \mu_Q) \exp\left(-\frac{r^2}{2R^2}\right)\delta(\tau-\tau_0)$$

Boost-invariant + Transverse Gaussian + Linear flow $v_T = \tanh^{-1}\eta_f \frac{r}{R}$

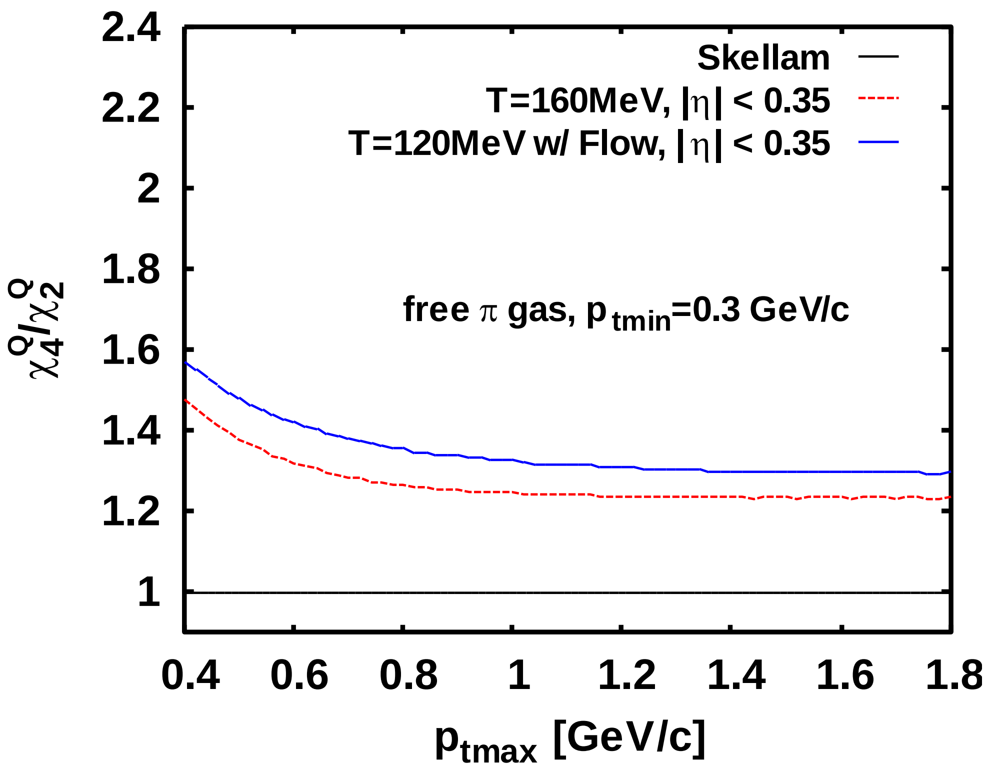




Similar χ_4/χ_2 in T=120MeV w/ Flow and T=160MeV w/o flow for p_{tmin} ~ 0.2GeV

HRG results found in P. Garg et al., '13

High p Cut Effects on Cumulant Ratios



p_{tmax} < 1GeV : stronger influences from Bose statistics

M/σ^2 in HRG

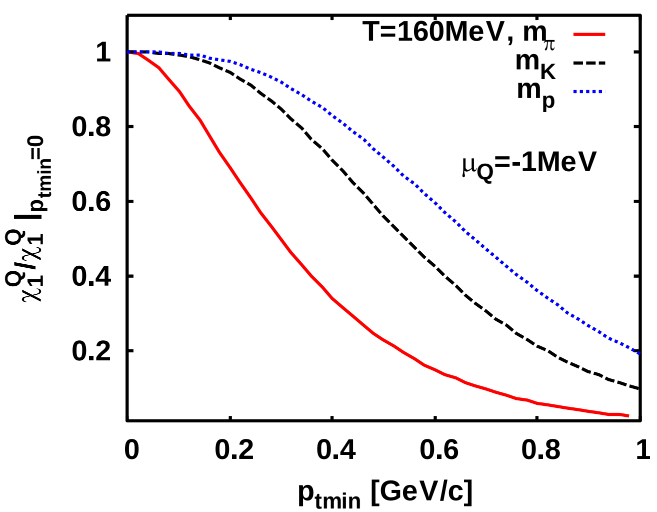
M = (π, ρ, etc) + (strange mesons) + (baryons) + (hyperons)

