Search for the QCD Critical Point -
Fluctuations of Conserved Quantities in High
Energy Nuclear Collisions at RHIC

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Nov. 2-6, 2015
QCD Phase Diagram (Conjectured)

Very rich phase structure in the QCD phase diagram.


Large Uncertainties in determining phase structure by theory at finite $\mu_B$. 
The QCD Critical Point

- Singularity of EOS: Diverges of the thermodynamics quantities, such as correlation length ($\xi$), Susceptibilities ($\chi$), heat capacity ($C_V$).


Search for CP in Heavy-ion Collisions
1. finite size/time. $\xi=2 \sim 3$ fm
2. Non-CP physics effects.
3. Need CFO closer enough to CP.
4. Signal didn’t wash out after evolution.

Very Challenging !!!
Location of QCD Critical Point: Theory

**Lattice QCD:**

1) Reweighting: Fodor & Katz, 2004:
\[ \mu_B^E/T^E \approx 2.2 \Rightarrow \sqrt{s_{NN}} \approx 9.5 \text{ GeV} \]

2) Tylor Expansion: Gavai & Gupta, 2013
\[ \mu_B^E/T^E \approx 1.7 \Rightarrow \sqrt{s_{NN}} \approx 14.5 \text{ GeV} \]

\[ \sqrt{s_{NN}} = 6 \sim 14.5 \text{ GeV}, \quad \mu_B^E = 266 \sim 496 \text{ MeV} \]

**DES:**

\[ \mu_B^E/T^E \approx 2.88 \Rightarrow \sqrt{s_{NN}} \approx 8 \text{ GeV} \]

2) C. S. Fischer et al., PRD90, 034022 (2014).
\[ \mu_B^E/T^E \approx 4.4 \Rightarrow \sqrt{s_{NN}} \approx 6 \text{ GeV} \]
Observables: Higher Moments (fluctuations)

\[ C_{2,x} \sim \xi^2 \quad C_{3,x} \sim \xi^{4.5} \quad C_{4,x} \sim \xi^7 \]

“Shape” of the fluctuations can be measured: non-Gaussian moments.

\[ C_{1,x} = \langle x \rangle, C_{2,x} = \langle (\delta x)^2 \rangle, \]
\[ C_{3,x} = \langle (\delta x)^3 \rangle, C_{4,x} = \langle (\delta x)^4 \rangle - 3 \langle (\delta x)^2 \rangle^2 \]

B. Friman et al., EPJC 71 (2011) 1694.

➤ Susceptibility ratios ⇔ Moments of Conserved Charges ⇔ Cumulant Ratios

\[ \frac{\chi_q^4}{\chi_q^2} = \kappa \sigma^2 = \frac{C_{4,q}}{C_{2,q}} \quad \frac{\chi_q^3}{\chi_q^2} = S \sigma = \frac{C_{3,q}}{C_{2,q}} \]

(q=B, Q, S)
If proton and anti-proton are independent Poissonian distributions, the distributions of net-protons is **Skellam distributions**, which is the case in Hadron Resonance Gas Model.

\[
P(N) = \left( \frac{N}{N_p} \right)^{N/2} I_N \left( 2\sqrt{N\bar{p}N_p} \right) e^{-(N_{\bar{p}} + N_p)}
\]

\(N_{\bar{p}}\) : Mean number of anti-protons  
\(N_p\) : Mean number of protons

**Contribute from statistical/thermal fluctuations.**

Then we have the skellam expectations for various moments/cumulants:

\[
C_{2n} = N_p + N_{\bar{p}} \\
C_{2n-1} = N_p - N_{\bar{p}}, (n = 1, 2, 3, ...)
\]

\[
S\sigma = \frac{C_3}{C_2} = \frac{N_p - N_{\bar{p}}}{N_p + N_{\bar{p}}}, k\sigma^2 = \frac{C_4}{C_2} = 1
\]
## RHIC Beam Energy Scan-Phase I

