Fluctuation Signatures of QCD Critical Point

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Outline

1. QCD phase diagram, critical point and fluctuations
   - Critical fluctuations and correlation length
   - Higher moments and universality

2. Beam energy scan
   - Mapping to QCD and observables
   - Understanding data
   - Acceptance dependence
QCD Phase Diagram (a theorist’s view)

Lattice at $\mu_B \lesssim 2T$

Critical point – a singularity of EOS, anchors the 1st order transition.
Why fluctuations are large at a critical point?

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\[ P(X) \sim e^{S(X)} \quad (Einstein \ 1910) \]
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$$\langle X^2 \rangle_c = - (S'')^{-1} = VT\chi$$

Susceptibility $$\chi$$ is finite in thermodynamic limit $$V \to \infty$$ — CLT.
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  Susceptibility \( \chi \) is finite in thermodynamic limit \( V \to \infty \) — CLT.

- At the critical point \( S(X) \) has a “flat direction” or “soft-mode”.
  Fluctuation measures such as \( \chi \) diverge as \( V \to \infty \).

CLT?
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ClT? Fluctuations are not averaging out, but add coherently: \( \xi \to \infty \)
Fluctuations of order parameter and $\xi$

- Fluctuations at CP – conformal field theory.
  Parameter-free $\rightarrow$ universality. Near CP $\xi = m_\sigma^{-1} < \infty$,

  $$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\},$$

  $$\Omega = \int d^3x \left[ \frac{1}{2} (\nabla \sigma)^2 + \frac{m_\sigma^2}{2} \sigma^2 + \frac{\lambda_3}{3} \sigma^3 + \frac{\lambda_4}{4} \sigma^4 + \ldots \right].$$

- Moments of order parameter $\sigma_V \equiv \int d^3x \sigma(x)$:

  Higher moments grow faster with $\xi$ with universal exponents:
  $$\langle \sigma^n_V \rangle \sim V \xi^k, \quad k = n(3 - [\sigma]) - 3, \quad [\sigma] = \beta/\nu \approx 1/2.$$
Higher moments also depend on which side of the CP we are

$$\kappa_3[\sigma_V] = 2VT^{3/2} \tilde{\lambda}_3 \xi^{4.5}; \quad \kappa_4[\sigma_V] = 6VT^2 \left[ 2(\tilde{\lambda}_3)^2 - \tilde{\lambda}_4 \right] \xi^7.$$  

This dependence is also universal.

2 relevant directions/parameters. Using Ising model variables:

- Far from CP: \( \kappa_4 = 0 \)
- Crossover side: \( \kappa_4 < 0 \)
- 1st order side: \( \kappa_4 > 0 \)
In QCD \((t, H) \rightarrow (\mu - \mu_{CP}, T - T_{CP})\)

the mapping is not universal

\[ \kappa_4[\sigma_V] < 0 \text{ means } \kappa_4[M] \langle M \rangle < 1 \]

NB: Sensitivity to \(M\) accepted: \((\kappa_4)\sigma \sim M^4\) (number of 4-tets).
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Observed fluctuations, e.g., multiplicity \(M\), are not the same as \(\sigma_V\), but related

\[
\kappa_4[M] = \langle M \rangle + \kappa_4[\sigma_V] \times g^4 \left( \frac{d\langle M \rangle}{d\sigma} \right)^4 + \ldots,
\]

\(\sim M^4\)

Physical picture – fluctuating \(\sigma\) background, \(m(\sigma)\).

\(g\) – coupling of the critical mode \((g = \frac{dm}{d\sigma})\).
Mapping to QCD and experimental observables

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NB: Sensitivity to \(M_{\text{accepted}}\): \((\kappa_4)_\sigma \sim M^4\) (number of 4-tets).
Coupling vs particle momentum

For $g \bar{\psi} \psi$ coupling, or $m(\sigma)$:

$$\mathcal{O} = \int_p \frac{\partial f_p(m(\sigma))}{\partial \sigma} = -\frac{g}{T} \int_p \frac{f_p(1 - f_p)}{\gamma_p}$$

All protons in thermal distribution contribute with weight $\sim f_p$. Fluctuations of $\sigma$ correlate particles with all (thermal) momenta.
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All protons in thermal distribution contribute with weight $\sim f_p$.

fluctuations of $\sigma$ correlate particles with all (thermal) momenta.
Why $\xi$ is finite

System expands and is out of equilibrium

In this talk – *equilibrium* fluctuations. The only dynamical effect we consider is the one which makes $\xi$ finite:

Critical slowing down. Universal scaling law:

$$\xi \sim \tau^{1/z},$$

where $1/\tau$ is expansion rate

and $z \approx 3$ (Son-MS).

Estimates: $\xi \sim 2 - 3$ fm (Berdnikov-Rajagopal, Asakawa-Nonaka).

Need full critical dynamics to take non-equilibrium into account

e.g., memory effect – Mukherjee-Venugopalan-Yin

For more see Nahrgang’s talk
What should we see in the BES?