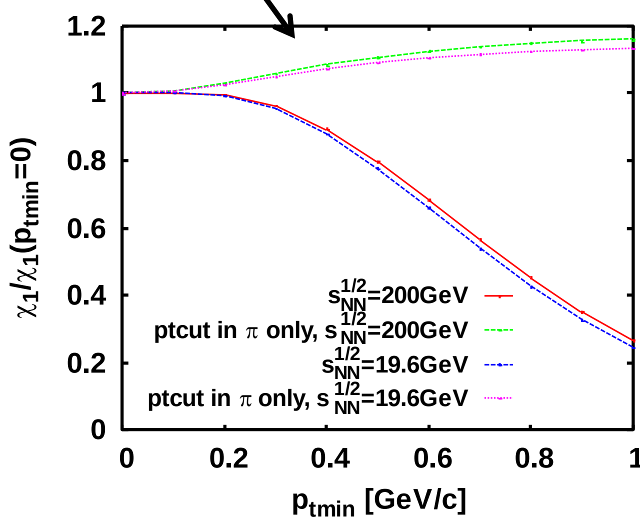
< 0 ($\mu_Q < 0$) > 0 ($\mu_S > 0$) > 0 ($\mu_B > 0$) > 0 (μ_B-(1~3)$\mu_S > 0$)

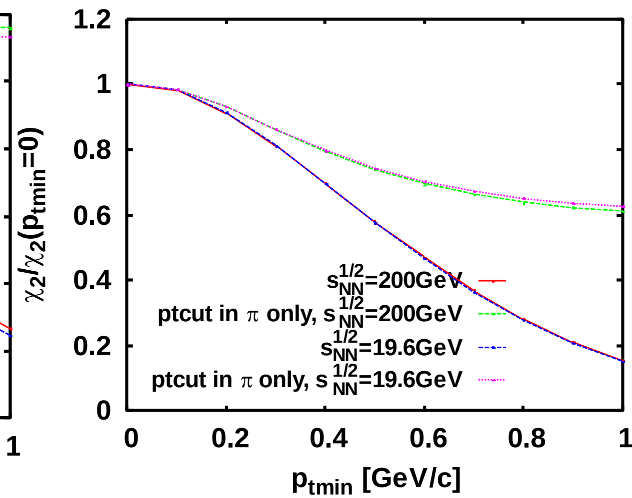
Negative meson contribution is reduced by pt cut

