STRANGENESS FLUCTUATIONS FROM BROAD RESONANCES

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EMMI WORKSHOP
02 NOV 2015 GSI

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THE CASE OF MISSING STRANGENESS

HADRON RESONANCE GAS MODEL

- Confinement

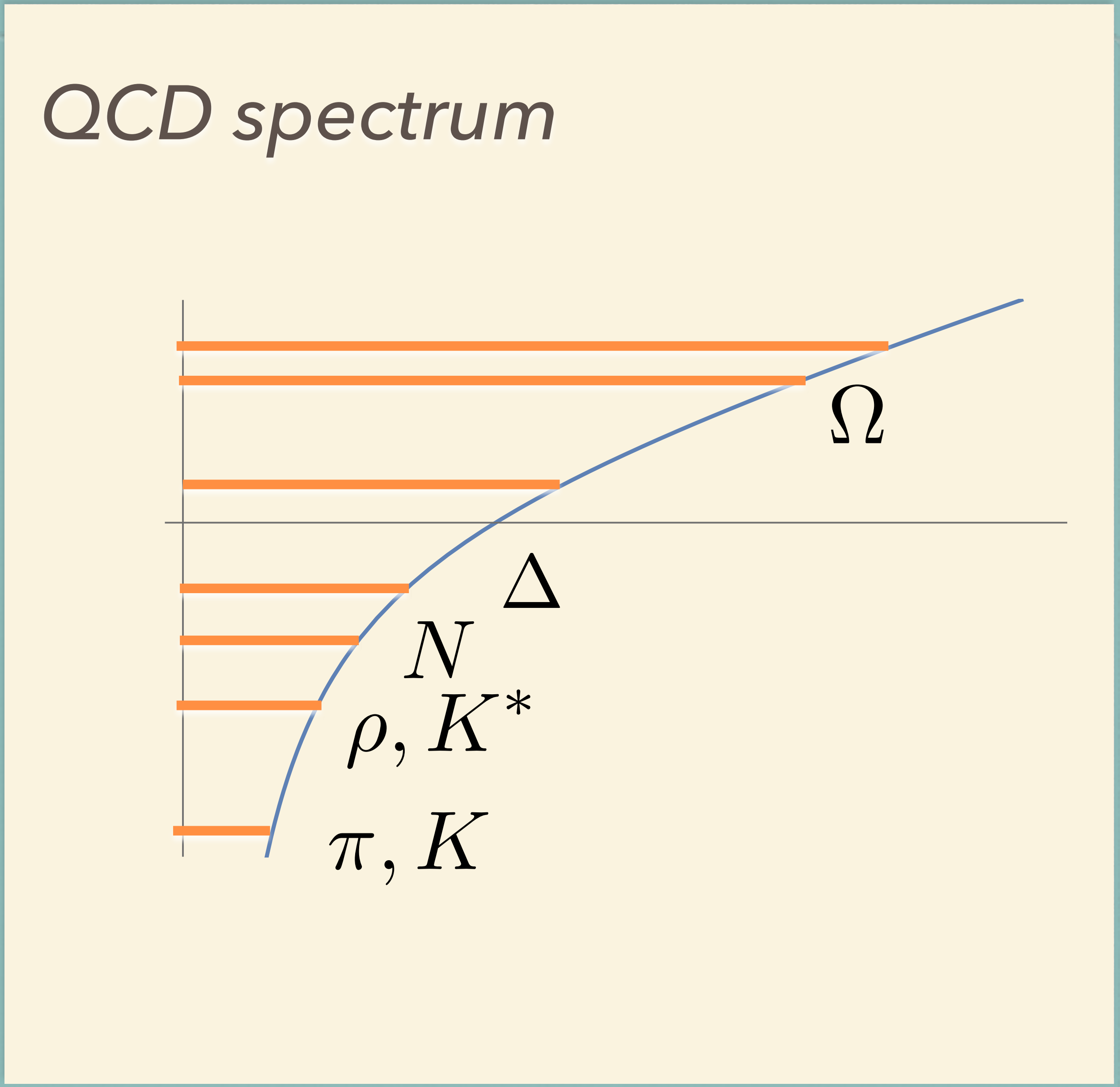
physical quantities → hadronic states representation

$$Z = \sum_{\alpha=B,M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

HADRON RESONANCE MODEL

- Confinement

physical quantities



$$Z = \sum_{\alpha=B,M} \langle \alpha | e^{-\beta H} | \alpha \rangle$$

HADRON RESONANCE GAS MODEL

- Ground states $\pi, K, P, N ...$

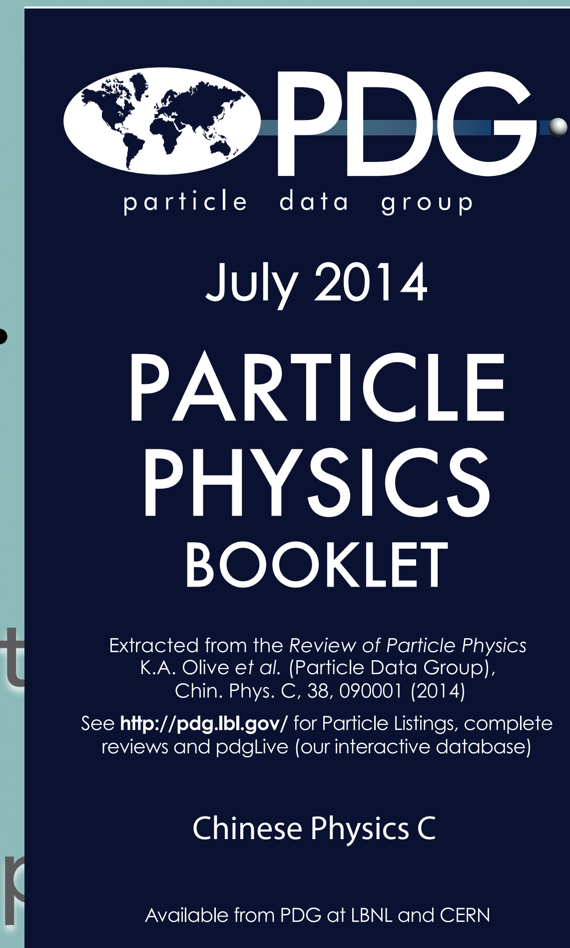
- Resonance formation dominates thermodynamics

- Resonances treated as point-like particles

$$P = T \sum_{\alpha=M,B} g_\alpha \int \frac{d^3k}{(2\pi)^3} \mp \ln(1 \mp e^{-\beta\sqrt{k^2+M_\alpha^2}})$$

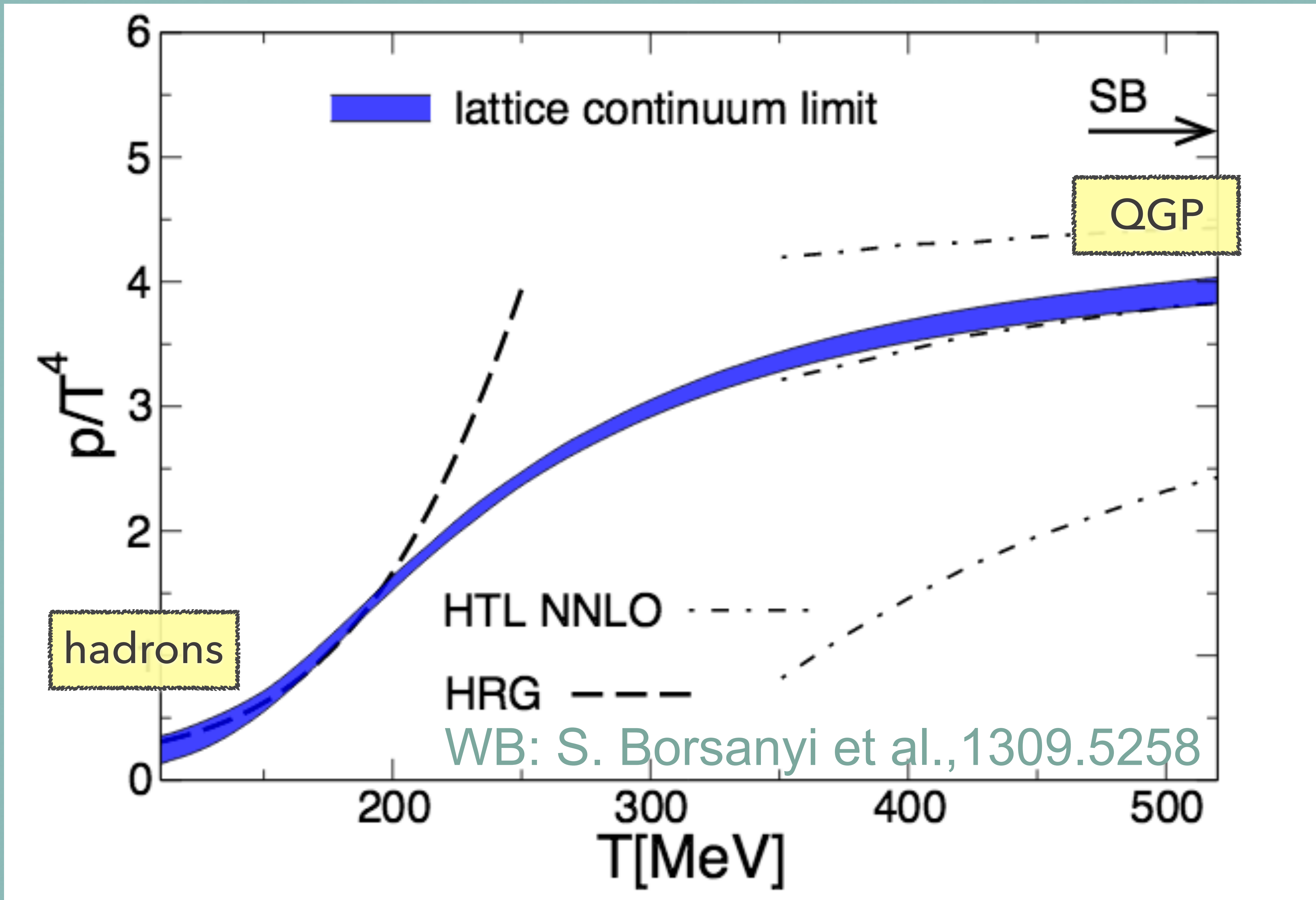
HADRON RESONANCE GAS MODEL

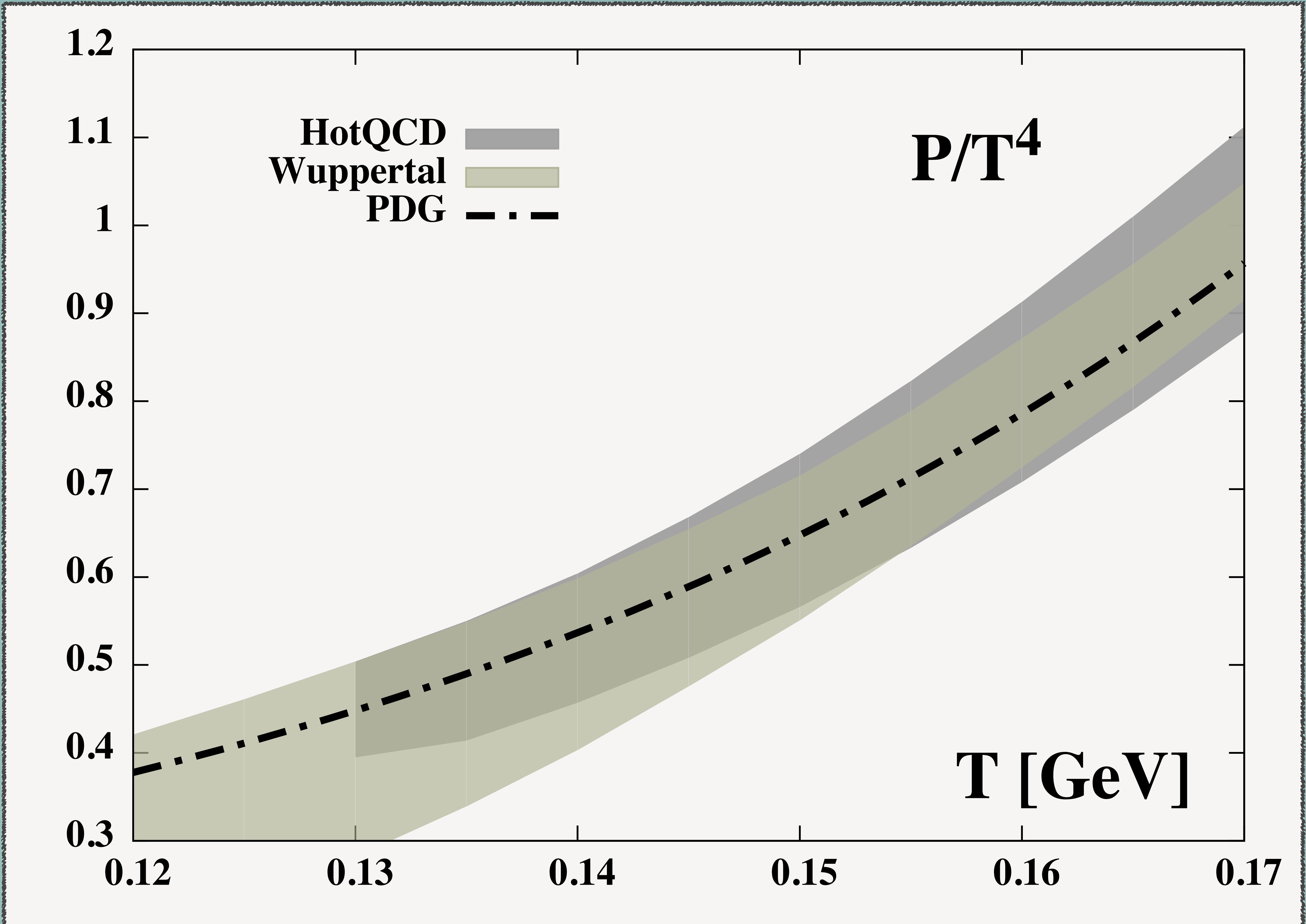
- Ground states $\pi, K, P, N \ldots$



- Resonance formation dominates t… nics
- Resonances treated as point-like p…

$$P = T \sum_{\alpha = M,B} g_\alpha \int \frac{d^3k}{(2\pi)^3} \mp \ln(1 \mp e^{-\beta\sqrt{k^2 + M_\alpha^2}})$$


lattice continuum limit
SB
QGP
hadrons
HTL NNLO
HRG
WB: S. Borsanyi et al.,1309.5258
p/T^4
T[MeV]
0
2
3
4
5
6
200
300
400
500



