Search for the QCD Critical Point -
Fluctuations of Conserved Quantities in High Energy Nuclear Collisions at RHIC

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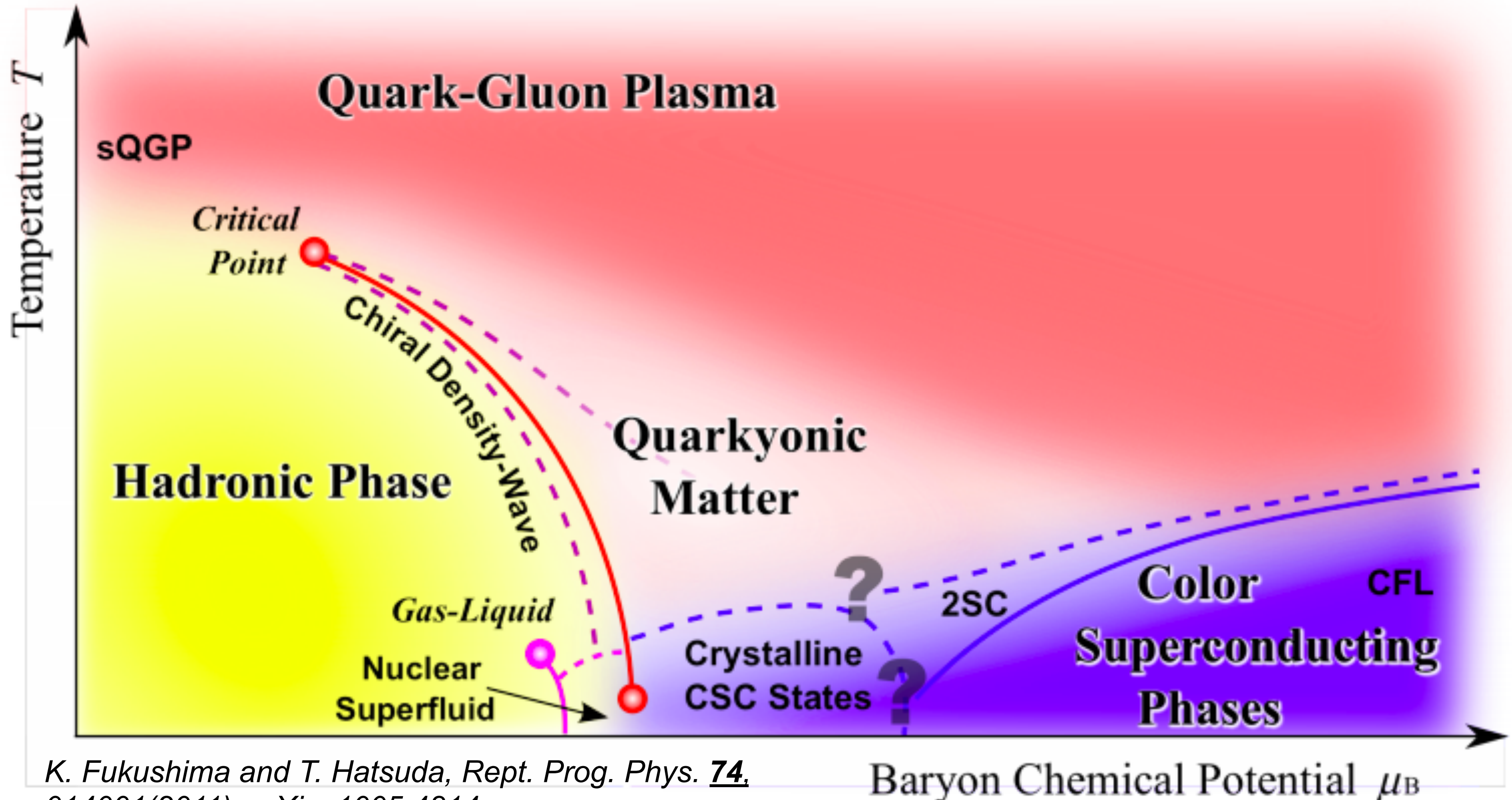
QCD Phase Diagram (Conjectured)

Very rich phase structure in the QCD phase diagram.

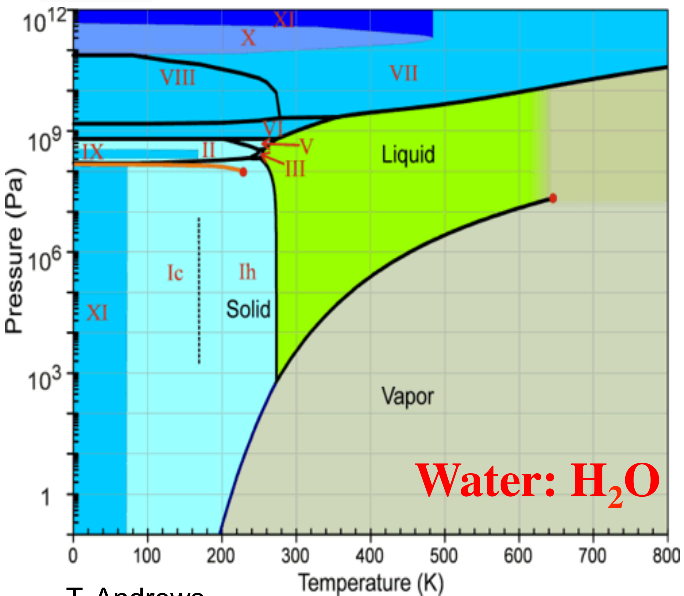


Hatsuda and Fukushima

*K. Fukushima and T. Hatsuda, Rept. Prog. Phys. **74**, 014001(2011); arXiv: 1005.4814*

Large Uncertainties in determining phase structure by theory at finite μ_B.

The QCD Critical Point



T. Andrews.
Phil. Trans. Royal Soc., 159:575, 1869

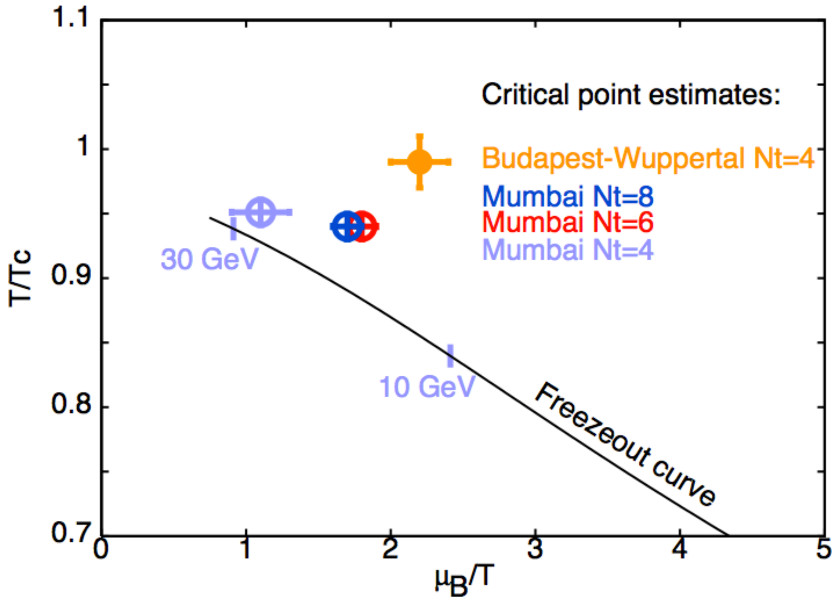
- Singularity of EOS: Diverges of the thermodynamics quantities, such as correlation length (ξ), Susceptibilities (χ), heat capacity (C_V).
- Long wavelength fluctuations of order parameter. Critical Opalescence.

Search for CP in Heavy-ion Collisions

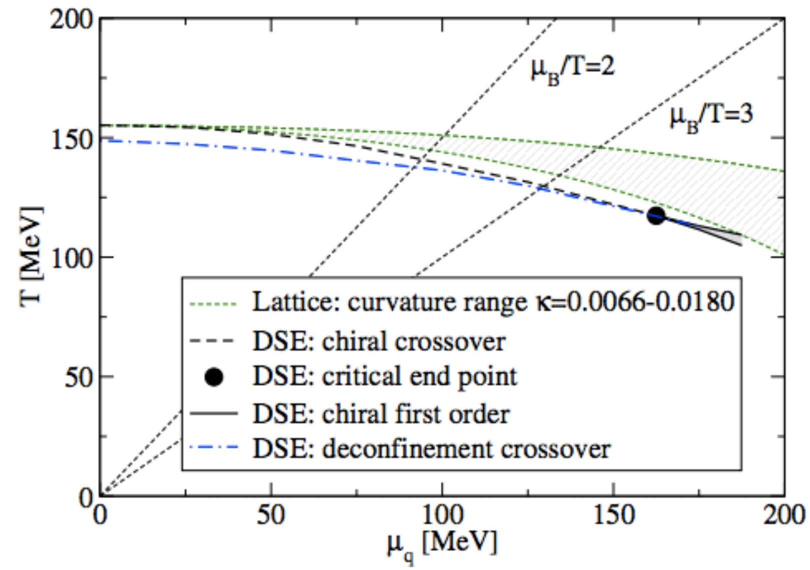
1. finite size/time. ξ=2 ~ 3 fm
2. Non-CP physics effects.
3. Need CFO closer enough to CP.
4. Signal didn't wash out after evolution.

Very Challenging !!!

Lattice QCD:



DES:



Lattice QCD：

1): Reweighting：Fodor&Katz,2004:

$\mu^E_B/T^E \sim 2.2 \rightarrow \sqrt{s_{NN}} \sim 9.5$ GeV

2): Tylor Expansion: Gavai&Gupta 2013

$\mu^E_B/T^E \sim 1.7 \rightarrow \sqrt{s_{NN}} \sim 14.5$ GeV

DSE:

1): Y. X. Liu, et al., PRD90, 076006 (2014).

$\mu^E_B/T^E \sim 2.88 \rightarrow \sqrt{s_{NN}} \sim 8$ GeV

2): C. S. Fischer et al., PRD90, 034022 (2014).

$\mu^E_B/T^E \sim 4.4 \rightarrow \sqrt{s_{NN}} \sim 6$ GeV

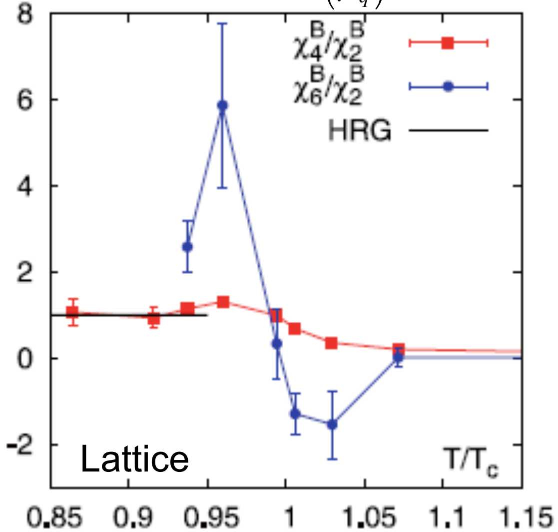
$\sqrt{s_{NN}} = 6 \sim 14.5$ GeV， $\mu^E_B = 266 \sim 496$ MeV

Observables: Higher Moments (fluctuations)

$$C_{2,x} \sim \xi^2 \qquad C_{3,x} \sim \xi^{4.5} \qquad C_{4,x} \sim \xi^7$$

M. A. Stephanov, PRL102, 032301 (2009); PRL107, 052301 (2011);
M. Akasawa, et al., PRL103,262301 (2009).

$$\chi_q^{(n)} = \frac{1}{VT^3} \times C_{n,q} = \frac{\partial^n (p/T^4)}{\partial(\mu_q)^n}, q = B,Q,S$$

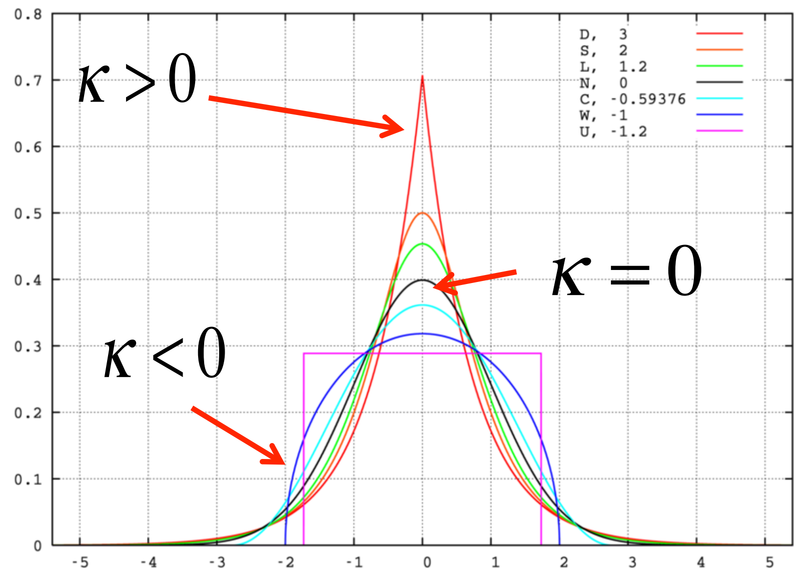


S. Ejiri et al, Phys.Lett. B633 (2006) 275-282
Cheng et al, PRD (2009) 074505.
B. Friman et al., EPJC 71 (2011) 1694.

"Shape" of the fluctuations can be measured: non-Gaussian moments.

