Fluctuation Signatures of QCD Critical Point

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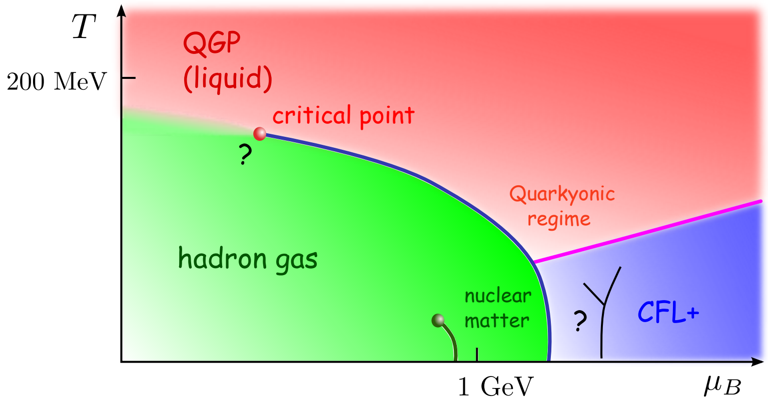
Outline

1. QCD phase diagram, critical point and fluctuations
   - Critical fluctuations and correlation length
   - Higher moments and universality

2. Beam energy scan
   - Mapping to QCD and observables
   - Understanding data
   - Acceptance dependence

QCD Phase Diagram *(a theorist's view)*



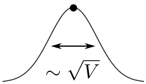
Lattice at $\mu_B \lesssim 2T$

Critical point – a singularity of EOS, anchors the 1st order transition.

Why fluctuations are large at a critical point?

- The key equation:

$$P(X) \sim e^{S(X)} \qquad \textit{(Einstein 1910)}$$



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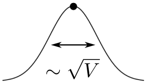

- For an extensive quantity $\langle X \rangle \sim V$:

$$\langle X^2 \rangle_c = -\left(S''\right)^{-1} = VT\chi$$

Susceptibility χ is finite in thermodynamic limit $V \to \infty$ — CLT.

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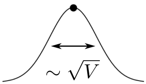

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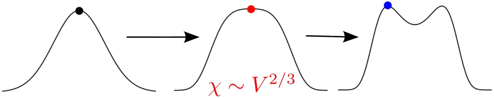

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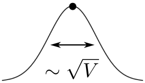

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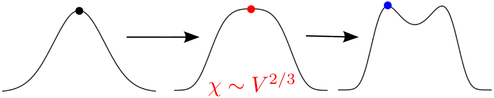
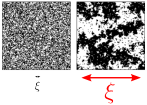




CLT? $\boxed{\text{Fluctuations are not averaging out, but add coherently: } \xi \to \infty}$

Fluctuations of order parameter and ξ

- Fluctuations at CP – conformal field theory.

  Parameter-free $\to$ universality. Near CP $\xi = m_\sigma^{-1} < \infty$,

$$P[\sigma] \sim \exp\left\{-\Omega[\sigma]/T\right\},$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\boldsymbol{\nabla}\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \dots\right].$$

  - Moments of order parameter $\sigma_V \equiv \int d^3x\, \sigma(x)$:

- Higher moments grow faster with ξ with universal

  exponents: $\boxed{\langle\sigma_V^n\rangle \sim V\xi^k}$, $k = n(3-[\sigma])-3$, $[\sigma] = \beta/\nu \approx 1/2$.

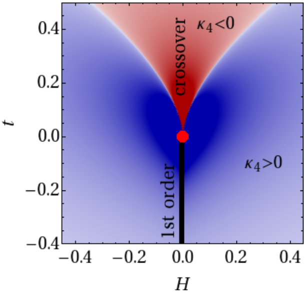
Sign

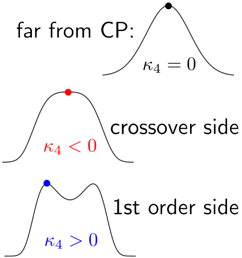
- Higher moments also depend on which side of the CP we are

$$\kappa_3[\sigma_V] = 2VT^{3/2}\,\tilde{\lambda}_3\,\xi^{4.5}\,; \quad \kappa_4[\sigma_V] = 6VT^2\,[\,2(\tilde{\lambda}_3)^2 - \tilde{\lambda}_4\,]\,\xi^7\,.$$

This dependence is also universal.