HotQCD
Wuppertal
PDG
P/T4
T [GeV]
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.12
0.13
0.14
0.15
0.16
0.17

FLUCTUATIONS

- Baryon sector

$$P = T \sum_{\alpha = \not{M}, B} g_\alpha \int \frac{d^3k}{(2\pi)^3} \mp \ln(1 \mp e^{-\beta\sqrt{k^2+M_\alpha^2}})$$

or introduce the chemical potential

$$P = T \sum_{\alpha = B, \bar{B}} g_\alpha \int \frac{d^3k}{(2\pi)^3} \ln(1 + e^{-\beta\sqrt{k^2+M_\alpha^2} \pm \bar{\mu}_B})$$

FLUCTUATIONS

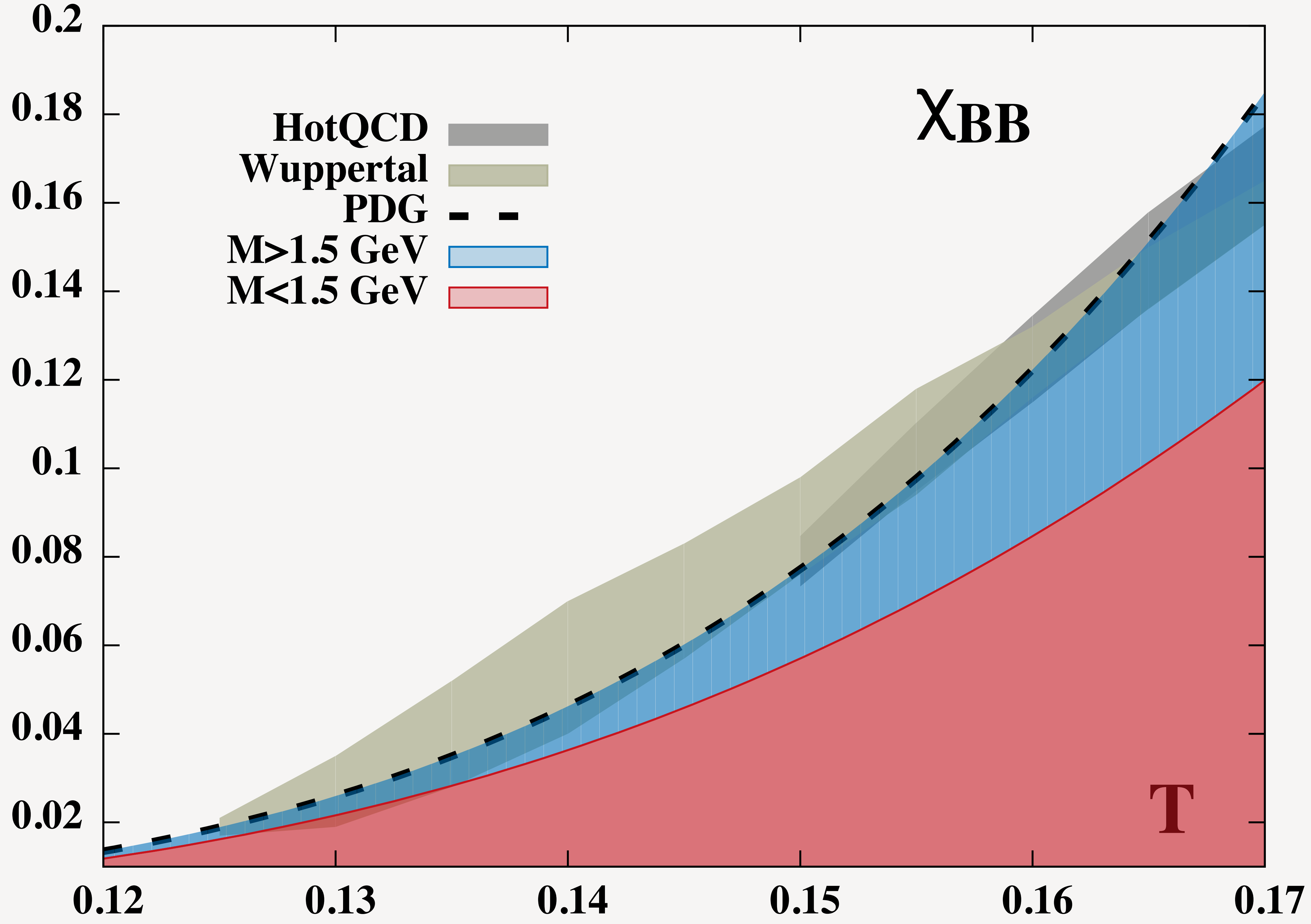
- taking derivative

$$\chi_B = \frac{\partial^2}{\partial\bar{\mu}_B \partial\bar{\mu}_B} P$$ *at the limit* $\mu_B \to 0$

probes fluctuations

$$\chi_B = \frac{1}{\beta V} \frac{\partial^2}{\partial\bar{\mu}_B \partial\bar{\mu}_B} \ln Z$$

$$= T^2 \langle\langle \int d^4x\, \bar{\psi}(x)\gamma^0\psi(x)\bar{\psi}(0)\gamma^0\psi(0)\rangle\rangle_c$$

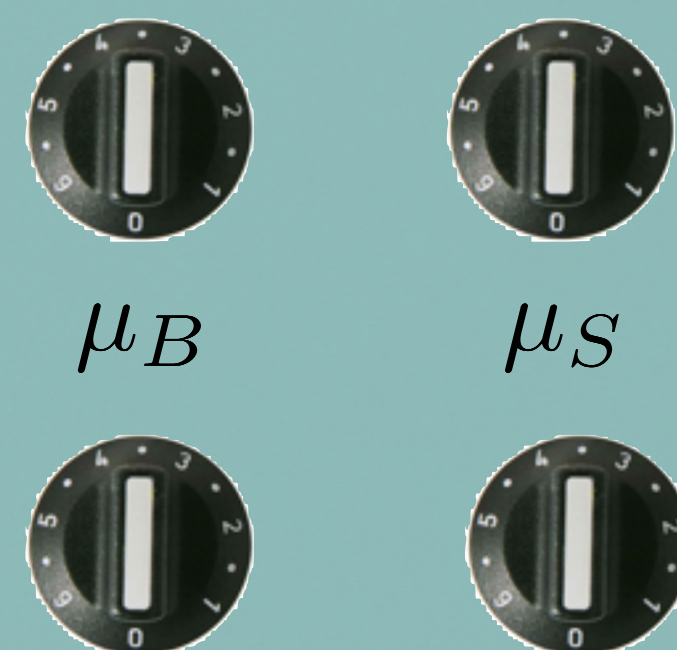

χBB
HotQCD
Wuppertal
PDG
M>1.5 GeV
M<1.5 GeV
T
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0.12
0.13
0.14
0.15
0.16
0.17

FLUCTUATIONS

- studying the system by linear response

$$\mu = \mu_B B + \mu_Q Q + \mu_S S$$

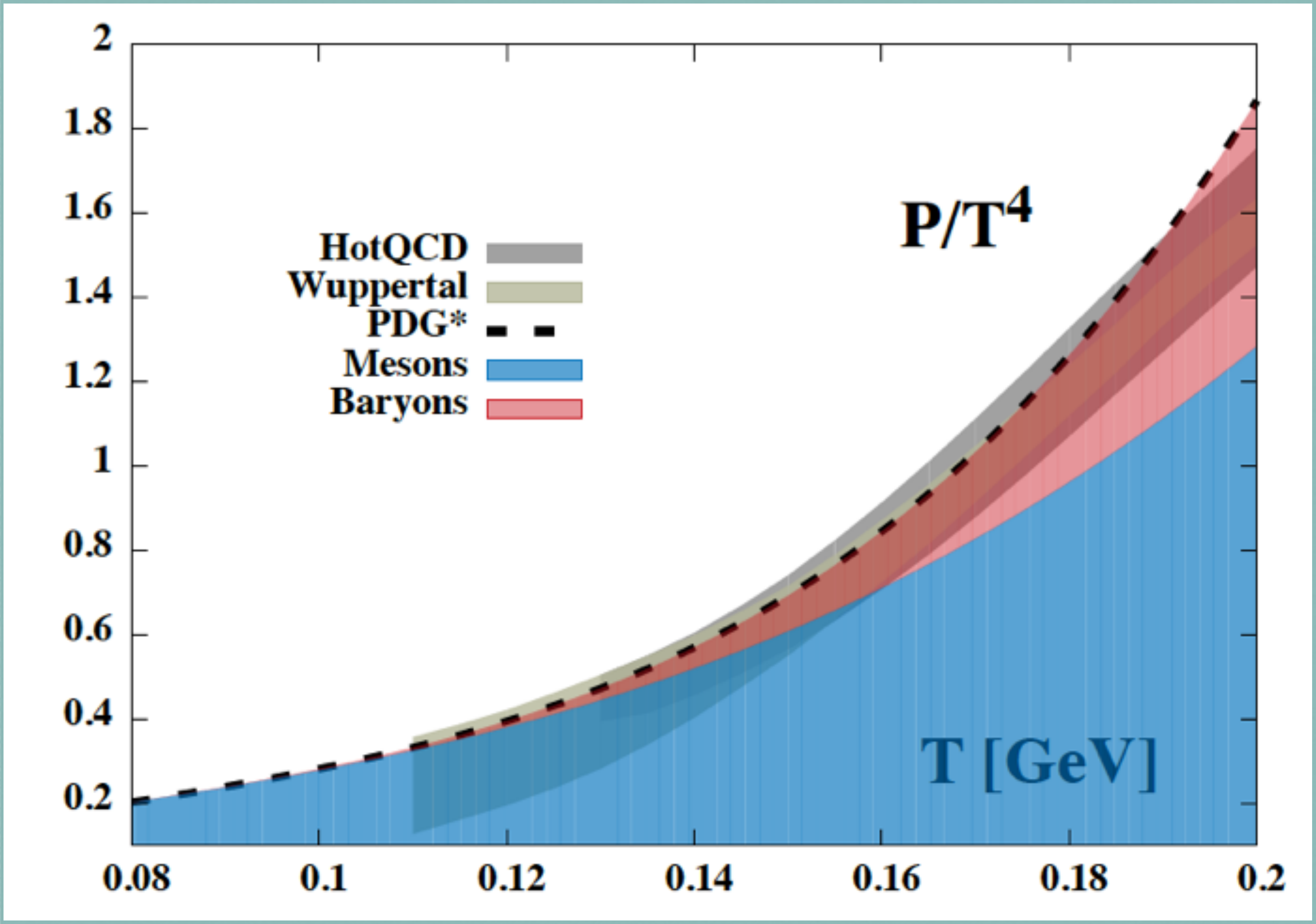
$$\chi_{\mathrm{B,S,\ldots}} = \frac{1}{VT^3} \frac{\partial^2}{\partial \bar{\mu_B} \partial \bar{\mu_S} \ldots} \ln Z$$