Total M can be non-monotonic in ptmin

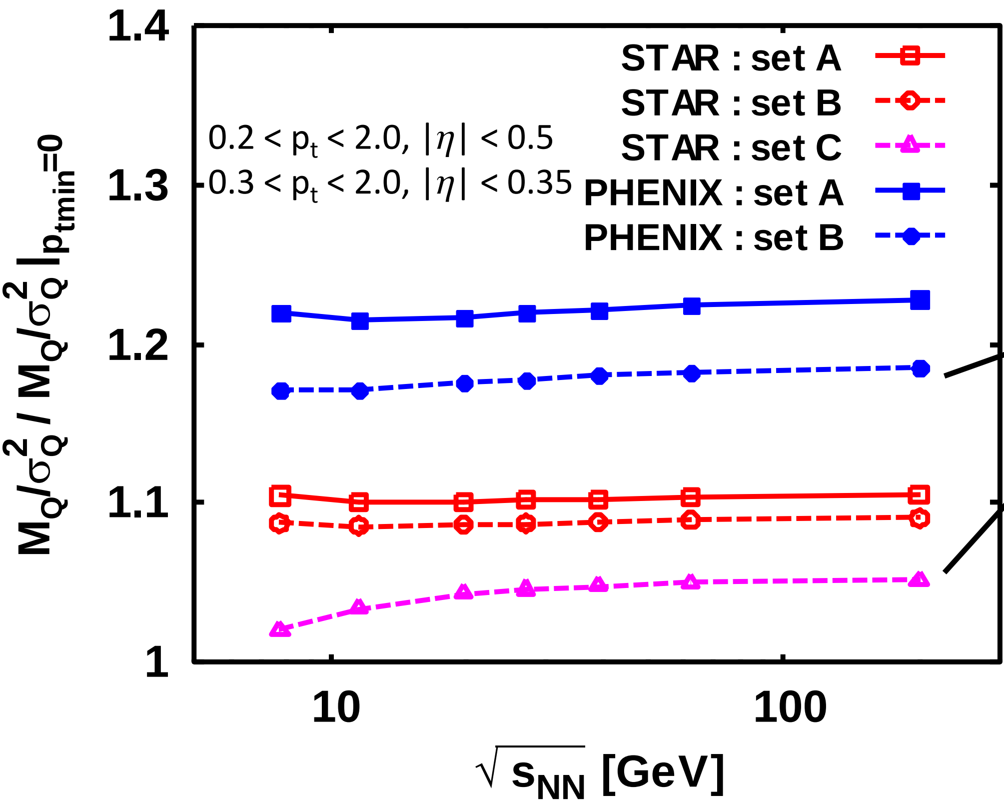
σ^2 simply decreases







M/σ^2 in HRG : STAR vs PHENIX

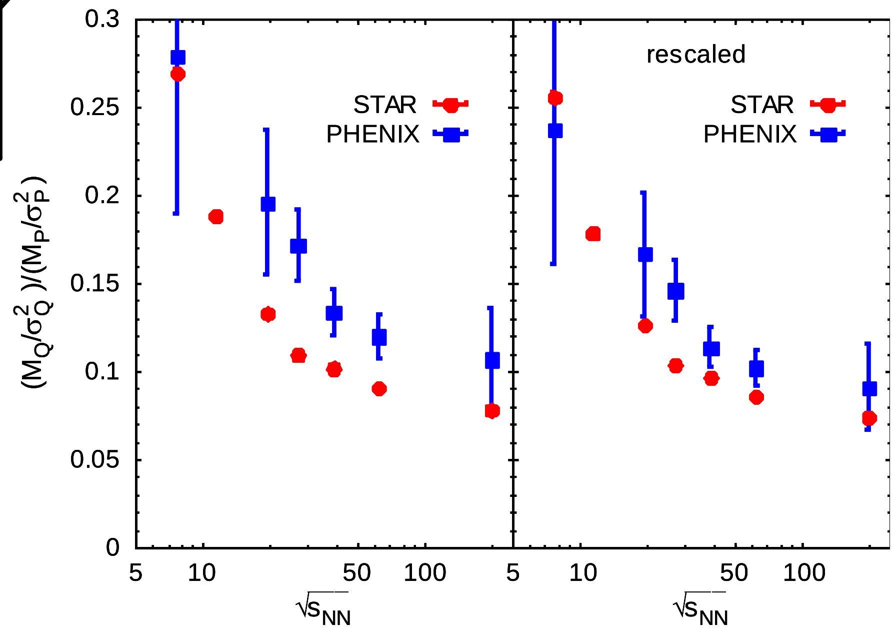


A : p_t cut for π only

B : Same p_t cut for all hadrons

C : p_{tmin} = 0.4 GeV for protons (STAR)