- 2 relevant directions/parameters. Using Ising model variables:

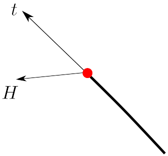




Mapping to QCD and experimental observables

- In QCD $(t, H) \to (\mu - \mu_{\text{CP}}, T - T_{\text{CP}})$

  the mapping is not universal



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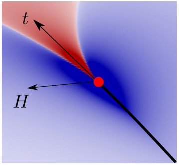

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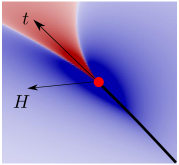

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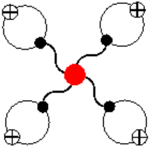
$$\kappa_4[M] = \underbrace{\langle M \rangle}_{\text{Poisson}} + \kappa_4[\sigma_V] \times g^4 \underbrace{\left(\quad \right)^4}_{\sim M^4} + \ldots,$$



Physical picture – fluctuating σ background, $m(\sigma)$.

g – coupling of the critical mode ($g = dm/d\sigma$).

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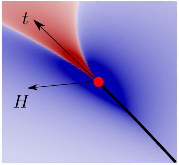

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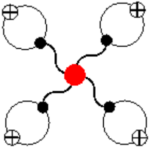

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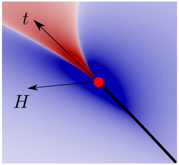

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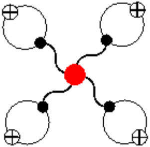

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- $\kappa_4[\sigma_V] < 0$ means $\dfrac{\kappa_4[M]}{\langle M \rangle} < 1$

- NB: Sensitivity to M_{accepted}: $(\kappa_4)_\sigma \sim M^4$ (number of 4-tets).

For $g\bar{\psi}\psi$ coupling, or $m(\sigma)$:

$$= \int_{\boldsymbol{p}} \frac{\partial f_{\boldsymbol{p}}(m(\sigma))}{\partial \sigma} = -\frac{g}{T} \int_{\boldsymbol{p}} \frac{f_{\boldsymbol{p}}(1 - f_{\boldsymbol{p}})}{\gamma_{\boldsymbol{p}}}$$

Coupling vs particle momentum

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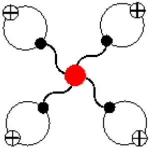
All protons in thermal distribution contribute with weight $\sim f_{\boldsymbol{p}}$.



fluctuations of σ correlate particles with all (thermal) momenta.

Why ξ is finite

$\boxed{\text{System expands and is out of equilibrium}}$

In this talk – *equilibrium* fluctuations. The only dynamical effect we consider is the one which makes ξ finite:

Critical slowing down. Universal scaling law:

$$\xi \sim \tau^{1/z}, \qquad \text{where } 1/\tau \text{ is expansion rate}$$

and $z \approx 3$ (Son-MS).

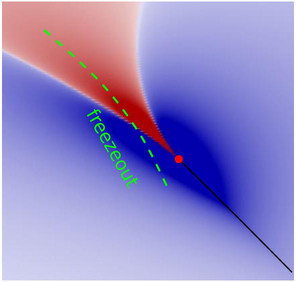
Estimates: $\xi \sim 2 - 3$ fm (Berdnikov-Rajagopal, Asakawa-Nonaka).

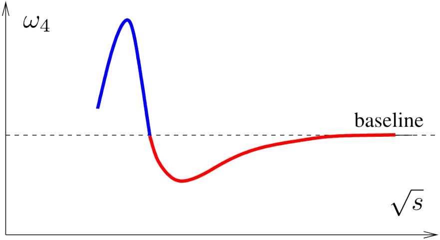
- Need full critical dynamics to take non-equilibrium into account

  e.g., memory effect – Mukherjee-Venugopalan-Yin

  For more see Nahrgang's talk

What should we see in the BES?