μ_B μ_S

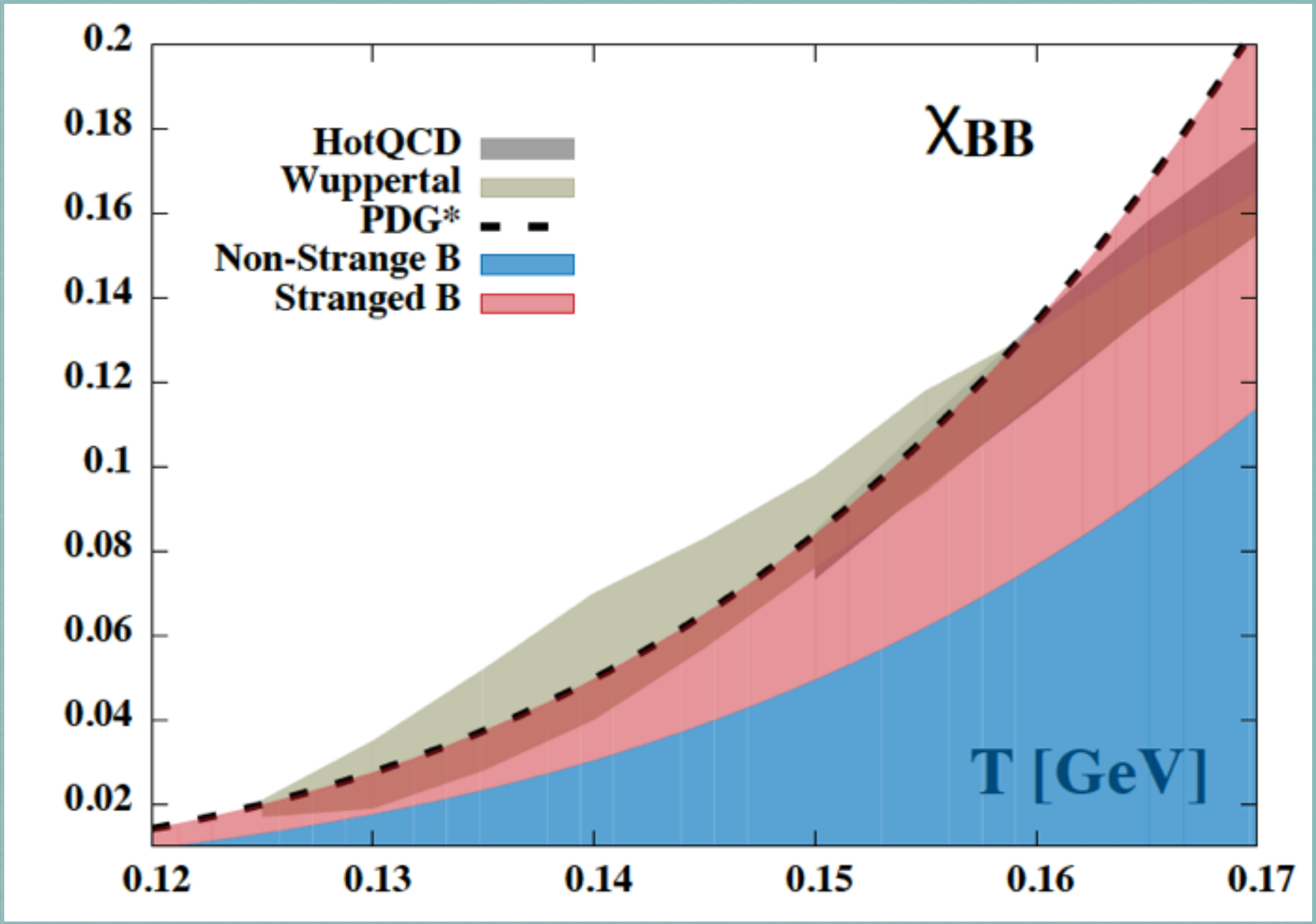
μ_Q m_q

COMPOSITION OF HADRONS



At 0.18 GeV
80% Mesons
20% Baryons

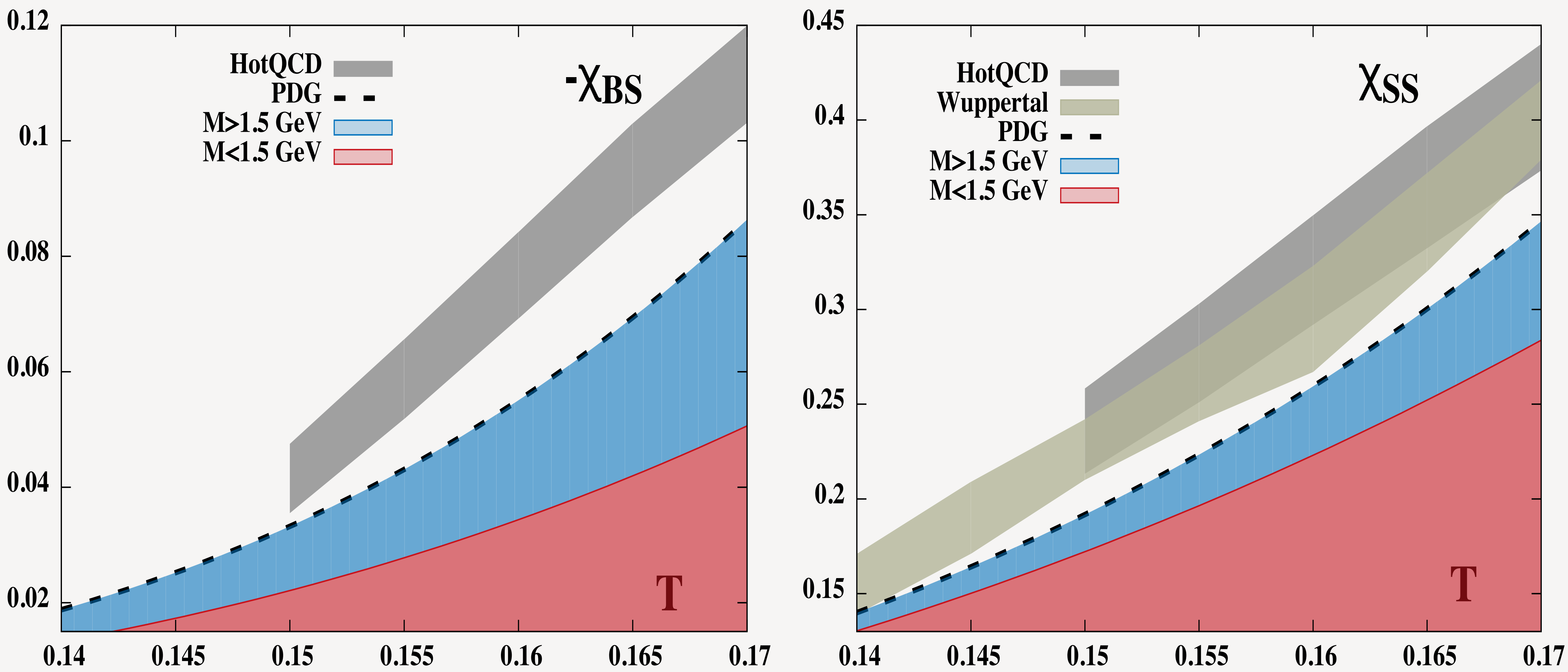
COMPOSITION OF HADRONS



At 0.18 GeV
80% Mesons
20% Baryons

For Baryons,
45% are
Stranged.

Missing resonances in the strange sector



WHERE TO FIND THE MISSING RESONANCES?

- unconfirmed light resonances in the strange sector

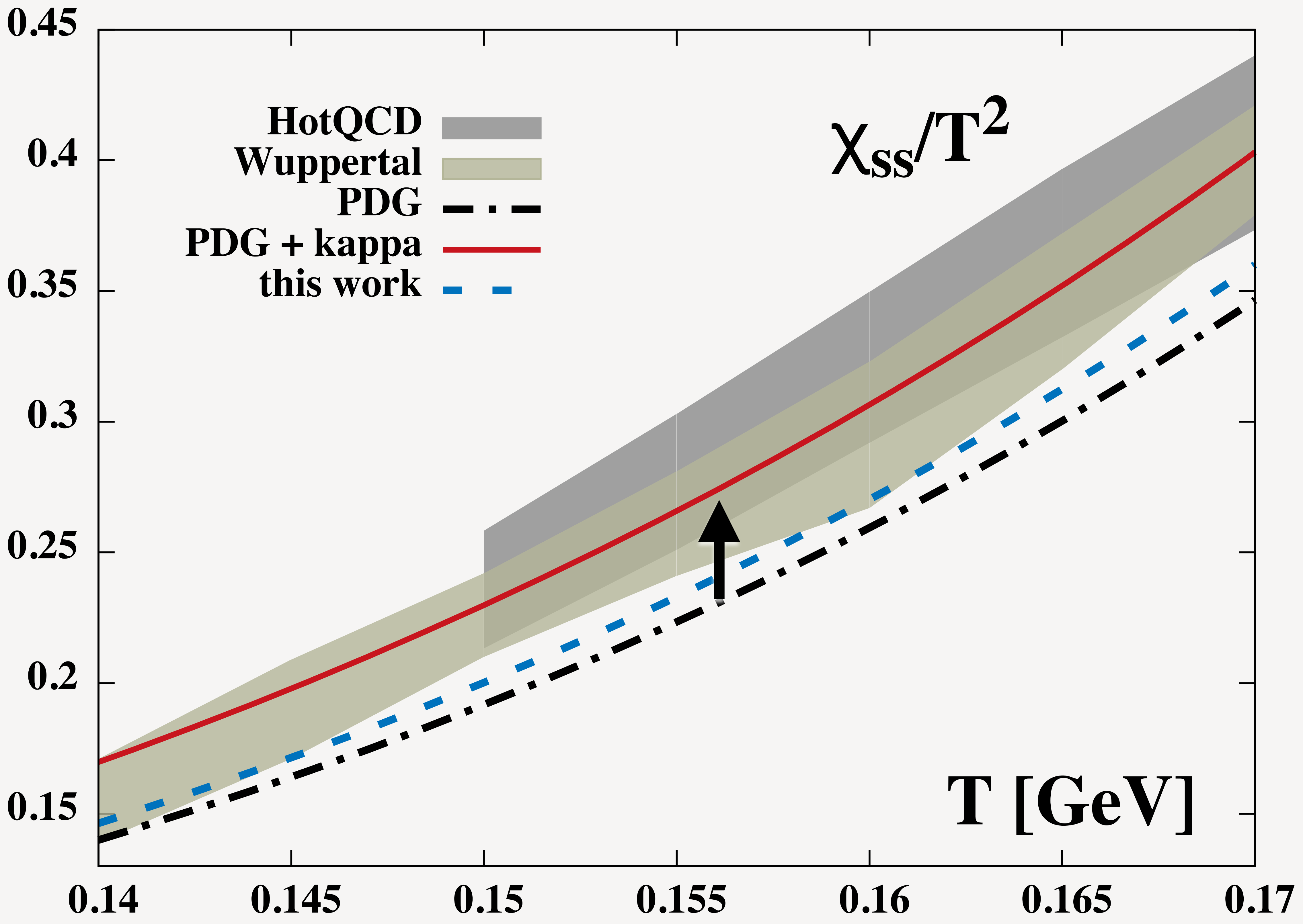
$K_0^*(800)$
or κ

$I(J^P) = \frac{1}{2}(0^+)$

OMITTED FROM SUMMARY TABLE

Needs confirmation. See the mini-review on scalar mesons under $f_0(500)$ (see the index for the page number).

and friends… K(1460)0-, K(1580) 2-, Sigma(1480) ?-



HotQCD
Wuppertal
PDG
PDG + kappa
this work
χ_{ss}/T^2
T [GeV]
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.14
0.145
0.15
0.155
0.16
0.165
0.17

WHERE TO FIND THE MISSING RESONANCES?

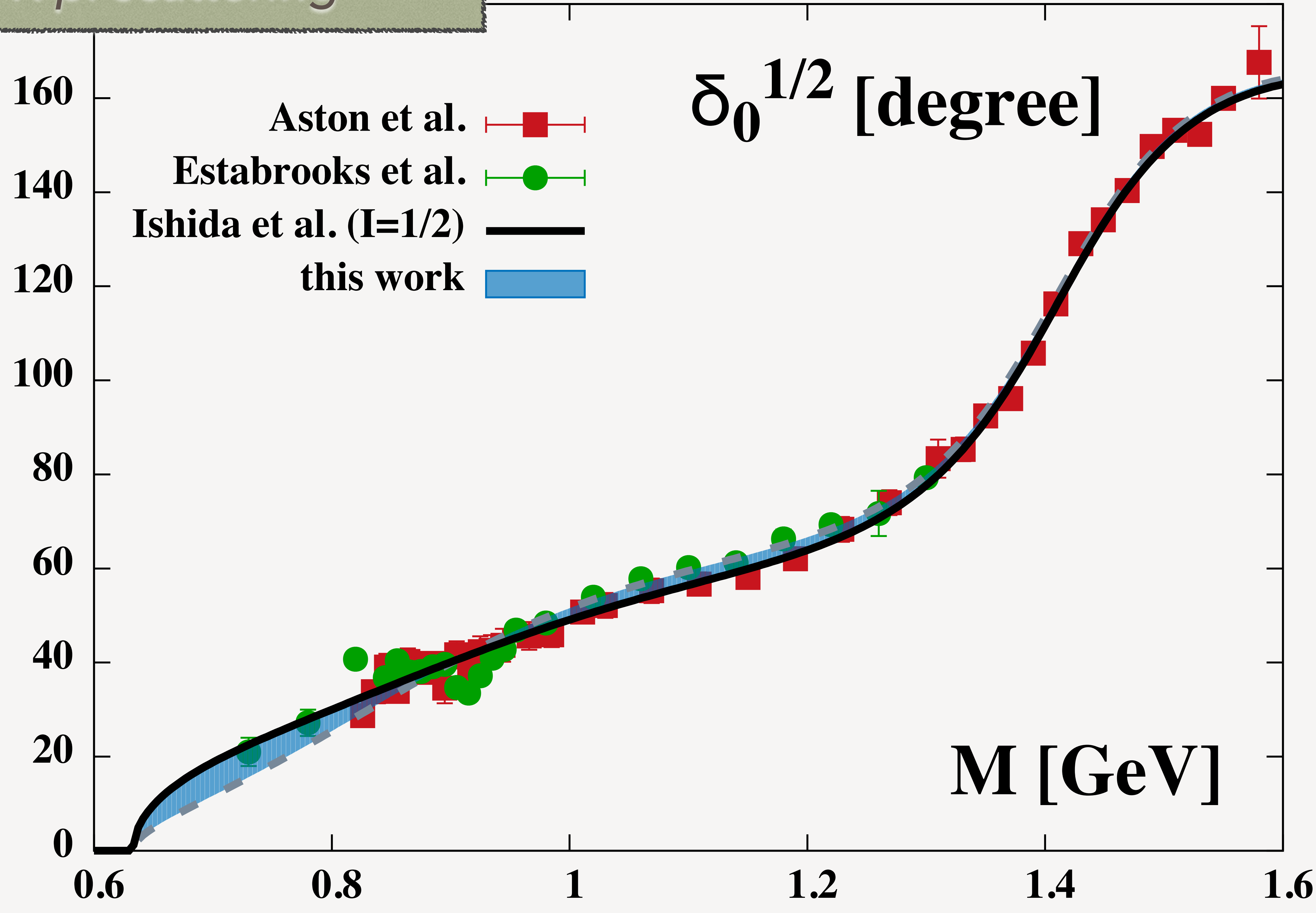
- The κ meson has the right mass range.
- But it also has a broad width!
- Question the assumption of HRG treatment for resonances: non-interacting and point-like.