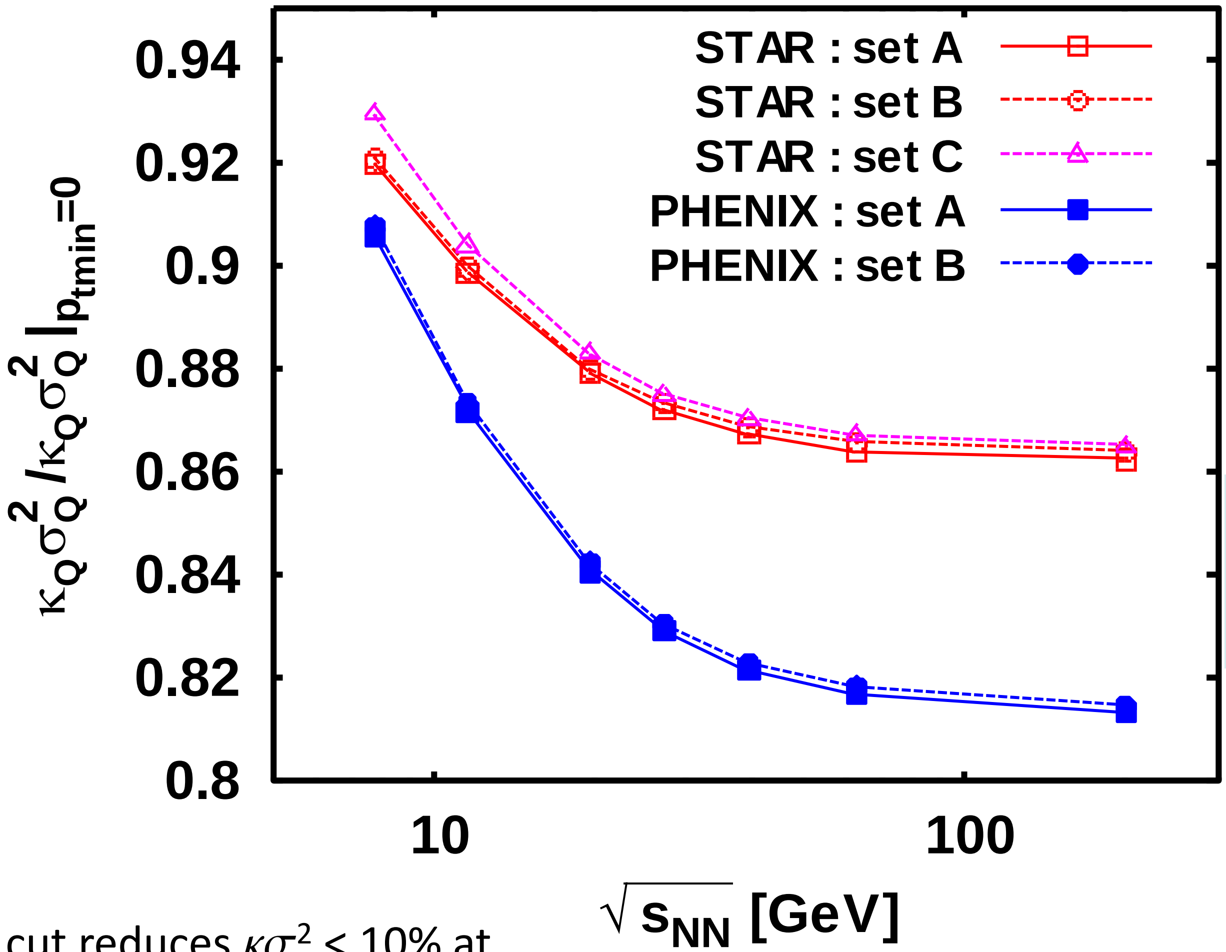
10% larger M/σ^2 in PHENIX acceptance



Data : PHENIX/STAR ~ 1.35

After rescaling, data become closer

$\kappa\sigma^2$ in HRG

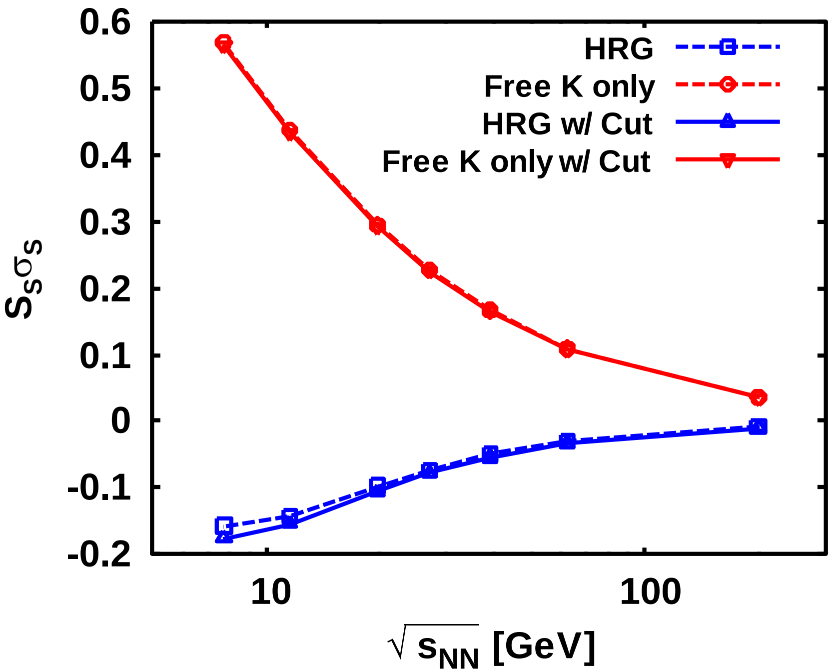


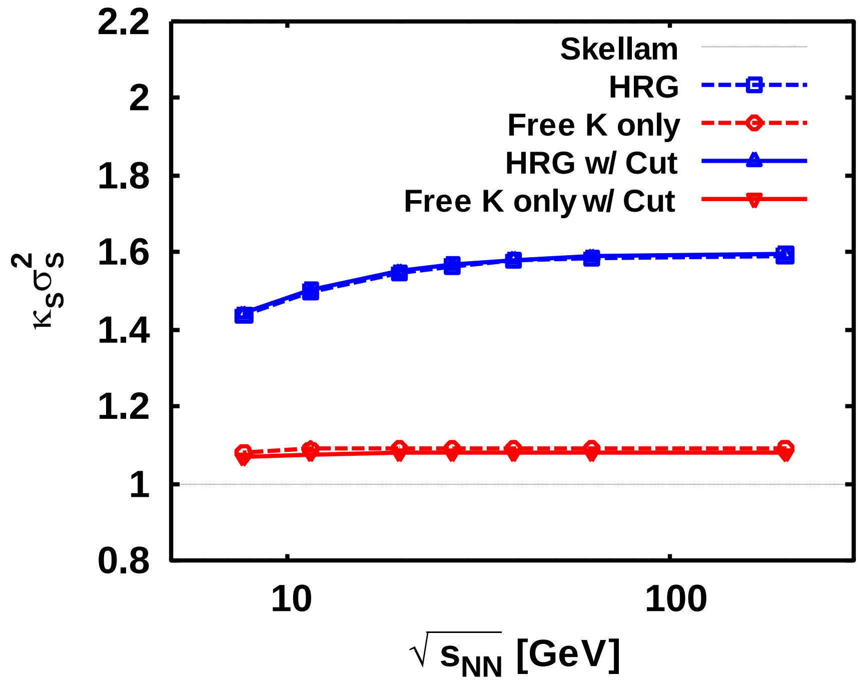
6% difference btw. STAR and PHENIX expected

p_t cut reduces $\kappa\sigma^2$ < 10% at low energies, due to more proton contribution