<table>
<thead>
<tr>
<th>√s (GeV)</th>
<th>Statistics(Millions) (0-80%)</th>
<th>Year</th>
<th>μ_B (MeV)</th>
<th>T (MeV)</th>
<th>μ_B /T</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.7</td>
<td>~4</td>
<td>2010</td>
<td>420</td>
<td>140</td>
<td>3.020</td>
</tr>
<tr>
<td>11.5</td>
<td>~12</td>
<td>2010</td>
<td>315</td>
<td>152</td>
<td>2.084</td>
</tr>
<tr>
<td>14.5</td>
<td>~20</td>
<td>2014</td>
<td>266</td>
<td>156</td>
<td>1.705</td>
</tr>
<tr>
<td>19.6</td>
<td>~36</td>
<td>2011</td>
<td>205</td>
<td>160</td>
<td>1.287</td>
</tr>
<tr>
<td>27</td>
<td>~70</td>
<td>2011</td>
<td>155</td>
<td>163</td>
<td>0.961</td>
</tr>
<tr>
<td>39</td>
<td>~130</td>
<td>2010</td>
<td>115</td>
<td>164</td>
<td>0.684</td>
</tr>
<tr>
<td>62.4</td>
<td>~67</td>
<td>2010</td>
<td>70</td>
<td>165</td>
<td>0.439</td>
</tr>
<tr>
<td>200</td>
<td>~350</td>
<td>2010</td>
<td>20</td>
<td>166</td>
<td>0.142</td>
</tr>
</tbody>
</table>

STAR Detector System

- EEMC
- Magnet
- MTD
- BEMC
- TPC
- TOF
- BBC

- Large, Uniform Acceptance at Mid-y
- Excellent Particle Identification
- Full TOF became available in 2010
First Order Phase Transition?

Indication of a First Order Phase Transition.

Methodology

1. Finite particle detection efficiency.

Need efficiency correction. It is not straight forward for higher moments.

2. Initial volume fluctuations.

Improve centrality resolution and apply centrality bin width correction.

3. Remove auto-correlation.

Particles used in the analysis are excluded in centrality definition.

4. Proper error calculations.

Delta theorem and Bootstrap

\[ \text{error} \propto O\left(\frac{\sigma^n}{\varepsilon^\alpha}\right) \]

STAR: PRL112, 32302(14)
STAR: PRL113, 092301(14)
X. Luo, JPG 39, 025008 (2012).
Higher Moments Results

Net-proton results:
All data show deviations below Poisson for \( \kappa \sigma^2 \) at all energies. Larger deviation at \( \sqrt{s_{NN}} \sim 20 \) GeV

Net-charge results:
Need more statistics.

\[ \text{Poisson: } \kappa \sigma^2 = 1 \]

Net-proton: STAR: \textbf{PRL112}, 32302(14)
Net-charge: STAR: \textbf{PRL113}, 092301(14)
New Net-proton results: Larger $p_T$ Acceptance

TOF is used for Identify $p/p\bar{p}$ in addition with TPC to extend the $p_T$ coverage.

Acceptance: $|y| \leq 0.5$, $0.4 \leq p_T \leq 2$ GeV/c

Efficiency corrections:
- TPC ($0.4 \leq p_T \leq 0.8$ GeV/c): $\epsilon_{TPC} \sim 0.8$
- TPC+TOF ($0.8 \leq p_T \leq 2$ GeV/c): $\epsilon_{TPC} \ast \epsilon_{TOF} \sim 0.5$
Efficiencies for Protons and Anti-protons

Au + Au Collisions at RHIC

Efficiencies

<table>
<thead>
<tr>
<th>Energy (GeV)</th>
<th>Proton Efficiency</th>
<th>Anti-proton Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.8</td>
<td>0.7</td>
</tr>
<tr>
<td>62.4</td>
<td>0.7</td>
<td>0.6</td>
</tr>
<tr>
<td>39</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>27</td>
<td>0.4</td>
<td>0.3</td>
</tr>
<tr>
<td>19.6</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>11.5</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>7.7</td>
<td>0.1</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Fraction of Collision Centralities (%)

Systematic errors: 1) Uncertainties on efficiency, 2) PID, 3) Track Cuts.
Efficiency Correlation and Error Estimation

We provide a unified description of efficiency correction and error estimation for higher moments analysis in heavy-ion collisions.

