Dynamical description of fluctuations in heavy-ion collisions

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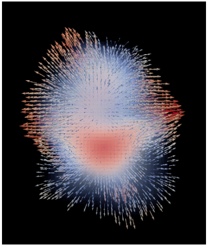
Transport models vs. grand-canonical ensemble

In a grand-canonical ensemble, the system is

- in thermal equilibrium ($=$ long-lifed),
- in equilibrium with a particle heat bath,
- spatially infinite
- and static.

Systems created in heavy-ion collisions are

- short-lived,
- spatially small,
- inhomogeneous,
- and highly dynamical!



plot by H. Petersen, madai.us

- Transport models take the microcanonical nature of individual particle scatterings into account.

Baryon-number conservation: toy model

Baryon-number conservation limits fluctuations of net-baryon number.

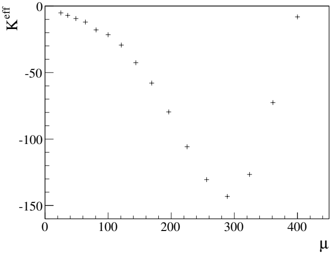
$$P_\mu(N,C) = \mathcal{N}(\mu,C)e^{-\mu}\frac{\mu^N}{N!} \quad \text{on} \quad [\mu - C, \mu + C]$$

μ: the expectation value of the original Poisson distribution, $\mathcal{N}(\mu,C)$: normalization factor, $C > 0$: cut parameter

$$C = \alpha\sqrt{\mu}\left(1 - \left(\frac{\mu}{N_{\text{tot}}}\right)^2\right).$$

$\alpha = 3$, $N_{\text{tot}} = 416$.

- An increase of the average net-baryon number does not lead to stronger fluctuations.
- At the upper limit of $N_{\text{tot}} = 416$ the distribution changes to a δ-function ($K_\delta^{\text{eff}} = 0$).

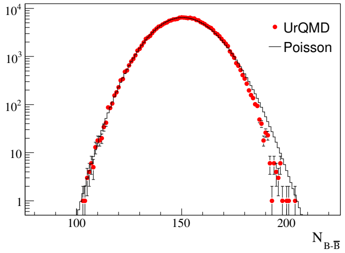


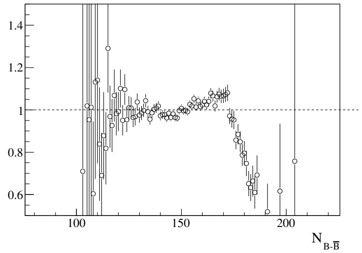
MN, T. Schuster, M. Mitrovski, R. Stock, M. Bleicher, EPJC72 (2012)

Net-baryon number distribution in UrQMD

- central Pb+Pb collisions at $E_{\mathrm{lab}} = 20A\mathrm{GeV}$
- fit to a Poisson distribution
- shoulders are enhanced
- tails are cut

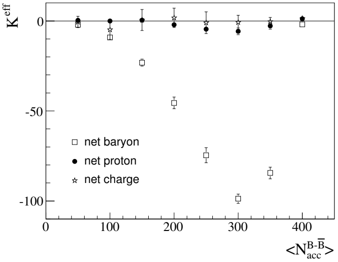
$\Longrightarrow$ decrease from $K^{\mathrm{eff}}_{\mathrm{Poisson}} = 1$ to $K^{\mathrm{eff}}_{\mathrm{UrQMD}} = -22.2$

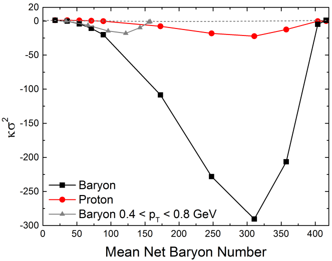




ratio of UrQMD to Poisson distribution

Rapidity window dependence of the effective kurtosis

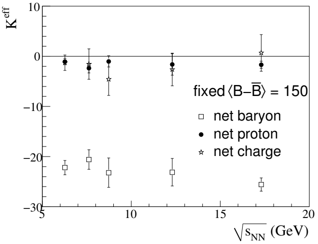


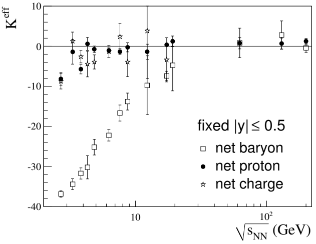


by J. Steinheimer

- Same qualitative behavior of the net-baryon kurtosis as expected from the toy model.
- $E_{\text{lab}} = 158A\text{GeV}$
- The net-proton kurtosis only slightly follows this trend.
- The net-charge kurtosis is not affected, but error bars are larger.