$$C_{1,x} = \langle x \rangle, C_{2,x} = \langle (\delta x)^2 \rangle,$$

$$C_{3,x} = \langle (\delta x)^3 \rangle, C_{4,x} = \langle (\delta x)^4 \rangle - 3 \langle (\delta x)^2 \rangle^2$$



➢ **Susceptibility ratios ⇔ Moments of Conserved Charges ⇔ Cumulant Ratios**

$$\frac{\chi_q^4}{\chi_q^2} = \kappa\sigma^2 = \frac{C_{4,q}}{C_{2,q}} \qquad \frac{\chi_q^3}{\chi_q^2} = S\sigma = \frac{C_{3,q}}{C_{2,q}},$$

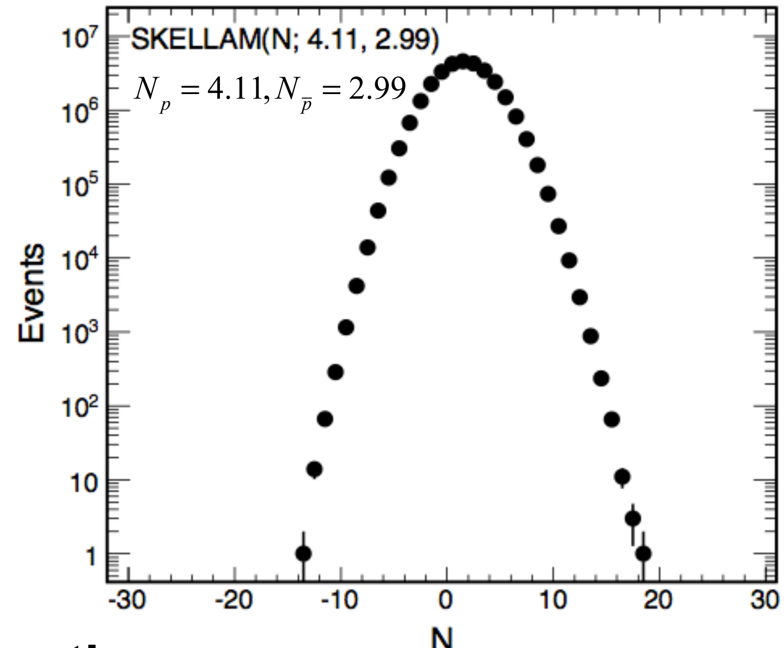
(q=B, Q, S)

Expectations from Skellam Distributions

- **If proton and anti-proton are independent Poissonian distributions, the distributions of net-protons is Skellam distributions, which is the case in Hadron Resonance Gas Model.**

$$P(N) = (\frac{N_{\bar{p}}}{N_p})^{N/2} I_N(2\sqrt{N_{\bar{p}} N_p}) e^{-(N_{\bar{p}}+N_p)}$$

N_{pbar} : Mean number of anti-protons
N_p : Mean number of protons



Contribute from statistical/thermal fluctuations.

- **Then we have the skellam expectations for various moments/cumulants :**

$$C_{2n} = N_p + N_{\bar{p}}$$
$$C_{2n-1} = N_p - N_{\bar{p}}, (n = 1,2,3...)$$

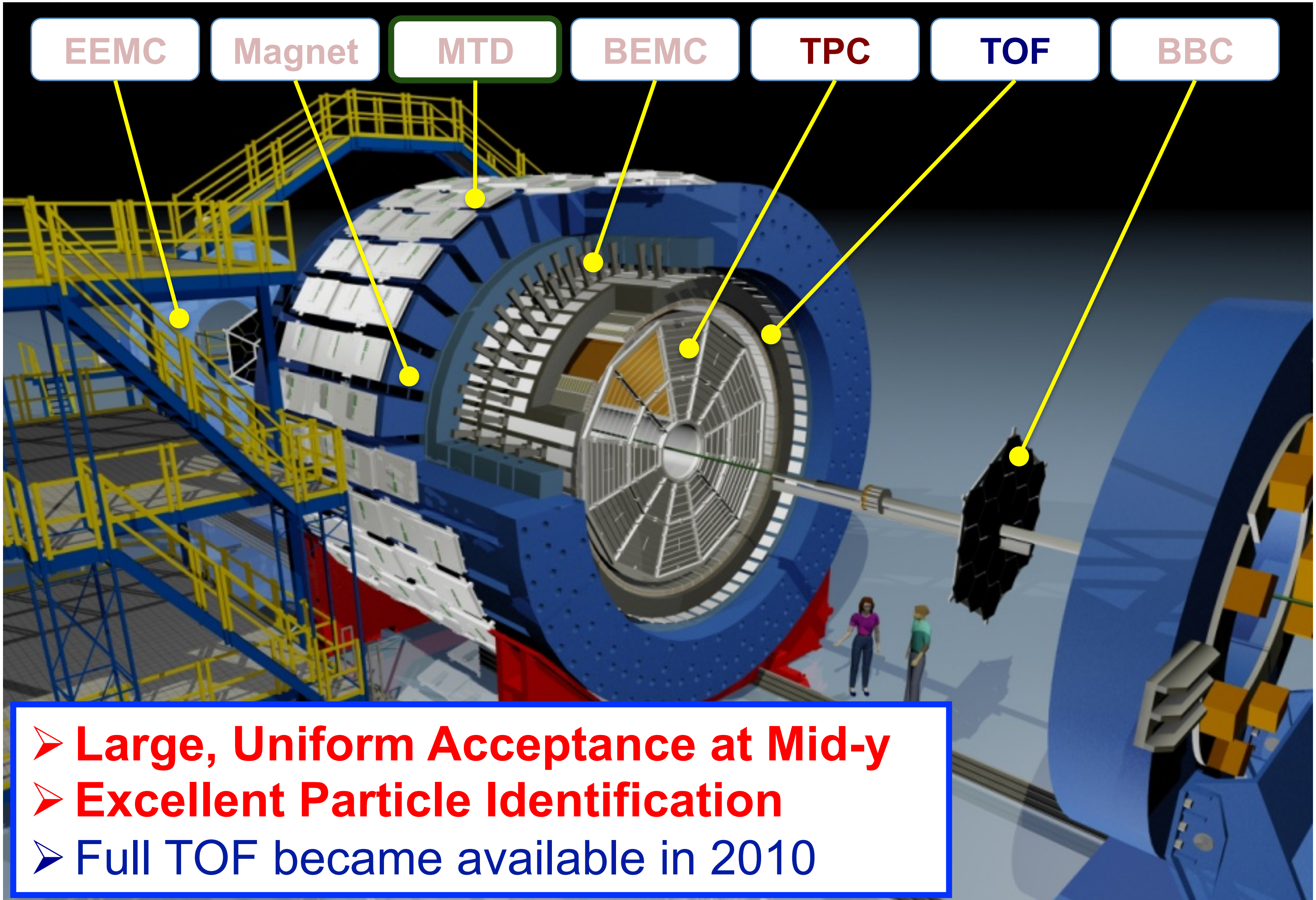
$$S\sigma = \frac{C_3}{C_2} = \frac{N_p - N_{\bar{p}}}{N_p + N_{\bar{p}}}, \kappa\sigma^2 = \frac{C_4}{C_2} = 1$$

RHIC Beam Energy Scan-Phase I

$\sqrt{s}$ (GeV)	Statistics(Millions) (0-80%)	Year	μ_B (MeV)	T (MeV)	μ_B /T
7.7	~4	2010	420	140	3.020
11.5	~12	2010	315	152	2.084
14.5	~ 20	2014	266	156	1.705
19.6	~36	2011	205	160	1.287
27	~70	2011	155	163	0.961
39	~130	2010	115	164	0.684
62.4	~67	2010	70	165	0.439
200	~350	2010	20	166	0.142

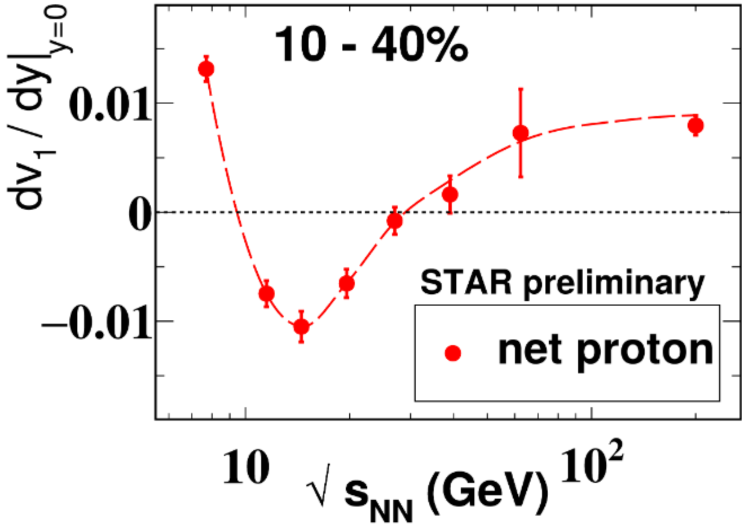
μ_B , T : J. Cleymans et al., Phys. Rev. C 73, 034905 (2006).

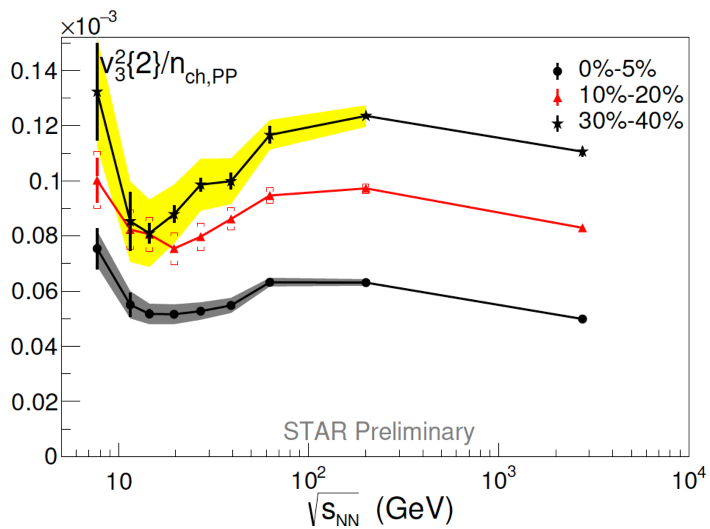
STAR Detector System

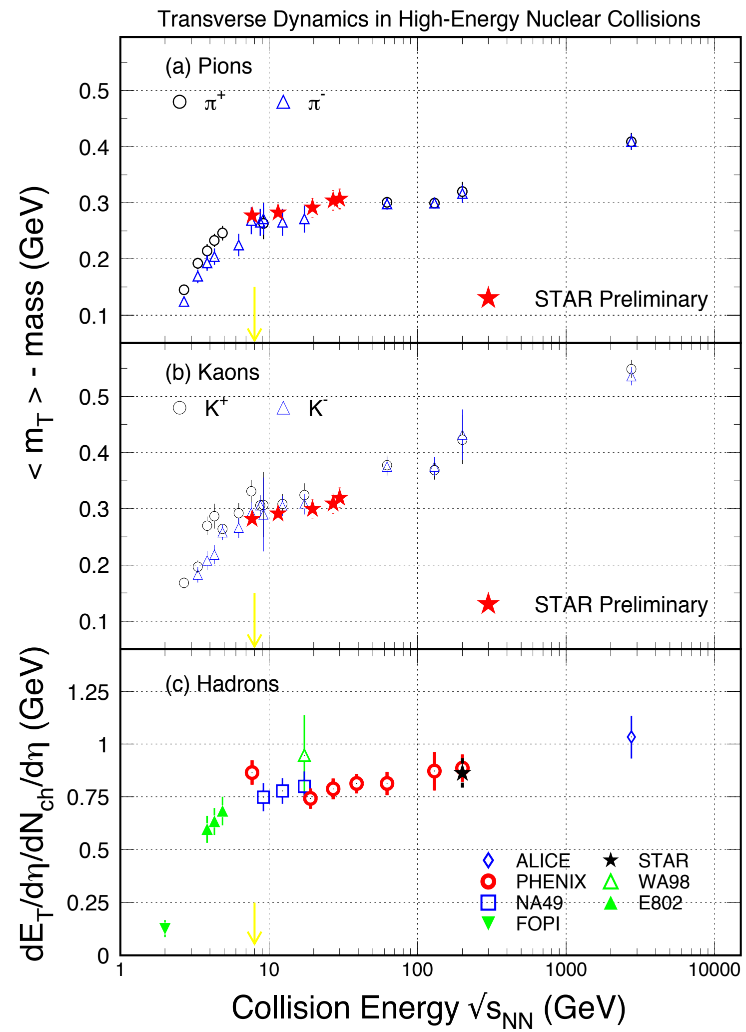


- **Large, Uniform Acceptance at Mid-y**
- **Excellent Particle Identification**
- Full TOF became available in 2010

First Order Phase Transition ?