p_t cut reduces $\kappa\sigma^2$ by 10-20% at high energies

Net-*S*, *K* Cumulant Ratios





Momentum cut effects negligible

Difference btw. Net-S and Net-K

Skewness : sign

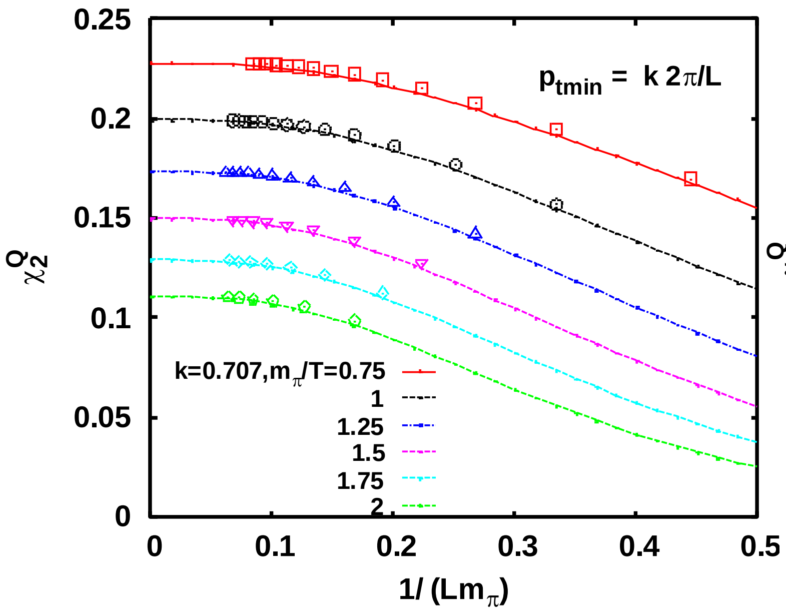
Kurtosis : magnitude

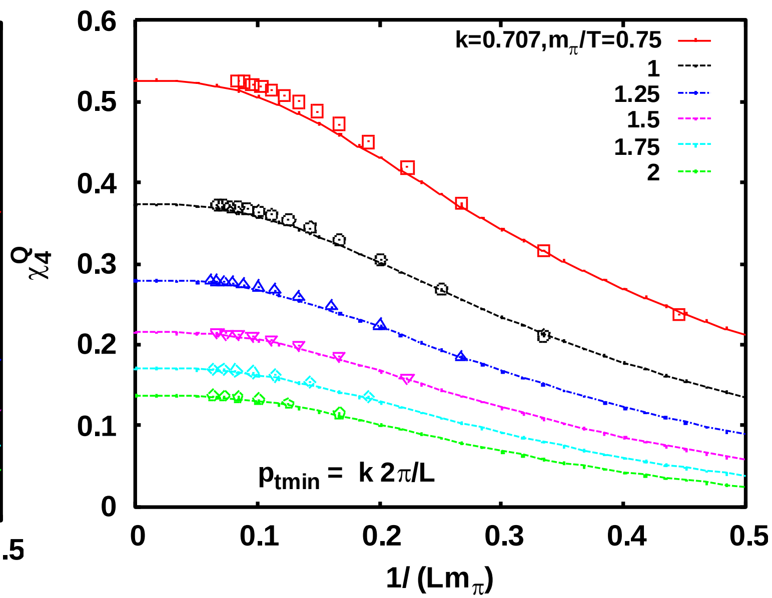
due to Multistrange hadrons

Finite Size and Low pt cut

Pion gas in a finite (L^3) box
Periodic boundary condition

? ⟺ $p_{t\min} = k\dfrac{2\pi}{L}$





$|\eta| < 0.35$

k = 0.707

[$|\eta| < 5$: k=0.5]

$$p_{t\min} = 0.2 - 0.3, \quad T = 0.16\,\text{GeV}$$

LQCD : $LT \simeq 4$

$$LT \simeq 2.4 - 3.6$$

Answers for Questions

1. **What causes kinematic momentum cuts dependence of cumulant ratios?**
   - **Quantum statistics for net electric charge**
2. **What is the role of the above mechanism in a multi-component system?**
   - **Different m and charge of hadrons produce non-trivial cut dependence of M/σ^2 and $\kappa\sigma^2$ in HRG, partly explaining difference btw. STAR and PHENIX**
3. **Can we compare theory and data with kinematic cuts?**
   - **Yes, finite V effect in LQCD is comparable with STAR low pt cut**