Energy dependence of the effective kurtosis





- adapting the rapidity window to fix the mean net-baryon number
- net-baryon effective kurtosis does not show an energy dependence
- for lower $\sqrt{s}$ K^{eff} becomes increasingly negative
- at $E_{\mathrm{lab}} = 2A$GeV: $\langle N_{B-\bar{B}} \rangle \simeq 240$

Difficulties in the transport calculations

- large statistics are needed (more than in our old analysis)
- very involved experimental analysis - crucial to repeat (some of) the same procedures
- error estimates in most theory calculations not very accurate

+ Skewness?
+ p_T-range dependence?
+ Fluctuations of strangeness? (Why do they not show the conservation laws?)

\- There is no phase transition in UrQMD...

Fluctuations at the phase transition

At a critical point

- correlation length of fluctuations of the order parameter diverges $\xi \to \infty$
- fluctuations of the order parameter diverge: $\langle \Delta\sigma^n \rangle \propto \xi^\alpha$ with higher powers of divergence for higher moments
- mean-field studies in Ginzburg-Landau theories, beyond mean-field: renormalization group
- relaxation time diverges $\Rightarrow$ critical slowing down!

$\Rightarrow$ fluctuations in equilibrated systems!

... and a first-order PT:

- at T_c coexistence of two stable thermodynamic phases
- metastable states above and below T_c $\Rightarrow$ supercooling and -heating
- nucleation and spinodal decomposition in nonequilibrium
- domain formation and large inhomogeneities

$\Rightarrow$ fluctuations in nonequilibrium!

... but also at the crossover:

- remnant of the $O(4)$ universality class in the chiral limit.

$\Rightarrow$ fluctuations in equilibrated systems!

Ref. by many people: M. Stephanov, E. Shuryak, K. Rajagopal, L. Csernai, J. Randrup, I. Mishustin, C. Sasaki, B. Friman, K. Redlich et al.

Nonequilibrium chiral fluid dynamics (NχFD)

IDEA: combine the dynamical propagation of fluctuations at the phase transition with fluid dynamical expansion!

(model-independent is nice, but in the end some real input is needed...)

- Langevin equation for the sigma field: damping and noise from the interaction with the quarks

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g\rho_s + \eta \partial_t \sigma = \xi$$

- Phenomenological dynamics for the Polyakov-loop

$$\eta_\ell \partial_t \ell T^2 + \frac{\partial V_{\mathrm{eff}}}{\partial \ell} = \xi_\ell$$

- Fluid dynamical expansion of the quark fluid = heat bath, including energy-momentum exchange

$$\partial_\mu T^{\mu\nu}_{\mathrm{q}} = S^\nu = -\partial_\mu T^{\mu\nu}_\sigma \,, \quad \partial_\mu N^\mu_{\mathrm{q}} = 0$$

$\Rightarrow$ includes a stochastic source term!

MN, S. Leupold, I. Mishustin, C. Herold, M. Bleicher, PRC **84** (2011); PLB **711** (2012); JPG **40** (2013)
C. Herold, MN, I. Mishustin, M. Bleicher PRC **87** (2013); NPA**925** (2014), C. Herold, MN, Y. Yan, C. Kobdaj JPG **41** (2014)

Dynamical slowing down

Phenomenological equation: $\frac{d}{dt} m_\sigma(t) = -\Gamma[m_\sigma(t)](m_\sigma(t) - \frac{1}{\xi_{eq}(t)})$
with input from the dynamical universality class $\Rightarrow \xi \sim 1.5 - 2.5$ fm
B. Berdnikov and K. Rajagopal, PRD **61** (2000))