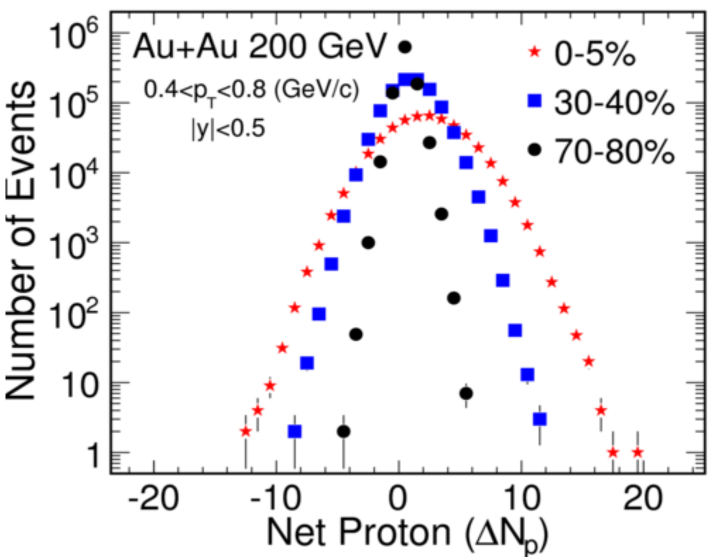


D.H. Rischke et al. HIP1, 309(1995) H. Stoecker, NPA750, 121(2005)
J. Steinheimer et al., arXiv:1402.7236 P. Konchakovski et al., arXiv:1404.276

Indication of a First Order Phase Transition.

Methodology

Raw net-p prob. distribution



$$\kappa\sigma^2 = \frac{C_{4,q}}{C_{2,q}} \qquad S\sigma = \frac{C_{3,q}}{C_{2,q}},$$

STAR: **PRL112**, 32302(14)
STAR: **PRL113**,092301(14)
X. Luo, J. Phys.: Conf. Ser. 316 012003 (2011).
X. Luo, JPG 39, 025008 (2012).
X. Luo, et al., JPG 40, 105104 (2013).
X. Luo, PRC 91, 043907 (2015).
A. Bzdak and V. Koch, PRC91,027901(2015), PRC86，044904(2012).

1. Finite particle detection efficiency.

Need efficiency correction. It is not straight forward for higher moments.

2. Initial volume fluctuations.
Improve centrality resolution and apply centrality bin width correction.

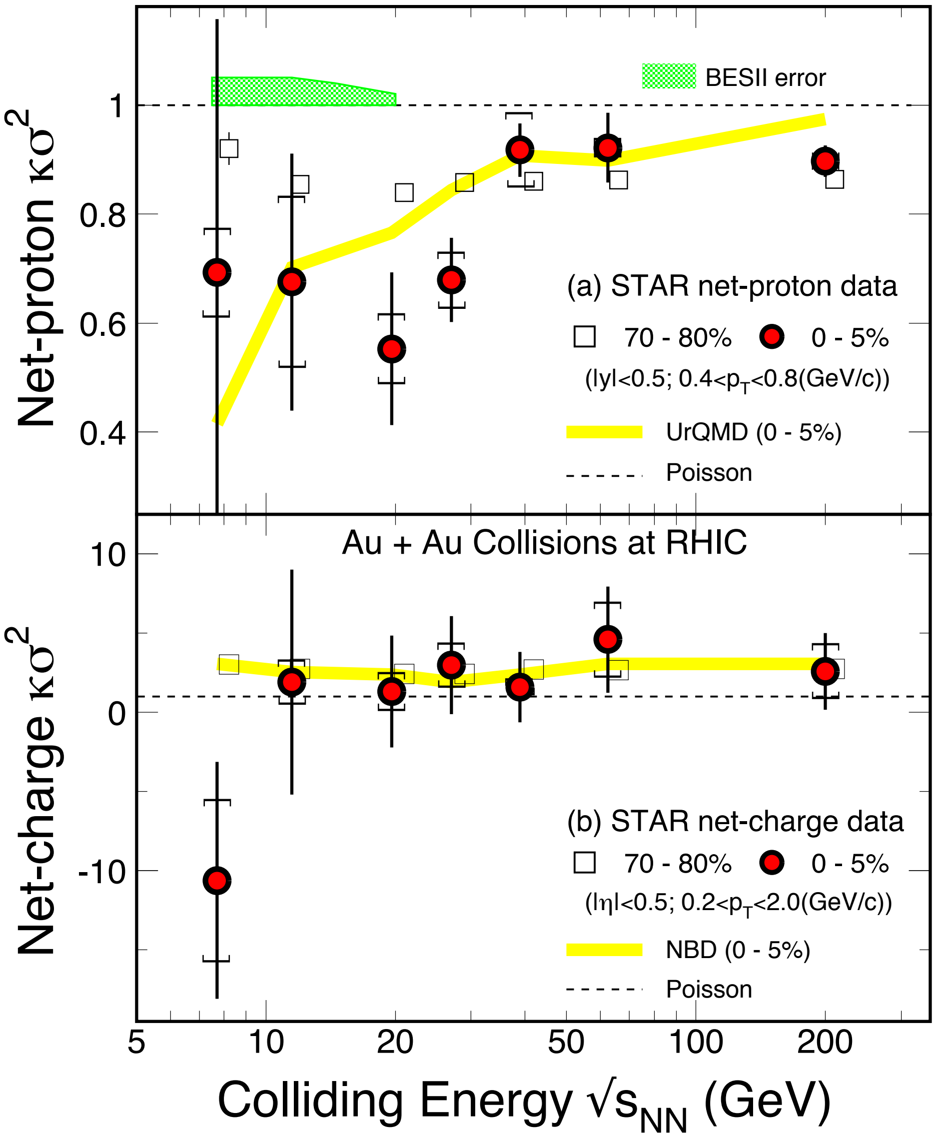
3. Remove auto-correlation.
Particles used in the analysis are excluded in centrality definition.

4. Proper error calculations.

Delta theorem and Bootstrap

$$error \propto O\left(\sigma^n / \varepsilon^\alpha\right)$$

Higher Moments Results



Net-proton results:

All data show deviations below Poisson for $\kappa\sigma^2$ at all energies. Larger deviation at $\sqrt{s_{NN}} \sim 20$ GeV

Net-charge results:

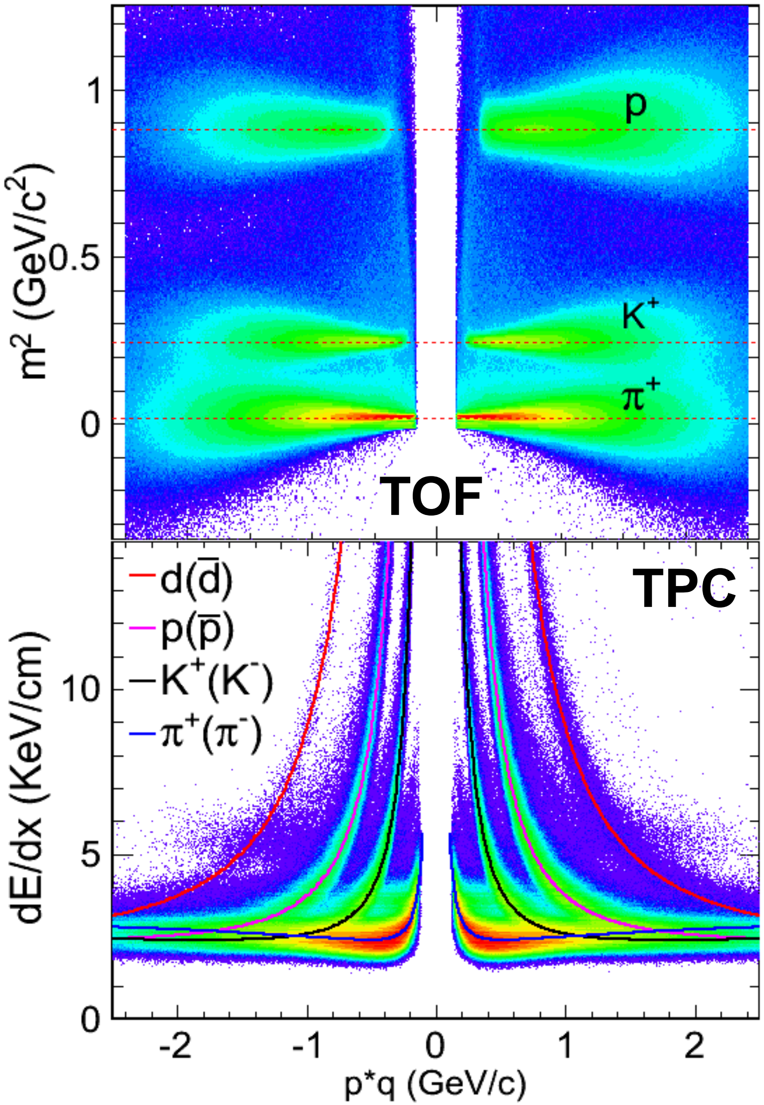
Need more statistics.

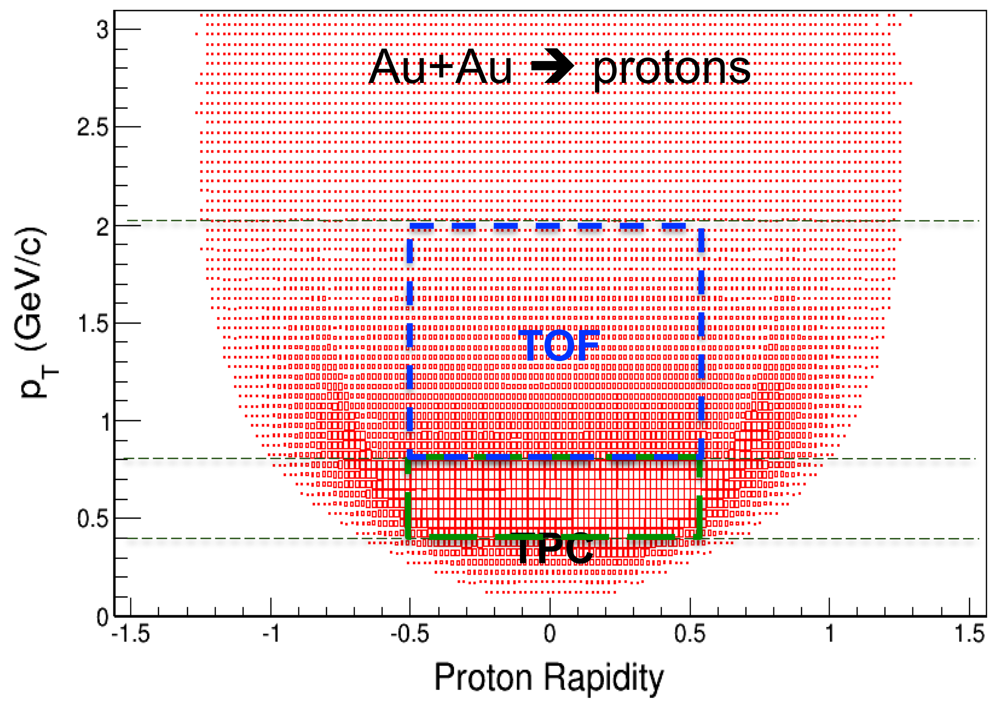
Poisson: $\kappa\sigma^2=1$

Net-proton: STAR: **PRL112**, 32302(14)
Net-charge: STAR: **PRL113**,092301(14)

New Net-proton results: Larger p_T Acceptance

TOF is used for Identify p/pbar in addition with TPC to extend the p_T coverage.