Backup

Finite Size and Low pt cut

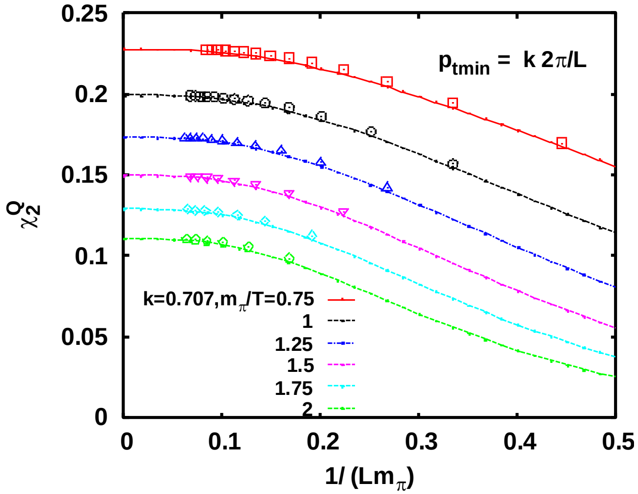
Pion gas in a finite (L^3) box
Periodic boundary condition

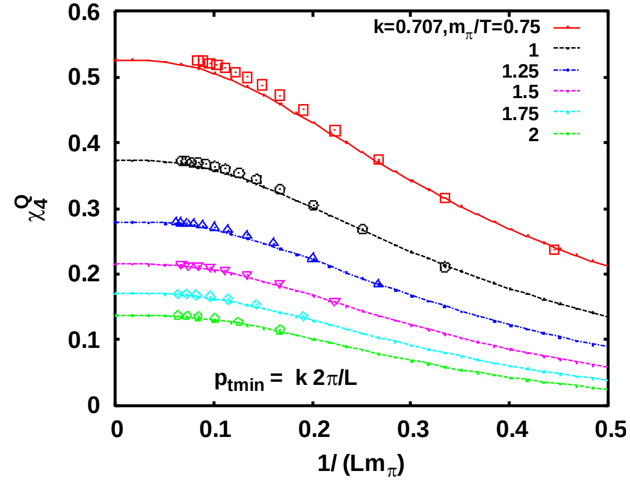
?