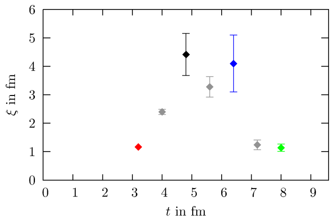
More advanced: Expanded Fokker-Planck dynamics by S. Mukherjee et al. 1506.00645

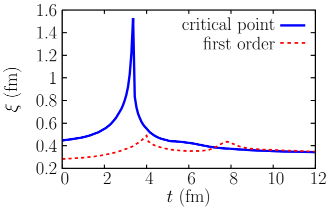
$$G(r) = \int d^3x d^3y \langle \sigma(x) - \sigma_0 \rangle \langle \sigma(y) - \sigma_0 \rangle$$
$$\sim \exp(-r/\xi)$$

Assume σ_0 is the volume averaged field.

From the curvature of $V_{\rm eff}$:

$$\langle \xi^2 \rangle = \langle 1/m_\sigma^2 \rangle = \left\langle \left(\frac{d^2 V_{\rm eff}}{d\sigma^2} \right)^{-1} \right\rangle$$





C. Herold, MN, I. Mishustin, M. Bleicher PRC **87** (2013)

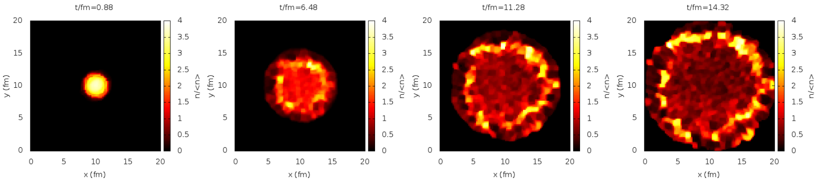
Definition of ξ in inhomogeneous systems involves averaging!

$\Rightarrow$ Similar magnitude of $\xi \sim 1.5 - 3$ fm!

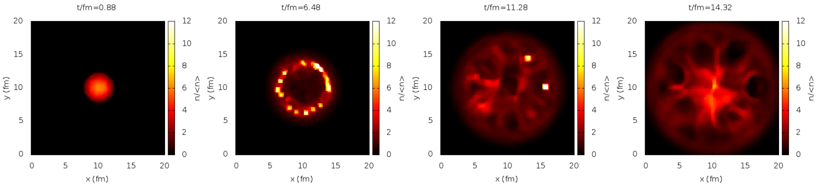
PQM vs. QH model: stability of droplets

See also: J. Steinheimer, J. Randrup, V. Koch PRC**89** (2014)

PQM EoS



QH EoS

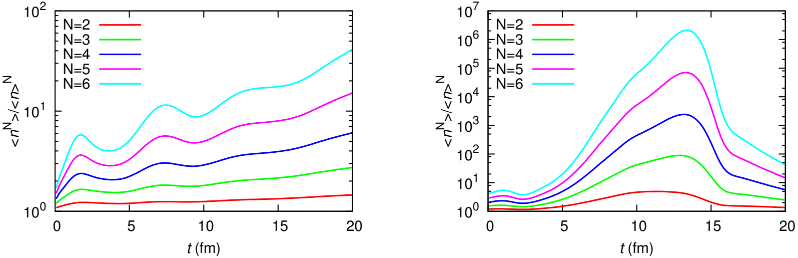


- Dynamical and stochastic droplet formation at the phase transition and subsequent decay in the hadronic phase.

PQM vs. QH model: moments of net-baryon density

Define normalized moments of the net-baryon density distribution as:

$$\langle n^N \rangle = \int \mathrm{d}^3x n(x)^N P_n(x) \quad \text{with} \quad P_n(x) = \frac{n(x)}{\int \mathrm{d}^3 x n(x)}$$



PQM EoS

QH EoS

- Infinite increase in the PQM.
- Increase in the HQ model around the phase transition followed by a rapid decrease due to pressure in the hadronic phase!
- REMEMBER: We started with smooth initial conditions and all inhomogeneities are formed dynamically!