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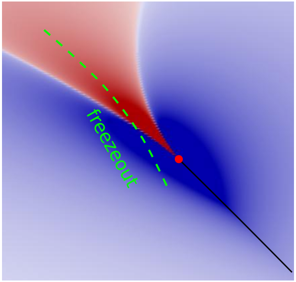

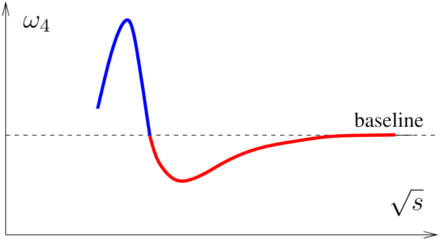

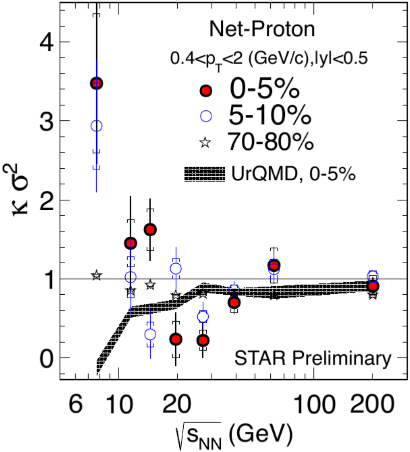

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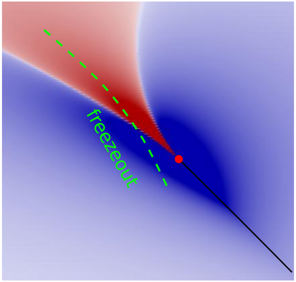

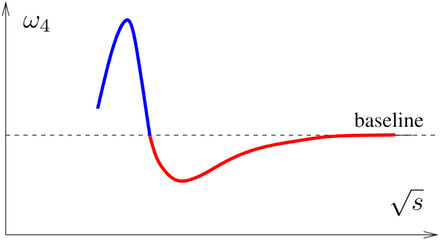

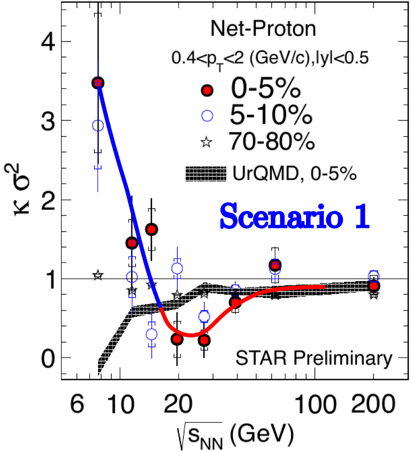

speculative

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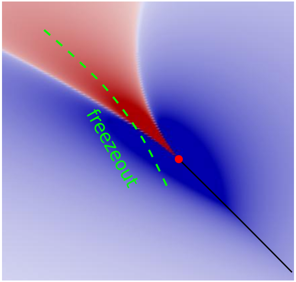

