Quark Deconfinement Transition of Hot Nuclear Matter in the Soliton Bag Model

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At high baryon density or temperature, one expects nuclear matter to undergo a phase transition to deconfined quark matter. Theories of the phase transition at finite baryon density have been based primarily on comparison of the free energies of the quark and hadronic phases, derived from quite inequivalent starting points,[1] rather than from a unified model. In this paper we present an attempt to derive the phase transition within a model field theory of quarks and "scalar gluons," that used by Friedberg and Lee[2,3] to construct "soliton models" for hadrons.

The model we consider is a system of ν (= number of flavors times the number of colors) kinds of massless spin-$\frac{1}{2}$ quarks interacting via a scalar "gluon" field, σ, with Lagrangian density

$$L = -\frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - U(\sigma) + \sum_{a=1}^{\nu} \bar{q}_a(i\gamma^\mu\partial_\mu - g\sigma)q_a, \tag{1}$$

where q_a is quark field, and g is the quark-gluon coupling constant. The potential $U(\sigma)$ is in general[2-6] taken to be a fourth order polynomial, with $U(0) > 0$ and a minimum at σ_o obeying $U(\sigma_o) = 0$; to be definite we take U here to have the form

$$U(\sigma) = \frac{\lambda}{4}(\sigma^2 - \sigma_o{}^2)^2, \tag{2}$$

as in the linear σ-model. The σ field acts in this model as a collective field coordinate, arising, in principle, from an underlying vector gluon field (see discussion in Refs. 3 and 5), and generating at least partial confinement of quarks.

In the vacuum σ has expectation value σ_o; the discrete chiral symmetry $\sigma \to -\sigma$, $q_a \to e^{i\gamma_5\pi/2} q_a$, is spontaneously broken and the quarks develop a finite mass $m_q^o = g\sigma_o$. More generally, one can imagine the present model to be a truncated version of one with continuous chiral symmetry rather than the discrete one of (1).

As the quark density is raised, $\langle\bar{q}q\rangle$ grows and the expectation value of the σ field is driven to smaller values. In hadronic models based on the Lagrangian (1),[2-4,6] the interior of the hadron is consequently in a phase with $\langle\sigma\rangle \ll \sigma_o$, while the exterior is the normal vacuum, $\langle\sigma\rangle = \sigma_o$. (Perfect confinement occurs only in the limit of infinite exterior quark mass, $g\sigma_o \to \infty$.) The potential U(0) plays the role of the bag constant in this model.

In uniform quark matter, increasing the quark density will, in the same way, tend, as Lee and Wick observed in their model of abnormal nuclear matter,[7] to make it energetically favorable for $\langle\sigma\rangle$ to be zero, or small (if there is no discrete chiral symmetry). The energy cost of having σ lose its large expectation value is compensated by the quarks losing their mass $g\sigma_o$.

Increasing the temperature will also tend to drive $\langle\sigma\rangle$ to smaller values, both through the increase of $\langle\bar{q}q\rangle$ arising from thermally excited quark - antiquark pairs, and the thermal fluctuations of the σ-field;[8] at a sufficiently high temperature a localized hadron in this model will dissociate into massless free quarks. Quite generally, any many hadron state will undergo a deconfinement phase transition through these mechanisms as either the temperature or baryon density is raised sufficiently. While construction of nuclear matter in this model is technically unfeasible at present, the existence of the deconfinement transition would be an intrinsic feature of the nuclear matter equation of state.

To illustrate the phase transition in the model, we neglect the localized structure within the hadronic phase, and study the thermodynamics of uniform phases only. Nuclear matter will, in this approximation, be described by a two phase system with a vacuum-like state, $\langle\sigma\rangle \approx \sigma_o$, coexisting with uniform quark matter with $\langle\sigma\rangle = 0$. By contrast, the true state of nuclear matter can be thought of as small units of quark matter coexisting with the vacuum. The physics that makes this latter state more favorable than the former is not contained within the framework of the present model, but on the other hand, it is probably not qualitatively important to the description of the phase transition.

In the approximation in which the fluctuations of the σ field are harmonic, and closed loop corrections involving quarks are neglected, the thermodynamic potential $\Omega(V,T,\mu,\langle\sigma\rangle)$, where V is the volume, T the temperature, and μ the quark chemical potential, is

$$\begin{aligned}\Omega &= F - \mu N \\ &= -2T\sum_{\underset{\sim}{p},a}\{\ln(1+e^{-(E_p-\mu)/T}) + \ln(1+e^{-(E_p+\mu)/T})\} \\ &\quad + T\sum_{\underset{\sim}{k}}\ln(1-e^{-\omega_k/T}) - \frac{3\lambda V}{4}\langle\tilde{\sigma}^2\rangle^2 + \frac{\lambda V}{4}(\langle\sigma\rangle^2-\sigma_o^2)^2, \end{aligned} \tag{3}$$