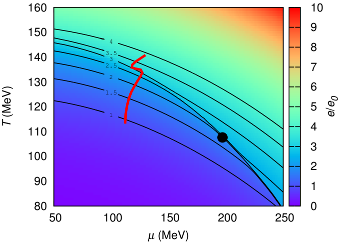
From densities fluctuations to particle fluctuations

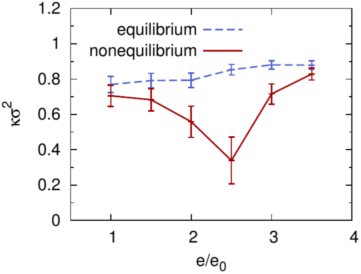
- From densities to particle via Cooper-Frye particlization: $E\frac{\mathrm{d}N_i}{\mathrm{d}^3p} = \int \mathrm{d}\sigma^\mu p_\mu (f_i^{\mathrm{eq}}(p) + \delta f)$
- Sampled from a single-particle distribution function $\Rightarrow$ (partially) destroys correlations.
- Modify the single-particle distribution functions or the entire particlization procedure?

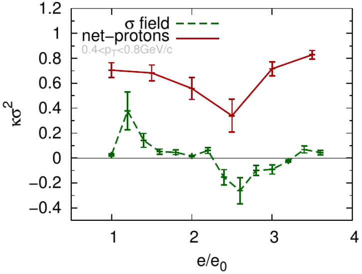
For single-particle distribution : $m_i \to m_{i,0} + g\sigma(x)$

First net-proton fluctuations from NχFD

- Here: Only couple the densities of the order parameter field with the fluid dynamical densities.







UrQMD initial conditions rescaled to the EoS of the effective model.

The critical mode

- At $\mu_B \neq 0$ σ mixes with the net-baryon density n (and e and $\vec{m}$)
- In a Ginzburg-Landau formalism:

$$V(\sigma, n) = \int \mathrm{d}^3x (\sum_m (a_m\sigma^m + b_m n^m) + \sum_{m,l} c_{m,l}\sigma^m n^l) - h\sigma - jn$$

- $V(\sigma, n)$ is flat in $(a\sigma, bn)$ direction
- Equations of motion (including symmetries in $V(\sigma, n)$):

$$\partial_t^2\sigma = -\Gamma\delta V/\delta\sigma + \ldots$$
$$\partial_t n = \gamma\vec{\nabla}^2\delta V/\delta n + \ldots$$

- two time scales (with diffusion $D \to 0$ at the critical point)

$$\omega_1 \propto -i\Gamma a$$
$$\omega_2 \propto -i\gamma D\vec{q}^2$$

- The diffusive mode becomes the critical mode in the long-time dynamics. These fluctuations need to be included at the critical point!

H. Fuji, M. Ohtani PRD**70** (2004); M. Stephanov, D. Son PRD**70** (2004)

Fluid dynamical fluctuations

Conventional fluid dynamics propagates thermal averages of the energy density, pressure, velocities, charge densities, etc.

However, ...

- ... already in equilibrium there are thermal fluctuations
- ... the fast processes, which lead to local equilibration also lead to noise!

Conventional ideal fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq}$$
$$N^{\mu} = N^{\mu}_{\rm eq}$$

Y. Minami, T. Kunihiro, PTP122 (2010); P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); J. Kapusta, B. Müller, M. Stephanov PRC**85** (2012); C. Chafin, T. Schäfer, PRA87 (2013); P. Romatschke, R. Young, PRA87 (2013); C. Young, J. Kapusta, C. Gale, S. Jeon, B. Schenke, PRC91 (2015)

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Conventional viscous fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\text{eq}} + \Delta T^{\mu\nu}_{\text{visc}}$$
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Fluctuating viscous fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \Delta T^{\mu\nu}_{\rm visc} + \Xi^{\mu\nu}$$
$$N^{\mu} = N^{\mu}_{\rm eq} + \Delta N^{\mu}_{\rm visc} + \mathrm{I}^{\mu}$$

Y. Minami, T. Kunihiro, PTP122 (2010); P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); J. Kapusta, B. Müller, M. Stephanov PRC**85** (2012); C. Chafin, T. Schäfer, PRA87 (2013); P. Romatschke, R. Young, PRA87 (2013); C. Young, J. Kapusta, C. Gale, S. Jeon, B. Schenke, PRC91 (2015)

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- $\langle T^{\mu\nu} T^{\mu\nu} \rangle$ give viscosities (Kubo-formula), consistently with dissipation-fluctuation theorem fluctuations need to be included as well!