\[ F_{r_1,r_2}(N_{p_1}, N_{\bar{p}_1}) = F_{r_1,r_2}(N_{p_1} + N_{p_2}, N_{\bar{p}_1} + N_{\bar{p}_2}) \]
\[
= \sum_{i_1=0}^{r_1} \sum_{i_2=0}^{r_2} s_1(r_1, i_1)s_1(r_2, i_2) < (N_{p_1} + N_{p_2})^{i_1}(N_{\bar{p}_1} + N_{\bar{p}_2})^{i_2} > 
\]
\[
= \sum_{i_1=0}^{r_1} \sum_{i_2=0}^{r_2} s_1(r_1, i_1)s_1(r_2, i_2) \sum_{s=0}^{i_1} \binom{i_1}{s} N_{p_1}^{i_1-s} N_{p_2}^{s} \sum_{t=0}^{i_2} \binom{i_2}{t} N_{\bar{p}_1}^{i_2-t} N_{\bar{p}_2}^{t} > 
\]
\[
= \sum_{i_1=0}^{r_1} \sum_{i_2=0}^{r_2} \sum_{s=0}^{i_1} \sum_{t=0}^{i_2} s_1(r_1, i_1)s_1(r_2, i_2) \binom{i_1}{s} \binom{i_2}{t} < N_{p_1}^{i_1-s} N_{p_2}^{s} N_{\bar{p}_1}^{i_2-t} N_{\bar{p}_2}^{t} > 
\]
\[
= \sum_{i_1=0}^{r_1} \sum_{i_2=0}^{r_2} \sum_{s=0}^{i_1} \sum_{t=0}^{i_2} \sum_{u=0}^{s} \sum_{v=0}^{t} \sum_{j=0}^{s} \sum_{k=0}^{t} s_1(r_1, i_1)s_1(r_2, i_2) \binom{i_1}{s} \binom{i_2}{t} \times F_{u,v,j,k}(N_{p_1}, N_{p_2}, N_{\bar{p}_1}, N_{\bar{p}_2}) 
\]

We can express the moments and cumulants in terms of the factorial moments, which can be easily efficiency corrected.

Assume Binomial response for efficiency

\[ F_{u,v,j,k}(N_{p_1}, N_{p_2}, N_{\bar{p}_1}, N_{\bar{p}_2}) = \frac{f_{u,v,j,k}(n_{p_1}, n_{p_2}, n_{\bar{p}_1}, n_{\bar{p}_2})}{(\varepsilon_{p_1})^u(\varepsilon_{p_2})^v(\varepsilon_{\bar{p}_1})^j(\varepsilon_{\bar{p}_2})^k} \]

Also see:
A. Bzdak and V. Koch, 
PRC91,027901(2015), 
PRC86, 044904(2012).
Verification of the Eff. Correction Method with Model

AMPT model: Au+Au 39 GeV.
Set different efficiency for two $p_T$ range.
$(0.4, 0.8): 80\%, (0.8, 2): 50\%$ for $p$ and $p\bar{p}$.

1. The eff. corrected results match the model inputs very well, which indicate the efficiency correction method works well.
2. The error estimation for eff. corrected results are based on the Delta theorem.

Statistical Errors Comparison Between for Net-Q, Net-P and Net-K

\[ \text{error}(k\sigma^2) \propto \frac{1}{\sqrt{N}} \frac{\sigma^2}{\varepsilon^2} \]

\[ \text{error}(S\sigma) \propto \frac{1}{\sqrt{N}} \frac{\sigma}{\varepsilon^{3/2}} \]

X. Luo, JPG 39, 025008 (2012).

<table>
<thead>
<tr>
<th>14.5 GeV: 0-5%</th>
<th>Net-Charge (Q)</th>
<th>Net-proton (B)</th>
<th>Net-Kaon (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical Width (( \sigma ))</td>
<td>12.2</td>
<td>4.2</td>
<td>3.4</td>
</tr>
<tr>
<td>Aver. Efficiency (( \varepsilon ))</td>
<td>65%</td>
<td>75%</td>
<td>38%</td>
</tr>
<tr>
<td>( \sigma^2 / \varepsilon^2 )</td>
<td>355</td>
<td>32</td>
<td>82</td>
</tr>
</tbody>
</table>

With the same # of events: \( \text{error(Net-Q)} > \text{error(Net-K)} > \text{error (Net-P)} \)
Net-proton as proxy for net-baryon.