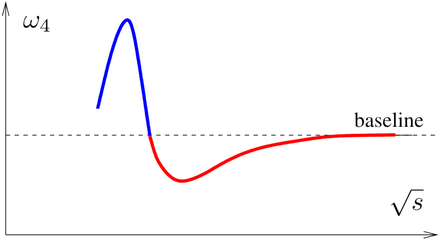

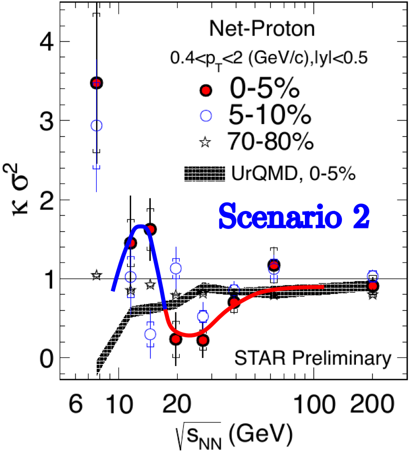

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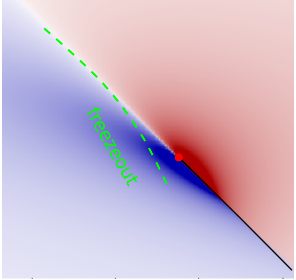
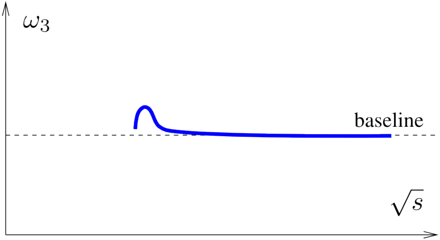
Skewness?





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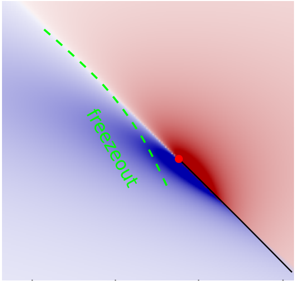
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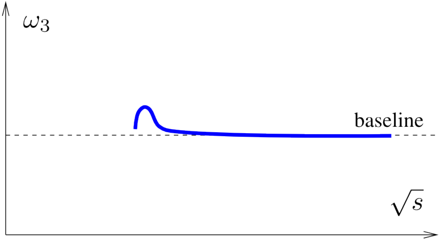
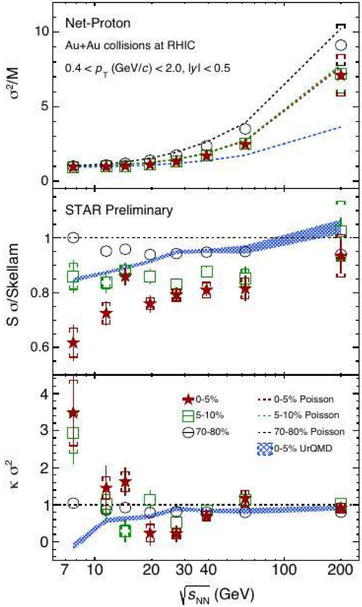

ω_3
baseline
$\sqrt{s}$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Au+Au collisions at RHIC
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σ^2/M
STAR Preliminary
S σ/Skellam
κ σ²
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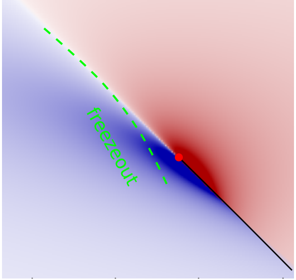
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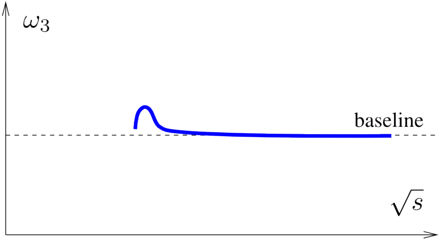
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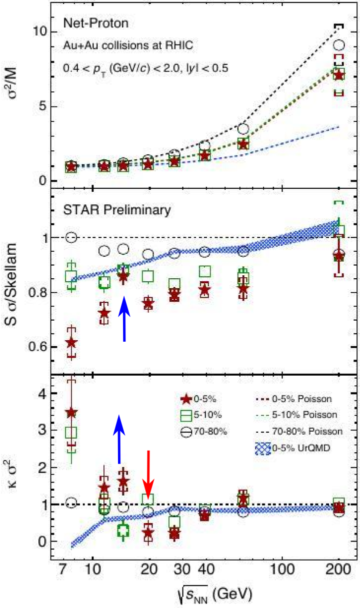
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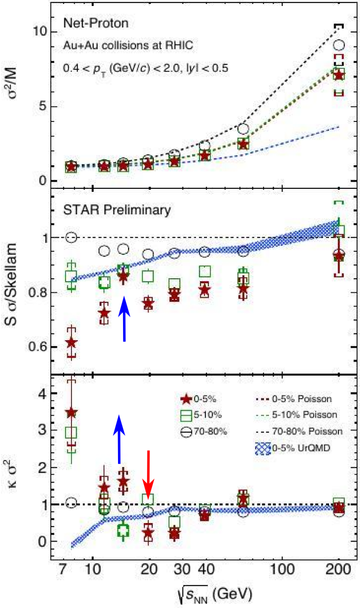
Questions

- 14.5 GeV: physics or detector issues?