Y. Minami, T. Kunihiro, PTP122 (2010); P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); J. Kapusta, B. Müller, M. Stephanov PRC**85** (2012); C. Chafin, T. Schäfer, PRA87 (2013); P. Romatschke, R. Young, PRA87 (2013); C. Young, J. Kapusta, C. Gale, S. Jeon, B. Schenke, PRC91 (2015)

Fluid dynamical fluctuations

The noise terms are such that averaged quantities exactly equal the conventional quantities:

$$\langle T^{\mu\nu} \rangle = T^{\mu\nu}_{\text{eq}} + \Delta T^{\mu\nu}_{\text{visc}} \qquad \text{with} \quad \langle \Xi^{\mu\nu} \rangle = 0$$

$$\langle N^{\mu} \rangle = N^{\mu}_{\text{eq}} + \Delta N^{\mu}_{\text{visc}} \qquad \text{with} \quad \langle \mathrm{I}^{\mu} \rangle = 0$$

The two formulations will, however, differ when one calculates correlation functions:

$$\langle T^{\mu\nu}(x) T^{\mu\nu}(x') \rangle$$

$$\langle N^{\mu}(x) N^{\mu}(x') \rangle$$

Fluid dynamical fluctuations

In linear response theory the retarded correlator

- $\langle T^{\mu\nu}(x)T^{\mu\nu}(x')\rangle$ gives the viscosities and
- $\langle N^{\mu}(x)N^{\mu}(x')\rangle$ the charge conductivities

via the dissipation-fluctuation theorem (Kubo-formula)!

It means that when dissipation is included also fluctuations need to be included!

CAUTION: If nonlinearities are included fluid dynamical fluctuations contribute to the transport coefficients!

$\Rightarrow$ absolut lower limit for the effective viscosity!

$\Rightarrow$ non-analytic contribution to τ_π, breakdown of gradient expansion!

P. Kovtun, G. D. Moore, P. Romatschke, PRD**84** (2011); C. Chafin, T. Schäfer, PRA**87** (2013); P. Romatschke, R. E. Young, PRA**87** (2013)

Fluid dynamical fluctuations

- Linearized fluid dynamical equations: small fluctuations $\bar{e} + \delta e$, $\bar{p} + \delta p$ and δv^i with: $\delta T^{00} = \delta e$ and $\delta T^{ij} = m^i = (\bar{e} + \bar{p}) v^i = \bar{w} v^i$

$$\begin{aligned} \partial_t \mathbf{m}_\perp + \eta / \bar{w} \mathbf{k}^2 \mathbf{m}_\perp &= 0 \\ \partial_t \delta e + i \mathbf{k} \cdot \mathbf{m}_{||} &= 0 \\ \partial_t \mathbf{m}_{||} + i v_s^2 \mathbf{k} \delta e + \gamma_v \mathbf{k}^2 \mathbf{m}_{||} &= 0 \end{aligned}$$

- retarded Green's function for δe and $\mathbf{m}_{||}$:

$$G^{\text{ret}}_{ab}(\omega, \mathbf{k}) = \frac{\bar{w}}{\omega^2 - v_s^2 \mathbf{k}^2 + i \omega \gamma_s \mathbf{k}^2} \begin{pmatrix} \mathbf{k}^2 & \omega |\mathbf{k}| \\ \omega |\mathbf{k}| & v_s^2 \mathbf{k}^2 - i \omega \gamma_s \mathbf{k}^2 \end{pmatrix}$$

- including the transverse momentum density:

$$G^{\text{ret}}_{m_i, m_j}(\omega, \mathbf{k}) = \left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2} \right) \frac{\eta \mathbf{k}^2}{i \omega - \gamma_\eta \mathbf{k}^2} + \frac{k_i k_j}{\mathbf{k}^2} \frac{\bar{w} (v_s^2 \mathbf{k}^2 - i \omega \gamma_s \mathbf{k}^2)}{\omega^2 - v_s^2 \mathbf{k}^2 + i \omega \gamma_s \mathbf{k}^2}$$