$$p_{t\min} = k\frac{2\pi}{L}$$

LQCD : $LT \simeq 4$

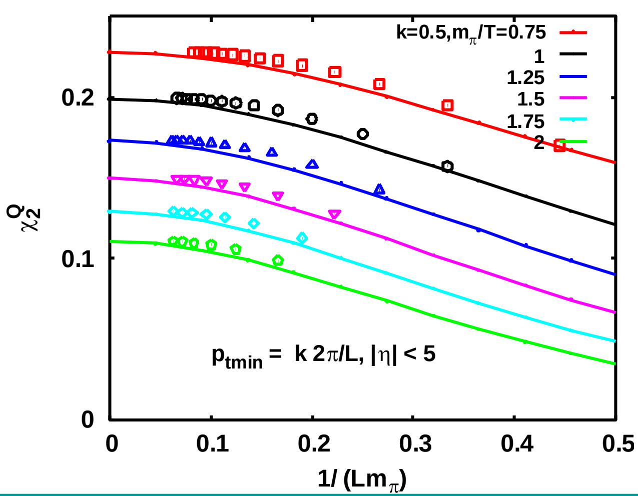
$|\eta| < 0.35$

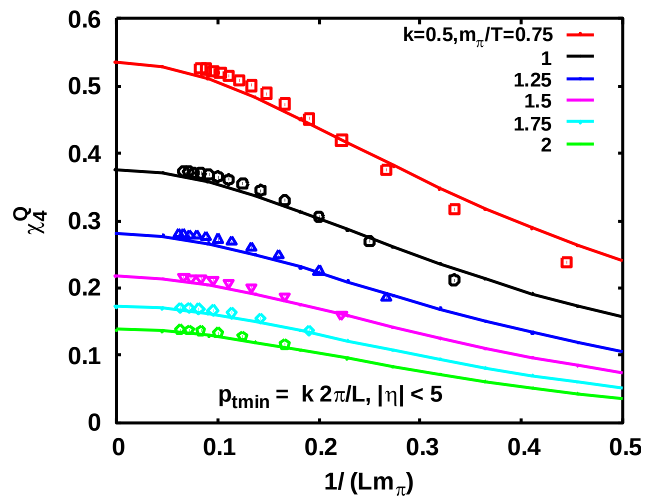




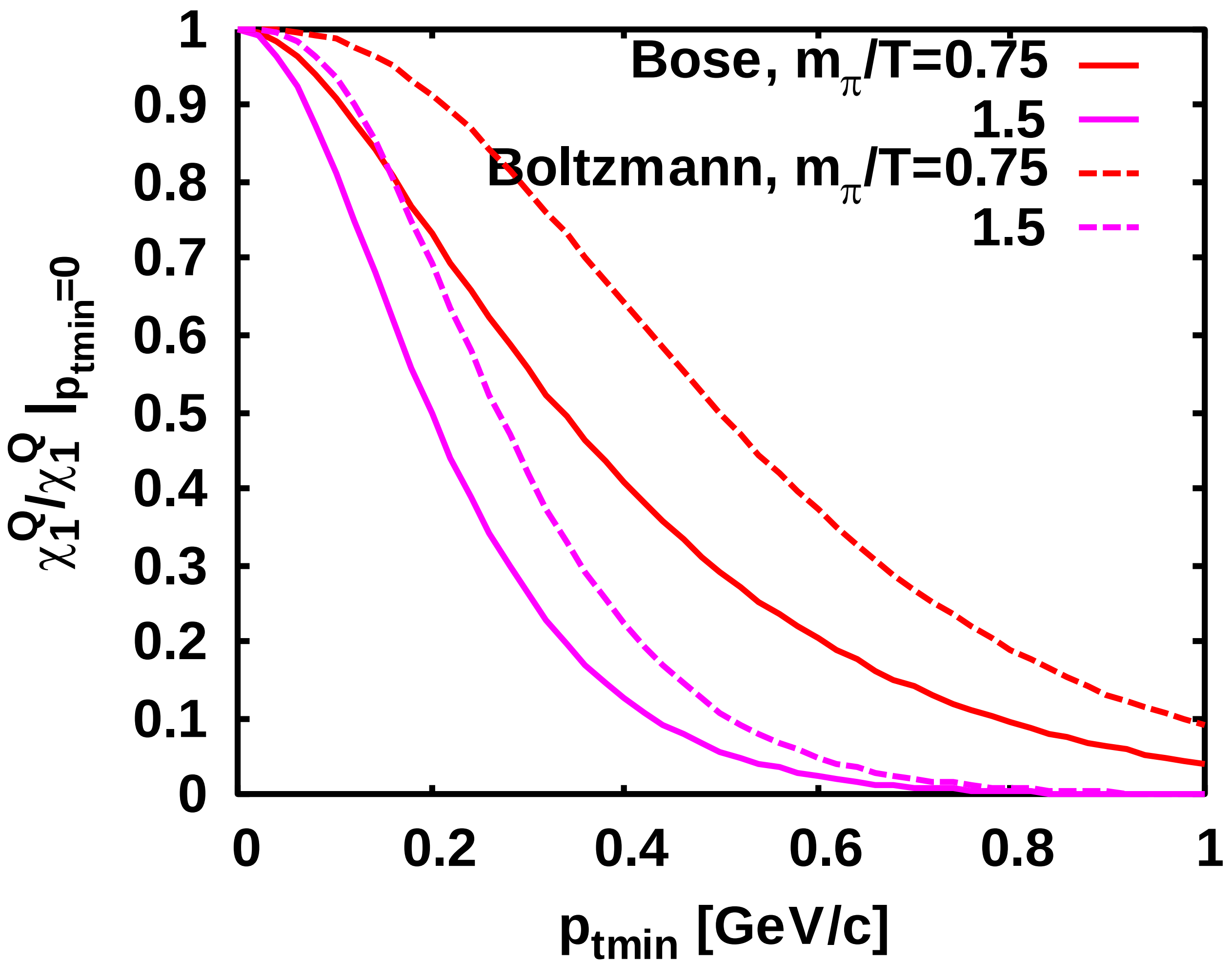
k = 0.707 (=1/√2)

$|\eta| < 5$





k = 0.5


Bose, m_π/T=0.75
1.5
Boltzmann, m_π/T=0.75
1.5
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.2
0.4
0.6
0.8
1
$\chi_1^Q/\chi_1^Q|_{p_{tmin}=0}$
p_{tmin} [GeV/c]

Skellam distribution

Poisson-Poisson ⟺ Boltzmann Statistics

$$P(N) = \left(\frac{b}{\bar{b}}\right)^{N/2} I_N(2\sqrt{b\bar{b}})e^{-(b+\bar{b})}$$

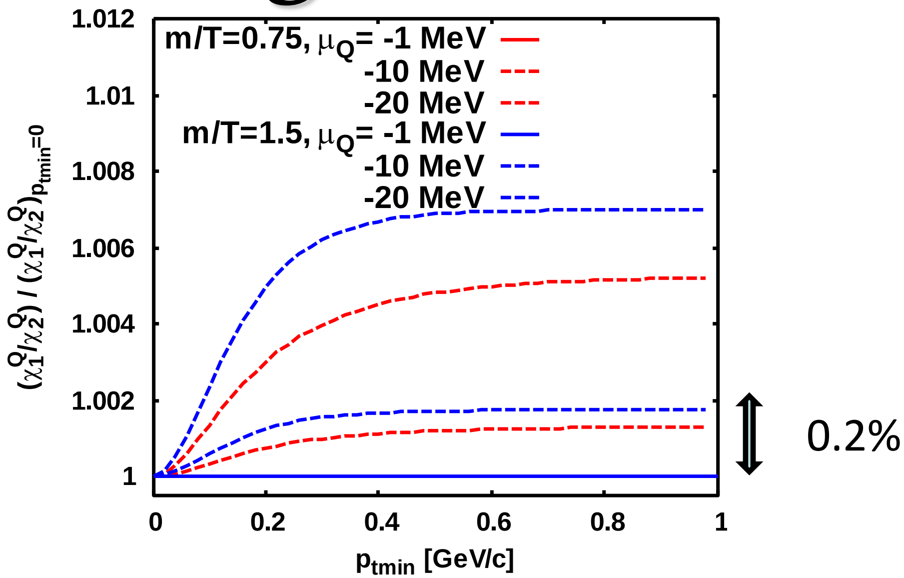
$$b = d\int \frac{d^3p}{(2\pi)^3} e^{-(E_p-\mu)/T}, \quad \bar{b} = d\int \frac{d^3p}{(2\pi)^3} e^{-(E_p+\mu)/T}$$