where $\tilde{\sigma} = \sigma - \langle\sigma\rangle$, $E_p = (p^2 + g^2\langle\sigma\rangle^2)^{1/2}$ is the quark energy, $\omega_k = (k^2 + m_\sigma^2)^{1/2}$ is the scalar gluon energy, with

$$m_\sigma^2 = \lambda\left(3\langle\tilde{\sigma}^2\rangle + 3\langle\sigma\rangle^2 - \sigma_o^2\right) + g\sum_a \frac{\langle\bar{q}_a q_a\rangle}{\langle\sigma\rangle}\ , \tag{4}$$

and

$$\langle\tilde{\sigma}^2\rangle = \sum_{\underset{\sim}{k}}\frac{1}{\omega_k}\frac{1}{e^{\omega_k/T}-1} \tag{5}$$

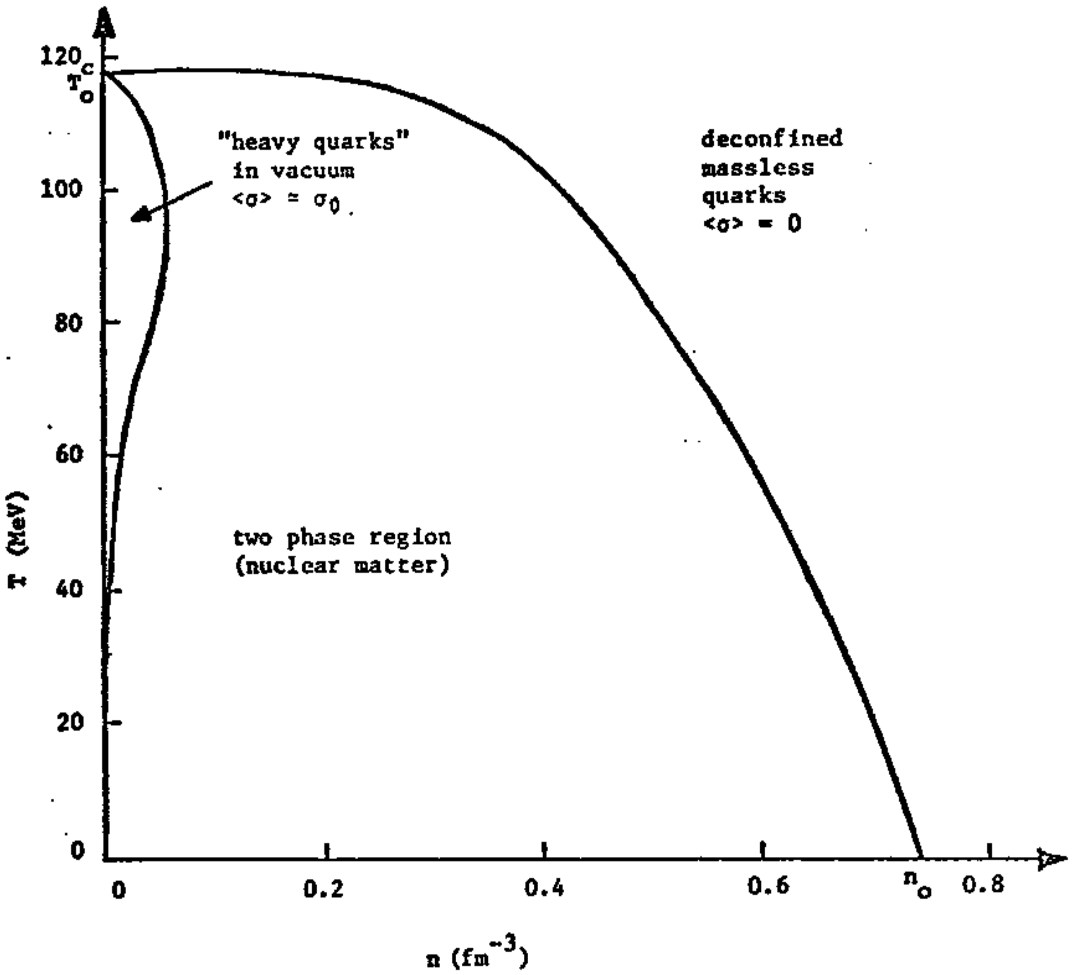
$$\langle\bar{q}_a q_a\rangle = \frac{2}{V}\sum_{\underset{\sim}{p}}\frac{g\langle\sigma\rangle}{E_p}\left(\frac{1}{e^{(E_p-\mu)/T}+1} + \frac{1}{e^{(E_p+\mu)/T}+1}\right)\ . \tag{6}$$

The equation for $\langle\sigma\rangle$ (the expectation value of the σ field equation, $\delta L/\delta\sigma = 0$) is

$$\frac{1}{V}\frac{\delta\Omega}{\delta\langle\sigma\rangle} = \lambda\langle\sigma\rangle\left(\langle\sigma\rangle^2+3\langle\tilde{\sigma}^2\rangle-\sigma_o^2\right) + g\sum_a\langle\bar{q}_a q_a\rangle = 0. \tag{7}$$

Equations (4) and (7) imply that for $\langle\sigma\rangle \neq 0$, $m_\sigma^2 = 2\lambda\langle\sigma\rangle^2$.