- Kubo-formulas for viscosities:

$$\eta = -\frac{\omega}{2 \mathbf{k}^2} \left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2} \right) \Im G^{\text{ret}}_{m_i m_j}(\omega, \mathbf{k} \to 0)$$

$$\zeta + \frac{4}{3} \eta = -\frac{\omega^3}{\mathbf{k}^4} \Im G^{\text{ret}}_{ee}(\omega, \mathbf{k} \to 0)$$

Fluid dynamical fluctuations

$$
\begin{aligned}
\frac{\partial}{\partial_x^\mu}\frac{\partial}{\partial_{x'}^\mu}\langle \Xi^{\mu 0}(x)\Xi^{\mu 0}(x')\rangle^S &= -\frac{\partial}{\partial_x^\mu}\frac{\partial}{\partial_{x'}^\mu}\langle T^{\mu 0}(x)T^{\mu 0}(x')\rangle^S \\
&= \int \frac{\mathrm{d}\omega}{2\pi}\int\frac{\mathrm{d}^3k}{(2\pi)^3} e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')}e^{-i\omega(t-t')}\times \\
&\quad \times \left(\omega^2 \underbrace{G^S_{ee}(\omega,\mathbf{k})}_{\text{FDT}} - 2\omega|\mathbf{k}|\underbrace{G^S_{em_{||}}(\omega,\mathbf{k})}_{\text{FDT}} + \mathbf{k}^2 \underbrace{G^S_{m_{||}m_{||}}(\omega,\mathbf{k})}_{\text{FDT}}\right) \\
&\qquad\qquad G^S_{ab}(\omega,\mathbf{k}) = -\frac{2T}{\omega}\Im G^{\mathrm{ret}}_{ab}(\omega,\mathbf{k}) \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial}{\partial_x^\mu}\frac{\partial}{\partial_{x'}^\mu}\langle \Xi^{\mu i}(x)\Xi^{\mu j}(x')\rangle^S &= -\frac{\partial}{\partial_x^\mu}\frac{\partial}{\partial_{x'}^\mu}\langle T^{\mu i}(x)T^{\mu j}(x')\rangle^S \\
&= 2T\left[\left(\zeta+\frac{4}{3}\eta\right)\partial_i\partial_j + \eta(\delta_{ij}\nabla^2 - \partial_i\partial_j)\right]\delta^4(x-x')
\end{aligned}
$$

Then boost to arbitrary frame:

Fluid dynamical fluctuations

$$T^{\mu\nu} = T^{\mu\nu}_{\text{eq}} + \Delta T^{\mu\nu}_{\text{visc}} + \Xi^{\mu\nu}$$
$$N^{\mu} = N^{\mu}_{\text{eq}} + \Delta N^{\mu}_{\text{visc}} + \mathrm{I}^{\mu}$$

with

$$\langle \Xi^{\mu\nu}(x)\Xi^{\alpha\beta}(x')\rangle = 2T[\eta(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}) + (\zeta - 2/3\eta)\Delta^{\mu\nu}\Delta^{\alpha\beta}]\delta^4(x - x')$$

- In second-order fluid dynamics there are relaxation equations for $\Xi^{\mu\nu}$:

$$u^{\gamma}\partial_{\gamma}\Xi^{\langle\mu\nu\rangle} = -\frac{\Xi^{\mu\nu} - \xi^{\mu\nu}_{\text{gauss}}}{\tau_{\pi}}$$

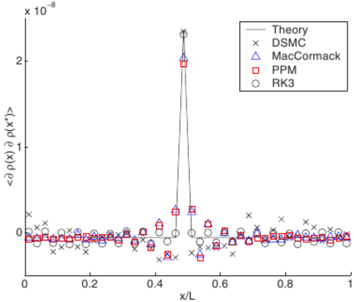
- In white noise approximation and ignoring bulk viscosity ($\zeta = 0$):

$$\langle \xi^{\mu\nu}_{\text{gauss}}(x)\xi^{\alpha\beta}_{\text{gauss}}(x')\rangle = 4T\eta\Delta^{\mu\nu\alpha\beta}\delta^{(4)}(x - x')$$

Fluid dynamical fluctuations

- In a numerical treatment → discretization: $\langle \xi^2 \rangle \propto \frac{1}{\Delta V}$
- ⇒ large fluctuations from cell to cell ⇒ coarse-graining, smearing, etc. compare to expectations from equilibrium and MC kinetic theory!