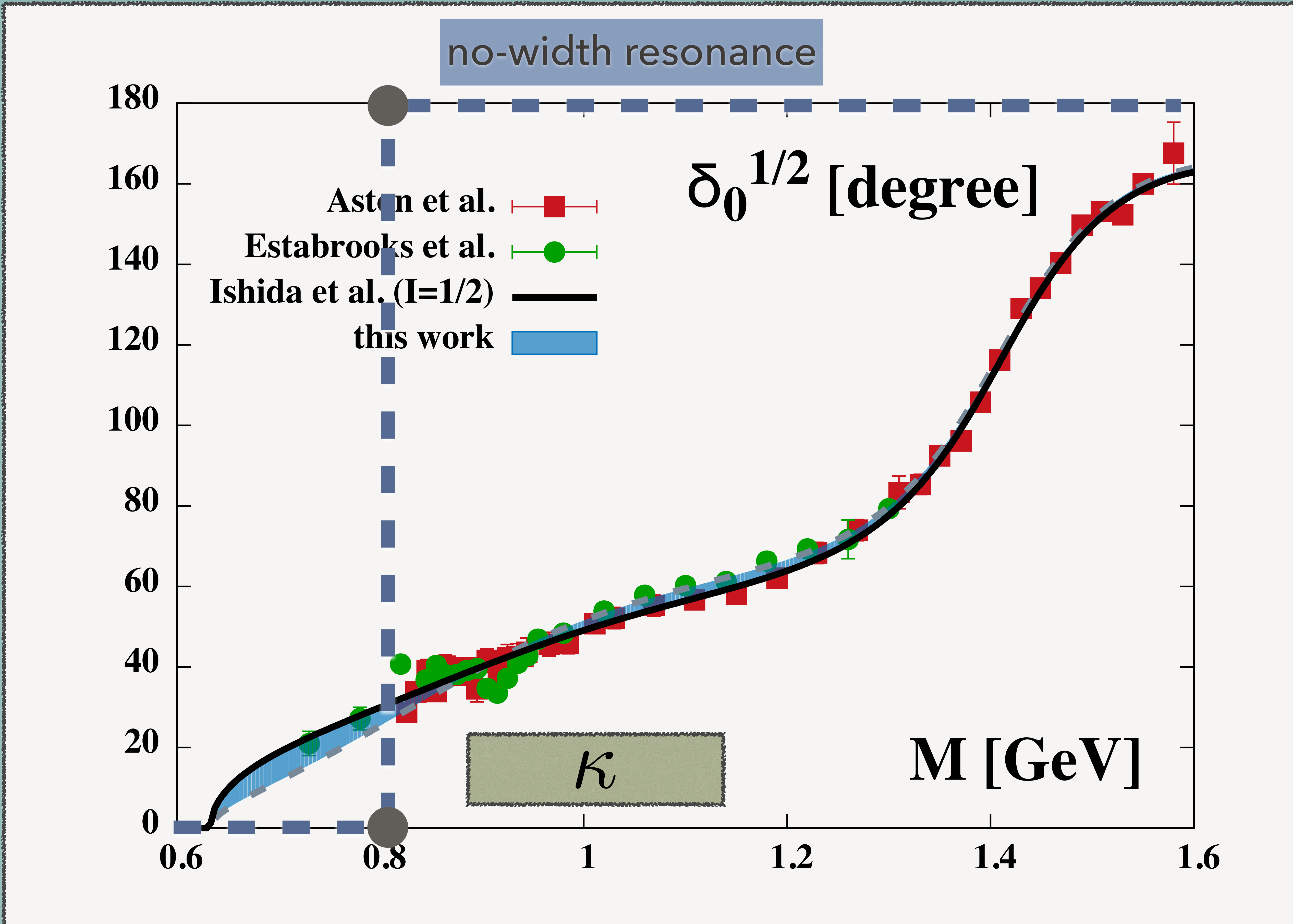
WHAT IS THE EFFECT OF RESONANCE'S WIDTH ON THERMODYNAMICS?

PHYSICS OF BROAD RESONANCES

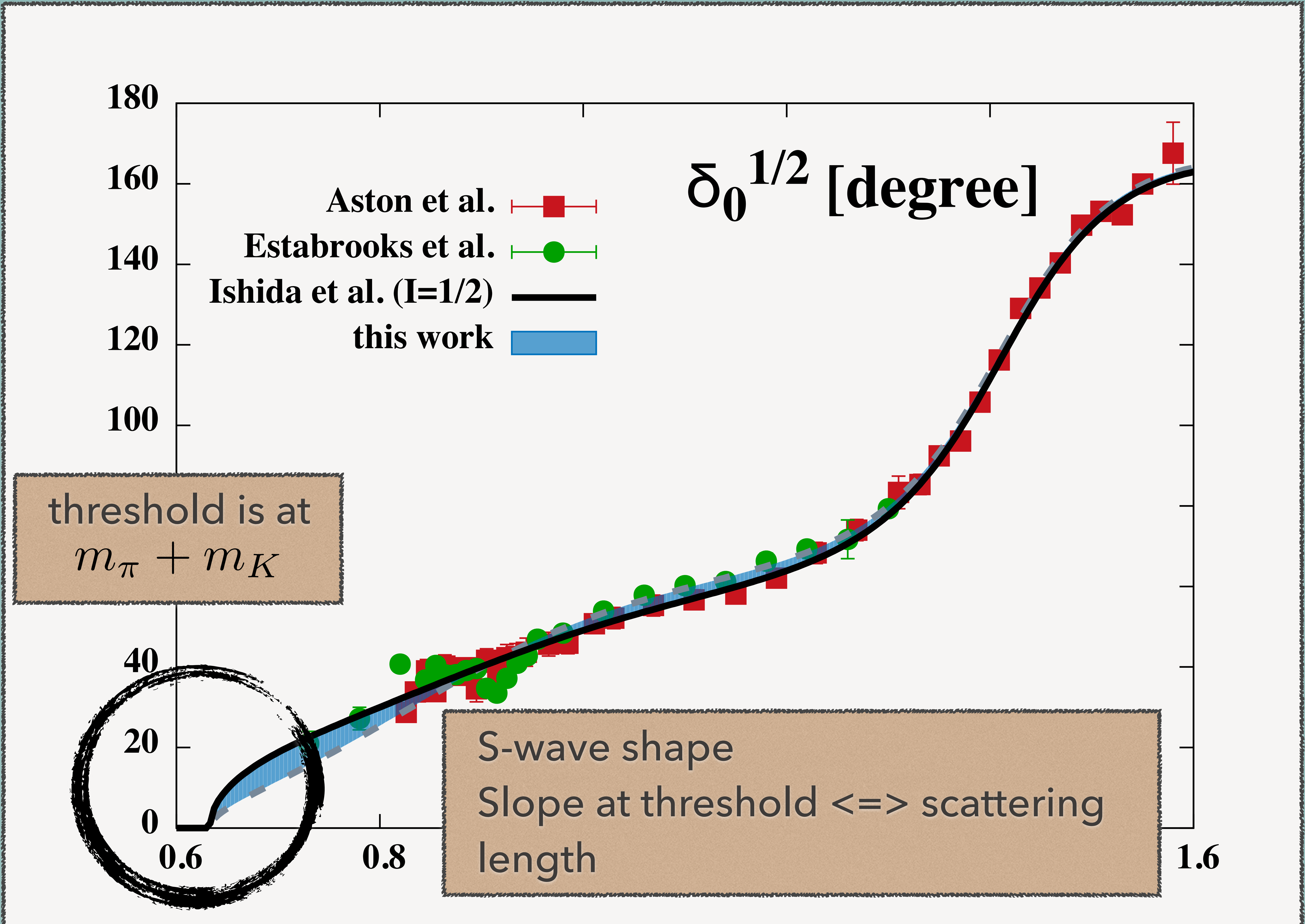
phase shift of
K pi scattering

S-wave





no-width resonance
δ0 1/2 [degree]
Aston et al.
Estabrooks et al.
Ishida et al. (I=1/2)
this work
κ
M [GeV]
180
160
140
120
100
80
60
40
20
0
0.6
0.8
1
1.2
1.4
1.6



$\delta_0^{1/2}$ [degree]
Aston et al.
Estabrooks et al.
Ishida et al. (I=1/2)
this work
180
160
140
120
100
40
20
0
0.6
0.8
1.6
threshold is at
$m_\pi + m_K$
S-wave shape
Slope at threshold <=> scattering length

S-MATRIX APPROACH

- from phase shift to thermodynamics

$$\langle \hat{O} \rangle = \int_{M_{thres}}^{\infty} \frac{dM}{2\pi}\, B(M)\, \hat{O}[M]$$

$$B = 2\frac{d}{dM}\delta(M)$$

$$\rightarrow 2M \frac{2M\gamma_{\mathrm{BW}}}{(M^2 - M_0^2)^2 + M^2\gamma_{\mathrm{BW}}^2}$$

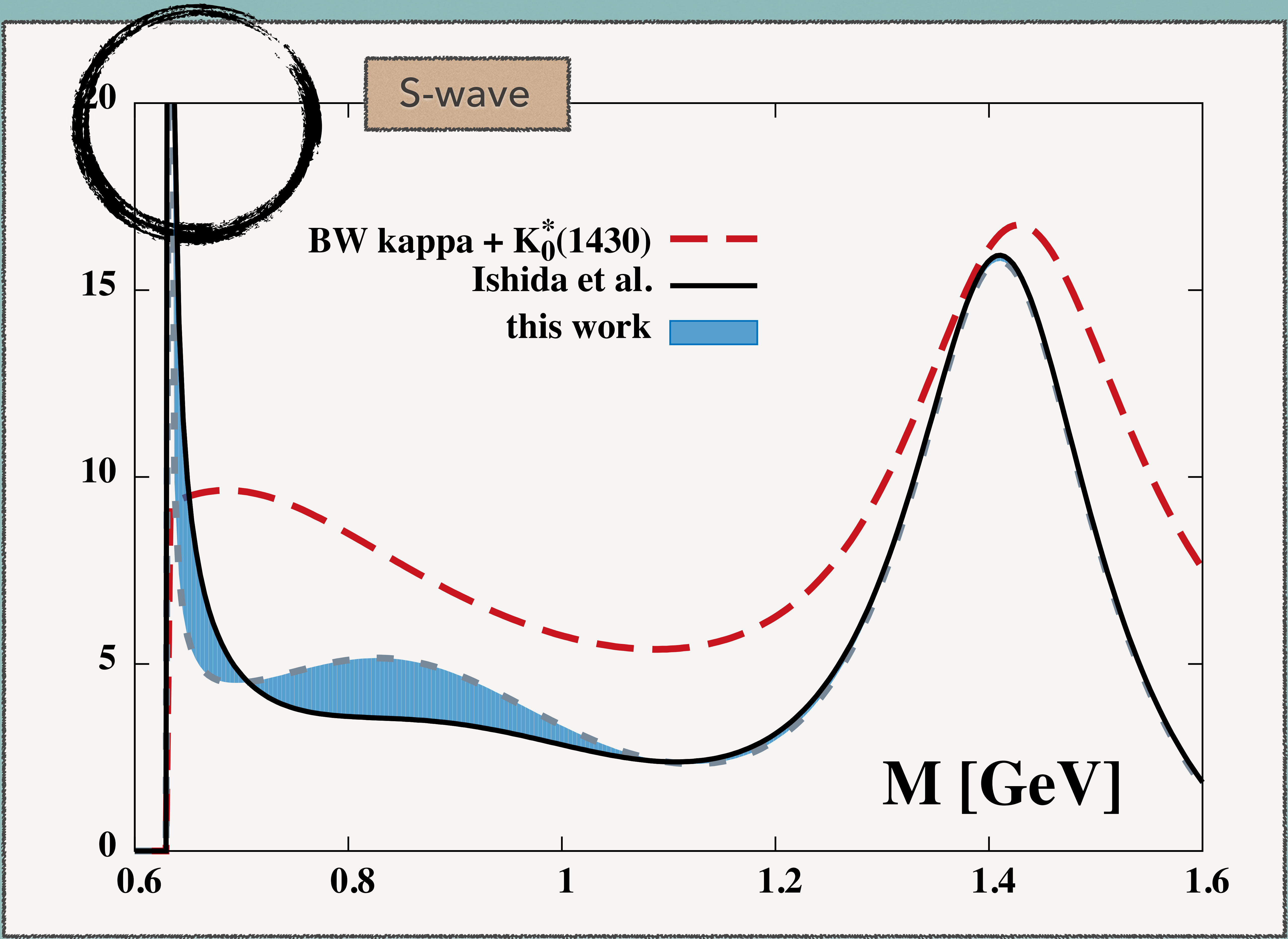
Breit-Wigner

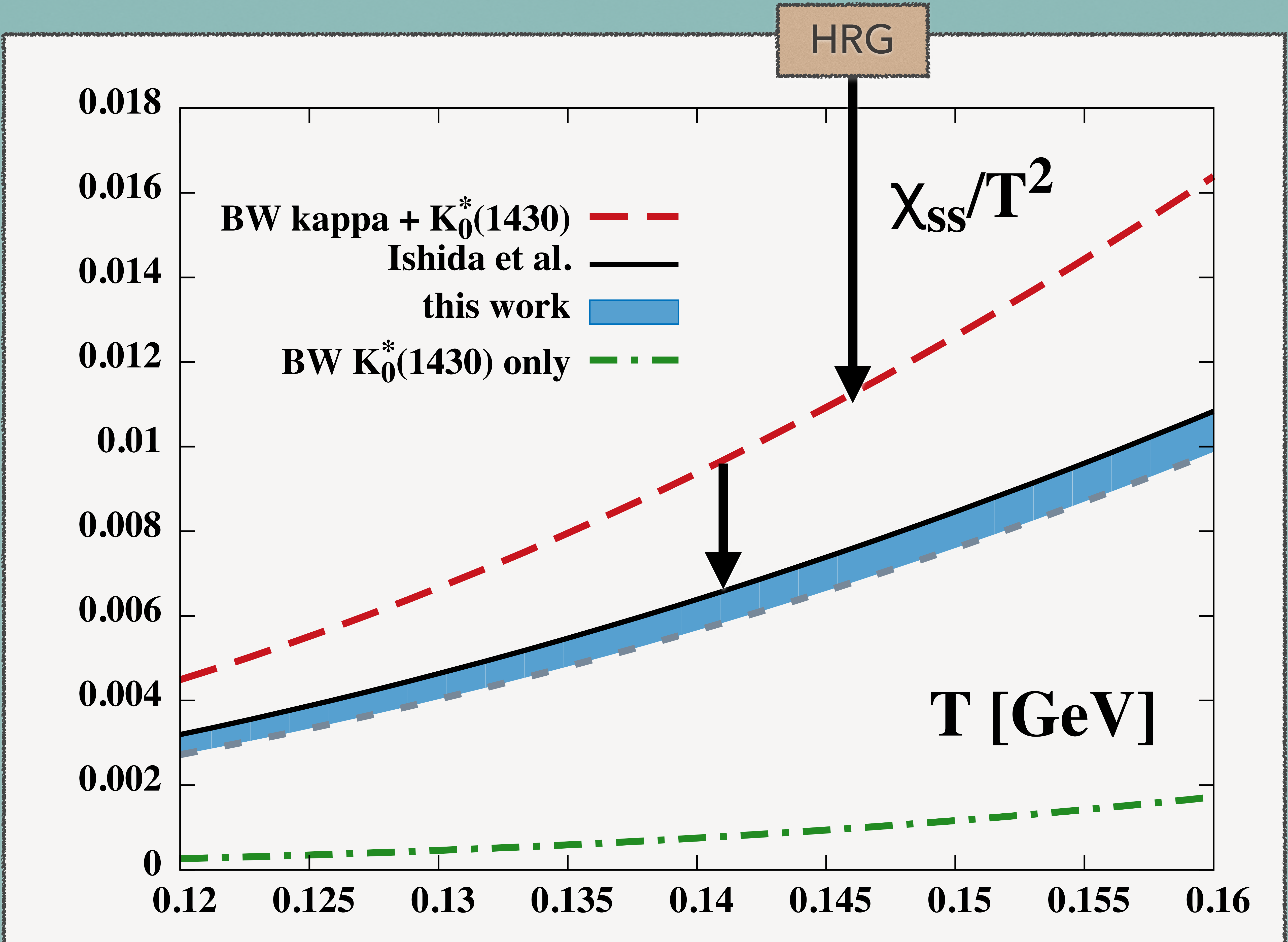
S-MATRIX APPROACH

$$\Omega^{\rm B}_{\rm int} \approx 2TV \int_{m_{th}}^{\infty} \frac{dM}{2\pi} \int \frac{d^3p}{(2\pi)^3}\, \mathcal{B}(M) \times \left\{ \ln[1 - e^{-\beta(\sqrt{p^2+M^2}+\mu_S)}] + \ln[1 - e^{-\beta(\sqrt{p^2+M^2}-\mu_S)}] \right\}.$$