At low temperatures and densities Eq. (7) has two solutions, $\langle\sigma\rangle = 0$ and $\langle\sigma\rangle \simeq \sigma_o$. The first corresponds to a maximum in Ω and hence an unstable phase. The thermodynamically stable phase at low temperature and quark chemical potential has $\langle\sigma\rangle \simeq \sigma_o$; this vacuum-like phase contains at finite T a small



density of heavy quarks $\left(m_q \approx g\sigma_o = m_q^o\right)$ which goes to zero as the vacuum quark mass $g\sigma_o \to \infty$. As T or μ is raised the central maximum becomes a minimum, and a new maximum develops between $\langle\sigma\rangle = 0$ and σ_o. Finally at high T or μ this maximum and the minimum at $\langle\sigma\rangle \approx \sigma_o$ merge, as detailed numerical calculations show, and the only solution is $\langle\sigma\rangle = 0$, corresponding to uniform massless quark matter. Thus the transition from the "heavy quark" phase to quark matter with $\langle\sigma\rangle = 0$ is first order.

In the Figure we show the resulting phase diagram, determined by equating pressures and chemical potentials in the two phases, in the T, $n = n_q - n_{\bar{q}}$ (the net quark density) plane. The parameters used were $\lambda = 1$, $g \approx 2.45$, $\sigma_o = 204$ MeV, and $\nu = 6$, which corresponds to $m_q^o = 500$ MeV, $m_g^o \equiv \sqrt{2\lambda}\sigma_o = 204\sqrt{2}$ MeV, an effective bag constant $\lambda\sigma_o^4/4 = 56$ MeV/fm^3, $n_o = 0.74$ fm^{-3}, and $T_c^o = 118$ MeV. In the region labelled $\langle\sigma\rangle = 0$ the equilibrium phase is uniform massless quark matter, consisting of q, $\bar{q}$ and thermally excited scalar gluons. The region to the left corresponds to the phase with $\langle\sigma\rangle \approx \sigma_o$, containing a low density (< 0.045 fm^{-3}) of massive quarks and scalar gluons (and a negligible density of anti-quarks). As $g\sigma_0 \to \infty$ (perfect confinement) the heavy quark region collapses towards $n = 0$, with only slight effect on the boundary between the two phase and massless quark regions.

The two phase region, consisting of massive quarks in phase equilibrium with (massless) quark matter, corresponds, as we indicated above, to nuclear matter in this model. Were we able to include the forming of the quarks into individual hadrons in this region we would find additional pressure in the "nuclear matter" phase, from the nucleons. However, the resulting shift in the phase boundary is not expected to be large, since the incompressibility of nuclear matter is only $\sim k_f/m_n$ times that of quark matter.

The phase boundary between "nuclear matter" and quark matter is by and large determined by the condition $P = 0$. This is because in the heavy quark phase the pressure (< 10 MeV fm^{-3}), due mostly to the fluctuations of the σ field, is small compared to the pressure scale in the massless quark matter phase. In the limit $m_q^o, \lambda \to \infty$, which implies $P = 0$ in the two phase region, and in the absence of σ fluctuations in the quark matter phase, the deconfinement curve is determined simply by

$$P = \frac{7\pi^2}{30} T^4 + \mu^2 T^2 + \frac{1}{2\pi^2} \mu^4 - U(0) = 0, \tag{8}$$

and the net quark density by

$$n = 2\,(\mu^3/\pi^2 + \mu\, T^2); \tag{9}$$

$U(0)$ is the only parameter that enters. Equations (8) and (9) imply that $T_c = (30U(0)/7\pi^2)^{1/4}$ and $n_o = (2/\pi^{1/2})(2U(0))^{3/4}$, which for $U(0) = 56$ Mev/fm^3 yields $T_c = 117$ MeV and $n_o = 0.74$ fm^{-3}.

In conclusion we remark that there are, of course, many corrections to our present numerical results, such as quantum fluctuations, and pionic and vector gluonic contributions, so that the present calculation should not be regarded as quantitative. Within the context of the soliton model, spontaneously broken chiral symmetry is restored in the quark deconfinement transition. On the other hand, as recently suggested by Pisarski[9] and by Shuryak,[10] the deconfinement transition may slightly precede the restoration of chiral symmetry as the density or temperature is raised. In the additional phase between the two transitions, matter is in the form of deconfined massive quarks. The present model can then be interpreted as a description of the chiral transition from the massive to the massless quark phase. Unlike in the former case, where one implicitly imagines the limit $m_q^o \to \infty$ to realize true confinement, the heavy quark mass in this case remains finite.

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