- Example: non-relativistic Navier-Stokes + fluctuations
- 1d, dilute gas, periodic boundary conditions



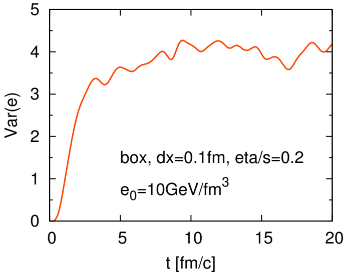
J. Bell, A. Garcia, S. Williams, PRE**76** (2007)

- Different algorithms treat fluctuations differently, third-order methods seem to work best.

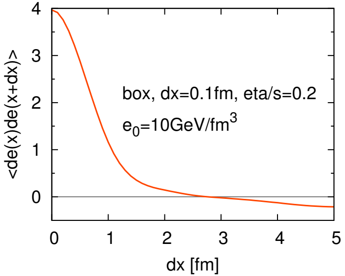
Fluid dynamical fluctuations

- Static box with periodic boundary conditions in relativistic $3+1$d fluid dynamics
  based on $3+1$d viscous fluid dynamical code by Y. Karpenko.
- Noise correlated over 1 fm^3

time evolution of the variance $\langle \delta e^2 \rangle$:



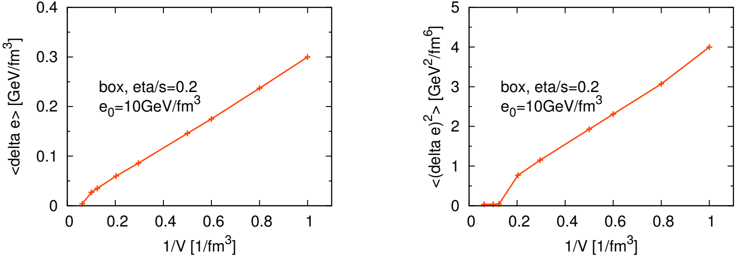
$\langle \delta e(x) \delta e(x+dx) \rangle$ correlation function:



- Initial increase of the variance $\rightarrow$ saturation (thermalization?) at later times.
- Noise-correlation length recovered.

work in progress with M. Bluhm and Th. Schäfer (NCSU)

Fluid dynamical fluctuations



- Reduction of the pressure due to the nonlinearities in the fluctuations.
- Variance of energy density fluctuations approximately $30 - 40\%$ of what is expected in a grandcanonical ensemble.
- NEXT: include net-baryon number density, diffusion and fluctuations.

Conclusions

- How to propagate fluctuations at the phase transition?
- How to propagate fluctuations at the phase transition consistently with an expanding, inhomogeneous medium?
- How to connect fluctuations/correlations at each stage consistently? Preequilibrium initial state $\rightarrow$ fluid dynamics $\rightarrow$ particles.
- Plea to experiments: please provide p_T-spectra, etc. for model tuning!

backup

The Kurtosis

The kurtosis is a measure of the deviation of fluctuations from Gaussian statistical fluctuations.