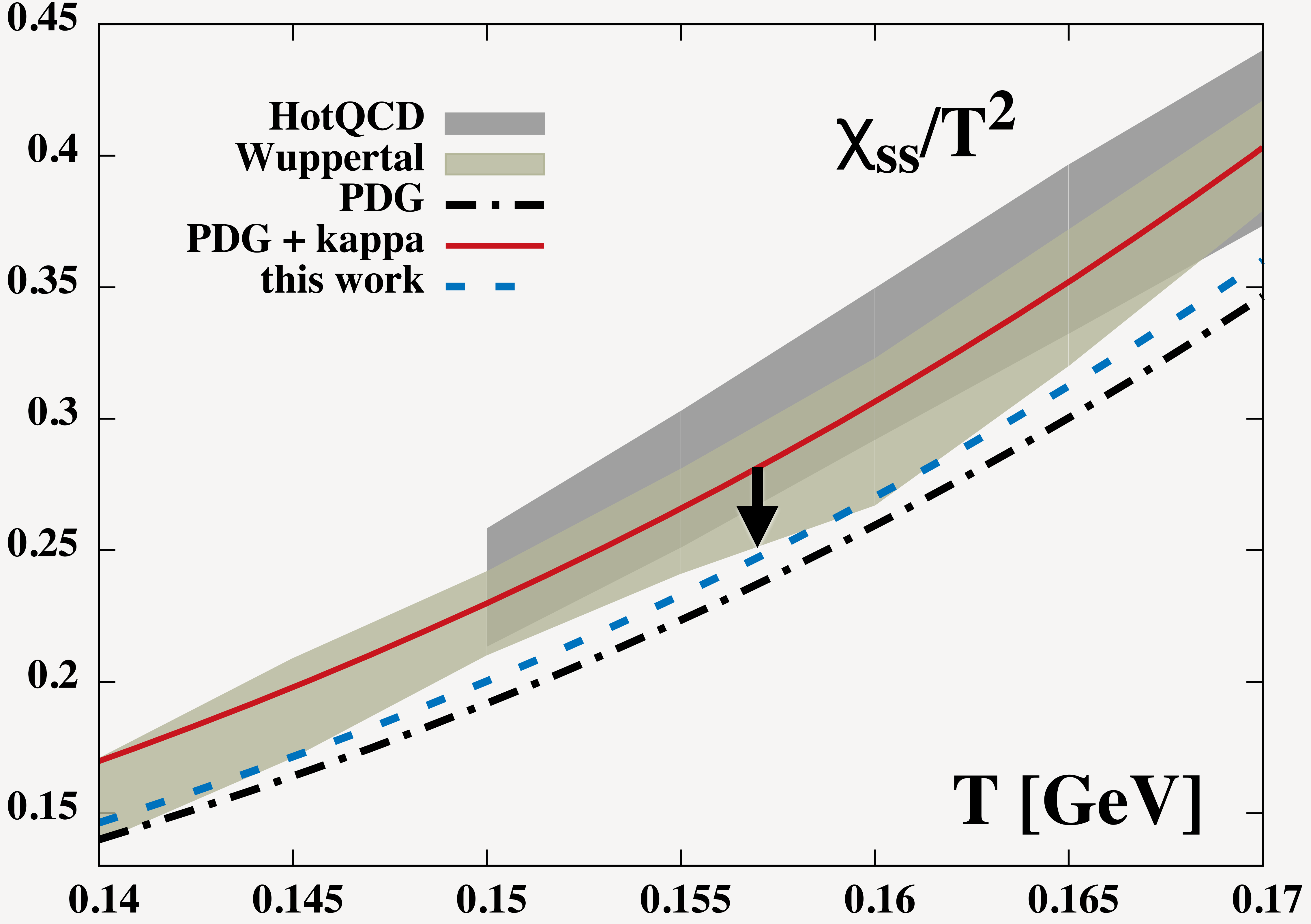
$$B = 2\frac{d}{dM}\delta(M) \to 2M \frac{2M\gamma_{\rm BW}}{(M^2 - M_0^2)^2 + M^2\gamma_{\rm BW}^2}$$

Breit-Wigner

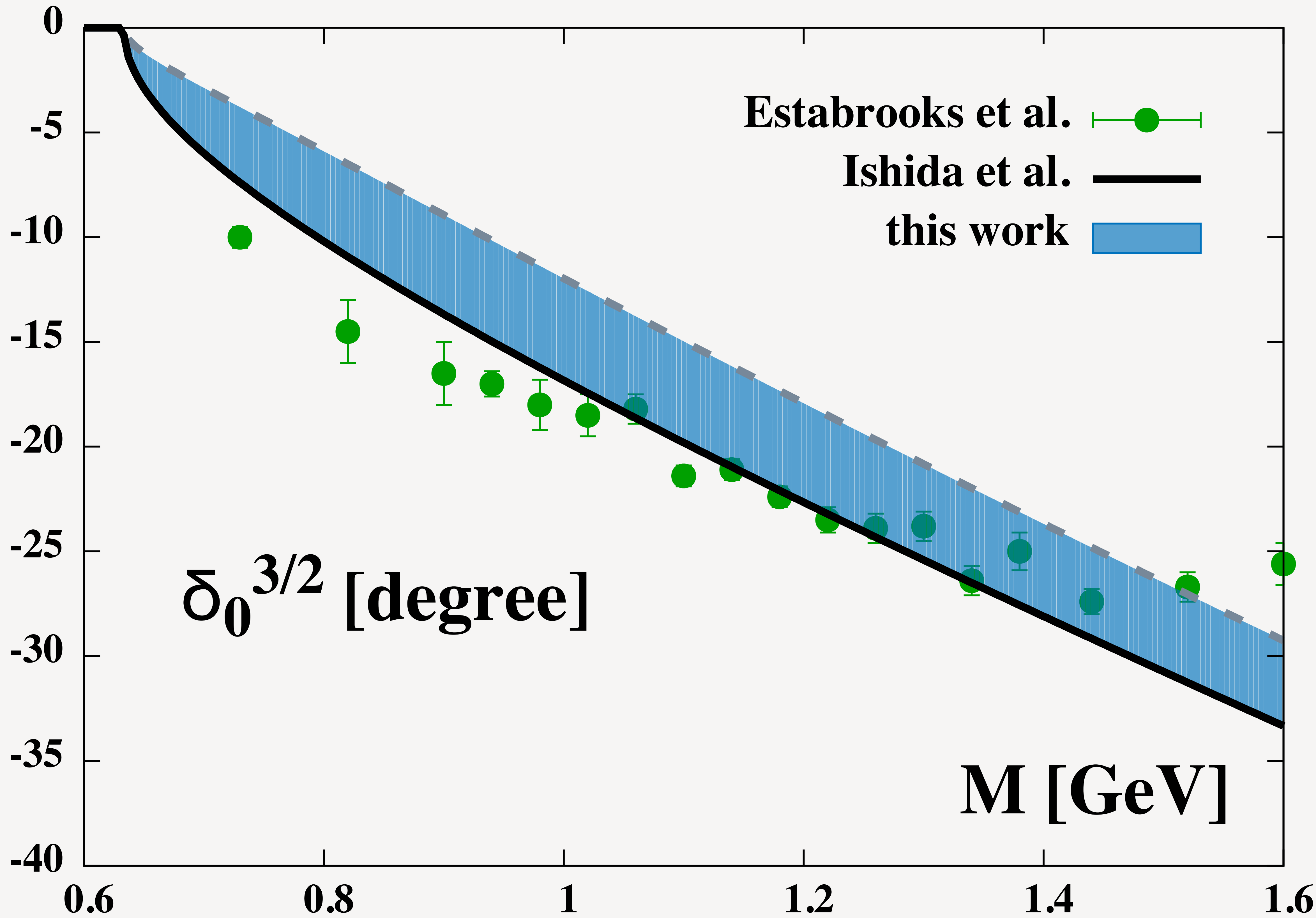

S-wave
BW kappa + $K_0^*(1430)$
Ishida et al.
this work
M [GeV]
0
5
10
15
20
0.6
0.8
1
1.2
1.4
1.6



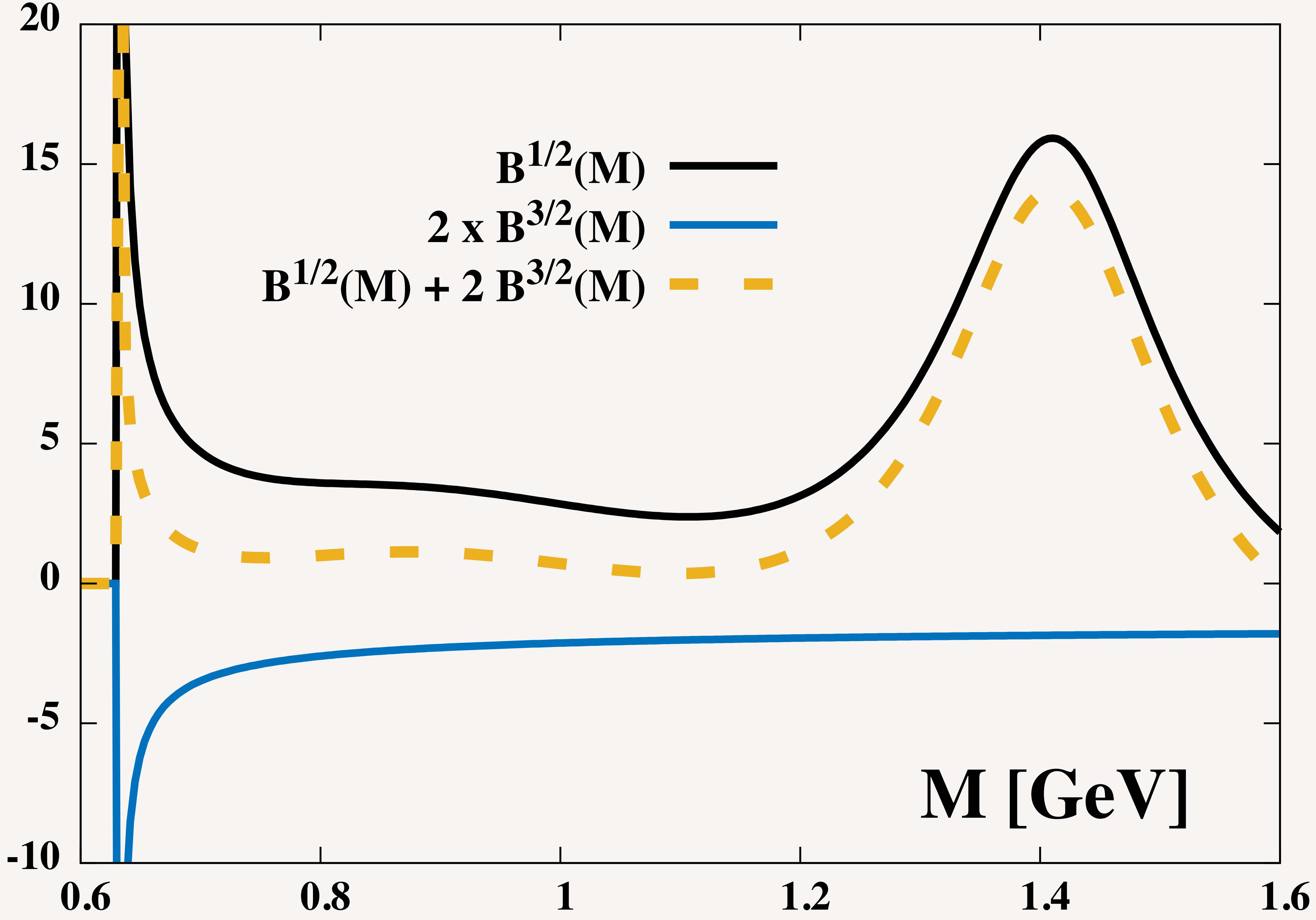
HRG
χ_{ss}/T^2
BW kappa + $K_0^*(1430)$
Ishida et al.
this work
BW $K_0^*(1430)$ only
T [GeV]
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0.12
0.125
0.13
0.135
0.14
0.145
0.15
0.155
0.16


HotQCD
Wuppertal
PDG
PDG + kappa
this work
χ_{ss}/T^2
T [GeV]
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.14
0.145
0.15
0.155
0.16
0.165
0.17