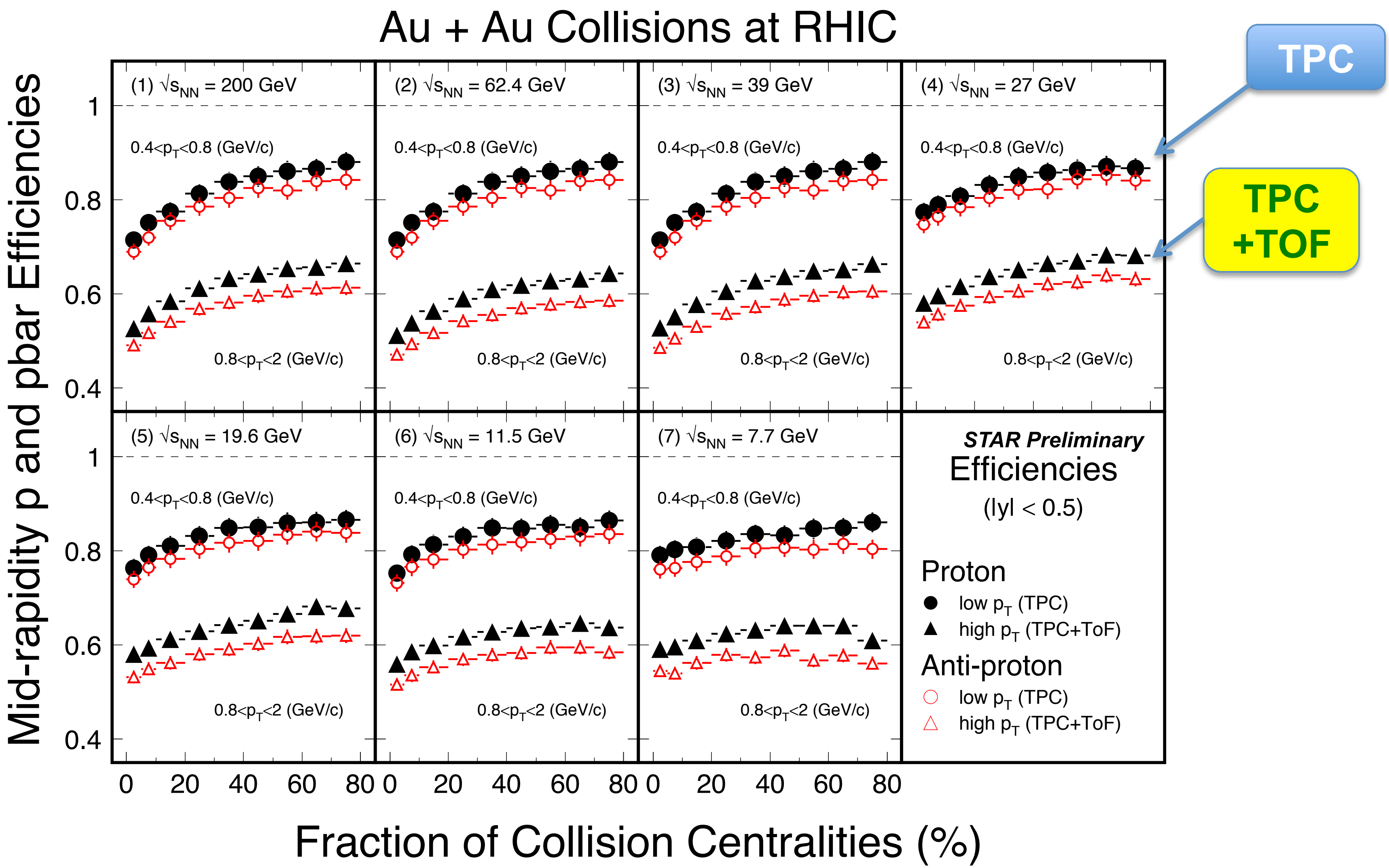
Acceptance: $|y| \leq 0.5$, $0.4 \leq p_T \leq 2$ GeV/c

Efficiency corrections:

TPC ($0.4 \leq p_T \leq 0.8$ GeV/c): $\varepsilon_{TPC} \sim 0.8$

TPC+TOF ($0.8 \leq p_T \leq 2$ GeV/c): $\varepsilon_{TPC} * \varepsilon_{TOF} \sim 0.5$

Efficiencies for Protons and Anti-protons

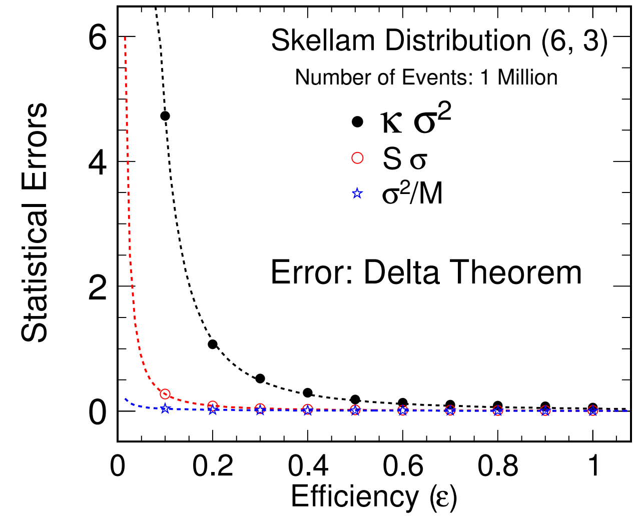


Systematic errors: 1) Uncertainties on efficiency, 2) PID, 3) Track Cuts.

Efficiency Correlation and Error Estimation

We provide a unified description of efficiency correction and error estimation for higher moments analysis in heavy-ion collisions.

$$F_{r_1,r_2}(N_p, N_{\bar{p}}) = F_{r_1,r_2}(N_{p_1}+N_{p_2}, N_{\bar{p}_1}+N_{\bar{p}_2})$$
$$= \sum_{i_1=0}^{r_1}\sum_{i_2=0}^{r_2} s_1(r_1,i_1)s_1(r_2,i_2) < (N_{p_1}+N_{p_2})^{i_1}(N_{\bar{p}_1}+N_{\bar{p}_2})^{i_2} >$$
$$= \sum_{i_1=0}^{r_1}\sum_{i_2=0}^{r_2} s_1(r_1,i_1)s_1(r_2,i_2) < \sum_{s=0}^{i_1}\binom{i_1}{s} N_{p_1}^{i_1-s}N_{p_2}^{s} \sum_{t=0}^{i_2}\binom{i_2}{t} N_{\bar{p}_1}^{i_2-t}N_{\bar{p}_2}^{t} >$$
$$= \sum_{i_1=0}^{r_1}\sum_{i_2=0}^{r_2}\sum_{s=0}^{i_1}\sum_{t=0}^{i_2} s_1(r_1,i_1)s_1(r_2,i_2)\binom{i_1}{s}\binom{i_2}{t} < N_{p_1}^{i_1-s}N_{p_2}^{s}N_{\bar{p}_1}^{i_2-t}N_{\bar{p}_2}^{t} >$$
$$= \sum_{i_1=0}^{r_1}\sum_{i_2=0}^{r_2}\sum_{s=0}^{i_1}\sum_{t=0}^{i_2}\sum_{u=0}^{i_1-s}\sum_{v=0}^{s}\sum_{j=0}^{i_2-t}\sum_{k=0}^{t} s_1(r_1,i_1)s_1(r_2,i_2)\binom{i_1}{s}\binom{i_2}{t}$$
$$\times\, s_2(i_1-s,u)s_2(s,v)s_2(i_2-t,j)s_2(t,k) \times F_{u,v,j,k}(N_{p_1},N_{p_2},N_{\bar{p}_1},N_{\bar{p}_2})$$



Fitting formula: $f(\varepsilon) = \frac{1}{\sqrt{n}}\frac{a}{\varepsilon^b}$

X. Luo, PRC91, 043907 (2015).

We can express the moments and cumulants in terms of the factorial moments, which can be easily efficiency corrected.

Assume Binomial response for efficiency

$$F_{u,v,j,k}(N_{p_1},N_{p_2},N_{\bar{p}_1},N_{\bar{p}_2}) = \frac{f_{u,v,j,k}(n_{p_1},n_{p_2},n_{\bar{p}_1},n_{\bar{p}_2})}{(\varepsilon_{p_1})^u(\varepsilon_{p_2})^v(\varepsilon_{\bar{p}_1})^j(\varepsilon_{\bar{p}_2})^k}$$

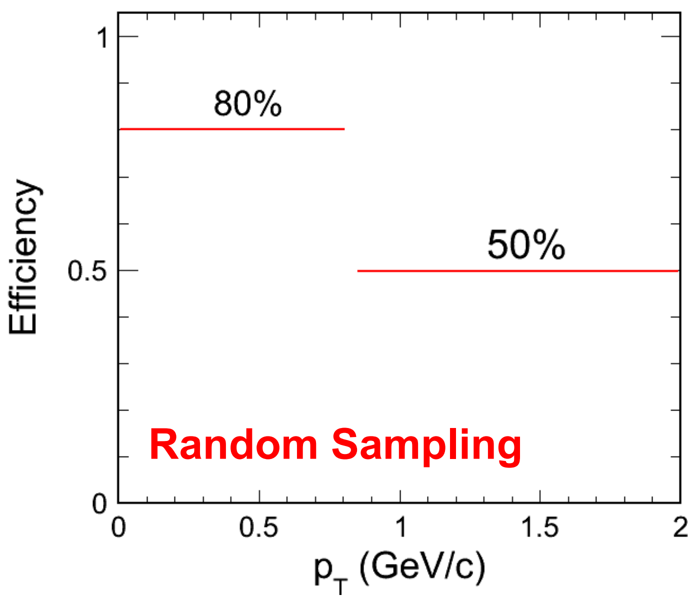
Also see:
A. Bzdak and V. Koch, PRC91,027901(2015), PRC86，044904(2012).

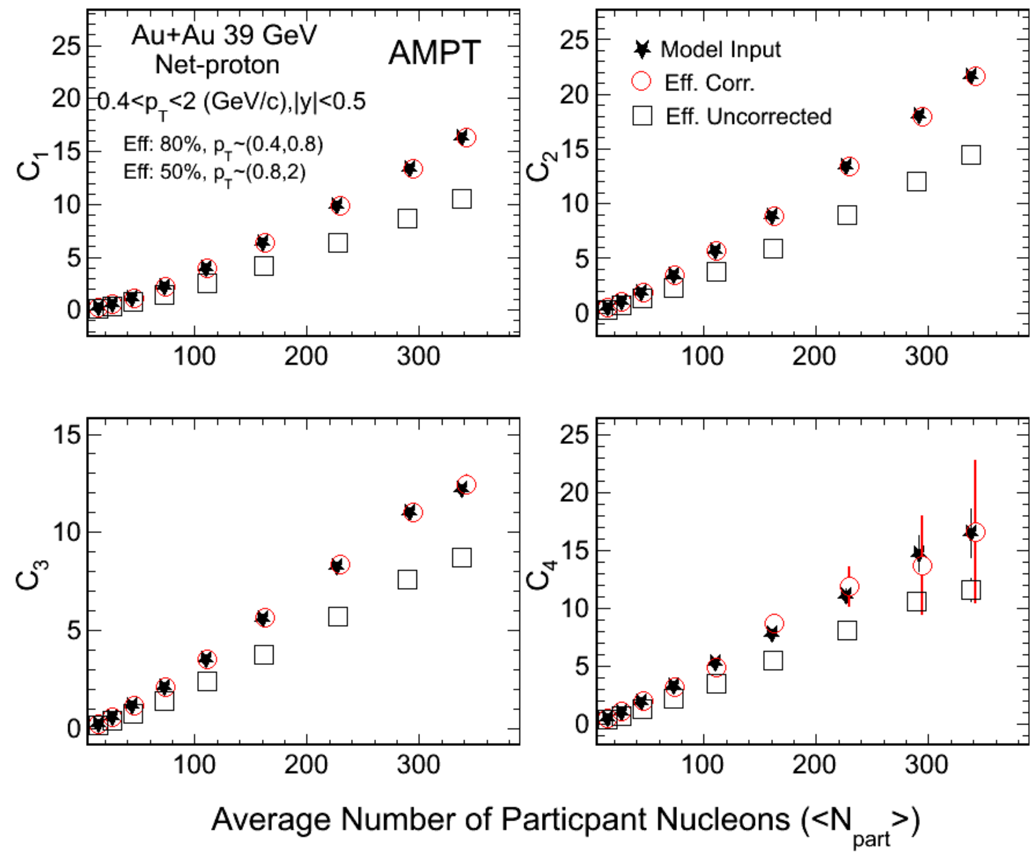
Verification of the Eff. Correction Method with Model

AMPT model: Au+Au 39 GeV.