M. Stephanov (UIC)
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Fluctuations and QCD Critical Point

Net-Proton
0.4<p_{T}<2 (GeV/c),|y|<0.5
- 0-5%
- 5-10%
- 70-80%

UrQMD, 0-5%

STAR Preliminary

\( \sqrt{s_{NN}} \) (GeV)

\( \kappa \sigma^2 \)
What should we see in the BES?

Scenario 1

Net-Proton
0.4<p_T<2 (GeV/c), lyl<0.5
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- 5-10%
- 70-80%

STAR Preliminary

K o^2

speculative

M. Stephanov (UIC)
Fluctuations and QCD Critical Point
GSI 2015
What should we see in the BES?

Scenario 2

Net-Proton
0.4<p_T<2 (GeV/c), |y|<0.5
- 0-5%
- 5-10%
- 70-80%
- \[\text{UrQMD, 0-5\%}\]

\(\kappa \sigma^2\)

\(\sqrt{s}\)

baseline

\(\omega_4\)

\(\sqrt{s_{NN}}\) (GeV)

STAR Preliminary

speculative
Skewness?
Skewness?

[Graph and text discussing skewness and freezeout]

M. Stephanov (UIC)  Fluctuations and QCD Critical Point  GSI 2015 11 / 17
Skewness?
Questions

- 14.5 GeV: physics or detector issues?
  
  BES II will help answer.

- If confirmed as physics –
  
  Finer measurements may be needed: 13 GeV, 16.5 GeV?

- Then 7.7 GeV – another physics effect? Perhaps, 1st order transition (non-equilibrium)?
Acceptance dependence
Correlations – spatial vs kinematic

$\xi \sim 1 - 3 \text{ fm}$

$\Delta \eta_{\text{corr}} = \frac{\xi}{\tau_f} \sim 0.1 - 0.3$

Particles within $\Delta \eta_{\text{corr}}$ have thermal rapidity spread. Thus

$\Delta y_{\text{corr}} \sim 1 \gg \Delta \eta_{\text{corr}}$
Acceptance dependence – two regimes

How do cumulants depend on acceptance?

Let $\kappa_n(M)$ be a cumulant of $M$ – multiplicity of accepted, say, protons.

- $\Delta y \gg \Delta y_{corr}$ – a theory limit (thermodynamic): CLT applies.

$$\kappa_n \sim M$$

or $\omega_n \equiv \frac{\kappa_n}{M} \to \text{const}$ – an “intensive”, or volume indep. measure.
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- $\Delta y \ll \Delta y_{\text{corr}}$ – more typical in experiment.

  Subtracting trivial (uncorrelated, Poisson) contribution:

  $\kappa_n - M \sim M^n$ – proportional to number of correlated $n$-plets;

  or $\omega_n - 1 \sim M^{n-1}$.
Critical point fluctuations vs acceptance

Proton multiplicity at 19.6 GeV: \( \omega_{n,\sigma} \equiv \omega_n - 1 \)

\[ PT \in (0, 2) \text{GeV} \]
\[ PT \in (0.4, 2) \text{GeV} \]
\[ PT \in (0.4, 0.8) \text{GeV} \]

\( \Delta y \)
\( \omega_{2,\sigma}(\Delta y) \)
\( \omega_{4,\sigma}(\Delta y) \)

\( p_T \) and rapidity cuts have qualitatively similar effects.
Critical point fluctuations vs acceptance

Proton multiplicity at 19.6 GeV: \( \omega_{n,\sigma} \equiv \omega_n - 1 \)

\[ \begin{align*}
\omega_{n,\sigma} &\quad \Delta y \\
\omega_{n,\sigma}(\infty) &\quad \Delta y \\
\end{align*} \]

\( P_T \in (0, 2) \text{ GeV} \)
\( P_T \in (0.4, 2) \text{ GeV} \)
\( P_T \in (0.4, 0.8) \text{ GeV} \)

\( P_T \) and rapidity cuts have qualitatively similar effects.

- Wider acceptance improves value/error:
  - errors grow slower than \( M^n \).
Fluctuations reflect universal features of the CP and could be used to discover it via the BES.

Interesting recent data. Needs better understanding.

14.5 GeV results: critical physics or detector issues? BESII + 13 GeV, 16.5 GeV?

What physics in 7.7 GeV?

As long as $\Delta y \ll \Delta y_{\text{corr}}$: $\kappa_n[M] - M \sim M^n$.

Wider acceptance improves relative precision.

Dynamical description of fluctuations is essential.
More ...
Critical slowing down.

Mukherjee-Venugopalan-Yin

\[
\frac{dP}{d\tau} = F[P] \\
\downarrow \\
\frac{d\kappa_n}{d\tau} = L[\kappa_n, \kappa_{n-1}, \ldots]
\]
Time evolution of cumulants (memory)

Critical slowing down.

Mukherjee-Venugopalan-Yin

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