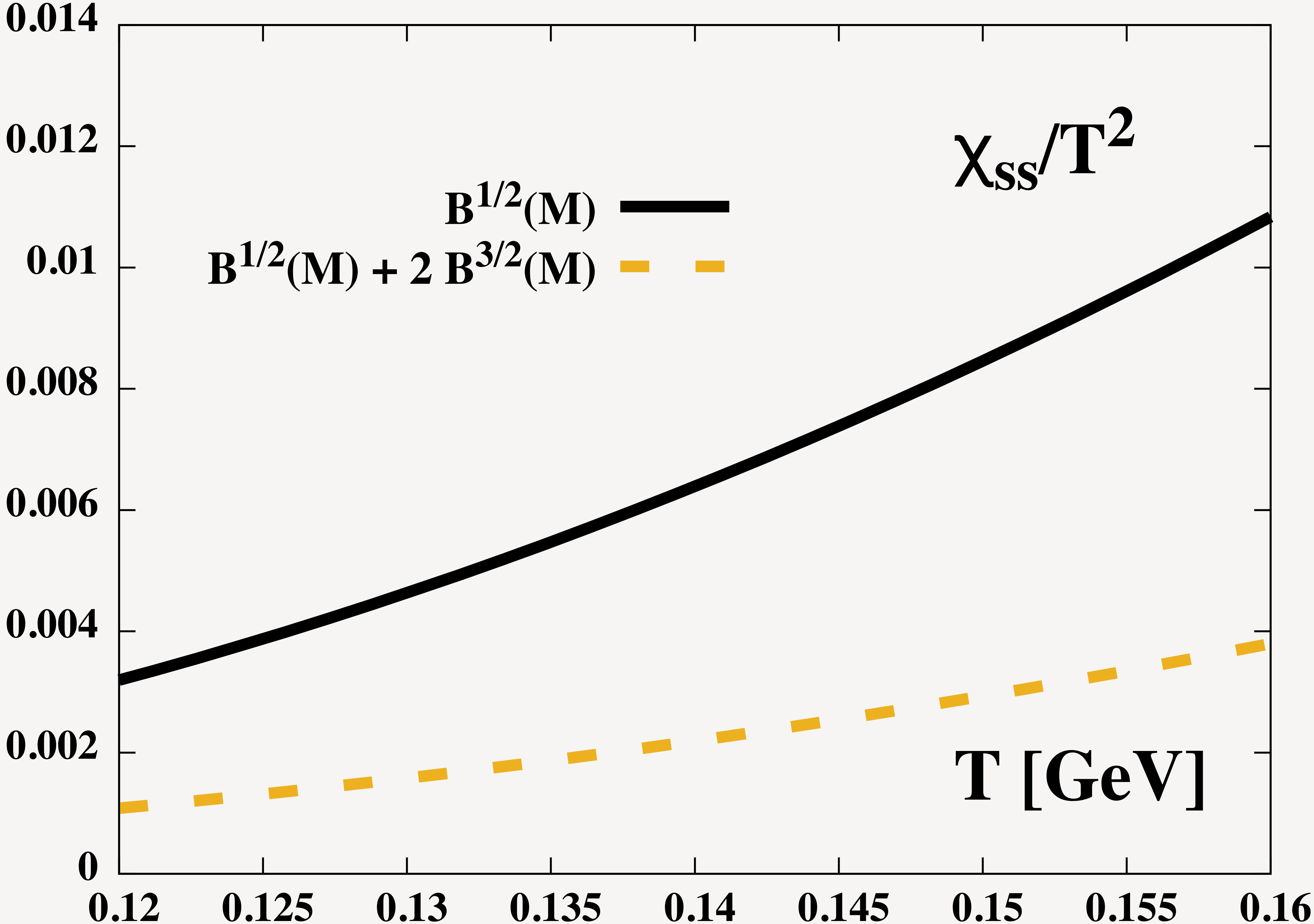
I=3/2 REPULSIVE CHANNEL



0
-5
-10
-15
-20
-25
-30
-35
-40
0.6
0.8
1
1.2
1.4
1.6
Estabrooks et al.
Ishida et al.
this work
δ0 3/2 [degree]
M [GeV]



20
15
10
5
0
-5
-10
0.6
0.8
1
1.2
1.4
1.6
$B^{1/2}(M)$
$2 \times B^{3/2}(M)$
$B^{1/2}(M) + 2\,B^{3/2}(M)$
M [GeV]



0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0.12
0.125
0.13
0.135
0.14
0.145
0.15
0.155
0.16
χ_{ss}/T^2
$B^{1/2}(M)$
$B^{1/2}(M) + 2\ B^{3/2}(M)$
T [GeV]

CONCLUSIONS

- HRG systematically overestimates the interaction contribution to strangeness fluctuation.
- S-matrix approach – a consistent treatment for low-mass, broad resonances.
  resonant + non-resonant contribution

THANK YOU

BACKUP SLIDES

WHERE TO FIND THE MISSING RESONANCES?

- refit the Hagedorn spectrum

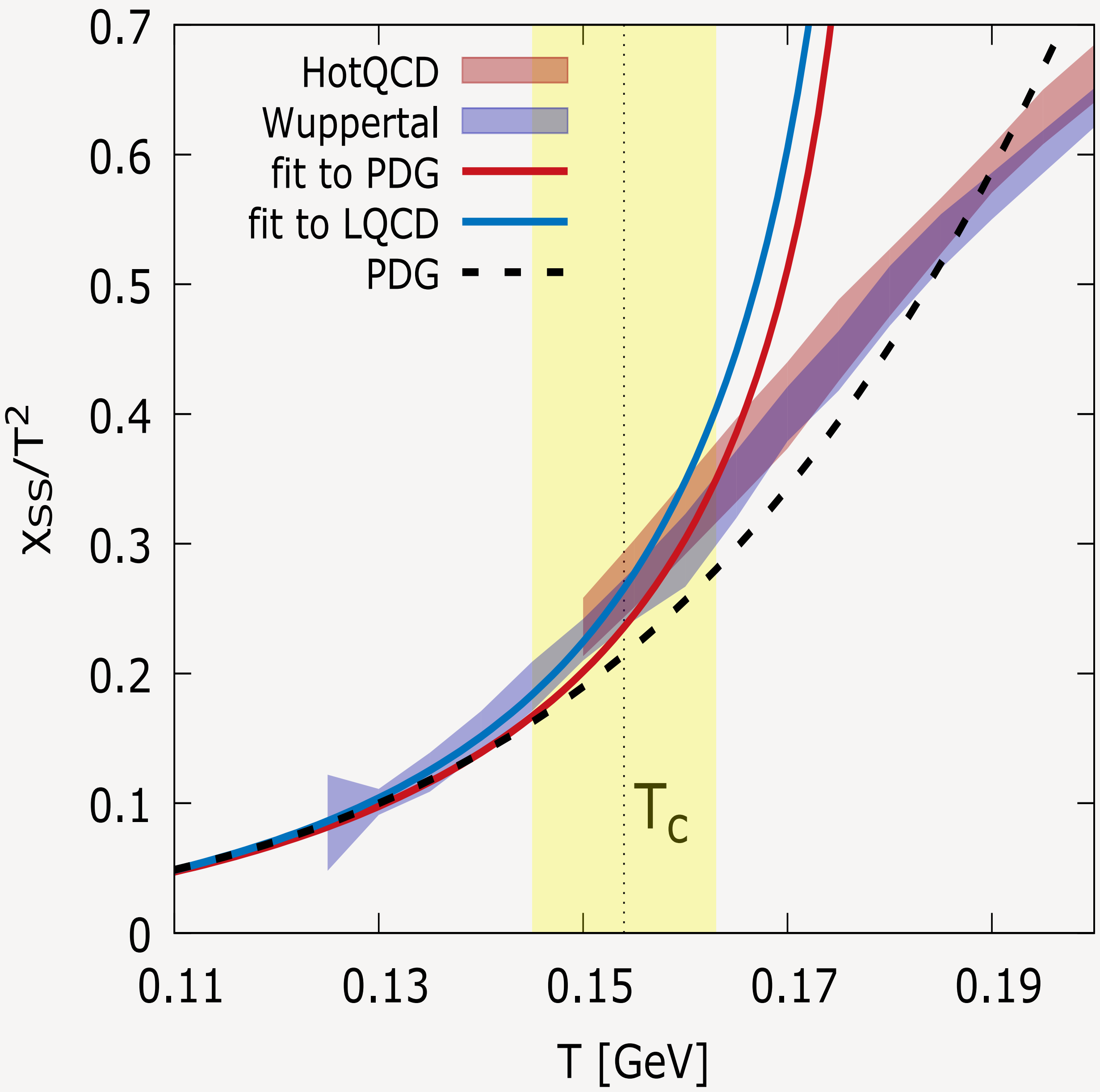
$$P = T \sum_{\alpha=M,B} g_\alpha \int \frac{d^3k}{(2\pi)^3} \mp \ln(1 \mp e^{-\beta\sqrt{k^2+M_\alpha^2}})$$

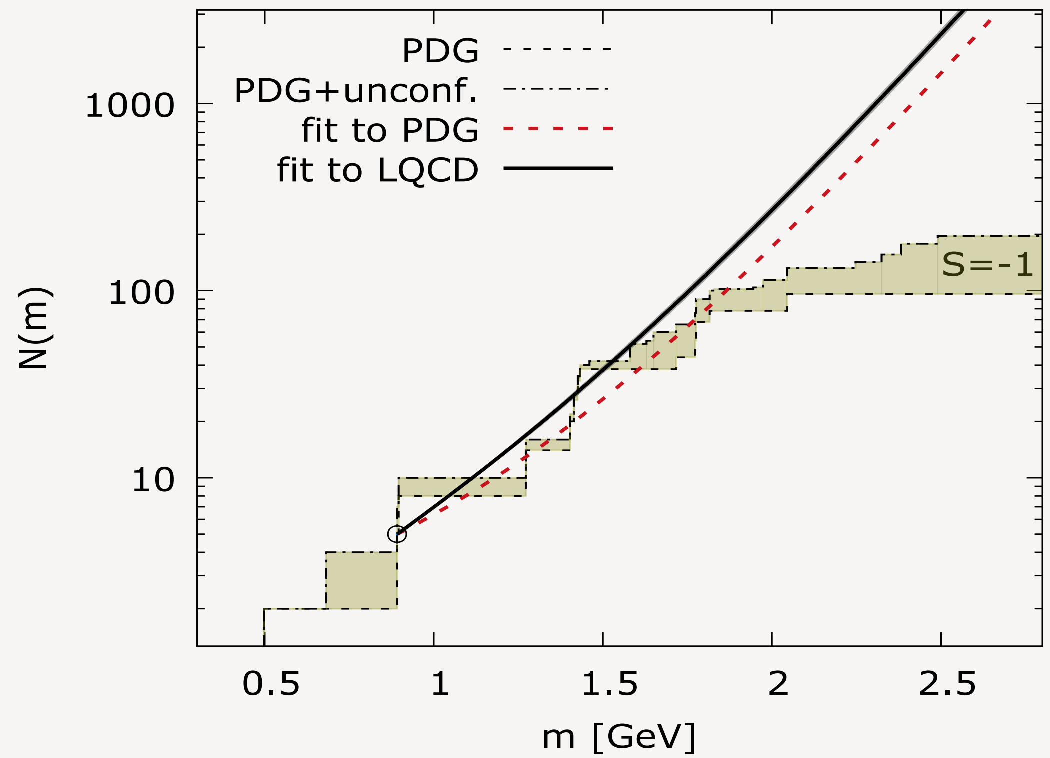
↓

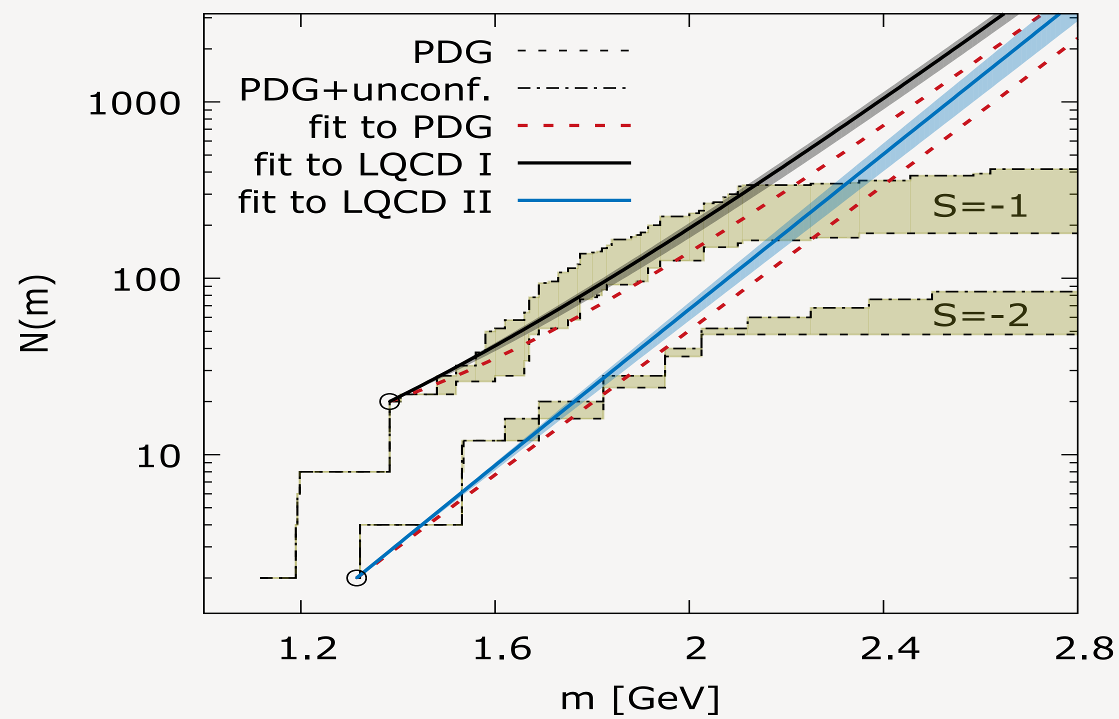
$$\sum_{\alpha=G.S} + \int dm\, \rho^H(m)$$

Improve the spectrum using lattice inputs

strange mesons to be discovered...