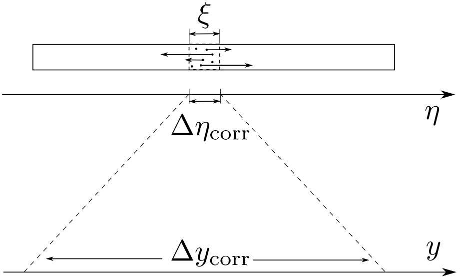
  BES II will help answer.

- If confirmed as physics –
  - Finer measurements may be needed: 13 GeV, 16.5 GeV?
  - Then 7.7 GeV – another physics effect? Perhaps, 1st order transition (non-equilibrium)?



Acceptance dependence

Correlations – spatial vs kinematic



$\xi \sim 1 - 3$ fm

$$\Delta\eta_{\rm corr} = \frac{\xi}{\tau_{\rm f}} \sim 0.1 - 0.3$$

Particles within $\Delta\eta_{\rm corr}$ have thermal rapidity spread. Thus

$$\Delta y_{\rm corr} \sim 1 \gg \Delta\eta_{\rm corr}$$

Acceptance dependence – two regimes

How do cumulants depend on acceptance?

Let $\kappa_n(M)$ be a cumulant of M – multiplicity of *accepted*, say, protons.

- $\Delta y \gg \Delta y_{\rm corr}$ – a theory limit (thermodynamic): CLT applies.

  $\kappa_n \sim M$

  or $\omega_n \equiv \dfrac{\kappa_n}{M} \to \mathrm{const}$ – an “intensive”, or volume indep. measure

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- $\Delta y \ll \Delta y_{\rm corr}$ – more typical in experiment.

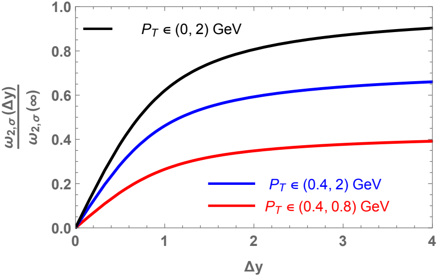
  Subtracting trivial (uncorrelated, Poisson) contribution:

  $\kappa_n - M \sim M^n$ – proportional to number of correlated n-plets;

  or $\omega_n - 1 \sim M^{n-1}$.

Critical point fluctuations vs acceptance

Proton multiplicity at 19.6 GeV: $\omega_{n,\sigma} \equiv \omega_n - 1$



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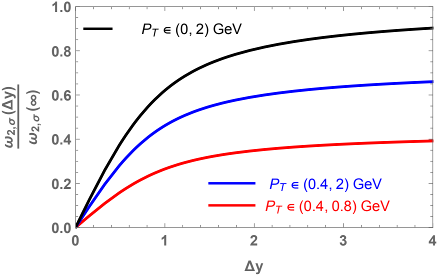
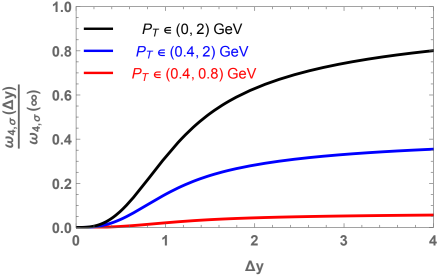
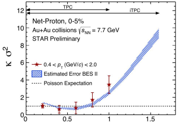




p_T and rapidity cuts have qualitatively similar effects.

- Wider acceptance improves value/error:

  errors grow slower than M^n.



Summary and conclusions

- Fluctuations reflect universal features of the CP and could be used to discover it via the BES.
- Interesting recent data. Needs better understanding.
  - 14.5 GeV results: critical physics or detector issues?