X. Luo, PRC91, 043907 (2015).

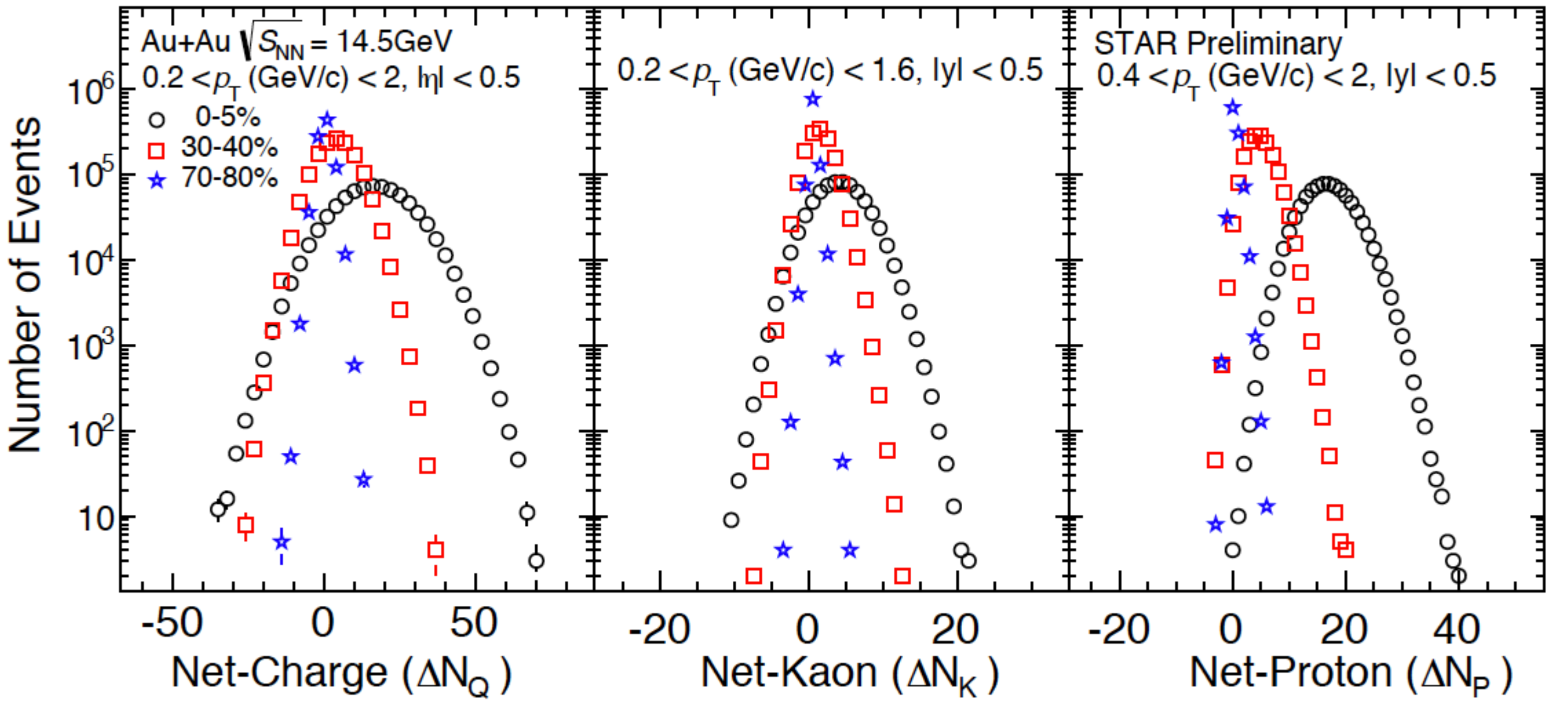
Set different efficiency for two p_T range. (0.4， 0.8) : 80%, (0.8， 2): 50% for p and pbar.





1. The eff. corrected results match the model inputs very well, which indicate the efficiency correction method works well.
2. The error estimation for eff. corrected results are based on the Delta theorem.

Statistical Errors Comparison Between for Net-Q, Net-P and Net-K



X. Luo, JPG 39, 025008 (2012).

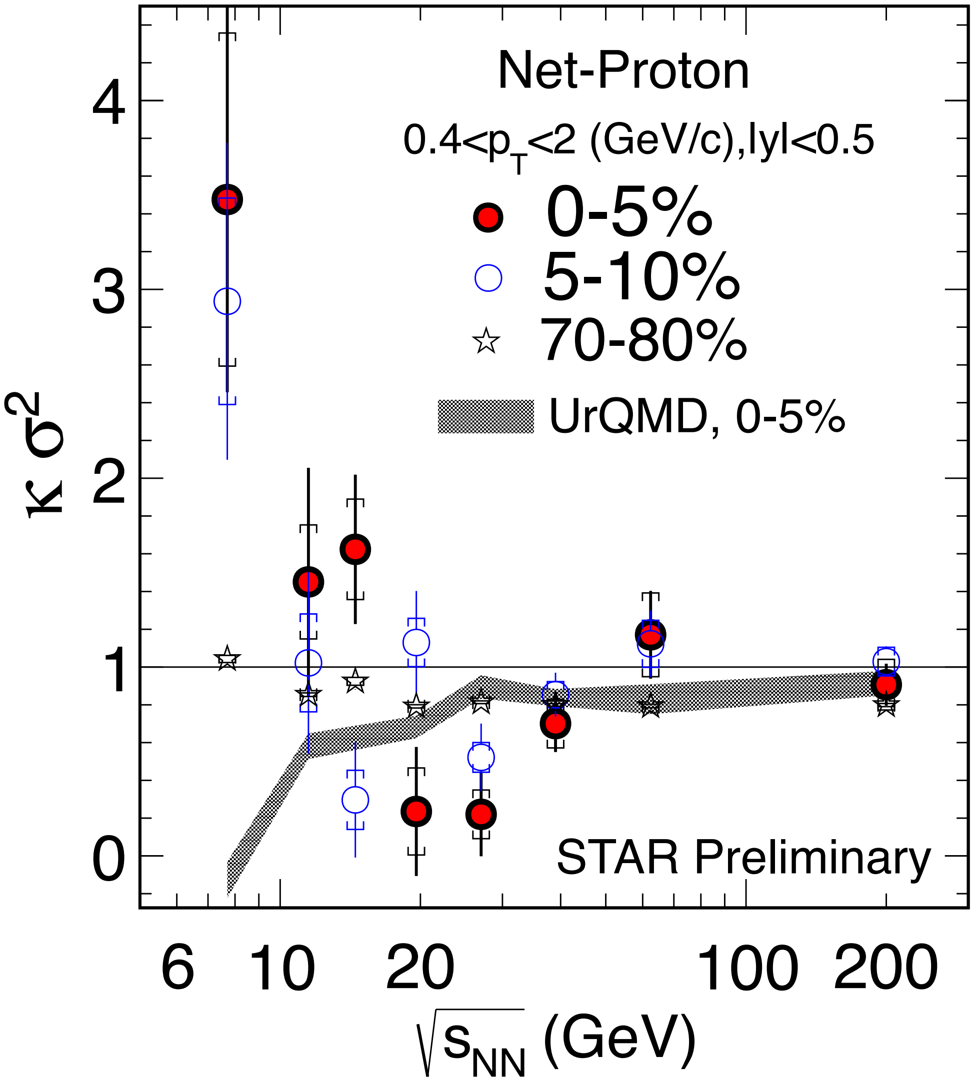
$$error(\kappa\sigma^2) \propto \frac{1}{\sqrt{N}}\frac{\sigma^2}{\varepsilon^2}$$

$$error(S\sigma) \propto \frac{1}{\sqrt{N}}\frac{\sigma}{\varepsilon^{3/2}}$$

14.5 GeV: 0-5%	Net-Charge (Q)	Net-proton (B)	Net-Kaon (K)
Typical Width (σ)	12.2	4.2	3.4
Aver. Efficiency (ε)	65%	75%	38%
σ^2 / ε^2	355	32	82

With the same # of events: error(Net-Q) > error(Net-K) > error (Net-P)

Energy Dependence for Net-proton $\kappa\sigma^2$

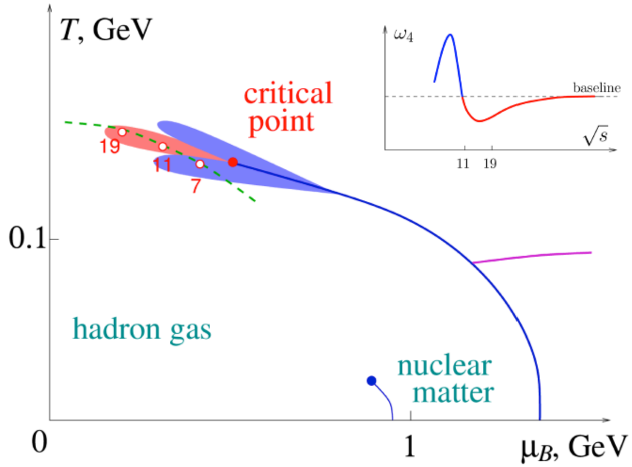


Net-proton as proxy for net-baryon.

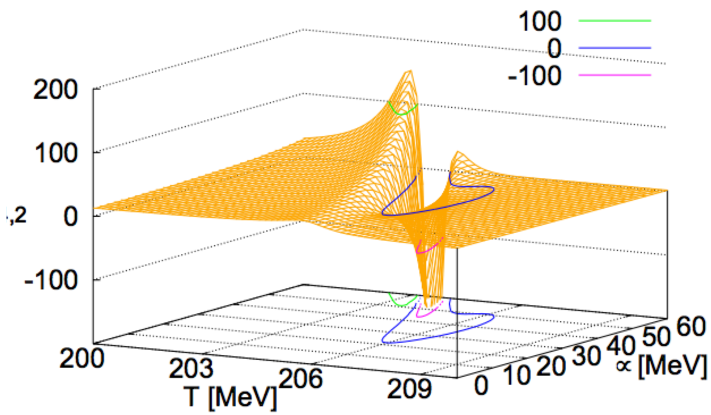
- Non-monotonic trend is observed for the 0-5% most central Au+Au collisions. Dip structure is observed around 19.6 GeV.
- Separation and flipping for the results of 0-5% and 5-10% centrality are observed at 14.5 and 19.6 GeV. (**Oscillation Pattern observed !**)
- UrQMD (no CP) results show suppression at low energies. Consistent with the effects of baryon number conservation.

Sign of Kurtosis :Model and Theoretical Calculations

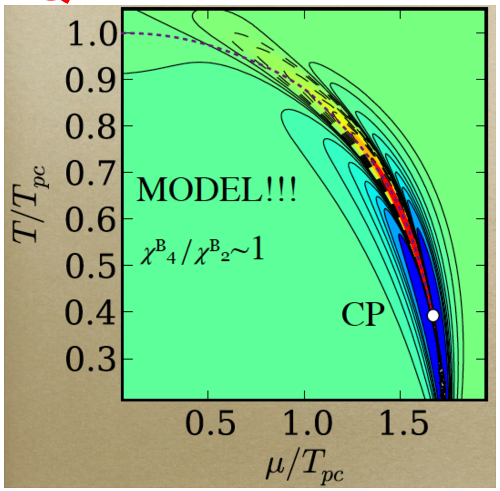
σ model M.A. Stephanov, PRL107, 052301 (2011).