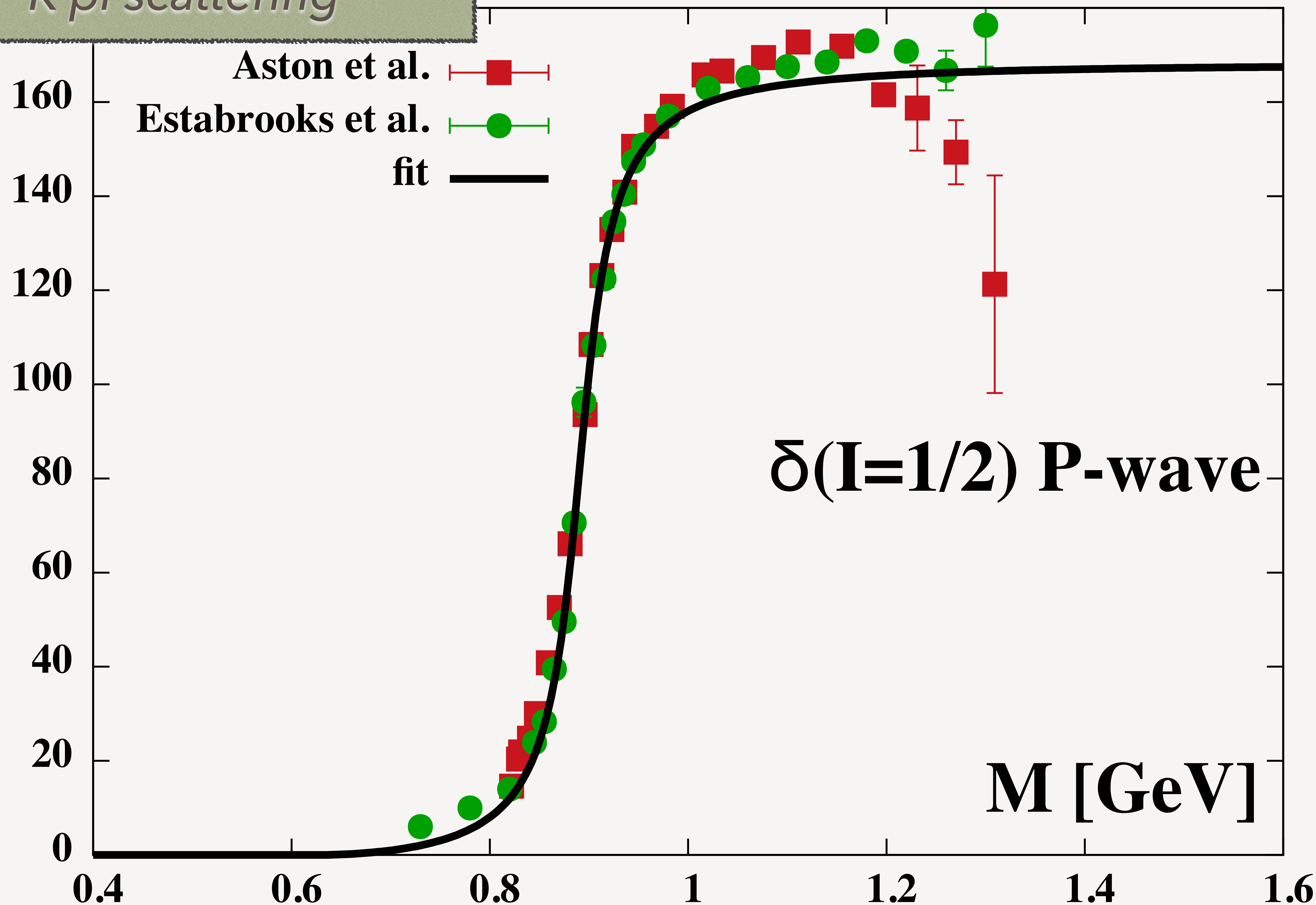


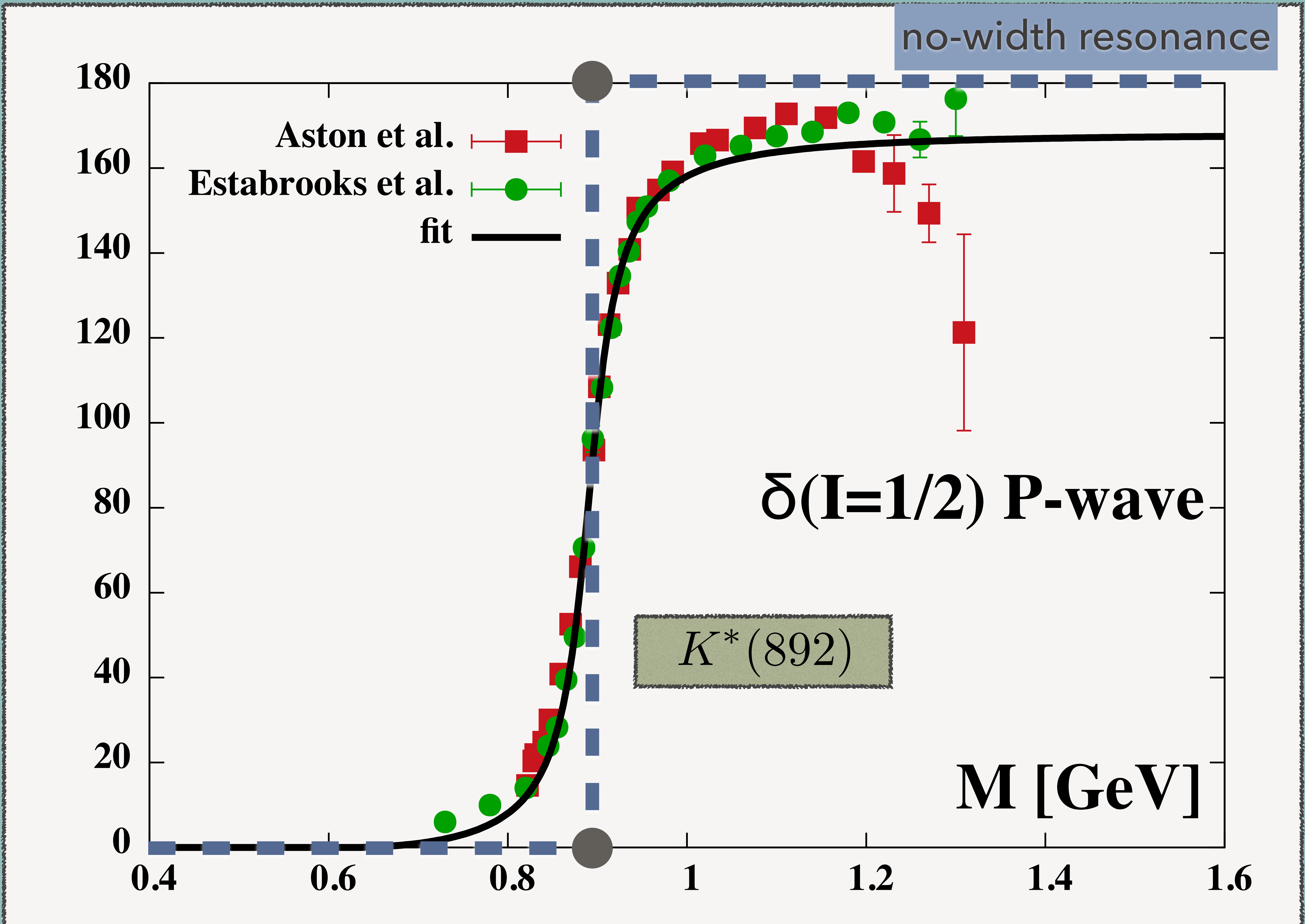




phase shift of
K pi scattering

P-wave





no-width resonance
180
160
140
120
100
80
60
40
20
0
0.4
0.6
0.8
1
1.2
1.4
1.6
Aston et al.
Estabrooks et al.
fit
δ(I=1/2) P-wave
$K^*(892)$
M [GeV]

WIDTH AND PHASE SHIFT

- Width => particle can decay => existence of an imaginary part in the self energy

$$G(t) \propto e^{-i\Sigma_R t + \Sigma_I t}$$

$$|G(t)|^2 \propto e^{2\Sigma_I t} => e^{-\Gamma t}$$

N.R.

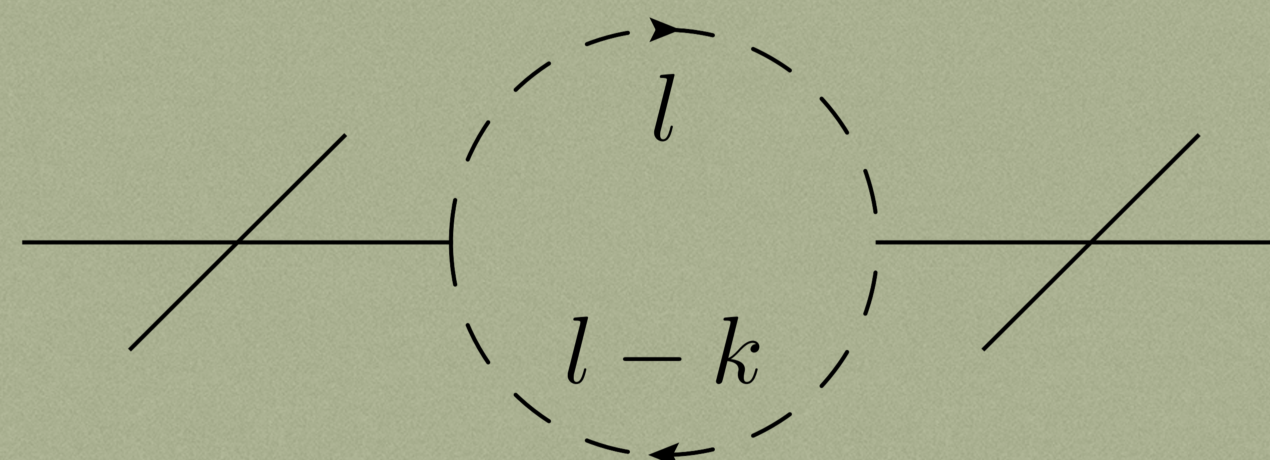
$$\Gamma = -2\Sigma_I$$

WIDTH AND PHASE SHIFT

- Width comes from interactions.
- illustration:

$$\mathcal{L}_{int} = -g\sigma\phi_\pi^2$$

$$-i\Sigma_\sigma =$$



WIDTH AND PHASE SHIFT

$$\Sigma_\sigma(k) = 2g^2 i \int \frac{d^4 l}{(2\pi)^4} \frac{1}{l^2 - m_\pi^2} \frac{1}{(l-k)^2 - m_\pi^2}$$

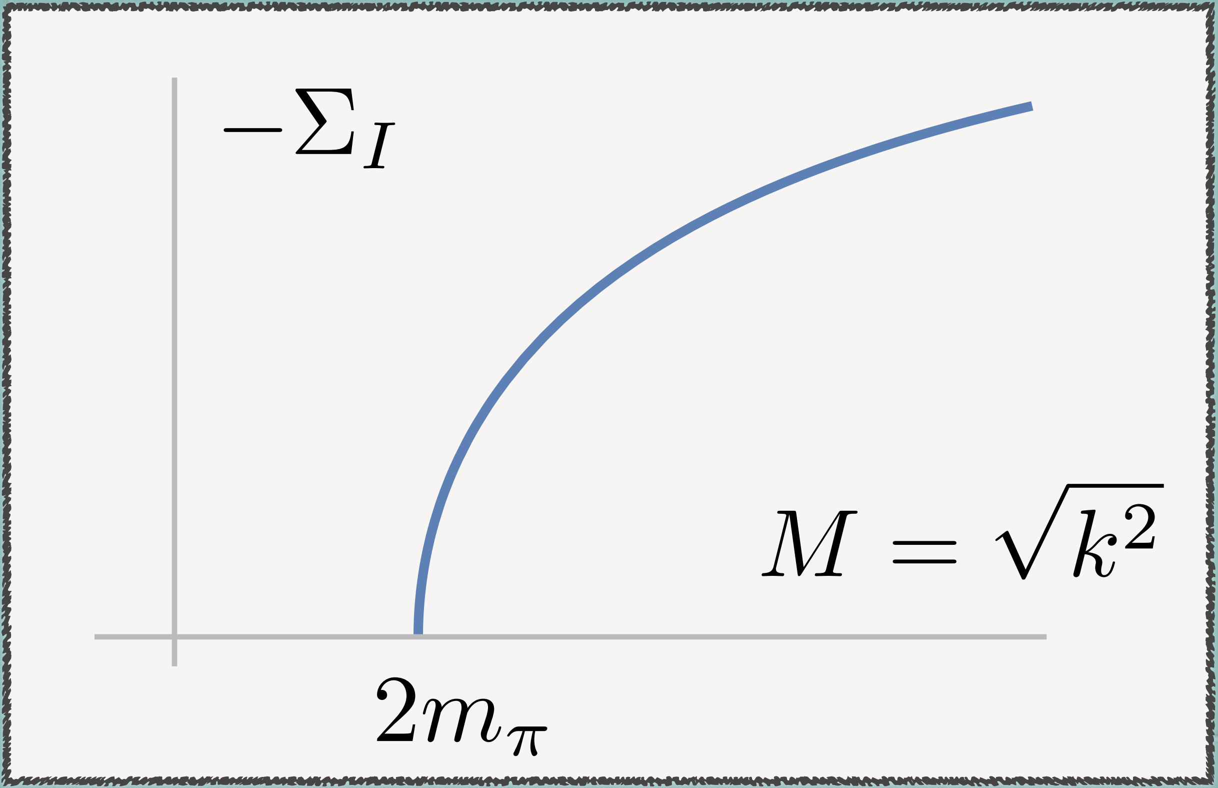
$$= \frac{1}{(2\pi)^4} 2g^2\pi^2 \int_0^1 dx \, \ln \left[(m_\pi^2 - x(1-x)k^2)\pi \right]$$

develops an imaginary part if

$$k^2 \geq (2m_\pi)^2$$

threshold

$$\ln(-1) = \pm i\pi$$



Rel.