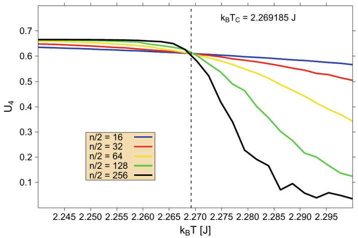
$$\begin{aligned}\langle \Delta X_i \Delta X_j \Delta X_k \Delta X_l \rangle \sim & \langle \Delta X_i \Delta X_j \rangle \langle \Delta X_k \Delta X_l \rangle \\ & + \langle \Delta X_i \Delta X_k \rangle \langle \Delta X_j \Delta X_l \rangle \\ & + \langle \Delta X_i \Delta X_l \rangle \langle \Delta X_j \Delta X_k \rangle\end{aligned}$$

$\Rightarrow \langle \Delta X^4 \rangle - 3 \langle \Delta X^2 \rangle^2 = 0$ in the Gaussian approximation.

compare to *Binder cumulant* for eg. 2d Ising model:

$$U = 1 - \frac{\langle M^4 \rangle}{\langle M^2 \rangle^2}$$

$= 0 + \mathcal{O}(1/V)$ in symmetric phase
$= U^* = 2/3$ at $T = T_c$
$= 2/3 + \mathcal{O}(1/V)$ in the broken phase



T. Preis et al., JCP228 (2009)

Kurtosis in thermal models (confined phase)

Hadron **R**esonance **G**as model:

contains hadrons as relevant d.o.f. & interactions that result in resonance formation

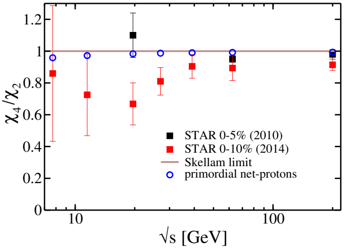
- good description of lattice QCD equation of state
- rather well description of continuum-extrapolated lattice QCD susceptibility results, cf. 1507.04627

in Boltzmann approximation net-numbers of conserved charges follow a Skellam distribution, i.e. $\kappa_B \sigma_B^2 = 1$

$\rightarrow$ cannot capture non-thermal fluctuations

early comparison with STAR net-proton number fluctuations F. Karsch, K. Redlich, PLB 695, 136 (2011)

but net-proton $\neq$ net-baryon ...



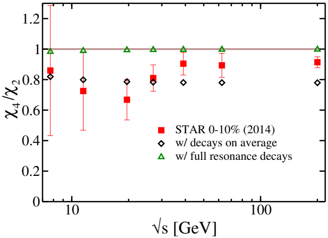
STAR, PRL 105, 22302 (2010); STAR, PRL 112, 032302 (2014)

Kurtosis in thermal models (confined phase)

- resonance decays affect net-distributions of identified particles:

  average influence (thermal fluctuations of resonance numbers) $\leftrightarrow$ HRG in partial chemical equilibrium

  in reality multinomial distribution
  $\rightarrow$ probabilistic effect erasing non-Gaussianities

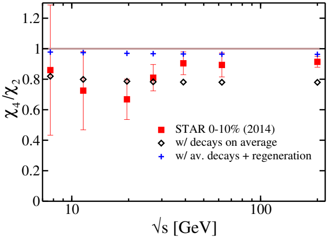


- regeneration of resonances:

  $$p(n) + \pi \rightarrow \Delta \rightarrow n(p) + \pi$$

  can lead to (complete) isospin randomization, M. Kitazawa, M. Asakawa, PRC 86, 024904 (2012)

  $\rightarrow$ similar effect on non-Gaussianities

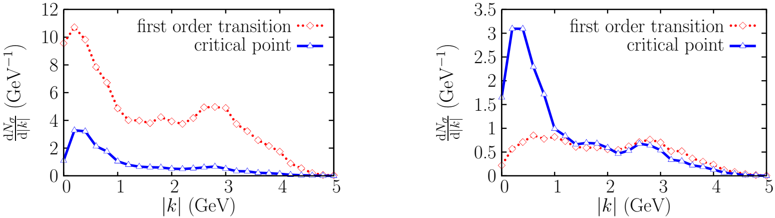


MN et al., 1402.1238

... (global) conservation of baryon number, A. Bzdak et al., PRC87, 014901 (2013); M. Sakaida et al., PRC90, 064911 (2014), ...

Dynamics versus equilibration

- Static box with temperature quench to $T < T_c$.
- Fluctuations of the order parameter:



- Strong enhancement of the intensities for a first-order phase transition during the evolution.
- Strong enhancement of the intensities for a critical point scenario after equilibration.

C. Herold, MN, I. Mishustin, M. Bleicher PRC **87** (2013)