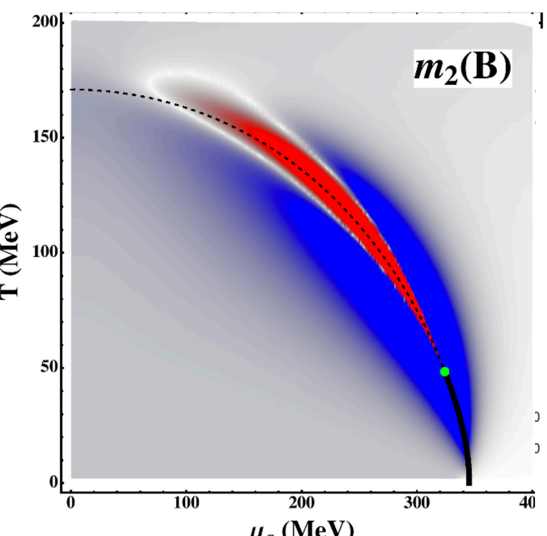
PQM Schaefer&Wanger, PRD 85, 034027 (2012)



PQM V. Skokov, QM2012

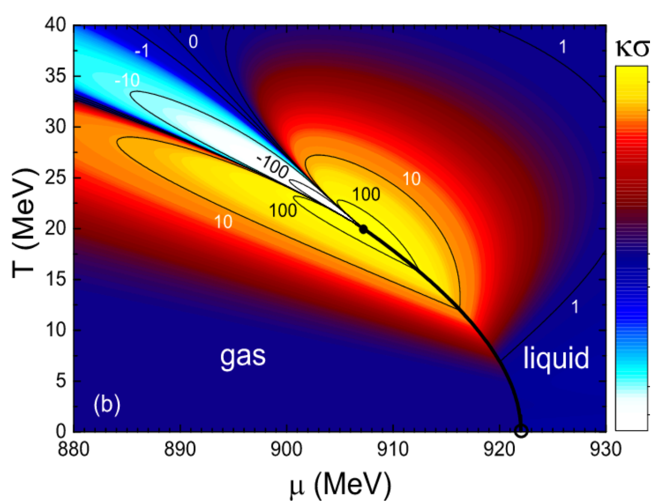


NJL



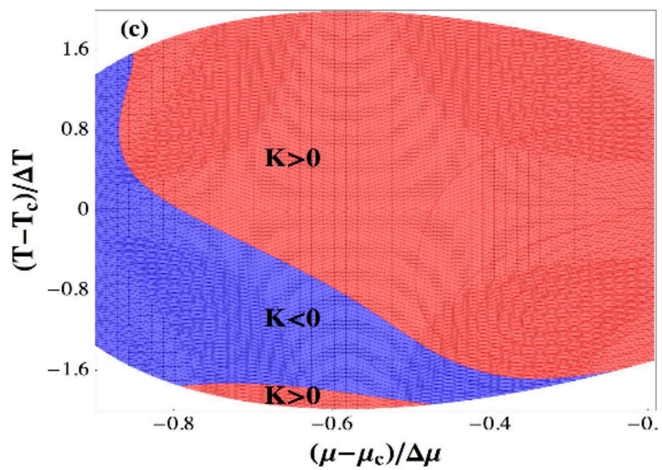
JW Chen et al., arXiv:1509.04968

VDW



Vovchenko et al., arXiv:1506.05763

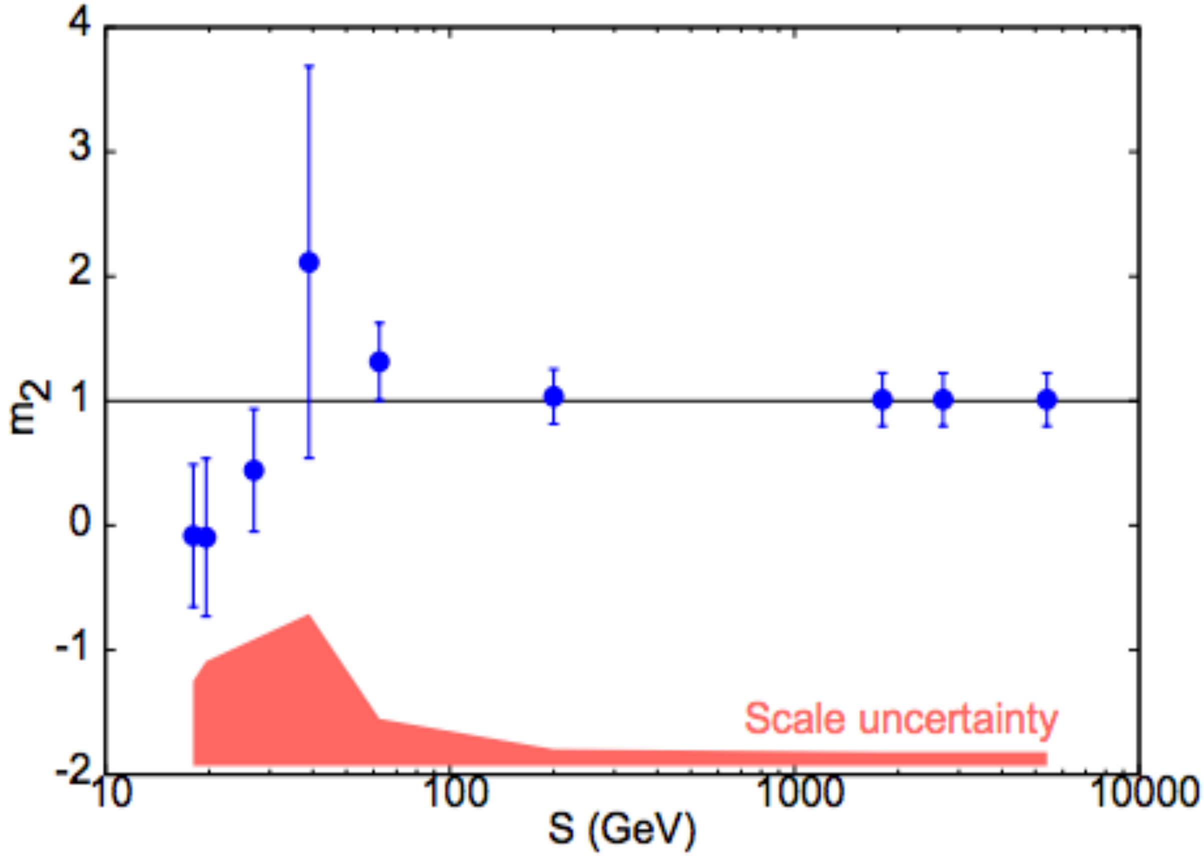
Memory Effects



Swagato, et al, PRC92,034912 (2015).

Lattice Calculation

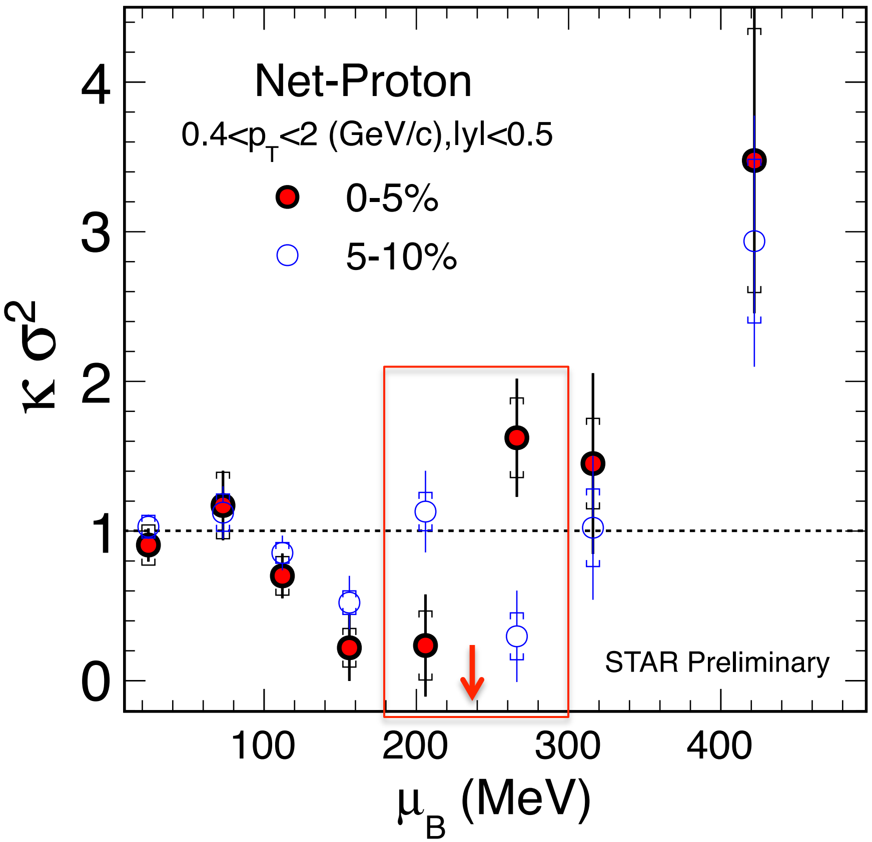
$$m_2 = \kappa\sigma^2$$

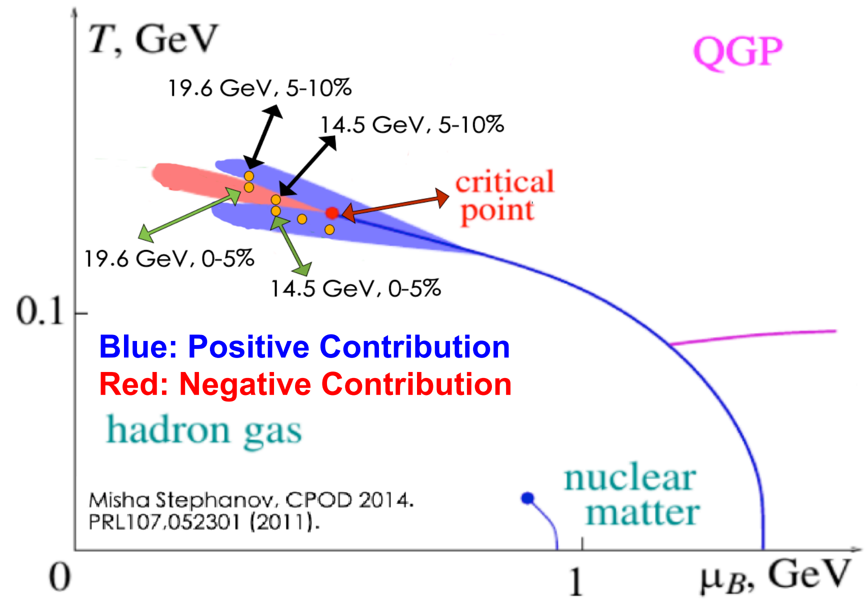


- $\mu^E_B/T^E \sim 1.7$.
- Large uncertainty will appear when approaching critical point.

S. Gupta et al., Lattice 2013

Oscillation Pattern: Signature of Critical Region ?





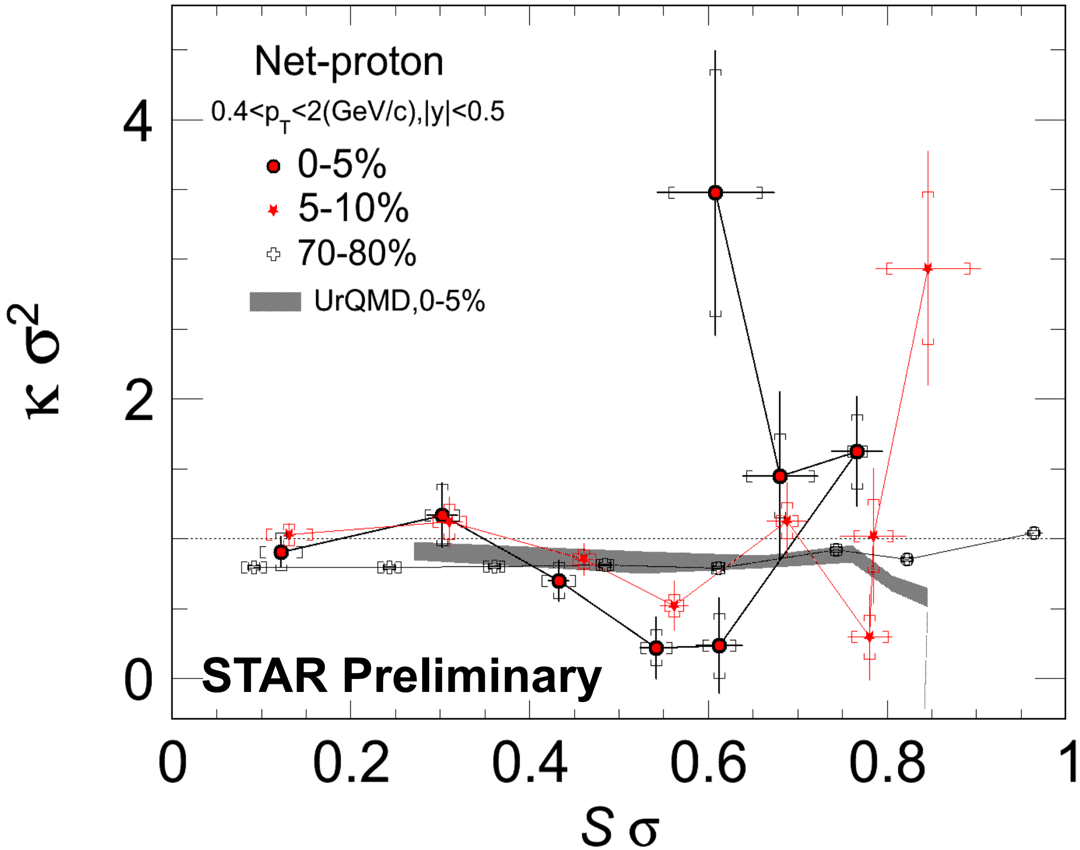
Depending on relative position between reaction trajectories/freeze out position and critical region.

"Oscillation pattern" around baseline for Kurtosis **may indicate a signature of critical region.**

$\kappa\sigma^2$	0-5%	5-10%
14.5 GeV	**1+Pos.**	**1+Neg.**
19.6 GeV	**1+Neg.**	**1+Pos.**

Propose to scan **16.5 GeV (μ_B =238 MeV)** or even finer step between 14.5 and 19.6 GeV, expect to see bigger dip and no separation for the results of the 0-5% and 5-10%.