- Non-monotonic trend is observed for the 0-5% most central Au+Au collisions. Dip structure is observed around 19.6 GeV.
- Separation and flipping for the results of 0-5% and 5-10% centrality are observed at 14.5 and 19.6 GeV. (Oscillation Pattern observed!)
- UrQMD (no CP) results show suppression at low energies. Consistent with the effects of baryon number conservation.
Sign of Kurtosis: Model and Theoretical Calculations

\(\sigma\) model
M.A. Stephanov, PRL 107, 052301 (2011).

PQM
Schaefer&Wanger, PRD 85, 034027 (2012)

PQM
V. Skokov, QM2012

\(\chi^4/\chi^2 \sim 1\)

\(T/T_{pc}\)

\(\mu/T_{pc}\)

Memory Effects


\(\sigma\) model

NJL

VDW

M.A. Stephanov, PRL 107, 052301 (2011).

JW Chen et al., arXiv:1509.04968

Vovchenko et al., arXiv:1506.05763


Nov. 2-6

Xiaofeng Luo, EMMI Workshop 2015, GSI, Germany
\[ m_2 = \kappa \sigma^2 \]

- \[ \mu_B^E/T_E \approx 1.7. \]
- Large uncertainty will appear when approaching critical point.

**S. Gupta et al., Lattice 2013**
Oscillation Pattern: Signature of Critical Region?

"Oscillation pattern" around baseline for Kurtosis may indicate a signature of critical region.

Propose to scan 16.5 GeV ($\mu_B = 238$ MeV) or even finer step between 14.5 and 19.6 GeV, expect to see bigger dip and no separation for the results of the 0-5% and 5-10%.

<table>
<thead>
<tr>
<th>$\kappa\sigma^2$</th>
<th>0-5%</th>
<th>5-10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.5 GeV</td>
<td>1+Pos.</td>
<td>1+Neg.</td>
</tr>
<tr>
<td>19.6 GeV</td>
<td>1+Neg.</td>
<td>1+Pos.</td>
</tr>
</tbody>
</table>
A structure is observed for 0-5% most central data while it is flat for peripheral collisions.

Can be directly compared with theoretical calculations.
Efficiency corrections are important not only for the values in the higher moments analysis, but also the statistical errors.

\[ \text{error} \propto O\left(\frac{\sigma^n}{\varepsilon^\alpha}\right) \]
Acceptance Study: $p_T$ and Rapidity

Significant $p_T$ and rapidity dependence are observed at low energies. Large acceptance is crucial for the fluctuation measurement.

Bjorken rapidity $\Delta \eta_{corr} = \xi/\tau_f \sim 0.1 - 0.3$

<< Kinematic rapidity $\Delta y_{corr} \sim 2$ (Misha)

arXiv: 1503.02558
Within current errors, Net-Kaon and Net-Charge $\kappa\sigma^2$ are consistent with unity.

More statistics are needed to make a conclusion.

UrQMD (no CP), show no energy dependent.

\[
\text{error}(\kappa\sigma^2) \propto \frac{1}{\sqrt{N}} \frac{\sigma^2}{\varepsilon^2}
\]

$\sigma$: Measured width of distributions.

$\varepsilon$: Efficiency.

In STAR, with the same # of events: error(Net-Q) > error(Net-K) > error(Net-P)
STAR Upgrades and BES Phase-II (2019-2020)

Electron cooling upgrade will provide increased luminosity ~ 3-10 times.

Inner TPC(iTPC) upgrade: $|\eta| < 1$ to $|\eta| < 1.5$. Better dE/dx resolution.

Forward Event Plane Detector (EPD): Centrality and Event Plane Determination. $1.8 < |\eta| < 4.5$

Larger rapidity acceptance crucial for further critical point search with net-protons


Summary

Intriguing structures are observed at low energies.

Discovery Potential at High Baryon Density:
First order phase transition and QCD Critical Point etc.
Exploring Phase Structure at High Baryon Density
(1) QCD Critical Point
(2) Quarkyonic Phase/Phase Boundary

Fixing Target Detector, CBM@FAIR

Center of Mass Energy $\sqrt{s_{NN}} \leq 12$ GeV per nucleon pair.

It allows us to explore the QCD phase structure at high baryon density region with high precision!
It is time to discover the QCD Critical Point!

Thank you for your attention!