$$\Gamma = \frac{-\Sigma_I}{M}$$

WIDTH AND PHASE SHIFT

- Field theory knows about the kinematics and phase space
- Width arises from interaction
- Angular momentum dependence $\propto k^{2l+1}$

$$\Gamma(M) = \frac{g^2_{\sigma\pi\pi}}{8\pi}\frac{P_{c.m.}(M)}{M^2}$$

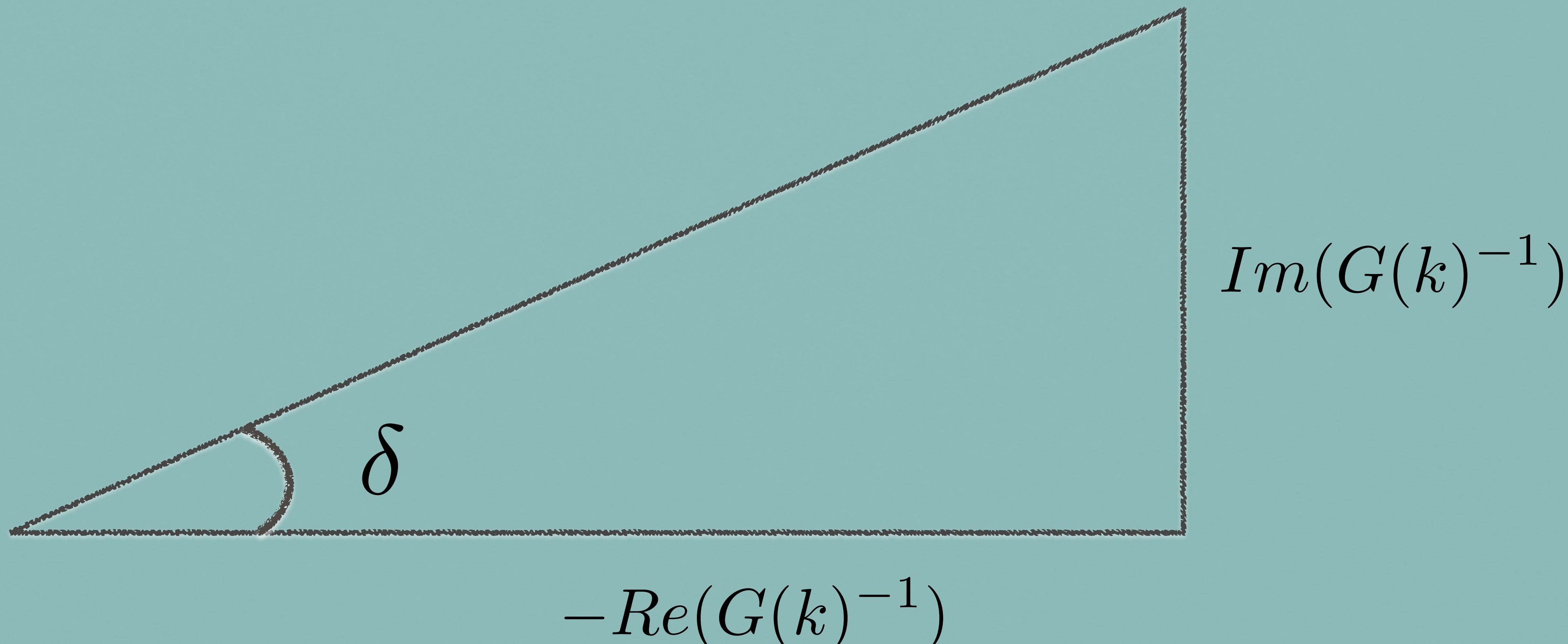
S-wave

$$\Gamma(M) = \frac{g^2_{\rho\pi\pi}}{6\pi}\frac{(P_{c.m.}(M))^3}{M^2}$$

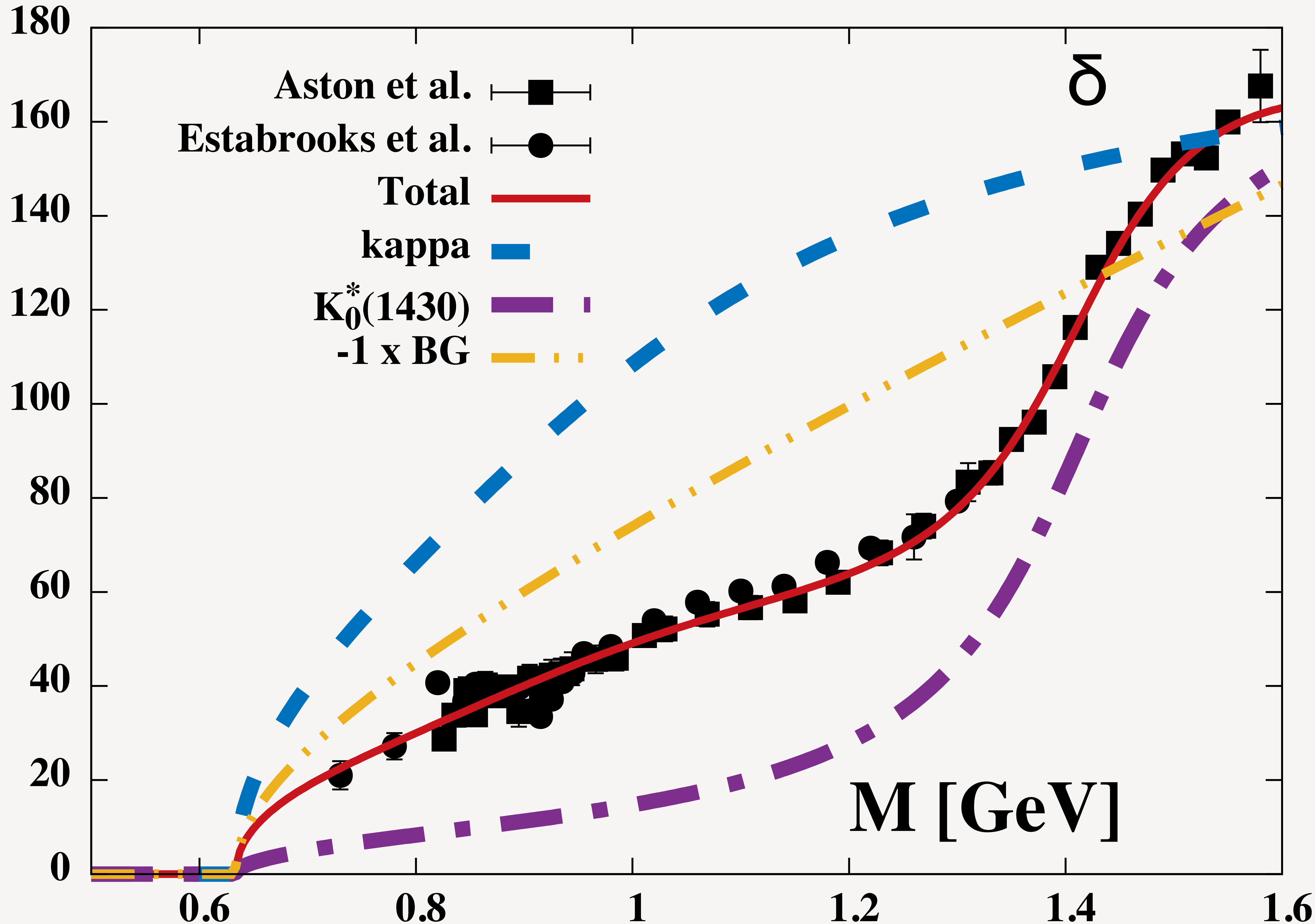
P-wave

WIDTH AND PHASE SHIFT

Green's function (QFT) ⟷ *Scattering theory*



DECOMPOSITION OF PHASE SHIFT IN CHIRAL MODELS

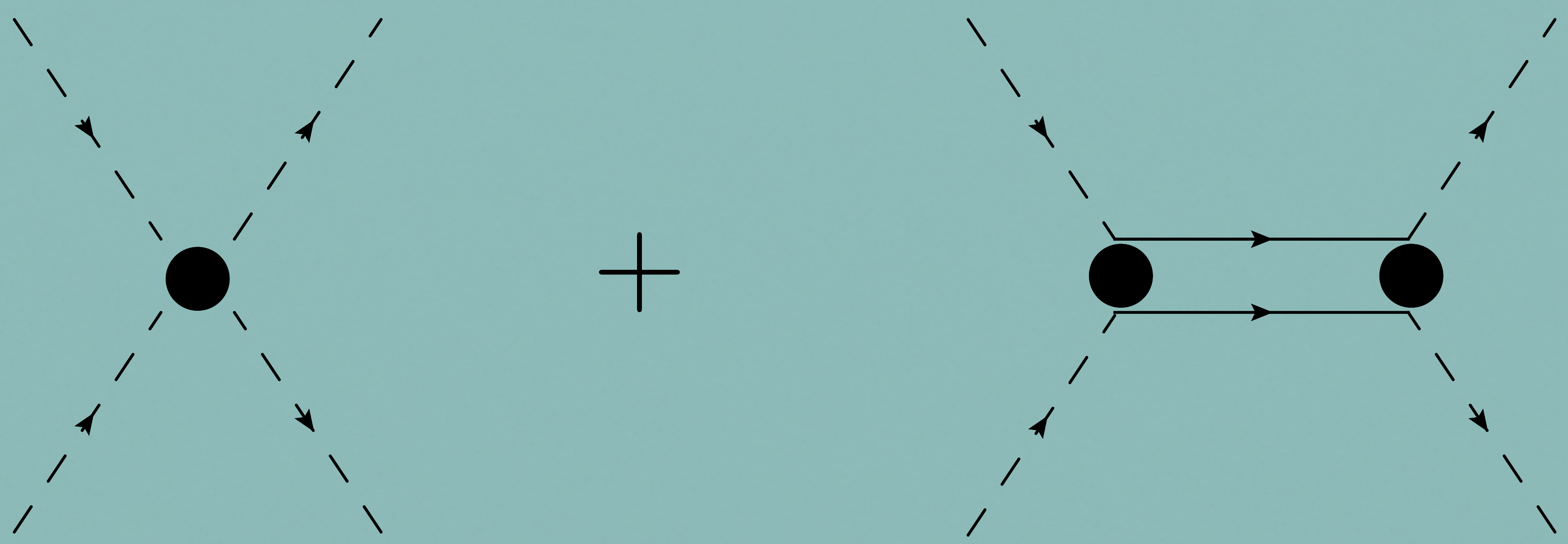


180
160
140
120
100
80
60
40
20
0
0.6
0.8
1
1.2
1.4
1.6
Aston et al.
Estabrooks et al.
Total
kappa
$K_0^*(1430)$
-1 x BG
δ
M [GeV]

CHIRAL SYMMETRY

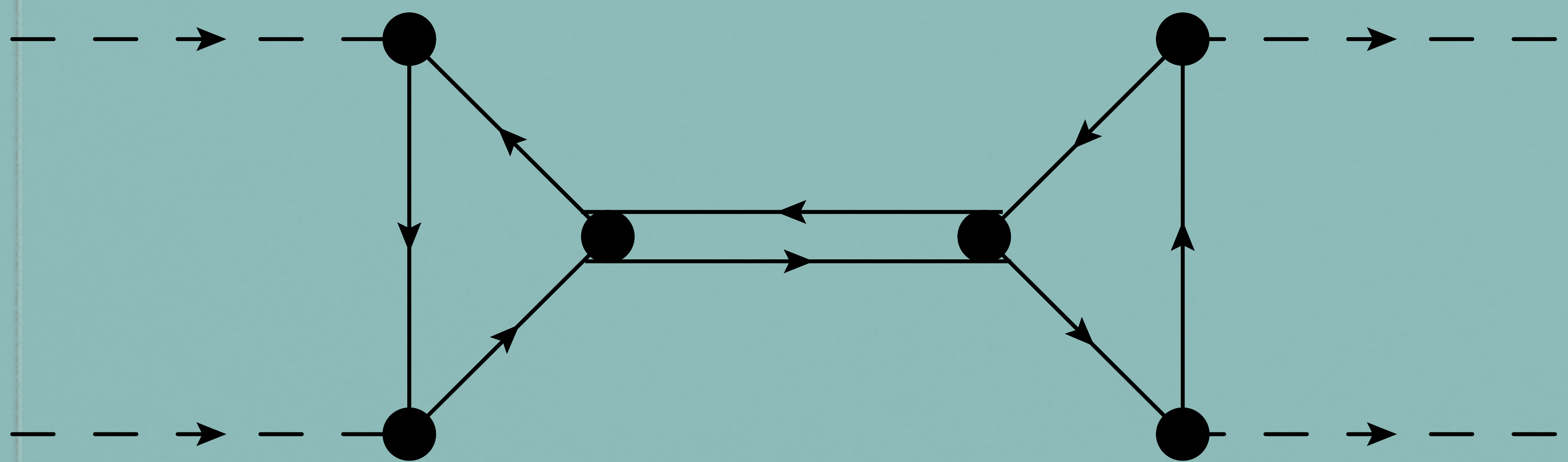
- Linear sigma model

$$U_{eff}(\sigma, \pi) = -\mu^2(\sigma^2 + \pi^2) + \lambda(\sigma^2 + \pi^2)^2$$

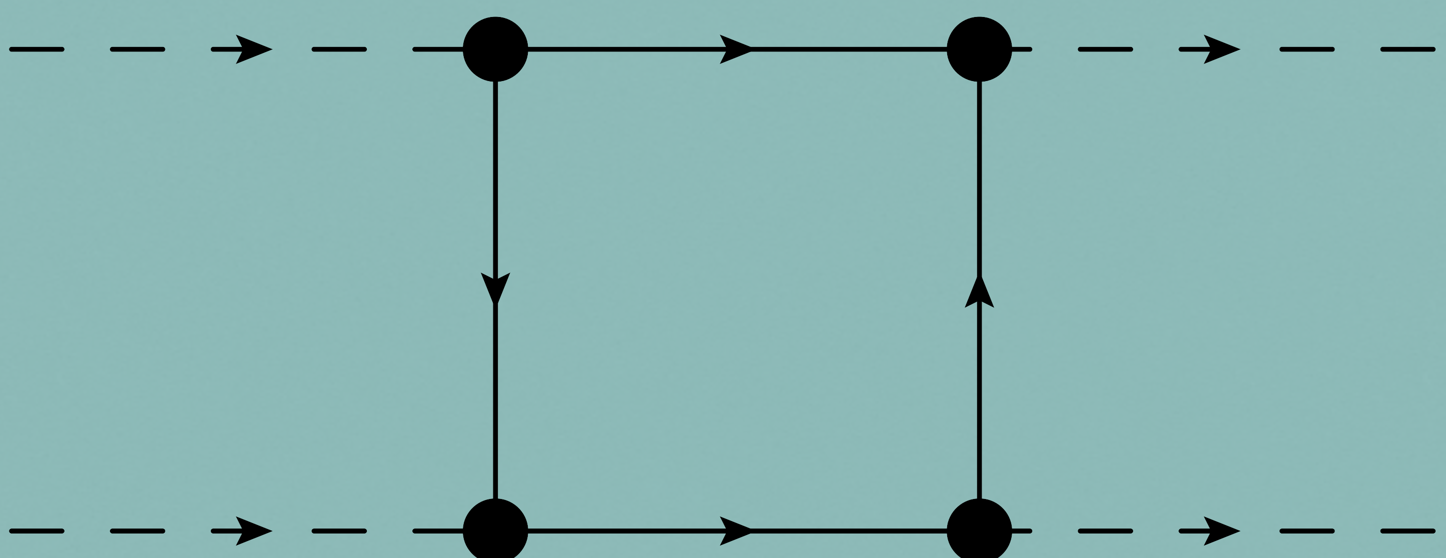


CHIRAL SYMMETRY

- NJL description



direct term



box diagram

negative scattering length -> B.G.?