$K\sigma^2$ Vs. $S\sigma$



Taylor expansion in Lattice :

THERMO-meter :

$$\kappa\sigma^2 \sim \frac{\chi_B^4}{\chi_B^2}(T,0)$$

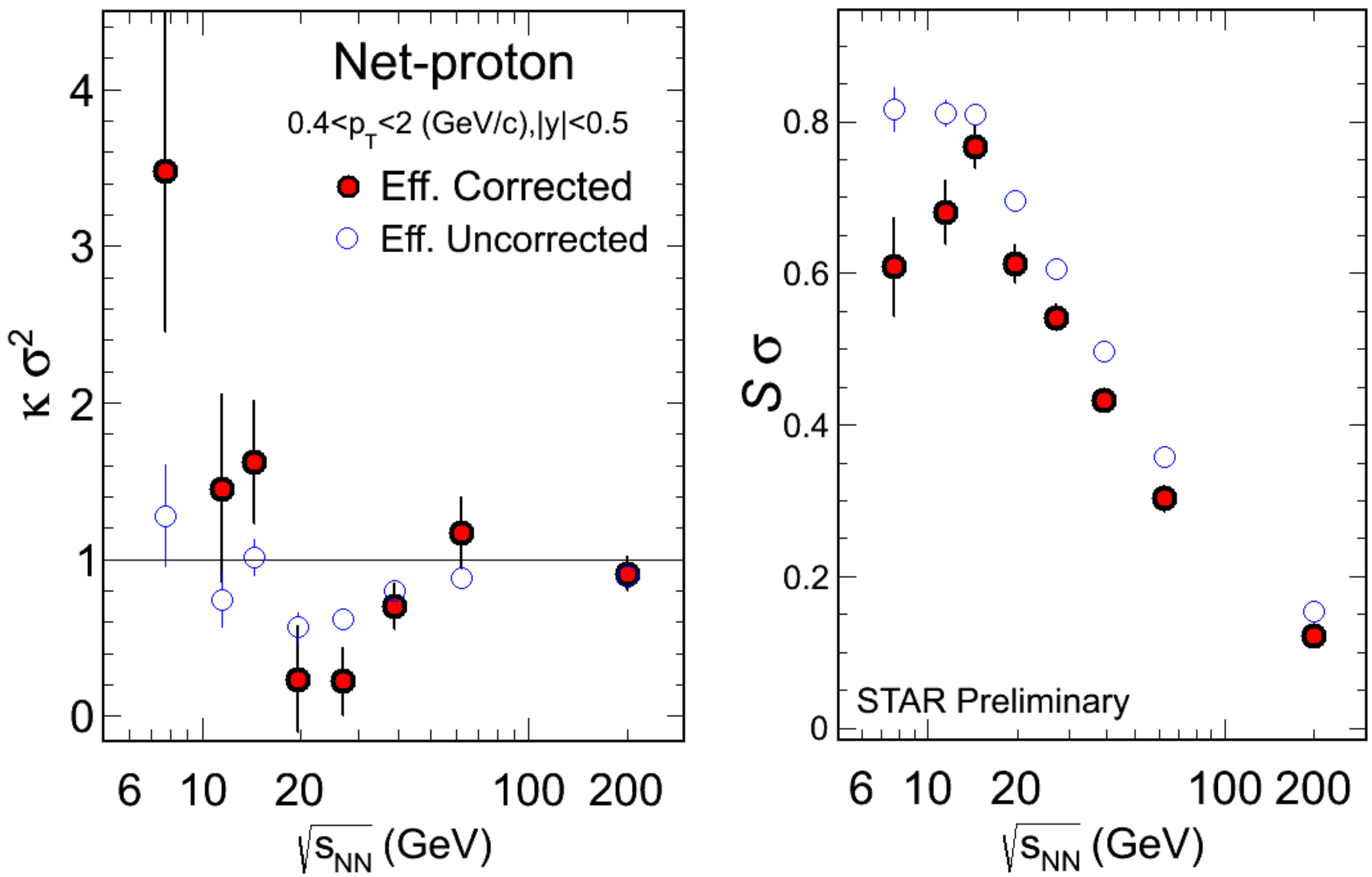
BARYO-meter :

$$S\sigma \sim \frac{\chi_B^3}{\chi_B^2}(T,\mu_B) \sim \tanh\left(\frac{\mu_B}{T}\right)$$

- A structure is observed for 0-5% most central data while it is flat for peripheral collisions.
- Can be directly compared with theoretical calculations.

Eff. Corrected and Eff. Uncorrected

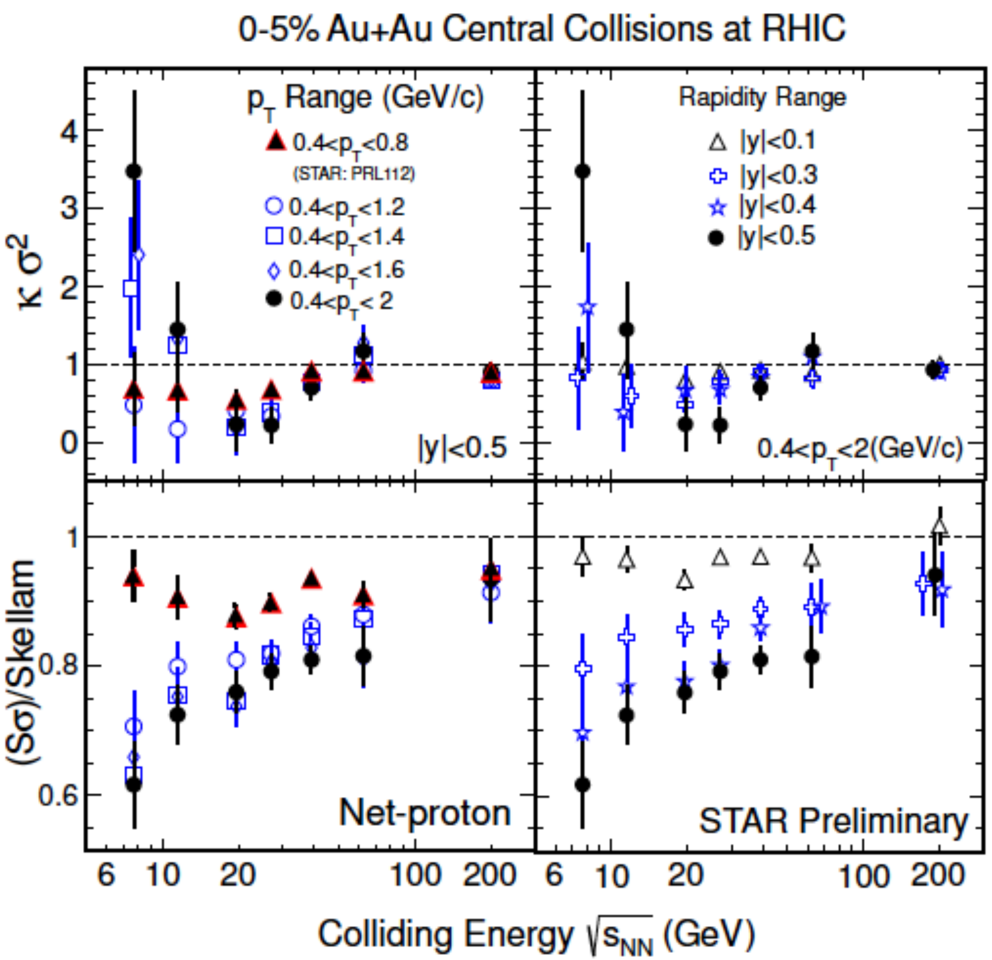
0-5% Au+Au Collisions at RHIC



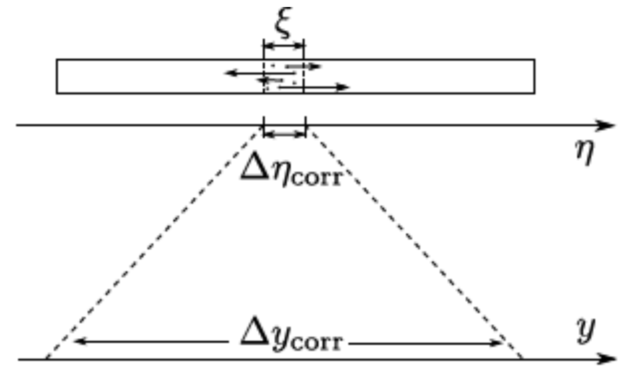
Efficiency corrections are important not only for the values in the higher moments analysis, but also the statistical errors.

$$error \propto O\left(\sigma^{n} / \varepsilon^{\alpha}\right)$$

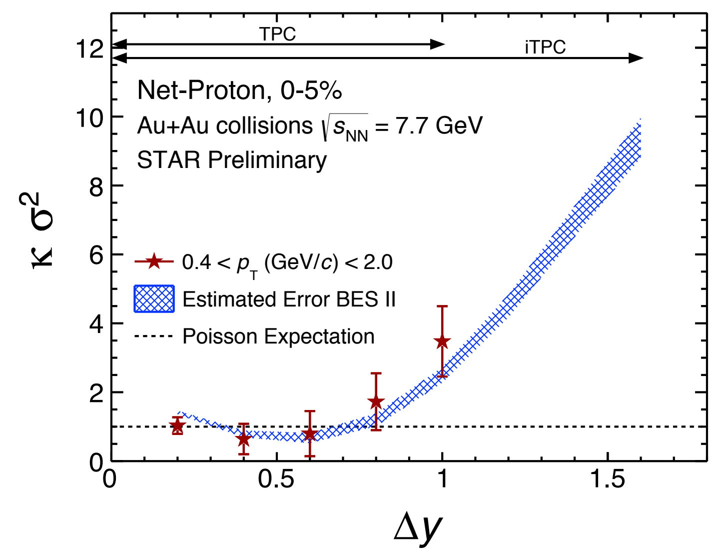
Acceptance Study: p_T and Rapidity



arXiv: 1503.02558

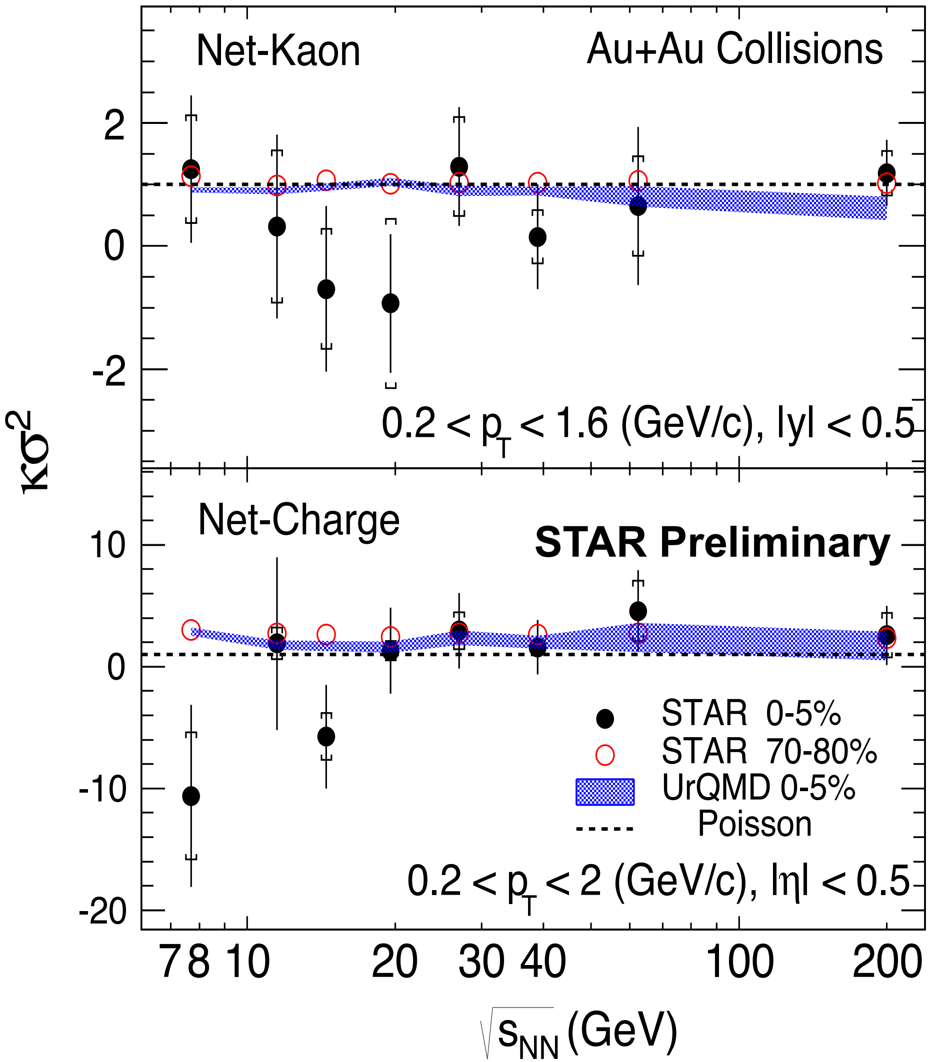


Bjorken rapidity $\Delta\eta_{corr}=\xi/\tau_f$~0.1-0.3
<< Kinematic rapidity Δy_{corr}~ 2 (Misha)



Significant p_T and rapidity dependence are observed at low energies.
Large acceptance is crucial for the fluctuation measurement.