    BESII + 13 GeV, 16.5 GeV?
  - What physics in 7.7 GeV?
- As long as $\Delta y \ll \Delta y_{\rm corr}$: $\kappa_n[M] - M \sim M^n$.

  Wider acceptance improves relative precision.
- Dynamical description of fluctuations is essential.

More ...

Time evolution of cumulants (memory)

Critical slowing down.

Mukherjee-Venugopalan-Yin

$$\frac{dP}{d\tau} = F[P]$$

$$\Downarrow$$

$$\frac{d\kappa_n}{d\tau} = L[\kappa_n, \kappa_{n-1}, \ldots]$$

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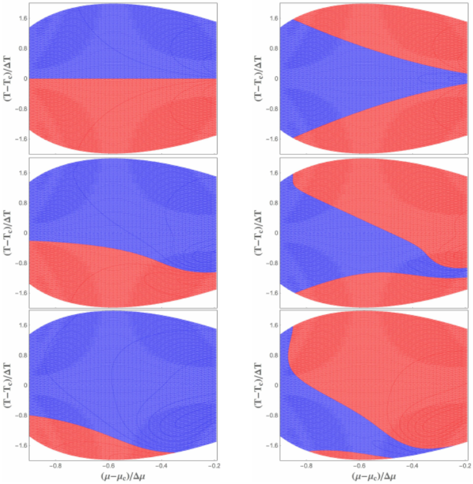

κ_3 κ_4

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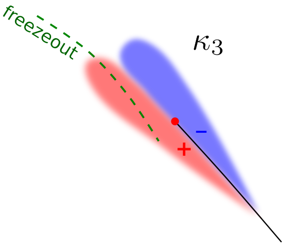
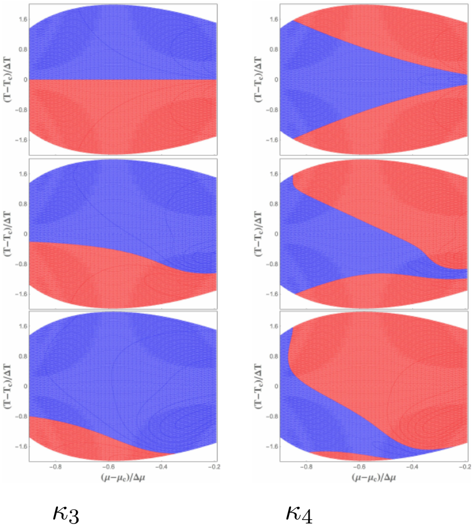
$$\Downarrow$$

$$\frac{d\kappa_n}{d\tau} = L[\kappa_n, \kappa_{n-1}, \ldots]$$





Time evolution of cumulants (memory)

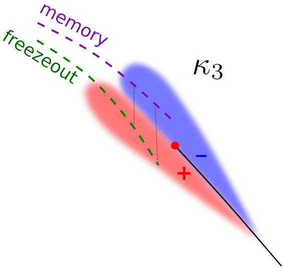
Critical slowing down.

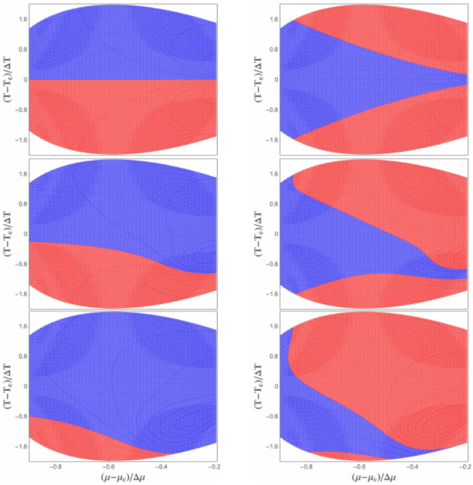
Mukherjee-Venugopalan-Yin

$$\frac{dP}{d\tau} = F[P]$$

$$\Downarrow$$

$$\frac{d\kappa_n}{d\tau} = L[\kappa_n, \kappa_{n-1}, \ldots]$$





κ_3 κ_4