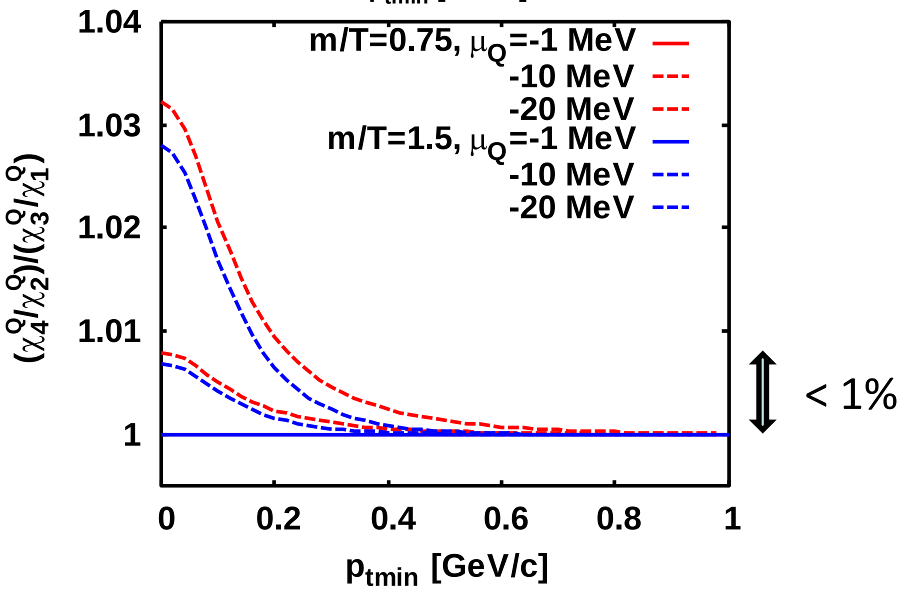
$$\chi_{2n+1} = b - \bar{b}$$

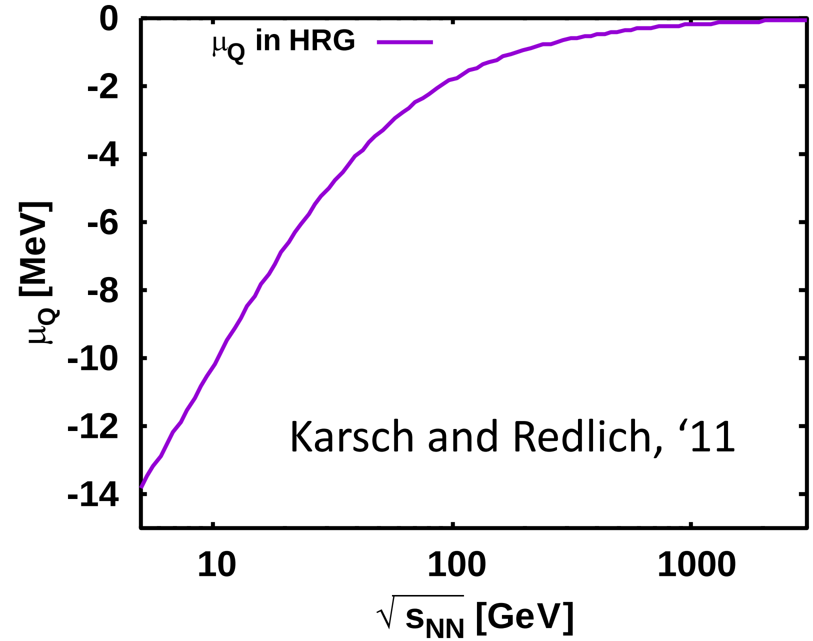
$$\chi_{2n} = b + \bar{b}$$

Effects of momentum cuts through b and $\bar{b}$ Do not affect the property of $P(N)$ and Cumulant ratios

Charge Chemical Potential







Deviation from the general relations is <u>small</u> for the BES energy range