STAR: Moments of Net-Charge and Net-Kaon Distributions



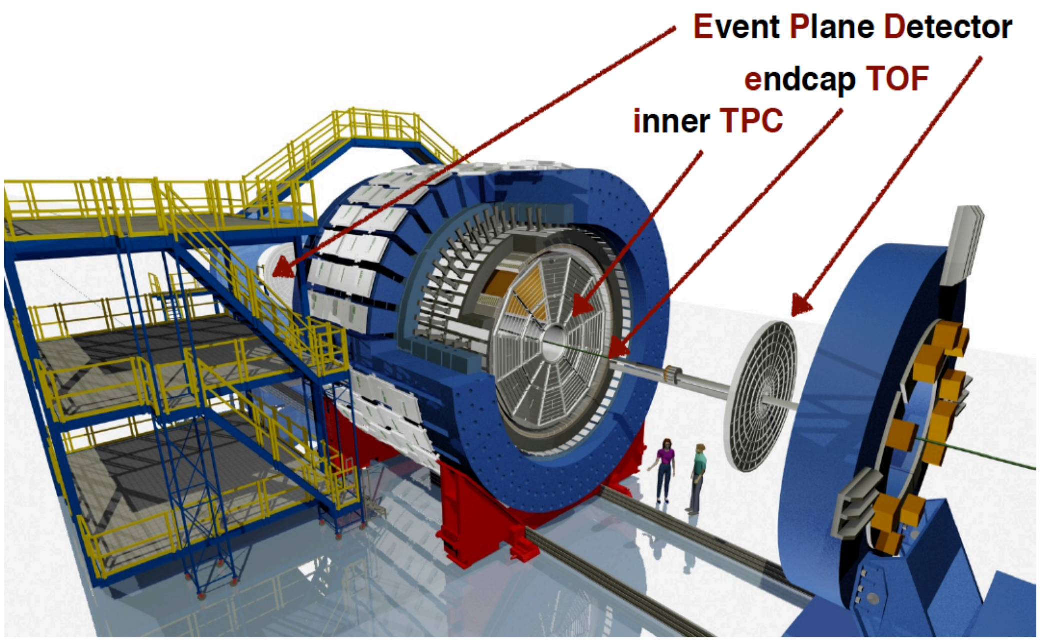
- Within current errors, Net-Kaon and Net-Charge $\kappa\sigma^2$ are consistent with unity.
- More statistics are needed to make a conclusion.
- UrQMD (no CP), show no energy dependent.

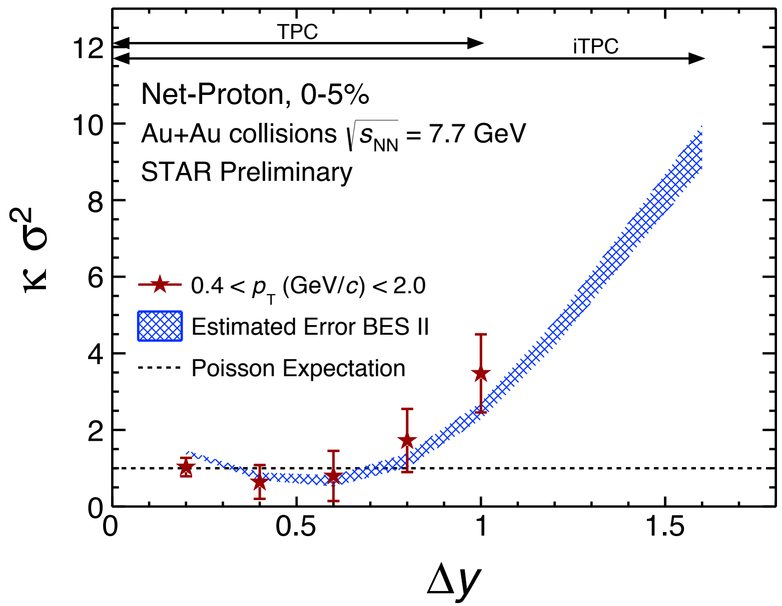
$$\boxed{error(\kappa\sigma^2) \propto \frac{1}{\sqrt{N}}\frac{\sigma^2}{\varepsilon^2}}$$

σ : Measured width of distributions.
ε : Efficiency.

In STAR, with the same # of events: error(Net-Q) > error(Net-K) > error (Net-P)

STAR Upgrades and BES Phase-II (2019-2020)





iTPC proposal: http://drupal.star.bnl.gov/STAR/starnotes/public/sn0619
BES-II whitepaper: http://drupal.star.bnl.gov/STAR/starnotes/public/sn0598

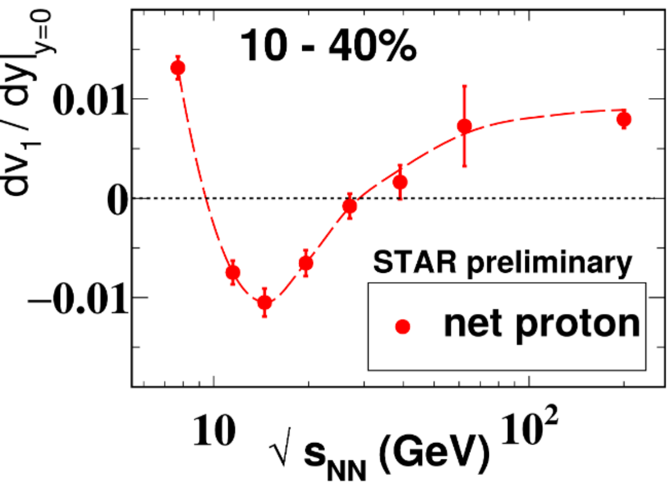
Larger rapidity acceptance crucial for further critical point search with net-protons

- Electron cooling upgrade will provide increased luminosity ~ 3-10 times.
- Inner TPC(iTPC) upgrade : |η| < 1 to | η |< 1.5. Better dE/dx resolution.
- Forward Event Plane Detector (EPD)： Centrality and Event Plane Determination. 1.8 < |η| < 4.5

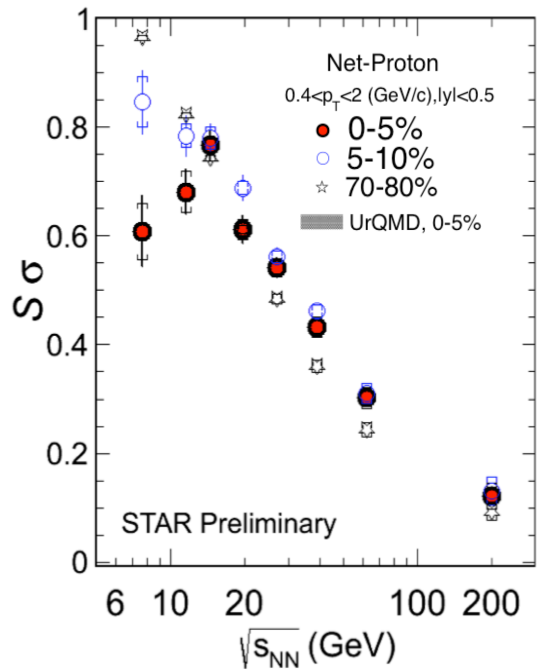
Summary

Intriguing structures are observed at low energies.

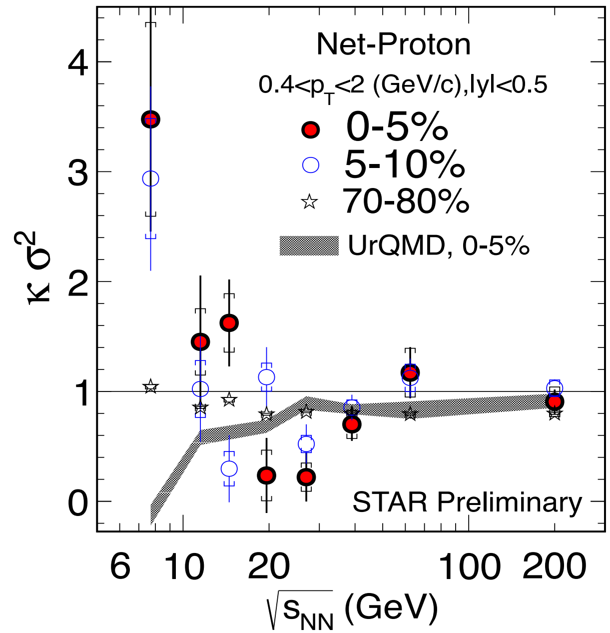
Dip



Peak



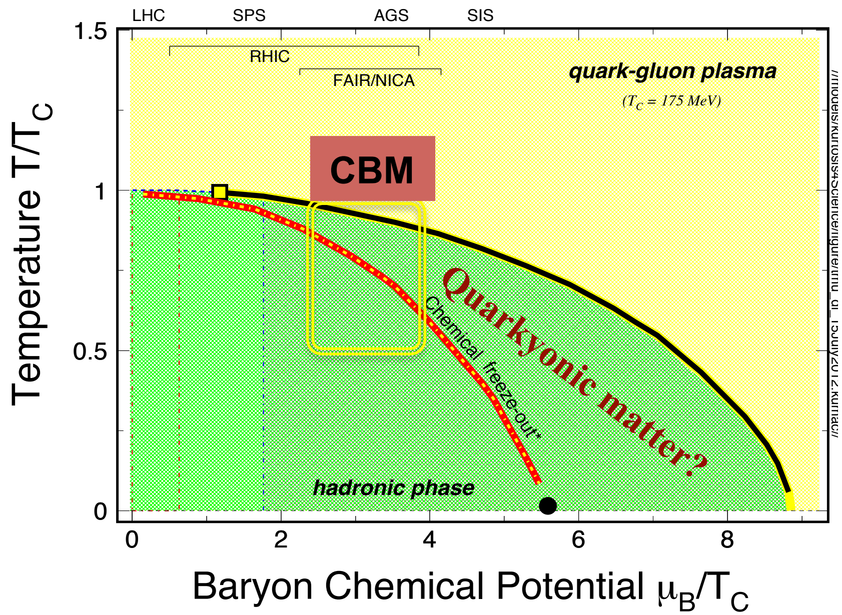
Oscillation



Discovery Potential at High Baryon Density:

First order phase transition and QCD Critical Point etc.

Experimental Study on Highly Compressed Baryonic Matter

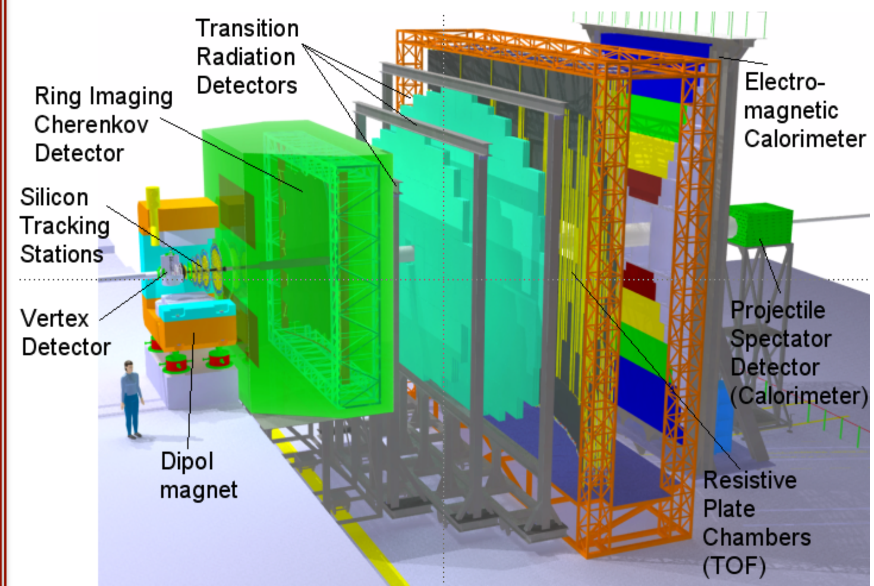


Exploring Phase Structure at High Baryon Density

（1） **QCD Critical Point**

（2） **Quarkyonic Phase/Phase Boundary**

Fixed Target Detector, CBM@FAIR



Center of Mass Energy $\sqrt{s_{NN}} \leqslant 12$ GeV per nucleon pair.

It allows us to explore the QCD phase structure at high baryon density region with high precision !

It is time to discover the QCD Critical Point !

Thank you for your attention !